Saturation effects in elastic scattering at the LHC

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Abstract

The problems linked to the extraction of the basic parameters of the hadron elastic scattering amplitude at the LHC are explored. The impact of the Black Disk Limit (BDL)− which constitutes a new regime of the scattering processes − on the determination of these values is examined.

1 Introduction

The diffraction processes will occupy an important place in the experimental program at the LHC. Firstly, we will need to know the luminosity and the total cross section with a high precision. Secondly, the diffraction processes will be directly explored at the LHC and will contribute to many different observable reactions. The planned analyses very clearly have problems from the theoretical view point. For example, the definition of the differential cross sections of the elastic proton-proton scattering, as presented in [1]

$$\frac{dN}{dt} = \mathcal{L}[\frac{4\pi\alpha^2}{|t|^2} - \frac{\alpha\rho\sigma_{tot}e^{-b|t|/2}}{|t|} + \frac{\sigma_{tot}^2(1+\rho^2)e^{-b|t|}}{16\pi}] \tag{1}$$

does not contain the electromagnetic form factor and the Coulomb-hadron interference phase Φ_{CH}. Such terms have to be included: all the corrections to ϕ_{CH} were calculated in [2]. More importantly, Eq. (1) is based on the assumption of an exponential behavior of the imaginary and real parts of the hadron scattering amplitude, which is at best an approximation.

Furthermore, the TOTEM experiment has announced the extraction of σ_{tot} from the experimental data, assuming a fixed value of $\rho(s, t = 0) = 0.15$. Indeed, the impact of ρ on σ_{tot} is connected with the term $(1 + \rho^2)$, and is very small when ρ is small. However the most important correlation between ρ and σ_{tot} enters the analysis through the Coulomb-hadron interference term, the size of which remains unknown if we do not know the normalization of dN/dt and the size and t-dependence of $\rho(s, t)$ and ϕ_{CH}.

In [1], it was shown that there would be large correlations between the value of ρ and that of σ_{tot}. However, these correlations and the error estimates were obtained using an exponential behavior of the imaginary and real part of the hadron scattering amplitude. Several models predict an increase in the slope $B(t)$ as $t \to 0$, which effectively leads to an additional term in the description of the hadron scattering amplitude. We shall return to this question later.

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Collaboration	σ_{tot} (mb)	σ_{el}/σ_{tot}	$\rho(t=0)$	$B(t=0)$
[3]	103	0.28	0.12	19
[4]	110.5	0.229		20.5
[5]	111		0.11	
[6]	123.3		0.103	
[7]	128	0.33	0.19	21
[8]	150	0.29	0.24	21.4
[9]	230	0.67		

Table 1: Predictions of different models at ($\sqrt{s} = 14$ TeV, $t = 0$)

$\bar{\rho}$ ($\sqrt{s} = 540$ GeV, $0.000875 \leq \|t\| \leq 0.12$ GeV^2)			
experiment	experimental analysis	global analysis I [10]	global analysis II [11]
UA4	0.24 ± 0.02	0.19 ± 0.03	-
UA4/2	0.135 ± 0.015	-	0.17 ± 0.02

Table 2: Average values of ρ, derived with fixed total cross section (first two columns), and from a global analysis (last two columns).

One should realise that the theoretical predictions are somewhat uncertain. We show in Table 1 recent estimates of the cross section at the LHC. This is partially due to the fact that the dispersion of the experimental data for σ_{tot} at high energy above the ISR energies is very wide. We must note that, except for the UA4 and UA4/2 collaborations, the other experiments have not published the actual numbers for dN/dt. We can only hope that the new results from the LHC experiments will not continue this practice. In this context, we must remember the eventual problems that may arise if one fixes σ_{tot} or ρ to decrease the size of the errors: indeed, this is what the UA4/2 Collaboration did when they extracted $\rho(0)$, fixing σ_{tot} from the UA4 Collaboration ($\sigma_{tot} = 61.9$ mb), or from their own measurement ($\sigma_{tot} = 63.0$ mb). As shown in Table 2, the resulting values for $\rho(0)$ appear inconsistent. A more careful analysis [10, 11] shows that there is no contradiction between the measurements of UA4 and UA4/2.

2 Fitting procedure for $\sigma_{tot}(s)$ and Black Disk Limit (BDL)

The situation is complicated by the possible transition to the saturation regime, as the Black Disk Limit (BDL) will be reached at the LHC [8, 12]. The effect of saturation will be a change in the t-dependence of B and ρ, which will begin for $\sqrt{s} = 2$ to 6 TeV, and which may drastically change $B(t)$ and $\rho(t)$ at $\sqrt{s} = 14$ TeV [8]. As we are about to explain, such a feature can be obtained in very different models.

