Measuring the time-varying systemic risks of hedge funds

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Abstract

With the rise of shadow banking, the threat posed by hedge funds to financial stability has become a major concern for regulators. In this paper, we propose a definition of hedge funds systemic risk based on the sensitivity of a banking index to hedge funds extreme losses. We then use regression methods based on extreme value theory to estimate this quantity, overcoming short reporting history encountered in commercial databases. This approach allows us to estimate the time-varying systemic risk contribution of hedge funds at the fund level, as well as the marginal effects of its determinants. We find that the fund's size, the use of leverage as well as market conditions associated with high uncertainty and low market liquidity all indicate higher systemic risk levels. Moreover, we show that hedge funds systemic risk significantly increased after 2008.

Keywords: systemic risk; hedge funds; extreme value regression.

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1. Introduction

Since the Global financial crisis, regulatory authorities have taken an interest in the threats hedge funds represent for the stability of financial markets. Due to increasingly strong capital constraints imposed on banks, many risk-taking activities have been transferred to lightly regulated non-banks entities such as hedge funds (Irani et al., 2021; Abad et al., 2022). This phenomenon is particularly prominent in credit lending activities, where loans granted by deregulated entities are funded by credit lines originating from banks, leading to an increase in counterparty risk (Jiang, 2023). Major concerns with shadow banking include fire-sales spirals amplified by illiquidity and highly leveraged positions of hedge funds (Boyson et al., 2010; Gennaioli et al., 2013; Bussière et al., 2015), or loss spillovers to the banking and insurance sectors (Adams et al., 2014). A striking illustration of this phenomenon can be found in the demise of Archegos Capital Management in March 2021, causing huge losses to banks such as Credit Suisse (FT, 2021). Hence, there is a strong motivation for regulatory authorities to develop proper tools to assess the systemic threat posed by hedge funds for the banking sector, and to document factors influencing their dynamics (Bellavite Pellegrini et al., 2022). The present paper contributes to these two objectives.

Throughout this paper, we define hedge fund systemic risk as the risk of adverse consequences, for the financial system, stemming from the failure of a hedge fund (Acharya et al., 2012, 2017). A common approach for quantifying systemic risks consists in computing indicators such as the CoVaR (Adrian and Brunnermeier, 2016) or the marginal expected shortfall (MES) (Acharya et al., 2012), which measure the joint occurrence of extreme losses. However, these computations are challenging to conduct for hedge funds, due to data scarcity and the need to account for rare events: since hedge funds data are available on a monthly basis at the fund level, and that funds generally live for a short period of time (e.g. a few years), the majority of the funds usually report for less than 100 months. Hence, it limits the range of statistical tools one could exploit. Several solutions have been considered to overcome this difficulty, such as data aggregation in indices (Adams et al., 2014; Bernal et al., 2014) or grouping funds per investment strategies (Adrian et al., 2013; Mhalla et al., 2022), but they suffer from a loss of accuracy and are unable to capture fund-level heterogeneity. Moreover, time-series methods such as quantile regressions (Greppmair, 2020) or multivariate GARCH models (Hwang et al., 2017) cannot be reliably applied due to the short length of the time series.

To address these shortcomings, we extend the systemic risk model of van Oordt and Zhou (2019a,b) to a regression context. In this model, we capture the impact of a shock generated by individual hedge funds on an index of representative banks. However, in contrast to van Oordt and Zhou (2019a), we assume that a common regression structure describes the tail distribution of hedge funds returns and its dependence with the banking sector. As such, we can estimate this model, and by extension the systemic risk measures, relying on univariate and multivariate extreme value regression (EVR) methods (Chavez-Demoulin et al., 2016; Hambuckers et al., 2018; Mhalla et al., 2019, 2022). Moreover, because we incorporate conditioning information to control for tail heterogeneity over time and across funds, we estimate the model by pooling all the returns of the funds, escaping small-sample issues. Hence, thanks to EVR, we tackle data scarcity issue and are able to draw conclusions using only the tail distribution of the financial entities, an important quality highlighted in Benoit et al. (2017).

