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To cite this article: D Fiandaca *et al* 2024 *J. Phys.: Conf. Ser.* **2647** 182016

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# A Vehicle Bridge Interaction based approach for the monitoring of bridges through an electric mobile platform

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**Abstract.** The main purpose of the Structural Health Monitoring (SHM) techniques applied in bridges is to monitor many structural parameters to prevent serious damage that may eventually lead to collapse of the entire structure. To date, Operational Modal Analysis (OMA) methods are widely seen as the most reliable SHM technique. These encompass a series of procedures for deriving the modal parameters of a structure using the data acquired under its operating conditions, without recording the external excitation. However, traditional OMA methods generally require expensive setup and time-consuming procedures. With the aim of overcoming these issues, in this paper, an innovative Vehicle-Bridge Interaction (VBI) based approach is investigated as a low-cost monitoring system for the identification of the structural modal parameters. These are obtained from the analysis in the frequency domain of the measured vehicle's vibration response during several passages on the structure. Therefore, the proposed approach has been adopted in an experimental campaign on a pedestrian bridge in the city of Palermo (Italy). In particular, a very low-cost setup comprising a scaled-up electric vehicle as a mobile platform and few accelerometers has been used.

## 1. Introduction

A widely used technique for the structural monitoring of bridges is Operational Modal Analysis (OMA) [1][2]. OMA-based methods make the identification of dynamic parameters such as frequencies, modal shapes, and damping ratios possible using data collected from sensors placed directly on the structure while it is under its operating conditions. These methods have gained increasing interest among researchers and engineers due to the possibility to provide valuable insights into the structural health of bridges. However, setting up a monitoring procedure based on OMA methods can be expensive and challenging in operating conditions. The sensors must therefore be connected to an acquisition system using long cables and access to the power grid is required. To overcome the challenges associated with traditional Structural Health Monitoring (SHM) procedures [3], researchers have proposed to use the properly acquired vehicle response and analyze it in the frequency domain to indirectly measure the dynamic parameters of bridges. This approach has been identified as a promising alternative to traditional SHM methods and it is based on Vehicle Bridge Interaction (VBI) theory [4]. The validity of this approach was first investigated by Yang et al. [5], who demonstrated its potential for accurately measuring the main bridge frequencies. Since then, several experimental verifications have been carried out using different types of test vehicles and many research efforts have focused on identifying bridge



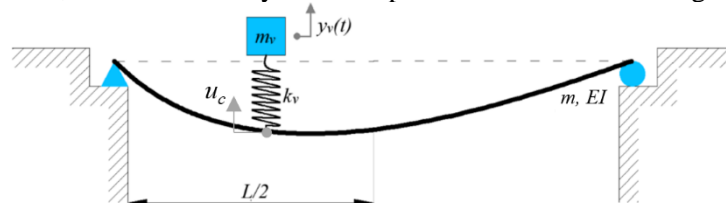
modal parameters such as structural frequencies, mode shapes, and damping ratios. Despite these promising developments, there are still some issues that can affect the accuracy of this approach. One major issue is pavement roughness [6] which adds random and uncontrollable high-frequency excitations to the vehicle response, reducing the reliability of these monitoring methods.

In addition to pavement roughness, another issue that can affect the accuracy of Vehicle Bridge Interaction (VBI) procedures is related to the frequency of the test vehicle [7]. It may occur, in fact, that some bridge frequencies may not be visible in the Power Spectral Density (PSD) function due to interference from the frequency of the test vehicle itself. To address these issues and improve accuracy, Yang et al. [8] recently proposed a “hybrid” procedure that consists of a vehicle in a parked configuration to acquire the vertical acceleration at various points on the structure obtaining the bridge’s modal parameters. However, it is important to note that this procedure requires an expensive setup, including a vehicle and a heavy trailer. Furthermore, due to the interaction between the vehicle and the bridge, the considerable mass of the vehicle-tractor system can influence the final results. In light of these challenges, this paper investigates a new VBI-based procedure using a low-cost scaled-up electric vehicle as a mobile platform properly instrumented. By alternately parking this platform at various measuring points on the structure and acquiring its vertical accelerations, it is possible to obtain accurate results, in terms of dynamic properties of the bridge, without requiring numerous acquisitions during the passage over the structure as in other VBI-based procedures.

