A Bilevel Programming Approach for Distribution Network Development Planning

Bailly Geoffrey*
Montefiore Institute
University of Liège
Liège, Belgium
gbailly@uliege.be

Cornet Manon*
Montefiore Institute
University of Liège
Liège, Belgium
mcornet@uliege.be

Glavic Mevludin
Montefiore Institute
University of Liège
Liège, Belgium
Mevludin.Glavic@uliege.be

Cornélusse Bertrand
Montefiore Institute
University of Liège
Liège, Belgium
Bertrand.Cornelusse@uliege.be

***Abstract*—When elaborating a distribution network development plan, it is paramount to jointly consider the multi-year network infrastructure development plan and the network users' energy infrastructure evolution. To this end, we provide a comprehensive formulation of a bilevel program in a one-leader multi-follower setting with the distribution network development plan as the upper level, while the lower level minimizes network users' energy costs. Solving this optimization problem allows for assessing the impact of exogenous factors, such as network users' demand on network development plans and network user investment in distributed energy sources and storage. Results are reported using a 23-node test system and a new open-source toolbox developed during this research. We also evaluate the CO2 footprint of the components of the system.**

***Index Terms*—Distribution network, network users, development planning, bilevel programming.**

I. Introduction

A huge energy infrastructure investment will be needed to transition towards a decarbonized energy sector [1]. As estimated [1], a small percentage of increased efficiency in this transition could create trillions of euros value. It appears that co-optimization, across different energy sectors and within each energy sector, is the key to an efficient transition.

An important aspect of the electricity sector transition is distribution network development planning (DNDP). A solution to this problem aims to establish an optimal and cost-effective plan that includes the enhancement of existing distribution feeders and substations and the installation of new ones. This plan must cater to the projected demand over the defined time horizon while adhering to the technical constraints of the network. The importance of this problem is recognized by the European Commission through setting the rule requiring distribution network operators (DNOs) to conduct their network development plan at least every two years for a time horizon of five to ten years [2]. A number of approaches have been considered to solve this problem [3], [4]. All these research works focus on the DNO's perspective, overlooking the behavior of network users resulting from their interactions with the DNO. From the users' standpoint, there is already a large body of research aimed at optimizing the sizing and operation of a microgrid or an energy community, as seen in [5], [6].

In this work, we demonstrate how bilevel programming [7] (an optimization approach that fits the co-optimization context) offers an efficient way to solve the DNDP problem. Bilevel programming has already been considered as an approach for the DNDP problem in [8] and [9]. In [8], generation and distribution network development is set as the upper-level problem, while the lower-level concentrates on demand response. In contrast, [9] sets the upper-level problem to optimize the DNDP, with the lower-level problems involving RES and demand aggregators.

We recently presented a bilevel program (BP) formulated in a one-leader multi-follower setting [10] as a solution of the DNDP. To the best of our knowledge, no other work considered the DNDP using a BP that incorporates a lower-level representing the behavior of users as a microgrid optimization problem, with their investment decisions directly coupled to the DNO's policy.

This work extends [10] by detailing the DNO and the network users' objectives and constraints, including electricity storage in the model, and performing an extended sensitivity analysis. The ultimate goal is to develop a comprehensive framework (as an open-source toolbox) to solve the DNDP problem with the following characteristics:

- It is in line with the European context and the recent European Commission directives [2],
- It allows the study of the exogenous factors' impact on the development plan,
- It is easily extensible with new network users.

The problem formulation is presented in Section II, while Section III presents the results using a 23-node test system. Section IV discusses the results. In Section V, some conclusions and future research are presented.

II. Problem formulation and mathematical model

The DNO, assumed here to be the leader, optimizes the DNDP, while the network users, assumed here to be the followers, optimize their investments to meet their electricity demand. The users' demand is considered fixed, and therefore drives investment in the network, renewable energy sources (RES) and storage. We make several assumptions (to be

*These authors contributed equally to this work.

relaxed in future work): investment decisions are made at the beginning of the planning horizon and remain fixed afterward (single-stage), users are considered to act independently and are modeled as perfectly rational agents with complete knowledge of the future, and users' demand is inflexible. The bilevel programming optimization problem is formulated as follows:

$$\min \quad \text{Upper-Level Objective Function (3a)} \tag{1a}$$

$$\text{s.t.}: \quad \text{Upper-Level Constraints (3b) to (3ad)} \tag{1b}$$

$$\text{Lower-level optimality:}$$

$$(p^{imp}, p^{exp}, q^{imp}, q^{exp}, c^{grid}) \in \text{argmin}\{(2a)| \text{ s.t.: } (2b) \text{ to } (2z)\} \tag{1c}$$

