Achromatization of solar concentrator thanks to diffractive optics

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Abstract
Refractive solar concentrators suffer from important chromatic aberration. To enhance solar cell performance and increase the concentrating ratio, we propose an hybrid (diffractive/refractive) lens which allows a more uniform flux density and an achromatized (in the sense of achromatic doublet) illumination of the cell. To solve the problem of high diffraction efficiency, we turn to multilayer diffractive lenses to have broadband high diffraction efficiency optimized on solar spectrum. Rigorous coupled wave analysis simulations were performed in order to check the validity of scalar theory, they led to a new order of magnitude for the ratio period/wavelength which has been found to be around 100 in place of 10.

Keywords: solar concentrator, diffraction efficiency, achromatization, hybrid lens, RCWA

1. Introduction
Photovoltaic (PV) energy suffers from payback time. High solar concentration (above 500 suns) is a very interesting way to reduce cost production, and the small size of the solar cell allows the use of high efficiency material. The concentration of solar beams could be performed either by reflection (mirror) or by transmission (lens). Currently, mirrors are usually used for solar thermal applications while lenses are used in PV. The most important reason why lenses are more suitable for PV is their ability to uniformize the flux on cell. In addition, lenses are less sensitive to manufacturing errors and such errors are reduced by the refraction while they are doubled in the case of reflection. Moreover, plastic lenses can be easily duplicated. Unfortunately, refractive index of optical materials are highly dependent on the wavelength, which causes important chromatic aberrations. Those aberrations restrict the size of the cell and negatively affect the efficiency of the cell. To solve those problems, one can think about achromatic refractive doublets but they are thick, heavy and require exotic glasses which make them expensive.

The solution presented in this paper consists in hybrid lens composed of a refractive Fresnel lens and a diffractive lens optimized to achieve a broadband high diffraction efficiency.

2. Chromatic aberration and consequences on the efficiency of MJ solar cell
Most common solar concentrating systems using lenses as primary optics operate in the range of 500 to 1000 suns (usually thanks to a secondary optics system). In HCPV (High concentrating for photovoltaic), the cost of the cell is drastically reduced so that multijunction (MJ) cells may be used to increase the efficiency of the module. Moreover the efficiency is enhanced by the factor of concentration. Indeed, the higher the concentration level the higher the gradient of the chemical potential and thus the higher the current, and also the open-circuit voltage. This is well illustrated by Nishioka et al. [Nis06]. So, why don't common concentrating systems operate at higher suns to enhance efficiency and to reduce cell cost? As explained by Feuermann et al. [Feu01] the restriction resides in aberrations, especially the chromatic aberration, leading to a restriction of the collected solar spectrum and the concentration ratio.

Considering the solar spectrum from 350 to 1800 nm, the refraction index $n(\lambda)$ of PMMA (polymethylmethacrylate) – a typical optical plastic used for Fresnel lenses – varies from 1.517 to 1.480 (see Fig. 1).
Let's consider a lens and two wavelengths $\lambda_1$ and $\lambda_2$. The longitudinal chromatic aberration corresponds to the difference of the focal distances $f(\lambda_1)$ and $f(\lambda_2)$. For an ideal (with only chromatic aberration) converging thin lens, the focal distance is given by

$$f(\lambda) = \frac{-\rho}{n(\lambda) - 1} \quad \text{with} \quad \rho = \left(\frac{1}{R_1} - \frac{1}{R_2}\right) > 0 \quad (1)$$

with $R_1$ and $R_2$, the radii of curvature of the lens. Hence, the difference of focal lengths is

$$f(\lambda_2) - f(\lambda_1) = f(\lambda_2)\frac{n(\lambda_1) - n(\lambda_2)}{n(\lambda_1) - 1} = f(\lambda_1)\frac{n(\lambda_1) - n(\lambda_2)}{n(\lambda_1) - 1} \quad (2)$$

The longitudinal chromatic aberration is thus 7.7% $f(350\text{ nm})$ or 7.2% $f(1800\text{ nm})$ for PMMA.