As a corollary, the final estimates of our risk measures are time-varying and fund-specific, and allows us to test if banks are affected by shocks stemming from particular hedge funds, or during specific economic conditions. One key component of our measure is the systemic linkage (SL) between banks and hedge funds. This component is mainly driven by the tail dependence between the two entities, a measure extensively considered in the systemic risk literature (Poon et al., 2004; Gravelle and Li, 2013; Balla et al., 2014; Mhalla et al., 2022). In our framework, this dependence measure is also estimated conditional to a set of covariates, and can be used to conduct time-varying and fund-specific analyses.

We use this approach to estimate systemic risk measures for 4,847 funds reporting over the period May 1995 - December 2020. We thus span both the Global Financial Crisis and the COVID crisis. To measure the sensitivity of banks to hedge funds, we build a bank index based on a list of 22 large banks. As for the covariates considered in the regression analysis, we use funds characteristics as well as market and macro-economic indicators such as the VIX, liquidity measures and credit spreads.

We show that banks' sensitivity to shocks stemming from hedge funds increases over time, especially after the 2008 crisis. This increase is particularly important for *Fixed Income* funds. In addition, we find evidence of a positive effect of the use of leverage and the size of the fund on systemic risks, independently from the investment strategy. The impact of other funds characteristics is more nuanced and varies across strategies. Coming to market conditions, periods of high uncertainty, captured by the VIX and the market realized volatility (RV), are characterized by an increase in systemic risk for all strategies. Decomposing the sources of this increase, we find that the intrinsic tail risk of the institutions is the main driver of systemic risk in high uncertainty periods. On the contrary, the tail dependence between hedge funds and banks tends to drop for all strategies during highly volatile times. In addition, we find that tail dependence is positively related to market illiquidity for three out of four strategies, suggesting that liquidity spirals might play an important role. This increase in tail dependence over time appears also to be a key driver in the overall increase in hedge funds systemic risk observed over the past 15 years.

Finally, we also provide several insights on the drivers of hedge funds downside risks. Although the average Value-at-Risk (VaR) of hedge funds, expressed as a percentage of the total asset, shows a decreasing trend over time, the increase in hedge funds individual sizes implies an overall increase of downside risk in nominal terms. As a result, the average dollar risk exposure per fund has increased dramatically for all strategies, illustrating the growing threat this industry represents for the financial sector (Abad et al., 2022).

The rest of the paper is structured as follows. In section 2, we detail the models and the estimation methods we use to measure the hedge funds systemic risk. In section 3, we provide a description of the data and the preliminary treatment we apply. In section 4, we present the empirical results, and develop our concluding remarks in section 5.

2. Methodology

In this section, we outline first the model used to infer systemic risk measures. Then, we detail our estimation method, before explaining the economic interpretation of the measures we derive.

2.1. Systemic risk model

To conduct our analysis, we rely on the model of van Oordt and Zhou (2019a) (hereafter VZ), originally developed for the banking sector. In the original application, the authors aim at capturing the effect of a shock from a broad equity index on banking entities. In our framework, we investigate the sensitivity of banks to a shock experienced by a particular hedge fund and the extremal interconnection between them. Hence, we measure the relationship between hedge funds extreme losses and the banking sector. Let R_{it} and R_{st} denote respectively the returns at time t of the hedge fund i and of the bank index. We assume that the following linear tail model holds

$$R_{st} = \beta_{sit} R_{it} + \epsilon_{sit}, \quad \text{where } R_{it} < -\text{VaR}_{it}(\bar{p}), \tag{1}$$

where $\text{VaR}_{it}(\bar{p})$ denotes the VaR of hedge fund i at level $\bar{p}$ and time t. β_{sit} is the quantity of interest and is interpreted as the sensitivity of the banking index to some hedge fund, conditional on this fund being shocked, i.e. when its return is below its VaR far in the tail. Provided that both returns distributions are heavy-tailed, VZ use extreme value theory (EVT) arguments to show that β_{sit} can be approximated by