A. Lower-level

Each user's optimization program can be seen as a single-bus microgrid optimization problem. It aims to identify the optimal investment decisions regarding generating units, storage, network connection capacity, and the optimal dispatch decisions over a planning horizon. By summing all network users individual objective functions and considering all of their individual constraints, we obtain the linear lower-level problem:

$$\min \sum_{i \in \mathcal{B}_u} \Big(c_i^{PV} + c_i^{st} + c_i^{grid} + \alpha \sum_{t \in \mathcal{T}} (c_{i,t}^{imp} + c_{i,t}^{grid} - c_{i,t}^{exp}) \Big) \tag{2a}$$

s.t. $\forall i \in \mathcal{B}_U,$

$$c_i^{PV} = 1/\gamma^{PVC} \left(\overline{s}_i^{PVC} \pi^{PVC}\right) + 1/\gamma^{PV} \left(\overline{p}_i^{PV} \pi^{PV}\right) \tag{2b}$$

$$c_i^{st} = 1/\gamma^{stC} \left(\overline{s}_i^{stC} \pi^{stC}\right) + 1/\gamma^{st} \left(\overline{e}_i^{st} \pi^{st}\right) \tag{2c}$$

$$c_i^{grid} = \overline{s}_i^{grid} \pi^{GC} \tag{2d}$$

$\forall i \in \mathcal{B}_U, \forall t \in \mathcal{T},$

$$c_{i,t}^{grid} = \left(p_{i,t}^{imp} \Pi^{EI} + p_{i,t}^{exp} \Pi^{EE}\right) \Delta t \tag{2e}$$

$$c_{i,t}^{imp} = p_{i,t}^{imp} \pi^{EI} \Delta t \tag{2f}$$

$$c_{i,t}^{exp} = p_{i,t}^{exp} \pi^{EE} \Delta t \tag{2g}$$

$$p_{i,t}^{imp} - p_{i,t}^{exp} = p_{i,t}^{D} - p_{i,t}^{PV} + p_{i,t}^{stc} - p_{i,t}^{std} \tag{2h}$$

$$q_{i,t}^{imp} - q_{i,t}^{exp} = q_{i,t}^{D} \tag{2i}$$

$$p_{i,t}^{imp} \le \overline{s}_i^{grid} \tag{2j}$$

$$q_{i,t}^{imp} \le \overline{s}_i^{grid} \tag{2k}$$

$$p_{i,t}^{exp} \le \overline{s}_i^{grid} \tag{2l}$$

$$q_{i,t}^{exp} \le \overline{s}_i^{grid} \tag{2m}$$

$$p_{i,t}^{PV} \le \overline{s}_i^{PVC} \tag{2n}$$

$$p_{i,t}^{PV} \le \overline{p}_i^{PV} P_{i,t}^{PV} \tag{2o}$$

$$SOC_{i,t} \le \overline{e}_i^{st} \overline{E} \tag{2p}$$

$$SOC_{i,t} \ge \overline{e}_i^{st} \underline{E} \tag{2q}$$

$$p_{i,t}^{stc} \le 1/\Delta t \left(\overline{e}_i^{st} R^{+}\right) \tag{2r}$$

$$p_{i,t}^{std} \le 1/\Delta t \left(\overline{e}_i^{st} R^{-}\right) \tag{2s}$$

$$p_{i,t}^{stc} \le 1/(\epsilon^{stc} \Delta t) \left(\overline{e}_i^{st} \overline{E} - SOC_{i,t}\right) \tag{2t}$$

$$p_{i,t}^{std} \le \epsilon^{std}/\Delta t \left(SOC_{i,t} - \overline{e}_i^{st} \underline{E}\right) \tag{2u}$$

$$p_{i,t}^{std} \le 1/\Delta t \left(\overline{e}_i^{st} R^{-}\right) - \left(R^{-}/R^{+}\right) p_{i,t}^{stc} \tag{2v}$$

$$p_{i,t}^{stc} \le \overline{s}_i^{stC} \tag{2w}$$

$$p_{i,t}^{std} \le \overline{s}_i^{stC} \tag{2x}$$

$\forall i \in \mathcal{B}_U, \forall t \in \mathcal{T} \setminus \mathcal{T}^1,$

$$SOC_{i,t} = SOC_{i,t-1} + \left(\epsilon^{stc} p_{i,t}^{stc} - p_{i,t}^{std}/\epsilon^{std}\right) \Delta t \tag{2y}$$

$\forall i \in \mathcal{B}_U, \forall t \in \mathcal{T}^1,$

$$SOC_{i,t} = SOC_{i,t+T-1} + \left(\epsilon^{stc} p_{i,t}^{stc} - p_{i,t}^{std}/\epsilon^{std}\right) \Delta t \tag{2z}$$

The objective function (2a) sums the total expenditures on local generation and storage investments, the network connection capacity expenses, the electricity import costs, and the energy grid tariffs. This sum is then diminished by the revenues obtained from the electricity injected into the grid. Equations (2b) to (2g) relate these costs to the decision variables. The factor α scales the operational costs estimated over d^{rep} representative days to account for the lifetime of the investments.

The primary decision variables of this model are the grid connection capacity ($\overline{s}_i^{grid}$), the PV installation size ($\overline{p}_i^{PV}$), the storage unit size ($\overline{e}_i^{st}$), and the active and reactive power exchanges ($p_{i,t}^{imp}$, $q_{i,t}^{imp}$, $p_{i,t}^{exp}$, $q_{i,t}^{exp}$) with the grid at each time step.