But rather than the chromatic aberration, what matters is the optical efficiency. Given a lens of diameter $d_l$ and a PV cell of diameter $d_c$ at a distance $D$ from the lens. Let's consider a wavelength $\lambda_0$ such that $f(\lambda_0) = D$.

The optical efficiency $\eta_o$ corresponds to the ratio of the flux reaching the cell divided by the incoming flux. It can be calculated using the geometry presented in Fig. 2.

$$\eta_o(\lambda) = \frac{T(\lambda, r) \frac{tg(\beta)}{tg(\alpha)}}{T(\lambda, r) \frac{d_c}{d_l} \frac{f(\lambda)}{\left| f(\lambda_0) - f(\lambda) \right|}} \quad (3)$$

$T(\lambda, r)$ is the transmission factor of the lens, depending on the wavelength and the point of impact (because of dead zones, absorption...)

It can then easily be shown that the optical efficiency $\eta_o$ is independent of the focal distance but depends only on the geometrical concentration and the dispersion of the refractive index

$$\eta_o(\lambda) = \frac{T(\lambda, r) \frac{d_c}{d_l} \left| \frac{1 - n(\lambda_0)}{n(\lambda_0) - n(\lambda)} \right|}{\left| f(\lambda_0) - f(\lambda) \right|} \quad (4)$$

Except the transmission, the only limiting factor for the concentration is thus the dispersion curve. In imaging lens designs, the simplest system used to reduce the chromatic aberrations is the “doublet”, called the “achromatic doublet” although it is still sensitive to the wavelength. Considering two lenses of focal distances $f_1$ and $f_2$ with Abbe numbers $v_1$ and $v_2$, the longitudinal achromatization condition is

$$f_1v_1 + f_2v_2 = 0 \quad (5)$$

and the effective focal length is given by

$$\frac{1}{f_1} = \frac{1}{f_2} - \frac{d}{f_1f_2} \quad (6)$$

where $d$ is the distance separating the two lenses.

Because the Abbe number is always positive in refractive optics, to satisfy the equation 5 one needs to combine a converging and a diverging lens made from very different materials (a flint one $v<50$ and a crown one $v>50$). Unfortunately, this technique cannot be applied to solar concentration for several reasons:

- to be cost-effective, glass materials cannot be used and since almost all optical plastic are flint [Kas07] the converging lens should have been of very high power (small radius of curvature);
- because of the need of a diverging lens, small $f$-number cannot be achieved;
- the thickness of a doublet is almost twice the one of a singlet which implies to use two materials and thus a heavier and more expensive system;
- a doublet made by combining a converging refractive Fresnel lens with a diverging refractive Fresnel lens will suffer from important shadowing.

However, the achromatization of the flux reaching the cell is of a very high importance. Indeed, if the total illumination of the cell is not uniform it would be similar to connecting $pn$-junctions in parallel leading to a loss in the open-circuit voltage [Dha81] (which is also similar to increasing the “effective series resistance of the cell” [Jaq07]) and thus a drop of efficiency. And even if the total illumination is uniform, the illumination spectrum has to be uniform too in order to ensure the current matching of each diode.
The solution we propose here to achromatize Fresnel lens without refractive doublet, consists in an hybrid doublet: combining refractive and diffractive optics (see Fig. 3).

3. Ideal diffractive lens

Over the years, several diffractive lenses have been invented, like the Fresnel zone plate, the phase Fresnel zone plate and sinusoidal Fresnel zone plate. Each of these lenses has several foci of low efficiency [Mor97]. In 1969, Lesem et al. [Les69] introduced a new way to reconstruct wavefront leading to a diffractive lens able to have 100% efficiency in one focus.

Let’s consider a wavelength $\lambda_0$. The ideal diffractive lens is a surface-relief structure – often called *kinoform* – see Fig. 4) obtained thanks to Fermat’s principle.

