IMAC XLI

IT'S NOT JUST MODAL ANYMORE

A time-variant/invariant equivalence for the transient response of rotor blades in resonance crossing

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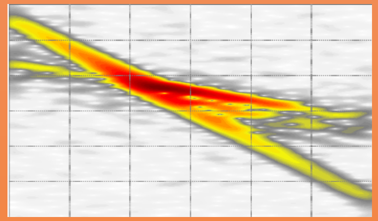
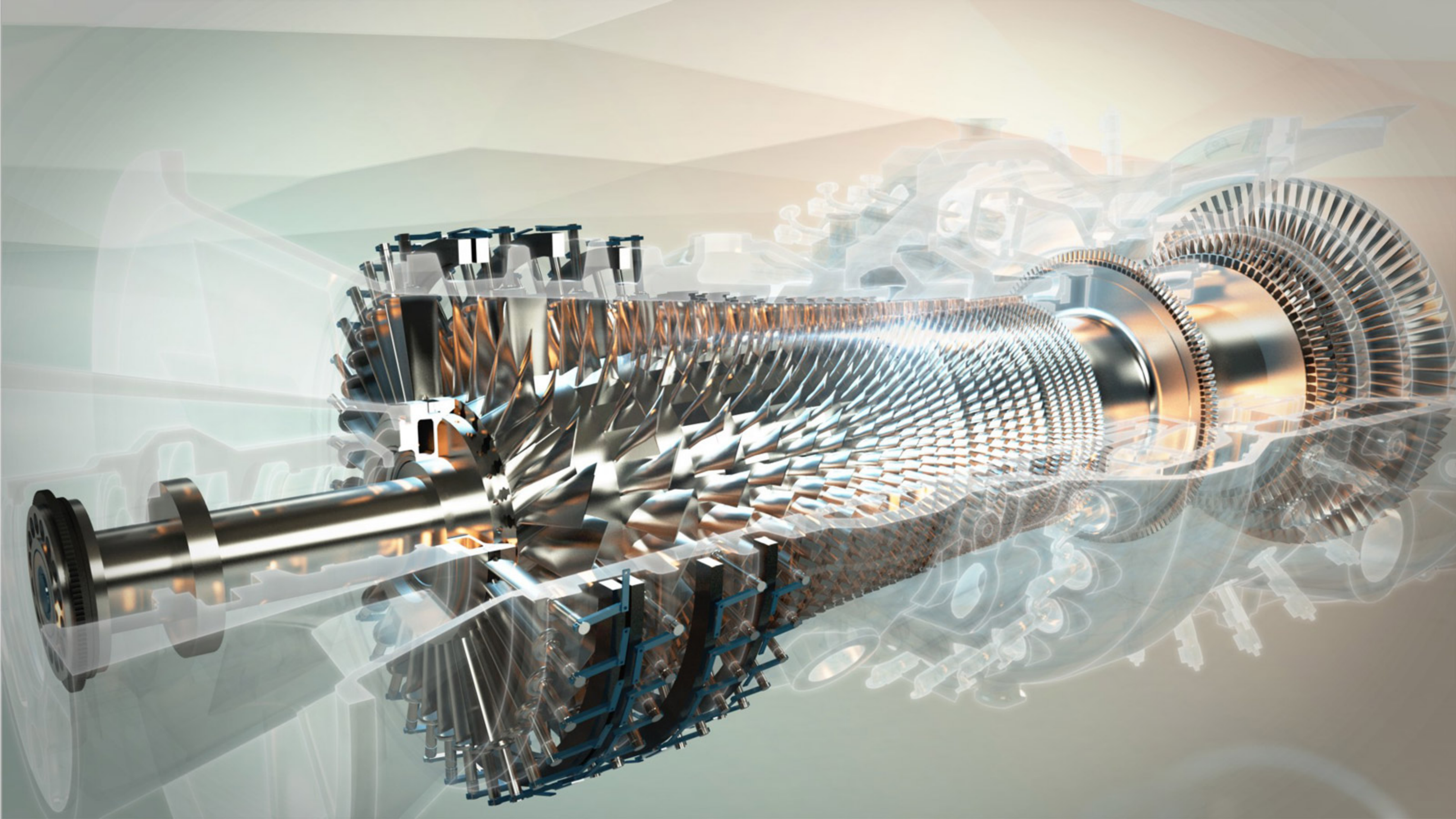
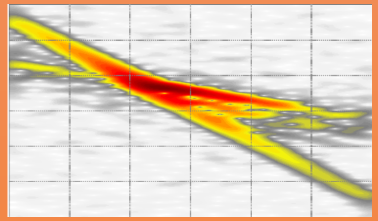


Summary

1. **Motivation.** *Modal identification of rotor components*
2. **Background.** *Why we like the Fourier transform*
3. Analytic singnals and **demodulation**. *The most brilliant ideas… never work*
4. **Fourier transform, short-time Fourier transform, wavelet transform.** *On the origin of species (by means of natural selection …)*
5. **Implementation and interpretation.** *How to make an elegant framework useful*
6. **Time-frequency spectral analysis.** *A numeric magnifying glass*
7. **System identification** by TFA. *Leveraging a 2D representation*

TO BE UPDATED





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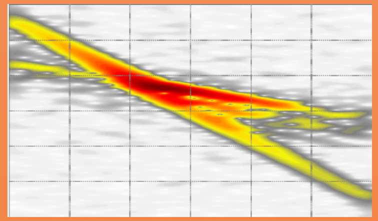

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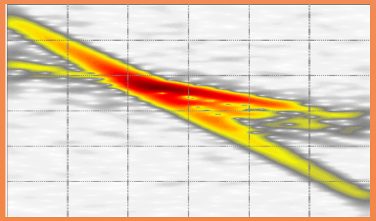

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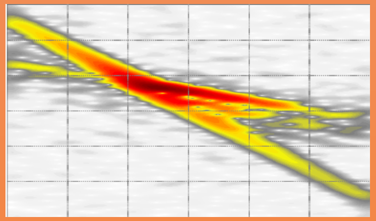

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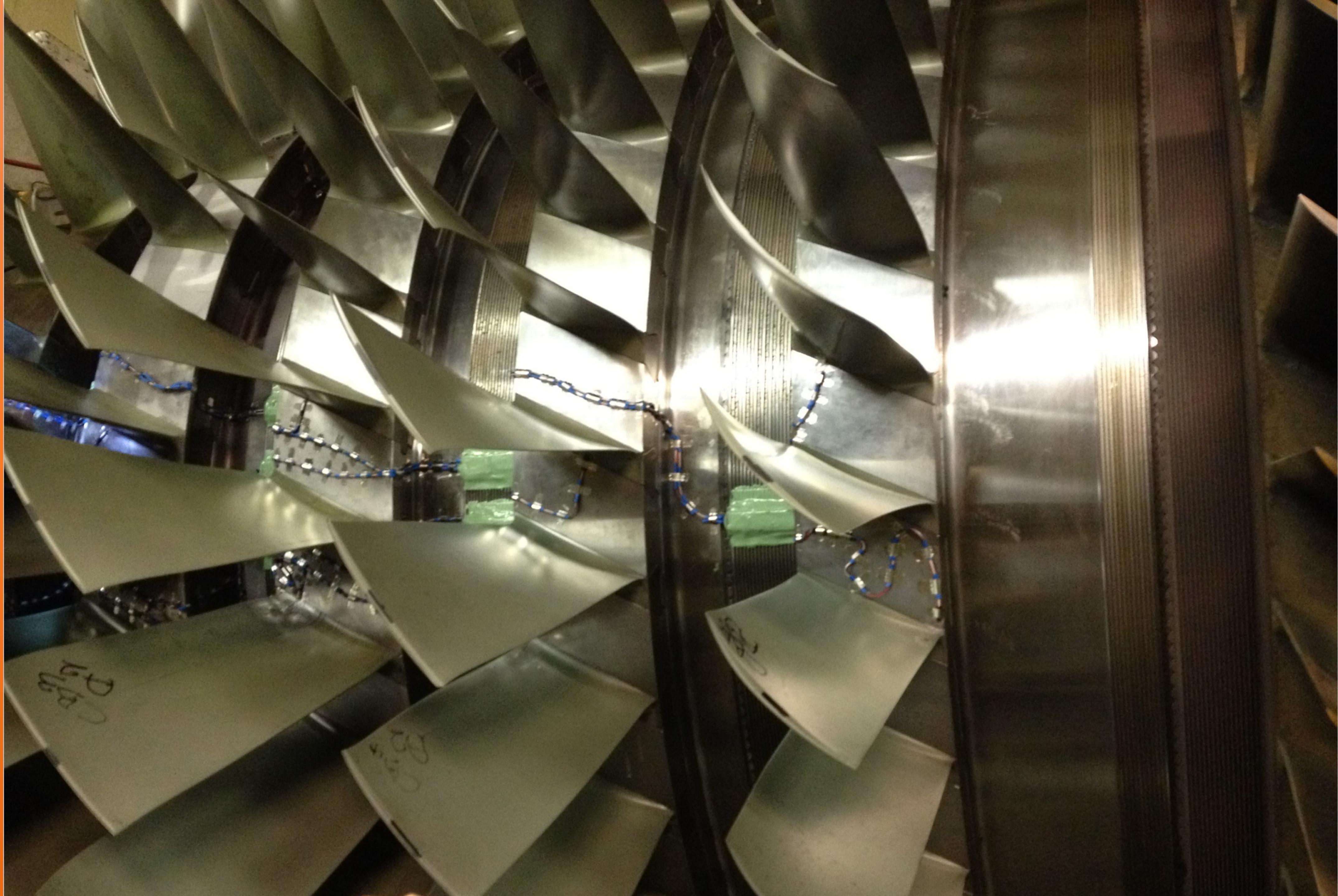