Constraints (2h) and (2i) are the bus active and reactive power balances for all time steps, respectively. The analysis of the effect of reactive power support from PV and storage devices is left for future work. This can be achieved by adding their contribution in the reactive power balance and restricting the active and reactive power of each device to stay within their P-Q diagram. Constraints (2j) and (2k) bound the active and reactive power imported from the grid at each time step, whereas (2l) and (2m) bound the active and reactive power injected to the grid at each time step. We use (2j) to (2m) to approximate linearly the grid connection capacity. Furthermore, (2n) limits the active power generated by the PV plant at each time step to the PV inverter capacity. The PV power generation of user i's PV installation is bounded in (2o) at each time step t by the PV generation forecast. The storage dynamics are modeled by constraints (2p) to (2z). Constraints (2p) and (2q) bound the state of charge of the storage device and constraints (2r) and (2s) limit its charging and discharging capacity rates. Constraints (2t) to (2v) provide limits to the $p_{i,t}^{stc}$-$p_{i,t}^{std}$ feasible region according to the Extn-LP formulation from [11]. Charging and discharging powers are limited by the converter capacity with constraints (2w) and (2x). Equality (2y) updates the state of charge $SOC_{i,t}$ based on its previous state and the charging and discharging powers of user i's storage unit at time period t. This requires knowing two additional parameters: the charging efficiency ϵ^{stc} and the discharging efficiency ϵ^{std}. These efficiencies are considered constant regardless of the storage unit state of charge and take values in the interval $[0,1]$. Equality (2z) is similar to (2y) but links the state of charge of the first time period to the one of the last time period for each representative day, and let this be a variable of the optimization problem.

B. Upper-level

The DNO solves the DNDP problem with the following objectives:

1) The DNO aims to identify the optimal investment decisions for conductors and substations within a specified planning horizon, typically spanning from 10 to 30 years. Optimal decisions correspond to strategies that reduce both the initial capital expenditure (CAPEX) associated with investments in conductors and substations and the continuous operational costs (OPEX), represented by the cost of losses in the network.
2) The distribution network topology resulting from these choices should ensure the connection of all grid users to a substation, and in operation, the network graph should have no loop. Mathematically, this requirement translates to seeking a distribution network graph that is a spanning tree.
3) The distribution network must be able to accommodate the predicted electricity demand from users throughout the planning period.
4) The distribution network must satisfy the operational constraints on bus voltages and line currents.
5) The money invested by the DNO, plus a margin to remunerate its activities, must be recovered through the network tariffs applied to the grid users. This is the so-called *budget balance* constraint.

Moreover, a series of assumptions are introduced to simplify the formulation of our upper-level model:

- we approach our problem as if we were creating an entirely new distribution network, although this can be easily adapted to model an existing network by fixing some decision variables,
- the DNO makes investment decisions only once at the beginning of the planning period,
- a single-phase equivalent network is considered, assuming a balanced three-phase regime.

Below is the MISOCP for the upper-level optimization program. It is based on a branch flow model (BFM) and a second-order conic programming (SOCP) relaxation of power flow equations. This formulation incorporates loop elimination constraints, guaranteeing the radial nature of the distribution network graph when distributed generation (DG) units are integrated into the network.

$$\min \quad \frac{1}{\Gamma}\left(C^{cond}+C^{sub}\right)+\alpha\sum_{t\in\mathcal{T}}\left(C_t^{loss}+\omega^I\Phi_t^I\right) \tag{3a}$$

$$\text{s.t.} \quad C^{cond}=\sum_{ij\in\mathcal{L}}\sum_{k\in K}\lambda_{ij,k}\Pi_{ij,k}^{cond} \tag{3b}$$

$$C^{sub}=\sum_{i\in\mathcal{B}_s}\overline{S}^{sub}\Pi^{sub} \tag{3c}$$

$$C_t^{loss}=\sum_{ij\in\mathcal{L}}\sum_{k\in\mathcal{K}}(R_{ij,k}I_{ij,k}^2)\Delta t\Pi^{loss},\forall t\in\mathcal{T} \tag{3d}$$

$$\Phi_t^I=\sum_{ij\in\mathcal{L}}\sum_{k\in\mathcal{K}}I_{ij,k,t}^{lim},\forall t\in\mathcal{T} \tag{3e}$$

$$\begin{aligned}&(1+\tau)^{\Gamma}\left(C^{sub}+C^{cond}\right)\\&+\Gamma\alpha\sum_{t\in\mathcal{T}}C_t^{loss}\leq\Gamma\sum_{i\in\mathcal{B}_u}\left(c_i^{grid}+\alpha\sum_{t\in\mathcal{T}}c_{it}^{grid}\right)\end{aligned} \tag{3f}$$

$$P_{ij,t}=\sum_{k\in\mathcal{K}}P_{ij,k,t},\forall ij\in\mathcal{L},\forall t\in\mathcal{T} \tag{3g}$$

