A probabilistic approach to the Poffenberger-Swart bending stress of conductors subject to aeolian vibrations

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Abstract. Overhead electrical transmission line conductors are prone to aeolian vibrations, resulting from the alternate shedding of vortices in the wake of the cable. Aeolian vibrations are characterized by small-amplitude high-frequency flexural oscillations and, whenever not properly controlled, can induce wear damage and fatigue failures of the conductor. The standard technical approach to the assessment of aeolian vibrations and residual life of overhead conductors is based on the Energy Balance Method (EBM) and the Poffenberger-Swart formula for bending stresses. This approach relies on the main simplifying assumption of mono-modal oscillations. Typical aeolian vibration records, however, clearly show that several modes can be simultaneously excited due to wind variations in time and along the span. In this work a new approach is proposed for the prediction of aeolian vibrations of conductors within a probabilistic framework. The proposed approach allows to account both for non-linearities typical of internal damping of metallic cables and multi-modal contributions to aeolian vibrations in a straightforward and mechanically sound way. The proposed approach paves the way to a full probabilistic description of the Poffenberger-Swart bending stresses, making a further step towards a more refined definition of the expected life of overhead conductors.

1. Introduction

Overhead electrical transmission line (OHL) conductors are prone to vortex induced vibrations (VIV), also known as aeolian vibrations. These vibrations can occur under a wide range of wind velocities, and are characterized by small-amplitude (typically less than one cable diameter) and frequencies in the range of 3-200 Hz, depending on the geometry and axial load of the cable. Whenever not properly controlled, aeolian vibrations can induce wear damage and fatigue failures of the conductor (see e.g. [16, 6]).

The standard technical approach to assess the severity of aeolian vibrations and the residual life of overhead conductors is based on the Energy Balance Method (EBM) and the Poffenberger-Swart (PS) formula for dynamic bending stresses (see e.g. [16, 23]). This approach relies on the main simplifying assumption of mono-modal oscillations, introduced by the EBM. Typical aeolian vibration records, however, clearly show that several modes can be simultaneously excited due to wind turbulence and wind velocity variations along the span [3]. Aiming at overcoming the mono-modal vibration assumption of the EBM, a stochastic model was recently proposed by the authors [7], inspired by the work of Hagedorn and coworkers [14]. In that model wind forces were modeled as a narrow band stochastic process, centered around the Strouhal frequency of the conductor, and with an arbitrary cross-correlation in space. The conductor dynamic response was obtained by exploiting the properties of Green functions of a taut string with linear viscous damping. The adopted damping model, however, is open to criticism, as it is not fully physically coherent with experimental results showing that internal damping in metallic cables depends non linearly on the vibration frequency, the vibration amplitude and the cable tension (see e.g. [9] and references therein).

In the present work a different approach is proposed to model aeolian vibrations of OHL conductors. The equations of motion are projected in the modal base, the damping term is linearized, and a fixed-point iterative scheme is adopted to find the modal amplitudes of vibration. The taut-string model is used to evaluate the response far from the supports (far-field response), while the approximate equation by Poffemberger and Swart is employed to estimate the maximum dynamic bending stress in the wires of the outermost layer of the conductor at a conventional distance $x_p = 89$ mm from the last point of contact of the supporting clamp edge.

The proposed formulation and solution strategy allows for several improvements over the current technical approach based on the EBM: a sound mechanical model of damping is included in a probabilistic approach for the cable response; the contributions of more modes to the VIV of conductors is accounted for in a simple and mechanically sound way; the proposed approach paves the way to a full probabilistic description of the Poffenberger-Swart bending stresses, making a further step towards a more refined definition of the expected life of overhead conductors.