$$\beta_{sit} \approx \lim_{p \to 0} \underbrace{\tau_{sit}(p)^{\gamma_{it}}}_{\text{systemic linkage}} \frac{\text{VaR}_{st}(p)}{\text{VaR}_{it}(p)}, \tag{2}$$

where $\tau_{sit}(p) = \mathbb{P}(R_{st} < -\text{VaR}_{st}(p) | R_{it} < -\text{VaR}_{it}(p))$ and γ_{it} is the inverse of the tail index of hedge fund i at time t. The expression (2) can be decomposed in two terms: one component is the systemic linkage (SL), which captures the effect of extremal dependence between the two entities and mainly depends on their tail dependence $\tau_{sit}(p)$. The remaining component relates to the intrinsic tail risk of the two considered entities and, contrary to VZ model, does not convey interpretable information in the present context. Although this model is close in spirit to the one presented by VZ, notice that here the conditioning event is dependent on the hedge fund i. Consequently, the ratio between the two VaR cannot be interpreted across entities, in the same way VZ do. For this analysis, our quantities of interest are therefore VaR_{it}, the tail dependence $\tau_{sit}(p)$, and β_{sit}. The advantage of this approach is twofold: first, we can easily introduce a regression structure to estimate the components that drive β_{sit} (see section 2.2); secondly, as we show in section 2.3, well-known systemic risk measures can be easily derived from β_{sit}.

2.2. Model estimation

The key challenge in estimating β_{sit} is the data scarcity of hedge funds. Hence, it considerably reduces the set of methodological alternatives to derive reliable estimates of $\text{VaR}_{it}(p)$ and $\tau_{sit}(p)$, which has to take into account time- and fund-heterogeneity. Thus, to estimate $\text{VaR}_{it}(p)$ in equation (2), we use the univariate EVR framework as introduced in Chavez-Demoulin et al. (2016). This approach was previously applied to the analysis of hedge funds extreme risks in Mhalla et al. (2022). To do so, we first filter out the conditional mean return $\hat{\mu}_{it}$ and conduct our analysis on the negative residuals, defined as

$$R^{\star}_{it} = -(R_{it} - \hat{\mu}_{it}). \tag{3}$$

The conditional mean model used for this preliminary filtering is based on the high-frequency asset pricing model of Patton and Ramadorai (2013), briefly discussed in section 3.1 and in Appendix C. Then, over some high threshold u_{it}, we assume that

$$R^{\star}_{it} \mid (R^{\star}_{it} > u_{it}) \sim GPD(u_{it}, \gamma(\mathbf{x}^{\gamma}_{it}), \sigma(\mathbf{x}^{\sigma}_{it})), \tag{4}$$

where $GPD(\cdot)$ is the cumulative distribution function (cdf) of the GPD with u_{it} as the location parameter. $\gamma(\mathbf{x}^{\gamma}_{it})$ and $\sigma(\mathbf{x}^{\sigma}_{it})$ are the time- and fund-specific shape and scale parameters, depending on vectors of covariates $\mathbf{x}^{\sigma}_{it}$ and $\mathbf{x}^{\gamma}_{it}$, respectively, for hedge fund i at time t. We impose

$\sigma(\mathbf{x}_{it}^{\sigma}) \geq 0$ and $\gamma(\mathbf{x}_{it}^{\gamma}) \geq 0$ by assuming

$$\log(\sigma(\mathbf{x}_{it}^{\sigma})) = \alpha_0^{\sigma} + \sum_{l=1}^{p_\sigma} \alpha_l^{\sigma} x_{it}^{\sigma}(l), \tag{5}$$

$$\log(\gamma(\mathbf{x}_{it}^{\gamma})) = \alpha_0^{\gamma} + \sum_{l=1}^{p_\gamma} \alpha_l^{\gamma} x_{it}^{\gamma}(l), \tag{6}$$

