

HARD POMERON CONTRIBUTION TO FORWARD ELASTIC SCATTERING ^a

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The introduction of a hard singularity in fits to total cross sections and to the ratio of real to imaginary parts enables to reproduce the data at $\sqrt{s} \leq 100$ GeV using only simple-pole parametrisations, both for the soft and for the hard pomerons.

1 Soft pomerons

Some time ago¹, it has been suggested that a pomeron model la Donnachie-Landshoff², where the leading singularity in the complex j plane is given by a simple pole, could not provide the best fits to the data for total cross sections, and for the ratio ρ of the real to the imaginary part of the forward elastic scattering amplitude. This conclusion was based on an analysis of all available data at $t = 0$ for $\bar{p}p$, pp , $\pi^\pm p$, $K^\pm p$, γp and $\gamma\gamma$ scattering, where three kinds of parametrisations were used:

- a triple-pole singularity, which makes total cross sections rise as $\log^2(s)$ at high energy;
- a double-pole singularity, which gives $\sigma_{tot} \sim \log(s)$;
- a simple-pole singularity, which gives a power rise s^ϵ .

In each case, non-leading exchanges were accounted for by two simple-pole contributions contributing to non-degenerate crossing-odd and crossing-even trajectories.

The conclusions were that the best fit was always given by the triple-pole pomeron, closely followed by the double pole, and that the simple-pole parametrisation was excluded if one went down in energy to $\sqrt{s} = 5$ GeV, or if one included the ρ parameter in the fit.

In³, we re-examined this question. First of all, we found that a few sub-leading effects improved the fits significantly:

- the use of the theoretical variable, $(s - u)/2 \propto \cos \theta_t$ rather than s , and of the flux factor $2m_{target} p_{beam}^{lab}$ instead of s ;
- the use of subtraction constants in the dispersion relations giving ρ from $\Im mA$.

These effects indicate that $\sqrt{s} = 5$ GeV is not really in the asymptotic region: the fit is affected by sub-leading terms, but these are under control. However, all the fits are improved, so that the triple-pole and the double-pole still give a better description of the data, as shown in Table 1.

However, these two parametrisations work better for rather peculiar reasons: the dipole becomes negative below $\sqrt{s} = 9.5$ GeV, whereas the tripole has a minimum at $\sqrt{s} = 5.8$ GeV, and so rises if s decreases. Both of these features significantly modify the fit at lower energy, by adding a more complicated s dependance to a power in the $C = +1$ sector. Hence it is natural to check whether a more complicated $C = +1$ exchange could lead to a better fit in the simple-pole case.

Given the hadronic amplitude \mathcal{A}^{ab} , we define the total cross section as

$$\sigma_{tot}^{ab} \equiv \Im mA^{ab} / (2m_b p_{lab}), \quad (1)$$

with p_{lab} the momentum of particle b in the a rest frame, and the model that we consider is

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Process	N_p	1 simple pole	dipole	tripole	2 simple poles	unitarised
$\sigma(pp)$	104	1.1	0.88	0.87	0.87	0.87
$\sigma(\bar{p}p)$	59	0.88	0.94	0.94	0.92	0.92
$\sigma(\pi^+p)$	50	1.2	0.68	0.68	0.70	0.69
$\sigma(\pi^-p)$	95	0.92	0.97	0.97	0.93	0.95
$\sigma(K^+p)$	40	0.97	0.73	0.71	0.72	0.72
$\sigma(K^-p)$	63	0.73	0.62	0.61	0.61	0.61
$\sigma(\gamma p)$	41	0.56	0.58	0.54	0.54	0.56
$\sigma(\gamma\gamma)$	36	0.88	0.80	0.73	0.70	0.82
$\rho(pp)$	64	1.6	1.6	1.7	1.7	1.7
$\rho(\bar{p}p)$	11	0.40	0.39	0.42	0.41	0.40
$\rho(\pi^+p)$	8	2.9	1.8	1.8	1.6	1.7
$\rho(\pi^-p)$	30	1.9	1.0	1.0	1.0	1.0
$\rho(K^+p)$	10	0.70	0.57	0.60	0.62	0.60
$\rho(K^-p)$	8	1.7	1.2	1.0	0.98	1.0
all, χ^2_{tot}	619	661	564	558	551	557
all, $\chi^2/\text{d.o.f.}$	619	1.10	0.94	0.93	0.924	0.933

Table 1: Values of the χ^2 per point for different processes in fits based on integral dispersion relations with subtraction constants. The third column corresponds to a simple-pole fit, the fourth to a double pole, the fifth to a triple pole. The last two columns show that the inclusion of a hard pomeron leads to a significant improvement of the fit.

then defined by the following equation:

$$\Im m\mathcal{A}^{ab} \equiv s_1 \left[\Im_{R^+}^{ab} \left(\frac{\tilde{s}}{s_1} \right) + \Im_S^{ab} \left(\frac{\tilde{s}}{s_1} \right) \mp \Im_-^{ab} \left(\frac{\tilde{s}}{s_1} \right) \right], \quad (2)$$

with $s_1 = 1 \text{ GeV}^2$, and the $-$ sign in the last term for particles, $\Im_{R^+}^{ab}$ and \Im_-^{ab} being the contributions of the crossing-even and crossing-odd reggeons. For the pomeron contribution \Im_S^{ab} , we allow two simple poles to contribute:

$$\Im_S^{pb} = S^b \left(\frac{\tilde{s}}{s_1} \right)^{\alpha_o} + H^b \left(\frac{\tilde{s}}{s_1} \right)^{\alpha_H} \quad (3)$$

We give in Table 1 the quality of the new fit. We can see that the inclusion of this new $C = +1$ singularity has a dramatic effect: the χ^2 drops from 661 to 551 for 619 points, nominally a 10σ effect! More surprisingly, the new singularity has an intercept of 1.39, very close to that obtained in DIS⁶.

