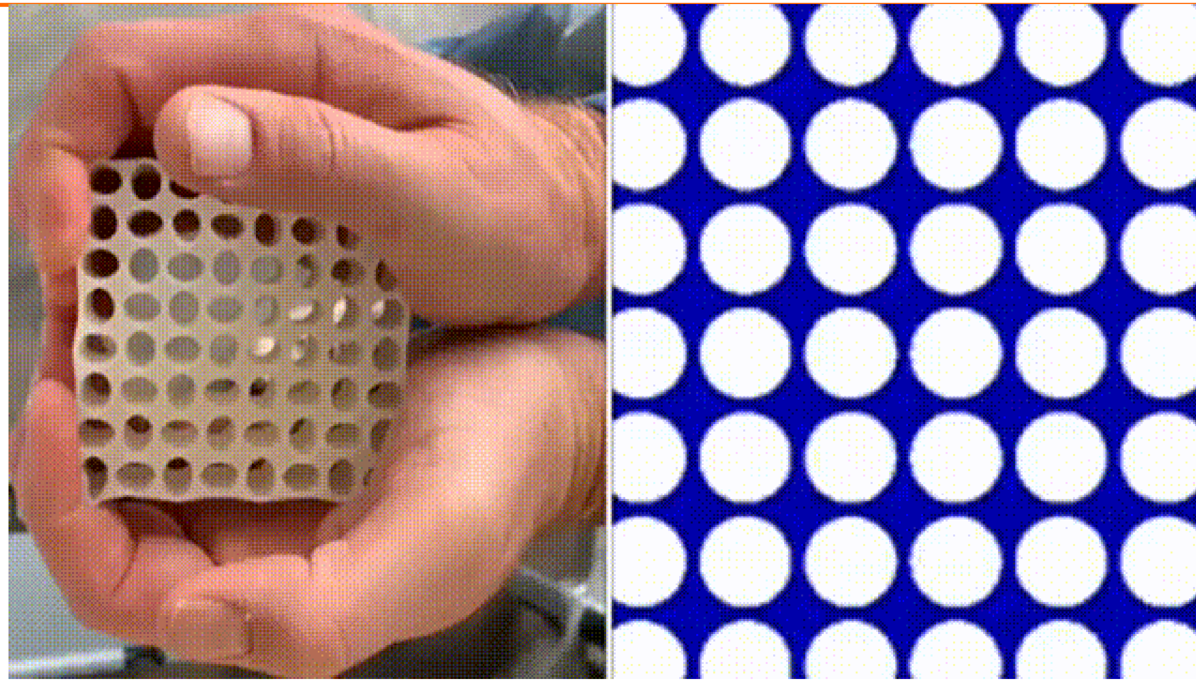
Multi-scale simulation of non-linear cellular- and meta-materials with body-force-enhanced second-order homogenisation



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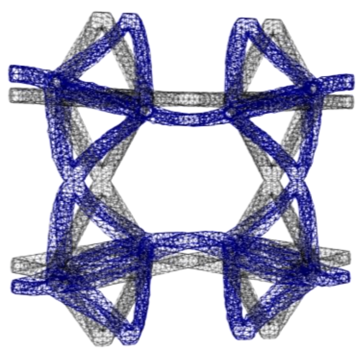
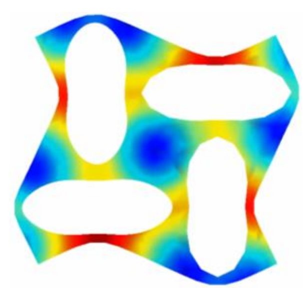
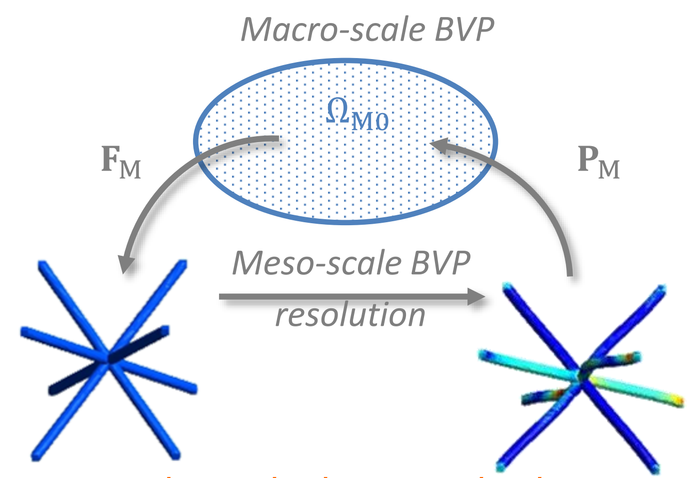
Second order homogenisation for cellular and metamaterials

- First vs. second order homogenisation
  - First order homogenisation
    - Does not prevent spurious localisation
    - No material length-scale
  - Second-order homogenisation
    - High order strain $\mathbf{G}_{\mathrm{M}}$ and stress $\mathbf{Q}_{\mathrm{M}}$ at macro-scale
    - Material length scale related to the RVE length

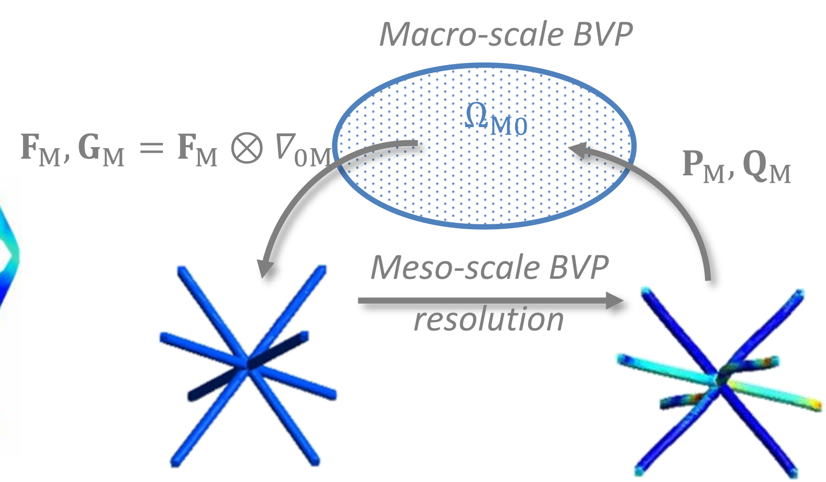




- Issue for metamaterial: RVE length is larger than unit cell because of patterning change