where $x_{it}^{\gamma}(l)$ and $x_{it}^{\sigma}(l)$ denote the l^{th} element of the covariate vectors $\mathbf{x}_{it}^{\gamma}$ and $\mathbf{x}_{it}^{\sigma}$, while p_γ and p_σ are the number of covariates respectively for the shape and scale parameters. The set of parameters to estimate is defined by $\Theta^{\text{marg}} = (\alpha_0^{\gamma}, \ldots, \alpha_{p_\gamma}^{\gamma}, \alpha_0^{\sigma}, \ldots, \alpha_{p_\sigma}^{\sigma})$. Under these assumptions, we estimate Θ^{marg} through penalized maximum likelihood estimation as introduced by Hambuckers et al. (2018) to automatically select relevant covariates. In this framework, the penalized log-likelihood function is defined as

$$\mathcal{L}_{pen}(\Theta^{\text{marg}}) = \mathcal{L}(\Theta^{\text{marg}}) - n^{\star}\mathcal{P}_{\nu}(\Theta^{\text{marg}}), \tag{7}$$

with

$$\mathcal{L}(\Theta^{\text{marg}}) = \sum_{t=1}^{T} \sum_{i \in I_t^*} \log(gpd(R_{it}^{\star}, u_{it}, \gamma(\mathbf{x}_{it}^{\gamma}), \sigma(\mathbf{x}_{it}^{\sigma}))),$$

where I_t^* is the set of hedge funds indices for which $R_{it}^{\star} > u_{it}$, $n^{\star} = \sum_{t=1}^{T} n_t^{\star}$, $n_t^{\star}$ is the number of observations present above the threshold at time t and $gpd(\cdot)$ is the pdf of the GPD. $\mathcal{P}_{\nu}(\Theta^{\text{marg}})$ is the penalty with parameters $\nu = (\nu_\sigma, \nu_\gamma)$. As penalty type, we use the adaptive Lasso (Zou, 2006). Simultaneous selection of the covariates and model estimation are obtained from maximizing (7). To select the penalty coefficients ν_σ and ν_γ, we conduct a sequential search starting with ν_σ using the BIC while not penalizing the covariates entering the shape parameter. In a second step, we select the value of ν_γ based on the same selection criterion while keeping the covariates determining the scale fixed to the ones selected in the first step. We then perform a final unpenalized estimation step using a standard maximum likelihood estimator (MLE) with the selected set. We thus obtained the so-called post-selection adaptive Lasso estimates. Thanks to the regression structure, we derive estimates for $\gamma(\mathbf{x}_{it}^{\gamma})$ and $\sigma(\mathbf{x}_{it}^{\sigma})$ that we denote $\hat{\gamma}_{it}$ and $\hat{\sigma}_{it}$, for every observation. From these quantities, the VaR of $R_{it}^{\star}$ is obtained from

$$\widehat{\text{VaR}}^{\star}_{it}(p) = \hat{u}_{it} + \frac{\hat{\sigma}_{it}}{\hat{\gamma}_{it}} \left(\left[\frac{1-p}{\mathbb{P}(R^{\star}_{it} > u_{it})} \right]^{-\hat{\gamma}_{it}} - 1 \right), \tag{8}$$

where $\hat{u}_{it}$ is the estimated location parameter of the GPD for hedge funds i at time t, namely the threshold above which the exceedances are assumed to follow a GPD. To select this threshold, we follow Mhalla et al. (2022) and use semi-parametric quantile regression with the time as covariate and level q. Hence, $1-q$ is thus an estimate for $\mathbb{P}(R^{\star}_{it} > u_{it})$ (details can be found in Appendix A). From $\widehat{\text{VaR}}^{\star}_{it}(p)$, we derive the final VaR estimates, given by

$$\widehat{\text{VaR}}_{it}(p) = \widehat{\text{VaR}}^{\star}_{it}(p) - \hat{\mu}_{it}. \tag{9}$$

Note also that $\text{ES}_{it}(p)$ can be estimated thanks to $\widehat{\text{VaR}}_{it}(p)$, $\hat{u}_{it}$, $\hat{\gamma}_{it}$ and $\hat{\sigma}_{it}$.

