

Advancing Soft QCD Understanding  
Revisiting Multi-Pomeron Exchange in String Models with U-  
Matrix Solutions

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# Hadron interaction and QCD

- Hadronic interactions : Involve particles that undergo strong interactions.
- QCD : Widely recognized as the theory of strong interactions.
- QCD is most applicable to processes in which the coupling constant is small.

$$\alpha = (Q^2)$$

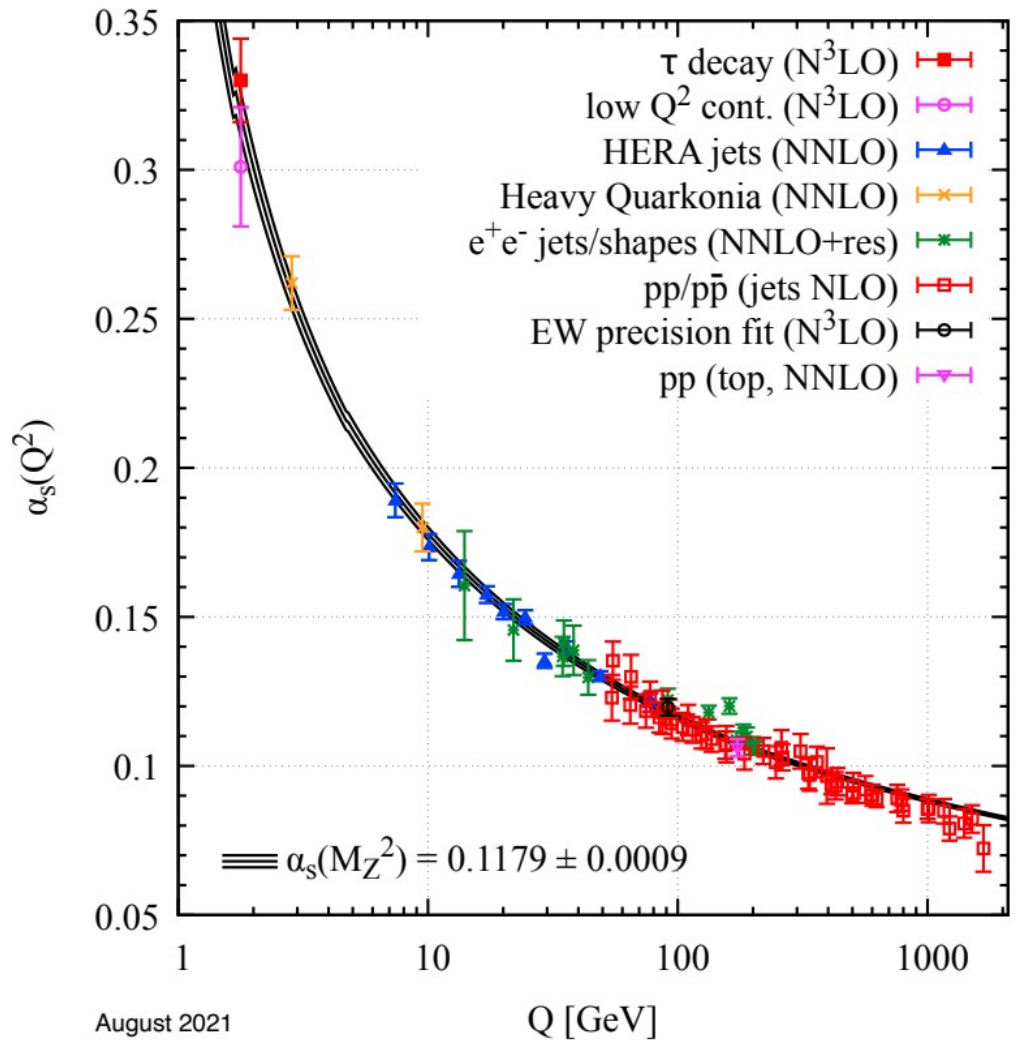
- Two QCD processes :

• At large  $Q$   $\longrightarrow$   $\alpha$  small

$\longrightarrow$  Hard QCD

• for small  $Q$   $\longrightarrow$   $\alpha$  large

$\longrightarrow$  Soft QCD



# Problem : soft qcd : perturbation theory breaks down

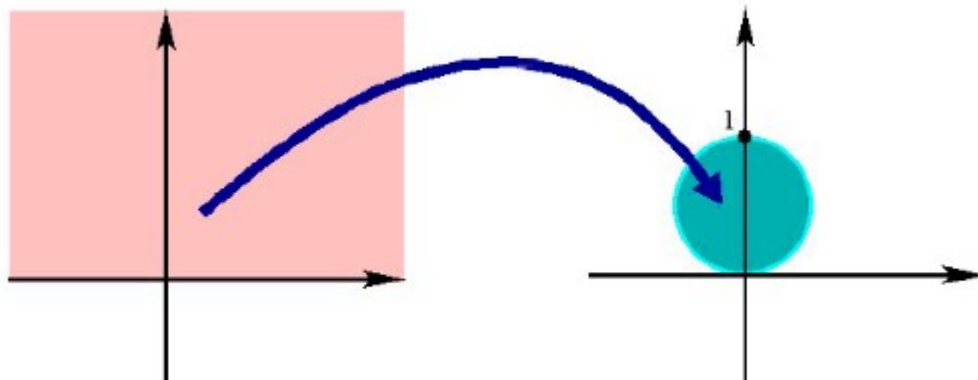
- Solution : Pheno. models
- abundant in the literature :
  - based on the Gribov-Regge phenomenology.
    - hinge on fundamental principles of S matrix theory : unitarity, analyticity and crossing
    - empirical parameterizations

## Unitarity Problem

$$S(s, \mathbf{b}) = 1 + iG(s, \mathbf{b})$$

Unitarity demands that  $|S(\mathbf{b})|^2 \leq 1 \implies$

Unitarity circle: the amplitudes must lay on the circle to satisfy the unitarity condition for elastic scattering.



