


L. Wu, J. Segurado, M. Mustafa, & L. Noels

University of Liege, Computational & Multiscale Mechanics of Materials

Universidad Politécnica de Madrid, Department of Materials Science &

IMDEA Materials Institute

ECCOMAS, 3-7 June 2024, Lisboa, Portugal



This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 862015



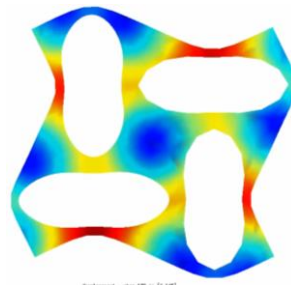
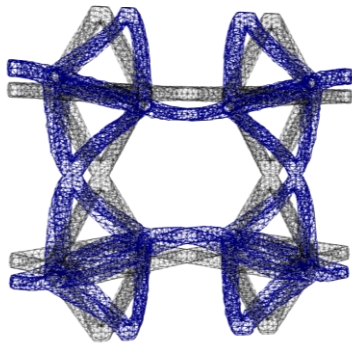
MOAMMM

www.moammm.eu

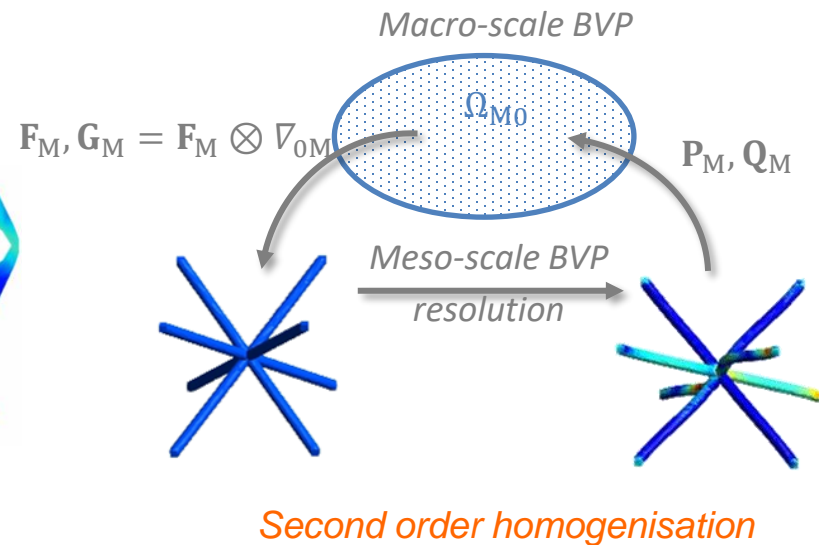
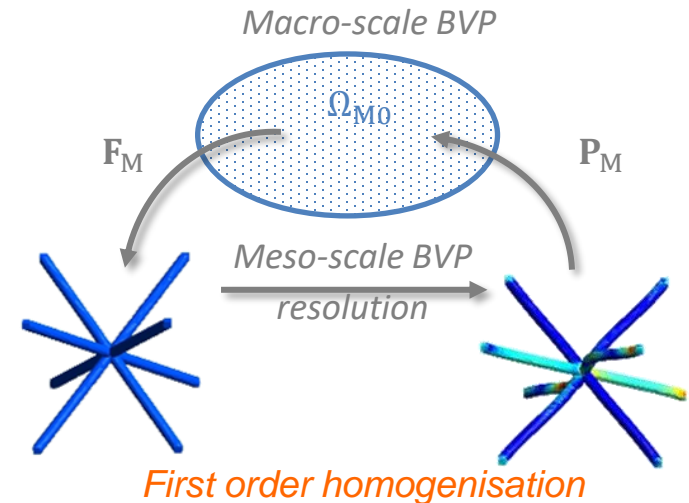
Second order homogenisation for cellular and metamaterials

- First vs. second order homogenisation

- First order homogenisation
 - Does not prevent spurious localisation
 - No material length-scale
- Second-order homogenisation
 - High order strain \mathbf{G}_M and stress \mathbf{Q}_M at macro-scale
 - Material length scale related to the RVE length



- Issue for metamaterial: RVE length is larger than unit cell because of patterning change



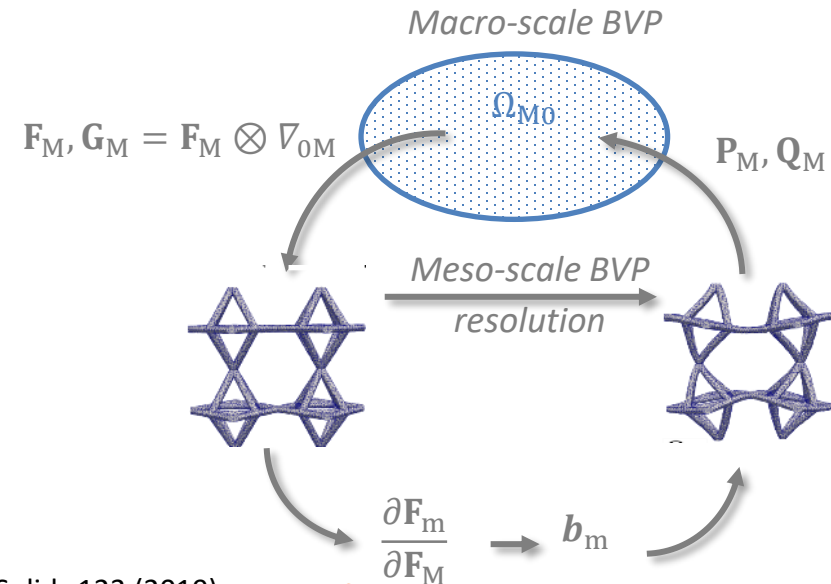
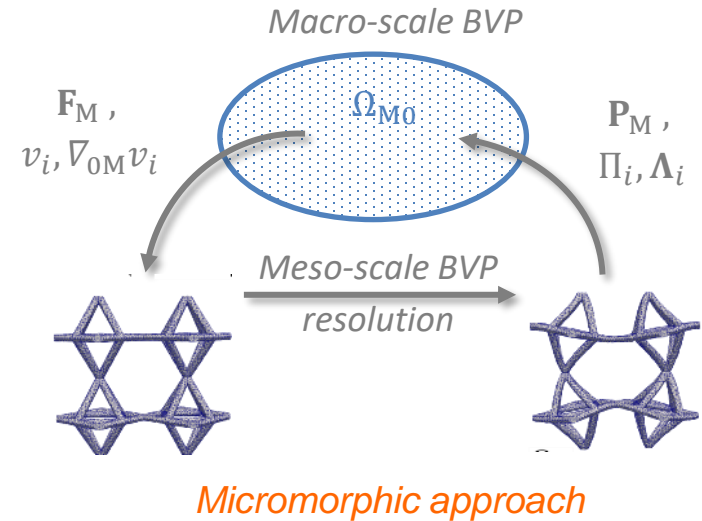
Second order homogenisation for cellular and metamaterials

- Account for patterning change

- Micromorphic approach*
 - Constrains change of patterning modes
 - Developed in elasticity

- Enhanced second-order homogenisation

- Remove cell size dependency using a body-force
- Arises from asymptotic homogenization in linear elasticity**
- How to account for finite strain, elasto-plasticity etc...?