Coming to estimating the tail dependence $\tau_{sit}(p)$, we rely on multivariate EVR (Mhalla et al., 2019). We focus on bivariate tail dependence as we are interested in pairs of financial entities. We assume that the two-dimensional random vector of losses $\mathbf{R}^{\star} = (R^{\star}_{st}, R^{\star}_{it})$ for the entities of interest (i.e. the banking index and the hedge funds) follows a joint (conditional) distribution $F_{si}(\cdot; \mathbf{x}^{\tau}_{sit})$ and marginal distributions F_s and F_i, respectively. The quantity of interest is defined by

$$\tau_{sit} = \lim_{p \to 0} \tau_{sit}(p) = \lim_{p \to 0} \mathbb{P}(F_s(R^{\star}_{st}) > 1-p | F_i(R^{\star}_{it}) > 1-p; \mathbf{x}^{\tau}_{sit}), \tag{10}$$

i.e. the limit probability that both entities i and s suffer an extremely large loss simultaneously at time t. For simplicity, we drop the explicit reference to t for the marginal distributions, and assume that this probability is time-varying because of its link with a p_{τ}-vector of covariates $\mathbf{x}^{\tau}_{sit}$, specific to time and to the two institutions under consideration.

To estimate (10), we use multivariate EVT. Namely, we assume that $F_{si}(\cdot; \mathbf{x}^{\tau}_{sit})$ belongs to the maximum domain of attraction of a multivariate extreme value distribution. Then, following Mhalla et al. (2022), we assume that

$$\tau_{sit} = 2 - \int_0^1 \max(\omega, 1-\omega) h(\omega, \lambda(\mathbf{x}_{sit}^{\tau})) d\omega,$$

where $h(\omega; \lambda(\mathbf{x}_{sit}^{\tau}))$, $\forall \omega \in [0,1]$, is the spectral density of the Hüsler-Reiss parametric family (Hüsler and Reiss, 1989) characterizing the tail dependence of the data; with parameter $\lambda(\mathbf{x}_{sit}^{\tau})$. For simplicity, we denote $\lambda(\mathbf{x}_{sit}^{\tau})$ by λ_{sit}. We restrict $\lambda_{sit} > 0$, excluding perfect (tail) independence. In this framework, we assume that λ_{sit} depends on $\mathbf{x}_{sit}^{\tau}$ via

$$\log(\lambda_{sit}(\Theta^{\text{mult}})) = \theta_0^{\text{mult}} + \sum_{l=1}^{p_\tau} \theta_l^{\text{mult}} x_{sit}^{\tau}(l), \tag{11}$$

where $x_{sit}^{\tau}(l)$ denotes the l^{th} covariate. As a result, a positive slope coefficient for θ_l^{mult} indicates a positive relationship between $x_{sit}^{\tau}(l)$ and the tail dependence of the two considered entities. We estimate $\Theta^{\text{mult}} = (\theta_0^{\text{mult}}, \ldots, \theta_{p_\tau}^{\text{mult}})$ through classical maximum likelihood procedures. Finally, notice that this ultimate step requires to transform first the components of $\mathbf{R}^{\star}$ to a unit-Fréchet scale. The unit-Fréchet transformation is defined by $f(R_{it}^{\star}) = \frac{-1}{\log(F_i(R_{it}^{\star}))}$ for $R_{it}^{\star}$ and conversely for $R_{st}^{\star}$. We do so using a combination of the empirical cumulative distribution function and the marginal conditional GPD detailed in the previous section (see Mhalla et al. (2022) for more details). Regarding $R_{it}^{\star}$, we assume that