# Several ways to represent the unit circle

- one can map the upper complex plane into a circle via a complex exponential

$$S(s, \mathbf{b}) = \exp(iz(s, \mathbf{b})) \quad \text{with} \quad \text{Im } z(s, \mathbf{b}) \geq 0.$$



$$G(s, \mathbf{b}) = i(1 - \exp(i\chi(s, \mathbf{b}))).$$

- Use a one-to-one map through a Möbius transform

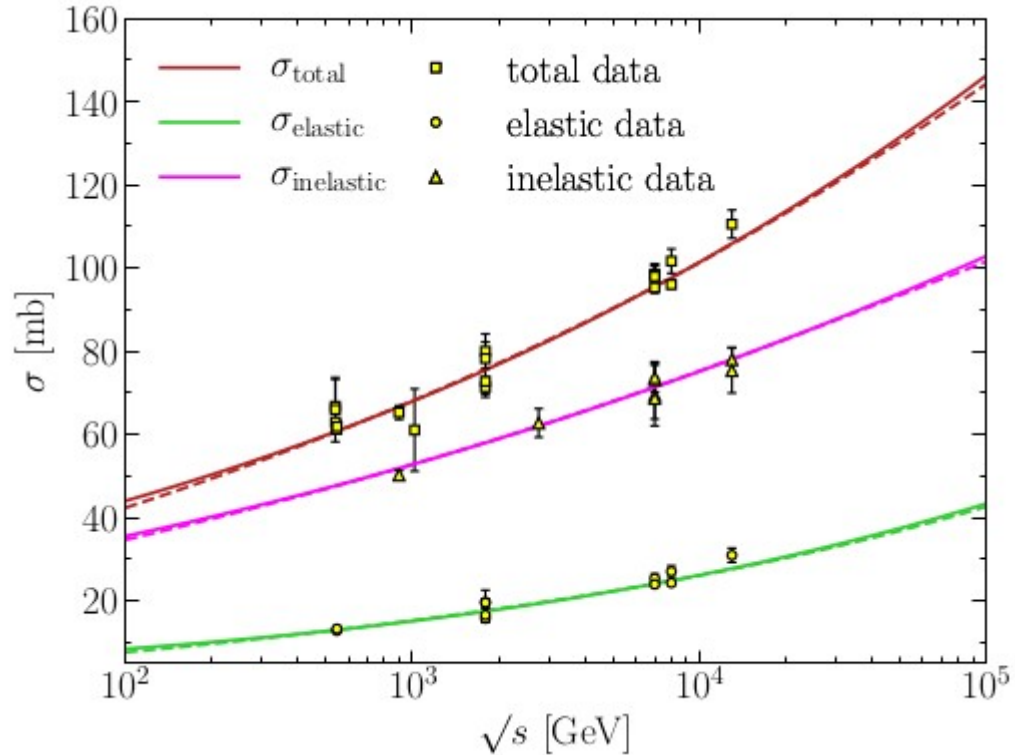
$$S(s, \mathbf{b}) = \frac{1 + iz'(s, \mathbf{b})}{1 - iz'(s, \mathbf{b})}, \quad \text{with} \quad \text{Im } z'(s, \mathbf{b}) \geq 0.$$