2. Formulation of the problem

2.1. Governing equations

Let us consider a cable suspended to horizontal rigid supports, with length L and linear density γ, subject to a distributed dynamic load $f_E = f_E(x,t)$ where $x \in [0, L]$ and t denote, respectively, an abscissa spanning the length of the cable and the time. The external force f_E describes the action of a space- and time-variable lift force induced by the alternate shedding of vortices in the wake of the cable cross sections. Focusing on overhead electrical line conductors, small planar vibrations can be described with a sufficiently good level of approximation by resorting to the classic taut-string model [13], i.e. by introducing the equation of motion:

$$\gamma \partial_t^2 w - T \partial_x^2 w + f_D = f_E, \; w = w(x,t), \; x \in (0, L), \; t \in \mathbb{R}^+ \tag{1}$$

where $w = w(x,t)$ is the transverse displacement of the cable centerline, T is the constant axial force, which can be conveniently expressed as a fraction η of the Rated Tensile Strenght (RTS) of the cable (i.e. $T = \eta \, RTS$), and f_D is a distributed damping force. The equation of motion (1) can be integrated under suitable initial conditions and boundary conditions:

$$\begin{cases} w(x=0,t) = 0 \\ w(x=L,t) = 0 \end{cases}, \; \forall t \tag{2}$$

Damping is mainly due to internal energy losses within the vibrating cable (also known as *cable self-damping*). Generally speaking, the damping term may depend on cable displacements through a non-linear and non-holonomic functional relation, which can be formally stated by introducing a non-linear operator $\mathcal{D}$ acting on the displacement function w, i.e.:

$$f_D = \mathcal{D}[V] \tag{3}$$

The distributed lift force f_E can be described, within the framework of "externally forced models" for VIV [19], as:

$$f_E = \frac{1}{2}\rho D U^2 q\left(x,t\right). \tag{4}$$

where ρ is the density of the air, D is the diameter of the cable, U is the mean wind velocity and $q\left(x,t\right)$ is a space- and time-variable lift coefficient. By following a classic approach of the literature (e.g. [22, 17, 1]), the lift coefficient is modeled as an incompletely homogeneous bidimensional random process with cross power spectral density function:

$$S_q\left(x_1,x_2,\omega\right) = Coh\left(x_1,x_2\right) S\left(\omega\right),\ x_1,x_2 \in \left[0,L\right],\ \omega \in \mathbb{R} \tag{5}$$

with the following definitions:

$$S\left(\omega\right) = \frac{\tilde{c}_{ls}}{2\sqrt{\pi}B\omega_s}\exp\left[-\left(\frac{1-\left|\frac{\omega}{\omega_s}\right|}{B}\right)^2\right],\ \omega \in \mathbb{R} \tag{6}$$

$$Coh\left(x_1,x_2\right) = \exp\left(-\frac{\left|x_1-x_2\right|}{l\,D}\right),\ x_1,x_2 \in \left[0,L\right] \tag{7}$$

where $\tilde{c}_{ls}$ is the root mean square (rms) of the lift coefficient, B is a non-dimensional spectral bandwidth parameter, ω_s is the Strouhal circular frequency: $\omega_s = 2\pi\frac{\mathsf{S}U}{D}$, S is the Strouhal number and l denotes a correlation length in diameters D of the cable.

The undamped natural circular frequencies (ω_n) and associated mode shapes ($\phi_n\left(x\right)$) of the cable can be easily evaluated by setting $f_D = f_E$ in Eq. (1) and read:

$$\omega_n = n\pi\omega_c,\ n \in \mathbb{N}^+ \tag{8}$$

$$\phi_n = \sqrt{2}\,\sin\left(n\pi\,\frac{x}{L}\right),\ n \in \mathbb{N}^+ \tag{9}$$

where $\omega_c = \frac{1}{L}\sqrt{\frac{T}{\gamma}}$.

Whenever the rotations of the end sections are restrained, as it is typically the case in overhead electrical line applications, boundary layers develop in the neighborhoud of the supports due to the effect of the cable bending stiffness (see e.g. [5, 4]) EI. Within this context, the outcomes of the taut-string model provide a good approximation (also known as *far-field* solution) of the cable response everywhere except in the boundary layers close to the supports, whose extent can be estimated as: $\varepsilon = \frac{1}{L}\sqrt{\frac{EI}{T}} \ll 1$.