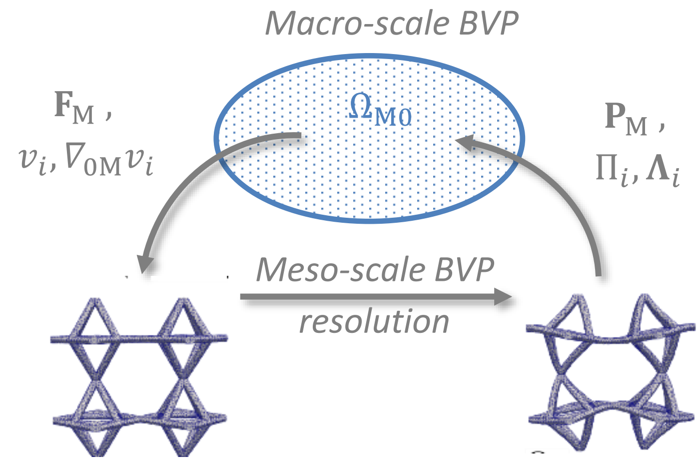
First order homogenisation



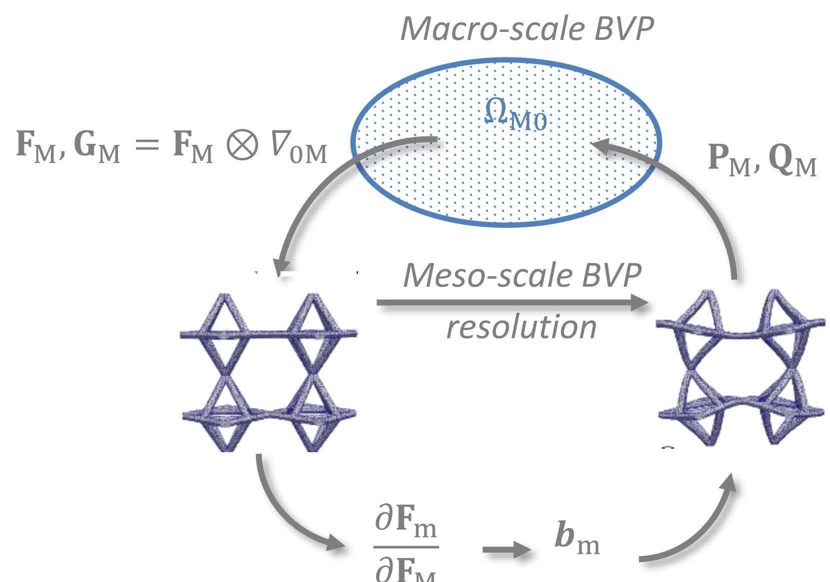
Second order homogenisation

Second order homogenisation for cellular and metamaterials

- Account for patterning change
  - Micromorphic approach*
    - Constrains change of patterning modes
    - Developed in elasticity
  - Enhanced second-order homogenisation
    - Remove cell size dependency using a body-force
    - Arises from asymptotic homogenization in linear elasticity**
    - How to account for finite strain, elasto-plasticity etc…?



Micromorphic approach

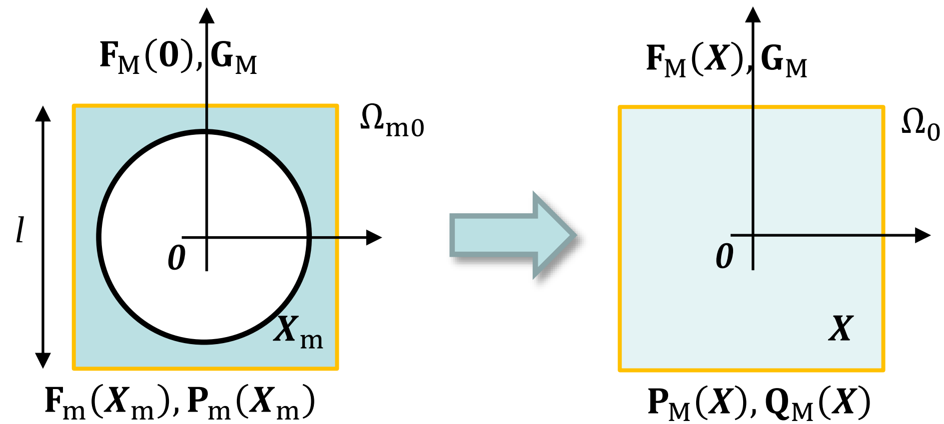


Body-force enhanced second order homogenisation

*O. Rokoš, M. Ameen, R. Peerlings, M. Geers, J. Mech. Phys. Solids 123 (2019)
**V. Monchiet, N. Auffray, J. Yvonnet, Mech. Mater. 143 (2020)
J. Yvonnet, N. Auffray, V. Monchiet, Int. J. Solids Struct. 191–192 (2020)

Second order homogenisation for cellular and metamaterials

- Second order homogenisation with body force enhancement
  - Consider an equivalent homogeneous volume element
    - Cauchy homogenous — Second order continuum



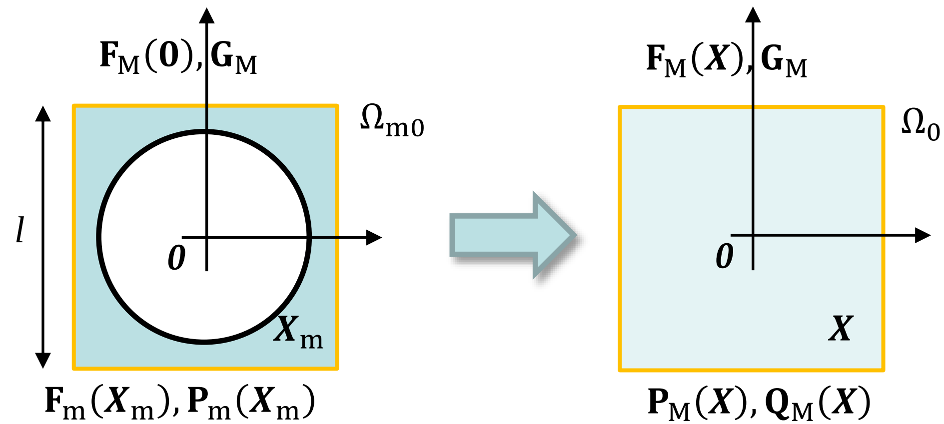
    - Development of the (no-longer) homogeneous field

$$\begin{cases} \mathbf{F}_M(\boldsymbol{X}) = \mathbf{F}_M(0) + \mathbf{G}_M \cdot \boldsymbol{X} \\ \mathbf{G}_M = \mathbf{F}_M(\mathbf{0}) \otimes \nabla_{0M} \end{cases}$$

$$\begin{cases} \mathbf{P}_M(\boldsymbol{X}) = \mathbf{P}_M(0) + \left.\dfrac{\partial \mathbf{P}_M}{\partial \mathbf{F}_M}\right|_0 : \mathbf{G}_M \cdot \boldsymbol{X} \\ \mathbf{Q}_M(\boldsymbol{X}) = \mathbf{Q}_M(0) + \left.\dfrac{\partial \mathbf{Q}_M}{\partial \mathbf{F}_M}\right|_0 : \mathbf{G}_M \cdot \boldsymbol{X} \end{cases}$$

$$\mathbf{B}_M(\boldsymbol{X})_M + \frac{\partial \mathbf{P}_M}{\partial \mathbf{F}_M}\Big|_0 : \mathbf{G}_M : \boldsymbol{I} = \mathbf{0}$$

- Second order homogenisation with body force enhancement
  - Consider an equivalent homogeneous volume element
    - The equivalence of energy (Hill-Mandel condition) with introduction of body forces $\boldsymbol{b}_{\mathrm{m}}(\boldsymbol{X}_{\mathrm{m}})$:



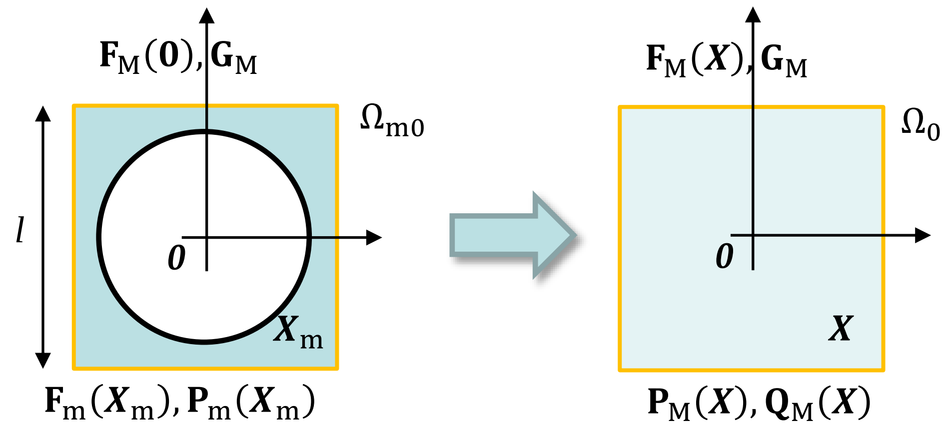
$$\int_{\Omega_{\mathrm{m0}}} [\mathbf{P}_{\mathrm{m}} : \delta\mathbf{F}_{\mathrm{m}} - \mathbf{b}_{\mathrm{m}} \cdot \delta\boldsymbol{u}_{\mathrm{m}}] d\Omega$$