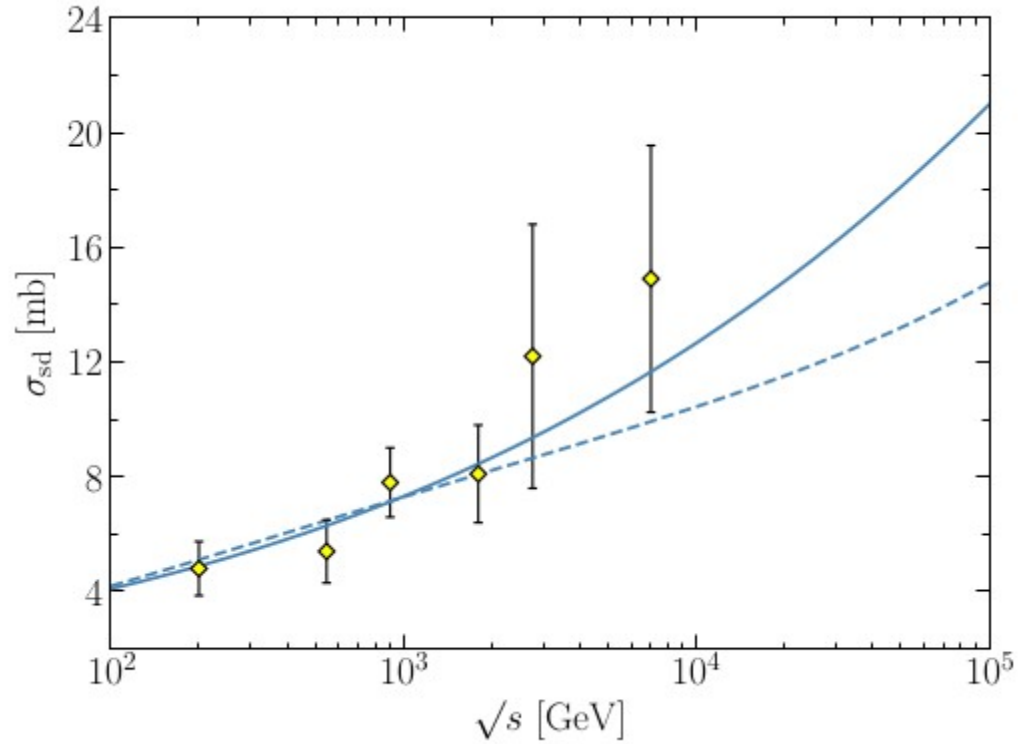
U Matrix scheme

$$G(s, \mathbf{b}) = \frac{\chi(s, \mathbf{b})}{1 - i\chi(s, \mathbf{b})/2}.$$

# High-energy collider data for pp and p $\bar{p}$ scattering

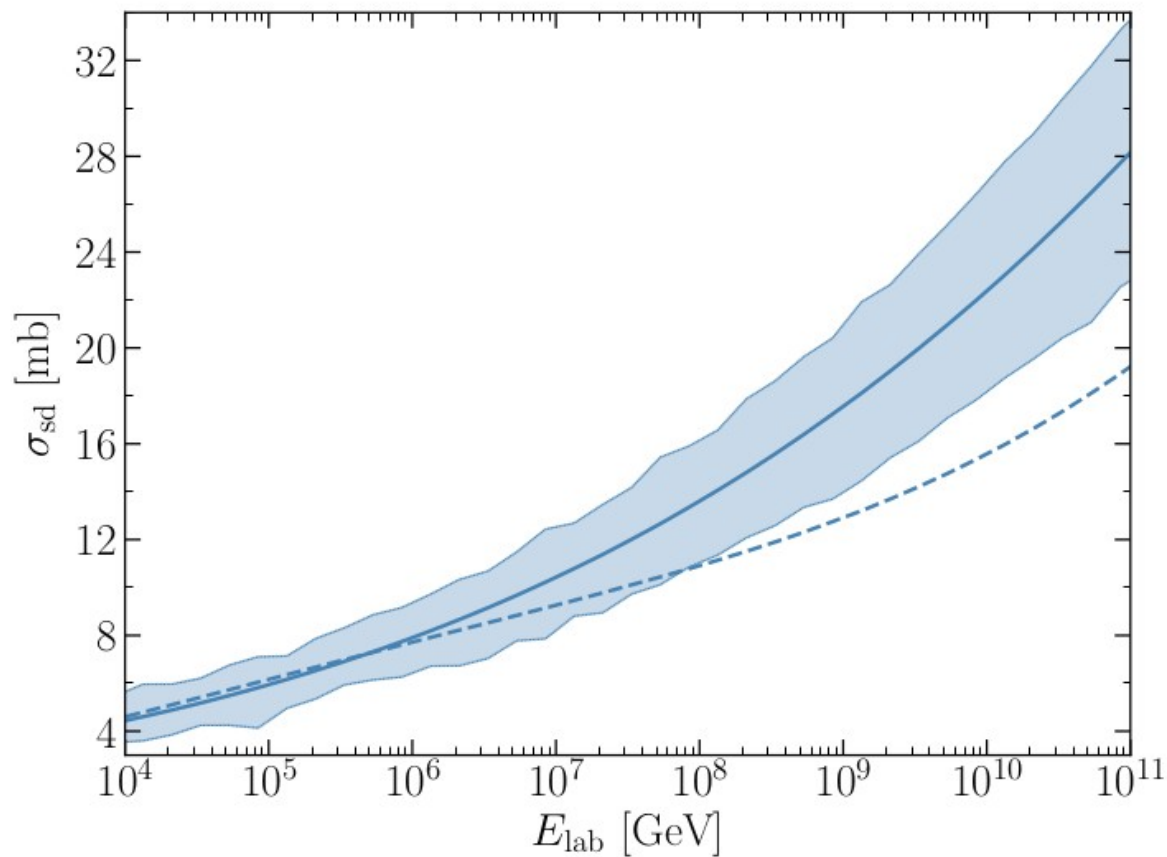


# Single diffractive cross-sections





# Single-diffractive cross section at ultra-high energy



# Unitarisation and multi-particle production

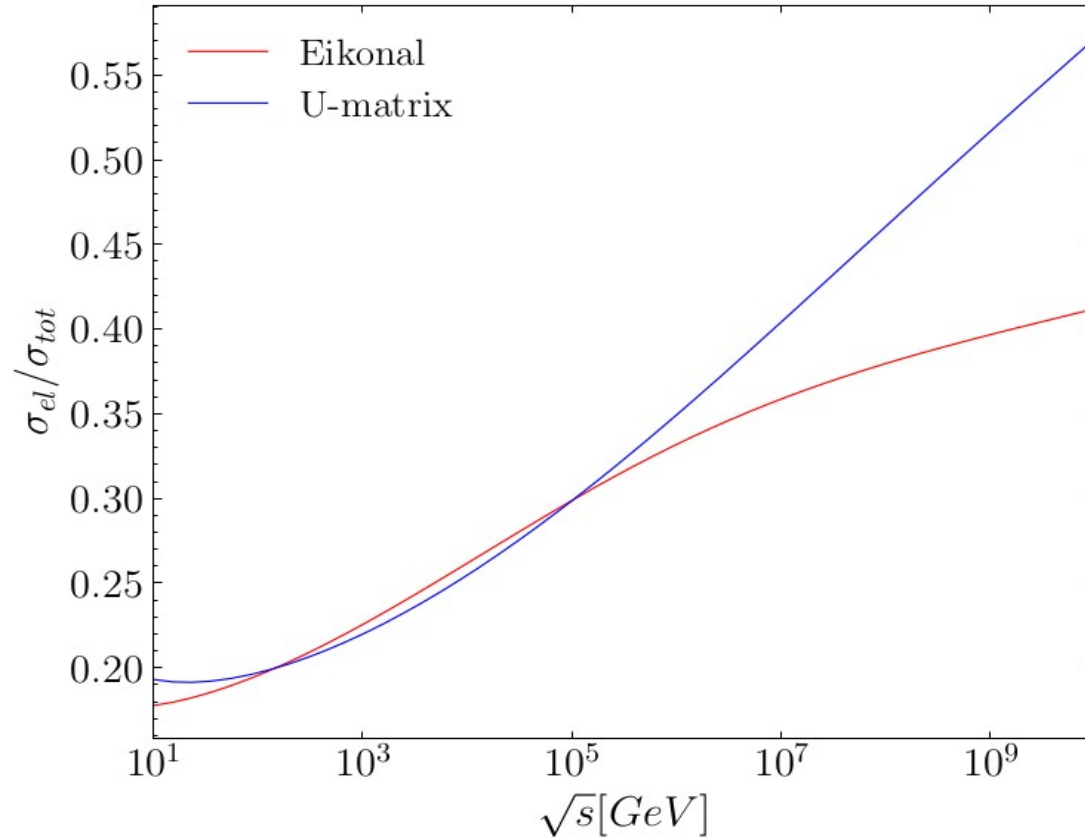
- Slight better description for cross-sections with the U matrix scheme than the eikonal
- What about the multiplicity distribution
- Probability of producing n char  $P_n(s) = \frac{\sigma_n(s)}{\sigma_{\text{in}}(s)}$  elastic p + p( $\bar{p}$ ) collision at the energy s

