SATURATION EFFECTS IN DIFFRACTIVE SCATTERING AT LHC ENERGIES

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Unitarization schemes can be reduced to non-linear equations which saturate at small b the evolution of the elastic amplitude with s, and which mimic parton saturation in the non-perturbative regime. These equations enable us to study the effect of saturation on total and elastic cross sections for various models, and to evaluate the uncertainties on σ_{tot} and $\rho(s,t)$ at the LHC in the presence of a hard pomeron.

Keywords: Elastic scattering; Saturation; Unitarization.

1. Introduction

The most important results on the energy dependence of diffractive hadronic scattering were obtained from first principles (analyticity, unitarity and Lorentz invariance), which lead to specific analytic forms for the scattering amplitude as a function of its kinematical parameters - s, t, and u. One of the important theorems is the Froissart-Martin bound which states that the high-energy cross section for the scattering of hadrons is bounded by $\sigma_{tot} \sim \log^2(s/s_0)$, where s_0 is a scale factor.

Experimental data reveal that total cross sections grow with energy. This means that the leading contribution in the high-energy limit is given by the rightmost singularity, the pomeron, with intercept exceeding unity. In the framework of perturbative QCD, the intercept is also expected to exceed unity by an amount proportional to α_s At leading-log s, one obtains the leading singularity at $J-1 \;=\; 12\ \log 2(\alpha_s/\pi)$. In this case, the Froissart-Martin bound is soon violated.

In a recent study,[1,2] we have found that forward data, including total cross sections and the ratios of the real part to the imaginary part of the scattering amplitude, are well fitted by a combination of a soft pomeron (which would be purely non perturbative) and a hard pomeron. The inclusion of these two pomerons, together with the use of the integral dispersion relations,[3] and the addition of the sub-leading meson trajectories, leads to a successful description of all pp, $\bar{p}p$, $\pi^{\pm}p$, $K^{\pm}p$, γp and $\gamma\gamma$ data for $\sqrt{s} \leq 100$ GeV. Indeed, the fast growing hard pomeron leads to a violation of unitarity for values of $\sqrt{s}$ around 1 TeV.

Now the saturation processes are actively studied in high-energy physics. They are connected with the saturation of the gluon density at small x, which may be related to the Black Disk Limit (BDL). However, one should note that the process of saturation has mostly been examined in the framework of the dipole model.[4] But, as noted by the Golec-Biernat and Wusthoff,[5] saturation in that case suppresses the soft contributions and the hard Pomeron plays the main role. That is why the dipole model describes so successfuly hard processes (such as vector-meson production). Contrarily, saturation in the BDL sense tames the hard Pomeron and increases the role of the long-distance processes. It is the latter that we shall study in this contribution.

The unitarity condition can be implemented via several different prescriptions. Two of them are based on particular solutions of the unitarity equation. The scattering amplitude in the impact parameter representation is defined as

$$T(s,t) = i\int_0^\infty bdbJ_0(b\Delta)f(b,s). \quad (1)$$

with $f(b,s) \leq 1$. It satisfies the unitarity equation

$$Imf(s,b) = [Im(f(s,b)]^2 + [Ref(s,b)]^2 + \eta_{inel}(s,b). \quad (2)$$

One of the possibilities is obtained in the $U-$matrix approach [6,7]:

$$f(s,b) = \frac{U(s,b)}{1 - i\,U(s,b)}. \quad (3)$$

The second possible solution of the unitarity condition corresponds to the eikonal representation

$$f(s,b) = [1 - \exp(-\chi(s,b)]. \quad (4)$$

One often takes the eikonal phase in factorised form

$$\chi(s,b) = h(s)\ f(b), \quad (5)$$

and one supposes that, despite the fact that the energy dependence of $h(s)$ can be a power $h(s) \sim s^\Delta$, the total cross section will satisfy the Froissart bound $\sigma_{tot} \leq a \log^2(s)$ We find in fact that the energy dependence of the imaginary part of the amplitude and hence of the total cross section will depend on the form of $f(b)$, *i.e.* on the t dependence of the slope of the elastic scattering amplitude. If $f(b)$ decreases as a power of b, the Froissart-Martin bound will always be violated. In the case of other forms of the b dependence, a special analysis[8] is required.