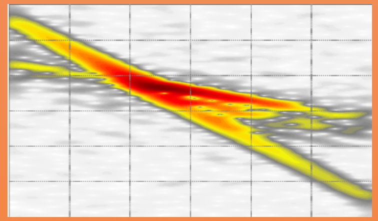

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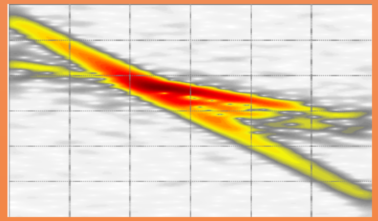

Modal identification of rotor components

Campbell diagram

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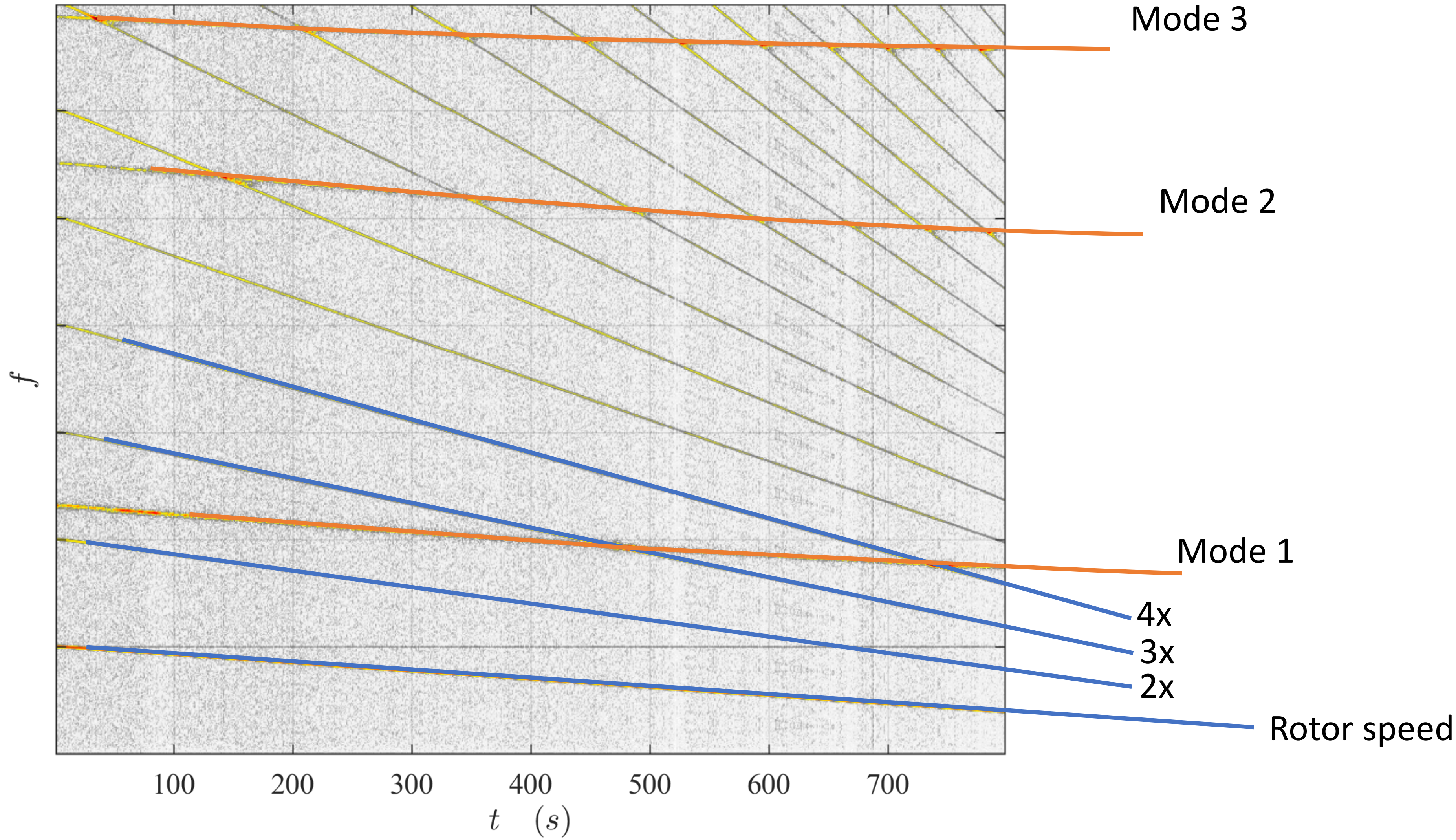
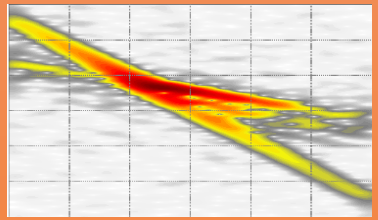
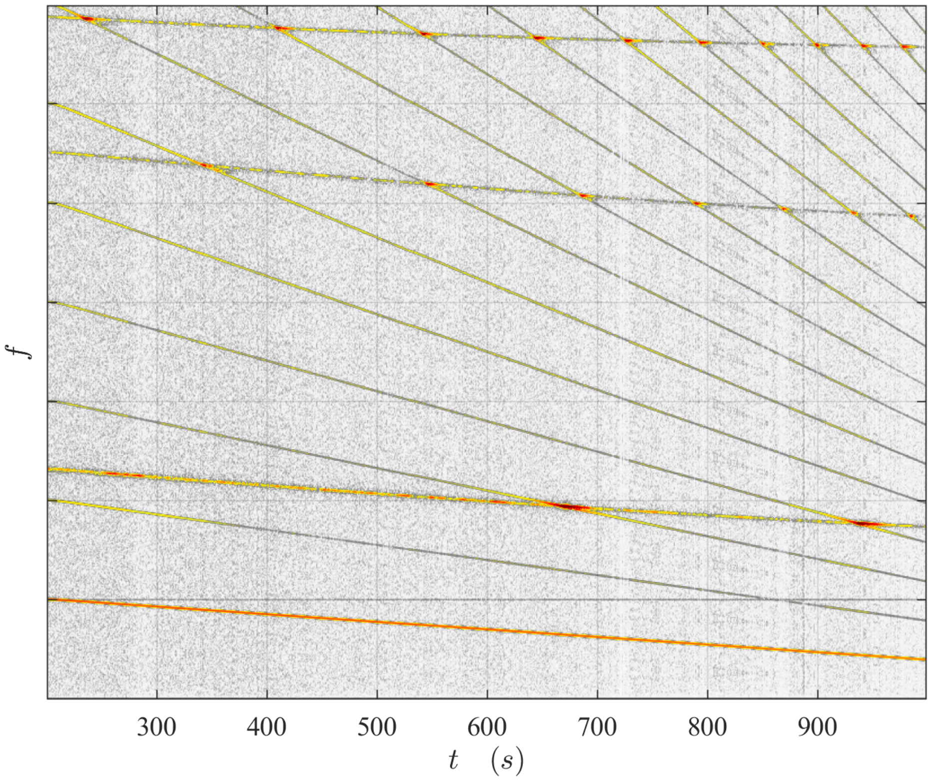
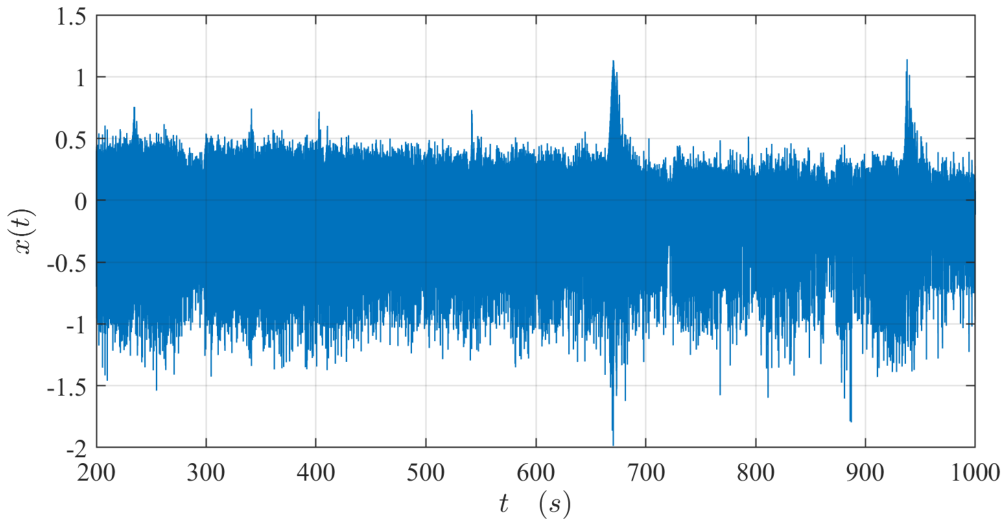