# The model

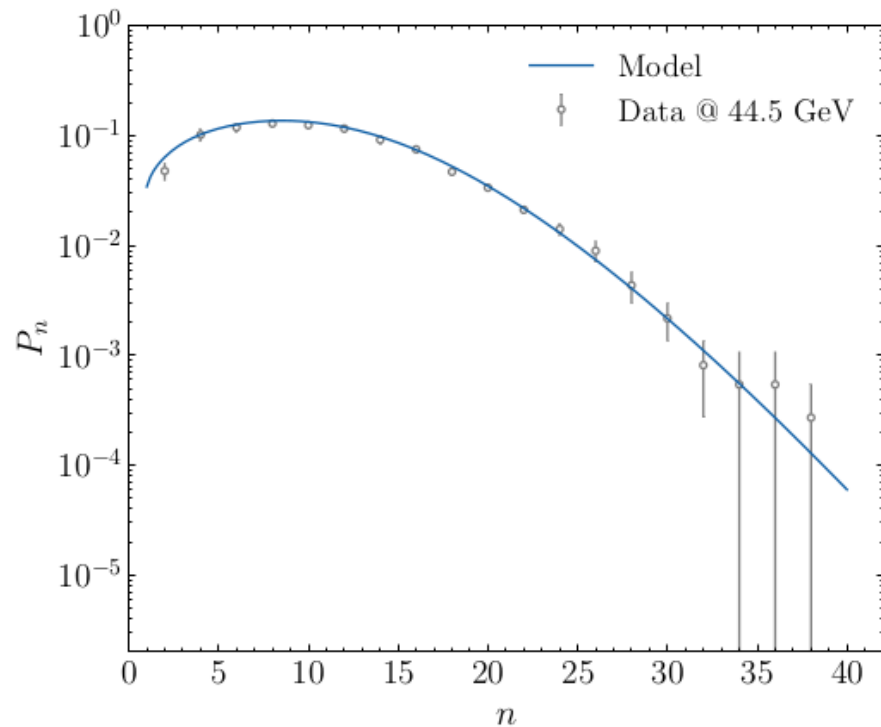
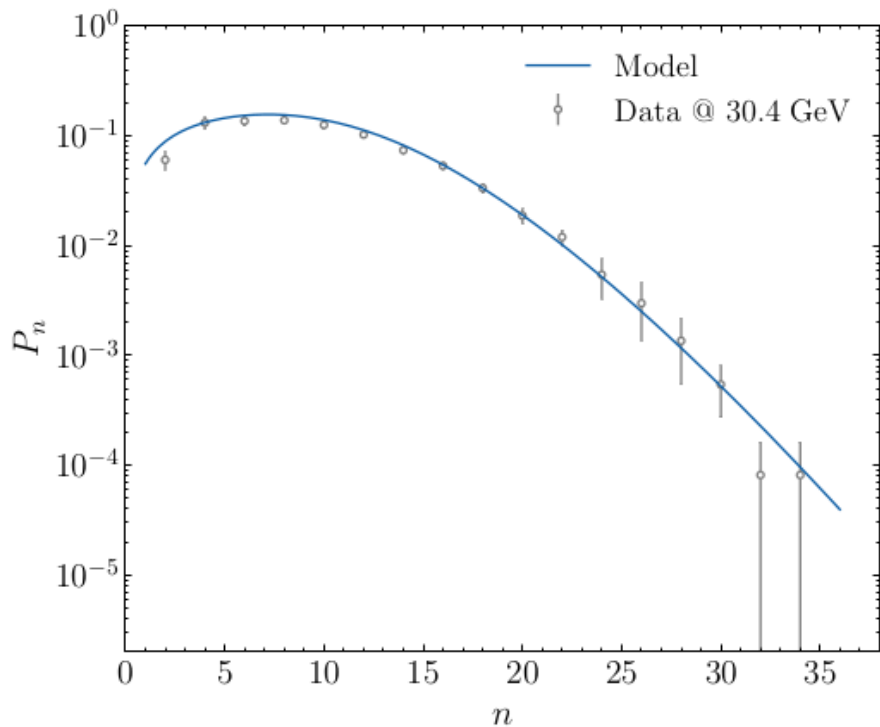
$$P_n(s) = \frac{1}{\langle n(s) \rangle} \int d^2b G_{\text{in}}(s, b) \int d^2b \frac{G_{\text{in}}(s, b)}{f(s, b)} \phi^{(1)} \left( \frac{z}{f(s, b)} \right),$$

- Superposition procedure : summing contributions from parton-parton collisions occurring at each impact parameter weighted by the inelastic overlap function, which dictates the unitarisation scheme.
- picture : the KNO scaling violation viewed as an extension of the geometrical scaling violation

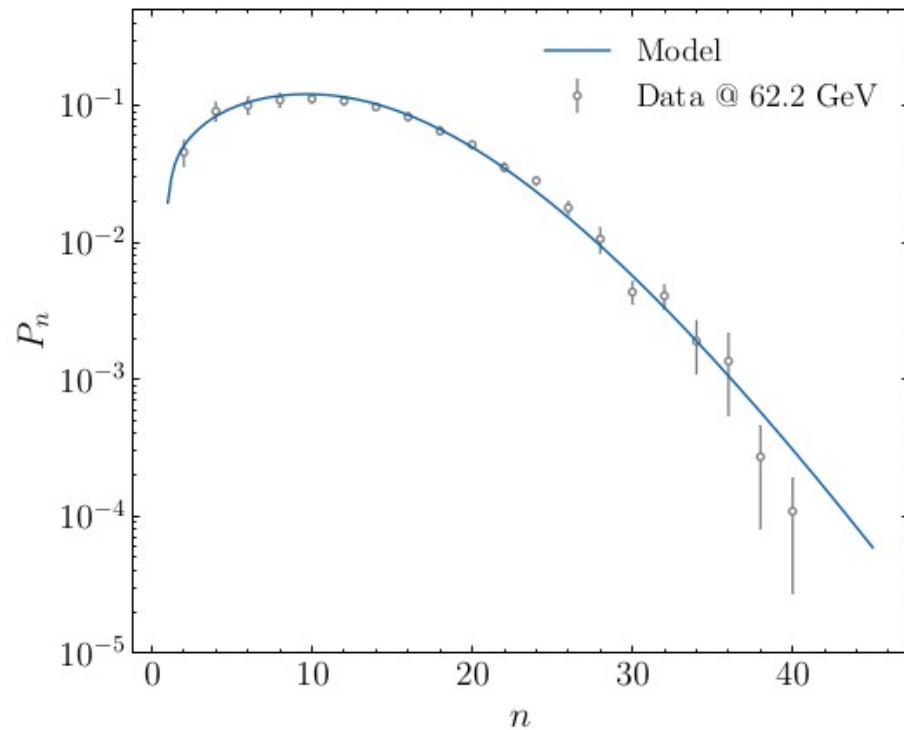
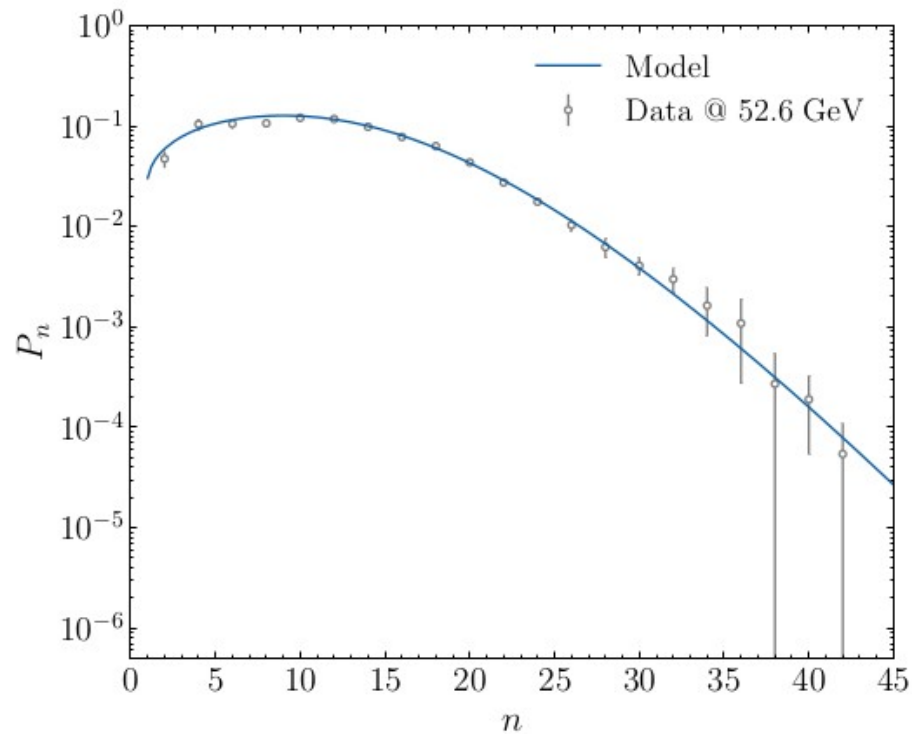
# Geometrical scaling violation



