Terahertz off-axis digital holography reconstruction using inverse problem resolution with the Alternating Direction Method of Multipliers

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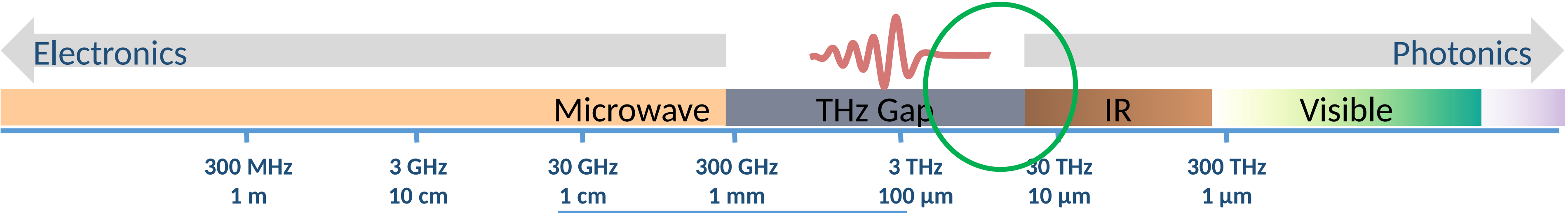
Outline

- **THz domain and applications**
- **THz coherent imaging methods**
- **Digital holography**
  - THz setup
  - Hologram formation
  - Usual method of reconstruction
- **Inverse methods reconstruction**
  - Main principles
  - Our method
- **Results**

The THz domain

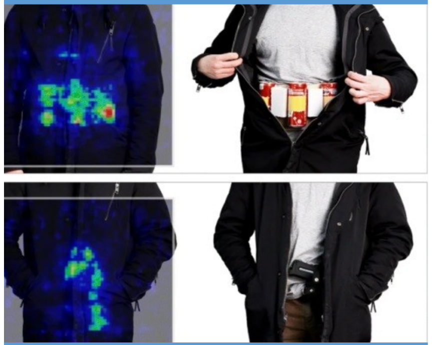


Terahertz (THz) Radiation

- Between microwave and infrared
- Frequency: 0.1-10 THz

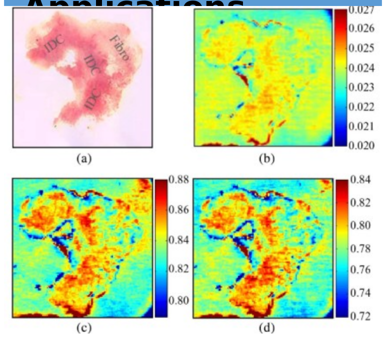
  Wavelength: 30 µm-3000 µm
- Unique features
  - ➢ Penetration of non polar materials
  - ➢ Water absorption
  - ➢ Ionization free
  - ➢ Spectral fingerprint
- Terahertz Imaging:
  - ➢ Various potential applications

Security and Defense



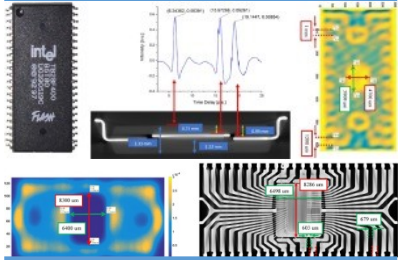
"New Real-Time Sub-Terahertz Security Body Scanner" by Tzydynzhapov, G. *et al.* 2020

Biomedical



"Terahertz Imaging of Excised Breast Tumor Tissue on Paraffin Sections" by T. C. Bowman et al., 2015

Quality control



"Quality control and authentication of packaged integrated circuits using enhanced-spatial-resolution terahertz time-domain spectroscopy and imaging" by Kiarash Ahi et al., 2018

Nondestructive inspection applications

THz coherent imaging methods

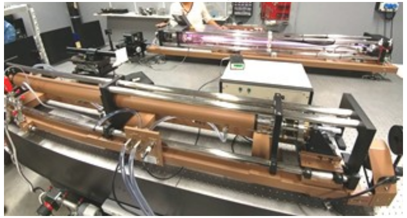
- **Alternative to TDS pulsed imaging**
  - No lengthy scanning with single point emitter-detector
- **They use**
  - Coherent CW light (e.g. in FIR: lasers)
  - THz focal plane array (FPA)
  - Reconstruction algorithm based on coherent diffraction imaging
  - Lensless imaging
- **Some examples**
  - Digital holography
  - Ptychography
  - Phase retrieval

L. Valzania, Y. Zhao, L. Rong, D. Wang, M. Georges, E. Hack, and P. Zolliker, "THz coherent lensless imaging," Appl. Opt. **58**, G256-G275 (2019)