Resonance crossing





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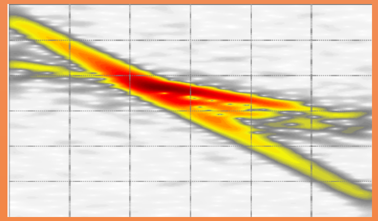

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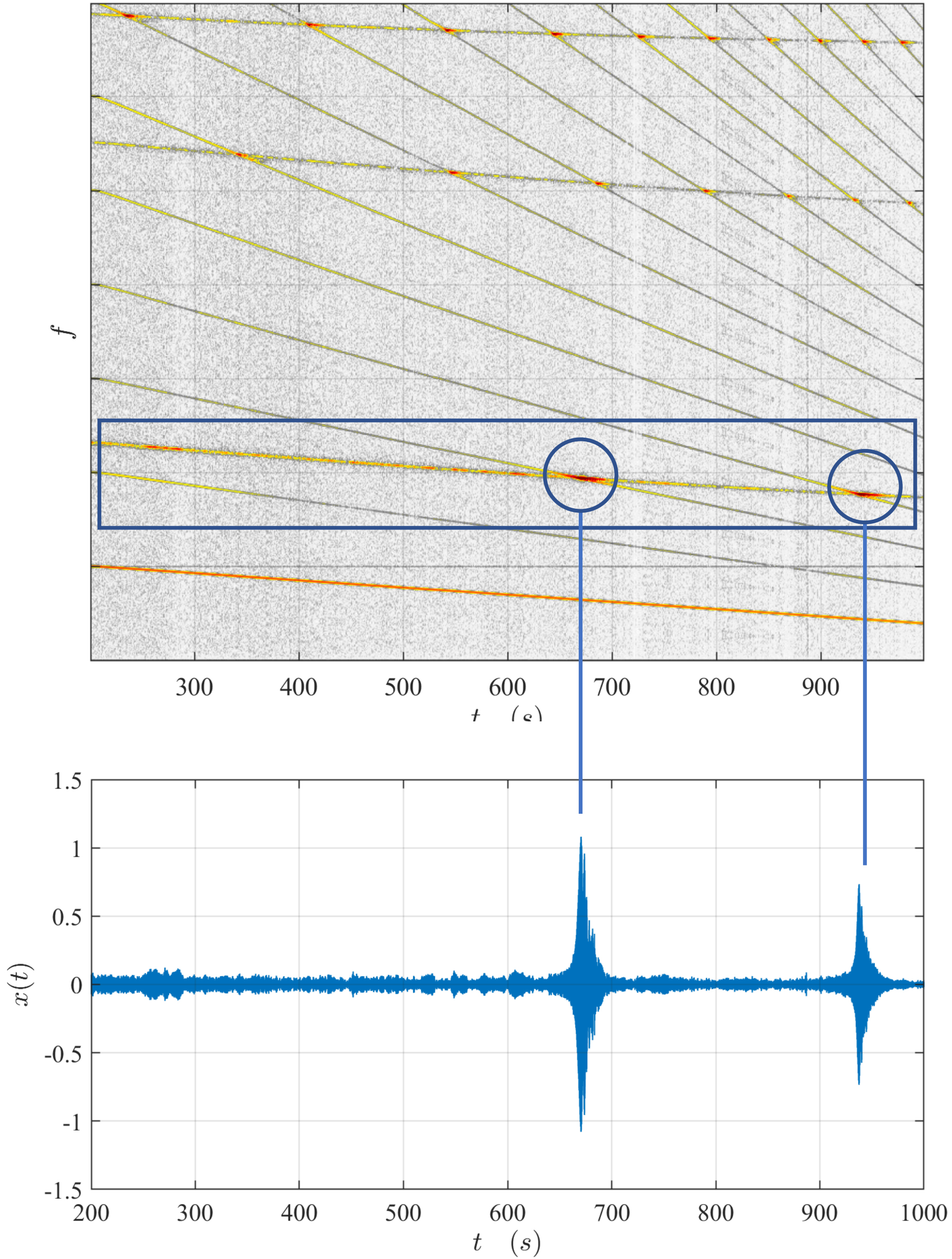

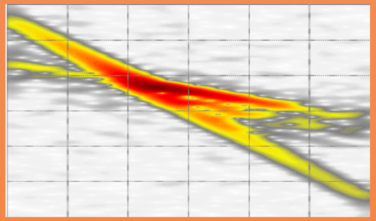

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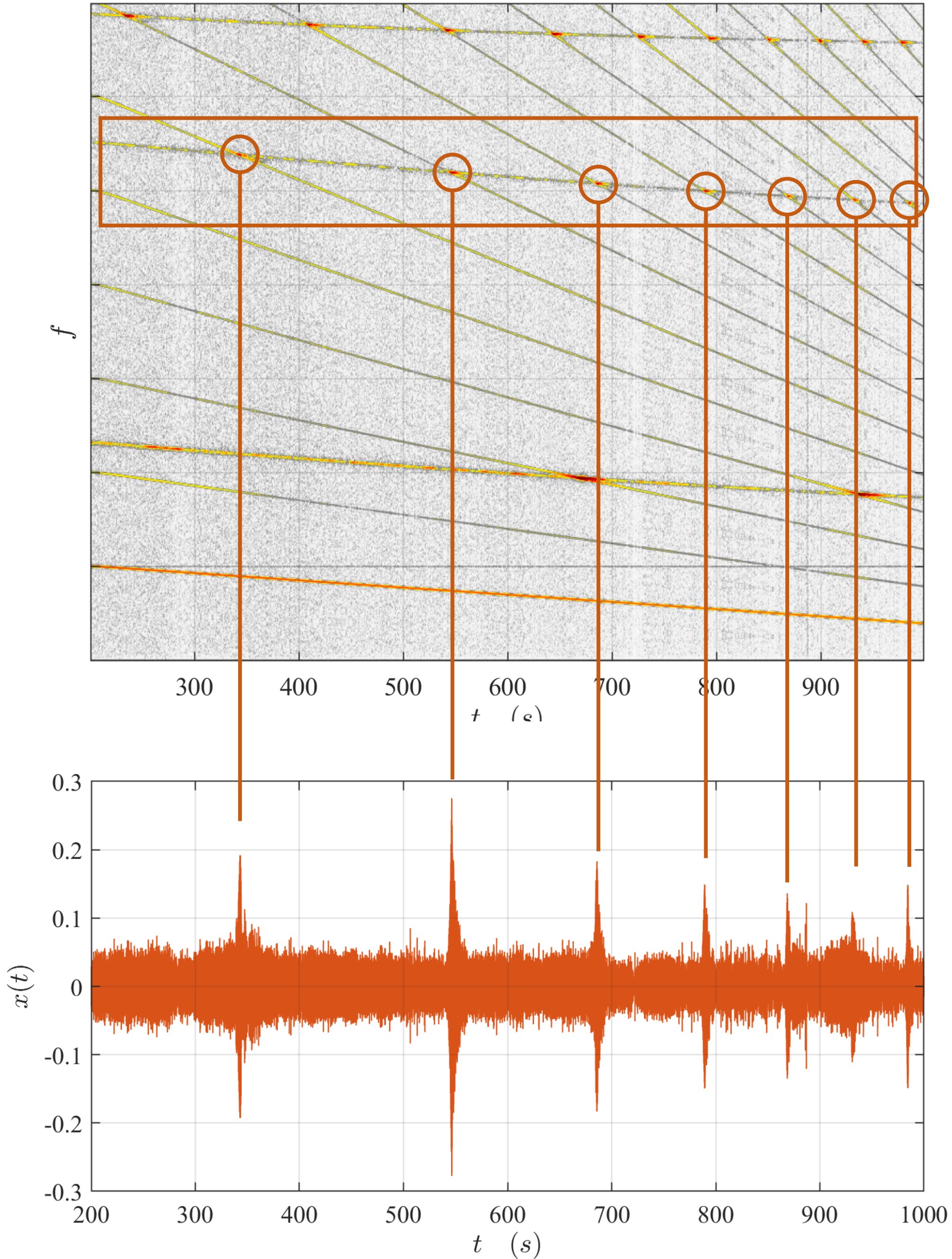