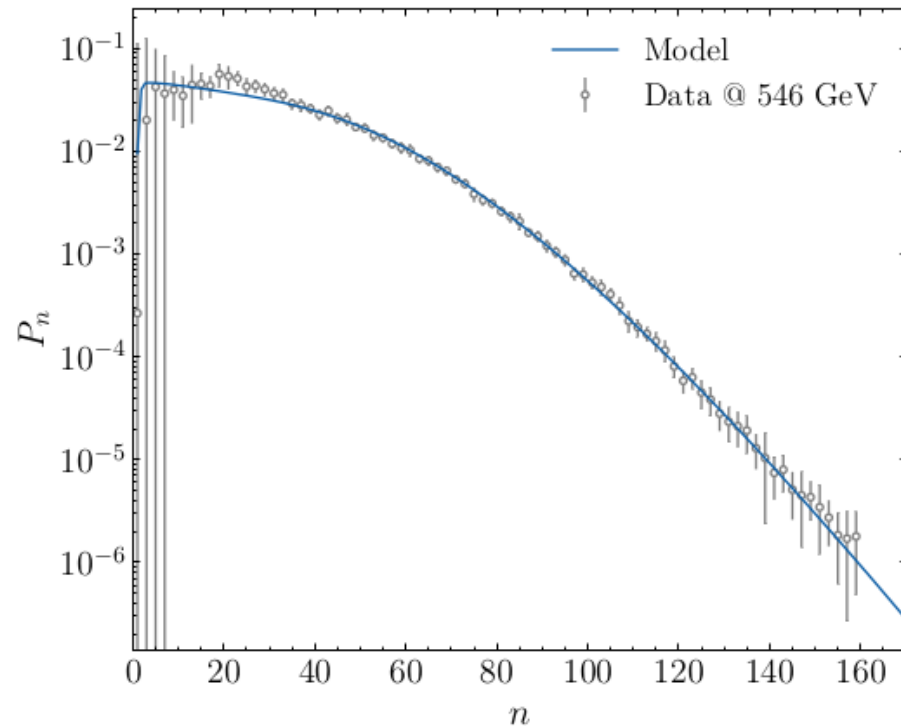
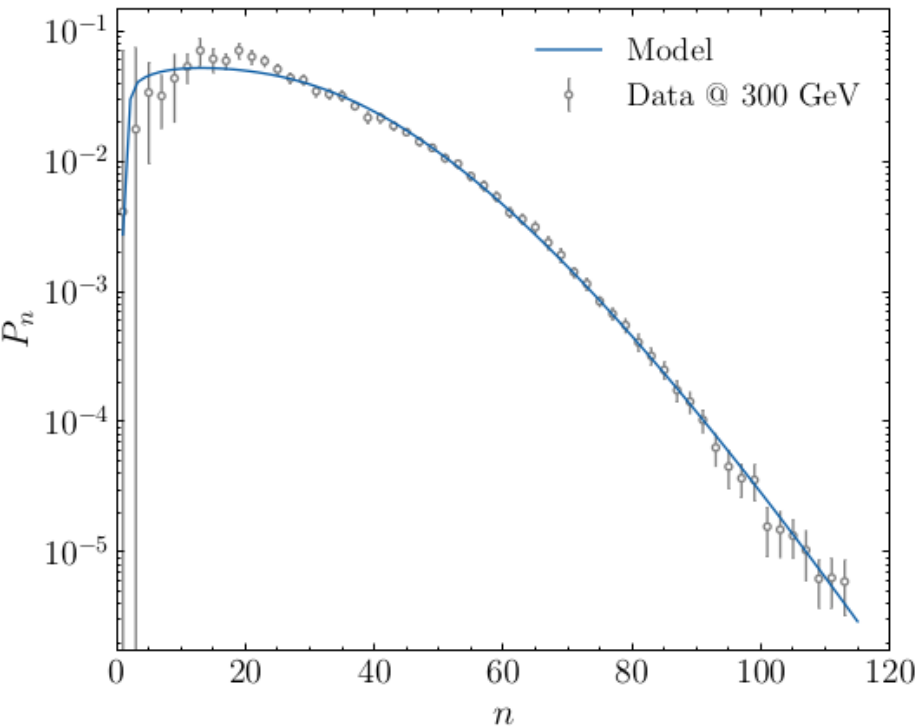
# Multiplicity distributions for inelastic pp collision



