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ORIGINAL ARTICLE

Simplified method for the vibration comfort assessment of steel-concrete floors induced by crowd-rhythmic activities

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Abstract

This paper presents a design-oriented method for a simplified prediction of the vibrational response of regular floors induced by crowd-rhythmic activities. The method is based on a frequency domain load model, along with the stochastic vibration theory. An application of the method is illustrated on an existing steel-concrete composite floor belonging to an open-plan office, with a total area of about 385m². Four rhythmic activities with experimentally identified load parameters are investigated. The individuals performing these activities are assumed to be uniformly distributed over the floor, with their numbers ranging between 1 and 64. The prediction of the floor response reveals that the floor presents tolerable levels of acceleration for up to 16 persons in general, while remedial measures should be undertaken to reduce the response of the floor subjected to larger crowd sizes. The proposed method might represent a first step in the development of the floor serviceability assessment for vibration comfort in the forthcoming editions of Eurocodes for practical application.

Keywords

steel-concrete composite floor, human-induced vibration, rhythmic load model, group effect, response prediction, human comfort assessment.

1 Introduction

Building floors are becoming increasingly lightweight, slender and flexible due to the continual progress of construction (use of lightweight materials, tendency to longer spans, etc.). This alters the dynamical performance of these structures making them more sensitive to human-induced vibrations. The loading producing the maximum response corresponds to a group of people performing coordinated rhythmic activities [1], where other floor occupants exposed to such a response may feel discomfort or even panic. The response prediction of floors using a reliable load model for rhythmic activities is thus a pre-requisite to tackle this issue.

Several design guidelines were drawn around the world to propose methods for the serviceability assessment of floors against human discomfort by means of floor response evaluation. Two of them are commonly used for rhythmic activities: SCI P354 guideline [2] based on RMS accelerations and AISC DG11 guideline [3] based on peak accelerations. As they offer a simplified response prediction of floors, these guidelines have two main limitations. First, rhythmic activities are characterized by time domain load models which were the most studied ones in this field of research [4]. However, these models do not consider the variation of rhythmic parameters (frequency, amplitude) during motion, leading to a spread of energy in the

vicinity of the load harmonics [1]. Few frequency domain models were established to consider this effect especially as random fields [5]. Second, both guidelines are only applied to the case of a single person performing rhythmic activities. The case of crowd-induced activities is not covered in spite of being prevalent in many floor structures (sports venues, gymnasiums, stadiums, etc.). Other guidance documents dealing with the subject provide more insight about crowd cases (such as UK recommendations [6] and ISO 10137 [7]) but are still not sufficient for a complete floor response calculation.

In this regard, a design-oriented method is proposed in this paper in order to handle simplified response prediction of regular floors due to rhythmic activities. This method is based on a frequency domain load model which accurately represents the load energy during rhythmic movement and could be applied for groups of people performing coordinated activities. A detailed presentation of the method based on the stochastic vibration theory is first provided. An existing steel-concrete floor in which this method is applied is introduced afterwards, followed by the description of investigated rhythmic activities.

Subsequently, the response of the analysed floor is calculated according to the simplified method steps and results are finally presented and discussed.

2 Simplified method for floor response prediction

A response calculation method is proposed for regular floors subjected to crowd-rhythmic activities in order to assess their behaviour against human discomfort. This method is based on a spectral crowd load model along with the random vibration theory.

2.1 Scope of the method

The proposed method is applied to rhythmic activities where individuals are losing contact with the ground while performing coordinated motion [4]. On the other hand, the floors for which this method is available should fulfil two conditions:

- Their response is dominated by only one natural mode in the frequency range between 0 and 10Hz (corresponding to human excitation [8]);
- They are characterized by a regular mode shape (close to the classical sinusoidal one).

These requirements should be agreed with the stakeholders after a modal analysis of the analysed floor (using experimental techniques, numerical modelling or simplified analytical formulations [2]).

2.2 Rhythmic load model

Consider a floor structure satisfying the aforementioned conditions where a single person or a group of individuals ($N \geq 1$) perform rhythmic movements at fixed positions, as shown in Figure 1. Each individual k is assumed to have a known position on the floor ($x_{0,k}$; $y_{0,k}$) and a known body mass m_k.