$$\Leftrightarrow$$

$$\int_{\Omega_0} (\mathbf{P}_{\mathrm{M}} : \delta\mathbf{F}_{\mathrm{M}} + \mathbf{Q}_{\mathrm{M}} \vdots \delta\mathbf{G}_{\mathrm{M}} - \mathbf{B}_{\mathrm{M}} \cdot \delta\boldsymbol{u}_{\mathrm{M}}) d\Omega$$

$$\mathbf{F}_{\mathrm{m}}(\boldsymbol{X}_{\mathrm{m}}) = \mathbf{F}_{\mathrm{M}} + \mathbf{G}_{\mathrm{M}} \cdot \boldsymbol{X}_{\mathrm{m}} + w(\boldsymbol{X}_{\mathrm{m}}) \otimes \boldsymbol{\nabla}_{\mathrm{m0}}$$

- Second order homogenisation with body force enhancement
  - Consider an equivalent homogeneous volume element
    - The equivalence of energy (Hill-Mandel condition) with introduction of body forces $\boldsymbol{b}_{\mathrm{m}}(\boldsymbol{X}_{\mathrm{m}})$:



$$\int_{\Omega_{\mathrm{m}0}} [\mathbf{P}_{\mathrm{m}} : \delta\mathbf{F}_{\mathrm{m}} - \mathbf{b}_{\mathrm{m}} \cdot \delta\boldsymbol{u}_{\mathrm{m}}] d\Omega$$

$$\Leftrightarrow$$

$$\int_{\Omega_0} (\mathbf{P}_{\mathrm{M}} : \delta\mathbf{F}_{\mathrm{M}} + \mathbf{Q}_{\mathrm{M}} \vdots \delta\mathbf{G}_{\mathrm{M}} - \mathbf{B}_{\mathrm{M}} \cdot \delta\boldsymbol{u}_{\mathrm{M}}) d\Omega$$

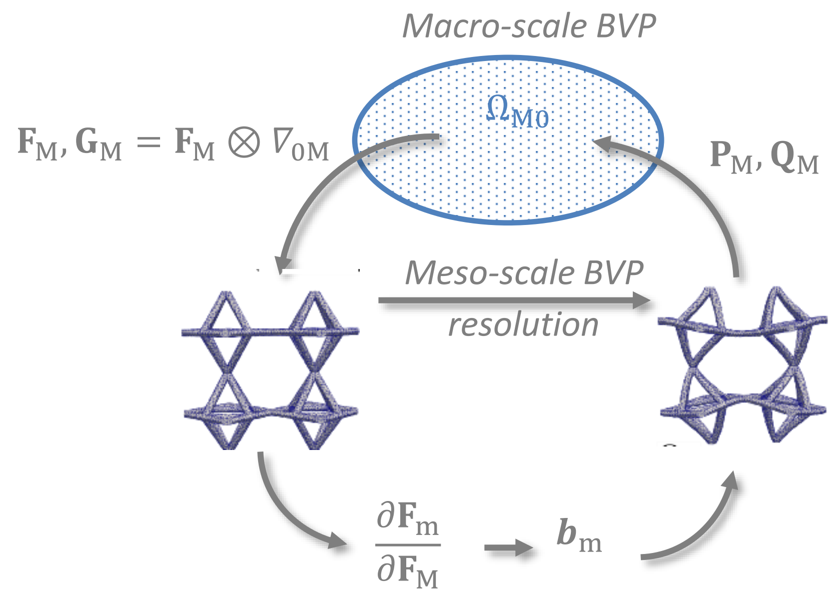
$$\mathbf{F}_{\mathrm{m}}(\boldsymbol{X}_{\mathrm{m}}) = \mathbf{F}_{\mathrm{M}} + \mathbf{G}_{\mathrm{M}} \cdot \boldsymbol{X}_{\mathrm{m}} + w(\boldsymbol{X}_{\mathrm{m}}) \otimes \boldsymbol{\nabla}_{\mathrm{m}0}$$

    - Is satisfied by the following introduction of micro-scale body forces and homogenised stresses

$$\begin{cases} \mathbf{P}_{\mathrm{M}} = \mathbf{P}_{\mathrm{M}}(0) = \dfrac{1}{V_0} \displaystyle\int_{\Omega_{\mathrm{m}0}} (\mathbf{P}_{\mathrm{m}} - \boldsymbol{b}_{\mathrm{m}} \otimes \boldsymbol{X}_{\mathrm{m}})\, d\Omega \\[2ex] \mathbf{Q}_{\mathrm{M}} = \mathbf{Q}_{\mathrm{M}}(0) = \frac{1}{2V_0} \int_{\Omega_{\mathrm{m}0}} [\mathbf{P}_{\mathrm{m}} \otimes \boldsymbol{X}_{\mathrm{m}} + (\mathbf{P}_{\mathrm{m}} \otimes \boldsymbol{X}_{\mathrm{m}})^T]\, d\Omega + \frac{1}{2V_0} \int_{\Omega_{\mathrm{m}0}} [\boldsymbol{b}_{\mathrm{m}} \otimes \boldsymbol{X}_{\mathrm{m}} \otimes \boldsymbol{X}_{\mathrm{m}}]\, d\Omega - \\ \qquad \frac{1}{2V_0} \left(\left[\frac{\partial \mathbf{P}_{\mathrm{M}}(0)}{\partial \mathbf{F}_{\mathrm{M}}} : \mathbf{G}_{\mathrm{M}} \cdot \boldsymbol{J}_{\mathrm{M}} + \left(\frac{\partial \mathbf{P}_{\mathrm{M}}(0)}{\partial \mathbf{F}_{\mathrm{M}}} : \mathbf{G}_{\mathrm{M}} \cdot \boldsymbol{J}_{\mathrm{M}} \right)^T \right] - \boldsymbol{B}_{\mathrm{M}} \otimes \boldsymbol{J}_{\mathrm{M}} \right) \\[2ex] \displaystyle\int_{\Omega_{\mathrm{m}0}} \mathbf{b}_{\mathrm{m}} d\Omega = \int_{\Omega_0} \mathbf{B}_{\mathrm{M}} d\Omega = -\int_{\Omega_0} \frac{\partial \mathbf{P}_{\mathrm{M}}}{\partial \mathbf{F}_{\mathrm{M}}} : \mathbf{G}_{\mathrm{M}} : \boldsymbol{I} d\Omega = -\int_{\Omega_{m0}} \left(\frac{\partial \mathbf{P}_{\mathrm{m}}}{\partial \mathbf{F}_{\mathrm{m}}} : \frac{\partial \mathbf{F}_{\mathrm{m}}}{\partial \mathbf{F}_{\mathrm{M}}} : \mathbf{G}_{\mathrm{M}} \right) : \boldsymbol{I} d\Omega \end{cases}$$

- Meso-scale problem
  - Micro-scale weak form

$$\int_{\Omega_{\mathrm{m}0}} \mathbf{P}_{\mathrm{m}} : (\delta \boldsymbol{w} \otimes \nabla_0) - \mathbf{b}_{\mathrm{m}} \cdot \delta \boldsymbol{w} \mathrm{d}\Omega = 0$$