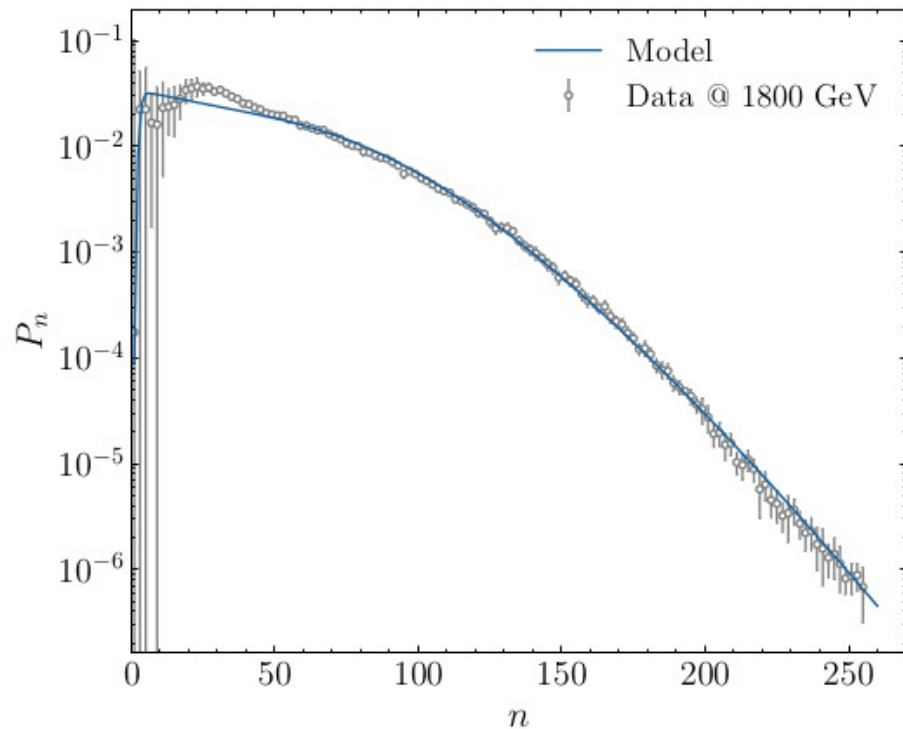
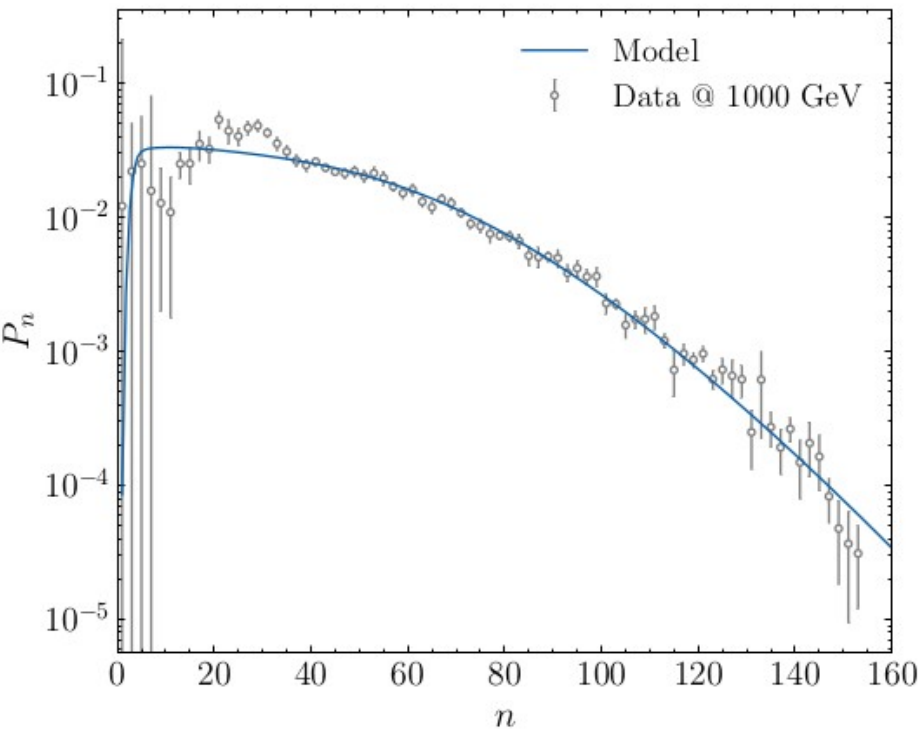
# Multiplicity distributions for inelastic pp collision



