

Fases de condensados de Bose-Einstein espinoriales a temperaturas finitas y sus propiedades físicas

Eduardo Serrano Ensástiga
IPNAS, CESAM, University of Liège, Belgium

En colaboración con John Martin (ULiège) y Francisco Mireles (CNyN, UNAM)

PRA **104**, 063308 (2021), PLA **492**, 129188 (2023)
PRA **108**, 055308 (2023), SciPost Phys. **15**, 120 (2023)

1 Introducción

- a Condensados de Bose-Einstein (BEC)
- b BEC espinoriales y sus diagramas de fase de espín
- c Estados base y metaestables

2 Metodología

- a Método Hartree-Fock (HF)
- b Método HF + grupo puntual de simetría

3 Resultados

- a Diagrama de fases
- b Aplicación: Evoluciones en procesos súbitos (quench)
- c Propiedades físicas: Magnetización, correlaciones cuánticas

4 Conclusiones

Bose-Einstein condensates (BEC)

New state of matter

First (scalar) BEC (^{87}Rb at $T = 170\text{nK}$)
(Science 269, 5221 (1995)).

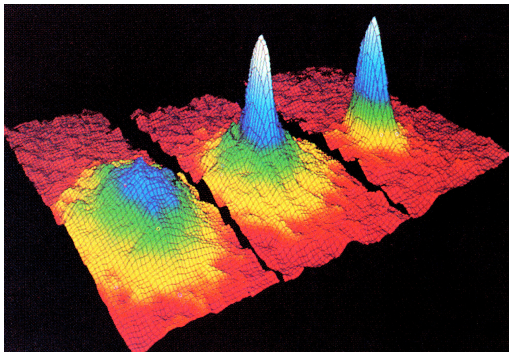
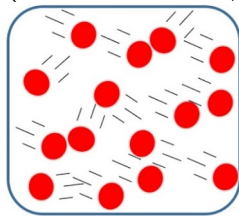
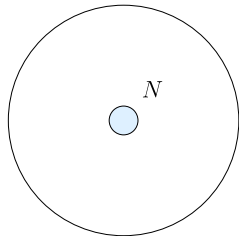


Image taken from nobelprize.org

Gas (General atoms at $T \neq 0$)



BEC (Bosonic atoms at $T < T_c$)



Condensados de Bose-Einstein (BEC)

Nuevo estado de la materia

Primer BEC escalar en 1995 (^{87}Rb a 170nK)
(Science 269, 5221 (1995)).

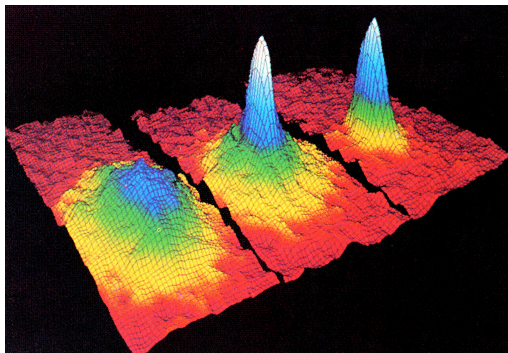
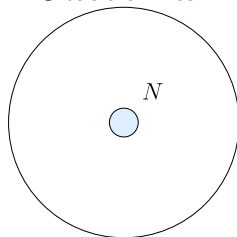
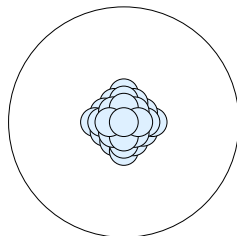


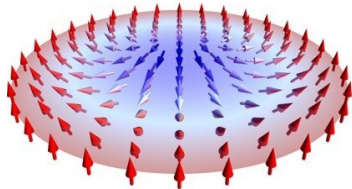
Imagen tomada de nobelprize.org

Situación real

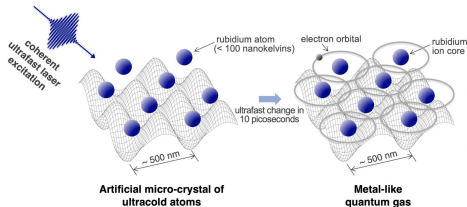


Dibujo esquemático

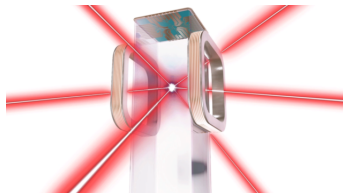




Skyrmions



PRL, **124**,253201(2020)



A BEC on the space (ISS)

Nature **582**, 193–197(2020)

- Systems that can be manipulated with great precision
- Quantum simulators of condensed matter
- Quantum computation
- High-precision experiments
- Topological defects; Spin textures, skyrmions

- Harvard Team: quantum simulator with 256 qubits, Nature **595** 227-232 (2021)

-New record set for lowest temperature- 38 pK PRL **127**, 100401 (2021)

BEC espinorial

Es un condensado en el que los átomos tienen espín total no cero y la trampa de confinamiento no desdobra los niveles hiperfinos (m_z).