  - Introduction of body forces $\mathbf{b}_{\mathrm{m}}(\mathbf{X}_{\mathrm{m}})$:

$$\mathbf{b}_{\mathrm{m}} = -\left(\frac{\partial \mathbf{P}_{\mathrm{m}}}{\partial \mathbf{F}_{\mathrm{m}}} \frac{\partial \mathbf{F}_{\mathrm{m}}}{\partial \mathbf{F}_{\mathrm{M}}} : \mathbf{G}_{\mathrm{M}}\right) : \boldsymbol{I}$$

Applied strain

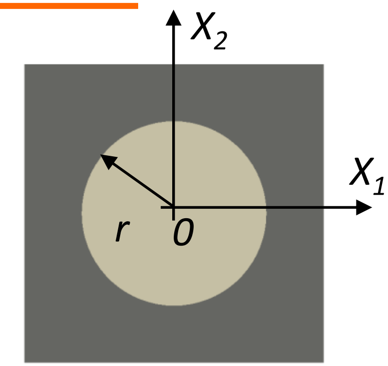
Instantaneous tangent

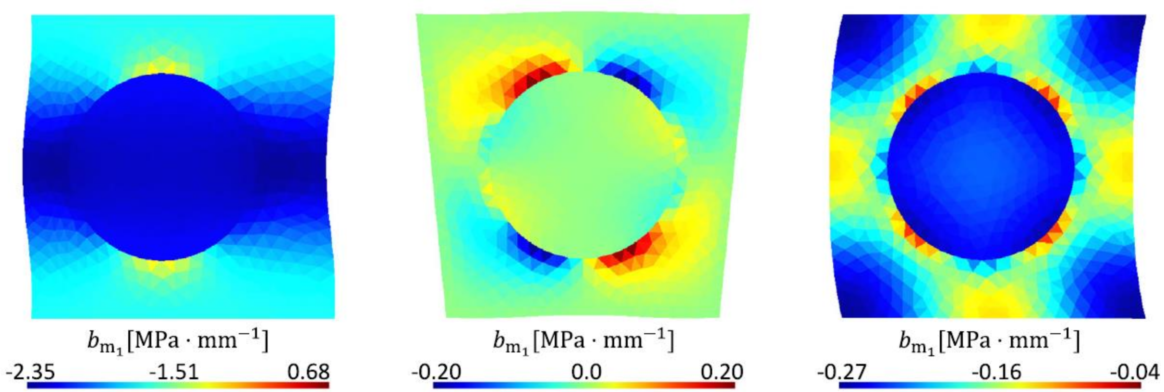
Strain concentration tensor

  - Approximation

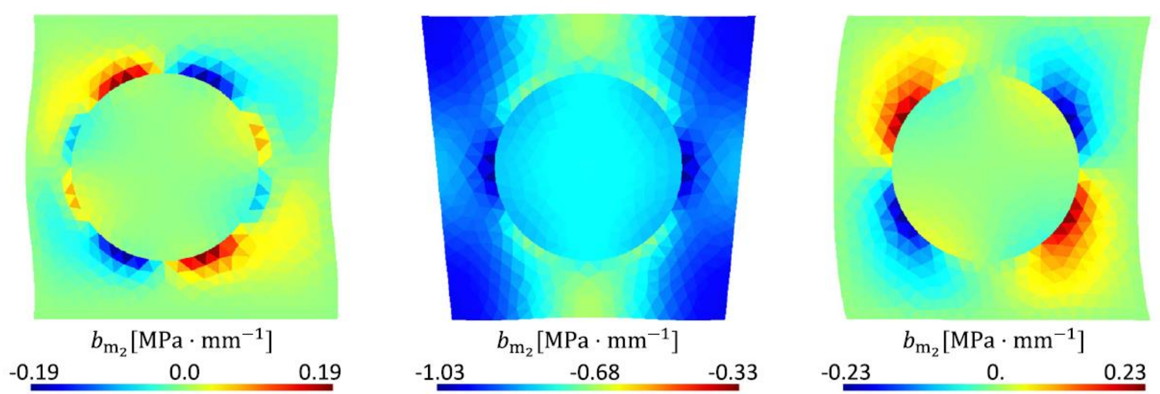
$$\mathbf{b}_{\mathrm{m}}^{n+1} = -\left(\left(\frac{\partial \mathbf{P}_{\mathrm{m}}}{\partial \mathbf{F}_{\mathrm{m}}}\right)^{n+1} : \left(\frac{\partial \mathbf{F}_{\mathrm{m}}}{\partial \mathbf{F}_{\mathrm{M}}}\right)^{n} : \mathbf{G}_{\mathrm{M}}^{n+1}\right) : \boldsymbol{I}$$

- Remove boundary effect
  - Linear elasticity
  - With the presented approach, the body forces are not uniform

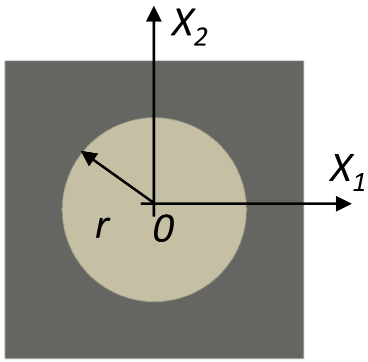


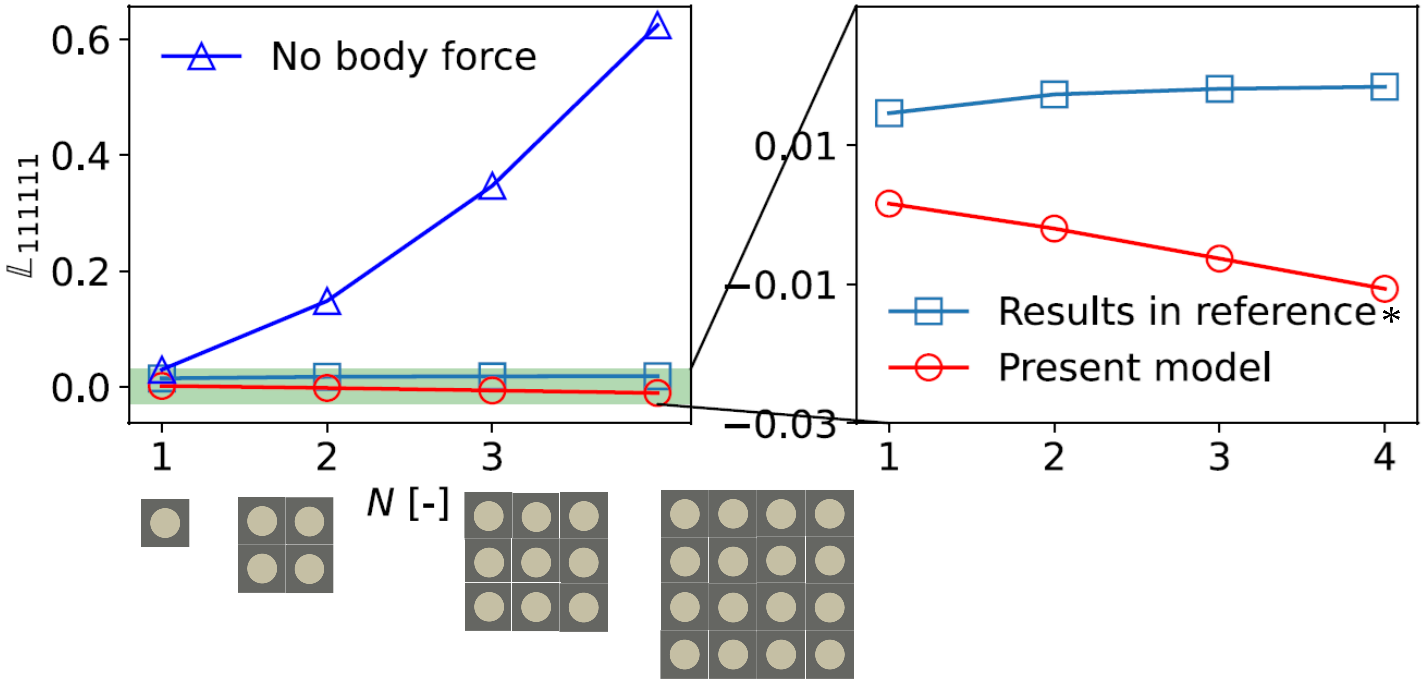


$G_{M_{xxx}} = 0.4$ /mm $G_{M_{xxy}} = 0.2$ /mm $G_{M_{xyy}} = 0.4$ /mm



- Remove boundary effect
  - Linear elasticity
  - Comparison with uniform body force*
  - Study of the higher-order operator: $\mathbb{L} = \frac{\partial \mathbf{Q}_M}{\partial \mathbf{G}_M}$