Using such a scaled-up electric vehicle for VBI-based procedures offers several advantages. For one, it provides a cost-effective mobile platform that can be used to monitor pedestrian bridges where vehicular transit is not allowed. Additionally, since the scaled-up vehicle has a low weight, its influence on the dynamic behavior of the bridge is negligible. Moreover, its characteristics (weight and stiffness above all) can be easily changed to adapt it to the structure being monitored.

## 2. Mathematical background

As previously mentioned, the research dealing with the feasibility of utilizing a VBI-based method to determine the modal characteristics of a bridge is relatively recent. Yang et al. [4] first proposed the idea of this indirect procedure, which is based on the concept that the response of a bridge, due to vehicular traffic and ambient noise, influences the dynamic response of a vehicle crossing that bridge.



**Figure 1.** Structural diagram of the vehicle-bridge interacting system.

The most straightforward model that can be utilized to examine the dynamic interaction between a moving vehicle and a bridge is that illustrated in Figure 1 in which a simply-supported beam of length  $L$ , with a smooth pavement, is crossed by a sprung mass  $m_v$  supported by a spring of stiffness  $k_v$ . In this regard, the equations of motion governing the vertical motion of the bridge and the moving vehicle are:

$$m_v \ddot{y}_v(t) + k_v (y_v(t) - u_c(t)) = 0 \quad (1.a)$$

$$m \ddot{u}(x,t) + EI u''''(x,t) = f_c(t) \delta(x - vt) + f_a \quad (1.b)$$

where the vertical deflection of the sprung mass is represented by  $y_v(t)$ , while its constant velocity and mass are denoted by  $v$  and  $m_v$ , respectively. The vertical displacement of the beam at the contact point is given by  $u_c(t)$ ,  $E$  represents the elastic modulus,  $I$  the moment of inertia, and  $m$  the per-unit-length mass. The ambient excitation force is given by  $f_a(t)$ , and the Dirac’s delta function is represented by

$\delta(\cdot)$ . It should be noted that a dot above a variable indicates derivation with respect to time  $t$ , while an apex indicates derivation with respect to  $x$ . Furthermore, the contact force between the beam and the sprung mass is expressed as follows:

$$f_c(t) = k_v \left( y_v(t) - u(x, t) \Big|_{x=vt} \right) - m_v g \quad (2)$$

where  $g$  is the acceleration of gravity.

As reported in [9], the response of the contact point can be expressed as:

$$u_c(t) = \sum_n \left[ A_{sn} + A_{dn} \cos(2\Omega_n)t + A_{bln} \cos(\omega_{bn} - \Omega_n)t + A_{brn} \cos(\omega_{bn} + \Omega_n)t \right] \quad (3)$$

where:

-  $A_{sn}, A_{dn}, A_{bln}, A_{brn}$  are amplitude coefficients;

-  $\omega_{bn} = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{EI}{m}}$  is the  $n$ -th bridge frequency;

-  $\Omega_n$  the driving frequency;

Further, the equation for the vertical displacement of the vehicle is obtained by substituting equation (3) into equation (1):

$$y_v(t) = \sum_n \left\{ A_{sn} + A_{vn} \cos(\omega_v)t + \frac{A_{dn} \omega_v^2}{\omega_v^2 - 4\Omega_n^2} \cos(2\Omega_n)t + \frac{A_{bln} \omega_v^2}{\omega_v^2 - (\omega_{bn} - \Omega_n)^2} \cos(\omega_{bn} - \Omega_n)t + \frac{A_{brn} \omega_v^2}{\omega_v^2 - (\omega_{bn} + \Omega_n)^2} \cos(\omega_{bn} + \Omega_n)t \right\} \quad (4)$$

where:

-  $A_{vn}$  is an amplitude coefficient related to the vehicle and is a function of the other amplitude coefficients introduced before;

-  $\omega_v = \sqrt{\frac{k_v}{m_v}}$  is the vehicle's natural frequency.