$$Q_{ij,t}=\sum_{k\in\mathcal{K}}Q_{ij,k,t},\forall ij\in\mathcal{L},\forall t\in\mathcal{T} \tag{3h}$$

$$I_{ij,t}^2=\sum_{k\in\mathcal{K}}I_{ij,k,t}^2,\forall ij\in\mathcal{L},\forall t\in\mathcal{T} \tag{3i}$$

$$\begin{aligned}-P_{i,t}^{sub}=&\sum_{k\in\mathcal{K}}P_{mi,k,t}-R_{mi,k}I_{mi,k,t}^2\\&-\sum_{k\in\mathcal{K}}P_{ij,k,t},\forall i\in\mathcal{B}_s,\forall t\in\mathcal{T}\end{aligned} \tag{3j}$$

$$\begin{aligned}-Q_{i,t}^{sub}=&\sum_{k\in\mathcal{K}}Q_{mi,k,t}-X_{mi,k}I_{mi,k,t}^2\\&-\sum_{k\in\mathcal{K}}Q_{ij,k,t},\forall i\in\mathcal{B}_s,\forall t\in\mathcal{T}\end{aligned} \tag{3k}$$

$$\begin{aligned}p_{i,t}^{imp}-p_{i,t}^{exp}=&\sum_{k\in\mathcal{K}}P_{mi,k,t}-R_{mi,k}I_{mi,k,t}^2\\&-\sum_{k\in\mathcal{K}}P_{ij,k,t},\forall i\in\mathcal{B}_u,\forall t\in\mathcal{T}\end{aligned} \tag{3l}$$

$$\begin{aligned}q_{i,t}^{imp}-q_{i,t}^{exp}=&\sum_{k\in\mathcal{K}}Q_{mi,k,t}-X_{mi,k}I_{mi,k,t}^2\\&-\sum_{k\in\mathcal{K}}Q_{ij,k,t},\forall i\in\mathcal{B}_u,\forall t\in\mathcal{T}\end{aligned} \tag{3m}$$

$\forall ij\in\mathcal{L},\forall t\in\mathcal{T},$

$$\begin{aligned}&V_{j,t}^2-V_{i,t}^2\leq\\&\quad\sum_{k\in\mathcal{K}}\left(-2\left(R_{ij,k}P_{ij,k,t}+X_{ij,k}Q_{ij,k,t}\right)\right.\\&\quad\left.+\left(R_{ij,k}^2+X_{ij,k}^2\right)I_{ij,k,t}^2\right)+M(1-\Lambda_{ij})\end{aligned} \tag{3n}$$

$$\begin{aligned}&V_{j,t}^2-V_{i,t}^2\geq\\&\quad\sum_{k\in\mathcal{K}}\left(-2\left(R_{ij,k}P_{ij,k,t}+X_{ij,k}Q_{ij,k,t}\right)\right.\\&\quad\left.+\left(R_{ij,k}^2+X_{ij,k}^2\right)I_{ij,k,t}^2\right)-M(1-\Lambda_{ij})\end{aligned} \tag{3o}$$

$$V_{i,t}^2I_{ij,t}^2\geq P_{ij,t}^2+Q_{ij,t}^2 \tag{3p}$$

$$(S_{i,t}^{sub})^2\geq(P_{i,t}^{sub})^2+(Q_{i,t}^{sub})^2,\forall i\in\mathcal{B}_s,\forall t\in\mathcal{T} \tag{3q}$$

$$S_{i,t}^{sub}\leq\overline{S}_i^{sub},\forall i\in\mathcal{B}_s,\forall t\in\mathcal{T} \tag{3r}$$

$$\overline{S}_i^{sub}\leq\beta_i\overline{S}_i^{sub,max},\forall i\in\mathcal{B}_s \tag{3s}$$

$$V_{i,t}^2-1\leq\left(\overline{V}^2-1\right)(1-\beta_i),\forall i\in\mathcal{B}_s \tag{3t}$$

$$V_{i,t}^2 - 1 \geq \left(\underline{V}^2 - 1\right)(1-\beta_i), \forall i \in \mathcal{B}_s \quad (3u)$$

$\forall ij \in \mathcal{L}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T}$

$$P_{ij,k,t} \leq \lambda_{ij,k}\overline{I}_{ij,k}\overline{V}, \quad (3v)$$

$$P_{ij,k,t} \geq -\lambda_{ij,k}\overline{I}_{ij,k}\overline{V}, \quad (3w)$$

$$Q_{ij,k,t} \leq \lambda_{ij,k}\overline{I}_{ij,k}\overline{V} \quad (3x)$$

$$Q_{ij,k,t} \geq -\lambda_{ij,k}\overline{I}_{ij,k}\overline{V} \quad (3y)$$

$$h_{ij,k,t} \leq \left(\overline{I}_{ij,k}\right)^2 I_{ij,k,t}^{lim} \quad (3z)$$

$$I_{ij,k,t}^2 - h_{ij,k,t} \leq \left(\overline{I}_{ij,k}\right)^2 \lambda_{ij,k} \quad (3aa)$$

$$\sum_{k \in \mathcal{K}} \lambda_{ij,k} = \Lambda_{ij}, \forall ij \in \mathcal{L} \quad (3ab)$$

$$\sum_{ij \in \mathcal{L}} \Lambda_{ij} = n_u, \forall t \in \mathcal{T} \quad (3ac)$$

single commodity flow constraints from [12] (3ad)

