Mixture Density Network for the Prediction of the Wall Shear Stress including its Statistical Moments for Turbulent Separated Flows

Machine Learning for Fluid Dynamics

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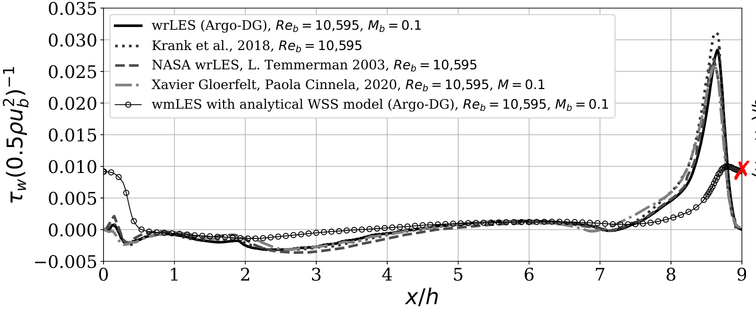
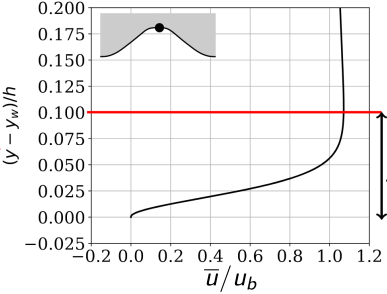
Motivation to improve WSS model for turbulent separation





Observation: Misprediction of (1) separation and (2) reattachment location, and (3) underestimation of friction peak. There is room for improvement.

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Data-driven WSS model - Motivations

Recent advances in **hardware** (mostly GPUs and now TPUs)

Exponential generation and accumulation of high-quality **data**

Now possible to train **deeper and deeper** neural networks

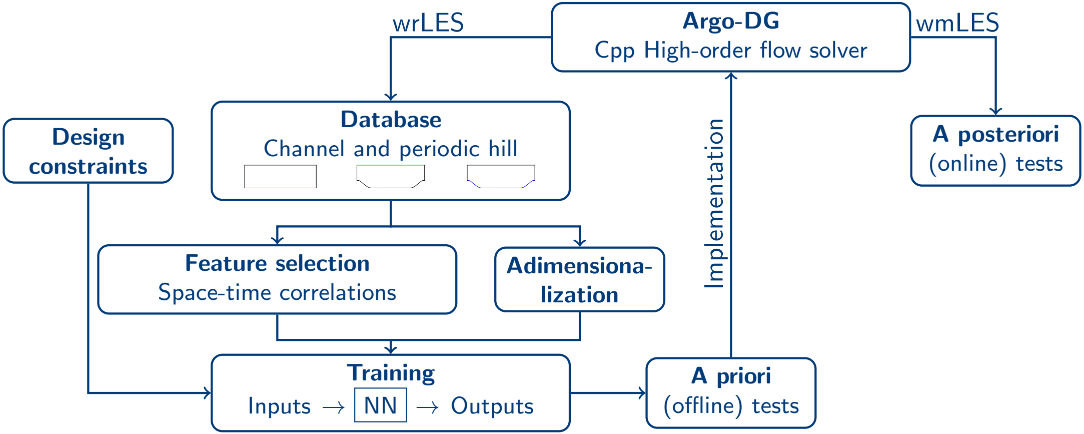
Making **no-prior assumptions** on the data

Among all the possible solutions to **improve WSS model**, we selected neural networks, the core element of Deep Learning.

