

1 Motivations and Challenges

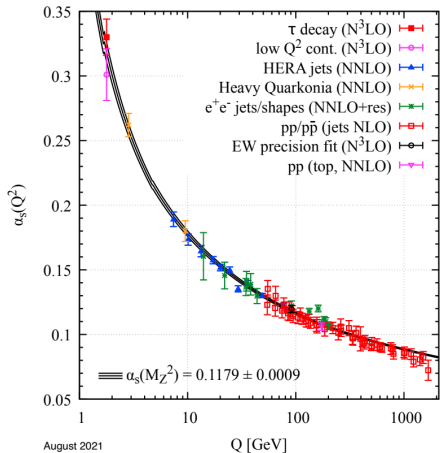
2 QCD at the LHC

- Soft and Hard QCD
- Phenomenological models for Soft QCD

3 Results: Papers

- The U -Matrix Geometrical Model. Rami Oueslati (Liege U.), Adel Trabelsi (Tunis El Manar U.), arXiv:2403.02263
- A Multi-Channel U -Matrix Model. Rami Oueslati (Liege U.), arXiv:2305.03424, **JHEP 08 (2023) 087**
- Unitarisation Dependence of Diffractive Scattering. Arno Vanthieghem (SLAC), Atri Bhattacharya (Liege U.), Rami Oueslati (Liege U.), Jean-René Cudell (Liege U.), arXiv:2104.12923, **JHEP 09 (2021) 005**
- Proton Inelastic Cross Section at Ultrahigh Energies. Atri Bhattacharya (Liege U.), Jean-René Cudell (Liege U.), Rami Oueslati (Liege U.), Arno Vanthieghem (SLAC), arXiv:2012.07970, **Phys.Rev.D 103 (2021) 5, L051502**

Soft and Hard QCD



- QCD is the fundamental theory of strong interactions.
- Two main aspects of strong interaction: Hard and Soft QCD.
- **Hard QCD:** Large momentum transfers, allowing perturbative QCD.
- **Soft QCD:** Small momentum transfers, requiring phenomenological models.
- **Soft hadronic interaction** \implies **Model dependant**

*Strong coupling constant as a function of the energy scale Q

Phenomenological models for Soft QCD

- A plethora of pheno models in the literature
- Based on fundamental principles of quantum field theory: unitarity, analyticity and crossing.
- Empirical parametrizations
- Test hypotheses, constrain the parametrizations by fine-tuning them parameters using data comparisons.

Regge phenomenology

- Scattering amplitudes at high energies based on the exchange of particles with specific spin and trajectory, known as Regge poles such as pomeron.

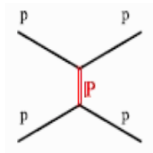


Figure: Lowest order Pomeron exchange graph

$$a(s, t) = g_p^2 \chi(t)^2 \left(\frac{s}{s_0} \right)^{\alpha(t)} \xi(t) \quad (1)$$

Summary Paper 1: verification of the unitarity condition

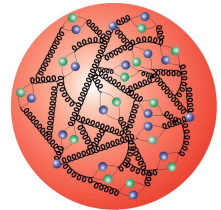
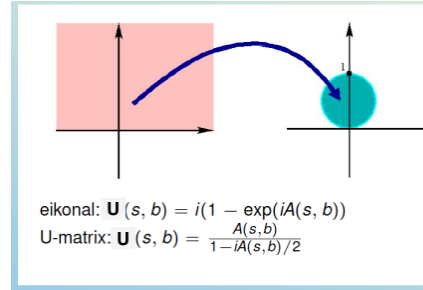
- Unitarization techniques are used to ensure that the scattering amplitudes satisfy unitarity
- Unitarization is achieved by summing the contributions from all multi-pomeron exchanges

$$\text{Im } T_{\text{el}} = \text{Diagram} = 1 - e^{-\Omega/2} = \sum_{n=1}^{\infty} \text{Diagram} \Omega/2$$

(s-ch unitarity)

Unitarity Condition

- The most common approach: the eikonal scheme.
- The eikonal scheme has limitations:
 - Less effective for interactions involving composite bodies.
 - Not the only possible solution, leading to the U matrix scheme.