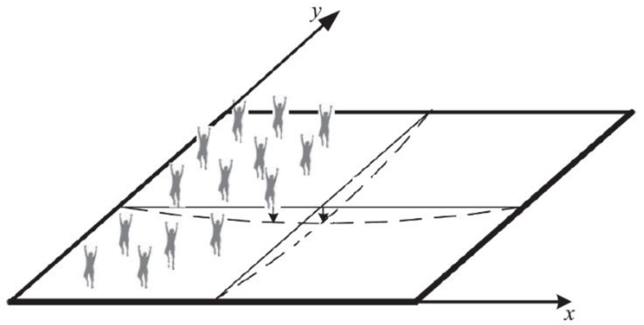

Figure 1 Single-mode floor subjected to crowd-rhythmic activities (modified from [5])

A spectral load model characterizing the rhythmic activity performed by each individual in the group has been already established [9]. The model is expressed by a Power Spectral Density (PSD) function for a single person, combined with coordination factors to take into account the lack of synchronization between multiple individuals.

The load model corresponding to the k^{th} individual $S_{p,N,k}(f)$ is computed by:

$$S_{p,N,k}(f) = \left[C(N) m_k g \alpha\right]^2 \sum_{i=1}^{3} \left[a_i^2 \exp\left(-\frac{(f - i f_p)^2}{(i\delta)^2} \right) \right] \quad (1)$$

where m_k is the body mass of the k^{th} individual, g the gravity acceleration (9.81m/s²), and for each activity: $C(N)$ is the coordination factor, f_p the excitation frequency, α the amplitude coefficient, δ the bandwidth coefficient and a_i the relative coefficient of the i^{th} harmonic (i=1, 2, 3).

An equivalent crowd load model is derived from the above model, based on the observation of experimental loads and responses against crowd size [10]. Indeed, instead of applying the PSD load model (given by Eq. (1)) at each of the N excitation positions, a single equivalent model is used in association with a modal amplitude corresponding to the mean of the N modal amplitudes.

The resulting equivalent load model is then expressed by:

$$S_{p,N}(f) = \left[NC(N)\bar{m}g\alpha\right]^2 \psi_p(f) \quad (2)$$

where $\bar{m}$ is the mean body mass of individuals and $\psi_p(f)$ is given by:

$$\psi_p(f) = \sum_{i=1}^{3} \left[a_i^2 \exp\left(-\frac{(f - i f_p)^2}{(i\delta)^2} \right) \right] \quad (3)$$

2.3 Calculation procedure

Using the same configuration as for Section 2.2, the vibrational response of the floor is calculated using the equivalent load model expressed by Eq. (2) together with the random vibration theory [11].

The PSD of generalized forces $S_{p^*}(f)$ is obtained by:

$$S_{p^*}(f) = \bar{\Phi}_p^2 S_{p,N}(f) \quad (4)$$

Here, $\bar{\Phi}_p$ is the mean of the fundamental modal amplitudes at the N excitation positions.

This results in the PSD of the fundamental modal coordinate $S_q(f)$ expressed as follows:

$$S_q(f) = \bar{\Phi}_p^2 \left|H_1(f)\right|^2 S_{p,N}(f) \quad (5)$$

where $H_1(f)$ is the transfer function related to the fundamental mode, such that:

$$\left|H_1(f)\right|^2 = \frac{1}{M_1^2\left[16\pi^4\left(f_1^2 - f^2\right)^2 + \left(8\pi^2\xi_1 f_1 f\right)^2\right]} \quad (6)$$

Here, f_1, M_1 and ξ_1 are the natural frequency, the modal mass and the damping ratio of the fundamental mode, respectively.