Primero de $f = 1$ con ^{23}Na en (1998)
PRL **80**, 2027(1998)

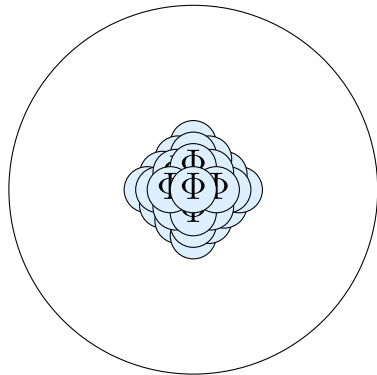
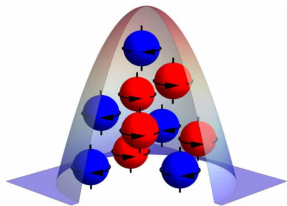


Imagen tomada de www.techexplorist.com ICFO P.Gomez

Estado base (de espín) del BEC espinorial

Estará dado por las interacción espín-espín entre los átomos.

Especies atómicas y valor total de espín (Phys. Rep. **520**, 253(2012).).

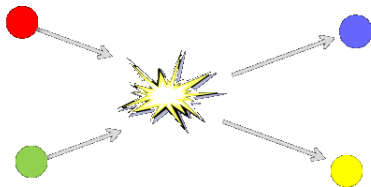
Atom	$f = j + i$
^1H	0, 1
^7Li	1, 2
● ^{23}Na	1, 2 ●
^{39}K	1, 2
^{41}K	1, 2
^{52}Cr	3
^{85}Rb	2, 3
^{87}Rb	1, 2
^{133}Cs	3, 4
^{164}Dy	8
^{168}Er	6

Hamiltoniano del BEC de espín-1

Aproximación: Gas suficientemente diluido

- Dispersión del tipo de orbital s
- Interacciones sólo por contacto
- Dos tipos de interacciones

$c_0 = \text{Int. espín-independiente}$, $c_1 = \text{Int. espín-dependiente}$



Hamiltoniano BEC de $f = 1$

$$H = \int d\mathbf{r} \left\{ h_s \hat{\Psi}^\dagger \hat{\Psi} + \frac{c_0}{2} \sum_{i,j} \hat{\psi}_i^\dagger \hat{\psi}_j^\dagger \hat{\psi}_j \hat{\psi}_i + \frac{c_1}{2} \sum_{\alpha, i, j, k, l} (F_\alpha)_{ij} (F_\alpha)_{kl} \hat{\psi}_i^\dagger \hat{\psi}_k^\dagger \hat{\psi}_l \hat{\psi}_j \right\}$$

Estado base de BECs espinorial

Ecuación de Gross-Pitaevskii para BEC de espín $f = 1$

Aproximaciones: Parte espinorial, gas diluido y teoría del campo medio $\langle \hat{\Psi} \rangle = \Phi$

$$E[\Phi] = \langle \hat{H} \rangle = \frac{c_0}{2} |\langle \Phi | \Phi \rangle|^2 + \frac{c_1}{2} \langle \Phi | \mathbf{F} | \Phi \rangle \cdot \langle \Phi | \mathbf{F} | \Phi \rangle, \quad \frac{\partial E[\Phi]}{\partial \Phi_i^*} = \mu \Phi_i,$$

$c_1 < 0$ Int. Ferromagnética

Estado base

Fase Ferromagnética (FM)

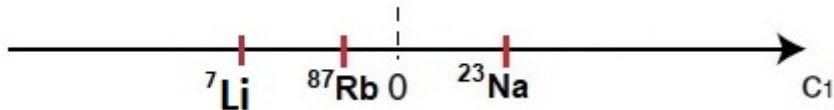
$$|\Phi\rangle = \sqrt{N}|1, 1\rangle = \sqrt{N}(1, 0, 0)^T$$

$c_1 > 0$ Int. Antiferromagnética

Estado base

Fase Polar (P)

$$|\Phi\rangle = \sqrt{N}|1, 0\rangle = \sqrt{N}(0, 1, 0)^T$$



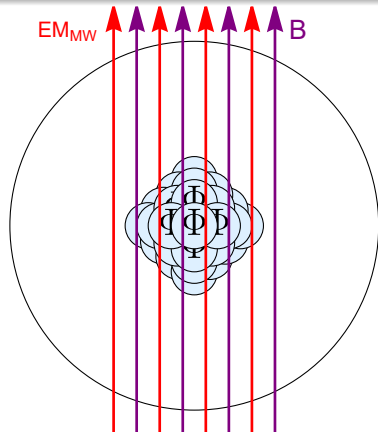
BEC antiferromagnético ($c_1 > 0$) de espín-1

Zeeman lineal F_z

$$p = -g\mu_B B,$$

Zeeman cuadrático F_z^2

$$q = q_B + q_{MW}$$
$$= \frac{(g\mu_B B)^2}{\Delta E_{hf}} + q_{MW}$$



EGP con int. Zeeman lineal y cuadrático (p, q)

$$E[\Phi] = \frac{c_0}{2} |\langle \Phi | \Phi \rangle|^2 + \frac{c_1}{2} \langle \mathbf{F} \rangle^2 - p \langle F_z \rangle + q \langle F_z^2 \rangle,$$

Diagrama de fases con los parámetros libres (q, p).