*O. Rokoš, M. Ameen, R. Peerlings, M. Geers, J. Mech. Phys. Solids 123 (2019)

**V. Monchiet, N. Auffray, J. Yvonnet, Mech. Mater. 143 (2020)

J. Yvonnet, N. Auffray, V. Monchiet, Int. J. Solids Struct. 191–192 (2020)



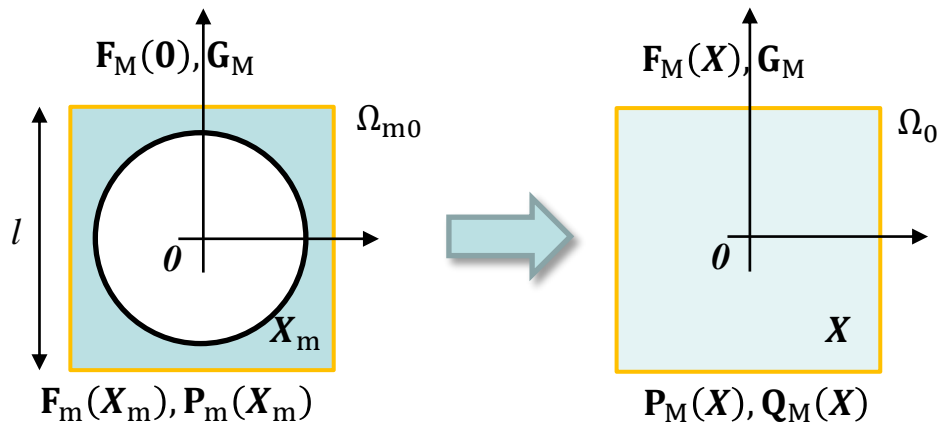
Second order homogenisation for cellular and metamaterials

- Second order homogenisation with body force enhancement

- Consider an equivalent homogeneous volume element

- Cauchy homogenous

- Second order continuum



- Development of the (no-longer) homogeneous field

$$\left\{ \begin{array}{l} \mathbf{F}_M(\mathbf{X}) = \mathbf{F}_M(\mathbf{0}) + \mathbf{G}_M \cdot \mathbf{X} \\ \mathbf{G}_M = \mathbf{F}_M(\mathbf{0}) \otimes \nabla_{\mathbf{0}M} \end{array} \right. \quad \left\{ \begin{array}{l} \mathbf{P}_M(\mathbf{X}) = \mathbf{P}_M(\mathbf{0}) + \left. \frac{\partial \mathbf{P}_M}{\partial \mathbf{F}_M} \right|_0 : \mathbf{G}_M \cdot \mathbf{X} \\ \mathbf{Q}_M(\mathbf{X}) = \mathbf{Q}_M(\mathbf{0}) + \left. \frac{\partial \mathbf{Q}_M}{\partial \mathbf{F}_M} \right|_0 : \mathbf{G}_M \cdot \mathbf{X} \end{array} \right.$$

$$\mathbf{B}_M(\mathbf{X})_M + \left. \frac{\partial \mathbf{P}_M}{\partial \mathbf{F}_M} \right|_0 : \mathbf{G}_M : \mathbf{I} = \mathbf{0}$$

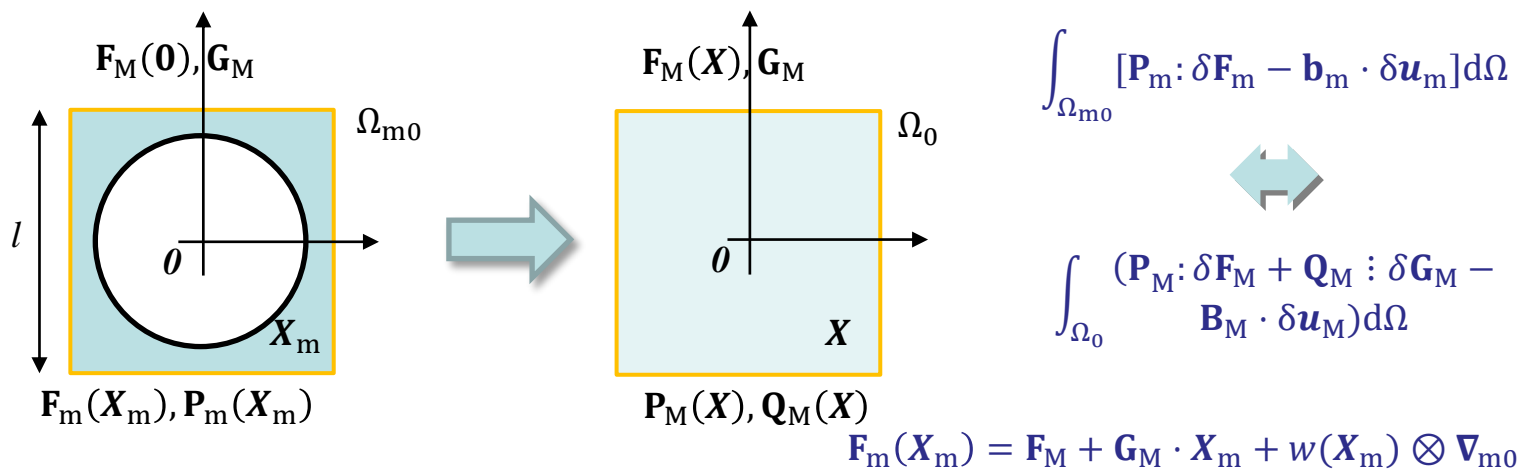


Second order homogenisation for cellular and metamaterials

- Second order homogenisation with body force enhancement

- Consider an equivalent homogeneous volume element

- The equivalence of energy (Hill-Mandel condition) with introduction of body forces $\mathbf{b}_m(\mathbf{X}_m)$:

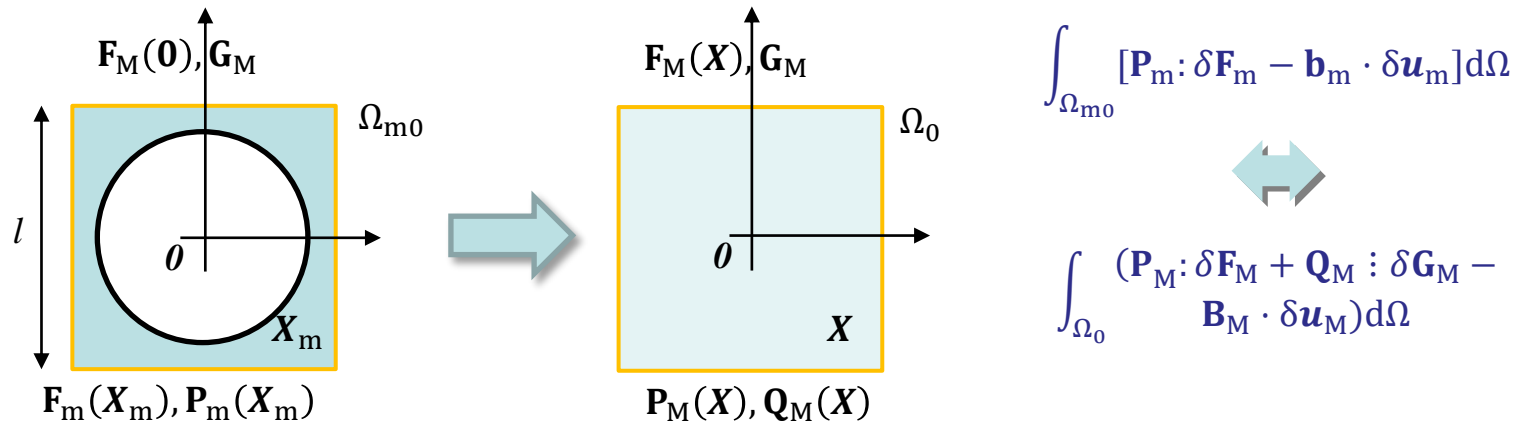


Second order homogenisation for cellular and metamaterials

- Second order homogenisation with body force enhancement

- Consider an equivalent homogeneous volume element

- The equivalence of energy (Hill-Mandel condition) with introduction of body forces $\mathbf{b}_m(\mathbf{X}_m)$:



$$\mathbf{F}_m(\mathbf{X}_m) = \mathbf{F}_M + \mathbf{G}_M \cdot \mathbf{X}_m + w(\mathbf{X}_m) \otimes \nabla_{m0}$$