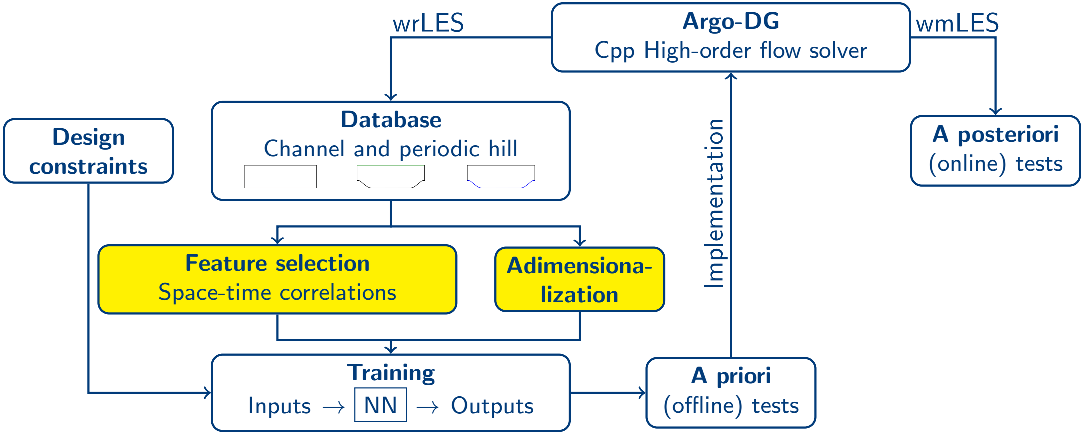
Better prediction of the **instantaneous** behaviors of the wall shear stress

Moreover, to cope with the lack of **variance**, the network is trained to predict a **distribution** rather than a point estimate.

Content


wrLES
Argo-DG
Cpp High-order flow solver
wmLES
Design constraints
Database
Channel and periodic hill
Implementation
A posteriori
(online) tests
Feature selection
Space-time correlations
Adimensiona-lization
Training
Inputs → NN → Outputs
A priori
(offline) tests

Content



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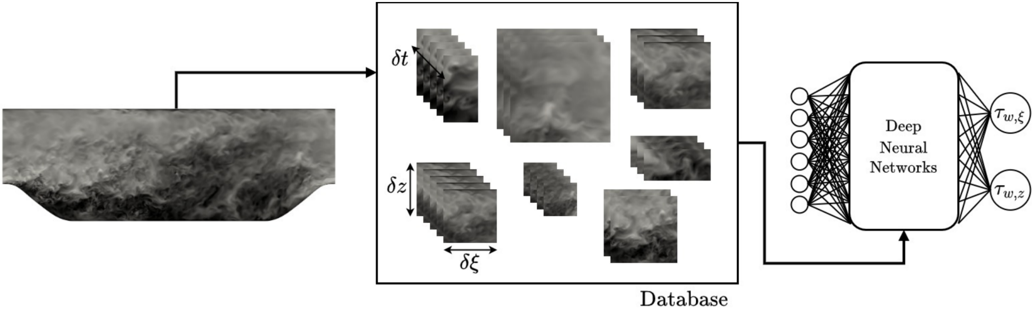
Feature Selection - Pearson and distance correlation

Prior to training the NN model on the collected data, need to evaluate the **relevance** of the input variables ($\mathbf{u}$, ∇p) w.r.t. the target variable (τ_w).



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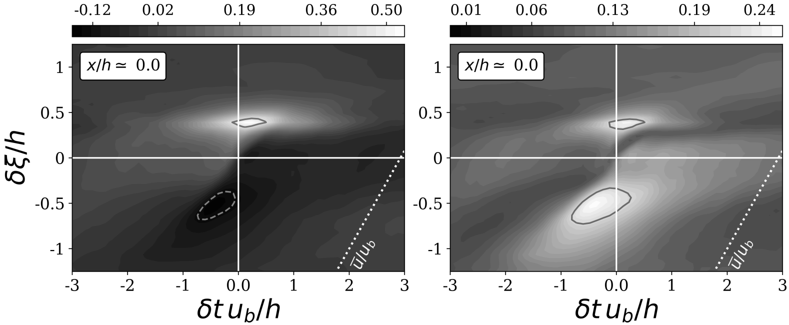
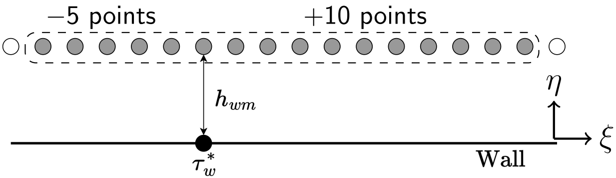
Feature Selection - Pearson and distance correlation

$R(u_\xi, \tau_\xi)$

at separation vicinity



This analysis results in an *appropriate* stencil:



Preprocessing - Non-dimensionalization

The non-dimensionalization of the input features is helpful to train a model on a limited dataset that will then be able to generalize to flows with different length scales, velocity scales, and fluid properties.