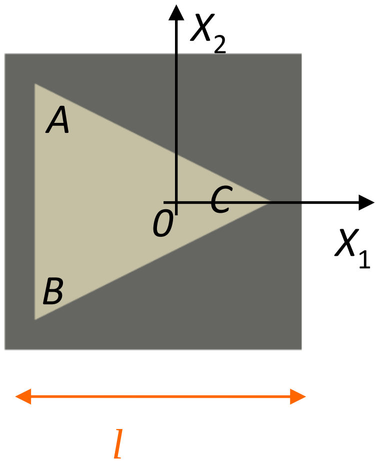


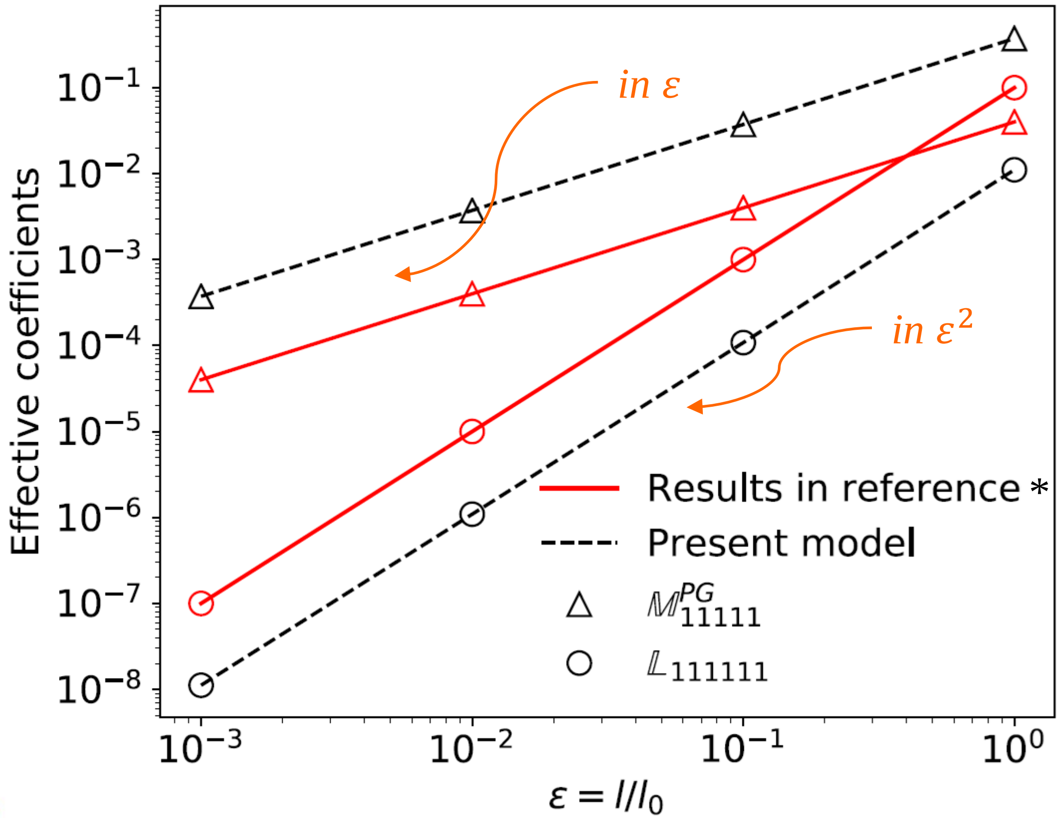


*J. Yvonnet, N. Auffray, V. Monchiet, Int. J. Solids Struct. 191–192 (2020)

Second order homogenisation for cellular and metamaterials

- Remove boundary effect
  - Linear elasticity
  - RVE with 4 × 4 unit cells, l_0=1,0 mm
  - Study of the higher-order operators: $\mathbb{L} = \frac{\partial \mathbf{Q}_\mathrm{M}}{\partial \mathbf{G}_\mathrm{M}}, \mathbb{M}^{PG} = \frac{\partial \mathbf{P}_\mathrm{M}}{\partial \mathbf{G}_\mathrm{M}}$





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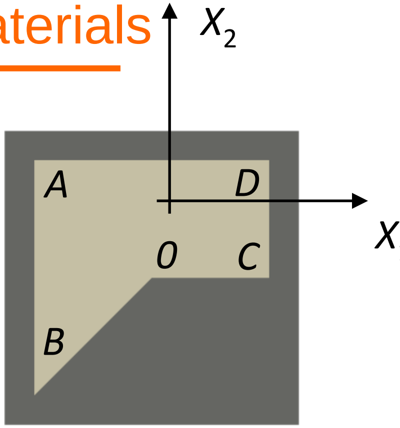
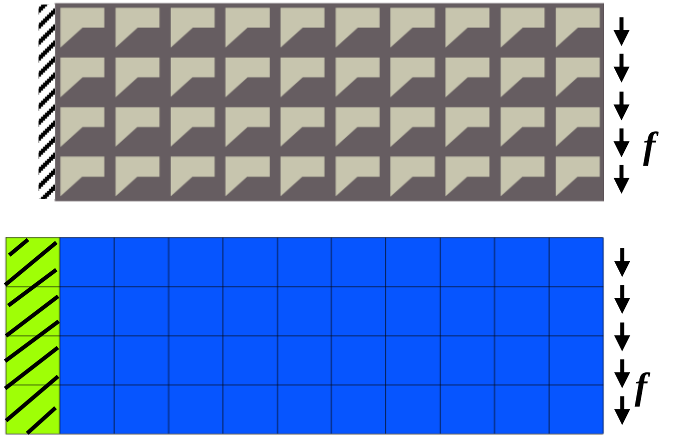
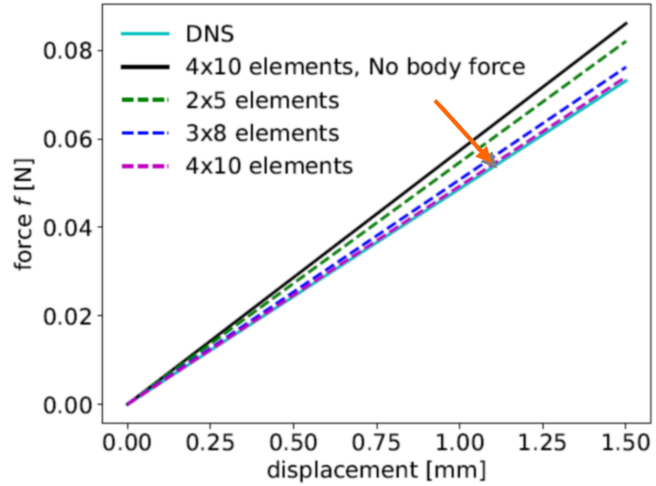
Second order homogenisation for cellular and metamaterials