$$F_i(R_{it}^{\star}) = \begin{cases} GPD(u_{it}, \gamma_{it}, \sigma_{it}), \text{ if } R_{it}^{\star} > u_{it}, \\ F_i^{\text{emp}} \text{ otherwise}, \end{cases} \tag{12}$$

where F_i^{emp} is the empirical cdf of $R_{it}^{\star}$, estimated at the fund-level. With this formulation, we rely on the EVR estimation we perform for the hedge funds VaR. For the negative log-returns of bank k at day d, we assume

$$F_k(R^\star_{kd}) = \begin{cases} GPD(u_{kd}, \gamma_{kd}, \sigma_{kd}) \text{ if } R^\star_{kd} > u_{kd}, \\ F_k^{\text{emp}} \text{ otherwise,} \end{cases} \tag{13}$$

where $R^\star_{kd}$ is the daily negative residual derived from a model describing the mean structure and the heteroscedasticity of the corresponding log-returns. We first model the conditional mean of each bank through a linear model with covariates: the `MKT`, `HML` and `SMB` factors built by Fama-French (Fama and French, 1995) and the `bond factor`, `credit spread factor`, `dLevel` and `TED spread`. We model the heteroscedasticity of the residuals with a student GARCH(1,1) model. For the penalized EVR, we use time and bank-level dummies in the quantile regression used to estimate u_{kd}. We also use these dummies to estimate γ_{kd} and σ_{kd} in addition to the variables used in the mean model, complemented by the MSCI returns, the VIX and the lagged absolute residuals from the mean-GARCH model (only for the scale). This approach shows satisfactory results to model the upper tail of $R^\star_{kd}$ (QQ-plot can be found in Appendix E). For each month t, we match the vth largest bank observation within the considered month with the vth largest one of the hedge funds to form bivariate observations, $v = 1, \ldots, V_t$, V_t being the minimum between the number of observations of hedge funds and banks at time t, an approach similar to Mhalla et al. (2022).

2.3. Interpretation and motivation

In the proposed model, β_{sit} can be interpreted as a sensitivity measure, namely the sensitivity of the expected return of the banking system to the realization of tail events for a given hedge fund. From this quantity, one can easily derive traditional systemic risk measures such as the Marginal Expected Shortfall (MES) of Acharya et al. (2017), denoted $\text{MES}_{sit}(\bar{p})$. Indeed, we have

$$\begin{aligned}\mathrm{MES}_{sit}(\bar{p}) =& -\mathbb{E}\left[R_{st}|R_{it} \leq -\mathrm{VaR}_{it}(\bar{p})\right], \\ =& -\beta_{sit}\mathbb{E}\left[R_{it}|R_{it} < -\mathrm{VaR}_{it}(\bar{p})\right], \\ =& \beta_{sit}\mathrm{ES}_{it}(\bar{p}), \end{aligned} \tag{14}$$

where $\mathrm{ES}_{it}(\bar{p})$ is the expected shortfall of hedge fund i at time t and level $\bar{p}$. As a result, we can also quantify the expected loss of the banking system, at any time, given that the hedge fund i suffers from a negative shock. Although more complex, a similar relationship exists with the CoVaR and ΔCoVaR of Adrian and Brunnermeier (2016). Defining $\mathrm{CoVaR}_{sit}(\bar{p})$, with conditioning event $R_{it} = -\mathrm{VaR}_{it}$, as

$$\mathbb{P}(R_{st} < -\mathrm{CoVaR}_{sit}(\bar{p})|R_{it} = -\mathrm{VaR}_{it}(\bar{p})) = \bar{p}, \tag{15}$$

and

$$\Delta\mathrm{CoVaR}_{sit}(\bar{p}) = \mathrm{CoVaR}_{sit}(\bar{p}) - \mathrm{VaR}_{st}(\bar{p}).$$