BEC antiferromagnético ($c_1 > 0$) de espín-1

Términos Zeeman

EGP con int. Zeeman lineal y cuadrático (p, q)

$$E[\Phi] = \frac{c_0}{2} |\langle \Phi | \Phi \rangle|^2 + \frac{c_1}{2} \langle \mathbf{F} \rangle^2 - p \langle F_z \rangle + q \langle F_z^2 \rangle,$$

Fase ferromagnética (FM)

$$|\Phi\rangle = \sqrt{N}(1, 0, 0)^T$$

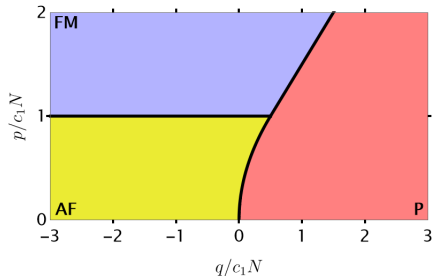
Fase polar (P)

$$|\Phi\rangle = \sqrt{N}(0, 1, 0)^T$$

Fase antiferromagnética (AF)

$$|\Phi\rangle = \sqrt{N}(\cos \chi, 0, \sin \chi)^T$$

$$\cos \chi = \sqrt{\frac{c_1 N + p}{2c_1 N}}$$



Representación estelar de Majorana

Una generalización de la esfera de Bloch

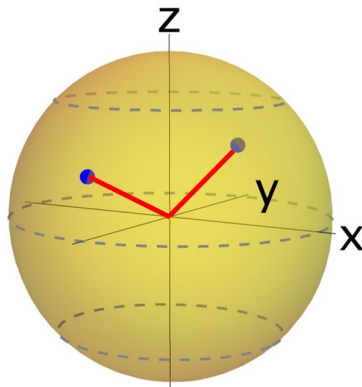
Representación estelar de Majorana, (Nuovo Cimento **9**, 43 (1932))

A un estado de espín $f = 1$ se puede representar con $2f = 2$ puntos (estrellas) en la esfera.

Cada estado de espín $f = 1$ tiene asociado 2 estrellas.



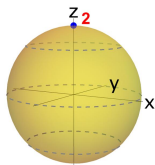
$$|\psi\rangle = (\lambda_1, \lambda_0, \lambda_{-1})^T$$



Propiedades de las fases (Simetrías)

Ferromagnético (FM)

$$|\psi\rangle = \sqrt{N}(1, 0, 0)^T$$

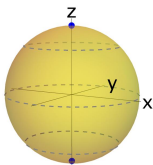


Simetrías

$$SU(2)$$

Polar (P)

$$|\psi\rangle = \sqrt{N}(0, 1, 0)^T$$

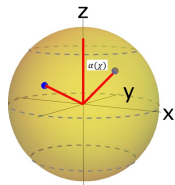


Simetrías

$$SU(2) \times \mathbb{Z}_2$$

Antiferromagnético (AF)

$$|\psi\rangle = \sqrt{N}(\cos \chi, 0, \sin \chi)^T$$



Simetrías

$$\mathbb{Z}_2 \times \mathbb{Z}_2$$

Propiedades de las fases (Magnetización por átomo)

Ferromagnético (FM)

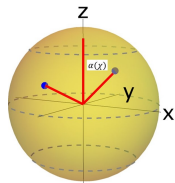
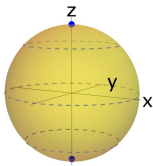
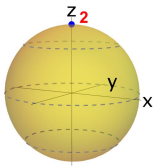
Polar (P)

Antiferromagnético (AF)

$$|\psi\rangle = \sqrt{N}(1, 0, 0)^T$$

$$|\psi\rangle = \sqrt{N}(0, 1, 0)^T$$

$$|\psi\rangle = \sqrt{N}(\cos \chi, 0, \sin \chi)^T$$



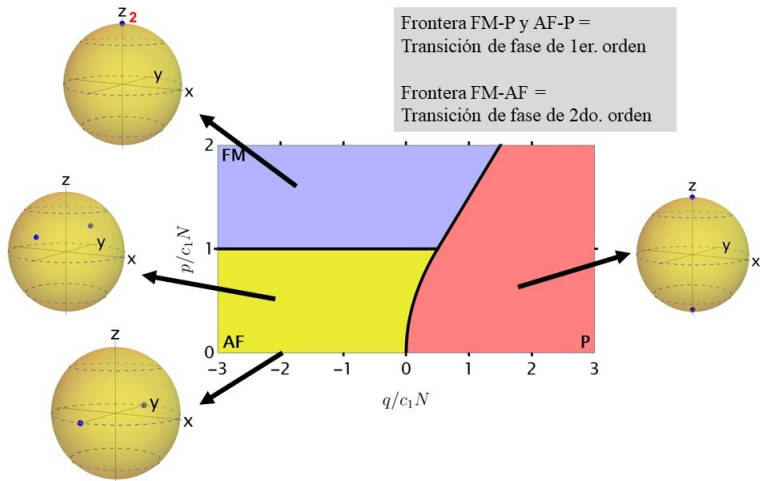
$$M_z = \langle F_z \rangle / N, \quad M_{\perp} = \sqrt{\langle F_x \rangle^2 + \langle F_y \rangle^2} / N$$

$$M_z = 1, \quad M_{\perp} = 0$$

$$M_z = 0, \quad M_{\perp} = 0$$

$$M_z = \cos(2\chi), \quad M_{\perp} = 0$$

Transiciones de fase y fases metaestables



Al cruzar una transición de fase de primer orden,
¿el estado se transforma en uno metaestable?

¿O se vuelve inestable y los átomos migran rápidamente al nuevo estado base?