2. Non-linear equations

The problem of the implementation of unitarity via saturation is that the matching procedure seems arbitrary. Hence we considered a different approach to saturate the amplitude. It is connected with the non-linear saturation processes which have been considered in a perturbative QCD context.[9,10] Such processes lead to an infinite set of coupled evolution equations in energy for the correlation functions of multiple Wilson lines.[11] In the approximation where the correlation functions for more than two Wilson lines factorise, the problem reduces to the non-linear Balitsky-Kovchegov (BK) equation.[11,12]

It is unclear how to extend these results to the non-perturbative region, but one will probably obtain a similar equation. In fact we found simple differential equations that reproduce either the U-matrix or the eikonal representation.

We shall consider saturation equations of the general form[13]

$$dN/d\xi = \mathcal{S}(N) \quad (6)$$

We shall impose the unitarity conditions $N \to 1$ as $s \to \infty$ and $dN/d\xi \to 0$ as $s \to \infty$. We shall also assume that $\mathcal{S}(N)$ has a Taylor expansion in N, and that the first term only the hard pomeron $N_{bare} = f(b)s^\Delta$, where Δ is the pomeron intercept minus 1. Similarly, we fix the integration constant by demanding that the first term of the expansion in s^Δ reduces to N_{bare}. We then need to take $\mathcal{S}(N) = \Delta N + O(N^2)$. The conditions at $s \to \infty$ then give $\mathcal{S}(N) = \Delta(N - N^2)$ as the simplest saturating function. The resulting equation

$$dN/dy = \Delta(N - N^2), \quad (7)$$

where $y = log(s) \sim log(1/x)$ This equation has the solution

$$N = \frac{f(b)s^\Delta}{f(b)s^\Delta + 1} \quad (8)$$

One may then wonder whether other unitarization schemes are possible. We give here a general algorithm to build such schemes:

$$\frac{dN}{dy} = \frac{dN_{bare}}{dy}(1-N) \qquad (9)$$

with N_{bare} the unsaturated amplitude. This will trivially obey the conditions at $s \to \infty$, and saturate at $N = 1$. It has as a solution

$$N(b,s) = 1 - \exp(-N_{bare}(b,s)) \qquad (10)$$

This is equivalent to the eikonal representation with $N_{bare}(b,s)$ corresponding the Born aplitude in the impact representation. The corresponding equation can also be written in the form

$$\frac{dN}{dy} = \Delta\ N_{bare}\ \exp[-N_{bare}(b,s)]. \qquad (11)$$

One can also find new unitarization schemes, *e.g.* from the equation

$$dN/dy = \frac{dN_{bare}}{dy}\ (1-N^2), \qquad (12)$$

one gets

$$N(b,s) = \tanh[N_{bare}(b,s)]. \qquad (13)$$

On Fig.1, the result of saturation is shown for these three unitarization procedures. It is clear that despite the essentially difference in the forms of the non-linear equations, the process of the saturation is very similar.

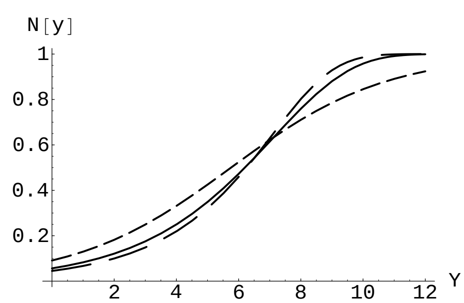

Fig. 1. The saturation of N(y) for different non-linear equations (hard line - eq.(11, long- dashed line - eq. (7), and short-dashed - eq. (12))