The maximum values of the bending stresses occur within the boundary layers and can be evaluated through classic perturbation techniques or resorting to the exact solution of a taut beam model as long as the bending behavior of the cable cross sections is assumed to be linearly elastic. The moment-curvature relation of metallic cables, however, is far from being linearly-elastic. Whenever a cable is bent, indeed, an axial force gradient is generated within the helical wires of the cable. This axial force gradient gives the wires the trend to slip. At small curvature values, internal friction forces are large enough to avoid any relative displacement between the wires, which can be considered as perfectly stuck together (*full-stick* state). The bending stiffness of the cable in full-stick state attains its maximum theoretical value EI_{max} . By increasing the curvature values, the effect of internal friction may be overcome leading to the onset and propagation of relative sliding between the wires. Progressive sliding of the wires is associated to a reduction of the tangent bending stiffness of the cable, which attains its minimum theoretical value EI_{min} when all wires of a cross section are in sliding state ($full-slip$ state). The transition from the full-stick to the full-slip state is a non-linear and non-holonomic phenomenon.

Several models have been proposed in the literature to characterize the nonlinear moment-curvature behavior of overhead line conductors (see e.g.[8] and references therein). A first attempt to investigate the characteristics of the cable boundary layers fully accounting for a realistic nonlinear moment-curvature relation has been presented in [10]. The most popular technical solution currently adopted to estimate the maximum bending stress close to the end sections of a vibrating conductor, however, relies on the approximate approach proposed by Poffenberger and Swart in 1965 [20].

Focusing on mono-modal vibrations with circular frequency ω_n and amplitude A_n, the peak-to-peak vibration amplitude Y_{bn} measured at the conventional reference distance $x_p = 89$ mm from the last point of contact of the supporting clamp edge is calculated through the approximate equation (cf. [18]):

$$Y_{bn} = A_n \upsilon_n,\ \upsilon_n = 2\left\{\sin(\beta_n x_p) - \frac{\beta_n}{\alpha_n}\left\{\sinh(\alpha_n x_p) + \tanh(\alpha_n L)\left[\cos(\beta_n x_p) - \cosh(\alpha_n x_p)\right]\right\}\right\} \tag{10}$$

with the definitions:

$$\alpha_n = \sqrt{\frac{T}{2EI_{min}} + \sqrt{\frac{\gamma\omega_n^2}{EI_{min}} + \left(\frac{T}{2EI_{min}}\right)^2}} \tag{11}$$

and

$$\beta_n = \sqrt{-\frac{T}{2EI_{min}} + \sqrt{\frac{\gamma\omega_n^2}{EI_{min}} + \left(\frac{T}{2EI_{min}}\right)^2}} \tag{12}$$

The maximum dynamic bending stress in the wires of the outermost layer ($\sigma_{a,n}$) is then evaluated through the approximate equation:

$$\sigma_{a,n} = K_{ps} Y_{bn} \tag{13}$$

where K_{ps} is a coefficient depending on the axial force T and on the mechanical properties of the cable cross section. Focusing on the widespread case of Aluminum Conductors Steel Reinforced (ACSR), and denoting with E_a and d_a, respectively, the Young modulus and the diameter of the aluminum wires, the following approximate equation can be used to evaluate K_{ps}:

$$K_{ps} = \frac{E_a d_a p^2}{4\left[\exp(-x_p p) - 1 + x_p p\right]} \tag{14}$$

with $p = \sqrt{\frac{T}{EI_{min}}}$.