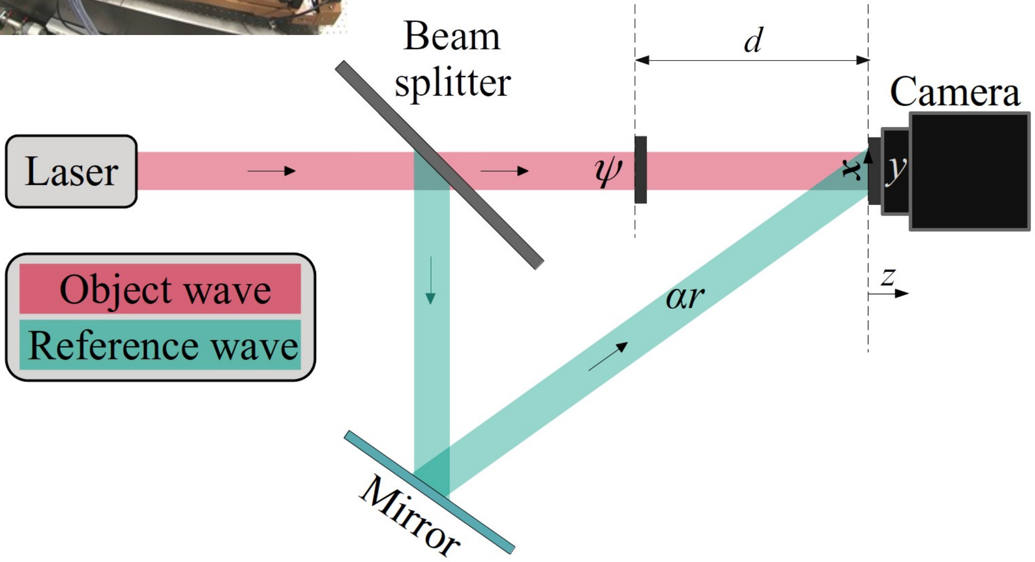
Digital Holography

- **Digital holography off-axis setup**

295-FIR, Edinburgh Instrument



Optically pumped far-infrared gas laser
118 µm up to 400-500 mW



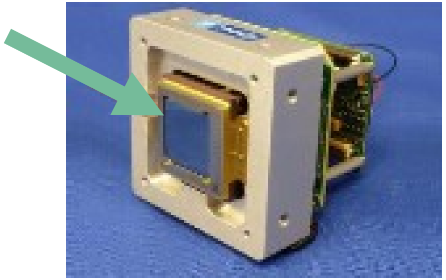
Uncooled THz microbolometer
388 × 284 pixels
Pitch 35 µm