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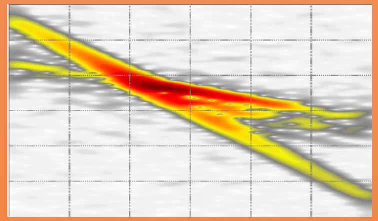

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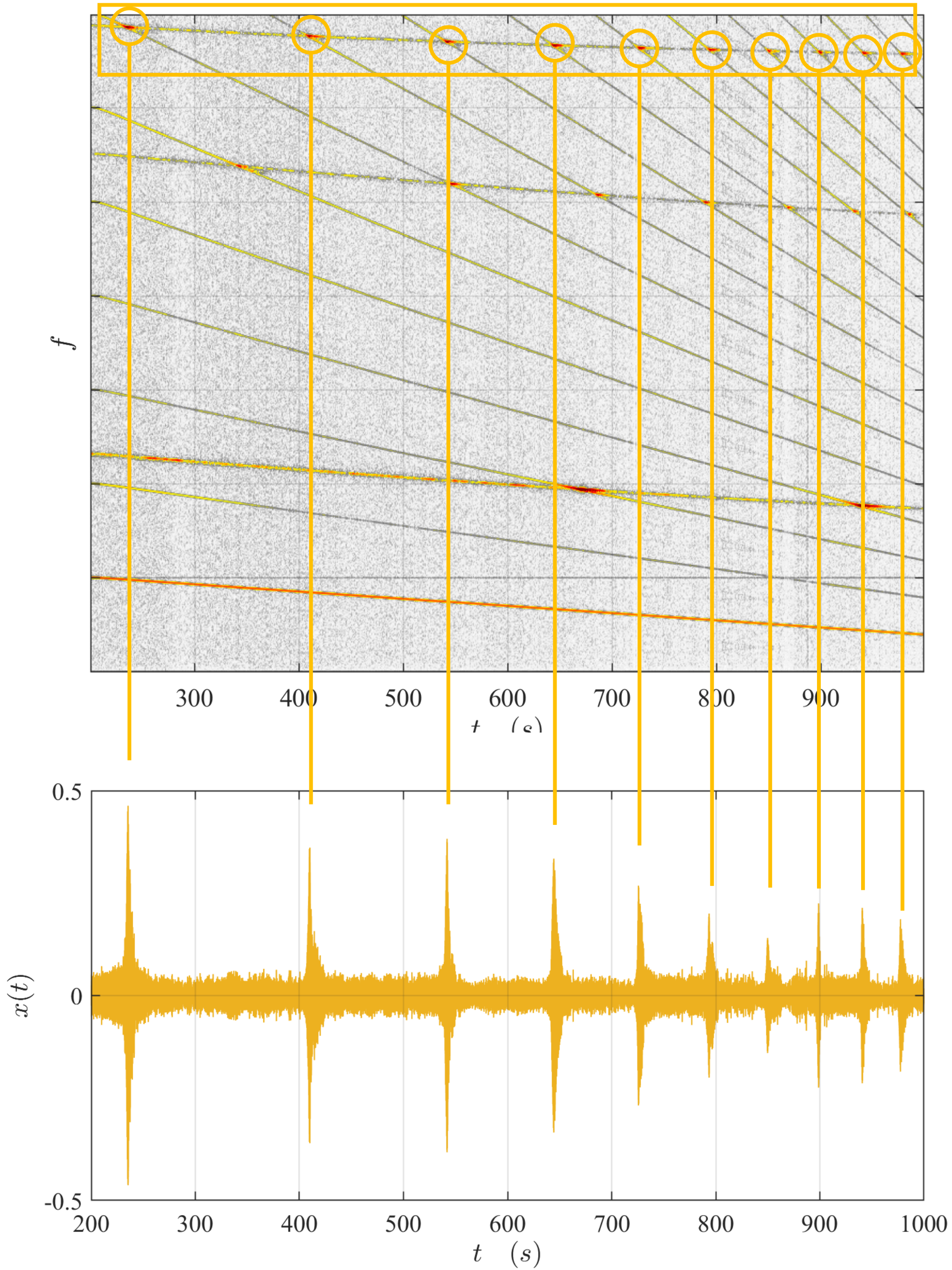

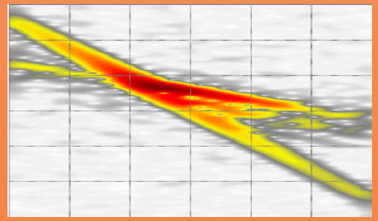

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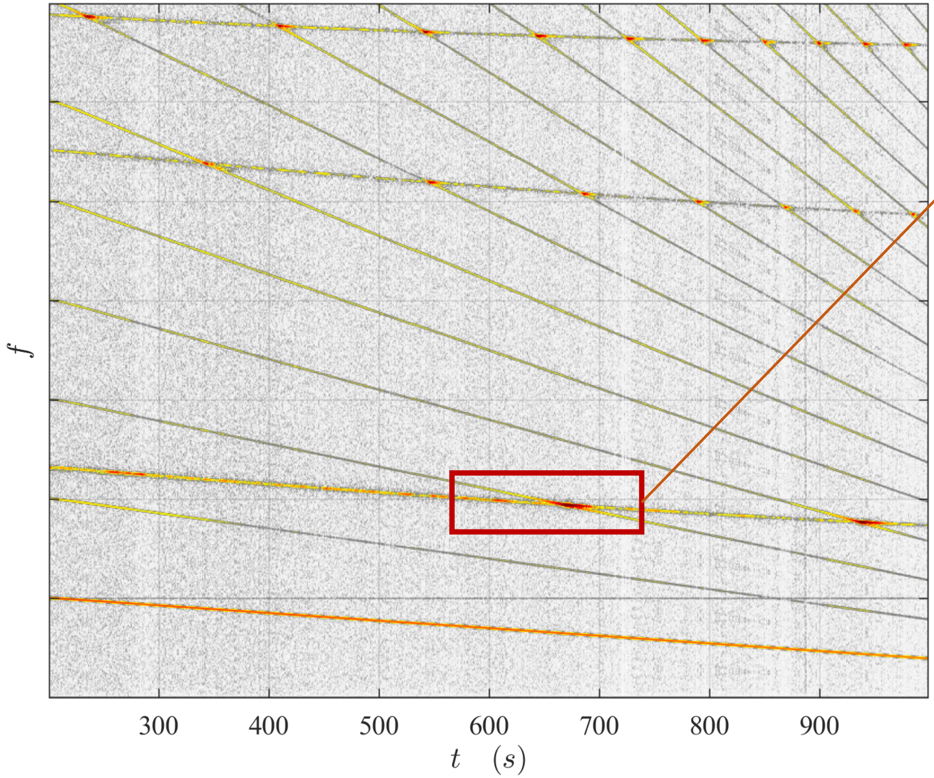