Once the general equations have been introduced, in this study the particular case of the vehicle parked at a random location  $x_0$ , consequently with zero speed  $v = 0$  m/s and driving frequency absent, was investigated. In this context, the responses of the contact point and the vehicle in terms of acceleration are herein reported:

$$\ddot{u}_c(t) = \sum_n \left[ - (A_{brn} + A_{bln}) \omega_{bn}^2 \cos(\omega_{bn}t) \right] \quad (5)$$

$$\ddot{y}_v(t) = \sum_n \left[ - A_{vn} \omega_v^2 \cos(\omega_v t) - \frac{\omega_v^2}{\omega_v^2 - \omega_{bn}^2} (A_{bln} + A_{brn}) \omega_{bn}^2 \cos(\omega_{bn} t) \right] \quad (6)$$

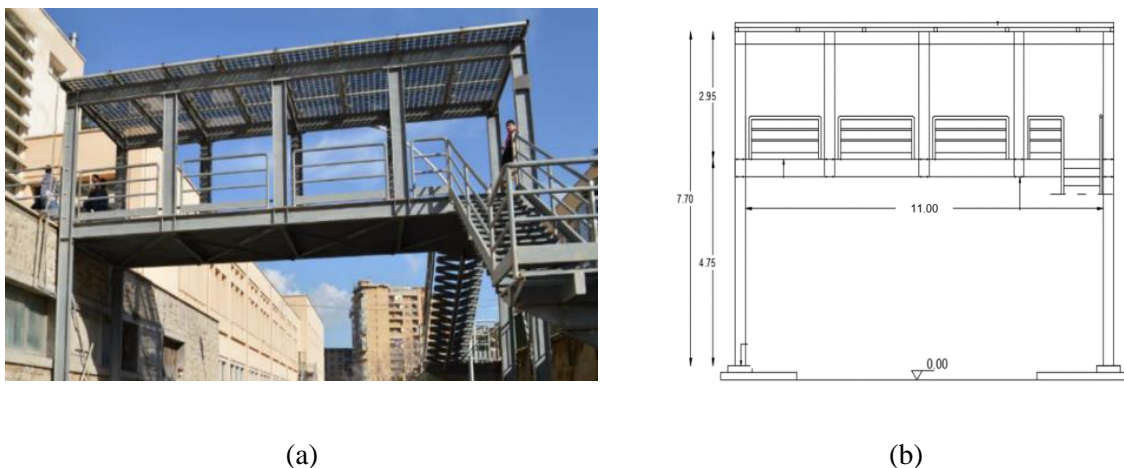
By observing these equations, it is important to note that the modal parameters of the bridge could be directly obtained from the structural response recorded at the contact point. Moreover, even from the vertical acceleration recorded by the vehicle, the modal parameters of the bridge can be obtained, but the modal parameters of the vehicle are also present. However, since it is experimentally complex to

deduce the structural response from the contact point [10], in this study the identification of the modal parameters of the bridge through the recording of the vertical acceleration of a vehicle positioned on the bridge is investigated. In particular, analyzing in the frequency domain all the  $\ddot{y}_v(t)$  acquired in various points of the structure, the bridge frequencies and the related modal shapes can be detected as will be shown in the following sections.

### 3. Tested structure

In order to evaluate the effectiveness of the proposed procedure, a pedestrian bridge located on the campus of the University of Palermo in Italy was selected for testing.

The structure (Figure 2) is made up of 4 IPE 300 steel columns that extend from the ground level and an additional 10 columns of the same dimensions that hold up the roof made of photovoltaic panels. The beams are also IPE 300 and are attached to the columns using bolted connections reinforced with extended flanges. The suspended floor is a composite type consisting of a galvanized corrugated steel sheet and a concrete layer, measuring 3 m in width and 11 m in length. On the underside of the floor, there are secondary IPE 200 beams that connect the columns and provide additional support through bracing.