The first model is based on a fit to soft data which includes a hard pomeron component [13] of intercept 1.4, which is linked to the growth of the gluon density at small x in inelastic processes [14]. This growth leads to non-linear effects, which saturate the BDL. Such effects

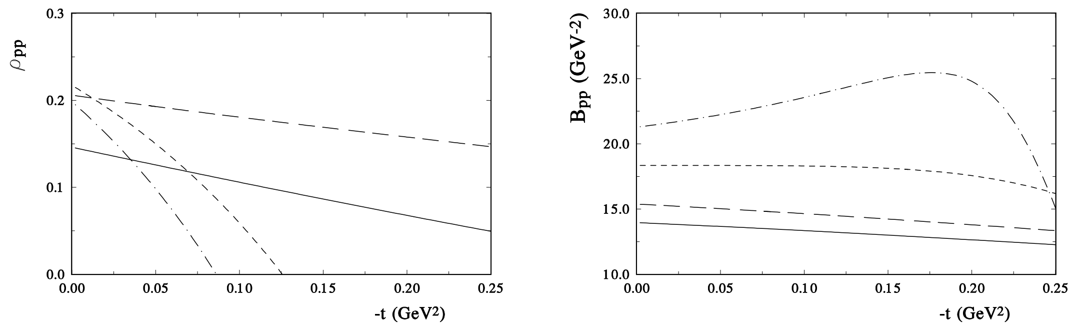

Fig. 1: Results of the ESHPM. Left panel: The ratio of the real to the imaginary part of the amplitude as a function of t, for the bare and the saturated amplitudes at various energies: 100 GeV (plain curve), 500 GeV (long dashes), 5 TeV (short dashes) and 14 TeV (dash-dotted curve). Right panel: The slope of the elastic differential cross section as a function of t, for the bare and saturated amplitudes at various energies: 100 GeV (plain curve), 500 GeV (long dashes), 5 TeV (short dashes) and 14 TeV (dash-dotted curve).

$t = 0$		$t = -0.1$ GeV ^{2}	
DDM	ESHPM	DDM	ESHPM
0.19	0.24	0.08	0.05

Table 3: Results of the DDM and of the ESHPM for ρ at $\sqrt{s} = 14$ TeV

were obtained in $[8, 12]$ and predict that $B(t)$ will increase with t at small t for LHC energies (see Fig. 1). We also show that the saturation of the BDL will heavily change the t-dependence of $\rho(t)$, as shown in Fig. 1. The hard pomeron component will lead to a decrease of the energy at which the BDL regime appears, and the effect on the growth of the total cross section in uncertain. We show in Fig. 1 and Table 3 the results coming from an eikonal unitarisation of the amplitude, and we shall refer to this model [8] as the Eikonalized Soft+Hard Pomeron Model (ESHPM).

The second model in which such effects appear is the Dubna Dynamical model (DDM) of hadron-hadron scattering at high energies [15]. It is based on the general principles of quantum field theory (analyticity, unitarity and so on) and takes into account basic information on the structure of a hadron as a compound system with a central region in which the valence quarks are concentrated, and a long-distance region filled with a color-singlet quark-gluon field. As a result, the hadron amplitude can be represented as a sum of a central and a peripheral part. The DDM predicts that the interaction of the Pomeron with the meson cloud of the hadrons will give an additional term growing like $\sqrt{s}$. This term will become important for energies $\sqrt{s} \geq 500$ GeV. This peripheric effect will lead to a saturation of the overlapping function $G(b)$, see Fig. 2. At small momentum transfer, the DDM predictions agree with the experimental data at $\sqrt{s} = 1.8$ TeV. Interestingly, as shown in Fig. 2, the DDM predicts that the differential cross sections at