2.2. Cable self-damping

Accurate modeling of the cable self-damping is essential to get reliable estimates of the VIV severity in overhead electrical line conductors. The most widespread technical approach to characterize self-damping relies on forced and/or decay vibration tests, which are performed on short laboratory spans (span length in the order of 30-90 m) according to international standards. Experimental evidences (see e.g. [2, 16, 6, 9]) allows to state that: (1) cable self-damping is a "proportional" or "classical" type of damping (i.e. no modal coupling is induced by the damping force); (2) the power dissipated per unit of length ($P_{d,n}$) during steady-state mono-modal vibrations of the cable is a non-linear function of the vibration frequency $f_n = \frac{\omega_n}{2\pi}$, of the modal vibration amplitude A_n (please notice that the subscript n refers to the number of the mode) and of the axial force T. Experimental measurements of the dissipated power are interpolated through the empirical power law:

$$P_{d,n} = K \frac{A_n^l f_n^m}{T^n} \tag{15}$$

where exponents l, m and n are typically in the ranges $l = 2 - 2.5$, $m = 4 - 6$, $n = 2 - 2.8$, while K is a proportionality coefficient that should be determined from tests.

Theoretical expressions to evaluate the dissipated power $P_{d,n}$ have also been proposed in the literature. Without loss of generality, in the present work the following expression derived by Foti and Martinelli [9] under the assumption of micro-slip conditions on the contact surfaces between adjacent wires of the vibrating cable will be adopted:

$$P_{d,n} = K_{ms} \frac{A_n^3 f_n^7}{T^4} \tag{16}$$

where K_{ms} is a proportionality coefficient that only depends on the mechanical and geometric properties of the conductor cross section (interested readers are referred to [9] for further details).

The equivalent viscous n-th modal damping ratio (ζ_n^{eq}) can be easily calculated starting from the knowledge of the dissipated power $P_{d,n}$ (both from the empirical (15) or from the theoretical power law (16)). The equivalent modal damping ratio turns out to be, in general, a function of the modal vibration amplitude A_n (i.e. $\zeta_n = \zeta_n(A_n)$) for an assigned value of axial force T, and can be expressed as:

$$\zeta_n^{eq} = \frac{2}{\gamma \omega_n^3 A_n^2} P_{d,n}\left(A_n, \omega_n, T\right) \tag{17}$$

3. Proposed solution strategy

A normal-mode approach is adopted to evaluate the dynamic response of the cable model defined in the previous Section. A spectral expansion of the cable displacement function is introduced as: $w(x,t) = \sum_{n=1}^{N} \phi_n(x)\, p_n(t)$, where N and p_n are, respectively, the truncation mode number and the nth modal displacement. Substitution of this series expansion in Eq. (1) yields after some standard manipulations the following set of equations of motion in the modal basis of the structure:

$$\ddot{p}_n + \omega_n^2 p_n + \frac{1}{\gamma L} \int_0^L f_D(x,t)\, \phi_n(x)\, dx = \frac{1}{\gamma L} \int_0^1 f_E(x,t)\, \phi_n(x)\, dx,\ n = 1, 2, ..., N \tag{18}$$

where a prime and a dot denote, respectively, derivation with respect to x and t.

By recalling the experimental evidences on cable self-damping and exploiting Eqs. (17), a linearized form of the n-th equation of motion (18) can be introduced as:

$$\ddot{p}_n + 2\omega_n \zeta_n^{eq} \dot{p}_n + \omega_n^2 p_n = \frac{1}{\gamma L} \int_0^1 f_E(x,t)\, \phi_n(x)\, dx,\ n = 1, 2, ..., N \tag{19}$$

where the equivalent modal damping ratio $\zeta_n^{eq} = \zeta_n^{eq}\left(\tilde{A}_n\right)$ is assumed to depend on the standard deviation $\tilde{A}_n$ of the modal amplitude, which is linearly related to the standard deviation $\tilde{p}_n$ of the n-th modal displacement through the equation: $\tilde{A}_n = \sqrt{2}\tilde{p}_n$.