Metodología

Método Hartree-Fock (HF)

Parámetro Φ + perturbación

$$\hat{\psi}_i = \phi_i + \hat{\delta}_i.$$

$$\rho = \rho^c + \rho^{nc},$$

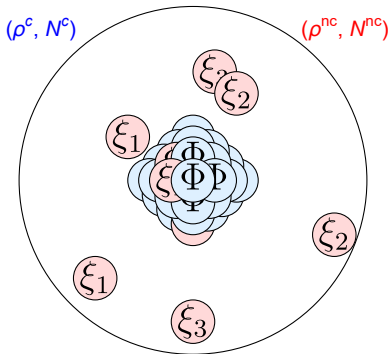
Fracción condensada $N^c = \text{Tr}(\rho^c)$

$$\rho^c = |\Phi\rangle\langle\Phi|$$

Fracción no-condensada $N^{nc} = \text{Tr}(\rho^{nc})$
(térmica)

$$\rho_{ij}^{nc} = \langle \hat{\delta}_i^\dagger \hat{\delta}_j \rangle$$

$$\rho^{nc} = \sum_{j=1}^3 \Lambda_j(\kappa_j) |\xi_j\rangle\langle\xi_j|$$



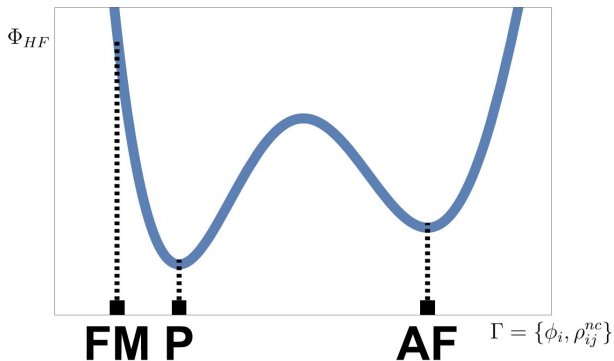
Fases permitidas o prohibidas

La fase asociada al estado base y los metaestables satisfacen que $\kappa_j > 0$ para toda j . De lo contrario es una fase prohibida.

Estado base minimiza $\Phi_{HF} = E_{HF} - TS_{HF}$.

Ejemplo de los tipos de fases

Base, metaestable y prohibido



En este caso:

- Fase **FM** = Fase prohibida
- Fase **P** = Fase del estado base
- Fase **AF** = Fase metaestable

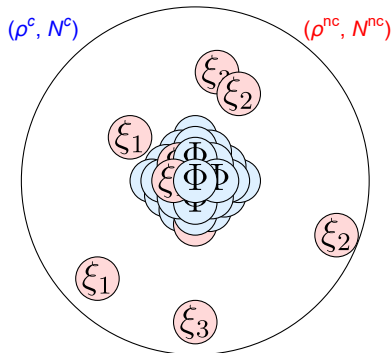
Método Hartree-Fock (HF)

Interacciones no-triviales

$$\rho^c \leftrightarrow \rho^c,$$

$$\rho^{nc} \leftrightarrow \rho^{nc},$$

$$\rho^c \leftrightarrow \rho^{nc},$$



Energía HF

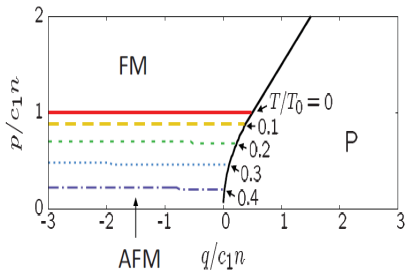
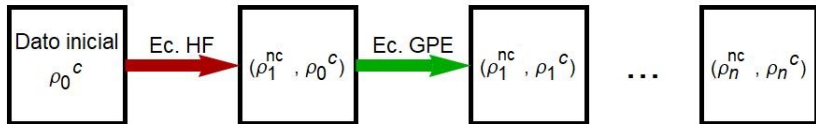
$$E_{HF} = E_s + \text{Tr} [\rho (-pF_z + qF_z^2)] + \frac{c_0}{2} (N^2 + \text{Tr} [\rho^{nc} (2\rho^c + \rho^{nc})]) \\ + \frac{c_1}{2} \sum_{\alpha} (\text{Tr} [\rho F_{\alpha}]^2 + \text{Tr} [F_{\alpha} \rho^{nc} F_{\alpha} (2\rho^c + \rho^{nc})]),$$

$$(\text{EGP}) \delta E_{HF} / \delta \phi_m^* = \mu \phi_m, \quad (\text{EHF}) A_{ij} = \delta E_{HF} / \delta \rho_{ji}^{nc}.$$

Método de Hartree-Fock (HF)

Método de HF en BEC spin-1 (PRA 85, 053611(2012))

- Método autoconsistente y numéricamente.



Simetrías heredadas (Blaizot y Ripka, *Quantum theory of finite systems*)

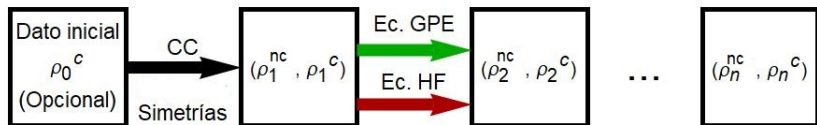
Si ρ^c y \hat{H} tienen simetrías en común, entonces ρ^{nc} los hereda.



Ventajas

- Método autoconsistente se reduce a un sistemas de tres ecuaciones algebro-trascendentales
- En varios casos reduce los parámetros libres de ρ^c y ρ^{nc} .
- Nuevos resultados: Estados base y metaestables
- Resultados analíticos a $T < T_0$.