The formulation of the ideal profile is described in several publications (e.g. [Bur89] and [Mor97]), each zone is designed to create a constant optical path length to the focal point $F$. The difference between two neighbouring zones is a $2\pi$-phase shift in order to create a continuity of the transmitted phase. To achieve this phase shifting, the depth ($h$) of the lens profile is given by

$$h = \frac{\lambda_0}{n(\lambda_0) - n_{\infty}} = \frac{\lambda_0}{n(\lambda_0) - 1}. \tag{7}$$

Respecting this condition, all the transmitted light will be diffracted in the 1$^{\text{st}}$ order of diffraction. In the case of PMMA, the ideal thickness of the lens for $\lambda_0=550$ nm will be about 1.1 $\mu$m.

If the system is now illuminated with a wavelength $\lambda$ other than the design wavelength $\lambda_0$, a modification of the focal length will occur following equation 8.

The corresponding Abbe number is given by [Dav93]

$$v_{\text{diff}} = \frac{\lambda_2}{\lambda_1 - \lambda_3} < 0, \quad \text{with } \lambda_1 > \lambda_2 > \lambda_3. \tag{9}$$

Four important properties have to be noticed:

- the Abbe number depends only on the wavelength;
- the same remark could be made for the focal length which depends only on the wavelength once the profile is designed but does not depend on the refraction index;
- if the wavelength increases, the focal length will decrease (the consequence is a negative Abbe number) contrary to refractive lenses;
- refractive Abbe number $v_{\text{ref}}$ and diffractive Abbe number $v_{\text{diff}}$ are strongly different as evidenced by the following equations in the case of the standard Abbe number $v_2$:

$$v_{\text{diff}} = \frac{d}{F - C} = -3.45 \quad \text{and} \quad v_{\text{ref}} = \frac{n(d) - 1}{n(F) - n(C)} \approx 50 \tag{10}$$

where $F, d$ and $C$ correspond to the wavelengths 486.13 nm, 587.56 nm and 656.27 nm respectively.

The condition of achromatization (equation 5) could be then achieved using two converging lenses which allows a small $f$-number. Moreover, since the Abbe number does not depend on the material (no dependency on the refraction index) an hybrid achromat could be manufactured in a single material.

The next figure compares the evolution of the focal length of a single refractive lens and a hybrid lens both designed to have the same focal distance at 600 nm.

Even if the difference of focal length may seem small, it has an important effect on the optical efficiency as shown in Fig. 6.
Unfortunately, if the system is illuminated with another wavelength than \( \lambda_0 \), a discontinuity of the transmitted phase will appear, leading to spurious orders of diffraction. The diffraction efficiency at the \( k \)th \( \ell \) order is given by

\[
\eta_k = \left( \frac{\sin \alpha}{\alpha} \right)^2 \frac{\sin^2 \alpha}{\alpha^2} , \tag{11}
\]

This function has been plotted in Fig. 7.

In diffractive lenses \( \alpha \) depends on the wavelength:

\[
\alpha(\lambda) = \frac{n_{\text{DOE}}(\lambda) - n_{\text{air}}}{\lambda} h = \frac{n_{\text{DOE}}(\lambda) - 1}{\lambda} h \frac{\lambda_0}{n_{\text{DOE}}(\lambda_0) - 1} \tag{12}
\]

The diffraction efficiency of a diffractive lens made of PMMA have been plotted in Fig. 8 for \( k=0,1 \) and 2. As can be observed in this figure, at the first order, the diffraction efficiency quickly drops away from the design wavelength. This is why, up to now, such a system has not been used in solar concentration.

4. Multilayer diffractive lens

4.1. Efficiency achromatized DOEs

Achromatization is generally understood as the diminution of longitudinal chromatic aberration. This section is not dedicated to the achromatization of focal length but to the achromatization of the efficiency, this is why we speak about “efficiency achromatized” DOEs (diffractive optical elements) in order to avoid the confusion with the common meaning.