- Is satisfied by the following introduction of micro-scale body forces and homogenised stresses

$$\left\{ \begin{aligned} \mathbf{P}_M &= \mathbf{P}_M(0) = \frac{1}{V_0} \int_{\Omega_{m0}} (\mathbf{P}_m - \mathbf{b}_m \otimes \mathbf{X}_m) d\Omega \\ \mathbf{Q}_M &= \mathbf{Q}_M(0) = \frac{1}{2V_0} \int_{\Omega_{m0}} [\mathbf{P}_m \otimes \mathbf{X}_m + (\mathbf{P}_m \otimes \mathbf{X}_m)^T] d\Omega + \frac{1}{2V_0} \int_{\Omega_{m0}} [\mathbf{b}_m \otimes \mathbf{X}_m \otimes \mathbf{X}_m] d\Omega - \\ &\quad \frac{1}{2V_0} \left(\left[\frac{\partial \mathbf{P}_M(0)}{\partial \mathbf{F}_M} : \mathbf{G}_M \cdot \mathbf{J}_M + \left(\frac{\partial \mathbf{P}_M(0)}{\partial \mathbf{F}_M} : \mathbf{G}_M \cdot \mathbf{J}_M \right)^T \right] - \mathbf{B}_M \otimes \mathbf{J}_M \right) \\ \int_{\Omega_{m0}} \mathbf{b}_m d\Omega &= \int_{\Omega_0} \mathbf{B}_M d\Omega = - \int_{\Omega_0} \frac{\partial \mathbf{P}_M}{\partial \mathbf{F}_M} : \mathbf{G}_M : \mathbf{I} d\Omega = - \int_{\Omega_{m0}} \left(\frac{\partial \mathbf{P}_m}{\partial \mathbf{F}_m} : \frac{\partial \mathbf{F}_m}{\partial \mathbf{F}_M} : \mathbf{G}_M \right) : \mathbf{I} d\Omega \end{aligned} \right.$$



Second order homogenisation for cellular and metamaterials

- Meso-scale problem

- Micro-scale weak form

$$\int_{\Omega_{m0}} \mathbf{P}_m : (\delta \mathbf{w} \otimes \nabla_0) - \mathbf{b}_m \cdot \delta \mathbf{w} d\Omega = 0$$

- Introduction of body forces $\mathbf{b}_m(\mathbf{X}_m)$:

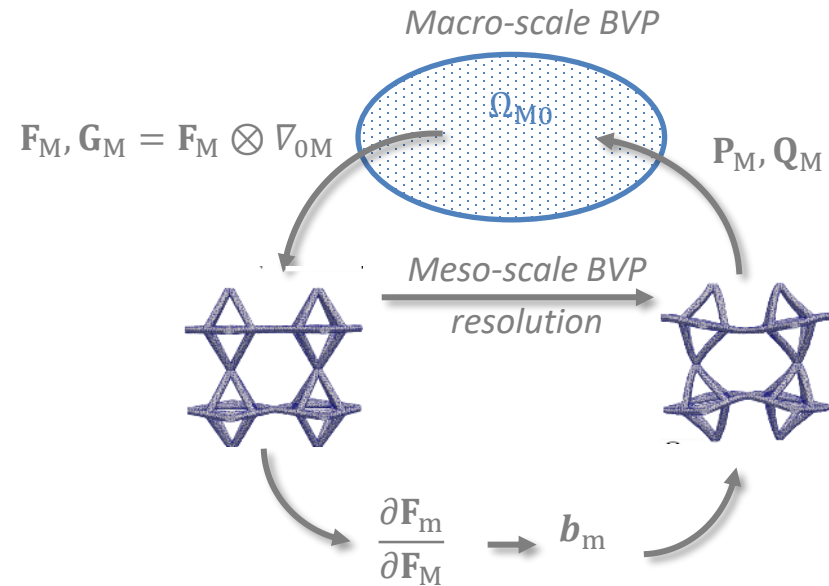
$$\mathbf{b}_m = - \left(\frac{\partial \mathbf{P}_m}{\partial \mathbf{F}_m} \frac{\partial \mathbf{F}_m}{\partial \mathbf{F}_M} : \mathbf{G}_M \right) : \mathbf{I}$$

Applied strain

Instantaneous tangent *Strain concentration tensor*

- Approximation

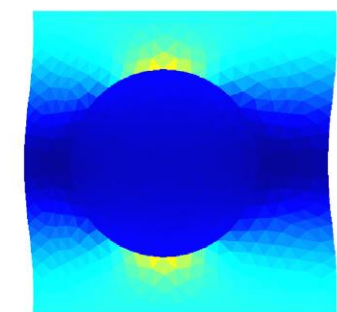
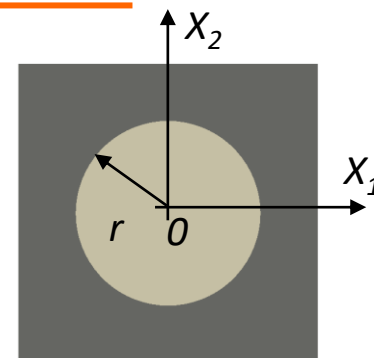
$$\mathbf{b}_m^{n+1} = - \left(\left(\frac{\partial \mathbf{P}_m}{\partial \mathbf{F}_m} \right)^{n+1} : \left(\frac{\partial \mathbf{F}_m}{\partial \mathbf{F}_M} \right)^n : \mathbf{G}_M^{n+1} \right) : \mathbf{I}$$



Second order homogenisation for cellular and metamaterials

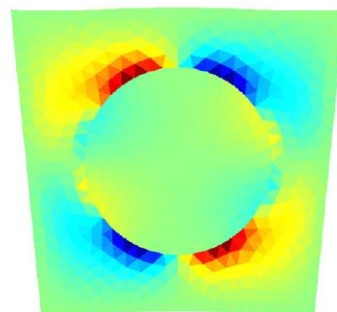
- Remove boundary effect

- Linear elasticity
- With the presented approach, the body forces are not uniform



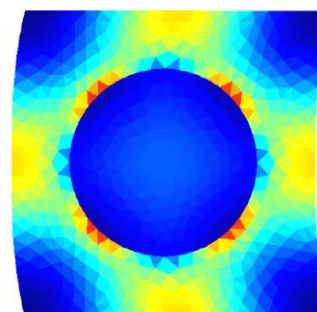
$b_{m_1} [\text{MPa} \cdot \text{mm}^{-1}]$
-2.35 -1.51 0.68

$G_{M_{xxx}} = 0.4 / \text{mm}$



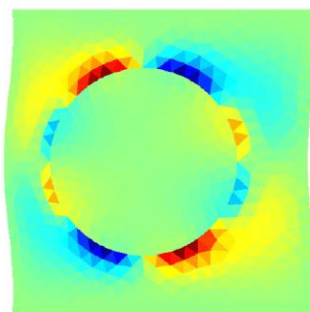
$b_{m_1} [\text{MPa} \cdot \text{mm}^{-1}]$
-0.20 0.0 0.20