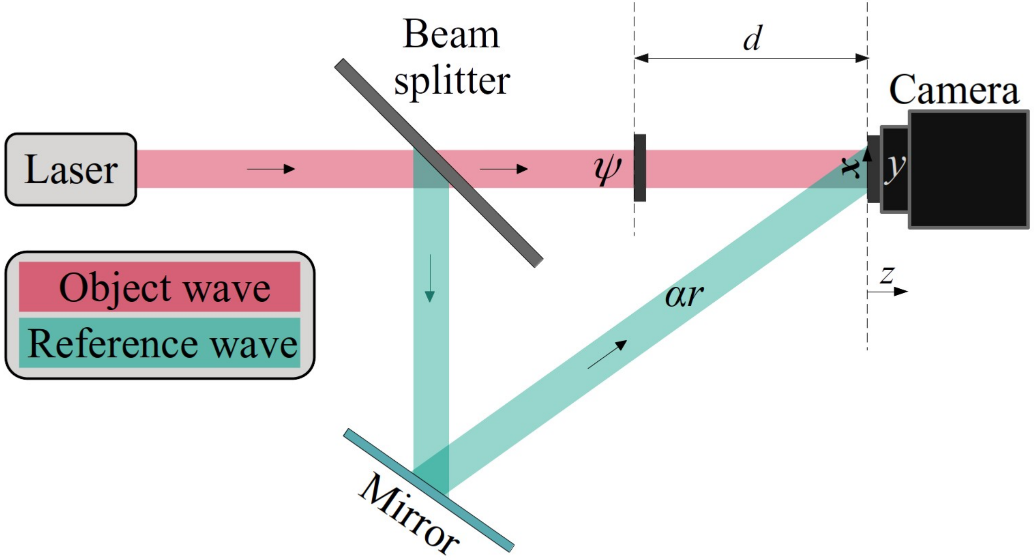
Digital Holography

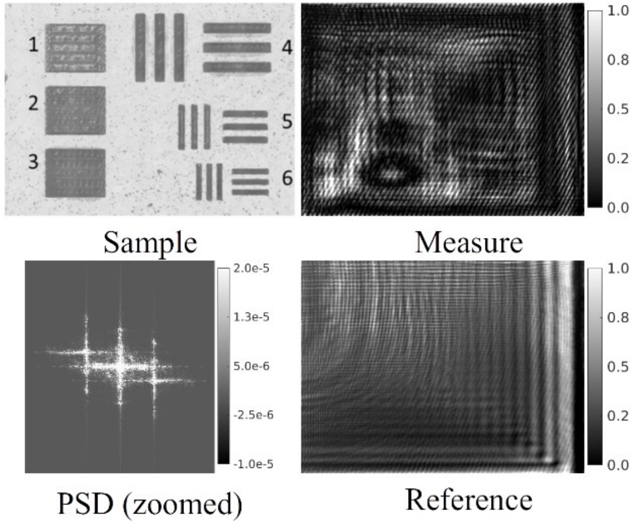
Problem

Obstruction of hologram pattern recorded because

- object close to sensor for increased resolution
- large angles between beams are used at such wavelengths





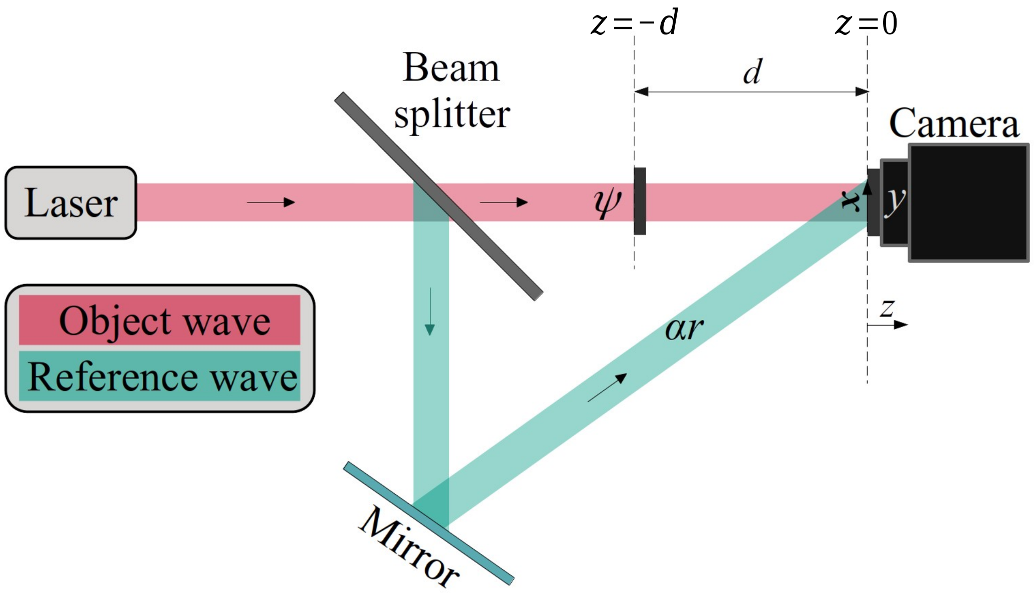


Model of hologram formation

also called here the "forward model"

we record $I(x,y,0) = |o(x,y,0) + r(x,y,0)|^2$

we want to estimate $\psi(x,y) = o(x,y,-d)$



The forward model of hologram formation can be written as

$$\boldsymbol{I} = \overline{\boldsymbol{I}} + \boldsymbol{n} = |\boldsymbol{A}_d \boldsymbol{\psi} + \alpha \boldsymbol{r}|^2 + \boldsymbol{n}$$

$\boldsymbol{I}$ *sampled version of by N pixels camera*

$\boldsymbol{r}$ *unit reference field*

α *relative amplitude of reference field*

The object wave is the propagated object field

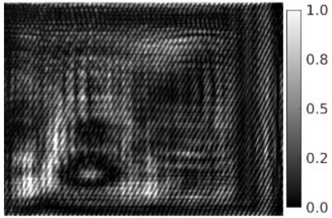
$\boldsymbol{o} = \boldsymbol{A}_d \boldsymbol{\psi}$

$\boldsymbol{A}_d$ *instrumental PSF + propagation at distance d*

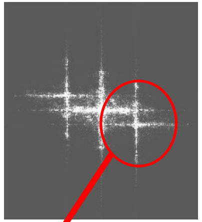
$\boldsymbol{n}$ *additive noise*

Hologram reconstruction

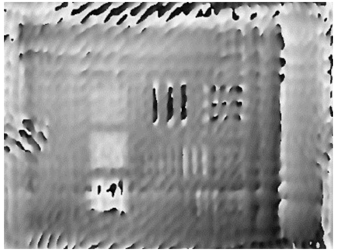
Usual method: direct Fourier Transform



FFT



$\boldsymbol{\psi}(phase)$



$$\boldsymbol{\psi} \propto FFT\left[\boldsymbol{I}\,.\,\alpha\,\boldsymbol{r}\,.\,exp\left(-i\frac{\pi}{\lambda\, d}\left(x^2+y^2\right)\right)\right]$$