However, as was already known⁷, the new trajectory, which we shall call the hard pomeron, almost decouples from pp and $\bar{p}p$ scattering. Nevertheless, it improves considerably the description of πp and Kp amplitudes, and parametrisation (3) becomes as good as the tripole fit advocated in¹. This solution is similar to that of ref.⁵, where only pp and $\bar{p}p$ scattering were considered. The residue for pp is about 20 times smaller than the residues that we get for πp and Kp , which seems impossible.

The cause of this suppression is easily understood: the hard pomeron generates a fast-rising contribution to the cross sections. Hence it must have a really small coupling to accommodate the data at the highest energies. In the pion and kaon cases, the data extend only to 63 GeV, so that a relatively large coupling is allowed. This leads to two conclusions: first of all, to study the hard contribution, one needs to limit oneself to low energy. We chose to limit all data to

	soft+hard poles		soft pole+ unitarised hard	
Parameters	value	error	value	error
α_o	1.0728	0.0008	1.0728	fixed
S_p	56.2	0.3	55	1
S_π	32.7	0.2	31.5	0.9
S_K	28.3	0.2	27.4	0.8
S_γ	0.174	0.002	0.174	0.003
$\alpha_h(0)$	1.45	0.01	1.45	fixed
G_p	–	–	0.18	0.06
G_γ	–	–	6×10^{-9}	1.5×10^{-8}
H_p	0.10	0.02	0.17	0.05
H_π	0.28	0.03	0.43	0.08
H_K	0.30	0.03	0.42	0.07
H_γ	0.0006	0.0002	0.0005	0.0002

Table 2: Parameters obtained in the fits. The second and third columns give the parameters and errors of the fit with a hard pole, Eq. (3) for \sqrt{s} from 5 to 100 GeV, the fourth and fifth columns give the parameters of a unitarised fit, Eq. (7) for $5 \text{ GeV} < \sqrt{s} < 2 \text{ TeV}$.

$\sqrt{s} = 100 \text{ GeV}$. Secondly, to describe the $Sp\bar{p}S$ and Tevatron data, one needs to unitarise the hard pomeron contribution.

Following this program, we obtained the parameters for the hard and soft pomerons which are summarised in Table 2. Two main conclusions can be drawn: the hard pomeron intercept is

$$\alpha_H(0) = 1.45 \pm 0.01 \quad (4)$$

and the couplings to protons, pions and kaons are

$$H_p : H_\pi : H_K = 1 : 2.8 : 3 \quad (5)$$

The origin of this hierarchy remains mysterious, although it could be a size effect, the hard pomeron being a short-distance effect.

Note that in this fit, we have used Regge factorisation for the couplings of the photon:

$$H_{\gamma\gamma} = H_{\gamma p}^2 / H_{pp}; \quad S_{\gamma\gamma} = S_{\gamma p}^2 / S_{pp} \quad (6)$$

and similar equations for the reggeon exchanges. Hence one can conclude⁵ that the hard pomeron obeys factorisation, although the quality of the $\gamma\gamma$ data leaves this point unsettled.

Finally, one needs to tackle the issue of unitarisation. Indeed, the hard singularity cannot be extended to energies beyond a few hundred GeV, where one reaches the black-disk limit. The problem of course is that nobody knows how to unitarise Regge exchanges unambiguously. We know that if 1-pomeron exchange is given by the amplitude

$$\Im m A(s, t) \approx g_1 \left(\frac{s}{s_1} \right)^{\alpha_H} e^{R_H^2 t}$$

with $R_H^2 = B_H + \alpha'_H \log s$ then, if the hadrons remain intact during multiple exchanges, the n -pomeron contribution at $t = 0$ will be proportional to

$$\Im m A^{(n)}(s) \propto (-1)^{n-1} s \frac{s^{n(\alpha_H-1)}}{[R_H^2]^{n-1}}$$

The coefficients of the successive terms, and the influence of triple pomeron vertices are unknown. In order to show that it is possible to reproduce the data via unitarisation, we chose³ the simplest form, which can be obtained in the U -matrix formalism, which reproduces simple-pole exchange at small s , and obeys the Froissart-Martin bound at high s :

$$\Im m A_+^H(s) = H_a s R^2 \left[\frac{1}{G} \log \left\{ 1 + G \frac{s^{\alpha_h - 1}}{R^2} \right\} \right]. \quad (7)$$

with $B_H = 4 \text{ GeV}^2$ and $\alpha' = 0.1 \text{ GeV}^{-2}$. We also assumed that G would be the same for all hadrons, and different for photons, and fixed the hard pomeron intercept to that obtained in the previous fit. We show in Table 1 that such a form can reproduce the Tevatron data well, while being almost identical to a simple-pole at low energy, G being of the order of 20 %. Table 2 gives the parameters corresponding to this unitarised form.

In conclusion, it could well be that the hard singularity that was predicted 30 years ago by BFKL was already present then in the data. It may have been observed in DIS, and it would be very surprising that no trace of it would subsist in soft data. Of course, if it is associated with short-distance fluctuations, then it can appear only rarely. But its inclusion does help the description of the data at $t = 0$. The hierarchy of its couplings is unexpected, and it seems compatible with the factorisation properties of a simple pole. Its presence in elastic cross sections has also now been motivated by the next contribution to these proceedings⁴, and it could well modify significantly the total cross section at the LHC⁸.

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