Inputs			**Outputs**
Velocity	Pressure gradients	Curvature	Wall shear stress
$\boldsymbol{u}^* = \dfrac{\boldsymbol{u}\, h_{wm}}{\nu}$	$\boldsymbol{u}_p^* = \dfrac{\boldsymbol{u}_p\, h_{wm}}{\nu}$	$\mathcal{K}^* = \mathcal{K} h_{wm}$	$\boldsymbol{\tau}_w^* = \text{sign}\left(\boldsymbol{\tau}_w\right) \dfrac{y}{\nu} \sqrt{\dfrac{\lvert \boldsymbol{\tau}_w \rvert}{\rho}}$

where $\boldsymbol{u}_p = \left(\frac{\nu}{\rho} \nabla p\right)^{1/3}$ is a velocity based on the pressure gradient.

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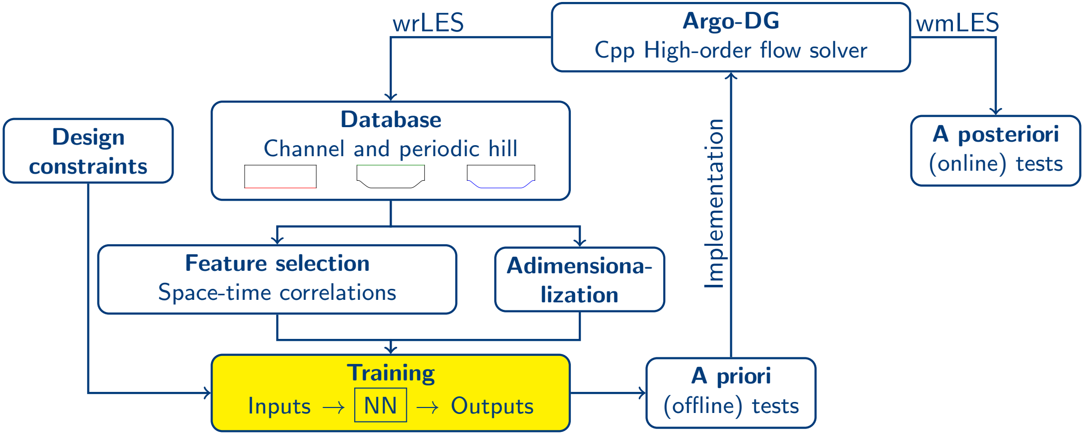

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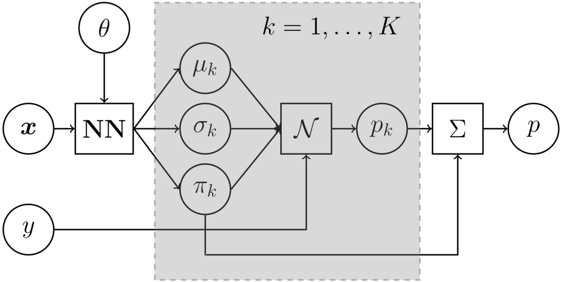
Mixture Density Networks (MDN) are the NN implementation of the Gaussian Mixture Models ...

$$p(\tau_w|\mathbf{x}) = \sum_{k=1}^{K} \pi_k p_k = \sum_{k=1}^{K} \pi_k \mathcal{N}(\tau_w|\mu_k(\boldsymbol{x}), \sigma_k(\boldsymbol{x}))$$



... and are trained with the **Negative Log-Likelihood.**

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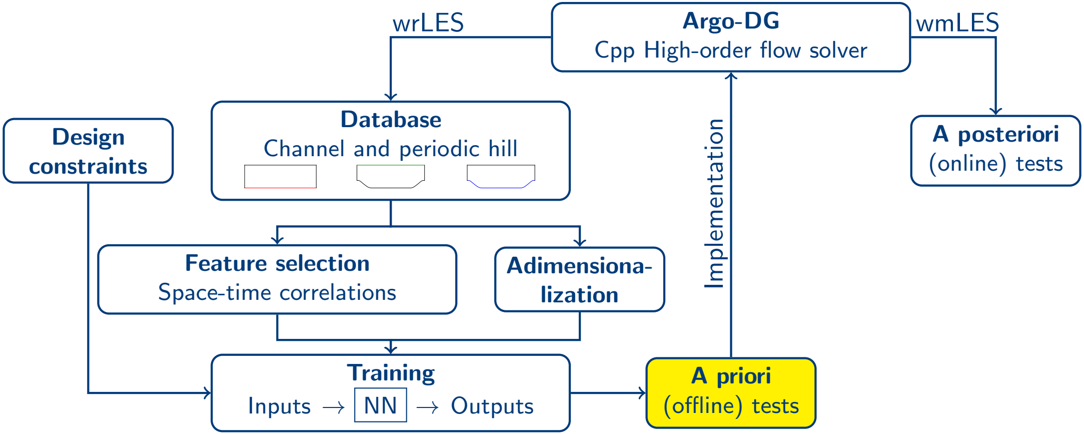
Content