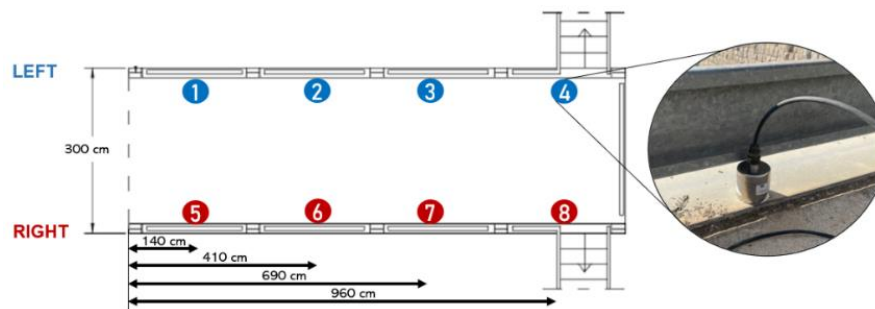


**Figure 2.** An external view (a) and the front elevation of the pedestrian bridge (b).

### 4. Identification using classical OMA procedure

In this section, the experimental results of the identification of the structural dynamic parameters of the pedestrian bridge are reported.

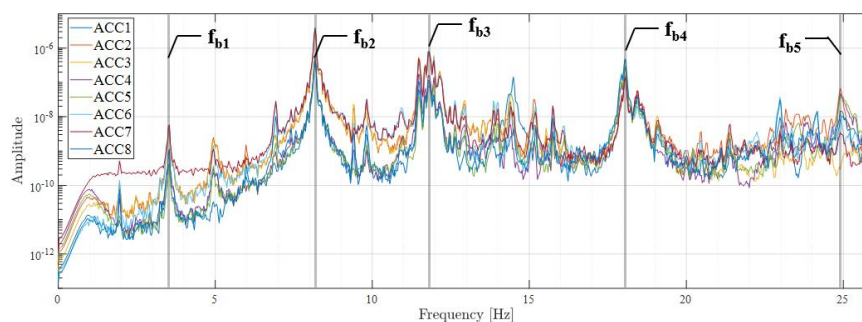
With the aim of validating the proposed procedure, whose experimental test and results are reported in section 5, a preliminary traditional OMA identification was performed. Using various accelerometers directly positioned over the bridge, the main frequencies of the structure were obtained. Specifically, eight high-sensitivity piezoelectric mono-axial accelerometers (PCB 393B31), were chosen to record the response of the bridge acquiring the vertical acceleration at various points of the structure, that is to say, 4 for each side of the structure as shown in Figure 3. It is worth highlighting that The number of accelerometers was determined by taking into account several factors, including the type of structure being monitored, the available equipment setup, and the expected modal shapes. In particular, we aimed to avoid placing accelerometers at the nodes of the modal shapes intended to be identified. Further, all the sensors were connected to the acquisition unit NI PXIe-1082, which acquired the signal with a sampling rate of 1000 Hz.



**Figure 3.** Position of the accelerometers on the deck and a picture of one of the sensors.

Data from the eight accelerometers were acquired and processed in the MATLAB environment. In particular, several records were performed with the bridge under ambient vibrations for a total duration of 15 min. It should be noted that throughout the duration of the analysis, an additional noise component was always present: the passage of pedestrians.

Once the signals were acquired, a band-pass filter was used, choosing a band of frequencies between 0.2 Hz and 40 Hz. Then, the respective PSD functions were determined, following Welch’s Method [11], by partitioning the signal recorded into sub-signals of 10 s. Finally, a Hanning window was employed, with an overlap of 50% between the segments.



**Figure 4.** Auto-PSD functions of all the accelerometers on the bridge.

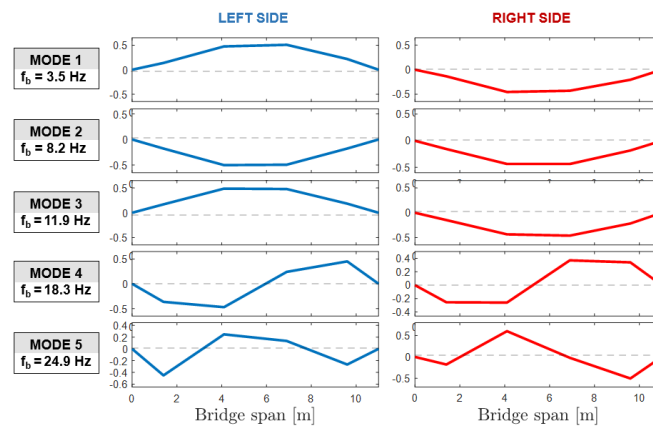
The PSDs obtained by the acquisition of each accelerometer are shown in Figure 4 and the frequencies accordingly obtained are reported in Table 1. It is worth noticing that, in this case, the discrepancies related to the results of each PSD are negligible.