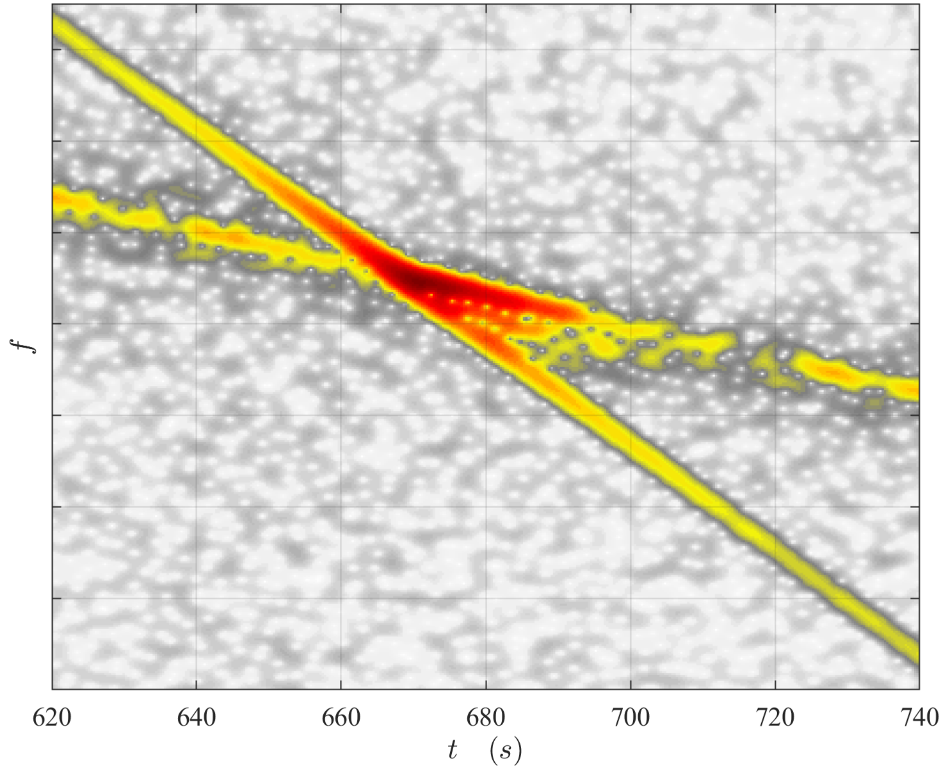

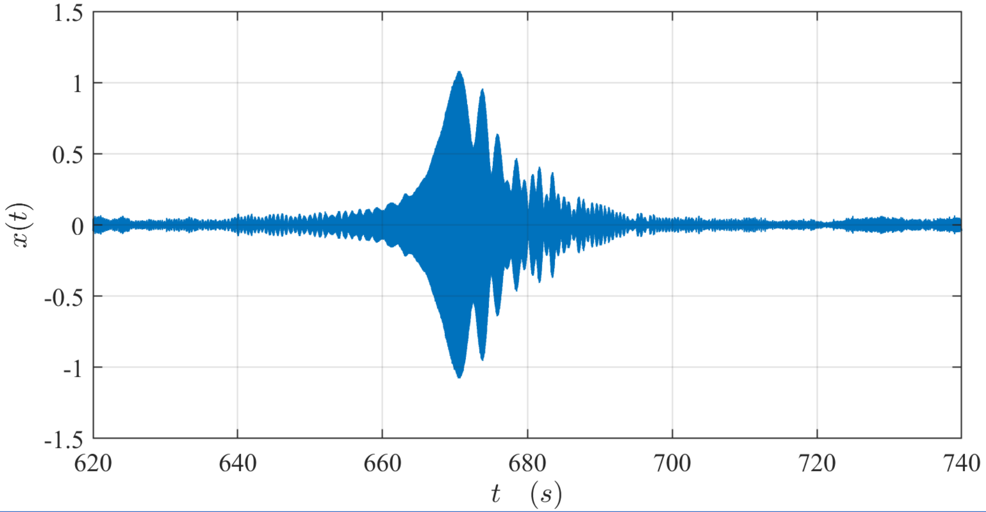

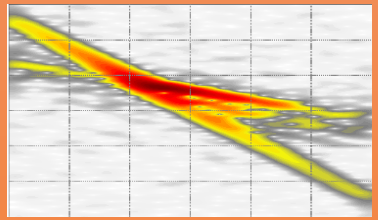

Single-dof model

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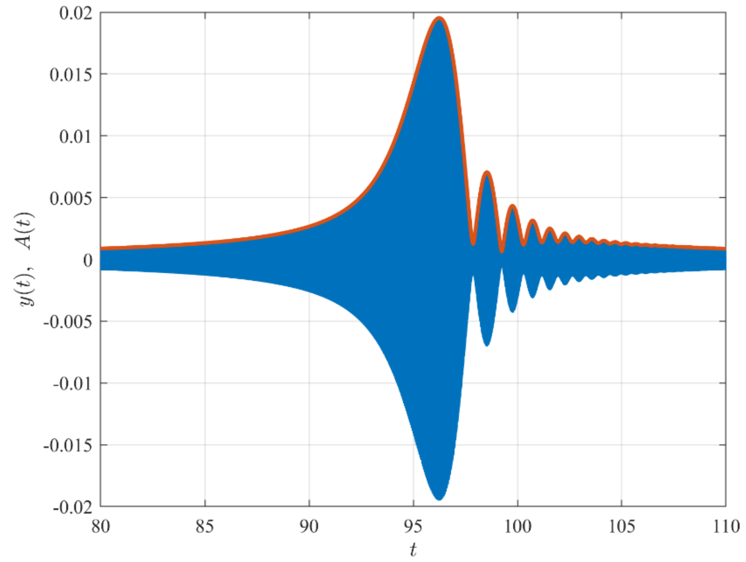
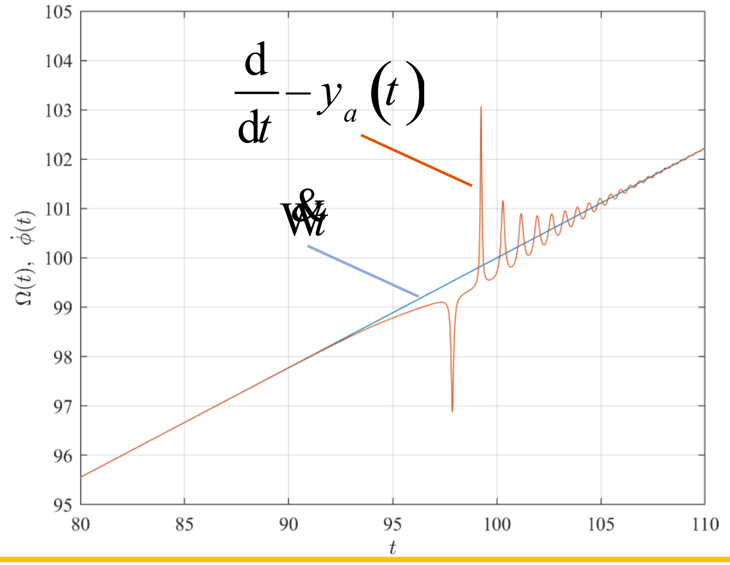
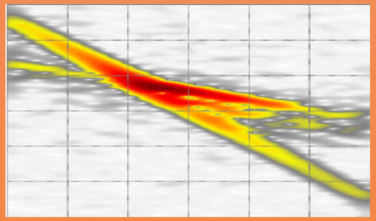
$$\ddot{x}+2\xi\omega_n\dot{x}+\omega_n^2 x=\frac{1}{m}p(t)$$