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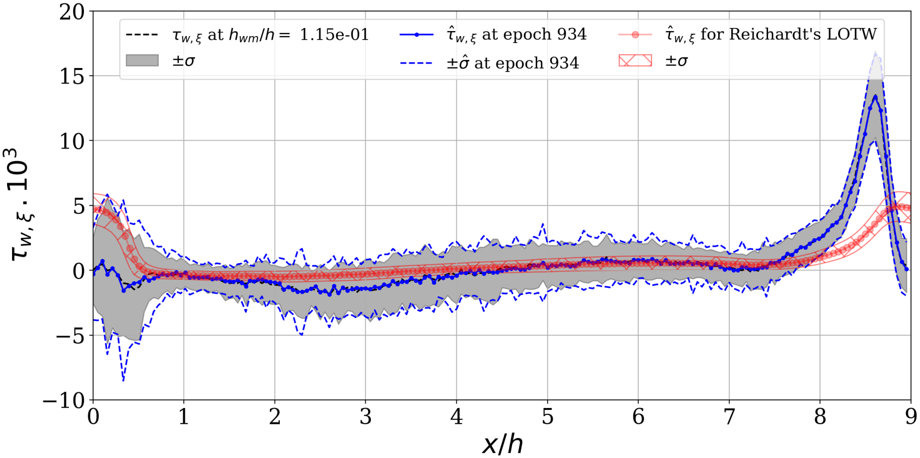
Data-driven WSS model - A priori testing

A priori prediction on the **lower wall** of the two-dimensional periodic hill,



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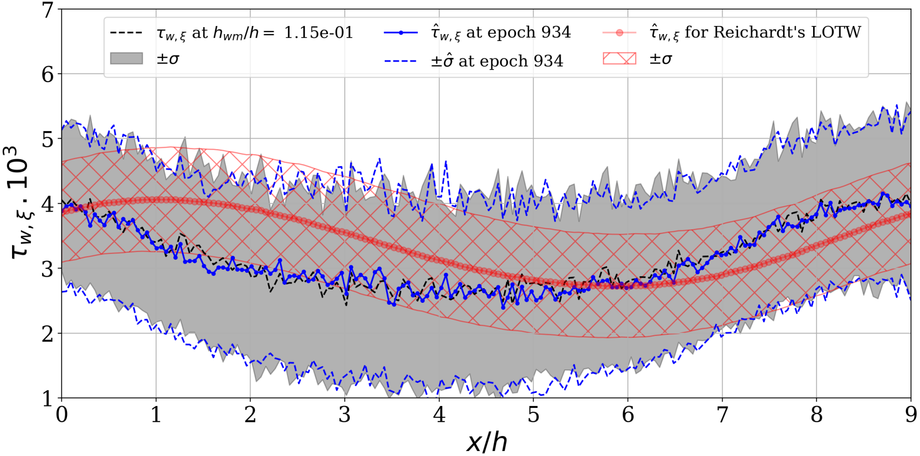
Data-driven WSS model - A priori testing