**Table 1.** Summary table of frequencies obtained by the OMA method.

$f_{b1}$ [Hz]	$f_{b2}$ [Hz]	$f_{b3}$ [Hz]	$f_{b4}$ [Hz]	$f_{b5}$ [Hz]
3.5	8.2	11.9	18.3	24.9

Once the Power Spectral Density (PSD) functions related to each sensor positioned on the structure have been derived, the data was analyzed using the Frequency Domain Decomposition (FDD) technique [12]. This allowed for the identification of the modal shapes of the structure associated with each frequency. The results are presented in Figure 5, where a distinction has been made between the modal shapes obtained from sensors located on the left side of the bridge and those obtained from sensors on the right side.

According to [13], to differentiate between the bending and torsional modes of a structure, an additional analysis of the cross-spectra phase can be taken between two sensors that are laterally spaced on opposite sides of the structure at the same longitudinal location. In the lateral plane, it is expected that the output signals from these two sensors will be in phase for all bending modes and 180° out of phase for all torsional modes, assuming that the modes are completely decoupled.



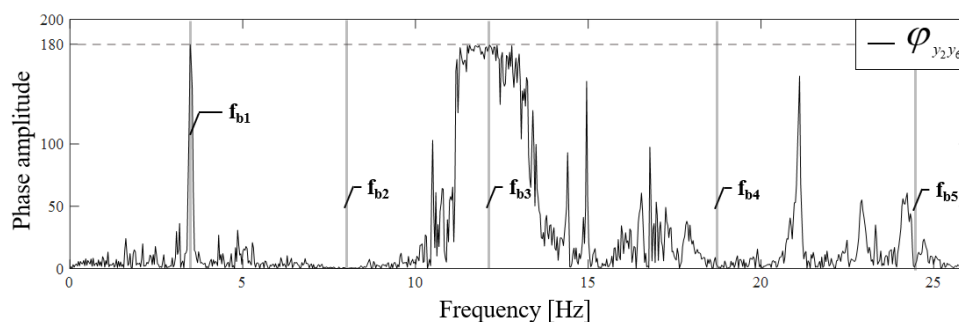
**Figure 5.** Modal shapes related to the first five frequencies of the pedestrian bridge (OMA-based method).

It is important to note that when a peak in the output spectrum is due to a spectral peak in the excitation rather than a resonant response of the structure, the phase data between two output measurements will typically be something other than zero or  $180^\circ$ . This means that a careful phase data analysis is necessary to accurately distinguish between bending and torsional modes.

In order to follow this approach, the cross-spectrum phase has been derived considering the following relation for each couple of sensors positioned in points of the structure with the same longitudinal distance:

$$\varphi_{y_i y_j} = \arctg \frac{\text{Im} \left\{ \mathcal{S}_{y_i y_j}(\omega) \right\}}{\text{Re} \left\{ \mathcal{S}_{y_i y_j}(\omega) \right\}} \quad i = 1 \div 4 ; j = 5 \div 8 \quad (7)$$

In Figure 6 is reported the cross-spectrum phase related to the cross-spectrum obtained from the analysis in the frequency domain of the structural response recorded in points 2 and 6 (OMA-based method). From this result, it is easily possible to distinguish the torsional modes with a phase value of about  $180^\circ$  ( $f_{b1}$ ,  $f_{b3}$ ) from the other ( $f_{b2}$ ,  $f_{b4}$  and  $f_{b5}$ ) with a value of  $0^\circ$  that are related to the first three bending modes of the bridge.



