Invariant Kalman Filtering with State Equality Constraints

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1 Introduction

The Extended Kalman Filter (EKF) is the *de facto* standard in many nonlinear filtering applications. Based solely on a dynamical and measurement model, this filter is unable to account for the geometry and symmetries of the considered problem. The latter concern progressively led to the development of geometrical filtering methods that became state-of-the-art in various navigation applications, like Simultaneous Localization And Mapping (SLAM) [1], inertial navigation [2], or attitude and pose estimation [3]. The invariance and equivariance properties of those methods are reminiscent of linear systems in many regards.

In various contexts, equality constraints can force the state to live in a subspace of the state space. Incorporating such deterministic information in the Kalman framework is challenging and constitutes a field of study on its own, see [4]. In this work, we advocate that nonlinear equality constraints can be treated as noise-free pseudo-measurements.

2 Noise-free pseudo-measurements in Kalman filtering

Consider the equality constraint $y_k = h(x_k)$, with x_k the state to estimate. The Kalman framework assumes $x_k \sim \mathcal{N}(\hat{x}_k, P_k)$, where the estimate $\hat{x}_k$ and error covariance P_k constitute the filter's belief. After incorporating noise-free pseudo-measurement y_k, we expect the following properties.

1. *State update consistency.* The updated state $\hat{x}_k^+$ satisfies the constraint, that is, $y_k = h(\hat{x}_k^+)$.

2. *Riccati update consistency.* The updated covariance P_k^+ exhibits null variance in the perfectly observed directions, that is, $\frac{\partial h}{\partial x}(\hat{x}_k^+)P_k^+\frac{\partial h}{\partial x}(\hat{x}_k^+)^T = 0_{m\times m}$.

3. *Update compatibility.* Assume the current estimate satisfies a previously enforced constraint $c_k = g(x_k)$, so that $c_k = g(\hat{x}_k)$ and $\frac{\partial g}{\partial x}(\hat{x}_k)P_k\frac{\partial g}{\partial x}(\hat{x}_k)^T = 0_{m\times m}$. After updating the belief with pseudo-measurement y_k, the updated state still satisfies the constraint defined by function g, that is, $c_k = g(\hat{x}_k^+)$.

In this work, we first show the EKF does not satisfy any of those properties. Then, we prove the invariant extended Kalman filter (IEKF) developed in [1] naturally satisfies the second and third properties, and we develop an improved update stage to ensure the first one.

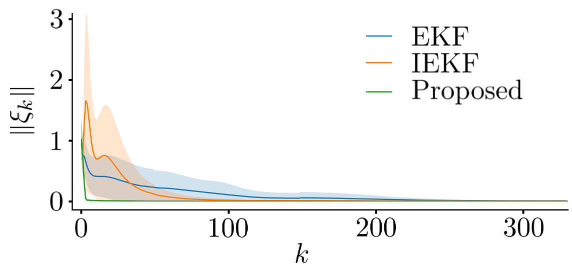

Figure 1: Error norm average and std over 30 simulations for the problem of estimating the extended pose of the hook of a crane mounted with an IMU. The cable length is perfectly known and is used as pseudo-measurement. Belief propagation and update are run at the same frequency of 100 Hz.

3 Application

We consider the concrete problem of the extended pose estimation of the hook of a crane evolving in two dimensions, to which an IMU is rigidly attached. The length of the cable is assumed to be perfectly known and is used as a pseudo-measurement. The invariant filtering technique developed in this work shows fast convergence and completely outperforms the EKF, as shown in Figure 1.

References

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