Following equation 11, the key to achromatizing the efficiency is keeping the parameter \( \alpha \) as close as possible to the desired diffraction order \( k \). First improvements have been obtained by filling the diffractive structure with an optical material (thus replacing a constant refractive index of 1.0 by a refractive index depending on the wavelength) [Ebs96] [Ima98]. This structure has a parameter \( \alpha \) given by

\[
\alpha(\lambda) = \frac{n_1(\lambda) - n_2(\lambda)}{\lambda} h \tag{13}
\]

If \( n_1(\lambda) - n_2(\lambda) \) increases with lambda then the drop of efficiency will be lower than without the second optical element.

Further improvements have been made by juxtaposing materials of different thicknesses \( (h_i) \). In this way \( \alpha \) has more flexibility to reduce the wavelength dependency:

\[
\alpha(\lambda) = \sum_i \frac{n_i(\lambda) - 1}{\lambda} h_i \tag{14}
\]

With two different materials one can achieve a diffraction efficiency almost independent of the materials refractive index [Kla08]. Moreover more than two optical elements will increase significantly the thickness of the layers leading to
manufacturing difficulties. Therefore, our studies are limited to two diffractive interfaces. In this context, the flexibility is such that one can achieve 100% diffraction efficiency for two selected wavelengths. Adjusting those two maxima allows to reach high diffraction efficiency over a broadband spectrum as shown by the orange curve on Fig. 9.

A more interesting way in the field of PV, is an optimization based on the solar flux itself (see Fig. 10). Note that ideally one might use the AM1.5d spectrum which was not done here, we used the black body spectrum at 5780K. The integrated transmitted flux could reach about 90% of the solar flux.

It is important to point out that if a ray is diffracted at the first order, it does not imply that it will reach the solar cell: it depends on the size of lens and the cell as discussed in section 2 (see equation 4). But claiming the opposite i.e. “the light not diffracted will not reach the cell” is also false, especially for short wavelengths. Indeed, the 0th order of diffraction does not affect the effective focal length. Fig. 5 shows that short wavelengths converge too quickly at the 1st order of diffraction, the 0th order of diffraction is then more convenient for a certain spectral band. The 2nd order of diffraction makes rays converging faster, which is beneficial to long wavelengths. The optical efficiency of 0th, 1st and 2nd order of diffraction are represented in the following figure.

4.2. Configurations

The function $\eta(\alpha)=\text{sinc}^2(\alpha-k)$ is even around $k$ (see Fig. 7). The consequence of such a parity is a flexibility in the configuration of multilayer DOEs. For example, the following configurations are equivalent to each other (but several others are possible). In section 5.2., some restrictions will be added to this assertion.

5. Limits of scalar theory

In sections 3 and 4, calculations have been performed with a scalar theory. But the width of Fresnel zones could be close to the wavelength and switching to a rigorous theory is essential to see the limit of the scalar theory. Based on Maxwell's equations, rigorous coupled wave analysis (RCWA) is a
powerful numerical tool to deal with diffraction gratings developed by Moharam and Gaylord [Mor81]. Although our software is limited by a periodic structure because of a decomposition of the permittivity in Fourier series, thanks to RCWA we are able to determine which part of energy will be diffracted in each order of diffraction. This allows us to:

– check the validity of scalar theory;
– rigorously study the influence of layer thickness;
– investigate the consequence of an angle of incidence other than zero.

Note that no apodization of the refraction index was used in our simulations, causing an important Higgs phenomenon at the interface between two media.