$$=\frac{P}{m}\cos\left(\frac{1}{2}\dot{\Omega}t^2\right)$$

$$\Omega(t)=\dot{\Omega}t$$







Minimalistic modeling : single-DOF model

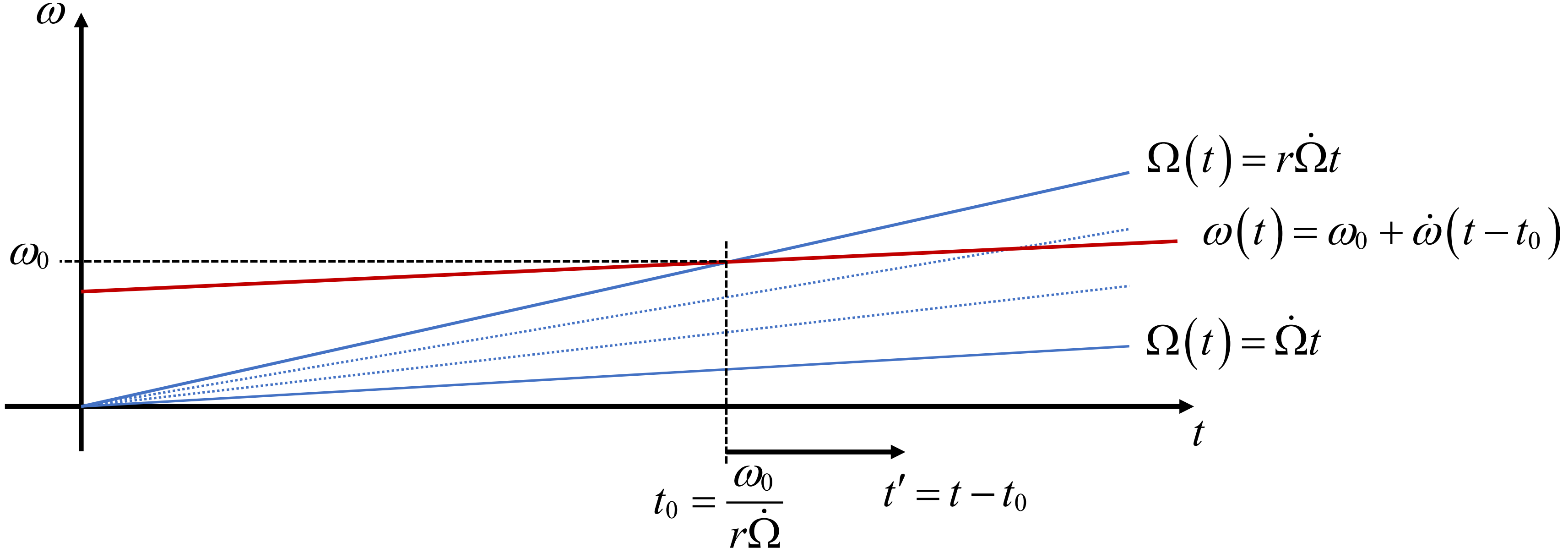
Governing equation

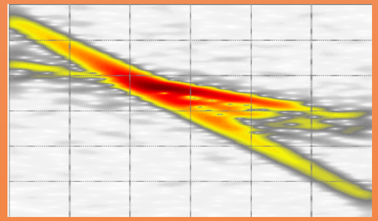
$$\ddot{q} + 2\xi\omega(t)\dot{q} + \boxed{\omega^2(t)}\,q = \frac{p_0}{m}\,\mathrm{e}^{\frac{\mathrm{i}}{2}r\dot{\Omega}t^2}$$

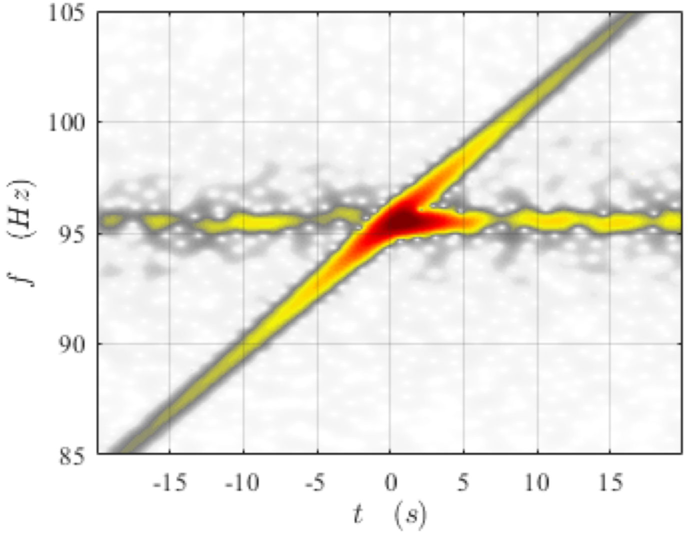
Time-varying natural frequency

$$\omega(t) = \omega_0 + \dot{\omega}(t - t_0) \qquad t_0 = \omega_0 / r\dot{\Omega}$$

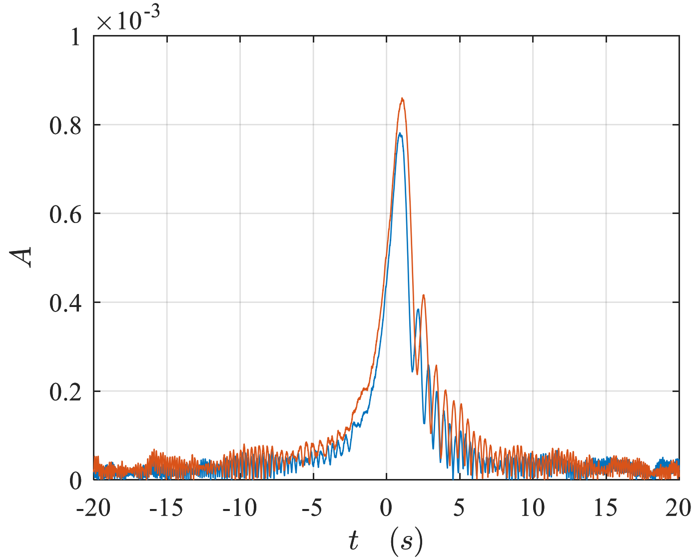
$$\ddot{q} + 2\xi(\omega_0 + \dot{\omega}t')\dot{q} + (\omega_0 + \dot{\omega}t')^2 q = \frac{p_0}{m}\,\mathrm{e}^{\frac{\mathrm{i}}{2}r\dot{\Omega}t'^2}\,\mathrm{e}^{\mathrm{i}r\dot{\Omega}t_0 t'}\,\mathrm{e}^{\frac{\mathrm{i}}{2}r\dot{\Omega}t_0^2} = \frac{p_0}{m}\,\mathrm{e}^{\frac{\mathrm{i}}{2}\cancel{r}\dot{\Omega}t'^2}\,\mathrm{e}^{\mathrm{i}\omega_0 t'}\,\cancel{\mathrm{e}^{\mathrm{i}\Phi_0}}$$



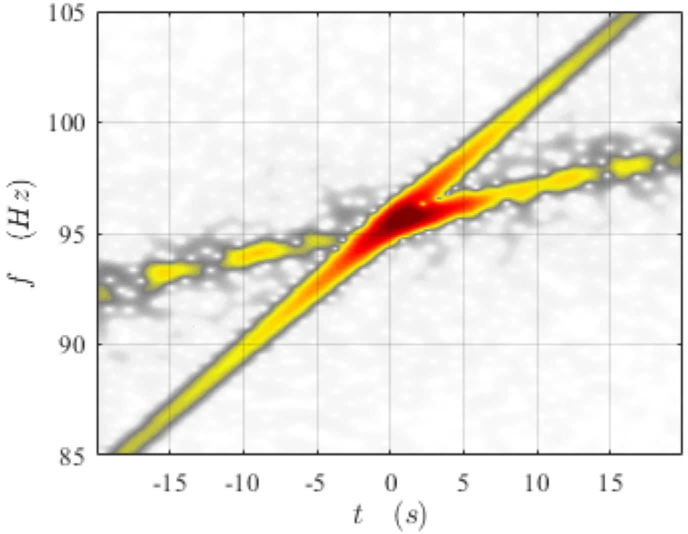




105
100
95
90
85
$f \quad (Hz)$
-15 -10 -5 0 5 10 15
$t \quad (s)$



$\times 10^{-3}$
1
0.8
0.6
0.4
0.2
0
A
-20 -15 -10 -5 0 5 10 15 20
$t \quad (s)$



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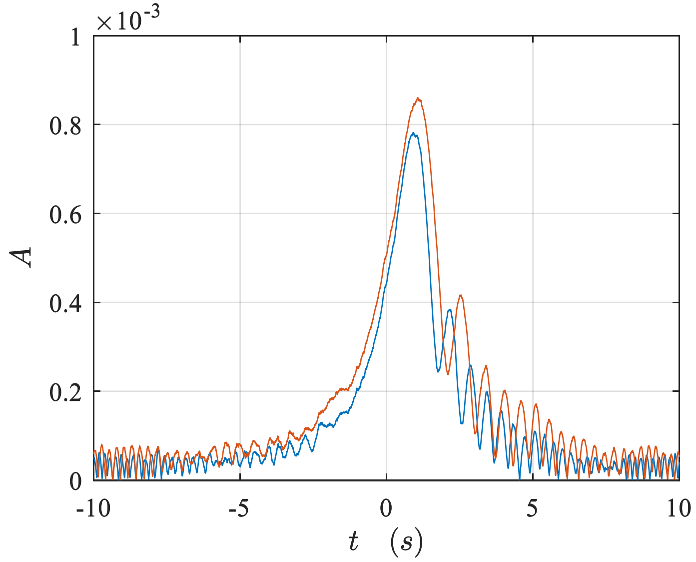

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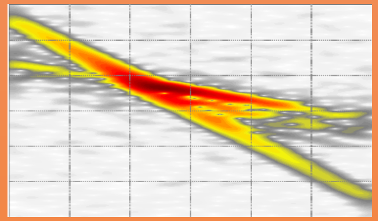

