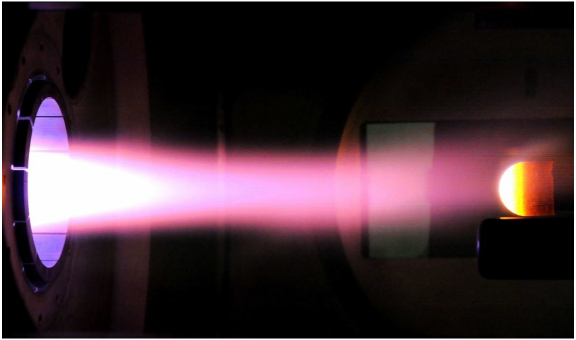
Development of a multi-domain hybridized discontinuous Galerkin solver



Author: Corthouts Nicolas

Promoters: Hillewaert Koen, May Georg

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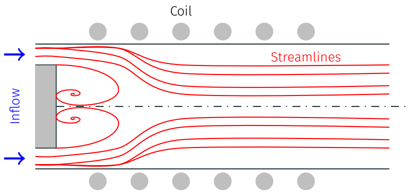
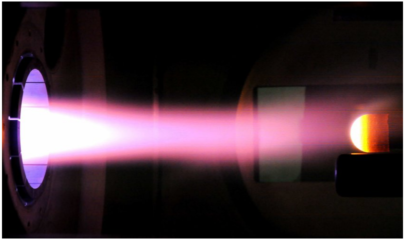
What is an inductively coupled plasma (ICP)?



Torch

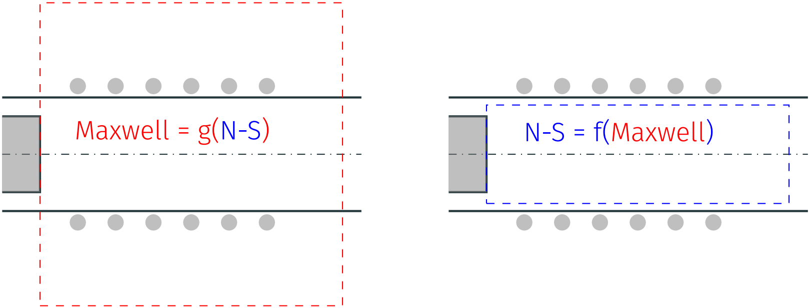


Test chamber

The most powerful ICP facility in the world, the Plasmatron, is located at VKI.

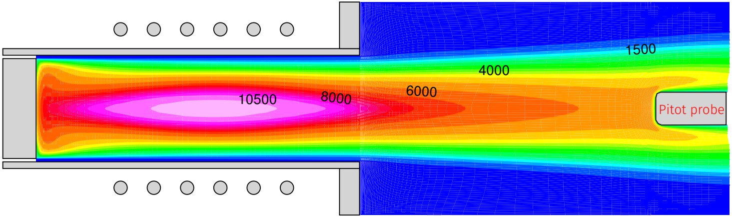
N.B. The governing equations are Maxwell + Navier-Stokes.

ICP: segregated approach of previous solvers



Segregated approach: Maxwell is solved, then used as a known field for N-S. Then, the solution for N-S is used as a known field for Maxwell.

Segregated approach: pros and cons



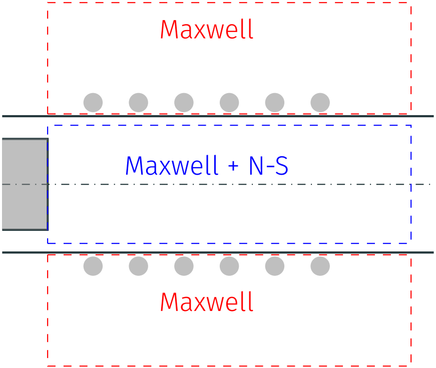
Pros

- It works.
- Allows to freeze the electric field in unsteady simulations.

Cons

- Convergence can be hard to achieve.

A multi-domain solver

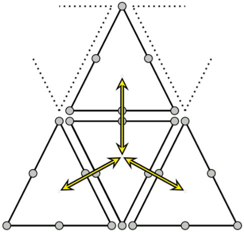


Two approaches

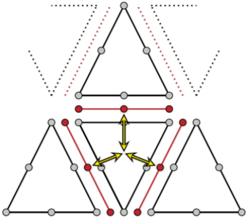
- MONOLITHIC: system solved as a whole.
- COUPLED: two solvers that exchange interface data.

The monolithic approach is chosen for its stability features.

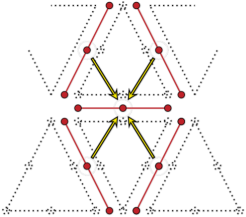
The numerical method: HDG



Classic DG.