$G_{M_{xxy}} = 0.2 / \text{mm}$

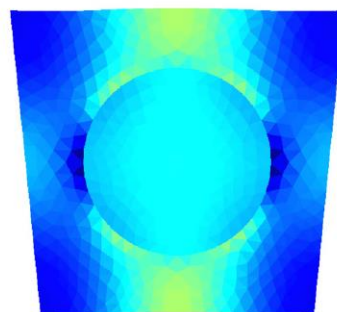


$b_{m_1} [\text{MPa} \cdot \text{mm}^{-1}]$
-0.27 -0.16 -0.04

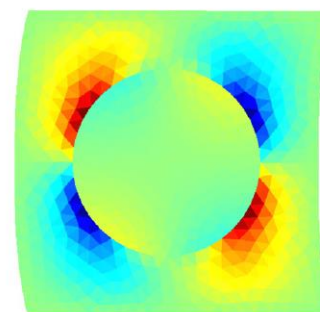
$G_{M_{xyy}} = 0.4 / \text{mm}$



$b_{m_2} [\text{MPa} \cdot \text{mm}^{-1}]$
-0.19 0.0 0.19



$b_{m_2} [\text{MPa} \cdot \text{mm}^{-1}]$
-1.03 -0.68 -0.33



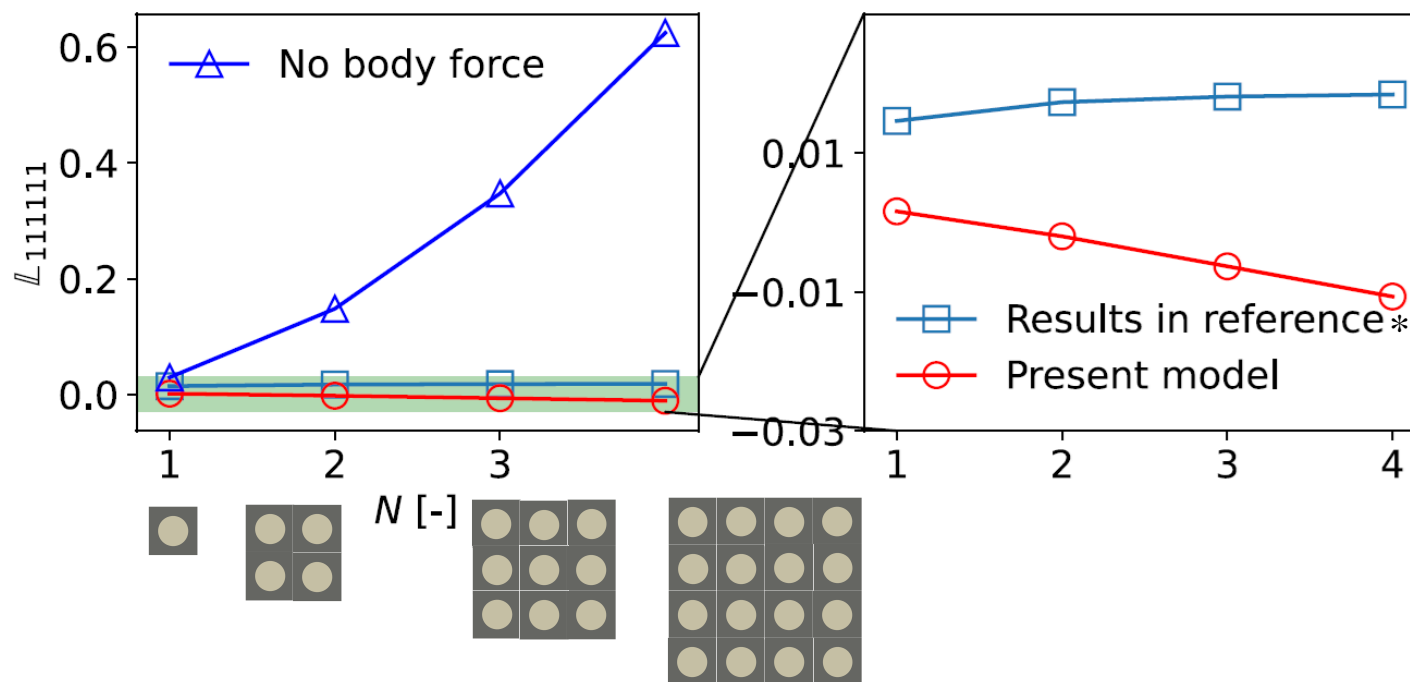
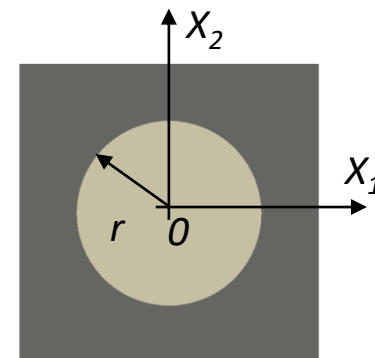
$b_{m_2} [\text{MPa} \cdot \text{mm}^{-1}]$
-0.23 0. 0.23



Second order homogenisation for cellular and metamaterials

- Remove boundary effect

- Linear elasticity
- Comparison with uniform body force*
- Study of the higher-order operator: $\mathbb{L} = \frac{\partial \mathbf{Q}_M}{\partial \mathbf{G}_M}$



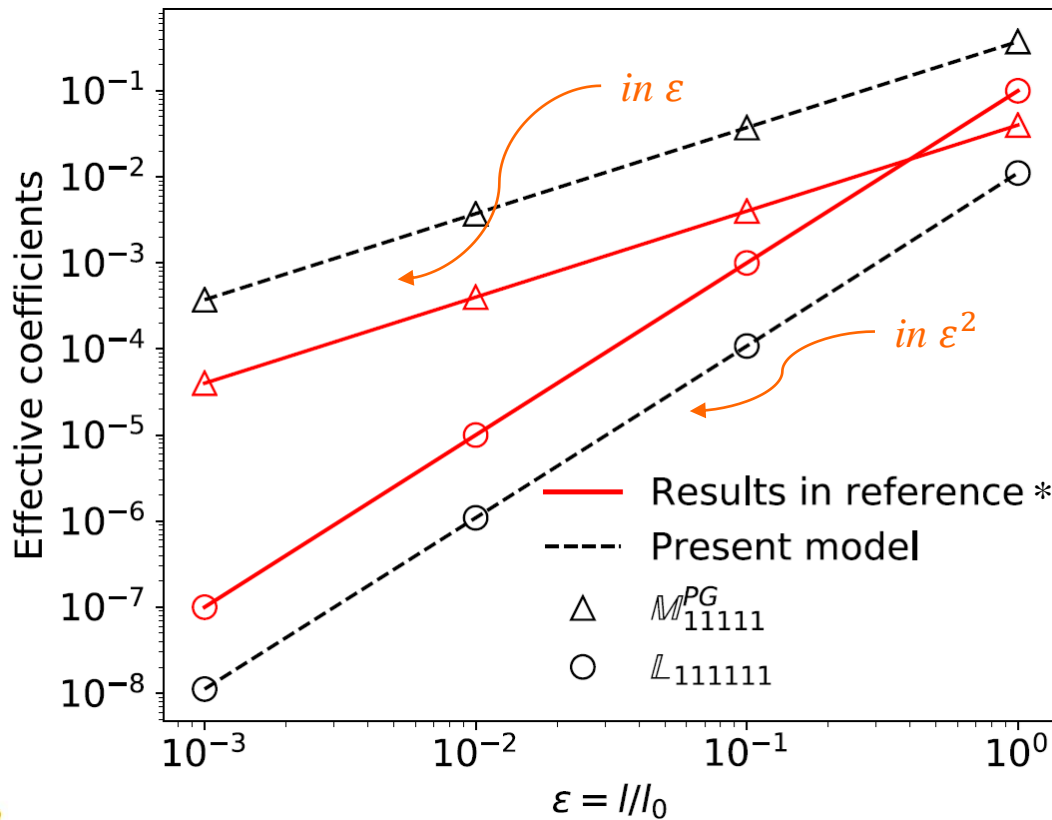
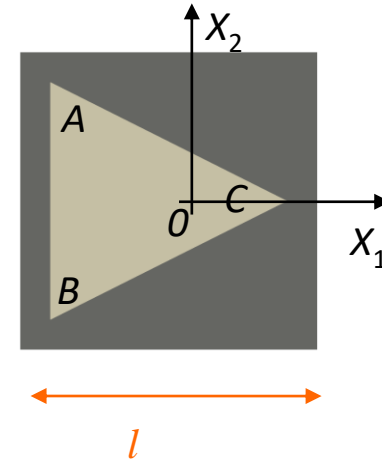
*J. Yvonnet, N. Auffray, V. Monchiet, Int. J. Solids Struct. 191–192 (2020)