A priori prediction on the **upper wall** of the two-dimensional periodic hill,



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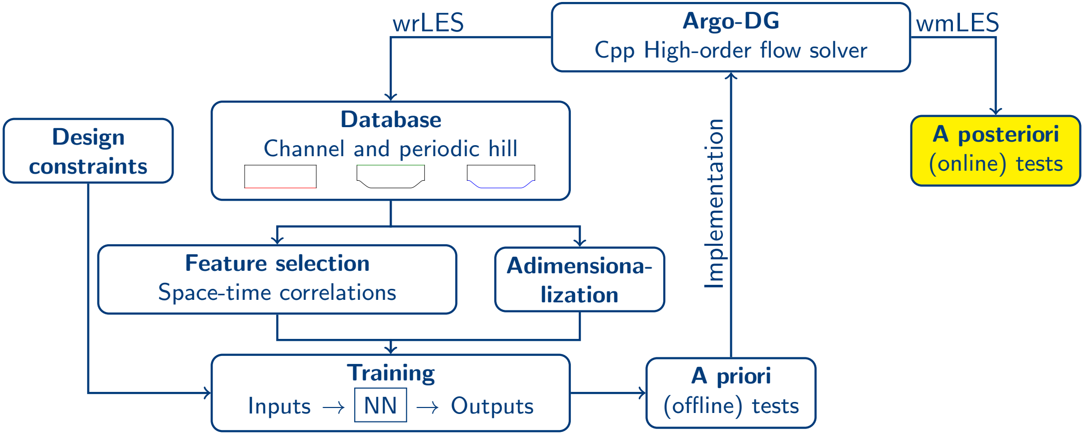

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Data-driven WSS model - A posteriori testing

"The production environment is different from the training environment because the wall model will interact with the resolved volume data. Moreover, the reattachment location is very sensitive to small errors."

	WSS model	**Align.**	p	**DOF**	h_{wm}/h	**Accum.**
`DD-NA-512-p3`	CNN-skip-GMH	✗	3	25,473,600	0.1	$\sim 36\, t_c$
`DD-NA-512-p4`	CNN-skip-GMH	✗	4	28,788,750	0.08	$\sim 25\, t_c$
`DD-A-512-p4`	CNN-skip-GMH	✓	4	28,788,750	0.1	$\sim 19\, t_c$
`Rcht-A-512-p4`	AWSSR	✓	4	28,788,750	0.1	$\sim 15\, t_c$

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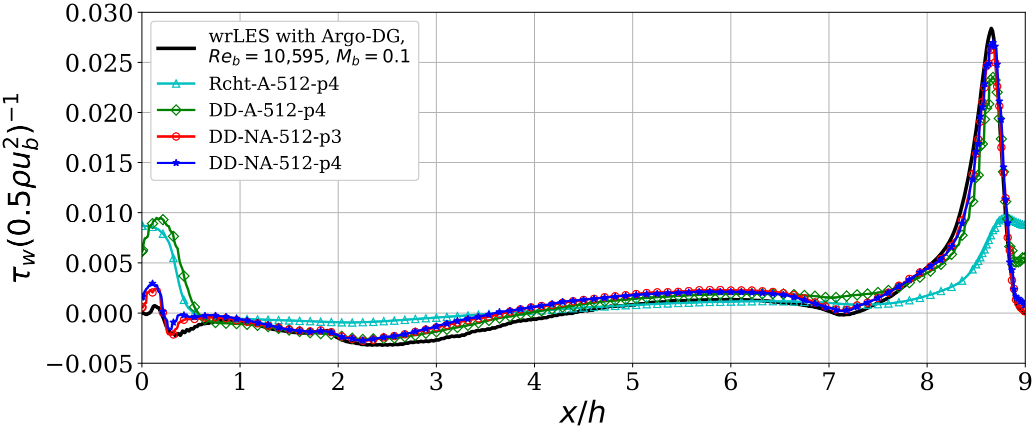
Friction coefficient on the **lower** wall,



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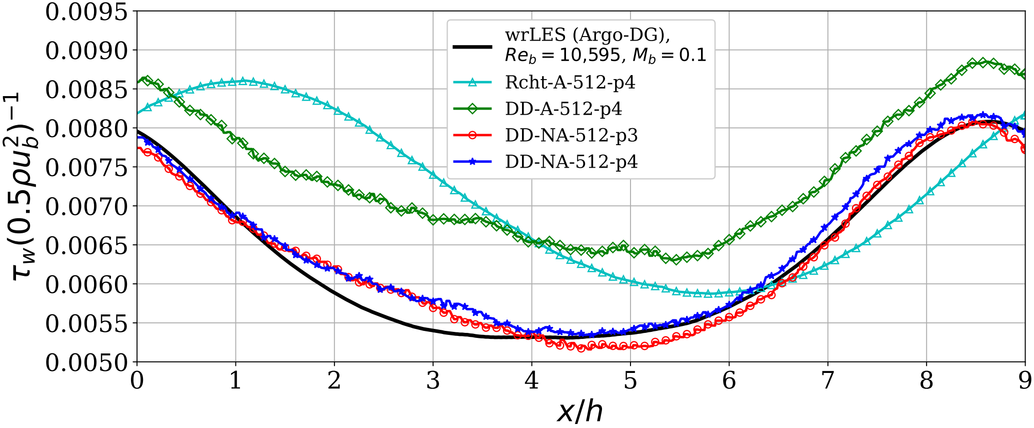
Friction coefficient on the **upper** wall,