**Figure 6.** Cross-spectrum Phase related to the sensors in points 2 and 6.

### 5. Identification using the VBI-based procedure

The proposed identification procedure based on VBI theory is here discussed. In this regard, tests were carried out using an electric mobile platform properly equipped to record the vertical accelerations in various and specific points of the same pedestrian bridge selected for the OMA-based monitoring test. It should be noted that an extremely simpler setup than that required for the traditional technique, was used to carry out the tests. These details are explained further below.

### 5.1. Experimental setup

An electric mobile platform and two high-sensitivity piezoelectric accelerometers (PCB 393B31) were utilized for these tests (see Figure 7). The choice to develop such a mobile platform was made for multiple reasons. On one hand, this setup is, with the same results, significantly less expensive than others, reported in the literature, in which is required a real vehicle and a trailer. On the other hand, this platform represents an adaptable setup that can be easily used not only on vehicular bridges but also on pedestrian bridges, allowing their operative conditions during the monitoring process.

The first sensor was placed directly on the structure, using it as a reference sensor, and the second one was installed on the mobile platform, that was alternately placed at 6 different points of the pedestrian bridge. It is worth noting that all piezoelectric accelerometers were connected to a PC through a signal conditioner, the Digital ICP - USB Signal Conditioner 485B39, and recorded using a self-developed acquisition software in the LabView environment by choosing a sampling frequency of 1000 Hz.

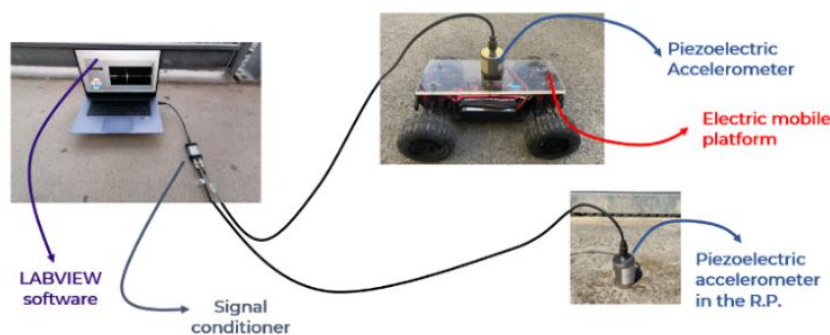


Figure 7. The setup used for the VBI-based procedure.

### 5.2. Effect of the vehicle frequency on the acquisitions

As already mentioned, the use of a vehicle in a parked configuration gives many advantages. One of these is the reduction of the influence of the vehicle frequency in the acquisition made from the vehicle itself. To demonstrate it, a simple experiment has been carried out.

In this experiment, the response of a vehicle, in terms of vertical acceleration, is acquired both in motion and in not-in-motion condition. Firstly, the vehicle is moved in a straight line and the response is acquired through a 10-minute acquisition. Then, the response of the vehicle is acquired in the other condition while it is subjected to ambient noise.

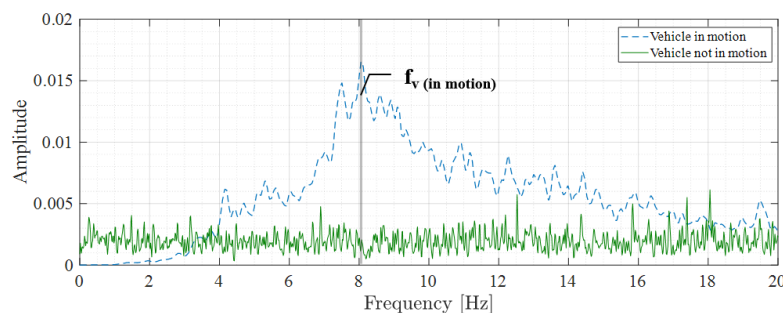
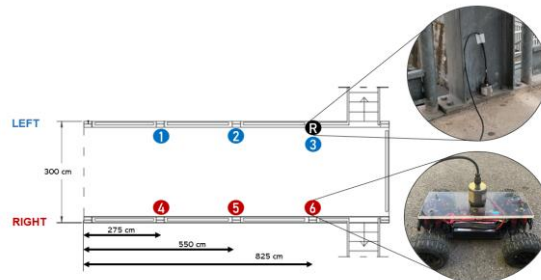


Figure 8. Comparison of the PSD functions of the response of the vehicle.