Ejemplo del metodo HF + simetrías



Para la fase FM

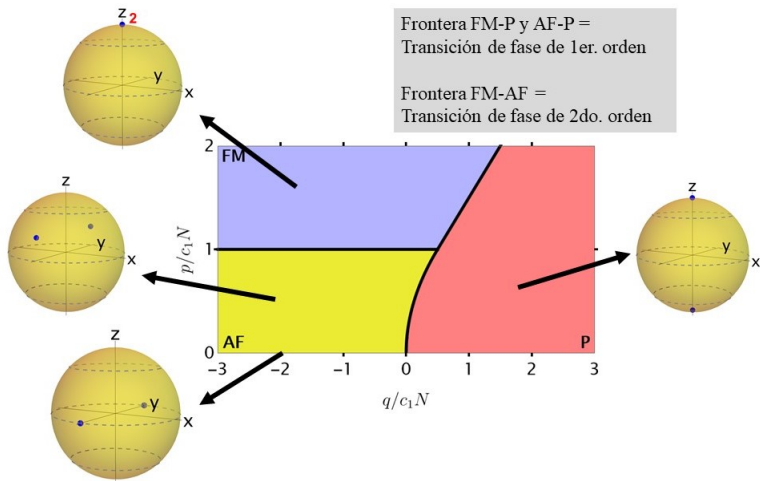
$$\rho^c = |\Phi\rangle\langle\Phi|, \quad |\Phi\rangle = \sqrt{N^c}(1, 0, 0)^T$$
$$\rho^{nc} = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{pmatrix} \xrightarrow{\text{Simetrías}} \rho^{nc} = \begin{pmatrix} \Lambda_1(\kappa_1) & 0 & 0 \\ 0 & \Lambda_0(\kappa_0) & 0 \\ 0 & 0 & \Lambda_{-1}(\kappa_{-1}) \end{pmatrix}$$

Resultado

$$\begin{aligned} \kappa_1 &= (c_0 + c_1)(N - \Lambda_1 - \Lambda_0 - \Lambda_{-1}), \\ \kappa_0 &= p - q - (c_0 + c_1)\Lambda_1 + (c_0 - c_1)\Lambda_0 + 2c_1\Lambda_{-1}, \\ \kappa_{-1} &= 2(p - c_1N) - (c_0 + c_1)\Lambda_1 + 2c_1\Lambda_0 + (c_0 + 5c_1)\Lambda_{-1}. \end{aligned}$$

Resultados

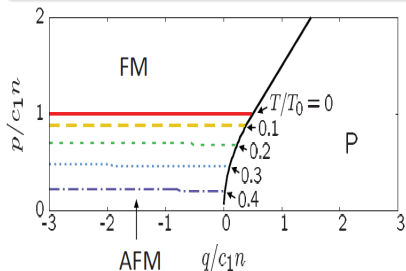
Transiciones de fase y fases metaestables



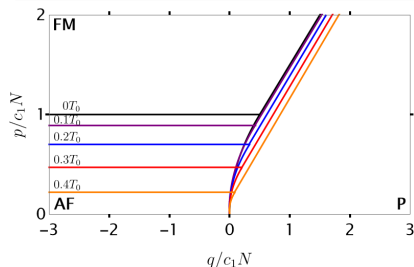
- Región del diagrama fase en el que la fase existe (base ó metaestable)
- Cambio de las transiciones de fase con respecto a la $T \neq 0$

Desviaciones de las transiciones de fase

PRA **85**, 053611(2012)



Método de HF y simetrías



Aproximación analítica de la frontera FM-P

$$\Phi_{HF}^{(FM)} - \Phi_{HF}^{(P)} = \frac{Ng}{2} (2(q - p) + c_1Ng),$$

donde $g = g(T) = 1 - AT^{3/2}$.

Resultados ($T = 0$)

Diagrama de fases base (ground)

- Mean-field theory
- PRA **85**, 053611(2012)

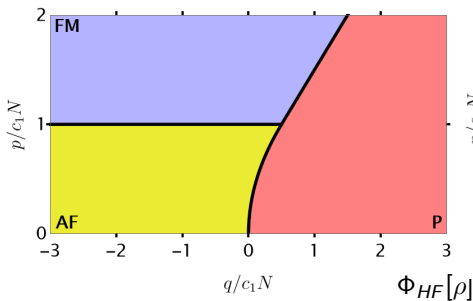
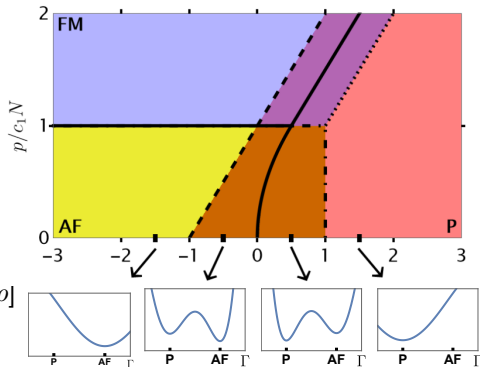
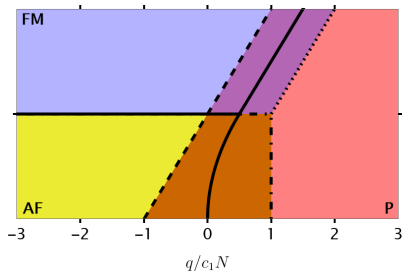