5.1. Period dependence

In scalar theory, the diffraction efficiency is totally independent of the period of the blazed grating. People usually relies on a publication of Swanson claiming that “The scalar theory is, in general, accurate when the grating period is greater than five wavelengths” [Swa89]. Kallioniemi et al. [Kal00] wrote: “The latter [scalar theory] is valid for structures whose smallest features are on the order of 10 wavelengths or more”. We performed several simulations in order to check the validity of the period size. As can be understood from Fig. 13 is that 10 times the wavelength is a good order of magnitude rather than 5 would have been. Note that the diffraction efficiency at 350 nm is lower than at 550 nm and 1050 nm because of Fresnel reflections given that the refractive index is higher.

But in the case of multilayers we observe that about a period of 100 times the wavelength is required. The diffraction efficiency depends not only on the period but also on the thickness of the layers (see Fig. 14).

![Fig. 13](image1.png)

Fig. 13: Period dependence of the diffraction efficiency in the case of monolayers of PMMA. Each simulation was performed for the design wavelength ( $\lambda = \lambda_0$ ). Circles indicate the efficiency for a period equal to 10 wavelengths.

![Fig. 14](image2.png)

Fig. 14: Diffraction efficiency for a bilayer DOE composed of PMMA (h=17.22 µm) and PC (h=13.43 µm).

Depending on the f-number of the lens, the width of the last zones could be smaller than 50 µm. One could think about using the second order of diffraction: with a phase shift of $4\pi$ radians between two neighbouring zones, and thus the thickness of the DOEs is twice the thickness for the first order of diffraction. But in this case, high diffraction occurs in a narrower band (see Fig. 15).

![Fig. 15](image3.png)

Fig. 15: Comparison between 1st and 2nd order of diffraction for the scalar and RCWA cases.

The same phenomenon occurs even if multilayer is used (see Fig. 16).
5.2. Thickness between layers

In section 4.2., it was said that the efficiency is independent of the configuration. If layers are not stuck together this is obviously false because of Fresnel reflection but it is easy to take heed of those reflections decomposing the incident light in TM and TE modes. Nevertheless, the bigger the distance between layers, the more the system behaves like two diffractive lenses instead of one bilayer lens.

A geometrical reason can be invoked to explain the drop of efficiency. A ray entering the first element will be deviated from its incident direction and in spite of reaching the second element at a point \( x \), it will strike at a point \( x' \) where it will suffer a phase shift other than expected. This phenomenon is even worse for the rays at the edges of the structure since they might not hit the right second element but the neighbouring one. Moreover, increasing the separating distance will also increase the shadow effect.

5.3. Angle of incidence

Since the very beginning, only normal incidence has been considered. Eq. 11 is independent of the angle of incidence. Although several extended scalar theories have been proposed (e.g. [Lev04] and [San06]), they all have their limitations and – as far as we know – were never tested for multilayer DOEs. The graph below shows that the angle of incidence could have a significant impact, especially for thick structures. But the effect is inferior to 1% in the sun angular size (~0.5°) range.

6. Linking blazed gratings and diffractive lens

A diffractive lens could be understood by two ways. The first one has already been presented: the Fresnel zones where each zone is designed such that the optical path to the focal point is the same for every beam of the zone.

The second way to “build” a diffractive lens is juxtaposing an element of a blazed grating (i.e. a sawtooth) of thickness determined by equation 7 and with a width \( \Lambda \) verifying the gratings formula:

\[
\Lambda \sin \theta_k = k \lambda
\]

such that \( \theta_k \) – the angle of diffraction at the \( k \)th order – corresponds to the angle made with the desired focal point.

Those two ways of conceiving a diffractive lens are almost the same. In general the really first zones are made curved, but the other ones are straight.

7. Conclusions

Hybrid multilayer lenses seem to be a promising solution to achromatize flux on solar cell and to increase the geometrical concentration. The main issue is the diffraction efficiency which is too low for an order of diffraction other than 1.
been shown that the scalar theory is not in good agreement for multilayer diffractive elements when the period is shorter than 100 times the wavelength. It has also been shown that the thickness separating the layers has to be as small as possible.

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9. References