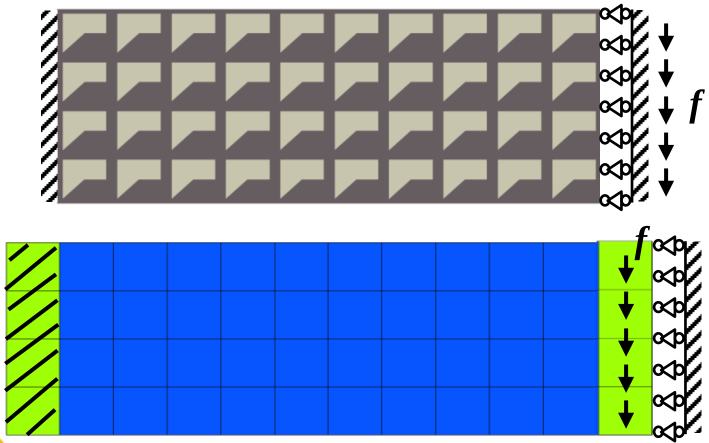


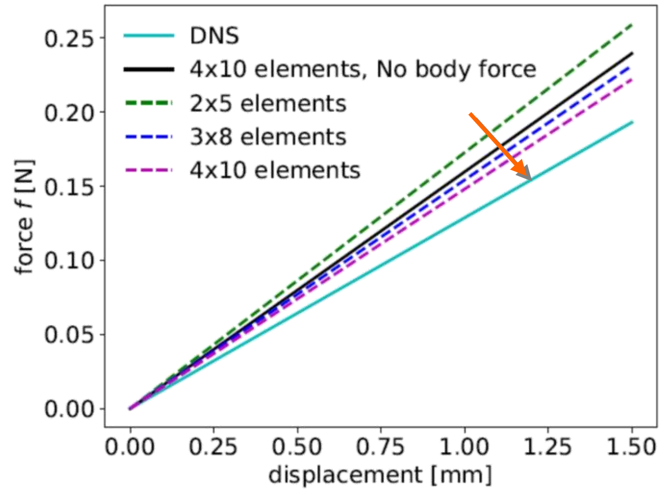
- Convergence toward DNS ?
  - Linear elasticity: Beam bending





  - Linear elasticity: Beam shearing

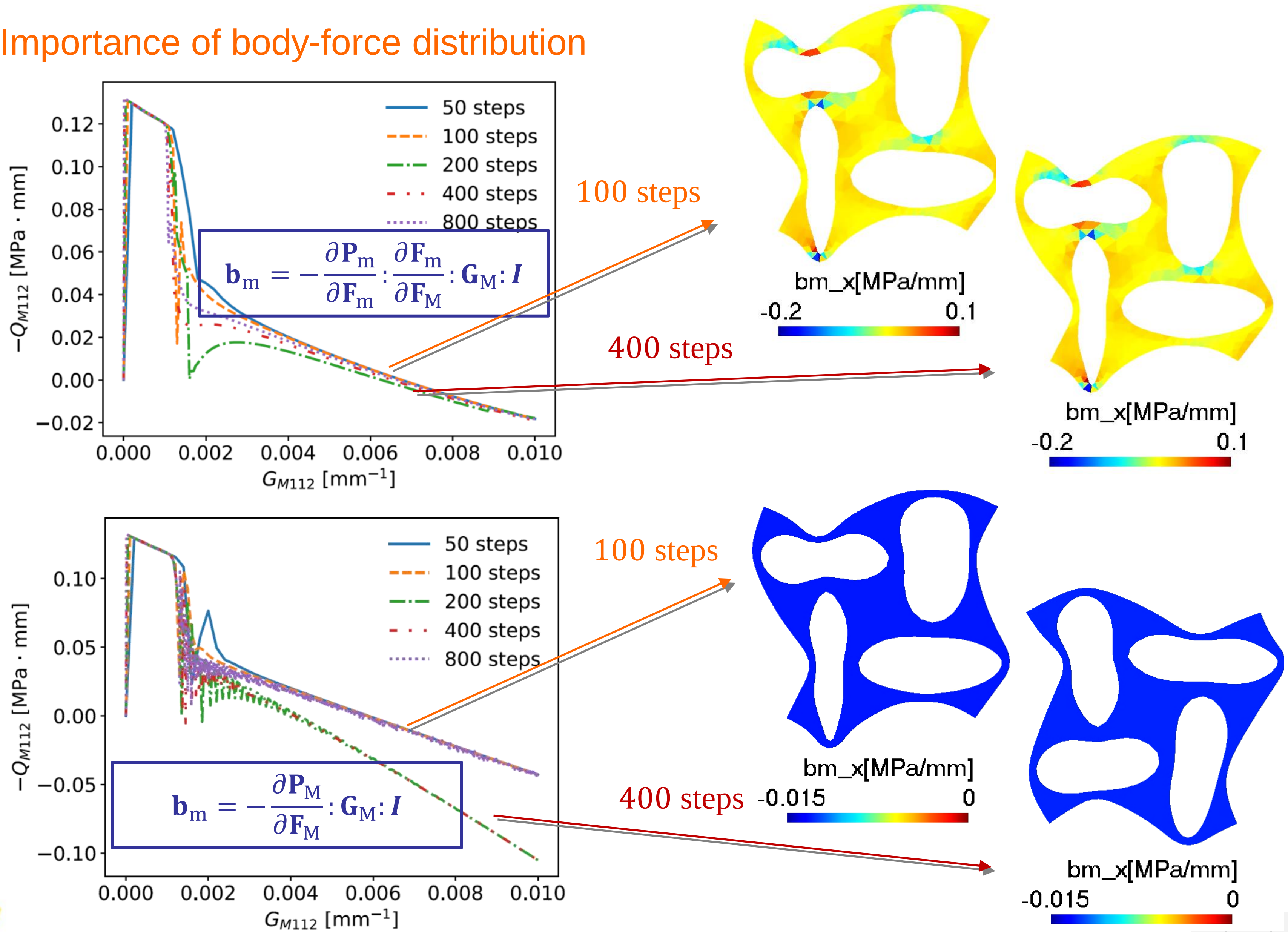




Second order homogenisation for cellular and metamaterials

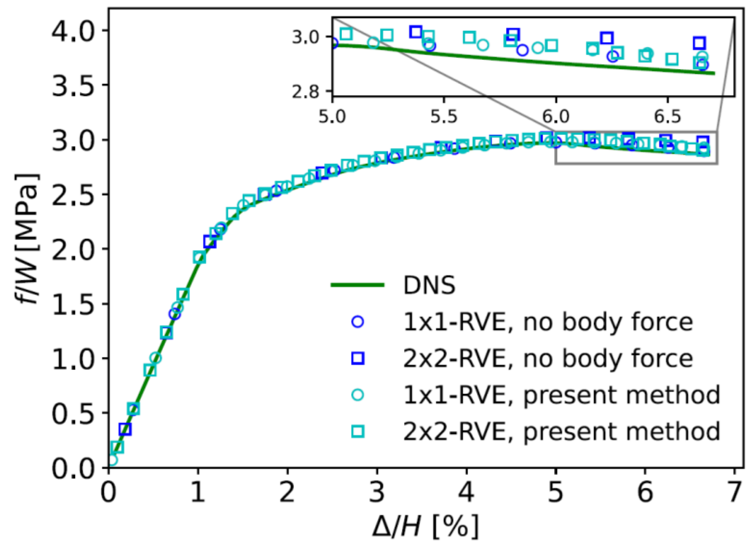
- Importance of body-force distribution

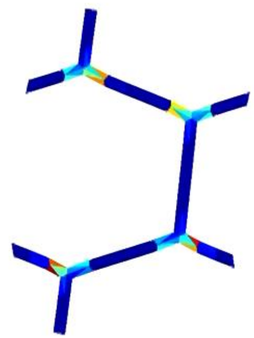
$\mathbf{F}_\mathrm{M} = 0.9\,\mathbf{I}$, $\mathrm{G}_{\mathrm{M}_{xxx}} = 0.02$ /mm, $\mathrm{G}_{\mathrm{M}_{xxy}} = 0.01$ /mm