Diagrama de fases metaestables

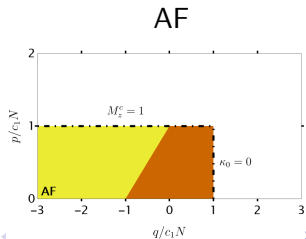
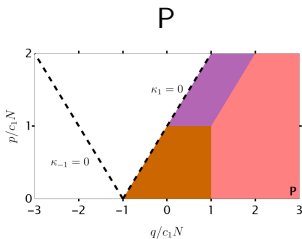
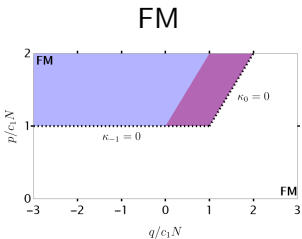
Método de HF y simetrías



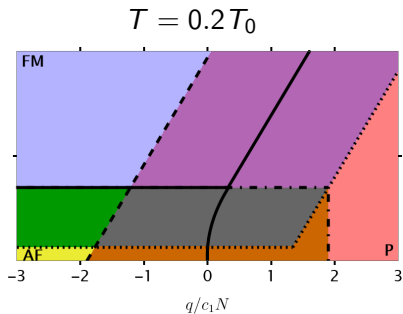
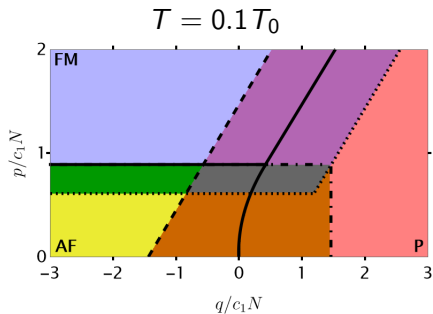
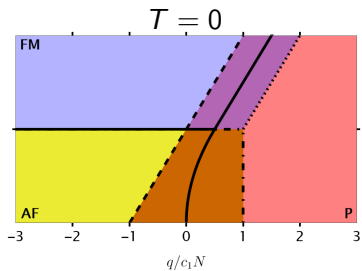
Región de cada fase por separado



La región de cada fase son



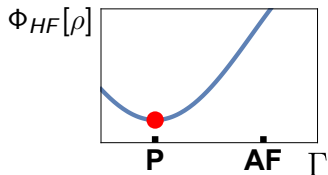
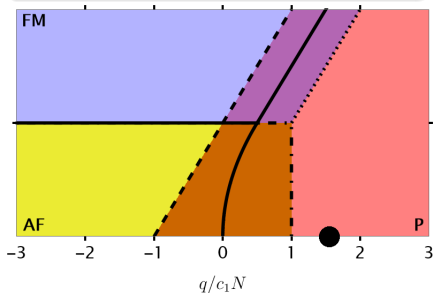
Resultados ($T \neq 0$)



Dinámica quench en BEC espinoriales

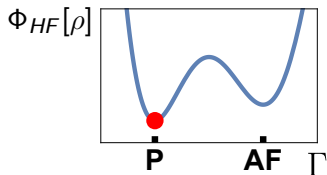
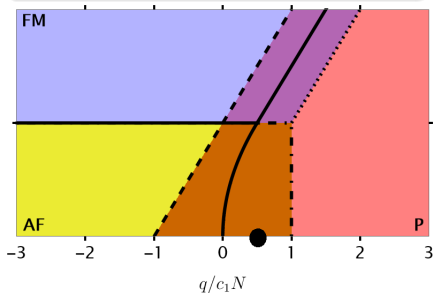
Hamiltoniano y estado inicial

$$(q, p) = (1.5, 0)c_1 N$$



Hamiltoniano final y evolución

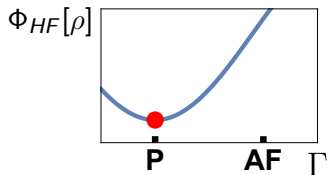
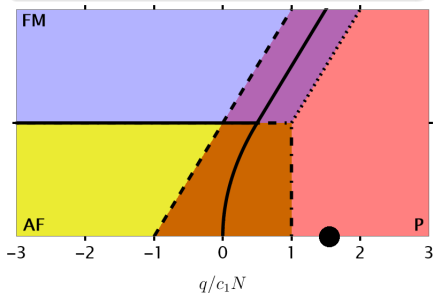
$$(q, p) = (0.5, 0)c_1 N$$



Dinámica quench en BEC espinoriales

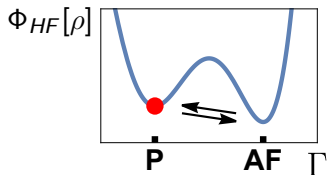
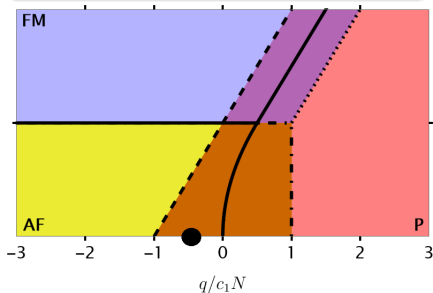
Hamiltoniano y estado inicial

$$(q, p) = (1.5, 0)c_1 N$$



Hamiltoniano final y evolución

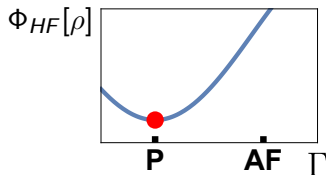
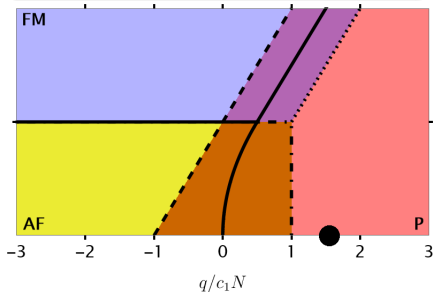
$$(q, p) = (-0.5, 0)c_1 N$$