Second order homogenisation for cellular and metamaterials

- Remove boundary effect

- Linear elasticity
- RVE with 4×4 unit cells, $l_0=1,0$ mm
- Study of the higher-order operators: $\mathbb{L} = \frac{\partial \mathbf{Q}_M}{\partial \mathbf{G}_M}$, $\mathbb{M}^{PG} = \frac{\partial \mathbf{P}_M}{\partial \mathbf{G}_M}$



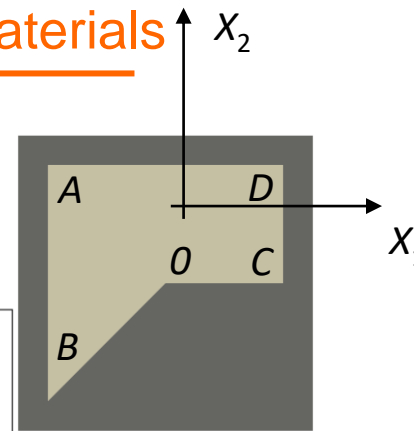
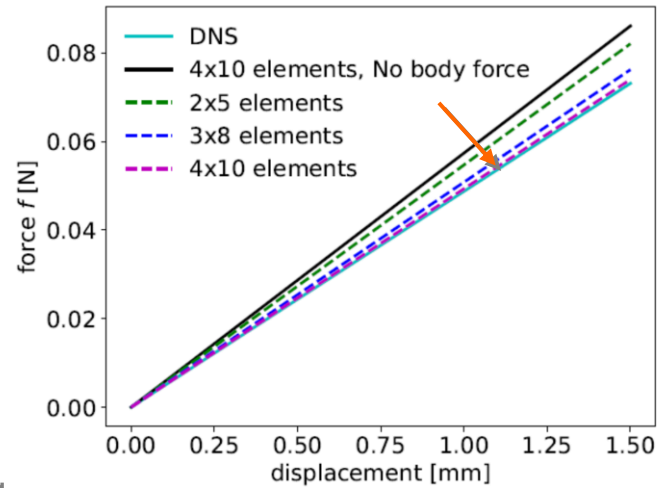
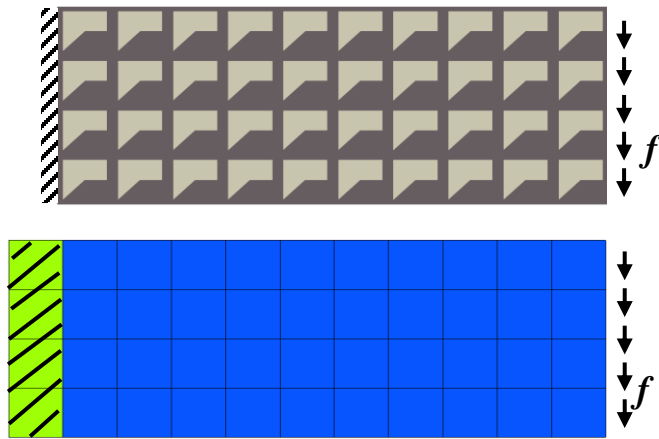
*J. Yvonnet, N. Auffray, V. Monchiet, Int. J. Solids Struct. 191–192 (2020)



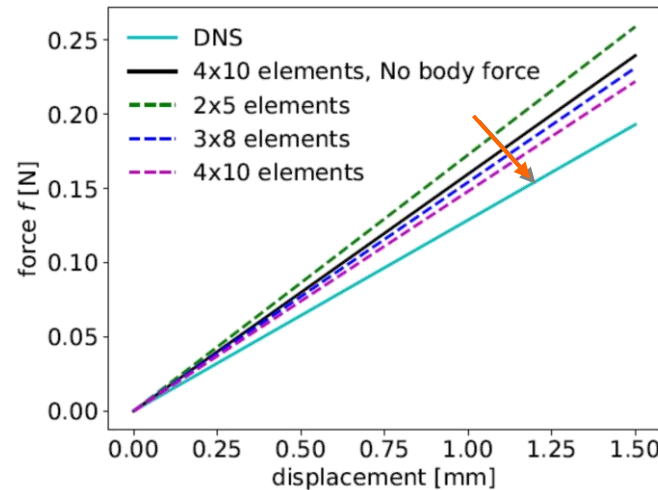
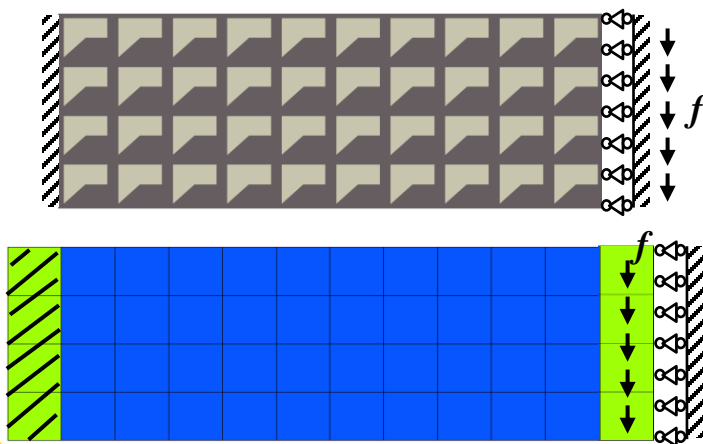
Second order homogenisation for cellular and metamaterials

- Convergence toward DNS ?