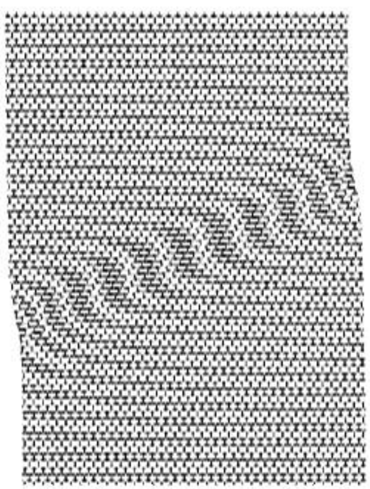
Second order homogenisation for cellular and metamaterials

- Multiscale simulation on honeycomb structures
  - Effect of RVE size disappears when considering body forces

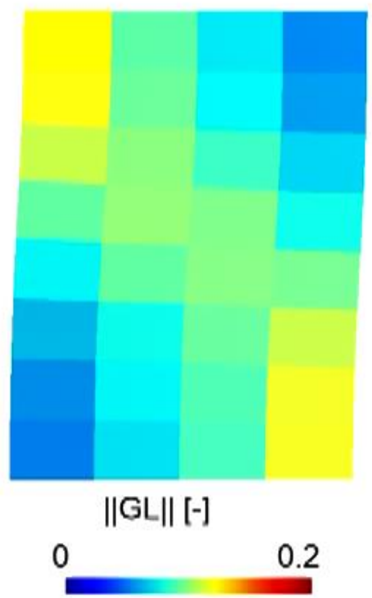


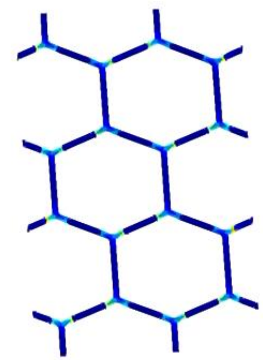


Direct simulation

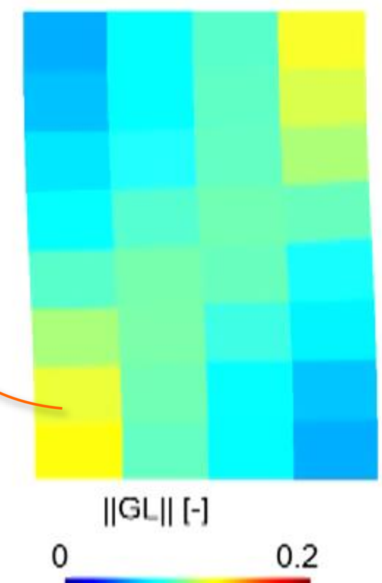


Body-force enhanced second-order homogenisation



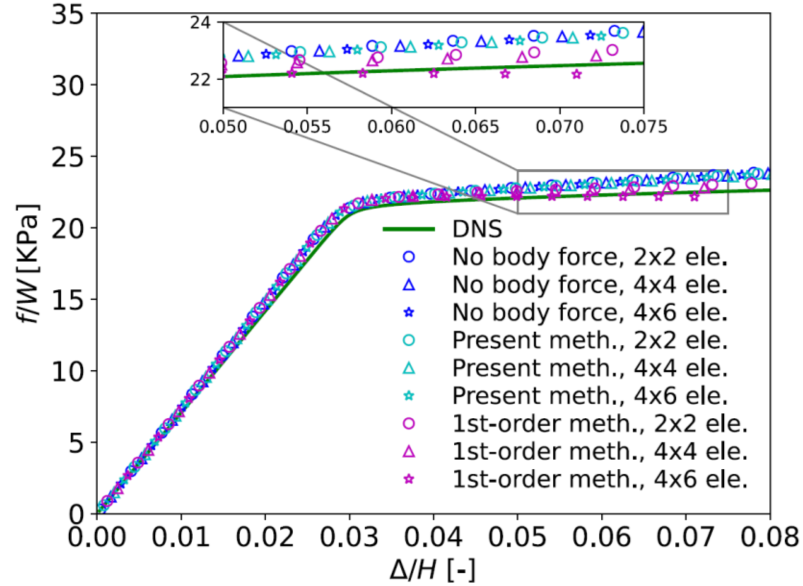


Body-force enhanced second-order homogenisation



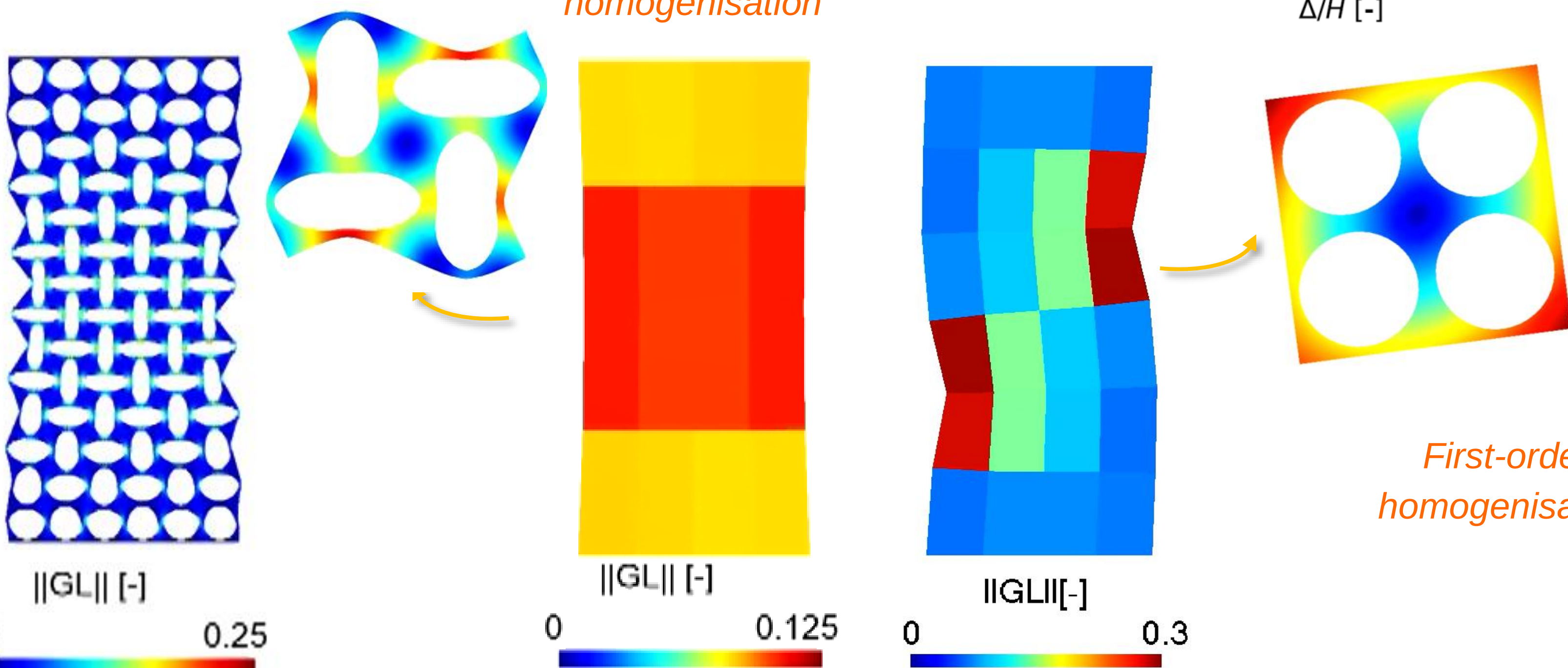
Second order homogenisation for cellular and metamaterials