Once the linearized equations of motion (19) are defined, the response can be calculated through an iterative solution strategy. At each iteration, the variance of the n-th modal displacement is first calculated as:

$$\tilde{p}_n^2 = \left(\frac{\rho D U^2}{2\gamma}\right)^2 2\rho_{nn} \int_0^{+\infty} \left|H_n(\omega)\right|^2 S(\omega)\, d\omega \tag{20}$$

where $S(\omega)$ is the power spectral density function of the lift coefficient (see Eq. (6)), $H_n = \left(-\omega^2 + j\, 2\zeta_n^{eq}\omega_n\omega + \omega_n^2\right)^{-1}$ and ρ_{nn} is defined as:

$$\rho_{nn} = \int_0^1 \int_0^1 Coh\left(x_1(\xi_1), x_2(\xi_2)\right) \phi_n\left(x_1(\xi_1)\right) \phi_n\left(x_2(\xi_2)\right) d\xi_1 d\xi_2,\ \xi_i = \frac{x_i}{L},\ i = 1, 2 \tag{21}$$

Once the variance of the modal coordinates is known, the standard deviation of the n-th modal amplitude $\tilde{A}_n$ can be easily computed and substituted back in Eq. (17) to have a new estimate of the equivalent modal damping ratio ζ_n^{eq}. Within the framework of a standard fixed-point algorithm, iterations are repeated until convergence on $\tilde{A}_n$ is obtained within a prescribed tolerance.

After convergence is achieved, the standard deviation of any response quantity of interest can be calculated, by approximately neglecting modal correlation, through a simple application of the SRSS modal combination rule. Focusing on the standard deviation of the cable displacements and of the Poffenberger-Swart dynamic bending stress, the following equations can be introduced:

$$\tilde{w}(x) = \sqrt{\sum_{n=1}^{N} \tilde{p}_n^2 \phi_n^2(x)} \tag{22}$$

$$\tilde{\sigma}_a = K_{ps} \sqrt{\sum_{n=1}^{N} \tilde{A}_n^2 \upsilon_n^2} = 2K_{ps} \sqrt{\sum_{n=1}^{N} \tilde{p}_n^2 \upsilon_n^2} \tag{23}$$

Should these be needed, higher-order statistics of the response parameters can also be calculated through standard techniques (e.g. [21]).

4. Application example

In this section, the proposed modeling approach and iterative solution strategy are applied to investigate the dynamic response of a benchmark overhead electrical line span already studied elsewhere [12, 11, 7]. The cable is a long ACSR Bersfort conductor (diameter $D = 0.0356$ m, linear density $\gamma = 2.375$ kg/m, Rated Tensile Strength $RTS = 180$ kN). The cable length and axial force are respectively equal to $L = 450$ m and $T = 0.2\, RTS = 36$ kN.

In order to characterize the wind input, the following assumptions have been made. The Strouhal number is set equal to $\mathsf{S} = 0.185$, as usual for Reynolds numbers and wind conditions typical of overhead electrical lines. The values of the rms lift coefficient $\tilde{c}_{ls}$ and of the non-dimensional bandwidth parameter B are site dependent and have not been explicitly reported in the literature with reference to OHL applications, at the best of authors' knowledge. Reference values for these parameters are herein defined by looking at the values reported by Vickery and Clark [22]for a slightly tapered cylindrical stack model, i.e.: $\tilde{c}_{ls} = 0.2$ and B is defined in the range $B = [0.08, 0.32]$, with smaller values associated to laminar flow conditions and the larger ones to a turbulence intensity of the order of 10%. The correlation length is assumed in the order

