

$$\begin{aligned}
f(\lambda) &= c_B^t (A_B + \lambda D_B)^{-1} b = \frac{1}{\lambda} \left(\frac{\det(I + \lambda(A^{-1}D + bc^t))}{\det(I + \lambda A^{-1}D)} - 1 \right) \\
&= \frac{1}{\lambda} \left(\prod_i \frac{1 + \lambda \alpha_i}{1 + \lambda \beta_i} - 1 \right)
\end{aligned}$$