The PSD of the acceleration response $S_a(f)$ is then calculated by:

$$S_a(f) = \left[NC(N)\bar{m}g\alpha\right]^2 \Phi_r^2 \bar{\Phi}_p^2 (2\pi f)^4 \left|H_1(f)\right|^2 \psi_p(f) \quad (7)$$

where Φ_r is the fundamental modal amplitude at the response location. Eq. (7) is rewritten as:

$$S_a(f) = \sum_{i=1}^{3} S_{a,i}(f) \tag{8}$$

Here, $S_{a,i}(f)$ is the PSD response due to the i^{th} harmonic of the load (i=1, 2, 3) obtained by:

$$S_{a,i}(f) = \gamma_N f^4 \left|H_1(f)\right|^2 a_i^2 \exp\left(-\frac{\left(f - if_p\right)^2}{(i\delta)^2}\right) \tag{9}$$

where $\gamma_N = (2\pi)^4 \left[NC(N)\bar{m}g\alpha\right]^2 \Phi_r^2 \bar{\Phi}_p^2$ (10)

The Root Mean Square (RMS) response is adopted due to its wide usage in human comfort assessment of floors [2],[7]. The RMS acceleration $a_{rms,i}$ due to the i^{th} harmonic is then expressed by:

$$a_{rms,i}^2 = \int_0^{+\infty} S_{a,i}(f)df \tag{11}$$

Which gives:

$$a_{rms,i}^2 = \gamma_N a_i^2 \int_0^{+\infty} \left[f^4 \left|H_1(f)\right|^2 \exp\left(-\frac{\left(f - if_p\right)^2}{(i\delta)^2}\right)\right] df \tag{12}$$

An approximate formulation of the integral provided by Eq. (12) is obtained using two principal simplifications. First, the exponential function of the PSD load model (see Eq. (9)) is replaced by a bilinear function $B_i(f)$ (i=1, 2, 3) given by:

$$B_i(f) = \begin{cases} 1 + \left(\dfrac{f - if_p}{2i\delta}\right), & i(f_p - 2\delta) \le f \le if_p \\ 1 - \left(\dfrac{f - if_p}{2i\delta}\right), & if_p \le f \le i(f_p + 2\delta) \end{cases} \tag{13}$$

For instance, Figure 2 illustrates the exponential and bilinear functions for the first harmonic with f_p=2Hz and δ=0.05Hz. It can be observed that the energy distribution is almost the same between the two functions with comparable load energy (variance). The same remarks hold true for higher load harmonics.

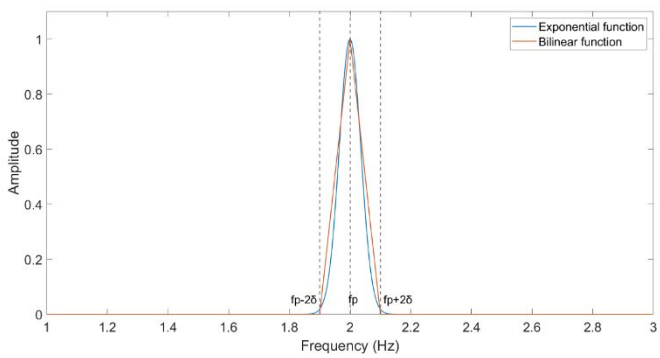

Figure 2 Comparison of exponential and bilinear functions for the PSD load model (first harmonic, excitation frequency of 2Hz)

Second, a closed-form expression of the bilinear function $B_i(f)$ is proposed. When the bandwidth coefficient δ is far below the excitation frequency f_p, it is assumed that the load energy of the i^{th} harmonic could be concentrated at the corresponding peak [12]. Since the enclosed area of the bilinear function curve equals $2i\delta$ (as illustrated in Figure 2), the following approximation is made:

$$\exp\left(-\frac{\left(f - if_p\right)^2}{(i\delta)^2}\right) \approx B_i(f) \approx \left(2i\delta\right)\, \Delta(f - if_p) \tag{14}$$

where Δ is the Dirac-delta function, having an integral property for any function $Z(f)$ continuous at $f=b$ (b>0) expressed by:

$$\int_0^{+\infty} Z(f)\Delta(f-b)df = Z(b) \tag{15}$$

Using the simplification of the exponential function provided by Eq. (14), the application of Eq. (15) provides the following approximation of the integral:

$$\int_0^{+\infty} \left[f^4 \left|H_1(f)\right|^2 \exp\left(-\frac{\left(f - if_p\right)^2}{(i\delta)^2}\right)\right] df \approx \left(2i\delta\right)(if_p)^4 \left|H_1(if_p)\right|^2 \tag{16}$$

This leads to a simplified expression of Eq. (12) given by:

$$a_{rms,i}^2 \approx \gamma_N a_i^2 \left(2i\delta\right)(if_p)^4 \left|H_1(if_p)\right|^2 \tag{17}$$

Finally, the total RMS acceleration of the floor a_{rms} is deduced by:

$$a_{rms} = \sqrt{\int_0^{+\infty} S_a(f)df} = \sqrt{\sum_{i=1}^{3} a_{rms,i}^2} \tag{18}$$

This method was further compared with the exact floor response prediction method in the frequency domain and acceptable results were found in terms of floor RMS acceleration [10].