Dinámica quench en BEC espinoriales

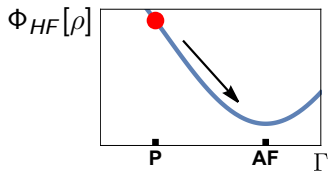
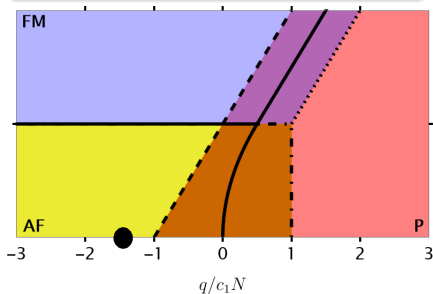
Hamiltoniano y estado inicial

$$(q, p) = (1.5, 0)c_1 N$$



Hamiltoniano final y evolución

$$(q, p) = (-1.5, 0)c_1 N$$



Evidencia experimental

Diferentes tipos de dinámica quench

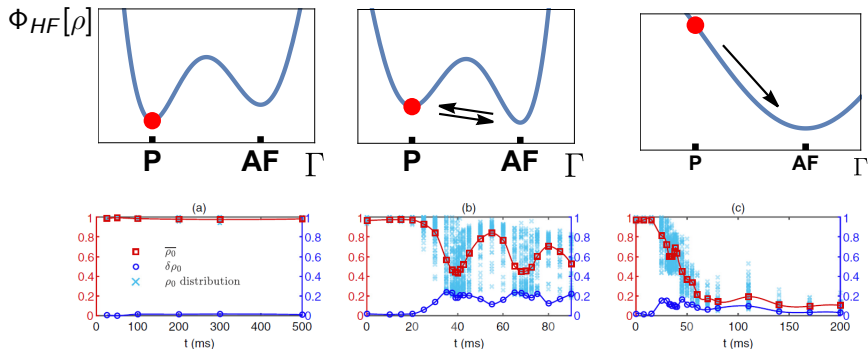
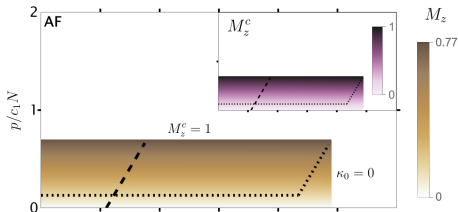
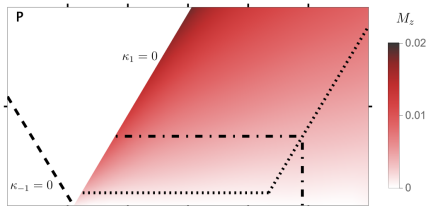
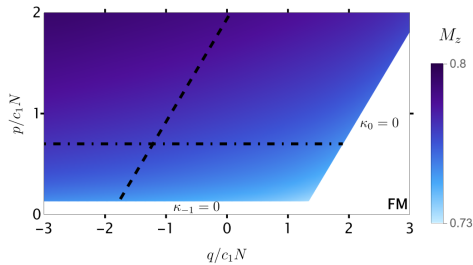
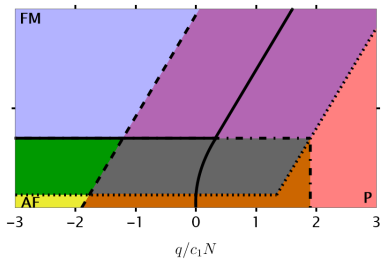


FIG. 2. Observed time evolutions of $\bar{\rho}_0$ and corresponding theoretical predictions derived from numerical simulations of the quench dynamics at three q_f/c_2 values (see text). (a, b, c) Experimental results at $c_2/h = 31$ Hz and $q_f/c_2 = 0.65, -0.48,$ and -0.97 , respectively.

Resultados experimentales publicados en PRA, **100**, 013622 (2019).

Magnetización $M_z = \text{Tr}(\rho F_z)$

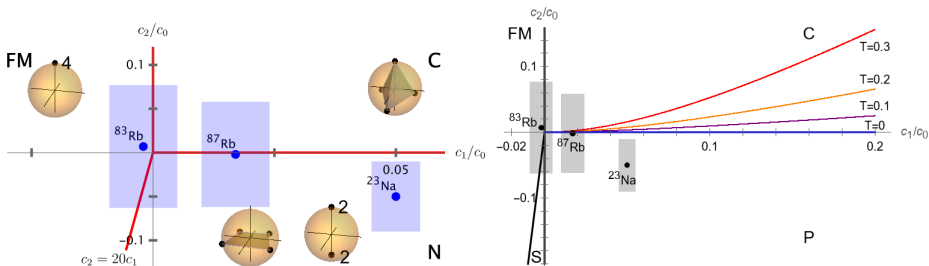


BEC de spín 2

Diagrama de fases base a diferentes temperaturas

Ecuación de MF de espín-2 BEC (ahora 3 interacciones)

$$E[\Phi] = \langle \hat{H} \rangle = \frac{c_0}{2} |\langle \Phi | \Phi \rangle|^2 + \frac{c_1}{2} |\langle \Phi | \mathbf{F} | \Phi \rangle|^2 + \frac{c_2}{10} |\Phi^\dagger \mathbf{T} \Phi|^2,$$