$\boldsymbol{\psi}(amplitude)$

Artifact fringes are observed and due to uncorrect knowledge of the reference field phase

To improve this, we already tried iterative phase-retrieval assisted reconstruction

Y. Zhao et al., *Appl. Opt.* 58(33), 9208-9216 (2019)

Inverse method reconstruction

- **The reconstruction problem-1**

from measurement of $\boldsymbol{I} = \underbrace{\left|\boldsymbol{A}_d \boldsymbol{\psi} + \alpha\, \boldsymbol{r}\right|^2}_{\bar{I}:\text{noise free}} + \boldsymbol{n}$ *deduce unknown* $\begin{cases} \boldsymbol{\psi} & \text{(complex object field)} \\ \alpha \end{cases}$

retrieve estimates $\begin{cases} \widetilde{\Psi} \\ \widetilde{\alpha} \end{cases}$ *which minimize* ***Data Fidelity***

$$\boxed{D(\boldsymbol{\psi}, \alpha) = \left\| \boldsymbol{I} - \left|\boldsymbol{A}_d \boldsymbol{\psi} + \alpha\, \boldsymbol{r}\right|^2 \right\|_2^2}$$

difference between measurement and noise-free model

Challenges:

- *Non-convex problem because of Many local minima*
- *Problem ill-posed: dimension of and larger than number of measurements* ***I***
- *High frequency distribution of leads to instabilities and noise sensitivity*

Mathematical regularization is mandatory

Previous works with inverse methods: Bourquard et al. *Optics Express* 21(3), 2013

$$Regularization = TV^{ph}(\boldsymbol{\psi}) + \delta . TV^{amp}(\boldsymbol{\psi})$$

Inverse method reconstruction

- **Our works**

Solution proposed:

- *represented by its wavelet components : use of Discrete Wavelet Transform*
- *Regularization in the wavelet domain: minimize -norm of wavelet coefficients*

We rewrite the noise-free model

$$\bar{\boldsymbol{I}}(\boldsymbol{\psi},\alpha)=\left|\boldsymbol{A}_d\boldsymbol{\psi}+\alpha\,\boldsymbol{r}\right|^2$$

$$\bar{\boldsymbol{I}}(\boldsymbol{c}_\psi,\alpha)=\left|\boldsymbol{A}_d\boldsymbol{W}^{-1}\boldsymbol{c}_\psi+\alpha\,\boldsymbol{r}\right|^2$$

W *Matrix of fast Discrete Wavelet Transform*

The data fidelity becomes

$$D(\boldsymbol{c}_\psi,\alpha)=\left\|\boldsymbol{I}-\bar{\boldsymbol{I}}(\boldsymbol{c}_\psi,\alpha)\right\|_2^2$$

The wavelet-based regularization of the off-axis DH reconstruction leads to the following regularization

$$\boxed{\left(\widetilde{\boldsymbol{c}_\psi},\tilde{\alpha}\right)=\underset{\boldsymbol{c}_\psi,\alpha}{argmin}\left(D(\boldsymbol{c}_\psi,\alpha)+\delta\left\|\boldsymbol{c}_\psi\right\|_1\right)}$$

Inverse method reconstruction

- **The reconstruction algorithm**
  - Iterative process
  - Inspired by ADMM (Alternating Direction Method of Multipliers)
    - It's a convex method which can be used in non-convex problem
    - ADMM used in many fields of image processing, machine learning, etc
    - Mostly used on real data, but was extended to complex numbers by Li et al. "ADMMCP" see Li et al. *Math. Probl. Eng.* (2015)
  - **Novelty of our works:**
    - We propose ADMMCP in Digital Holography inverse reconstruction
    - We propose extension of ADMMCP for optimizing variable in addition to