Optical Theorem

- Unitarity condition of the S matrix \implies Optical Theorem

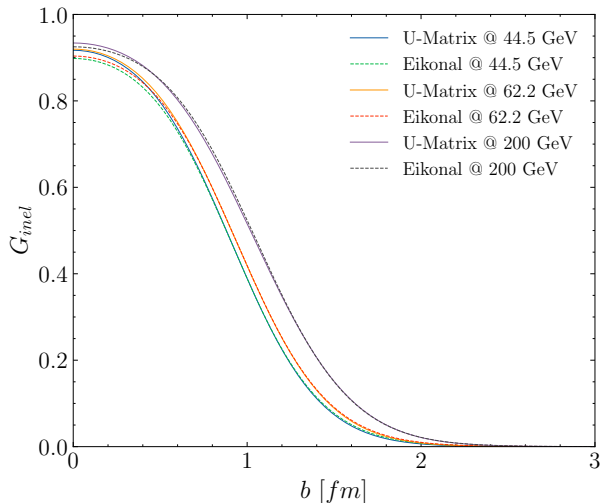
$$2 \operatorname{Im}[\Gamma(s, b)] = |\Gamma(s, b)|^2 + G_{in}(s, b), \quad (2)$$

- $\Gamma(s, b)$ denotes the profile function, i.e., the elastic hadron scattering amplitude
- $G_{in}(s, b)$ represents the inelastic overlap function fixed by the unitarity condition
- By employing the optical theorem, we can obtain various cross-sections

$$\sigma_{tot}(s) = 2 \int d^2 b \operatorname{Im}[\Gamma(s, b)], \quad (3)$$

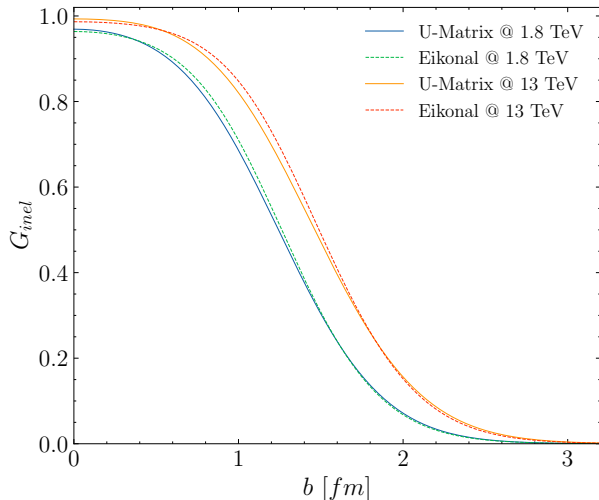
$$\sigma_{in}(s) = \int d^2 b G_{in}(s, b). \quad (4)$$

Results: Impact Parameter Space Evolution of the Inelastic Overlap Function



- **Pattern:** Generally similar across both schemes, with minor differences.
- **Concentration:** The inelastic overlap function is primarily central.
- **Energy Effect:** The function declines more slowly with increasing energy.

Results: Impact Parameter Space Evolution of the Inelastic Overlap Function



- **Energy Dependence:** The magnitude increases with energy, especially at central impact parameter $b = 0$.
- **Scheme Comparison:** The U -matrix scheme shows a greater magnitude than the eikonal scheme at the central impact parameter.
- **Consistency Across Energies:** These trends are consistent across energies from ISR to LHC, as observed in the figures.

What is Geometrical Scaling?

- Observed in ISR experiments involving proton-proton and proton-antiproton scattering.
- Refers to a consistent ratio of elastic to total cross-sections.

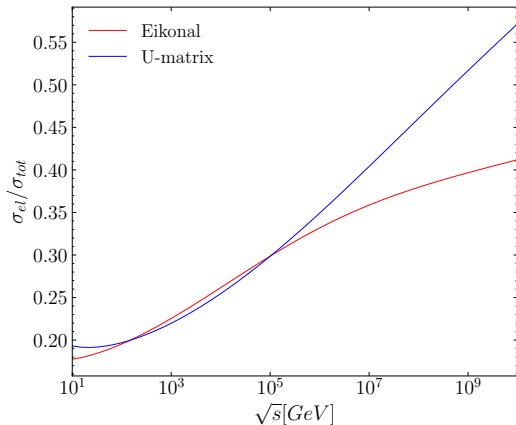
When Does Violation Occur?

- Experiments at CERN's SPS indicate that the regularity of geometrical scaling breaks when energy levels surpass those of ISR.
- This suggests that G. SC. is not upheld at higher energies.

Implications and Consequences

- These experimental findings may require exploring new theoretical models and could influence future experiments.