3 Investigated steel-concrete floor

The application of the simplified method detailed earlier is illustrated on an existing steel-concrete floor, where crowd-rhythmic activities are assumed to take place.

3.1 Presentation of the structure

The investigated structure is a steel-concrete floor belonging to a one storey open-plan office building. The total area of the floor is 28.5×13.5m². It comprises a composite concrete slab with a 12cm thickness supported underneath by steel beams and columns (see Figure 3). The connection between secondary beams and slabs is achieved by shear studs.

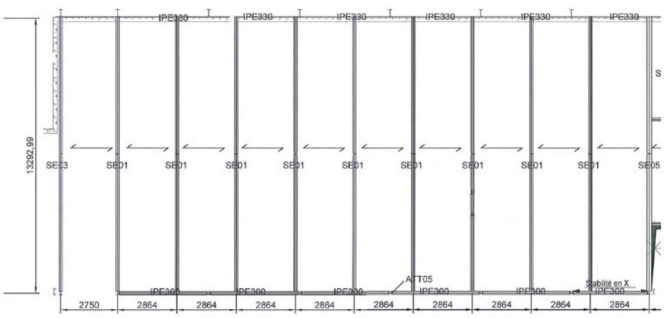

Figure 3 Plan view of the steel-concrete floor (dimensions in mm)

3.2 Modal analysis

A numerical model was established in order to determine the modal properties of the analysed floor. This was done by the Finite Element Method using ANSYS. All beams were considered as simply supported in the FEM model shown in Figure 4.

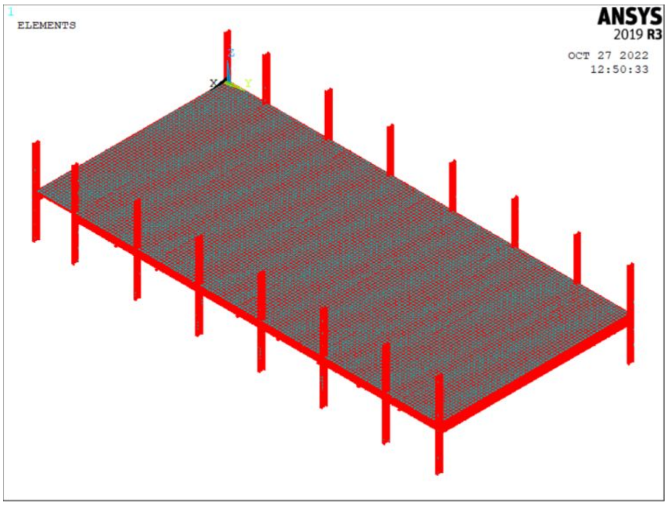

Figure 4 FEM model of the investigated floor

The floor modal analysis was carried out using Block-Lanczos extraction method. Table 1 presents results obtained for the fundamental mode. The damping ratio was taken as 3% for the steel-concrete floor according to design guidelines prescriptions [2],[3].

Table 1 Modal properties of the analysed floor

Parameter	Natural frequency (Hz)	Modal mass (t)	Damping ratio (%)
Value	3.40	24.62	3

The fundamental mode shape (see Figure 5) is close to a classical sinusoidal one. Assuming that the response of the floor is dominated by that mode of vibration, the simplified method could then be applied to the investigated floor (see Section 2.1).