Results

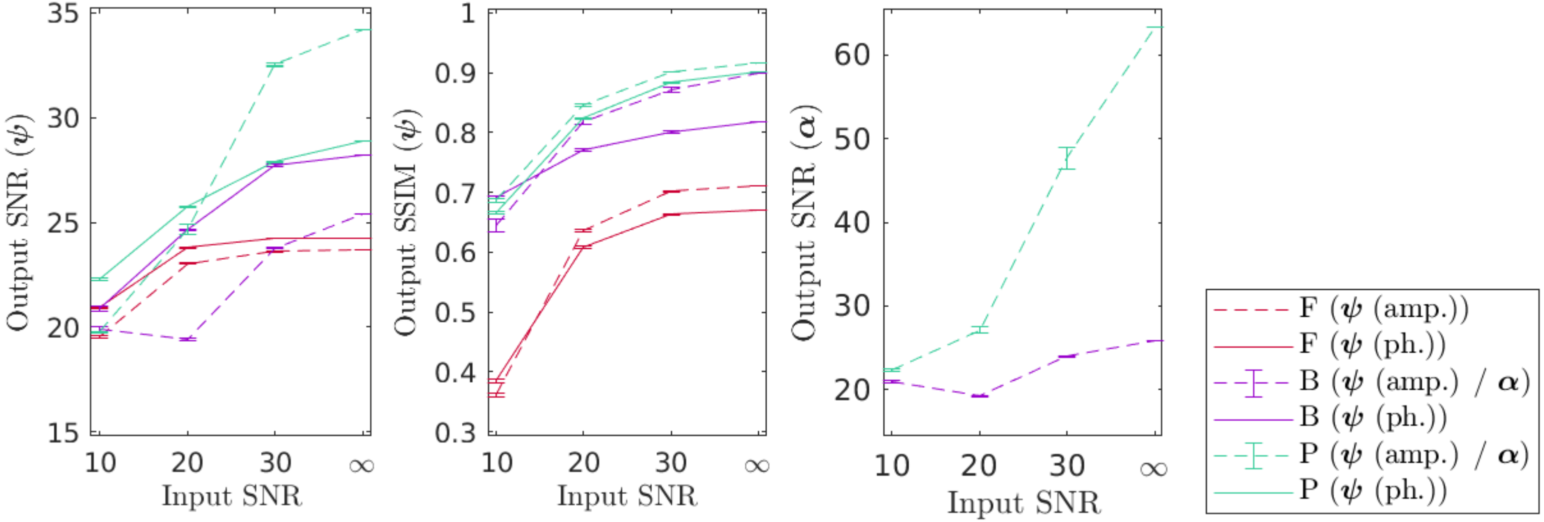
- **Experimenting the algorithm**
  - On simulated data
  - On real data
- **Comparison with**
  - Direct Fourier Transform
  - Inverse method by Bourquard et al.
  - Our method
- **Consider**
  - the effect of noise
  - with preprocessing or not
- **Details can be found in** Kirkove et al. *JOSA A* 41(3) (2024)

Results

- **Experimenting the algorithm on simulated data**
  - Study of noise effect on results