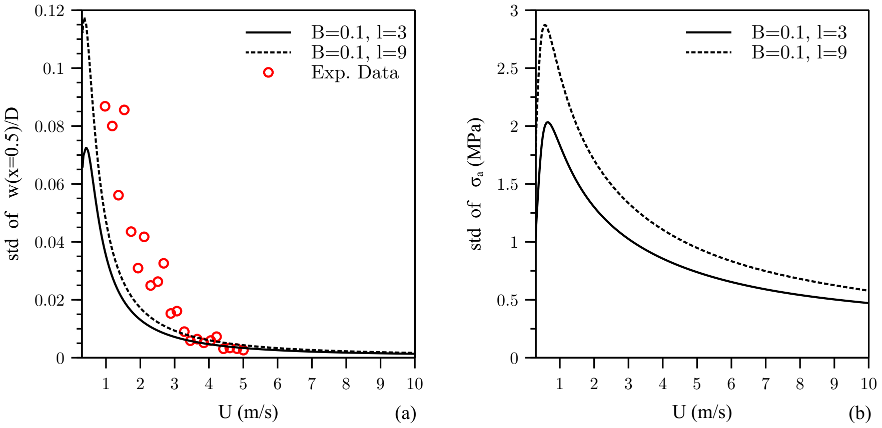

Figure 1: ACSR Bersfort strung at $T = 0.2\,RTS$. (a) Standard deviation of the non-dimensional displacement at midspan. Experimental data are from [15]. (b) Standard deviation of the Poffenberger-Swart dynamic bending stress. Results are plotted for a non-dimensional bandwidth parameter $B = 0.1$ and two different values of correlation length in diameters, i.e. $l = 3$ and $l = 9$.

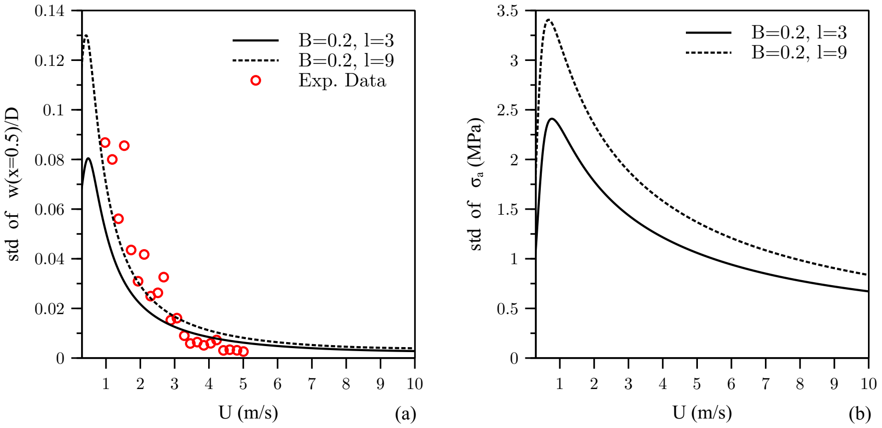

Figure 2: ACSR Bersfort strung at $T = 0.2\,RTS$. (a) Standard deviation of the non-dimensional displacement at midspan. Experimental data are from [15]. (b) Standard deviation of the Poffenberger-Swart dynamic bending stress. Results are plotted for a non-dimensional bandwidth parameter $B = 0.2$ and two different values of correlation length in diameters, i.e. $l = 3$ and $l = 9$.

of few diameters of the cable, i.e. l is taken in the range $l = [1, 10]$. A more detailed calibration of the input parameters of the wind input model could be obtained through in-situ experimental measurements or wind tunnel tests on scaled models.

Figures 1 and 2 report the standard deviation of the non dimensional displacement at midspan (Figs. 1(a) and 2(a)) and the standard deviation of the Poffemberger Swart dynamic bending stress (Figs. 1(a) and 2(a)) for two different values of the bandwidth parameter B and correlation length l. The vibration amplitudes obtained from the proposed procedure are compared to experimental data recorded in the field by Hardy and Van Dyke [15] for a Bersfort conductor strung at the same tension $T = 0.2RTS$ tension considered in the numerical applications. The agreement on the predicted vibration amplitude can be considered as good, taking also into account that some of the parameters that describe the wind input are affected by a degree of uncertainty.

5. Conclusions

In this work a new theoretical and computational stochastic framework is proposed for the prediction of aeolian vibrations of OHL conductors. The proposed approach, which includes a sound mechanical model of damping, can easily take into account the contributions of more modes to the cable vibrations and dynamic stresses induced by vortex shedding phenomena. This is a main difference with respect to current technical approaches based on the EBM.

The qualitatively and quantitatively good predictions for the standard deviation of the midspan displacement obtained for a Bersfort conductor, whenever confirmed by more extensive comparisons to experimental data, would pave the way to a full probabilistic description of the Poffenberger-Swart dynamic bending stresses, allowing to advance toward a more refined definition of the expected life of overhead conductors.

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