# Multiplicity distributions for inelastic $\bar{p}p$ data



# Multiplicity distributions for inelastic $\bar{p}p$ data



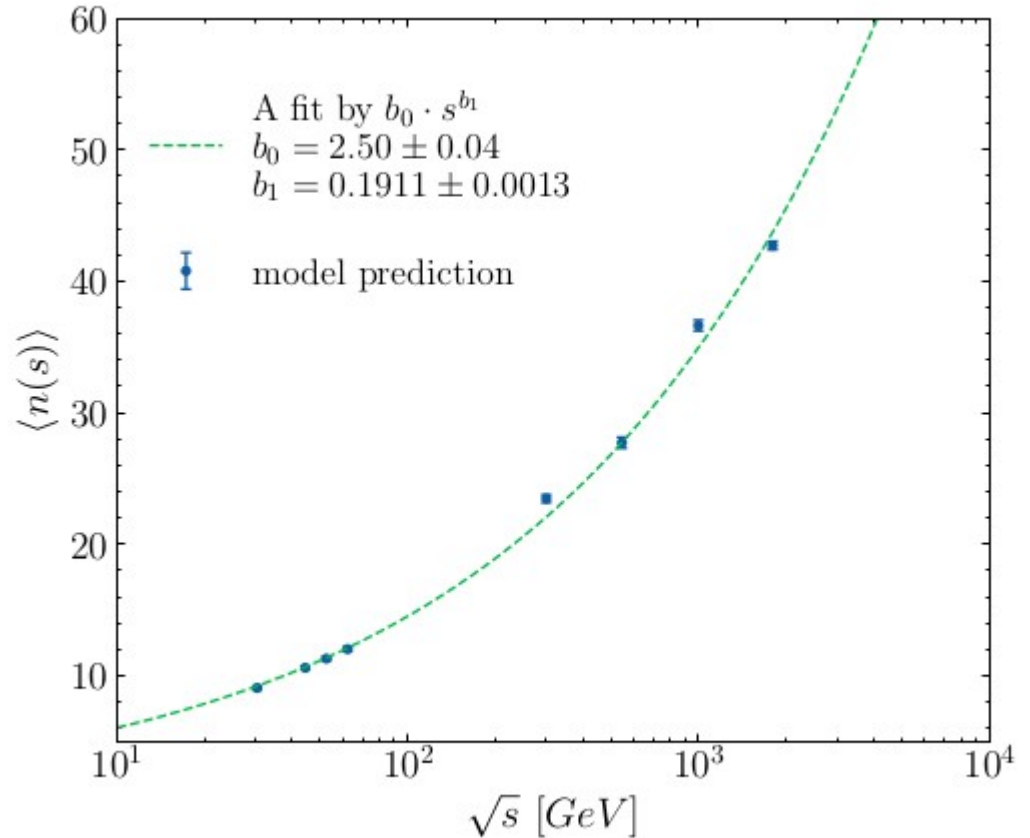


# Hadron mean multiplicity

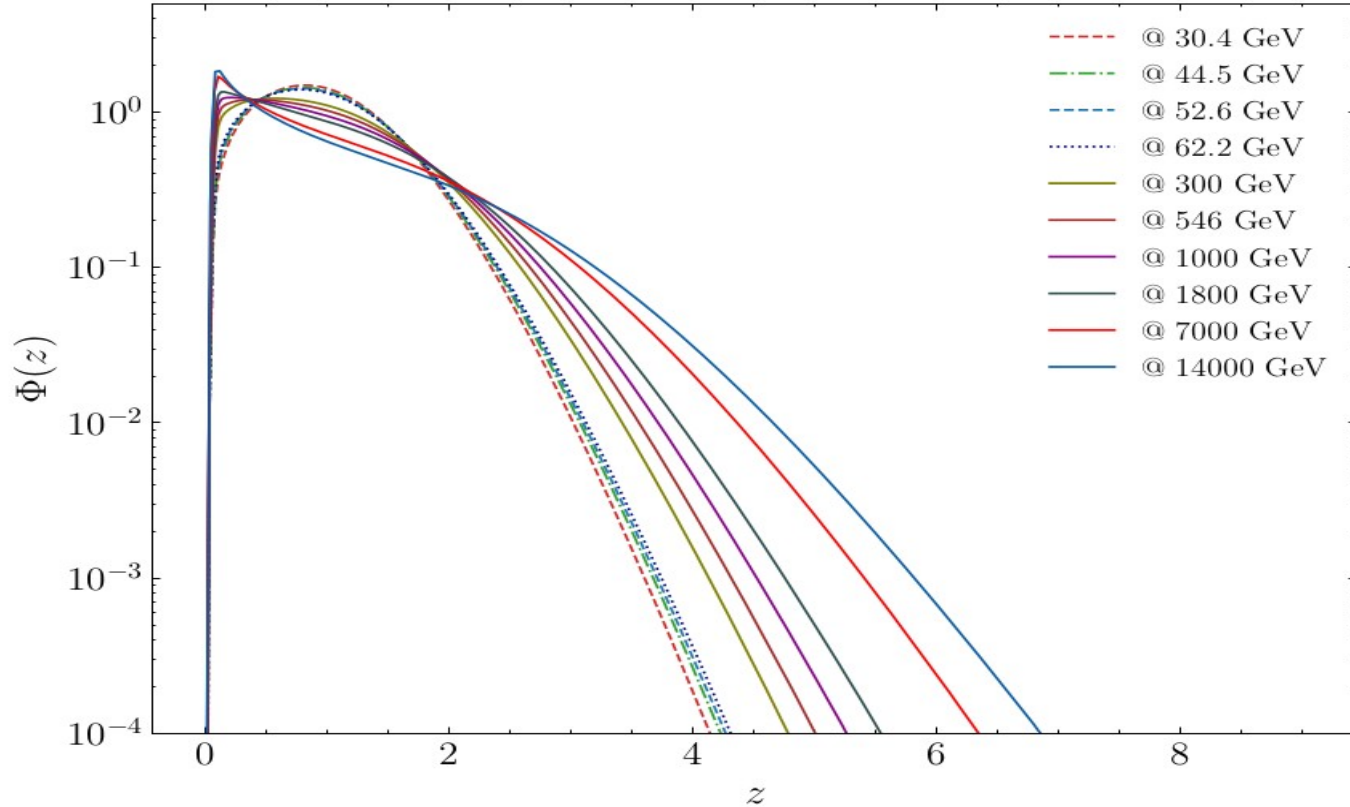
- In line with Troshin and Tyurin :

$$\langle n(s) \rangle = 2.328 s^{0.201},$$

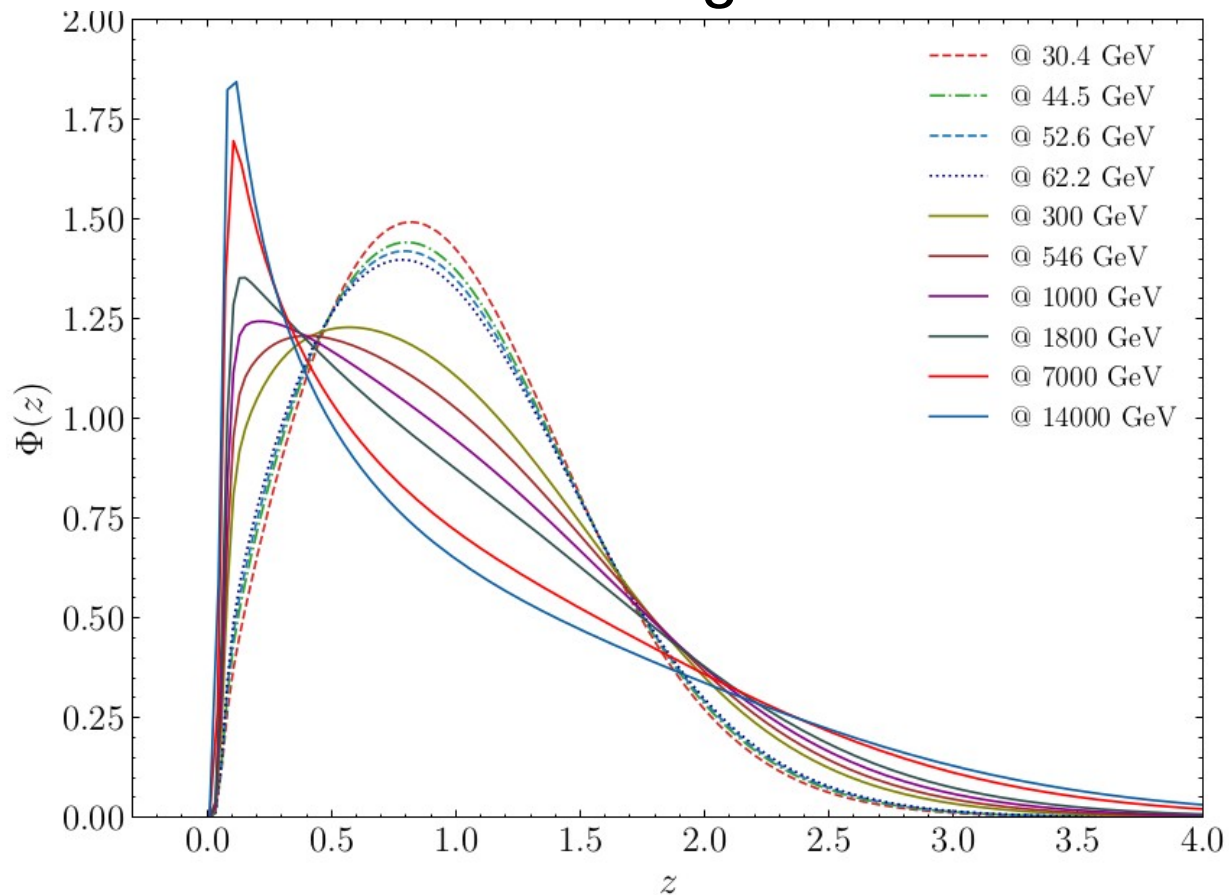
- this alignment further support the use of The U Matrix scheme