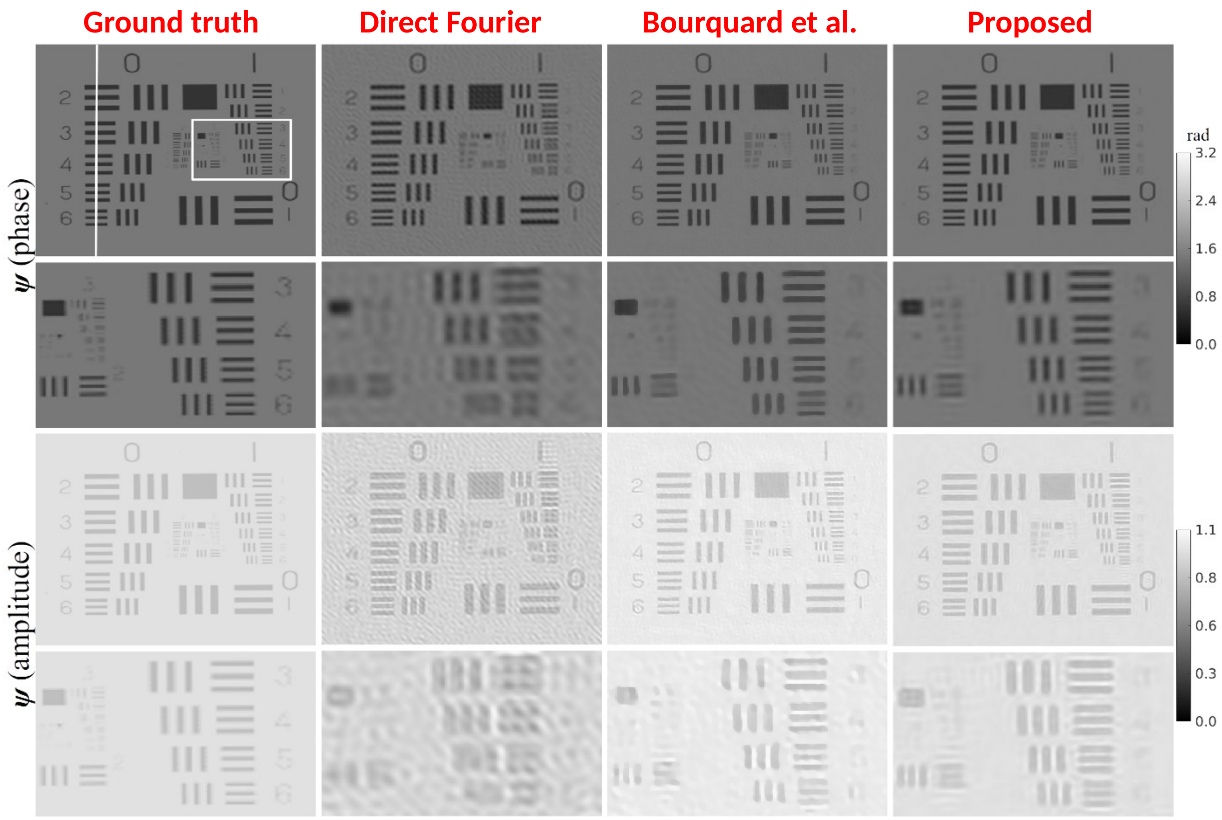
noise on

noise on



Results

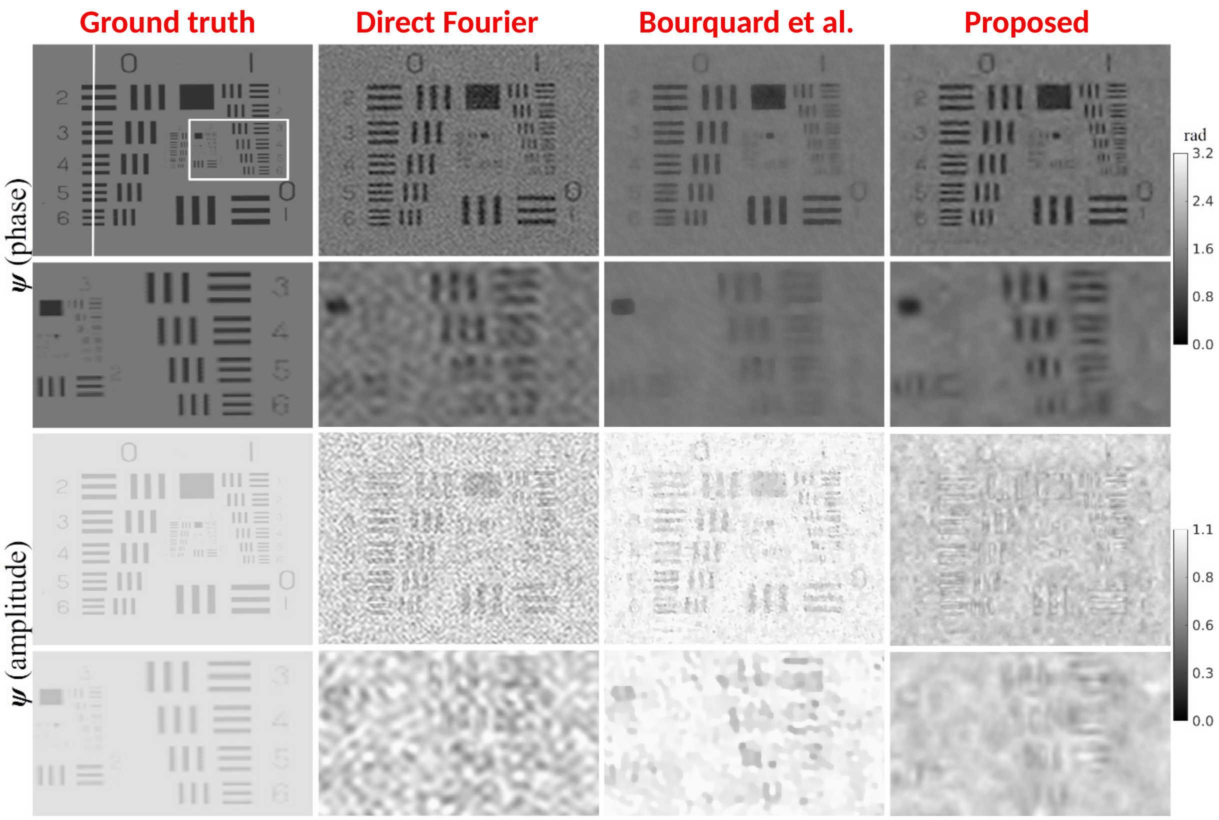
- **Experimenting the algorithm on simulated data**