The optimization problem spans a planning horizon of T time steps of equal size Δt. The objective function (3a) of the DNO is to minimize the total costs of new conductors C^{cond} plus the total cost of substations C^{sub}, the yearly values of the cost of the losses, and a term capturing the penalty for violations of current constraints. The last term is a relaxation of the problem to ensure the feasibility and tightness of cone constraints. Moreover, investment costs, i.e. $C^{cond}+C^{sub}$, are scaled by the amortization period Γ to express the objective function to one year.

(3f) is the *budget balance*. (3g) to (3i) define branch physical quantities. (3j) and (3k) represent active and reactive power balances at substation nodes, whereas (3l) and (3m) consider the lower-level variables p^{imp} and p^{exp} to write the active and reactive power balances at grid users' nodes. Regarding power flow constraints, the BFM relaxation of power flow constraints is adapted in (3n) to (3p) to account for conductor choices. Specifically, (3p) is the rotated SOCP constraint, which guarantees optimality when it is tight.

(3q) to (3s) model the decision of building a substation by introducing the binary variables β_i. $\overline{S}_i^{sub}$ represents the allocated substation capacity. Reference voltages are set to 1 pu at substation nodes through the introduction of (3t) and (3u). (3v) to (3y) impose a zero active and reactive power flow when conductor k is not selected for the line ij. This is done by considering the binary variable $\lambda_{ij,k}$ that equals one when conductor k is selected for line ij, and 0 otherwise. The relaxation of the current limit is expressed by introducing in (3z) the slack variable $h_{ij,k,t}$. (3z) introduces the binary variable $I_{ij,k,t}^{lim}$ equal to one when the current limit constraint is violated. The binary variable $\Lambda_i j$ is introduced in (3ab). Finally, the radiality constraints comprise the simple radiality constraint (3ac) and the loop elimination constraints (3ad).

III. Results

We conduct a detailed analysis of the results on a base case, then a sensitivity analysis on several important parameters.

A. The UDinet.jl toolbox

All the results were obtained with the UDinet.jl toolbox, which represents an initial effort to provide DNOs with a customizable tool for the formulation of DNDP. The tool was implemented in Julia using the JuMP and BilevelJuMP optimization packages. It enables the creation of multiple alternative formulations of the BP defined in (1). The required input data are related to network topology, load, and PV forecasts for several representative days spanning the planning horizon, costs and tariffs, and the configuration of the formulation to be tested. Based on these inputs, the module generates a BP formulation and conducts simulations across the representative days. The results of these simulations include key performance indicators, a radial network topology, and the network's state at each time step during the simulation. In its current version, the tool incorporates various features, such as experimentation with two convex relaxation formulations for power flow equations, relaxation of constraints related to current and voltage, the possibility of allowing for dynamic network reconfiguration and exploring multiple loop elimination constraints.

B. Test case description

We consider the 23-node test system of [13], illustrated in Fig. 1. There are two possible substations and 21 nodes with loads that we consider independent users of the network. The available conductors are listed in Table I. Demand and generation profiles come from [15]. In the base case, the demand is scaled so that the peak load is 7 MVA on a five-minute time scale, pro-rata of the load data in Table VI of [13]. The total energy consumed in the network is 16440 MWh/year. The amortization periods are 50 years for the DSO investment, 30 years for the PV panels, and ten years for the PV inverters. The other relevant parameters are summarized in Table II, where case 0 is the base case.

C. General sensitivity analysis

A sensitivity analysis is performed on some key parameters: the PV installation, the storage, the energy import price and the load. From cases 0 and 2 in Table III, PV and storage installations allow for a lesser grid usage, leading to a lower grid connection capacity and DNO investments. In case 1, DSO investments increase due to users' energy exportation and grid usage. In these three cases, self-sufficiency and self-consumption ratios show higher values. Also, users' investment cost in PV and storage installations is compensated by a cost decrease of the grid connection capacity and the energy

TABLE I
ALL ALUMINUM CONDUCTORS (DATA FROM [14]).