HDG: traces.

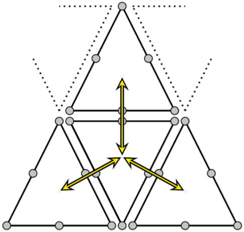


HDG: elements as transmitters.

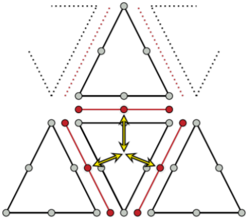
This method requires solving 2 types of systems:

1. **Local systems** solved directly & in parallel.
2. **A global system** smaller than the global DG system. Can also be run in parallel.

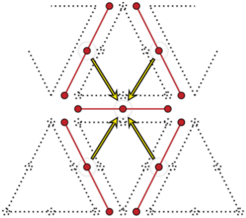
The numerical method: HDG



Classic DG.



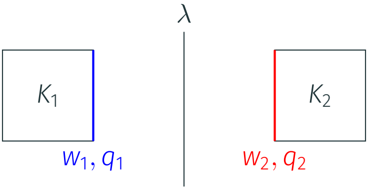
HDG: traces.



HDG: elements as transmitters.

The code (**Unified Framework**) used has been developed by the group of Prof. May.

HDG: Mathematical insights

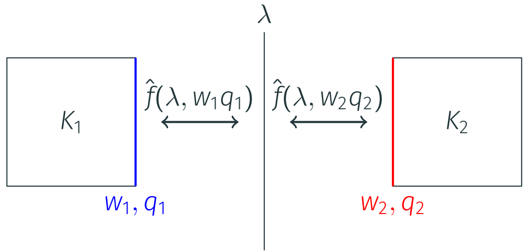


$$\begin{pmatrix} A & B & R \\ C & D & S \\ L & M & N \end{pmatrix} \begin{pmatrix} \delta Q \\ \delta W \\ \delta \Lambda \end{pmatrix} = \begin{pmatrix} F \\ G \\ H \end{pmatrix}$$

Local system equation **Global system equation**

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \delta Q \\ \delta W \end{pmatrix} = \begin{pmatrix} F \\ G \end{pmatrix} - \begin{pmatrix} R \\ S \end{pmatrix} \delta \Lambda \qquad \begin{pmatrix} L & M \end{pmatrix} \begin{pmatrix} \delta Q \\ \delta W \end{pmatrix} + N \delta \Lambda = H \tag{1}$$

HDG: Conservativity



Conservativity of the normal numerical flux

$$\int_\Gamma [[\hat{f}(\lambda, w, q)]]\mu dS = 0.$$

Rem: one equation for equation per λ.

Design constraints of multi-domain HDG

Conservativity of numerical flux

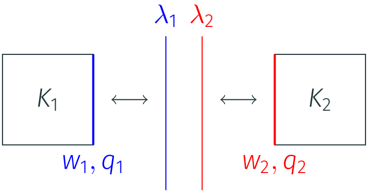
$$\int_{\Gamma} [[\hat{f}(\lambda, w, q)]] \mu dS = 0.$$

Local system equation

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \delta Q \\ \delta W \end{pmatrix} = \begin{pmatrix} F \\ G \end{pmatrix} - \begin{pmatrix} R \\ S \end{pmatrix} \delta \Lambda$$

Representation of discontinuous solution across interfaces

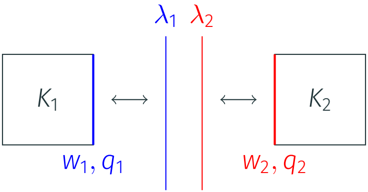
Multi-domain HDG: conservativity



New conservativity equation

$$\int_\Gamma \left[\hat{f}(\lambda_1, w_1, q_1) - \hat{f}(\lambda_2, w_2, q_2)\right] dS = 0$$

Not enough! There are twice as many λ, need additional equations.



Kinematic conditions

$$\int_\Gamma \mathcal{F}(\lambda_1, \lambda_2) dS = 0$$

λ_1 and λ_2 are usually correlated.

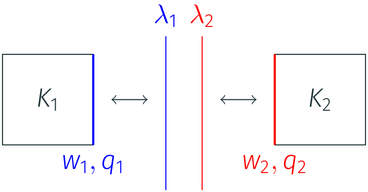
Now, we have the right amount of equations.