# KNO scaling violation



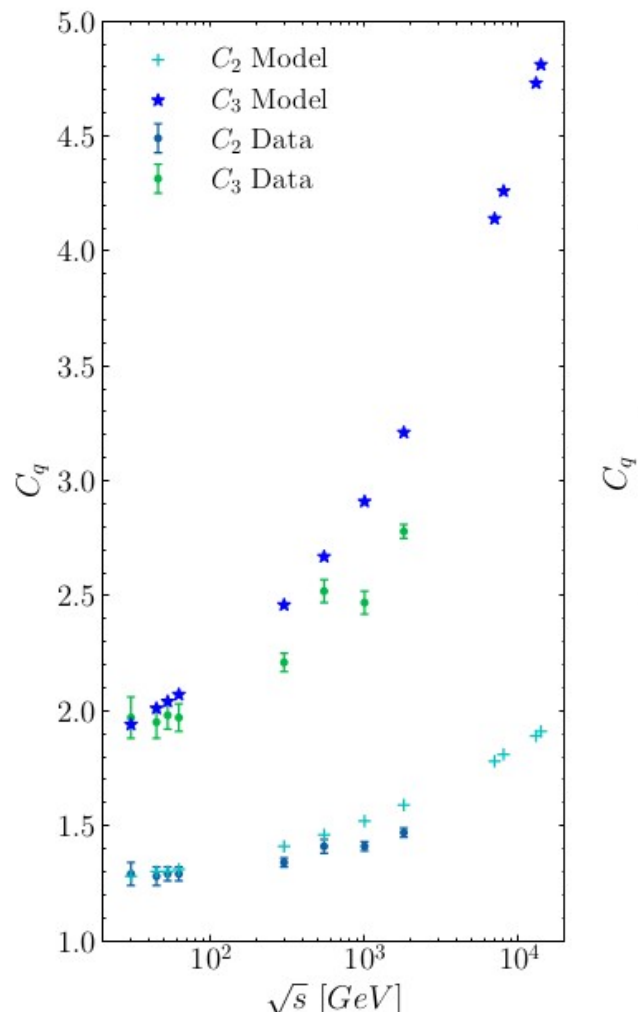
# KNO scaling violation



# Particles correlation and fluctuation

$$C_q = M_q / M_1^q,$$

$$M_q = \sum_{n=0}^{\infty} n^q P_n,$$

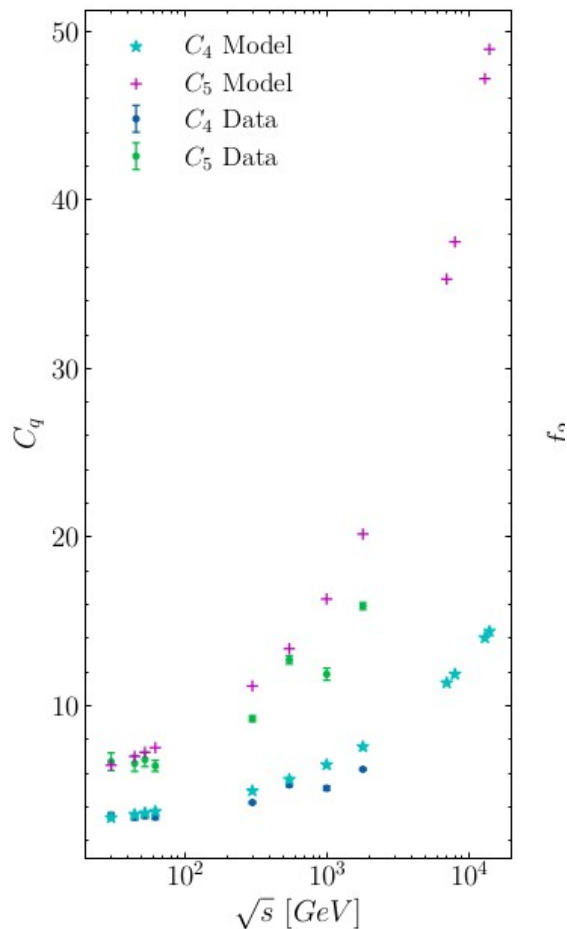


# Particles correlation and fluctuation

- Predictions match with the data points

within the ISR energy range

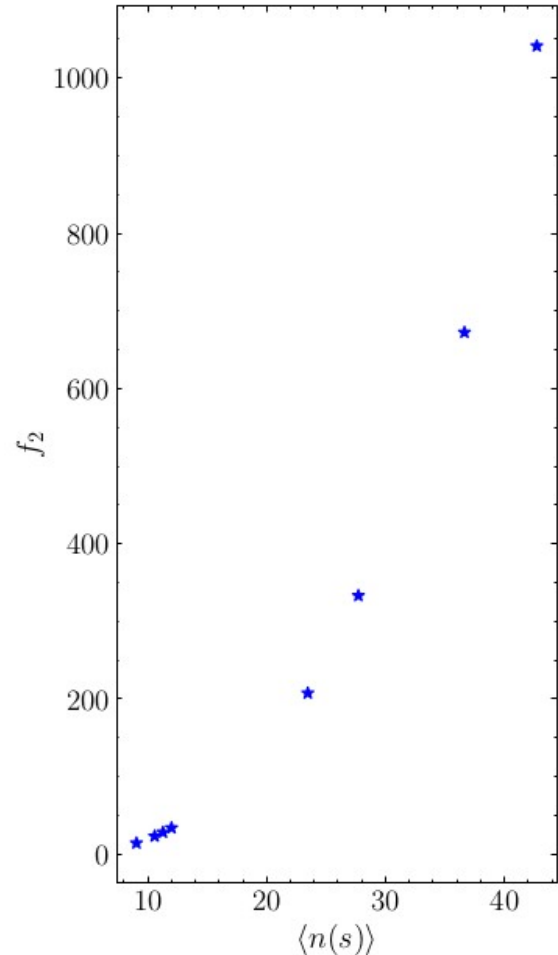
- Overestimates the fluctuations and correlations in the multiplicity distribution with rising energy notably above LHC energy.



# Particles correlation and fluctuation

- In order to further illustrate this overestimation
- The  $f_2$  moment  
(or the two-particle correlation parameter)

$$f_2 = \langle n(n-1) \rangle - \langle n \rangle^2$$



# Model's Outcomes

- Pronounced KNO scaling violation resulting from a strong Geometrical scaling violation
- Unexpected overestimation of the fluctuations and correlations with increasing energy
- Attributed to statistical fluctuations related to the inelastic overlap function and hence to the U Matrix scheme
- What is the distribution of pomerons in the U-Matrix scheme ?

**Thank you for your attention**