By comparing the PSDs (Figure 8), it is noted that a frequency at around 8 Hz is very clear in the response of the vehicle, whereas such a frequency is not visible with the mobile platform in a “parked” state. This means that the device can be used in parked state condition for monitoring purpose without interfering with the response of the bridge acquired.

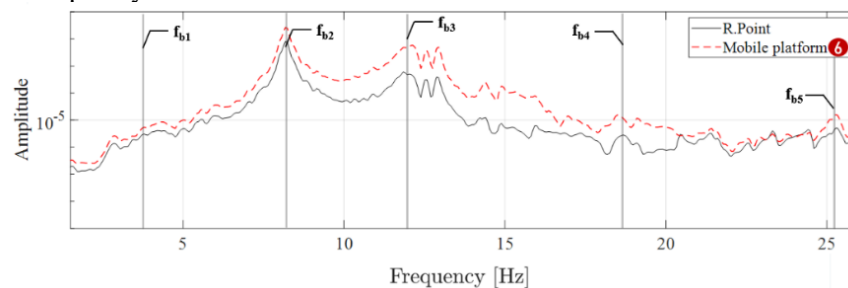
### 5.3. Experimental test

The proposed monitoring procedure involves acquiring data simultaneously, for a duration of 180 seconds, from an accelerometer placed at a designated location of the structure used as the reference point (identified as point R in Figure 9) and from a sensor installed on the mobile platform. This is alternately positioned at six measurement points on the structure (points 1-6 in Figure 9).



**Figure 9.** Arrangement of measuring points for the vehicle (Points 1-6) and reference point on the bridge surface (point R).

Once the data accelerations have been acquired, they have been filtered using a band-pass filter between 0.5 and 30 Hz. Then, analyzing the data in the frequency domain, the auto-PSD and cross-PSD functions were derived according to the Welch method. In order to apply it, all the signals were divided into 10-second segments and a Hanning window with an overlap of 50% between the segments was employed. It should be stressed that these parameters have been considered the optimal balance between achieving accurate results and minimizing analysis time after conducting several tests using different window lengths and overlaps. Once these parameters have been set, the PSD functions related to the acquisition from each point of the structure were derived and, for example, PSD functions obtained from the acceleration data recorded in the reference point and those recorded at the same time with the mobile platform positioned in point 6 are reported in Figure 10. Further, the related frequencies identified from the analysis in the frequency domain are shown in Table 2.



**Figure 10.** PSD functions obtained from the acceleration data recorded in the reference point (solid line) and from the mobile platform positioned in point 6 (dashed line).

**Table 2.** Summary table of frequencies obtained by the OMA method.

	$f_{b1}$ [Hz]	$f_{b2}$ [Hz]	$f_{b3}$ [Hz]	$f_{b4}$ [Hz]	$f_{b5}$ [Hz]
<b>OMA</b>	3.5	8.2	11.9	18.3	24.9
<b>VBI</b>	3.4	8.2	11.9	18.9	25.1
discrepancy	3%	0%	0%	3%	1%

It is possible to notice that the results, in terms of peaks related to specific structural frequencies, are very similar to those obtained following the traditional method.

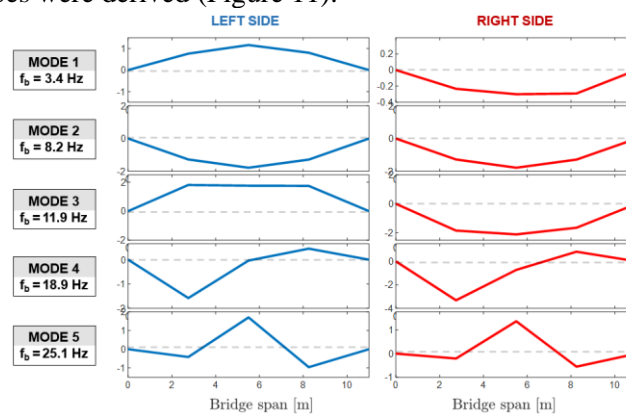
Furthermore, to obtain the modal shape from this acceleration data, the modal amplitude ratios of the cross-spectra were calculated following the relation introduced in [8] and here reported:

$$A_{ir} = \frac{S_{ir}(f_k)}{S_{rr}(f_k)} \quad (8)$$

where:

- $f_k$  is the k-th frequency of the bridge identified by analyzing all the auto-PSD functions;
- $S_{ir}$  is the cross-spectrum calculated considering the i-th point recorded from the mobile platform and the data acquired from the reference point;
- $S_{rr}$  is the auto-spectrum derived from the acceleration data of the reference point.

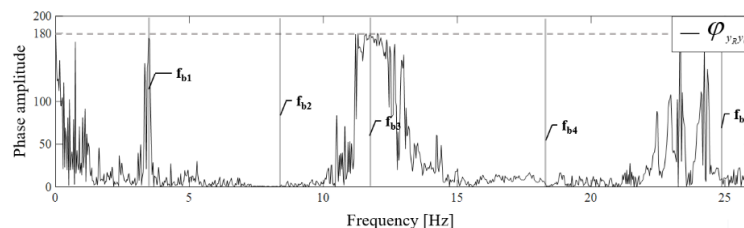
Once the modal amplitude ratio values have been obtained considering all the structural frequencies, the related modal shapes were derived (Figure 11).



**Figure 11.** Modal shapes related to the first five frequencies of the bridge (VBI-based procedure).

From the figures it is shown that the results in terms of modal shapes are perfectly matching those obtained previously (Figure 5).

Furthermore, to distinguish torsional and flexural modal shapes, as already shown depicting the OMA-based method, the cross-spectrum phase has been considered. In this case, since not each signal from the structure was simultaneously acquired, the phase function was calculated considering the data obtained from the sensor on the mobile platform positioned at point 6 of the pedestrian bridge and that from the sensor positioned at the reference point. Thus, the result, that is shown in Figure 12, validates the possibility to identify the flexural modal shapes ( $f_{b2}$ ,  $f_{b4}$  and  $f_{b5}$ ) adopting the innovative VBI-based procedure. According to these results, the fundamental mode of the bridge has a correspondent frequency of about 8.2 Hz as obtained in the tests carried out using the OMA-based procedure.



**Figure 12.** Cross-spectrum Phase related to the sensors in points 6 and R (VBI-based procedure).

## 6. Concluding remarks

In this paper, an innovative VBI-based procedure was experimentally validated to obtain the modal parameter of a pedestrian bridge using an extremely cost-effective setup with only two sensors. In this regard, it is important to note that traditional methods would require a much larger number of sensors to achieve comparable results. This procedure therefore leads to a substantial reduction in both the cost associated with the setup required and the time necessary to carry the tests out.

Further, the use of a modular mobile platform allows for easy placement of all necessary sensors. Additionally, for the first time to the best authors' knowledge, a method to distinguish between frequencies related to flexural and torsional modes has been implemented in a VBI-based procedure. The findings of the study are summarized in the following section.

Data analysis revealed that there is a satisfactory level of agreement, in terms of frequencies and modal shapes identified, between the innovative procedure based on VBI theory and traditional methods that rely on sensors positioned directly on the structure. Moreover, the new mobile platform developed for the experimental tests represents a convenient and modular platform in which various sensors can be inserted depending on the needs of the monitoring tests being conducted.

In addition, it should be stressed that the accuracy of identifying structural dynamic parameters using the proposed procedure is significantly higher, also considering the experimental tests carried out in previous studies by the authors [7], than that achieved by the VBI methods based on in-motion sensors in which no more than the first frequency of the structure could be easily identified.

In conclusion, this study demonstrates that the innovative VBI-based procedure can be considered a reliable and cost-effective tool for identifying structural dynamic parameters.

### Acknowledgment

Authors gratefully acknowledge the financial support of the project PRIN 2017 funding scheme (project 2017J4EAYB 002 - Multiscale Innovative Materials and Structures "MIMS")

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