Results: Geometrical Scaling G. SC. Violation



Observations:

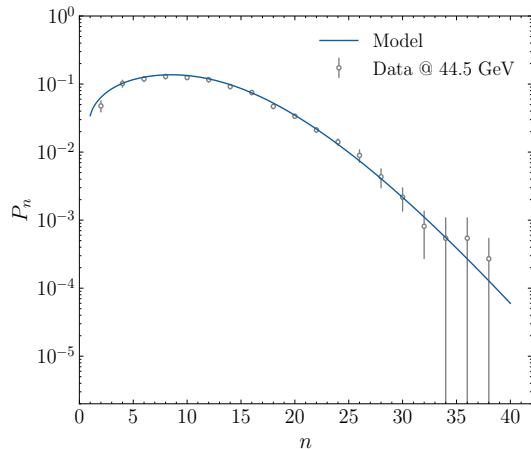
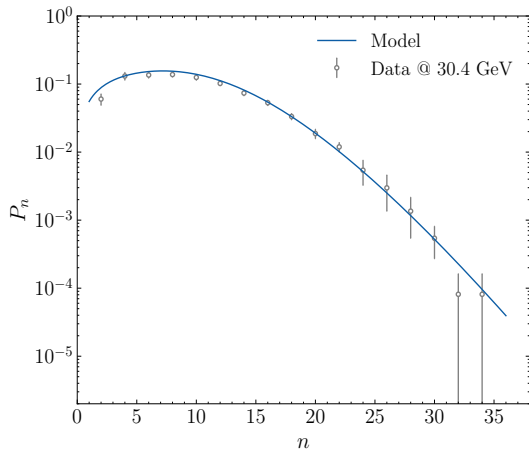
- As energy rises from 10 GeV to 10 TeV, the elastic-to-total cross-section ratio increases.
- Both U -matrix and eikonal schemes show non-linear increases, but the U -matrix scheme has a steeper rise.
- The steeper increase in the U -matrix scheme suggests a stronger violation of G. SC.
- This could be due to the inelastic overlap function's behavior at $b = 0$.
- **Implication:** Given its steeper trend, the U -matrix scheme might be more suitable for describing G. SC. at higher energies, indicating a potential direction for theoretical exploration.

- The multiplicity distribution in QCD is defined as:

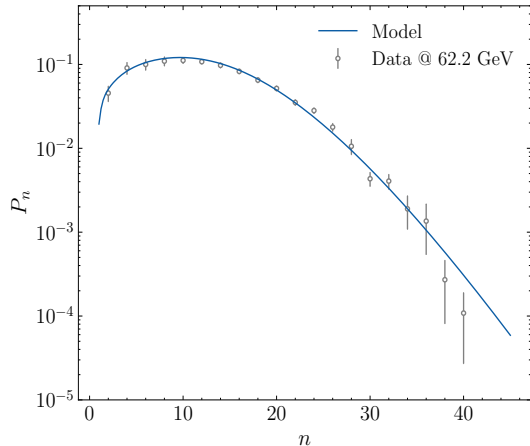
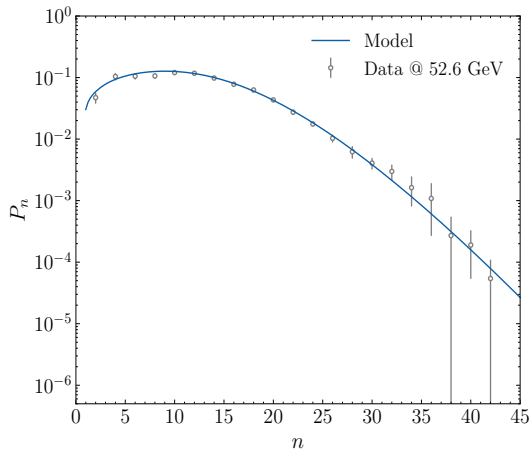
$$P_n(s) = \frac{\sigma_n(s)}{\sigma_{in}(s)} \quad (5)$$

- $\sigma_n(s)$ is n -particle topological cross-sections
- $\sigma_{in}(s)$ is the inelastic cross-section.
- P_n is **scheme** dependent