- Linear elasticity: Beam bending

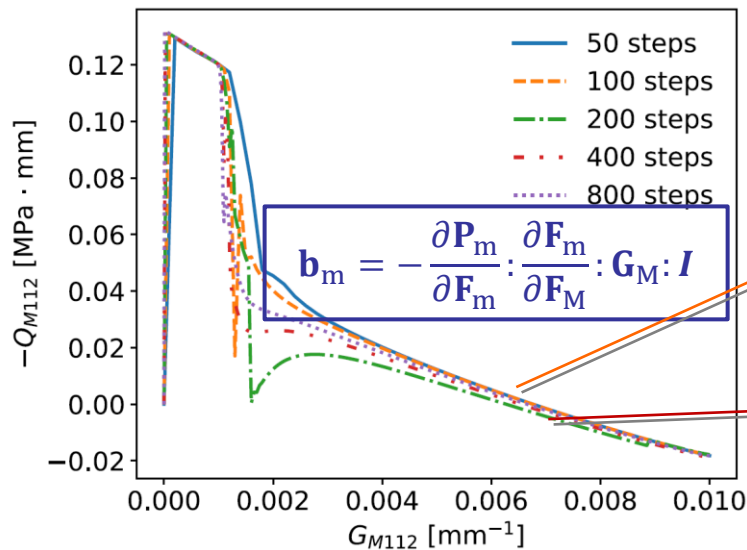


- Linear elasticity: Beam shearing



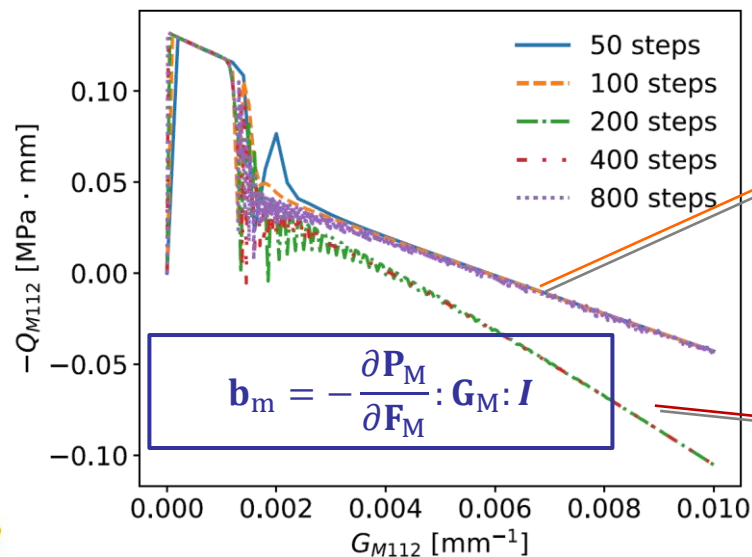
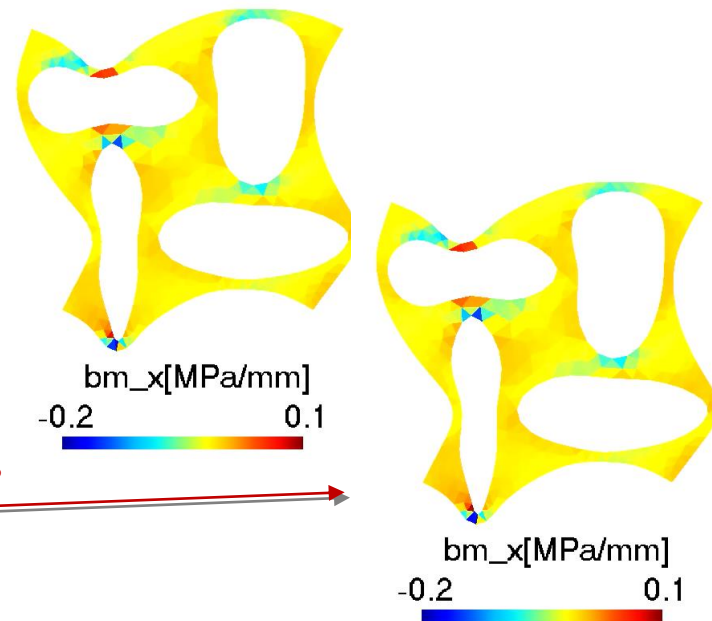
- Importance of body-force distribution

$\mathbf{F}_M = 0.9 \mathbf{I}$, $G_{M_{xxx}} = 0.02 / \text{mm}$, $G_{M_{xyy}} = 0.01 / \text{mm}$



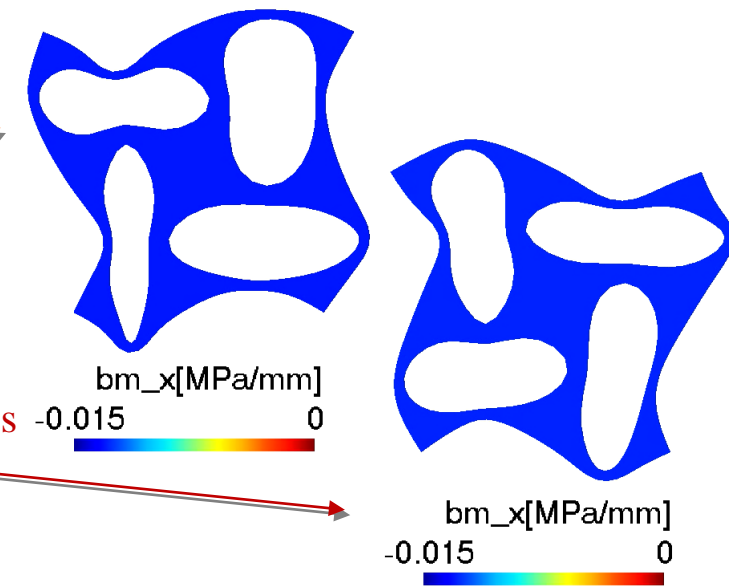
100 steps

400 steps



100 steps

400 steps



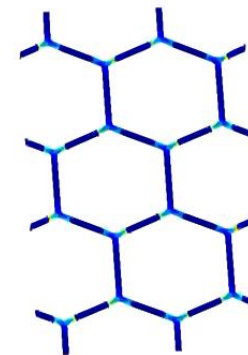
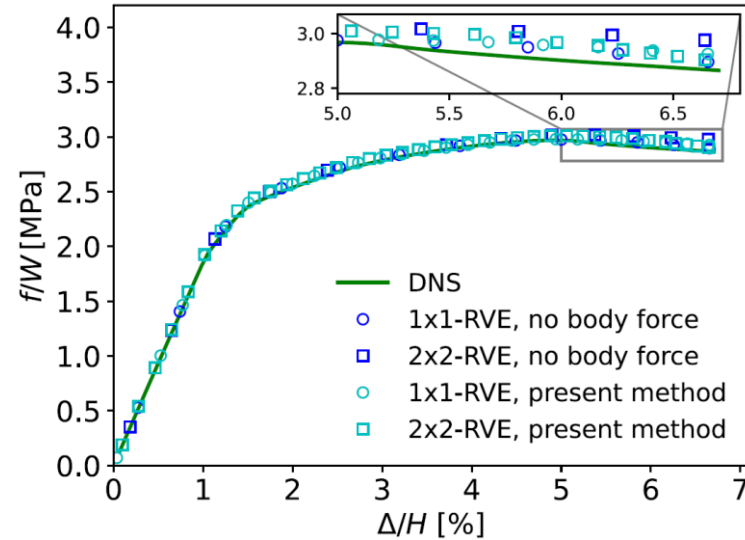
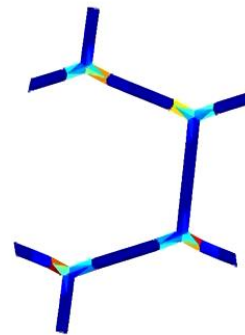
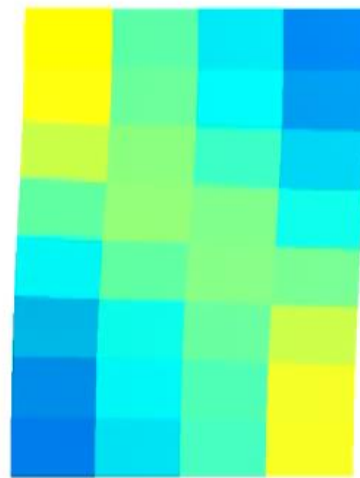
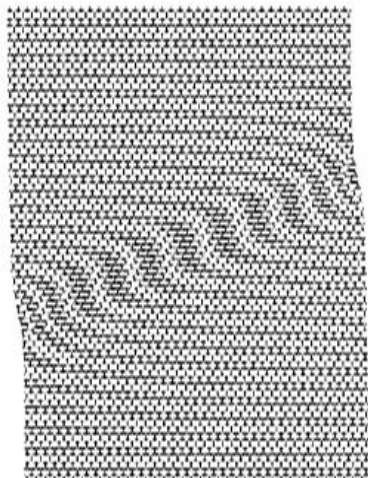
Second order homogenisation for cellular and metamaterials