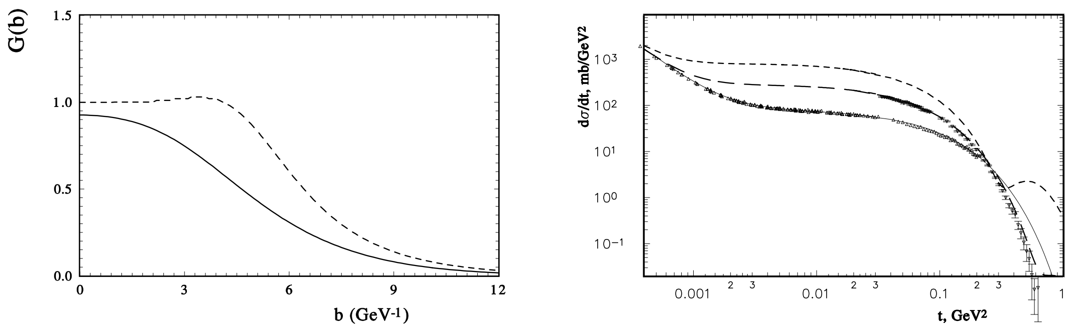

Fig. 2: Predictions of the DDM. Left panel: Overlapping function at $\sqrt{s} = 2$ TeV (solid line) and at $\sqrt{s} = 14$ TeV (dashed line). Right panel: Differential cross sections at $\sqrt{s} = 23.4$ GeV (solid thin line) and at $\sqrt{s} = 1.8$ TeV (dashed line) and at $\sqrt{s} = 14$ TeV (solid thick line).

$\sum \chi_i^2$	$\sigma_{tot}(mb)$	$\rho(t=0)$	B(0) (GeV^{-2})	normalization coefficient
91.2	82.3 ± 0.3	0.15_{fixed}	18.1 ± 0.2	1_{fixed}
88.3	$85. \pm 1.7$	0.15_{fixed}	18.16 ± 0.2	0.94 ± 0.04
89.3	82.3 ± 0.3	0.18 ± 0.02	18.3 ± 0.2	1_{fixed}
88.1	$85.2 \pm 3.$	0.147 ± 0.04	18.1 ± 0.25	0.93 ± 0.07

Table 4: Fits at $\sqrt{s} = 2$ TeV [Input $\rho(0) = 0.23$; $\sigma_{tot} = 82.7$ mb; $B(0) = 18.3$ GeV^{-2})].

$-t \approx 0.3$ GeV^2 will coincide for all high energies. Here again, the t-dependence of the slope-$B(s,t)$ will change its behavior at LHC energies because of saturation effects.

Let us now examine what the standard fitting procedure might give at the LHC in the case of saturation of the BDL, which leads to a behaviour of the scattering amplitude very far from an exponential. As an input, we shall use the predictions for the differential cross sections in the framework of the DDM for two energies $\sqrt{s} = 2$ TeV and $\sqrt{s} = 14$ TeV. For the first energy, the deviation from an exponential is small, whereas it becomes essential at the LHC. We can simulate the future experimental data from this theoretical differential cross sections and assume that 90 points will be measured in a t interval identical to that of the UA4/2 experiment. We then randomize the theoretical curve assuming Gaussian errors similar to those of UA4/2. After that, we can fit the simulated data with an exponential amplitude. The results of this exercise are shown in Tables 4 and 5. It is clear that at $\sqrt{s} = 14$ TeV, the simulated data differ significantly from the results of the fit, especially if one allows for a refitting of their normalisation.

Saturation of the profile function will surely control the behaviour of σ_{tot} at higher energies and will result in a significant decrease of the LHC cross section. However, it is clear that the simple saturation considered here is not enough, as the total cross section at the Tevatron will be 85 mb, which is 2 standard deviations from the CDF result. However, the increase of the slope

$\sum \chi_i^2$	σ_{tot} (mb)	$\rho(t=0)$	B(0) (GeV^{-2})	normalization coefficient
133	155.3 ± 0.5	0.15_{fixed}	23.1 ± 0.2	1_{fixed}
120	$180. \pm 8.6$	0.15_{fixed}	23.2 ± 0.15	0.74 ± 0.07
109	153.4 ± 0.7	0.26 ± 0.03	23.5 ± 0.17	1_{fixed}
108	142.3 ± 2.8	0.29 ± 0.05	23.6 ± 0.2	1.15 ± 0.05

Table 5: Fits at $\sqrt{s} = 14$ TeV [Input $\rho(0) = 0.24$; $\sigma_{tot} = 152.5$ mb; $B(0) = 21.4$ GeV^{-2}].

with t at small t is a generic feature of all saturating models.

3 Oscillations and additional method

As the standard fitting procedure can give misleading results, we need to find an additional method to define or check the basic parameters of the hadron scattering amplitude. Especially as there can be additional specific features in the t-dependence of the different parts of the amplitude. For example, there can be some oscillations in the differential cross sections which can come from different sources. It was shown [16] that if the Pomeranchuk theorem is broken and the scattering amplitude grows to a maximal possible extent, the elastic scattering cross section would exhibit a periodic structure in $q = \sqrt{|t|}$ at small $-t$. It was shown [17] that the oscilations in the UA4/2 data over $\sqrt{|t|}$ can be connected with a rigid-string potential or with residual long-range forces between nucleons. These small oscillations in the differential cross section are difficult to detect by the standard fitting method. Another method was proposed, which consists in the comparison of two statistically independent samples built by binning the whole t-interval in small intervals, proportional to $\sqrt{|t|}$, and by keeping one interval out of two. The deviations of the experimental values from theoretical expectations, weighted by the experimental error, are then summed for each sample k: [18].

$$\Delta R^k(t) = \sum_{|t_i|<|t|} \Delta R_i^k = \sum_{|t_i|<|t|} [(d\sigma^k/dt_i)^{exp} - (d\sigma/dt_i)^{th}]/\delta_i^{exp}, \quad (2)$$

where δ_i^{exp} is the experimental error. This method gives two curves which statistically coincide if oscillations are absent and which grow apart with t if the oscillations are present.