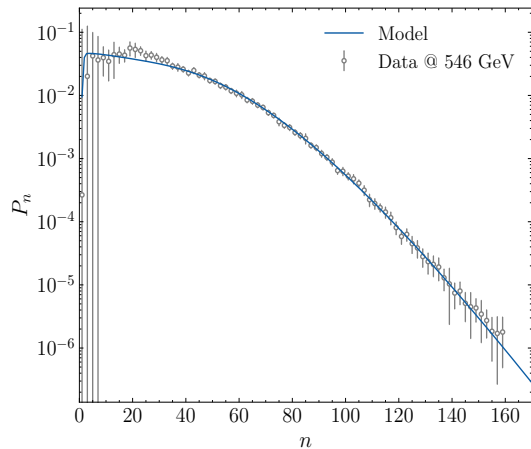
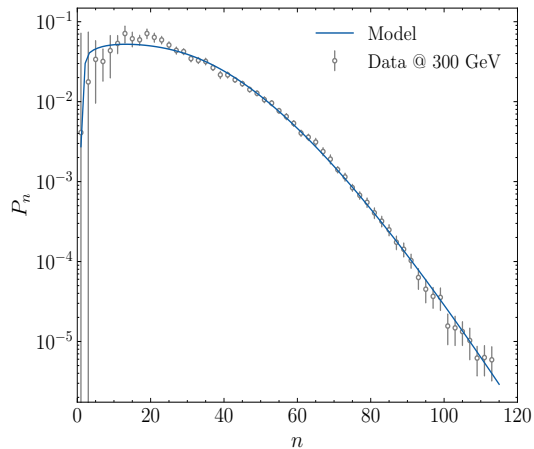
Results : Multiplicity distributions for inelastic pp data compared with theoretical expectations.



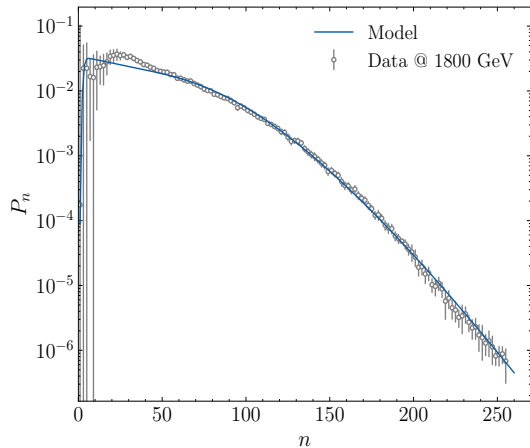
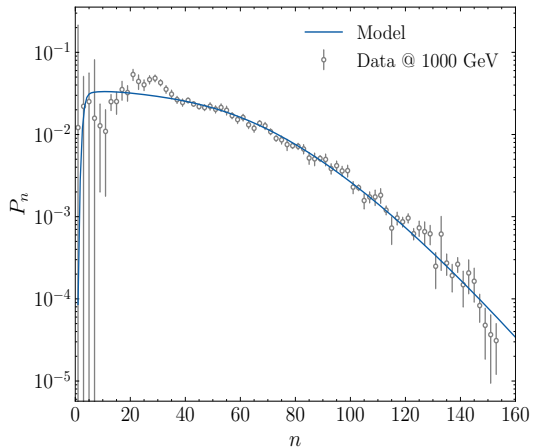
Results : Multiplicity distributions for inelastic $p\bar{p}$ data compared with theoretical expectations.



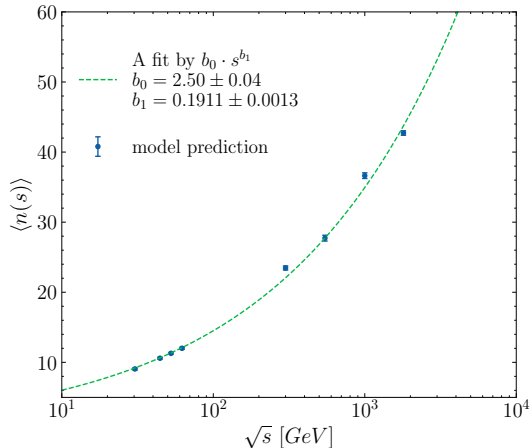
Results : Hadronic Multiplicity Distributions