Code Word	q	i_{max}	r	x_l	cost
	mm^2	kA	Ω/km	Ω/km	k€/km
Poppy	53.5	0.23	0.5502	0.429	10
Oxlip	107.3	0.34	0.2747	0.402	12
Daisy	135.3	0.46	0.2180	0.394	15
Tulip	107.3	0.53	0.1732	0.381	20

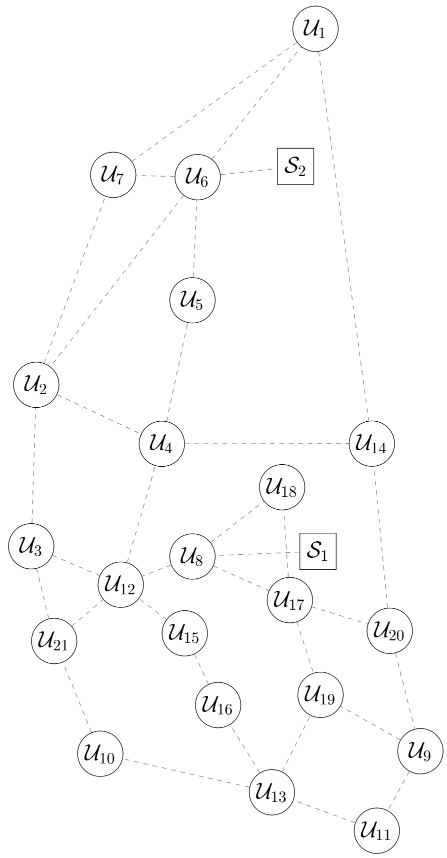

Fig. 1. Diagram illustrating a test network featuring bus IDs and potential line routes. Dashed lines denote the potential line routes, squares indicate potential substation locations, and circles represent grid users' buses.

TABLE II
DESCRIPTION OF TEST CASES.

MPV: Maximum PV capacity per bus (MVA), STO: add storage capability, EIP: energy import price (k€/MWh), EV: add electric vehicles' consumption, HP: add heat pumps' consumption. False (F), true (T).

Case	MPV	STO	EIP	EV	HP
0	0	F	0.3	F	F
1	**0.4**	F	0.3	F	F
2	**0.4**	**T**	0.3	F	F
3	**0.4**	**T**	**0.6**	F	F
4	0	F	0.3	**T**	**T**
5	**0.4**	F	0.3	**T**	**T**
6	**0.4**	**T**	0.3	**T**	**T**
7	**0.4**	**T**	**0.6**	**T**	**T**

drawn from the network. The same observations are made with higher loads from cases 4, 5 and 6.

In cases 2 and 6, since the PV installation size is fixed, the higher cost of PV installation is due to an increased converter capacity.

Increasing the energy import price in cases 3 and 7 encourages to invest in storage in order to cushion users' cost increase.

D. PV and storage sensitivity analysis

A second analysis focuses on the impact of the PV installation size in Table IV, based on cases 0, 1 and 2 from Table II.

TABLE III
RESULTS OBTAINED WITH THE BILEVEL MODEL

DNO: DNO's total annual amortized cost (M€/y), Users: Users' total annual amortized cost(M€/y), UPVC: Users' PV annual amortized cost of investments (M€/y), UStoC: Users' storage annual amortized cost of investments (M€/y), UGCC: Users' annual grid connection cost (M€/y), USS: Users' average self-sufficiency (%), USC: Users' average self-consumption (%).

Case	DNO M€/y	Users M€/y	UPVC M€/y	UStoC M€/y	UGCC M€/y	USS %	USC %
0	1.05	7.28	0.00	0.00	2.35	0	-
1	1.09	5.82	0.24	0.00	1.95	25	32
2	0.90	5.07	0.25	0.96	1.20	46	60
3	0.89	7.70	0.25	1.03	1.18	47	61
4	2.40	23.30	0.00	0.00	7.38	0	-
5	2.40	21.70	0.25	0.00	6.97	9	37
6	1.64	20.20	0.32	2.15	5.24	22	93
7	1.62	32.70	0.32	2.26	5.20	23	95

TABLE IV
RESULTS OBTAINED WITH THE BILEVEL MODEL

MPV: Maximum PV capacity per bus (MVA), UPVP: Users' average PV production ratio (%), USS: Users' average self-sufficiency (%), USC: Users' average self-consumption (%), CO2: CO2 emissions of the whole system. Boldface numbers correspond to cases with storage.

MPV	UPVP		USS		USC		CO2	
0.1	78	**99**	13	**19**	64	**98**	1989	**2017**
0.2	55	**88**	19	**34**	47	**87**	2009	**1978**
0.4	36	**60**	25	**46**	32	**60**	2159	**2084**
0.8	25	**37**	33	**57**	22	**37**	2480	**2423**

The indicator UPVP is the fraction of the PV production actually used, i.e. consumed, stored or exported. What is not used is lost and can be seen as a consequence of an oversized PV installation. Then the more the PV, the less useful the last panel becomes.

With a constant level of self-sufficiency, the addition of storage enables a decrease of the PV installation size. With an average level of 33% self-sufficiency for example, adding storage allows to divide the PV installation by four.

Adding storage increases also self-consumption and allows to better profit from an already existing PV installation.

Adding PV panels increases the CO2 emissions of the whole grid since PV panels have a large contribution on the CO2 impact of users' installations as described in the next section.

E. CO2 analysis

CO2 emissions are computed using Table V for case 2 from Table II. Substations are considered transformers, lines are considered three-cable aluminum conductors, loss emissions are computed using the grid electricity CO2 costs for Belgium, PV and storage CO2 emissions come from the installation sizes i.e. the peak power of the PV installation and the size of the storage battery, and the CO2 emissions of electricity drawn from the grid is based on the net consumption of grid users.