If the theoretical curve does not precisely describe the experimental data, (for example, if the physical hadron amplitude does not have an exactly exponential behavior with momentum transfer), the sum $\Delta R^k(t)$ will differ from zero, going beyond the size of a statistical error. This method thus gives the possibility to check the validity of the model assumptions and of the parameters which describe the hadron scattering amplitude. Note that another specific method was proposed in [19, 20].

4 Conclusion

As the cross section of proton elastic scattering will be measured at the LHC, we need to know more about the behaviour of the hadron scattering amplitude at small t. The analysis of soft data, taking into account the integral dispersion relations, shows a contribution of the hard pomeron in elastic scattering. In this case, it is very likely that at the LHC we shall reach the saturation regime called the BDL It will manifest itself in the behavior of $B(t)$ and of $\rho(t)$ and lead to a non-exponential behavior of the hadron scattering amplitude at small t, which will depend on the form of the unitarization procedure. In other words, different impact parameter dependences of the scattering amplitude will lead to different energy dependences of the ratio of the elastic to the total cross sections.

The regime of the BDL may correspond to parton saturation in the interacting hadrons, which is described by a non-linear equation. Indeed, there is a one-to-one correspondance between non-linear equations and the different forms of the unitarization schemes.

The possibility of a new behaviour of $\rho(s,t)$ and $B(s,t)$ at LHC energies has to be taken into account in the procedure extracting the value of the total cross sections by the standard fitting method. It is needed to use additional specific methods for the determination of the size of the total cross section and of $\rho(s,t)$, such as calculating ΔR and comparing independent choices.

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References

[1] C. Royon, Proceedings , ed. R. Fiore, 15-20 June 2006, Milos, Greece.

[2] O. Selyugin, Mod. Phys. Lett. **A11**, 2317 (1996).

[3] C. Bourrely, J. Soffer, T.T. Wu, Nucl. Phys. **B 247**, 15 (1984).

[4] E. Gotsman,E. Levin, and U. Maor, [hep-ph 0708.1506] (2007).

[5] J.-R. Cudell et al., Phys. Rev. **D 65**, 074024 (2002).

[6] B. Nicolescu, [hep-ph 0707.2923] (2007).

[7] S.V. Goloskokov, and S.P. Kuleshov, and O.V. Selyugin, Fiz. Elem. Chastits At. Yadra[Sov. J. Part. Nucl.] **18**, 14 (1987).

[8] O.V. Selyugin, J.-R. Cudell, [hep-ph 0612046] (2006).

[9] V.A. Petrov, A.V. Prokudin, S.M. Troshin, N.E. Tyurin, J. Phys. G **27**, 2225 (2001).

[10] O. Selyugin, Sov. J. Nucl. Phys. **55**, 466 (1992).

[11] O. Selyugin, Phys. Lett. **B333**, 245 (1994).

[12] O.V. Selyugin, and J.-R. Cudell, Czech. J. of Phys. **54**, A441 (2004).

[13] J.-R. Cudell, E. Martynov, O. Selyugin, A. Lengyel, Phys. Lett. **587**, 78 (2004).

[14] A. Donnachie and P. Landshoff, Phys. Lett. **B 550**, 160 (2002).

[15] S.V. Goloskokov, S.P. Kuleshov, O.V. Selyugin, Z. Phys. **C 50**, 455 (1991).

[16] G. Auberson, T. Kinoshita, A. Martin, Phys. Rev. **D3**, 3185 (1971).

[17] O. Selyugin, Ukr. J. Phys. **41**, 296 (1996).

[18] P. Gauron, B. Nicolescu, O.V. Selyugin, Phys. Lett. **B397**, 305 (1997).

[19] O. Selyugin, Proceedings of VIIth Blois Workshop on Elastic and Diffractive Scattering, ed. P. Chapetta, M. Haguenauer, L.T. Thanh Van, 20-24 June, Blois, France, p. 87 (1995).

[20] P. Gauron, B. Nicolescu, O.V. Selyugin, Phys. Lett. **B629**, 83 (2005).