Results : Hadronic Multiplicity Distributions



Results: Energy dependence of the Hadron mean multiplicity



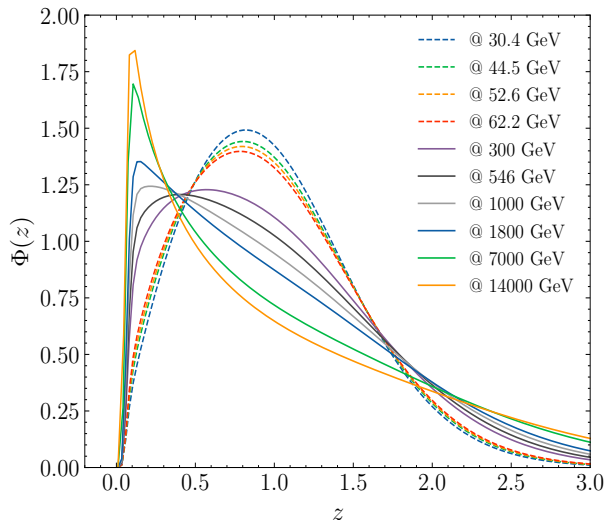
- It exhibits a rapid increase with the energy \sqrt{s} .
- Its energy dependence can be described using : equation
- This power-law energy dependence is often regarded as a prominent feature observed in different models
- The approach of Troshin and Tyurin developed in their specific model within the U -Matrix framework
 $\langle n(s) \rangle = 2.328 s^{0.201} \implies$ in line with our result.

\implies Supporting the fundamental principles

Results: Prediction and investigation of various phenomena

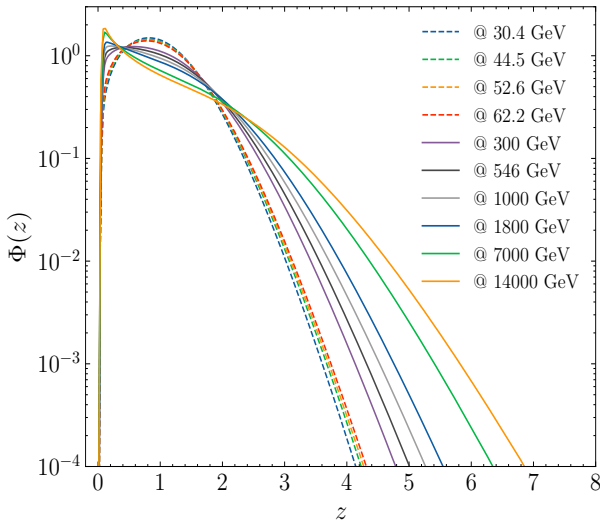
- Model fine-tuned \implies
 - Reliable prediction in extrapolating to novel collision energy regimes.
 - Investigating various phenomena, such as the KNO scaling violation and particle correlation.

Results: KNO scaling violation



- Predictions for the full-phase space multiplicity distribution in $p + p(\bar{p})$ collision, in KNO form at various energies, spanning from ISR to LHC.
- The maximum of the distribution shifts towards smaller values of z .

Results: KNO scaling violation



- The high-multiplicity tail rises with increasing energy.
- These features validate the violation of the KNO scaling.
- Width gets larger with increasing energy \implies A strong violation of the KNO scaling.
- Strong G. S. violation \implies Strong KNO S. violation.
- Interconnected nature of these phenomena within the U -matrix representation.
- Pivotal role in describing collision geometry and the processes of multi-particle production in hadron collisions.

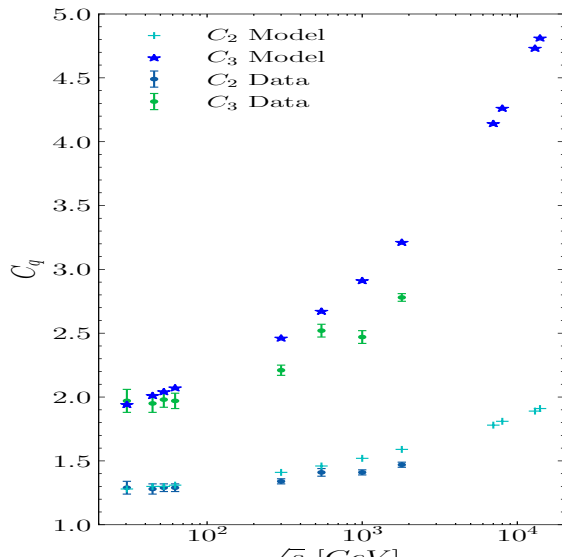
Results: Fluctuations and particle correlations