Multi-domain HDG: Solution strategy

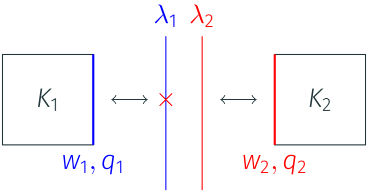
$$
\left(\begin{array}{ccc|ccc|ccc}
A_1 & B_1 & R_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
C_1 & D_1 & S_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
L_1 & M_1 & N_1 & L_{12} & M_{12} & N_{12} & L_{13} & M_{13} & N_{13} \\
\hline
0 & 0 & 0 & A_2 & B_2 & R_2 & 0 & 0 & 0 \\
0 & 0 & 0 & C_2 & D_2 & S_2 & 0 & 0 & 0 \\
L_{21} & M_{21} & N_{21} & L_2 & M_2 & N_2 & L_{23} & M_{23} & N_{23} \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & A_3 & B_3 & R_3 \\
0 & 0 & 0 & 0 & 0 & 0 & C_3 & D_3 & S_3 \\
L_{31} & M_{31} & N_{31} & L_{32} & M_{32} & N_{32} & L_3 & M_3 & N_3
\end{array}\right)
\left(\begin{array}{c}
\delta Q_1 \\ \delta W_1 \\ \delta \Lambda_1 \\ \hline \delta Q_2 \\ \delta W_2 \\ \delta \Lambda_2 \\ \hline \delta Q_3 \\ \delta W_3 \\ \delta \Lambda_3
\end{array}\right)
=
\left(\begin{array}{c}
F_1 \\ G_1 \\ H_1 \\ \hline F_2 \\ G_2 \\ H_2 \\ \hline F_3 \\ G_3 \\ H_3
\end{array}\right)
$$

The coupling terms only affect the $\delta\Lambda$ lines, so same solution strategy as before!

Continuous field with same equations on both sides

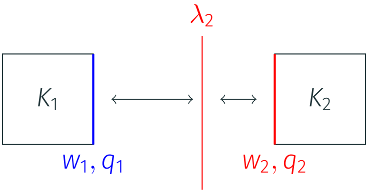


Continuous field with same equations on both sides



We enforce $\lambda_1 = \lambda_2$

Continuous field with same equations on both sides



We enforce stringly $\lambda_1 = \lambda_2$

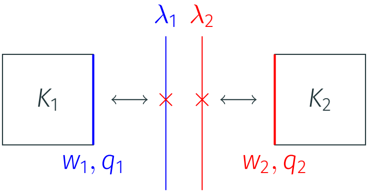
Conservativity of the normal numerical flux retrieved

$$\int_\Gamma [[\hat{f}(\lambda, w, q)]]\mu dS = 0.$$

Original scheme retrieved!

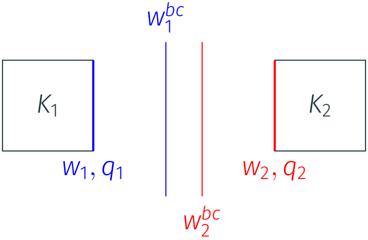
This is the case for ICP!

Boundary condition imposition



The hybrid unknowns are destroyed. A value is imposed like a classic BC.

Boundary condition imposition



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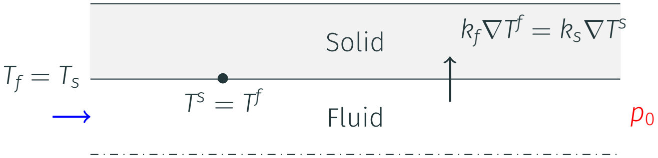
Validation: Conjugate heat transfer

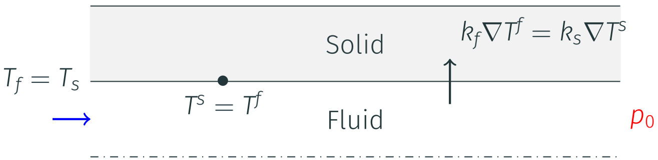


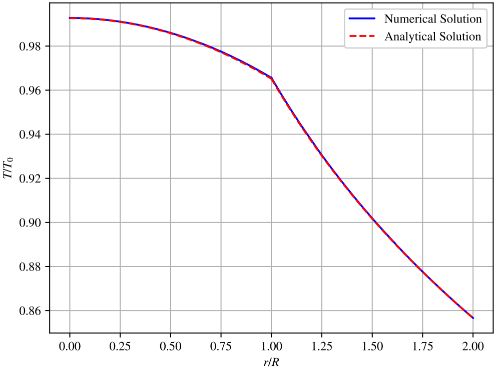
At the solid/fluid interface

$$T^f = T^s$$
$$k_s \nabla T^f = k_f \nabla T^s$$

There exists analytical solutions for $M \ll 1$, $\Delta T \ll 1$.

Validation: Conjugate heat transfer





Conclusions and future work

- A nice tool has been implemented in the HDG code.
- Possibility of extending to various physical situation.
- Need to be properly analysed.
- Opens the gate to ICP in the HDG code.