- Multiscale simulation on honeycomb structures

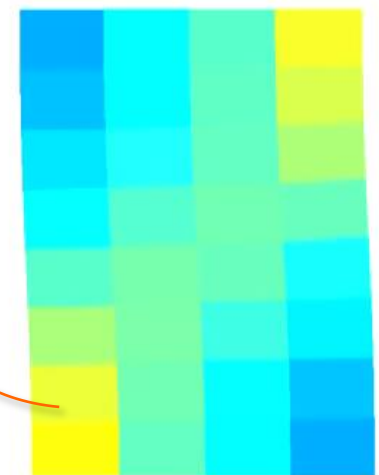
- Effect of RVE size disappears when considering body forces

Direct simulation

Body-force enhanced second-order homogenisation



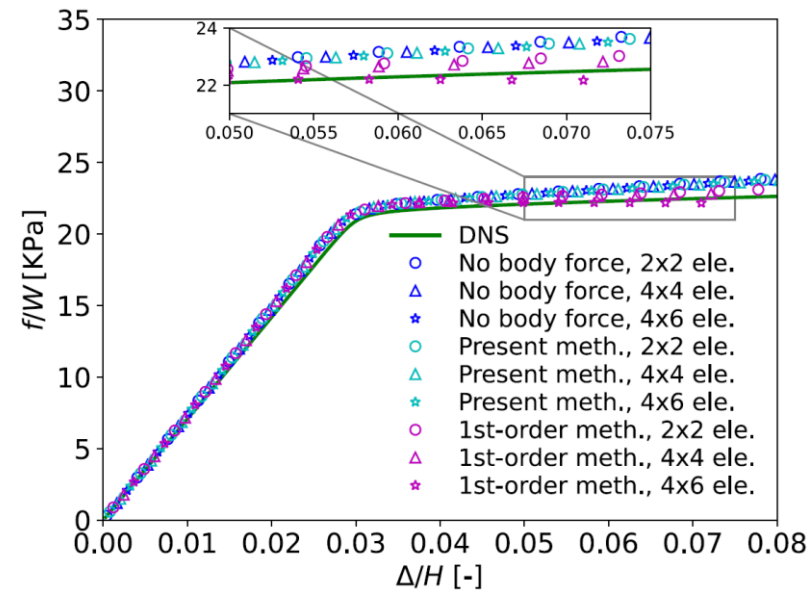
Body-force enhanced second-order homogenisation



Second order homogenisation for cellular and metamaterials

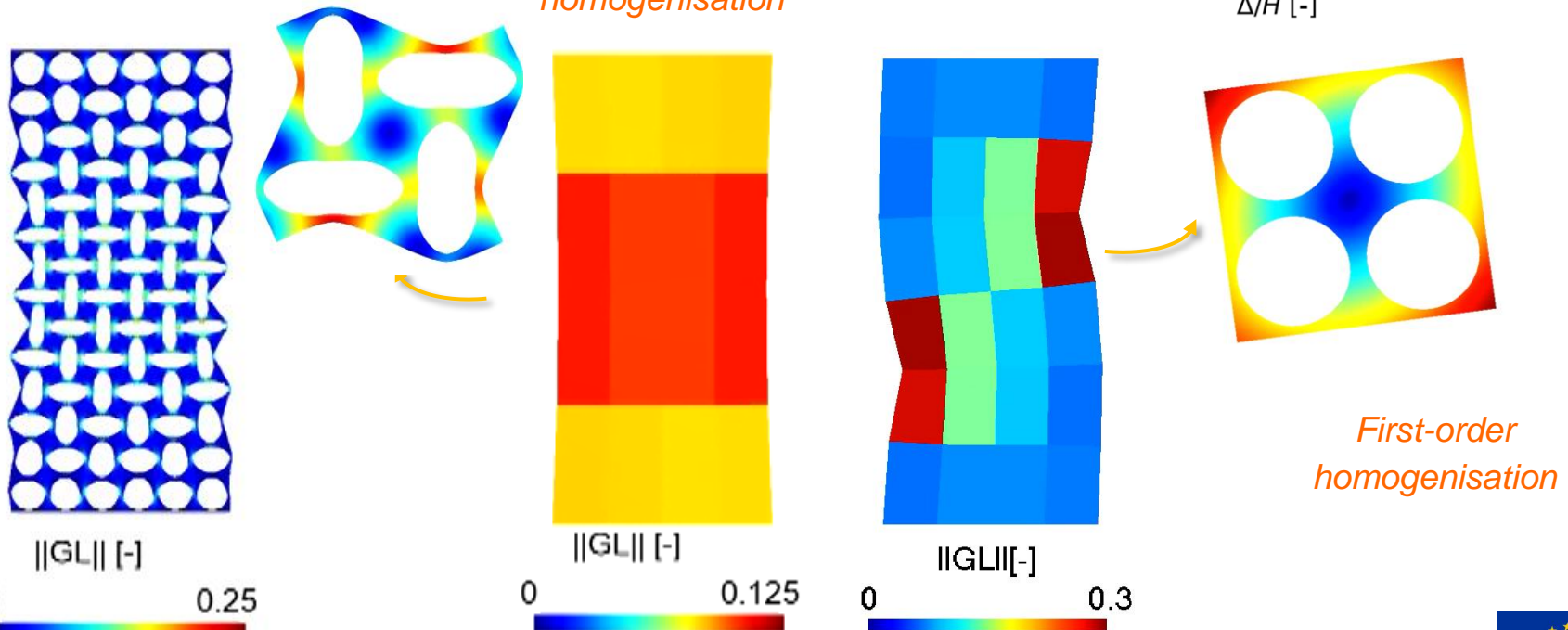
- Multiscale simulation on metastructures

- Local instability under compression
 - 6x14 holes \rightarrow local instability
- Limit of first-order homogenisation