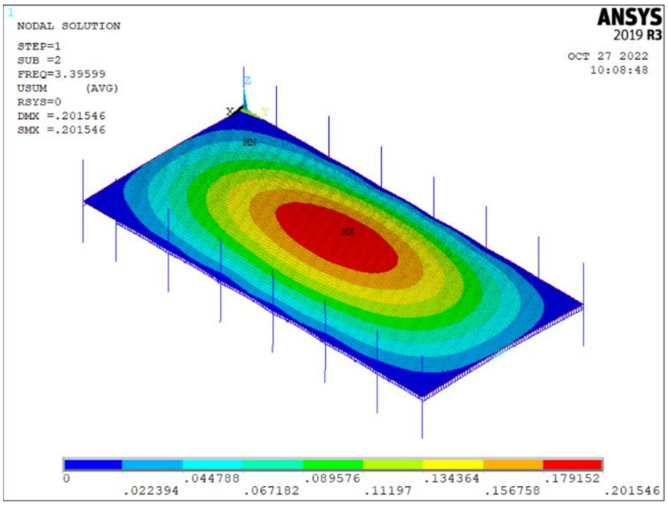

Figure 5 Fundamental mode shape of the analysed floor

4 Human comfort assessment due to rhythmic activities

The floor presented in Section 0 was assessed against human discomfort when subjected to crowd-rhythmic activities. Various activities performed by a multiple number of individuals were adopted for that purpose.

4.1 Investigated rhythmic activities

Two particular types of rhythmic activities were investigated in this study, where each type was represented by two activities: jumping (jumping jack, quick jumping) and skipping (on feet toes, on feet soles). The equivalent crowd load model (given by Eq. (2)) was used, while corresponding parameters had been identified in a single experimental vibration test campaign using force measurements [10]. Those parameters, namely mean load parameters along with coordination factors used in the present analysis, are summarized in Table 2. Relative parameters a_i (i=1 ,2, 3) (see Eq. (2)) were equal to [1, 0.6, 0.25] for jumping activities and [1, 0.4, 0.15] for skipping activities.

Table 2 Specific crowd load model parameters for the investigated rhythmic activities [10]

Activity	f_p (Hz)	α	δ (Hz)	$C(N)$
Jumping jack	2.36	4.64	0.035	$C(N) = 0.44 + 0.56 / N$
Quick jumping	2.81	3.64	0.100	$C(N) = 0.30 + 0.70 / N$
Skipping on toes	3.26	3.80	0.033	$C(N) = N^{-0.53}$
Skipping on soles	2.62	3.53	0.041	$C(N) = N^{-0.43}$

The vibrational response was evaluated at the floor centre using MATLAB. A number of 1, 2, 4, 8, 16, 32 and 64 individuals was considered for the rhythmic activities in virtue of the sufficiently large surface. All groups of people were assumed to be uniformly distributed in a central zone of the floor as illustrated in Figure 6 for 16 persons. The body mass of individuals was taken equal to 75kg, close to the nominal mass proposed by several design guidelines (SCI P354 for example [2]). The duration of each activity was considered equal to 30s with a maximum sampling frequency of 10Hz.

The identified values of the bandwidth coefficient δ are all below 0.15Hz (see Table 2), such that $\delta \ll f_p$. The application of the simplified method is then possible for the four investigated activities.

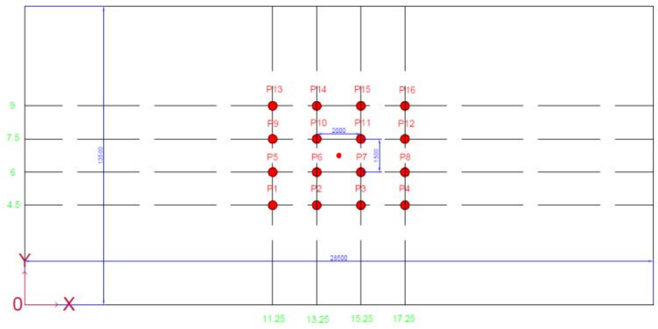

Figure 6 Positions of 16 individuals on the floor (dimensions in mm)

4.2 Illustration of the simplified method steps

In order to illustrate the steps for the application of the simplified method, the response of the steel-concrete floor was first predicted for the case of 16 individuals performing "jumping jack".

The calculation procedure of the floor RMS acceleration can be subdivided into six steps as follows:

(1) Floor parameters:

- Dimensions: L_x=28.5m, L_y=13.5m;
- Modal properties: f_1=3.40Hz, M_1=24.62t, ξ_1=3%.