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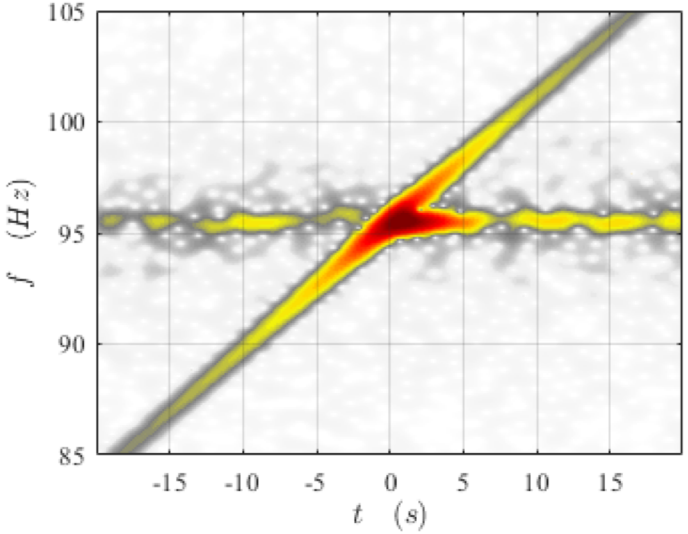
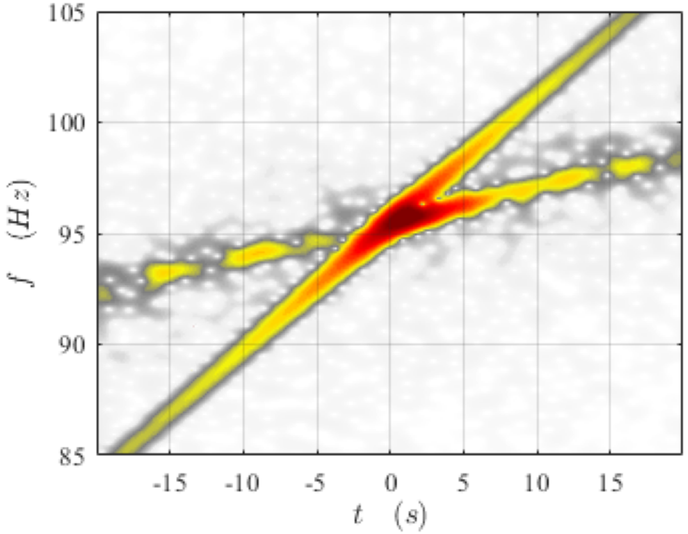
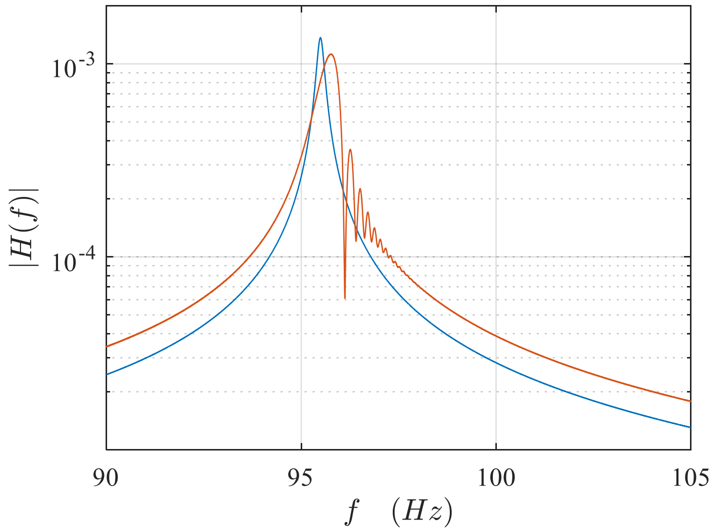
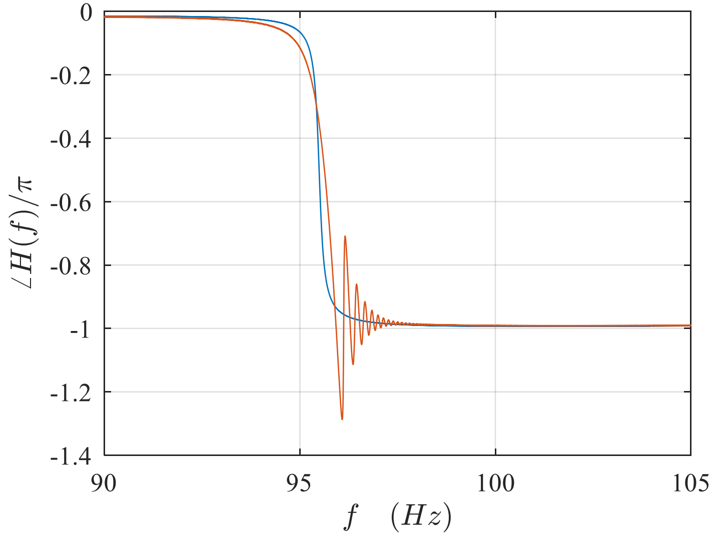
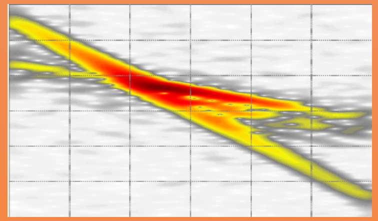




$$H(f)=\frac{F_{\omega}[q]}{F_{\omega}[p]}$$







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Single-DOF model

Governing equation

Sweeping rate

$$\ddot{q} + 2\xi \left(\omega_0 + \dot{\omega} t'\right) \dot{q} + \left(\omega_0 + \dot{\omega} t'\right)^2 q = \frac{p_0}{m} \mathrm{e}^{\frac{\mathrm{i}}{2}\dot{\Omega} t'^2} \mathrm{e}^{\mathrm{i}\omega_0 t'}$$

Time-varying natural frequency

Define a dimensionless time and a dimensionless response : $\tau = \omega_0 t'$ $\quad x = \frac{p_0}{2\xi \mathrm{i} m \omega_0^2} q$

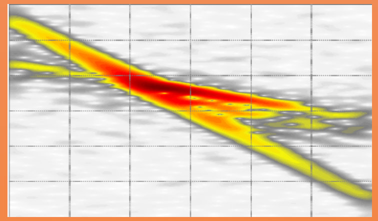
$$x'' + 2\xi \left(1 + \frac{\dot{\omega}}{\omega_0^2}\tau\right) x' + \left(1 + \frac{\dot{\omega}}{\omega_0^2}\tau\right)^2 x = 2\xi \mathrm{i} \mathrm{e}^{\mathrm{i}\tau} \mathrm{e}^{\frac{\mathrm{i}}{2}\frac{\dot{\Omega}}{\omega_0^2}\tau^2}$$

Small dimensionless parameters : $\xi = \varepsilon$ $\quad \frac{\dot{\Omega}}{\omega_0^2} = \kappa\varepsilon^2$ $\quad \frac{\dot{\omega}}{\omega_0^2} = \alpha\frac{\dot{\omega}}{\dot{\Omega}} = \alpha\kappa\varepsilon^2$

$$x'' + 2\varepsilon \left(1 + \kappa\alpha\varepsilon^2\tau\right) x' + \left(1 + \kappa\alpha\varepsilon^2\tau\right)^2 x = 2\varepsilon \mathrm{i} \mathrm{e}^{\mathrm{i}\tau} \mathrm{e}^{\frac{\mathrm{i}}{2}\kappa\varepsilon^2\tau^2}$$

Slowly time-varying oscillator subject to slow sine sweep →

- ~~Analytical solution~~
- Numerics
- Multiple timescales



Single-DOF model

Multiple timescale solution

$$x'' + 2\varepsilon \left(1 + \kappa\alpha\varepsilon^2\tau\right) x' + \left(1 + \kappa\alpha\varepsilon^2\tau\right)^2 x = 2\varepsilon \mathrm{i} \mathrm{e}^{\mathrm{i}\tau} \mathrm{e}^{\frac{\mathrm{i}}{2}\kappa\varepsilon^2\tau^2}$$

Fast time : τ Slow time : $T = \varepsilon\tau$ Ansatz : $x\left(\tau, T\right) = x_0\left(\tau, T\right) + \varepsilon x_1\left(\tau, T\right) + \cdots$

Leading order solution : $x_0 = A\left(T\right) \mathrm{e}^{\mathrm{i}\tau}$ (slowly modulated harmonic response)

Secularity condition : $A'(T) + (1 - \mathrm{i}\kappa\,\alpha\, T)\, A(T) = \mathrm{e}^{\frac{\mathrm{i}}{2}\kappa T^2}$ (linear time varying 1st order ODE)

<u>General solution</u>

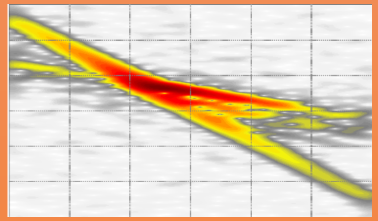
$$A\left(T\right) = \mathrm{e}^{-T} \mathrm{e}^{\frac{\mathrm{i}}{2}\kappa\,\alpha T^2} \mathcal{D}\left(T; \kappa\left(1 - \alpha\right)\right)$$

<u>Particular solution</u> $\alpha = 0$

$$A\left(T\right) = \mathrm{e}^{-T} \mathcal{D}\left(T; \kappa\right)$$

$$\mathcal{D}\left(T; \kappa\right) = \sqrt{\frac{\mathrm{i}\pi}{2\kappa}} \mathrm{e}^{\frac{i}{2\kappa}} \operatorname{erfc}\left[\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}} \frac{\mathrm{i} - \kappa T}{\sqrt{2\kappa}}\right]$$

Carassale, L., Denoël, V., Martel, C., & Panning-von Scheidt, L. (2021). Key features of the transient amplification of mistuned systems. *Journal of Engineering for Gas Turbines and Power*, *143*(3).