Table VI shows great discrepancy between the cost items. The largest contributions to CO2 emissions come from the grid electricity and the PV installation, whereas emissions from

TABLE V
CO2 DATA

Transformer	600	ton/MVA [16]
Aluminum	16	ton/ton Al [17]
Al density	2.7	ton/m³
PV	1700	ton/MWp [18]
Storage	200	ton/MWh [19]
Energy from grid	0.128	ton/MWh [20]

TABLE VI
CO2 RESULTS

Substations	62.7	T/year
Lines	0.3	T/year
Losses	1.7	T/year
PV	571.0	T/year
Storage	16.7	T/year
Net grid consumption	555.0	T/year
PV	47	kg/MWh
Grid	128	kg/MWh

lines, losses, and storage are comparatively small. The last two items show the emission per kWh consumed. In the case of Belgium and for the specific case considered, for a fixed energy consumption value, it can be concluded that installing PV panels would result in a sharp decrease in CO2 emission if it was the objective of our optimization problem.

IV. CONCLUSION

Bilevel programming naturally fits the co-optimisation context as the key for a successful energy transition. In this work:

- We illustrated how bilevel programming fits the problem of DNDP,
- A detailed problem formulation and used models are presented,
- We presented, using a small 24-bus medium voltage standard test system, the results supporting the viability of the proposed methodology for DNDP, and
- We developed an open source tool, in Julia, implementing the proposed methodology.

Future work will focus on extensions to network users' non-rational (bounded rationality) behavior, using consensus bilevel programming where a consensus rather than an optimal solution is sought, testing the methodology on larger cases, and better representative days considerations.

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APPENDIX: NOMENCLATURE

A. Notation for the lower-level problem

Sets and indices:

t	index of a time period
T	number of time periods per representative day
$\mathcal{T}$	set of time periods, with $\mathcal{T} = \{1, 2, \ldots, Td^{rep}\}$
$\mathcal{T}^1$	set of initial time periods from each representative day
i	index of a grid user
n_u	number of grid users
$\mathcal{B}_U$	set of grid users, with $\mathcal{B}_U = \{1, 2, \ldots, n_u\}$

Parameters:

$p^D_{i,t}$ active power demand of user i at time period t, in kW

$q^D_{i,t}$ reactive power demand of user i at time period t, in kVar

$P^{PV}_{i,t}$ shape of the forecast power generation profile of the PV installation of user i at time period t, in $[0,1]$

R^+ fraction of storage capacity charged per time step

R^- fraction of storage capacity discharged per time step

$\overline{E}$ maximum state of charge, as a fraction of the storage capacity

$\underline{E}$ minimum state of charge, as a fraction of the storage capacity

Δt duration of a time period, in hours

γ^{PVC} amortization period of a power converter, in years

γ^{PV} amortization period of a PV installation, in years

γ^{stC} amortization period of a storage device converter, in years

γ^{st} amortization period of a storage device, in years

α scaling factor of the simulation, such that $\alpha = \frac{d^{year}}{d^{rep}}$

d^{rep} number of representative days

d^{year} number of days in a year

π^{PVC} unitary cost for PV converter capacity, in €/kW

π^{PV} unitary cost for installed PV peak power capacity in €/kWp

π^{stC} unitary cost for storage converter capacity, in €/kW

π^{st} unitary cost for storage device, in €/kWh

π^{GC} unitary cost for grid connection capacity, in €/kW

π^{EI} unit price of energy imported from the grid at time period t, in €/kWh

π^{EE} unit price of energy exported to the grid at time period t, in €/kWh

Π^{EI} grid tariff imposed by the DNO on the energy imported from the grid at time period t, in €/kWh

Π^{EE} grid tariff imposed by the DNO on the energy exported to the grid at time period t, in €/kWh

Variables:

c^{PV}_i total PV investment costs of user i, in €

c^{st}_i total storage investment costs of user i, in €

c^{grid}_i total grid capacity costs of user i, in €

$c^{grid}_{i,t}$ grid tariff costs of user i at time period t, in €

$c^{imp}_{i,t}$ total cost of user i for the energy imported from the grid at time t, in €

$c^{exp}_{i,t}$ total revenue of user i for the energy exported to the grid at time period t, in €

$\overline{s}^{PVC}_i$ maximum power capacity of the PV converter of user i, in kW

$\overline{p}^{PV}_i$ peak power capacity of the PV installation of user i, in kW

$\overline{s}^{stC}_i$ maximum power capacity of the storage device converter of user i, in kW

$SOC_{i,t}$ state of charge of user i's storage device at time period t, in kWh

$\overline{e}^{st}_i$ storage device capacity of user i, in kWh

$\overline{s}^{grid}_i$ grid connection capacity of user i, in kVA

$p^{imp}_{i,t}$ active power imported from the grid for user i at time period t, in kW

$q^{imp}_{i,t}$ reactive power imported from the grid for user i at time period t, in kVar

$p^{exp}_{i,t}$ active power exported to the grid by user i at time period t, in kW