3. Minimum saturation of the Soft plus Hard Pomeron model

Let us take, as an example, the Soft plus Hard pomeron model [2,14] which includes two simple poles to describe pp and $\bar{p}p$ scattering at high s. In this case, the pp-elastic scattering amplitude is proportional to the hadron form factors and can be approximated at small t by:

$$T(s,t) \sim \ [\ h_1\ (s/s_0)^{\epsilon_1} e^{\alpha'_1\ t\ \log(s/s_0)} + h_2\ (s/s_0)^{\epsilon_2}\ \ e^{\alpha'_2\ t\ \log(s/s_0)}]\ F^2(t). \qquad (14)$$

where $h_1 = 4.7$ and $h_2 = 0.005$ are the coupling of the soft and hard pomerons, and $\epsilon_1 = 0.072$, $\alpha'_1 = 0.25$ GeV^{-2}, and $\epsilon_2 = 0.45$, $\alpha'_2 = 0.20$ GeV^{-2} are the intercepts and the slopes of the two pomeron trajectories, and where s contains implicitly the phase factor $\exp(-i\pi/2)$. $F^2(t)$ is the square of the Dirac elastic form factor. It can be approximated by sum of three exponentials.

We then obtain in the impact parameter representation a specific form for the profile function $\Gamma(b,s)$.[8] One finds that for $\sqrt{s} \approx$ 1.5 TeV and at small b, $\Gamma(b,s)$ reaches the black disk limit.

Note that one cannot simply cut the profile function sharply as this would lead to a non-analytic amplitude, and to specific diffractive patterns in the total cross section and in the slope of the differential cross sections. Furthermore, we have to match at large impact parameter the behaviour of the unsaturated profile function. We have to use some specific matching patters which softly interpolate between both regimes. The interpolating function gives unity in the large impact parameter region and forces the profile function to approach the saturation scale b_s as a Gaussian.

We show in Fig. 2 the possible behaviours of the total cross section at very high energies, depending on $\epsilon = \alpha(0) - 1$ and on the unitarization scheme. We also show there the result of a simple eikonalisation, where we took

$$T_{pp}(s,t=0) \sim \int d^2b[1-e^{h_{eik}G(s,b)}] \quad (15)$$

and where $h_{eik} = 1.2$ was chosen so that the values σ_{tot} determined by the overlapping function (14), and by eikonalisation (15) are equal at $\sqrt{s} = 50$ GeV.

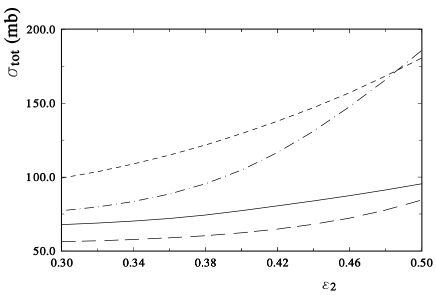

Fig. 2. The total cross section at $\sqrt{(s)} = 1.8$ TeV and $\sqrt{s} = 14$ TeV (lower two and upper two lines) for the saturated amplitude (plain and short dashes) and for an eikonalized amplitude (long dashes and dash-dots).

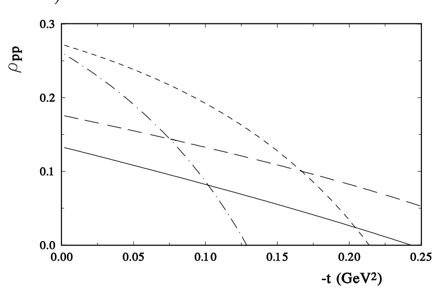

Fig. 3. The ratio of the real to the imaginary part of the amplitude as a function of t for the saturated amplitudes at various energies: 100, 500, GeV, and 5, 14 TeV (plain, short-dashed, dot-dashed lines correspondingly

To demonstrate the presence of saturation at the LHC, one can also consider the slope of the differential elastic cross section, which we show in Fig.3. We see that saturation increases its value at small t (this is in fact unavoidable for any saturation scheme), and predicts a fast drop around $|t| = 0.25$ GeV^2, when one enters the dip region.

In conclusion, we have shown that the most usual unitarization schemes could be cast into differential equations which are reminiscent of the saturation equation .[11,12] Such an approach can be used to build new unitarization schemes and may also shed some light on the physical processes responsible for the saturation regime.

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