(2) Characteristics of participants:

- Number and body mass: 16 individuals, each having a body mass $\overline{m}$ = 75kg;
- Distribution: uniform in a central zone of the floor (corresponding positions are shown in Figure 6).

(3) Properties of the rhythmic activity:

- Type: jumping jack;
- Parameters of the equivalent crowd load model (see Table 2): f_p=2.36Hz, α=4.64, δ=0.035Hz, a_i=[1, 0.6, 0.25], $C(N)$=0.44+0.56/N.

(4) Excitation modal amplitude:

A sinusoidal mode shape was used for the fundamental mode, which resulted in amplitudes given in Table 3. The mean amplitude is $\overline{\Phi}_p$ = 0.90.

Table 3 Excitation modal amplitudes for 16 individuals (coordinates in m)

y \ x	11.25	13.25	15.25	17.25
9	P13 (0.82)	P14 (0.86)	P15 (0.86)	P16 (0.82)
7.5	P9 (0.93)	P10 (0.98)	P11 (0.98)	P12 (0.93)
6	P5 (0.93)	P6 (0.98)	P7 (0.98)	P8 (0.93)
4.5	P1 (0.82)	P2 (0.86)	P3 (0.86)	P4 (0.82)

(5) Harmonic RMS acceleration at the floor centre (Φ_r=1):

The RMS acceleration $a_{\text{rms,i}}$ for each harmonic i is calculated by Eq. (17):

$$a_{\text{rms,i}}^2 \approx \gamma_N \, a_i^2 \left(2i\delta\right)\left(if_p\right)^4 \left|H_1(if_p)\right|^2$$

where:

$$\begin{cases} \gamma_N = (2\pi)^4 \left[16\, C(16) \overline{m} g \alpha\right]^2 \Phi_r^2 \, \overline{\Phi}_p^2 \\ \left|H_1(if_p)\right|^2 = \dfrac{1}{M_1^2 \left[16\pi^4 \left(f_1^2 - (if_p)^2\right)^2 + \left(8\pi^2 \xi_1 f_1 if_p\right)^2\right]} \end{cases}$$

which gives, after numerical substitution:

$$a_{\text{rms,i}}^2 \approx \frac{3028\, a_i^2\, i^5}{\left[\left(457 - 220 i^2\right)^2 + 361 i^2\right]}$$

(6) Total RMS acceleration:

Using the above equation for the three harmonics of the load, the total RMS acceleration a_{rms} is deduced by Eq. (18):

$$a_{\text{rms}} = \sqrt{\sum_{i=1}^{3} a_{\text{rms,i}}^2} \approx \sqrt{0.0536 + 0.1934 + 0.0198} = 0.52 m/s^2$$

4.3 Evaluation of floor acceleration

The simplified method detailed above for 16 individuals performing "jumping jack" was applied to the other rhythmic activities and crowd sizes (presented in Section 4.1). Obtained RMS responses for the steel-concrete floor are summarized in Table 4. Among existing acceptability criteria regarding human comfort, the criterion of the SCI P354 guideline [2] was selected as it is one of the few guidelines dealing with crowd-rhythmic activities. The acceptability limit for the RMS response proposed by this guideline is 0.60m/s² for floors subjected to vertical vibrations induced by crowd movements.

Table 4 RMS acceleration by crowd size (in m/s²) for the four investigated activities (regarding human comfort to vibrations, green accelerations are acceptable, red accelerations are unacceptable)

Activity	Crowd size						
	1	2	4	8	16	32	64
Jumping jack	0.08	0.11	0.17	0.30	0.52	0.90	1.35
Quick jumping	0.13	0.16	0.23	0.37	0.62	1.05	1.55

Skipping on toes	0.27	0.37	0.50	0.69	0.90	1.63	2.49
Skipping on soles	0.05	0.08	0.11	0.17	0.23	0.42	0.64

4.4 Discussion

Although having lower load amplitudes than jumping activities (see Table 2), it is found that "skipping on toes" produces the maximum responses for the analysed floor among all investigated activities. Indeed, this activity is characterized by an excitation frequency (3.26Hz) close to the fundamental natural frequency of the floor (3.4Hz), thus leading to a near resonant regime at the first harmonic of the load. This results in unacceptable accelerations starting from 8 persons. If resonance was assumed for this activity (f_1=f_p=3.40Hz), floor accelerations would be even more important (4.46m/s² for 64 persons). Hence, when a conservative design is agreed by the stakeholders, they should select an excitation frequency (belonging to the frequency range of the studied activity) causing resonance at one of the harmonics of the load whenever possible.