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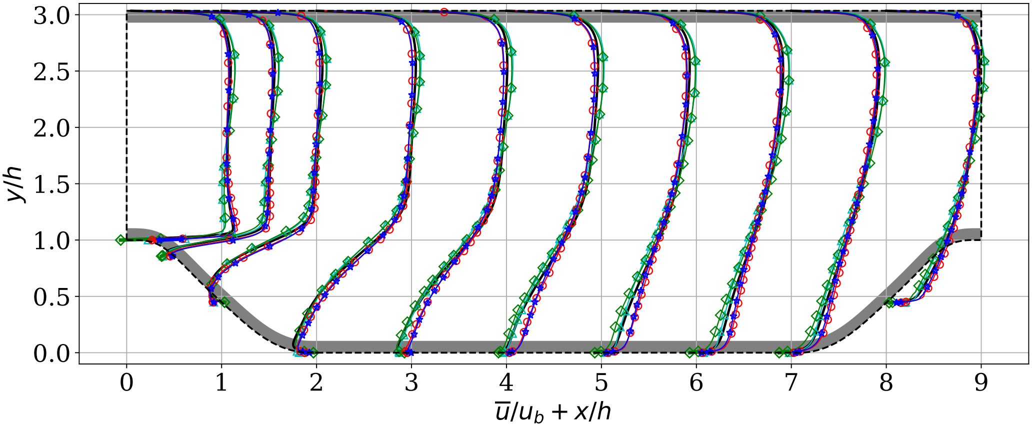
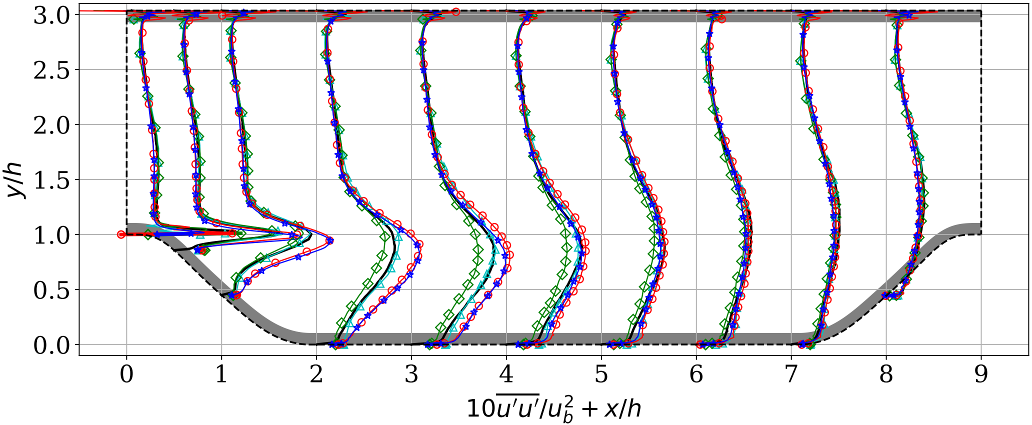
Mean velocity profile



Mean Reynolds stress profile



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Data-driven WSS model - Conclusion

- **Objective.** Development of a novel WSS model for the separation/reattachment phenomenon.
- **Scientific contribution.** Generate a data-driven WSS model to predict a distribution that better captures the instantaneous behaviour of wall shear stress.
- **Positive impact.** A great improvement in the WSS curve is observed on both the upper and lower walls of the two-dimensional periodic hill.
- **Points to be improved.** The reattachment location is underestimated and this affects the physics in the whole domain. Dupuy *et al.* [1, 2] have also observed this underestimation on other test cases featuring separation. The volume data may be more influenced by the direction of the wall shear stress (which is currently randomly generated) than its amplitude.

Bibliography

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