- Multiscale simulation on metastructures
  - Local instability under compression
    - 6x14 holes ➡ local instability
  - Limit of first-order homogenisation



Direct simulation

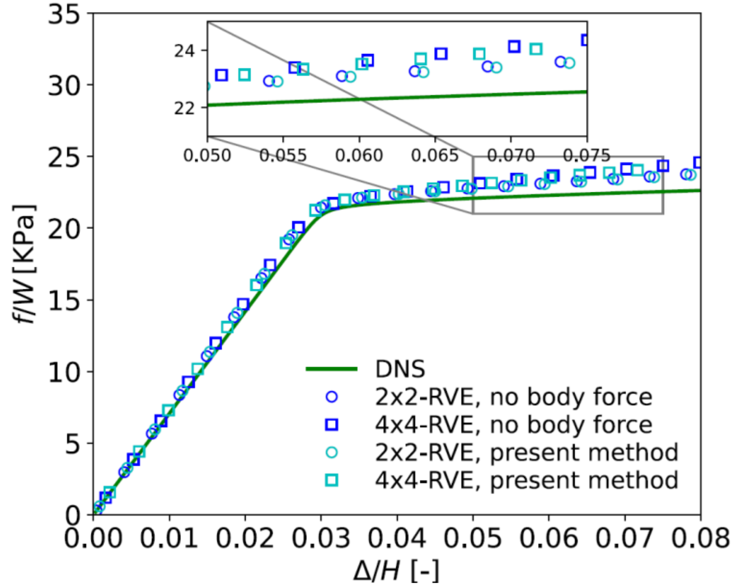
Body-force enhanced second-order homogenisation



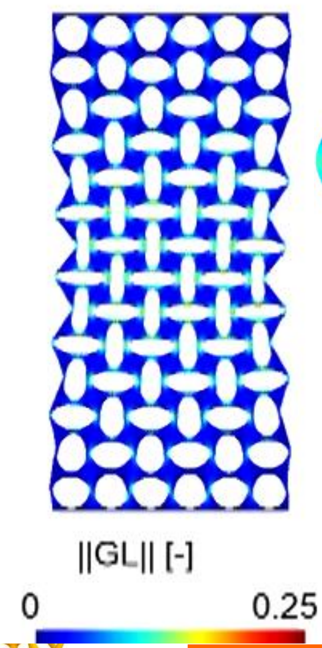
First-order homogenisation

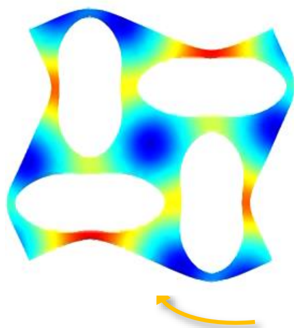
Second order homogenisation for cellular and metamaterials

- Multiscale simulation on metastructures
  - Local instability under compression
    - 6x14 holes → local instability
  - RVE size effect

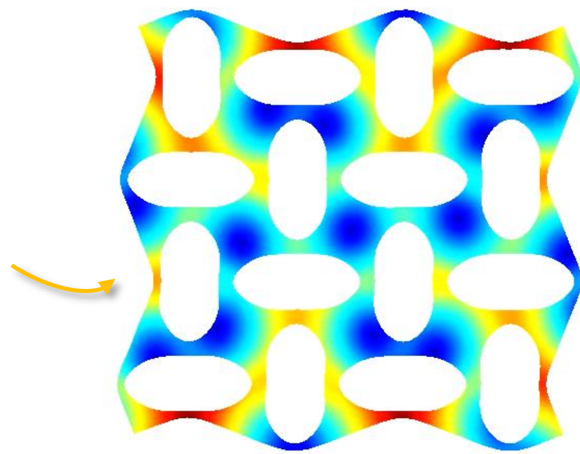


Direct simulation





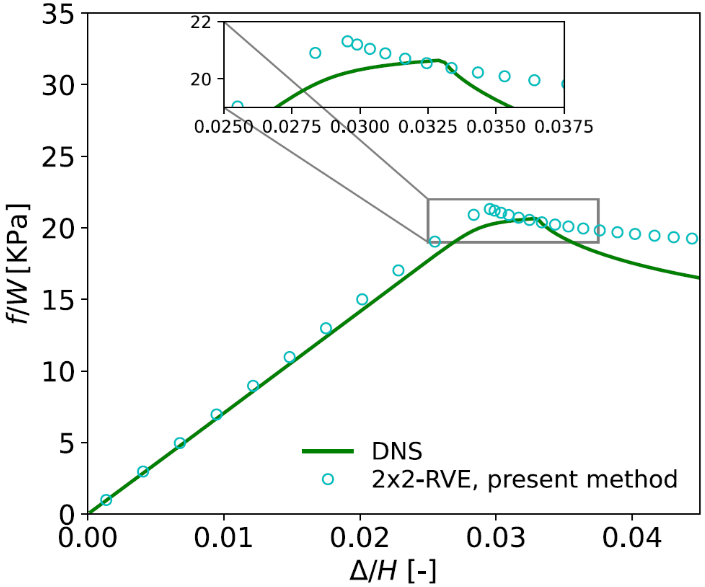
Body-force enhanced second-order homogenisation

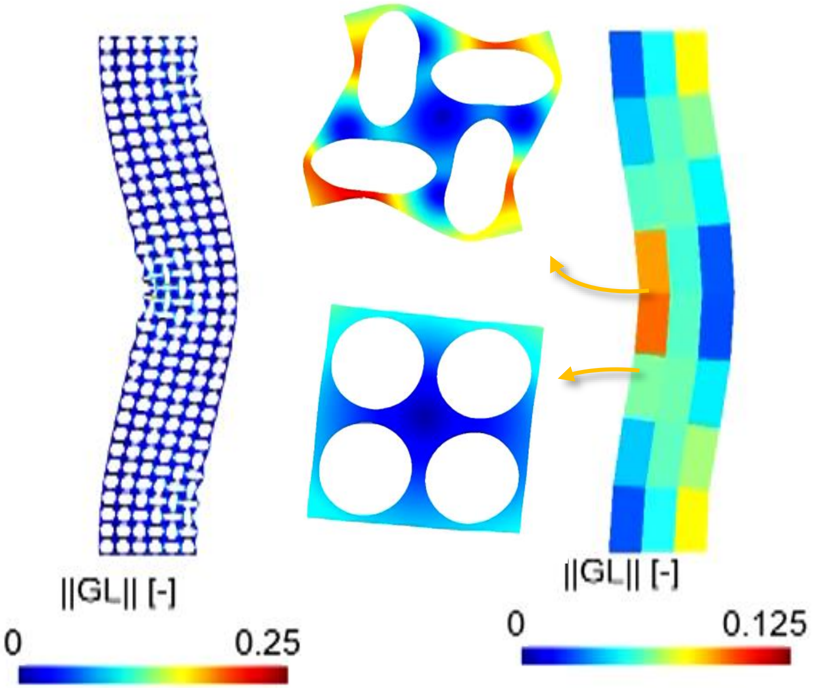


Body-force enhanced second-order homogenisation

Second order homogenisation for cellular and metamaterials

- Multiscale simulation on metastructures
  - Compression samples of different sizes
    - 6x34 holes ➡ global instability





Conclusions

- Effect of RVE size largely reduced
- Applicable to
  - Finite strain formulation
  - Elasto-plasticity
  - Local instabilities
  - Global instabilities
- More on
  - L. Wu, S. M. Mustafa, J. Segurado, and L. Noels. « Second-order computational homogenisation enhanced with non-uniform body forces for non-linear cellular materials and metamaterials. » Computer Methods in Applied Mechanics and Engineering, 407 (2023): 115931, doi: 10.1016/j.cma.2023.115931