"Skipping on soles" presents the minimum accelerations as it has lower amplitudes and coordination factors than "jumping jack" besides lower excitation frequency than "quick jumping". All accelerations were tolerable for this activity except for 64 individuals. Accelerations due to jumping activities lie between those obtained for skipping ones, which were not acceptable for a group size beginning from 16 and 32 for "quick jumping" and "jumping jack", respectively.

In general, the floor presents allowable accelerations regarding the comfort of occupants due to rhythmic activities for up to 16 persons, while remedial measures should be carried out to reduce the floor response above this crowd size (tuned mass damper, additional partitions, etc.).

5 Conclusions

A design-oriented method was proposed in this paper in order to perform simplified prediction of the response of regular floors subjected to crowd-rhythmic activities. A frequency domain load model characterizing such activities was used in this method together with the random vibration theory. The simplified method was then applied to an existing steel-concrete floor, involving four different rhythmic activities performed by a number of up to 64 individuals. The results revealed that the floor had unacceptable accelerations with regards to human comfort for crowd sizes starting from 16 persons, where corrective measures should be undertaken.

After defining design load parameters along with acceptability criteria by the stakeholders, a quick human comfort assessment could then be made by engineers using the proposed method for a wide category of floors (gymnasiums, fitness centres, stadiums, etc.). This paves a way for the development of a design approach on the floor vibration analysis to be adopted in the forthcoming editions of Eurocodes. As a perspective, the scope of the simplified method could be extended to cover structures with multiple dominant modes of vibration (such as multi-span or multi-panel floors) and characterized by non-regular mode shapes.

References

[1] Racic, V.; Pavic, A. (2010) *Mathematical model to generate near-periodic human jumping force signals*. Mechanical Systems and Signal Processing 24, H. 1, pp. 138-152.

[2] Smith, A.L.; Hicks, S.J.; Devine, P.J. (2009) *Design of Floors for Vibration: A New Approach*. SCI publication P354, the Steel Construction Institute (Revised Edition, February 2009).

[3] Murray, T.M.; Allen, D.E.; Ungar, E.E. (2003) *Floor Vibrations Due to Human Activity*. Steel Design Guide Series N°11, American Institute of Steel Construction and Canadian Institute of Steel Construction.

[4] Jones, C.A.; Reynolds, P.; Pavic, A. (2011) *Vibration serviceability of stadia structures subjected to dynamic crowd loads: A literature review*. Journal of Sound and Vibration 330, H. 8, pp. 1531-1566.

[5] Xiong, J.; Chen, J. (2019) *Random field model for crowd jumping loads*. Structural safety 76, pp. 197-209.

[6] *Dynamic Performance Requirements for Permanent Grandstands: Recommendations for Management Design and Assessment* (2008) IStructE/DCLG/DCMS Joint Working Group, Institution of Structural Engineers, London.

[7] ISO 10137 (2007) *Bases for design of structures - Serviceability of buildings and walkways against vibration*, International Organisation for Standardization.

[8] Ellis, B.R.; Ji, T. (2002) *Loads generated by jumping crowds: experimental assessment*. BRE IP4/02, CRC Ltd., London, pp. 1-12.

[9] El Asri, Y.; Lukić, M.; Hjiaj, M.; Couchaux, M.; Denoël, V. (2023) *Identification of a load model for crowd-rhythmic activities based on acceleration measurements of a building floor.* Structures 50, pp. 1980-1993.

[10] El Asri, Y. (2023) *Vibration of floor structures subjected to crowd-rhythmic activities* [PhD thesis]. INSA Rennes and University of Liège.

[11] Preumont, A. (1994) *Random vibration and spectral analysis*. Dordrecht: Kluwer Academic Publishers.

[12] Xiong, J.; Duan, S.; Qian, H.; Pan, Z. (2022) *Multi-mode response spectrum for multi-harmonic crowd jumping loads*. Applied Mathematical Modelling 111, pp. 696-712.