$q^{exp}_{i,t}$ reactive power exported to the grid by user i at time period t, in kW

$p^{PV}_{i,t}$ active power produced by the PV installation of user i at time period t, in kW

$q^{PV}_{i,t}$ reactive power produced by the PV installation of user i at time period t, in kW

$\mathcal{P}^{PV}_{i,t}$ polyhedron defining the PV installation PQ diagram of user i at time period t

$p^{stc}_{i,t}$ active power charging the storage device of user i at time period t, in kW

$p^{std}_{i,t}$ active power discharging the storage device of user i at time period t, in kW

B. Notation for the upper-level problem

Sets and indices:

t	index of a time period
T	number of time periods
$\mathcal{T}$	set of time periods, with $\mathcal{T} = \{1, 2, \ldots, T\}$
i/j	index of an electrical bus
n	number of electrical buses
n_s	number of substation buses
n_u	number of grid users' buses
$\mathcal{B}$	set of electrical buses
$\mathcal{B}_s$	set of substation buses, with $\mathcal{B}_s = \{1, \ldots, n_s\}$
$\mathcal{B}_u$	set of grid users' buses, with $\mathcal{B}_u = \{n_s + 1, \ldots, n\}$
ij	index of a line route connecting the sending bus i to the receiving bus j
l	index of a line route
L	number of possible line routes
$\mathcal{L}$	set of possible line routes, with $\lvert\mathcal{L}\rvert = L$
K	number of conductor choices
$\mathcal{K}$	set of possible conductor choices, with $\mathcal{K} = \{1, \ldots, K\}$

Parameters:

τ	interest rate of the DNO, in $[0, 1]$
Δt	duration of a time period, in hours
Γ	amortization period of investments for new line routes and substations, in years
α	scaling factor of the simulation, such that $\alpha = \frac{d^{year}}{d^{rep}}$
d^{rep}	number of representative days
d^{year}	number of days in a year
ω^{I}	unit cost for violating a current operational constraint, in k€
$\Pi^{cond}_{ij,k}$	unit cost for building the new line route ij with conductor k, in k€
Π^{sub}	unit cost for a new substation, in k€/MVA
Π^{loss}	unit cost of losses, in k€/MWh
$R_{ij,k}$	resistance of the line route ij when built with conductor k, in Ω
$X_{ij,k}$	reactance of the line route ij when built with conductor k, in Ω
M	big M constant used in CONSTRAINTS 3m and 3n
$\overline{S}^{sub,max}_i$	maximum allowed substation capacity at node i, with $i \in \mathcal{B}_s$, in MVA
$\overline{V}_i$	maximum voltage magnitude allowed at bus i, with $i \in \mathcal{B}$, in kV
$\underline{V}_i$	minimum voltage magnitude allowed at bus i, with $i \in \mathcal{B}$, in kV
$\overline{I}_{ij,k}$	maximum current magnitude allowed in line ij with conductor k, in kA

Variables:

C^{cond}	total DNO conductor investement costs of, in k€
C^{sub}	total DNO substation investement costs of, in k€
C^{loss}_t	total DNO loss costs at time period t, in k€
Φ^{I}_t	amount of overloaded lines at time t
$\lambda_{ij,k}$	binary variable equal to 1 when the conductor k is selected for line ij, and 0 otherwise
Λ_{ij}	binary variable equal to 1 when the line route ij is built, and 0 otherwise
$\overline{S}^{sub}_i$	substation power capacity at bus i, with $i \in \mathcal{B}_s$, in MVA
S^{sub}_i	apparent power supplied by the substation at bus i, with $i \in \mathcal{B}_s$, in MVA
$P^{sub}_{i,t}$	active power supplied by substation i at time t, with $i \in \mathcal{B}_s$, in MW
$Q^{sub}_{i,t}$	reactive power supplied by substation i at time t, with $i \in \mathcal{B}_s$, in MVar
β_i	binary variable equal to 1 if the substation at bus i is built, and 0 otherwise, with $i \in \mathcal{B}_s$
$I^2_{ij,k,t}$	square magnitude of the current in line ij at time period t when the conductor k is selected, in $(\text{kA})^2$
$I^2_{ij,t}$	square magnitude of the current in line ij at time period t, in $(\text{kA})^2$
$I^{lim}_{ij,k,t}$	binary variable equal to 1 when the current in line ij at time period t when the conductor k is selected violates the current operational limits, and 0 otherwise.
$h_{ij,k,t}$	slack variable of current CONSTRAINT 3aa
$P_{ij,k,t}$	active power flow at sending bus i in line ij at time period t when conductor k is selected, in MW
$P_{ij,t}$	active power flow at sending bus i in line ij at time period t, in MW
$Q_{ij,k,t}$	reactive power flow at sending bus i in line ij at time period t when conductor k is selected, in MVar
$Q_{ij,t}$	reactive power flow at sending bus i in line ij at time period t, in MVar
$V^2_{i,t}$	square magnitude of the voltage at bus i, at time period t, with $i \in \mathcal{B}$, in $(\text{kV})^2$