Noiseless data

Results

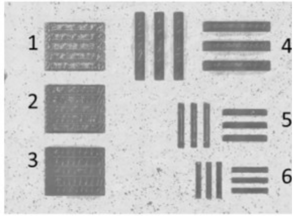
- **Experimenting the algorithm on simulated data**



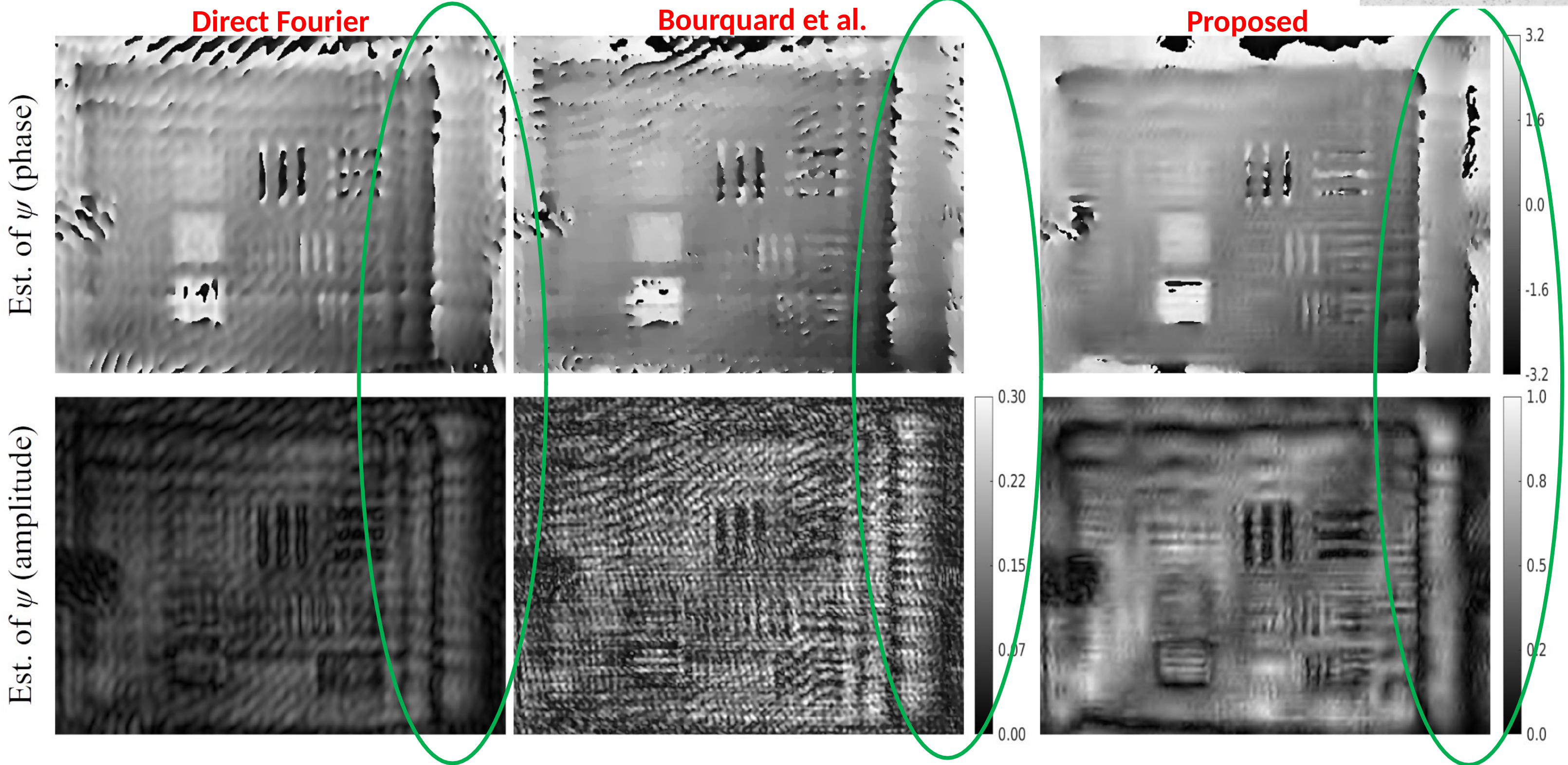
Input SNR 10dB

Results

- **Experimenting the algorithm on real data**



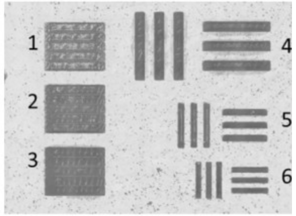
still some artifacts

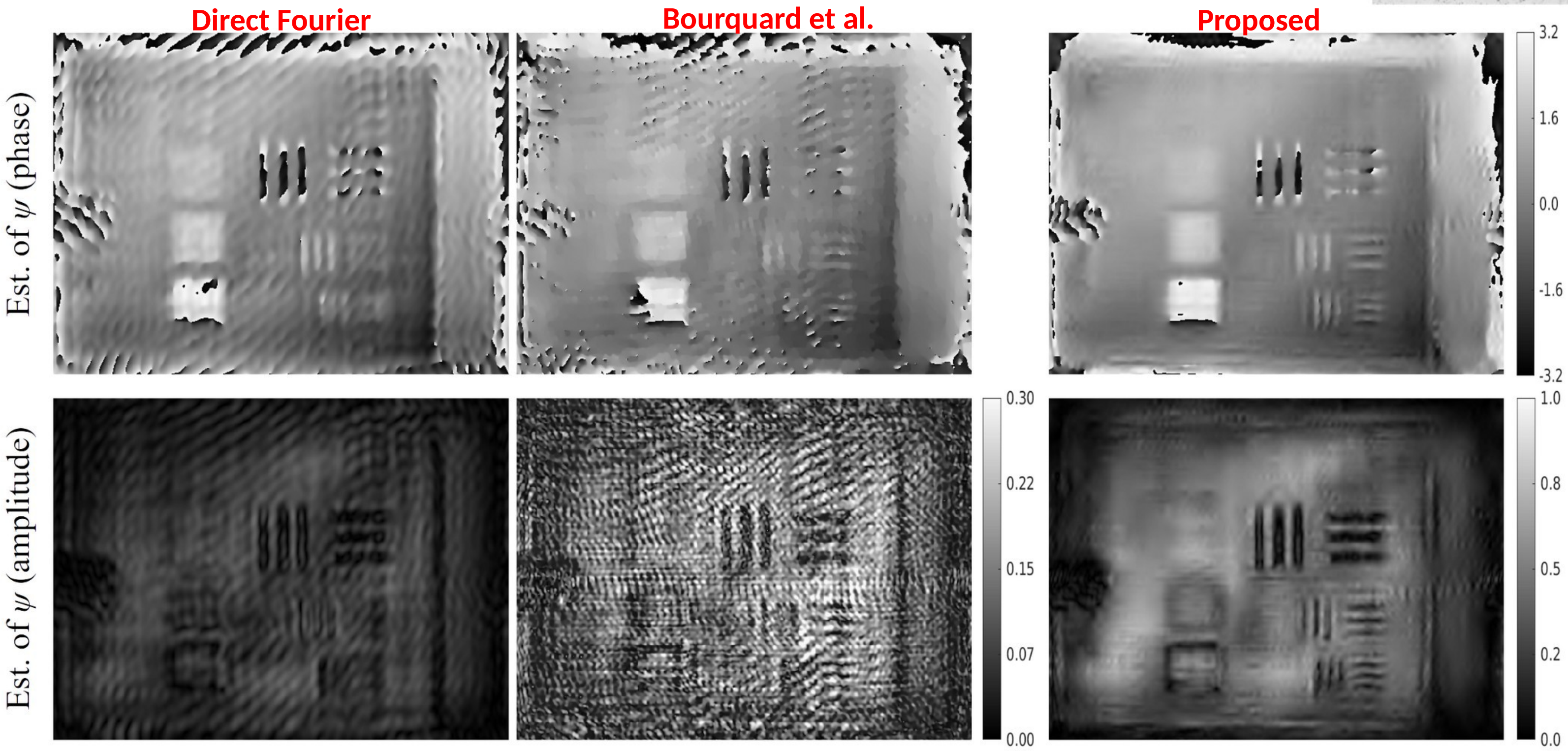


Results

- **Experimenting the algorithm on real data**

  + preprocessing by apodization





Conclusion

- We have demonstrated a new inverse method approach to reconstruct digital holography measurement
- In particular it addresses images obtained in THz where a lot of artifacts are present
- Our inverse approach is based on
  - Wavelet decomposition
  - Minimizing Data fidelity + regularization term using wavelet coefficient
  - iterative ADMM algorithm
    - adapted to complex numbers (typical of DH)
    - modified by us to cope with 2 variables
- The methods better performs compared to
  - usual direct Fourier transform approach
  - other inverse approach

Conclusion

Details can be found in

Research Article Vol. 41, No. 3 / March 2024 / *Journal of the Optical Society of America A* **A1**

Journal of the Optical Society of America A

OPTICS, IMAGE SCIENCE, AND VISION

ADMM-inspired image reconstruction for terahertz off-axis digital holography

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Received 25 August 2023; revised 24 October 2023; accepted 20 November 2023; posted 20 November 2023; published 8 December 2023