Correlaciones cuánticas en BEC

Entrelazamiento

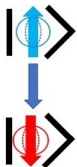
Sistemas de espín-1 = Qubit-qubit system $\mathcal{H}_2^{\otimes 2}$

Maximally entangled state (N=1)

$$| \uparrow \uparrow \rangle + | \downarrow \downarrow \rangle$$

Measurement of qubit A

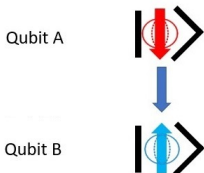
Case 1



Qubit A

Qubit B

Case 2



Qubit A

Qubit B

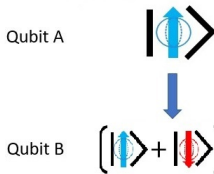
Qubit B is completely determined
[Correlation between A and B]

Separable state (N=0)

$$\left(| \uparrow \uparrow \rangle + | \downarrow \downarrow \rangle \right) \left(| \uparrow \uparrow \rangle + | \downarrow \downarrow \rangle \right)$$

Measurement of qubit A

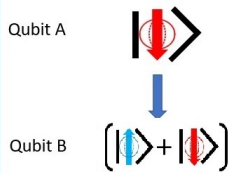
Case 1



Qubit A

Qubit B

Case 2

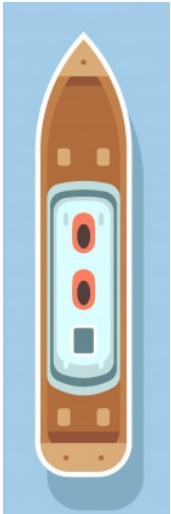


Qubit A

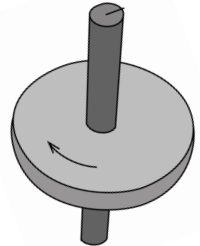
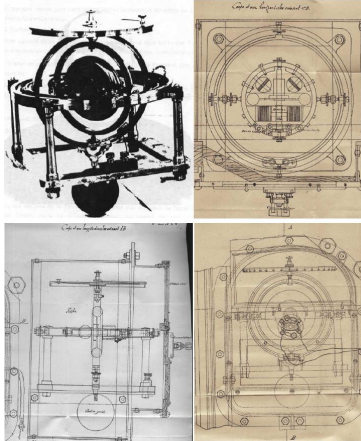
Qubit B

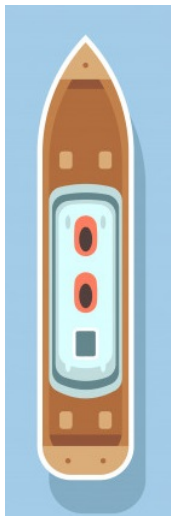
Qubit B is independent of the result
[No correlation between A and B]

Metrology: The science of measurements



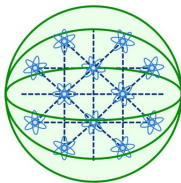
How can we measure a rotation on a boat in the sea?





Rotosensores cuánticos

Quantum System



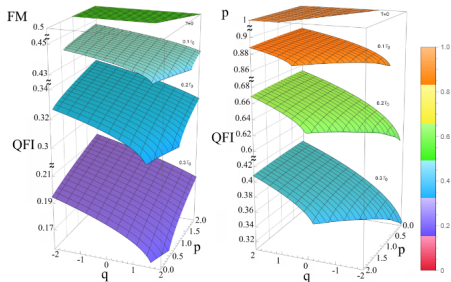
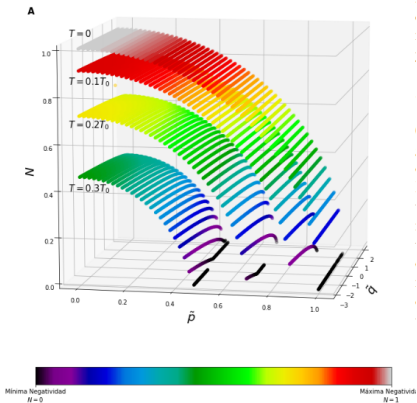
¿Cuál es el estado cuántico más óptimo para medir una rotación? Para cuantificar la susceptibilidad, se usa la Información Cuántica de Fisher (QFI)

$$QFI(J_z, \rho) = \left. \frac{\partial^2 F(\rho, e^{-i\eta J_z} \rho e^{i\eta J_z})}{\partial \eta^2} \right|_{\eta=0}$$

Correlaciones cuánticas

Entrelazamiento y QFI para fases de BEC espín-1

AF



Física de fases de BEC de espín-1 con interacción antiferromagnética

ESE and Francisco Mireles, PRA 104, 063308 (2021)

Caracterización de fases de BEC espinoriales

ESE and Francisco Mireles, PLA 492, 129188 (2023), PRA 108, 055308 (2023)

Entrelazamiento máximo para sistemas de espín 1

ESE and John Martin, SciPost Phys. **15**, 120 (2023)

Future work

Diagrama de fases de otras especies de BEC: BEC $f=1$ ferromagnético ^{87}Rb , y ^7Li

Dinámica de BEC espinoriales

Correlaciones cuánticas de BEC de espín más alto

¡Muchas gracias por su atención!

Optimización del entrelazamiento

Maximización de entrelazamiento después de una compuerta unitaria