- Correlation between the produced particles in the final state \implies to better understand the dynamics of particle production processes in hadron collisions.
- The P_n 's moments of order q

$$C_q = M_q / M_1^q \quad (6)$$

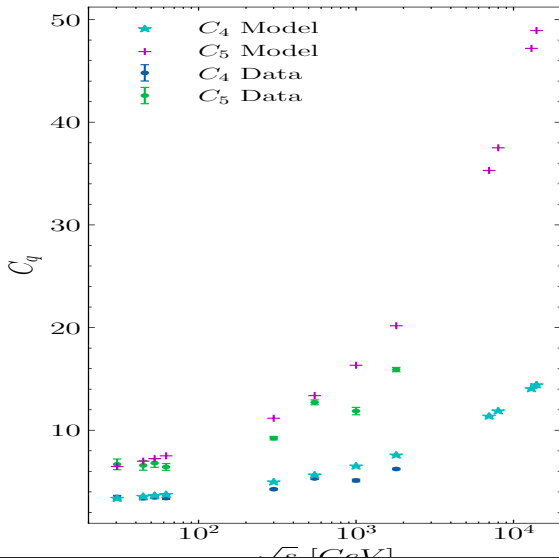
$$M_q = \sum_{n=0}^{\infty} n^q P_n \quad (7)$$

Results: Fluctuations and particle correlations

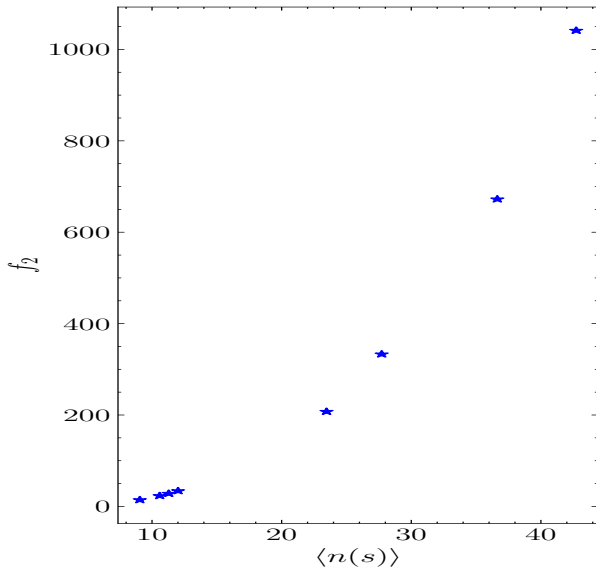


- Our results along with their comparison with the experimental data
- A gradual increase in the ordinary higher-order moments.

Results: Fluctuations and particle correlations



- Predictions match with the data points within the ISR energy.
- Model overestimates the fluctuations and correlations in the multiplicity distribution as energy rises, specifically for energies above the LHC.



- The f_2 moment (or the two-particle correlation parameter), as a means of examining the correlation between pairs of particles during a collision event

$$f_2 = \langle n(n-1) \rangle - \langle n \rangle^2 \quad (8)$$

- A noteworthy and sudden increase \implies Existence of strong correlations among the charged particles. \implies Infer that the model incorporates correlations in the final state, despite being constructed on the basis of independent particle production.

Results: Fluctuations and Particle Correlations

- A key question arises: Where does this particle correlation come from?

Correlation Formula:

$$P_n(s) = \frac{1}{\langle n(s) \rangle \int d^2b G_{in}(s, b)} \int d^2b \frac{G_{in}(s, b)}{f(s, b)} \phi^{(1)} \left(\frac{z}{f(s, b)} \right) \quad (9)$$

- This correlation is related to the construction of the overall hadronic multiplicity distribution.
- As it is derived by summing contributions from parton-parton collisions at each impact parameter, weighted by the inelastic overlap function.
- The correlation's overestimation might be due to the superposition model's weighting system, directly linked to the unitarization scheme. This differs from the predictions of an eikonal geometrical independent string model.

U-Matrix Scheme Advantages:

- Offers a better description of various phenomena in high-energy proton collisions.
- More suitable for complex interactions, especially with composite bodies.
- Addresses limitations of other schemes, like eikonal.

Implications for LHC Simulations:

- Provides a more accurate model for Monte Carlo simulations.
- Could lead to improved predictions for LHC data analysis.
- A step towards more precise LHC modeling.

Future Directions:

- Integrate the U -matrix scheme into existing simulation frameworks.
- Collaborate with experimentalists to validate the scheme against real LHC data.
- Explore further theoretical developments to refine the model.

Thank you for your attention!