Direct simulation

Body-force enhanced second-order homogenisation



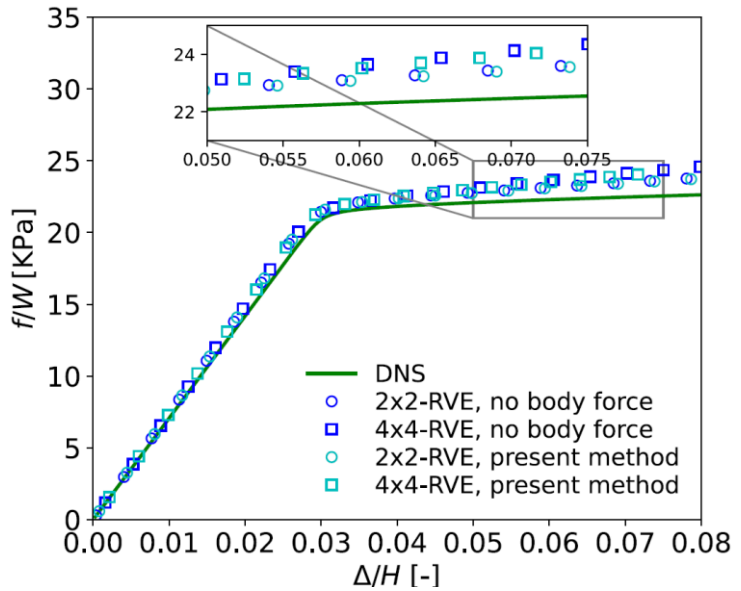
First-order homogenisation



Second order homogenisation for cellular and metamaterials

- Multiscale simulation on metastructures

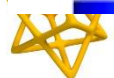
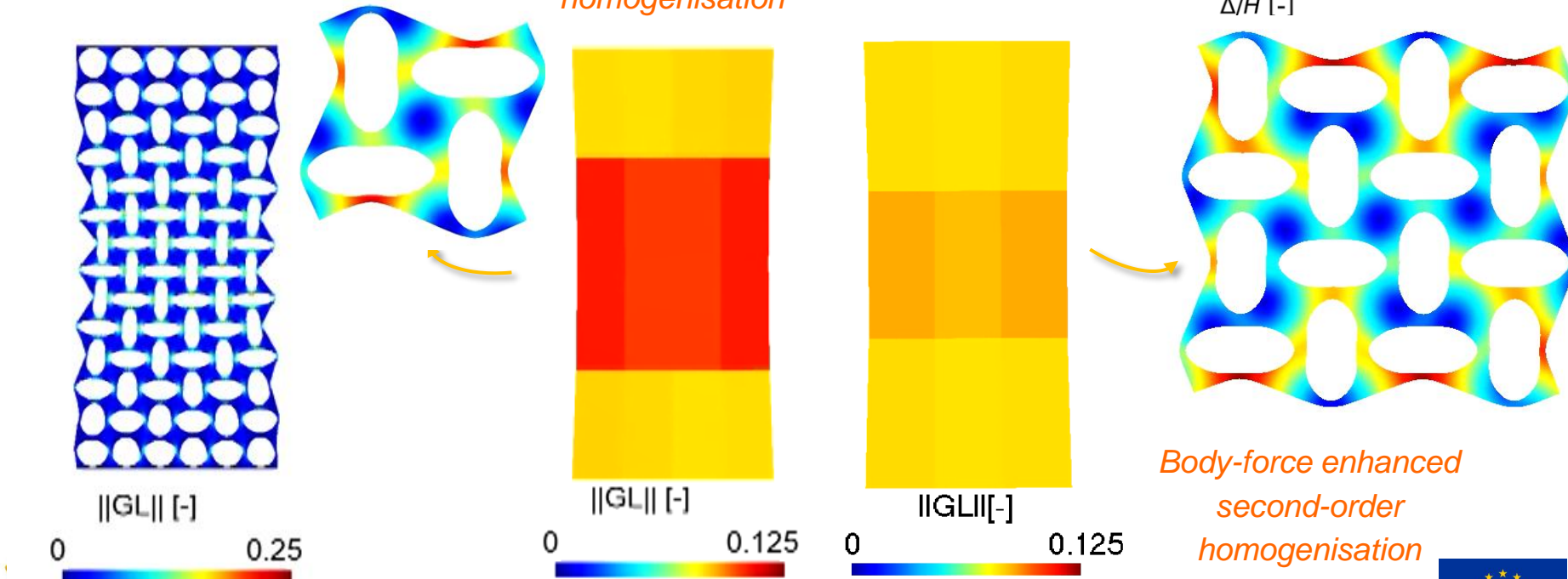
- Local instability under compression
 - 6x14 holes \rightarrow local instability
- RVE size effect



Direct simulation

Body-force enhanced second-order homogenisation

Body-force enhanced second-order homogenisation

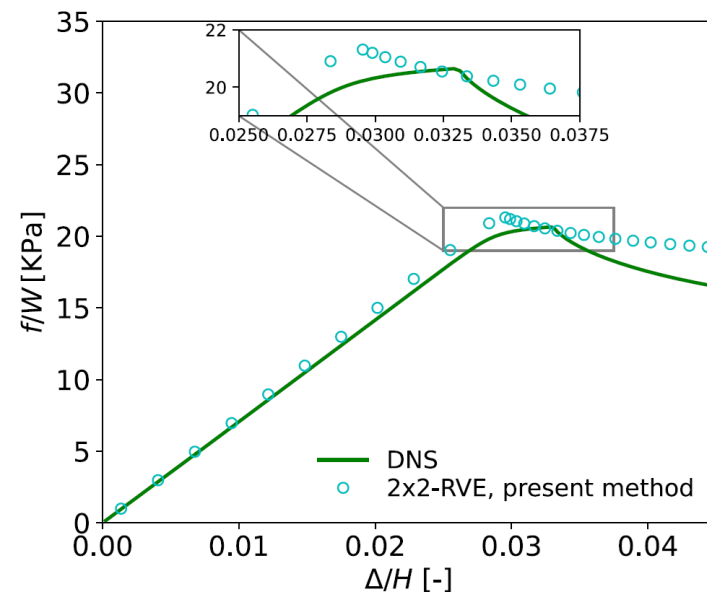
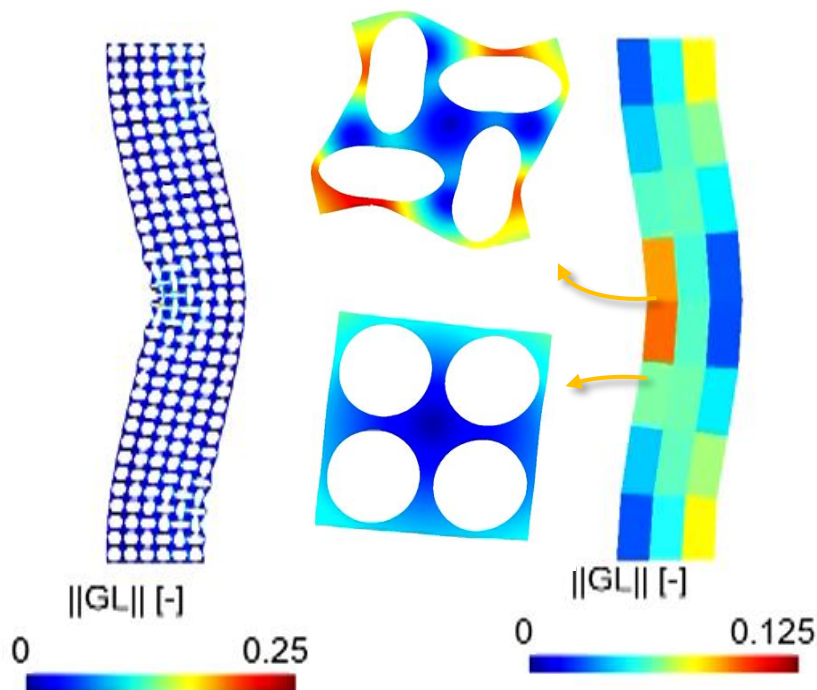


Second order homogenisation for cellular and metamaterials

- Multiscale simulation on metastructures

 - Compression samples of different sizes

 - 6x34 holes \rightarrow global instability



Conclusions

- Effect of RVE size largely reduced
- Applicable to
 - Finite strain formulation
 - Elasto-plasticity
 - Local instabilities
 - Global instabilities
- More on
 - L. Wu, S. M. Mustafa, J. Segurado, and L. Noels. « Second-order computational homogenisation enhanced with non-uniform body forces for non-linear cellular materials and metamaterials. » *Computer Methods in Applied Mechanics and Engineering*, 407 (2023): 115931, doi: 10.1016/j.cma.2023.115931