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Single-DOF model

Multiple timescale solution - illustration

General solution

$$A\left(T\right) = \mathrm{e}^{-T}\cancel{\mathrm{e}^{\frac{\mathrm{i}}{2}\kappa\alpha T^2}}\mathcal{D}\left(T;\kappa\left(1-\alpha\right)\right)$$

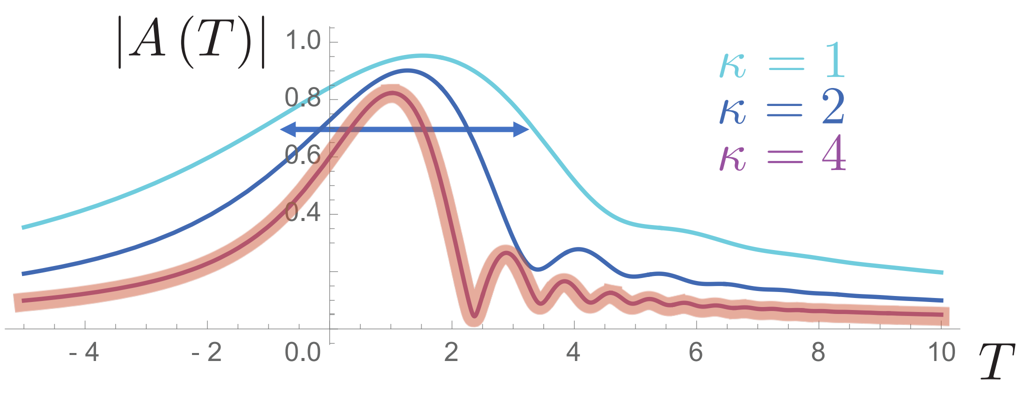
$$\alpha = 0.5$$

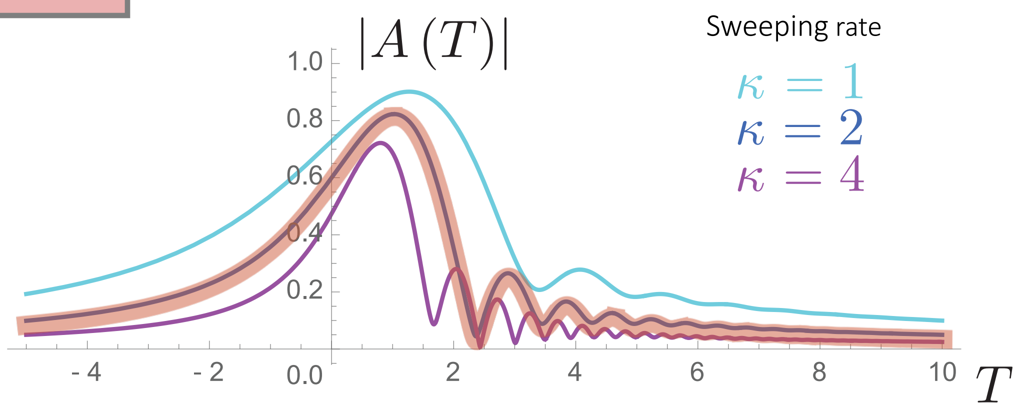
Particular solution

$$A\left(T\right) = \mathrm{e}^{-T}\mathcal{D}\left(T;\kappa\right)$$

$$\alpha = 0$$

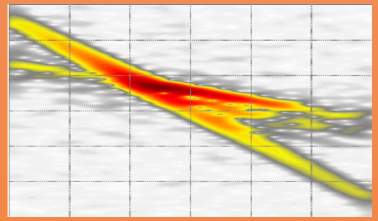
$$\kappa(1-\alpha) = 2$$





The width of the peak increases. Translates an apparently larger damping. More time spent near resonance

Norm of envelope only depends on $\kappa(1-\alpha)$



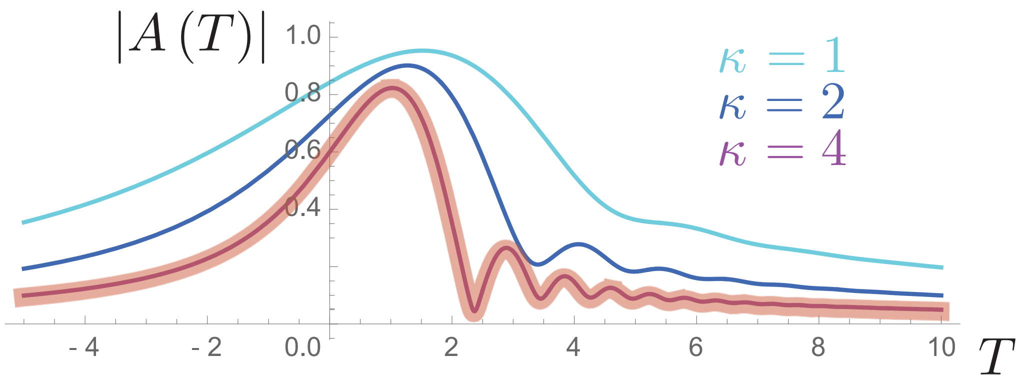
Equivalence

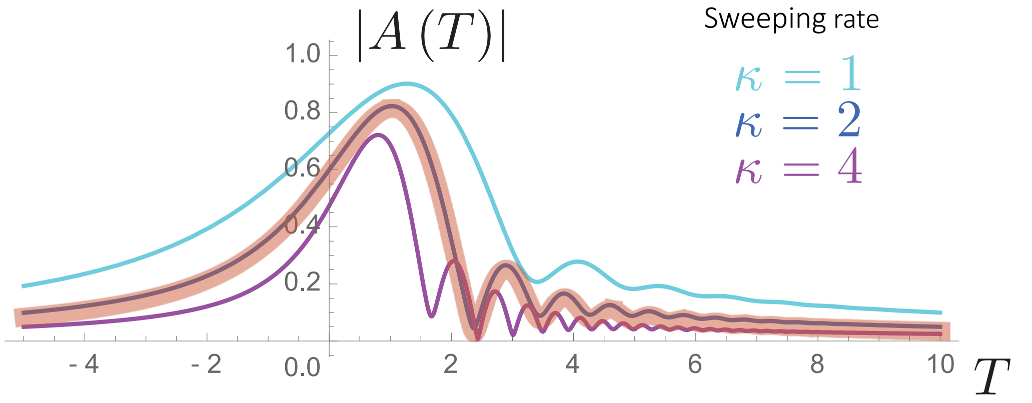
General solution

$$A(T) = \mathrm{e}^{-T} \cancel{\mathrm{e}^{\frac{\mathrm{i}}{2}\kappa\alpha T^2}} \mathcal{D}(T;\kappa(1-\alpha))$$

Particular solution

$$A(T) = \mathrm{e}^{-T} \mathcal{D}(T;\kappa)$$





As long a we focus on the **envelope of the response :**

An LTV system can be replaced by an equivalent LTI system subjected to a (smaller) sweeping rate

$$\kappa_{\mathrm{LTV}}(1-\alpha) = \kappa_{\mathrm{LTI}}$$

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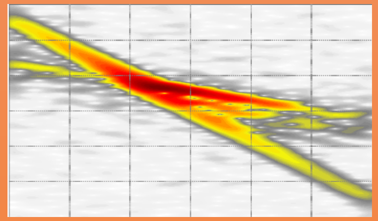

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Conclusions

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