This latter quantity can be interpreted as the additional VaR the banking system is facing when the hedge fund i exhibits extremely negative returns. As for $\mathrm{MES}_{sit}(\bar{p})$, VZ show that

$$\mathrm{CoVaR}_{sit}(\bar{p}) = \beta_{sit}\ \mathrm{VaR}_{it}\left[1 + \left(\frac{1}{\tau_{sit}} - 1\right)^{\gamma_{it}}\right], \tag{16}$$

which leads to

$$\Delta\mathrm{CoVaR}_{sit}(\bar{p}) = \beta_{sit}\ \mathrm{VaR}_{it}(\bar{p})\left[1 + \left(\frac{1}{\tau_{sit}} - 1\right)^{\gamma_{it}}\right] - \mathrm{VaR}_{st}(\bar{p}). \tag{17}$$

Hence, it only requires the estimation of β_{sit} and some of its components to estimate these systemic risk measures. While β_{sit} captures the strength of interconnection between two entities, MES_{sit} and $\Delta\mathrm{CoVaR}_{sit}(\bar{p})$ allow us to quantify the impact of this sensitivity in terms of returns, which eases economic interpretation.

Additionally, one should bear in mind that β_{sit} is a conditional measure, reflecting the sensitivity of the banking sector's returns only when the return of hedge fund i at time t is below its VaR. Concerning the estimation procedure, the consequence is that one should not use observations that do not satisfy this particular conditioning event to infer on β_{sit}. Otherwise, we would assume that data observed in normal conditions convey information about what happens under extreme conditions. In our case, the methodology is consistent with respect to this principle, since we use only observations that are far in the tail to infer the different components of β_{sit}. Thus, we avoid the criticism pointed out by Benoit et al. (2017): systemic risk measures are often estimated with parametric assumptions that do not explicitly model the tail distribution of the concerned entities. As such, when using a model focusing on the body of the distribution such as a bivariate GARCH, the ranking of institutions based on the MES or CoVaR with respect to an equity benchmark is akin to ranking these entities based on their systematic risk. They conclude that, since the marginal and joint tail dynamic of the returns cannot be described by their marginal and joint bodies, one should not use models aiming at capturing the center of the distribution to infer systemic risk measures. For these reasons, our methodology is of interest not only for hedge funds, where we suffer from data scarcity, but also for other more classical studies on systemic risk, as it allows us to simultaneously estimate and condition the risk measures while avoiding the pitfalls underlined by Benoit et al. (2017).

3. Data

To conduct our analysis, we use the returns and characteristics of hedge funds reporting in the Eurekahedge database. We first describe the hedge funds data, before describing the sample of banks selected to build a representative index of the banking sector.

3.1. Hedge funds data

We use the cross-sections of hedge funds returns provided by Eurekahedge, covering the period May 1995 - December 2020. We keep both dead and alive funds to address survivorship bias. We select funds based on a minimum lifespan of 48 months and remove the first 12 months of returns data for each fund to take into account backfilling bias. We use funds whose reporting currency is labeled in USD and require information availability for several important covariates in our analysis. We remove all fund-month observations for which the AUM is missing, in order to include this latter variable in our regression framework. The complete filtering procedure is detailed in the Appendix B. Our final sample consists of 4,847 funds, with a median number of reported observations per month of 86, leading to 475,339 fund-month observations. Table 1 provides some summary statistics. Performance fees and management fees are comparable to the ones reported by Patton and Ramadorai (2013) and Agarwal et al. (2009).

Long-Short Equities funds are the most important ones in terms of number of reporting funds, followed by *CTA/Managed Futures*. These are also the funds for which the highest average level of performance fee is observed, along with *Event Driven* and *Long-Short Equities* funds. The return distribution of hedge funds is leptokurtic and left-skewed for most strategies, which shows the importance of their extreme losses (Table 1). We also observe an overall increase in the average fund size over time, while the number of funds reporting has been decreasing since 2015 (Figure 1). These elements suggest a growth in the concentration of AUM within the industry.

Mean by strategy		Bottom-Up	Long Short Equities	Event Driven	Fixed Income	CTA/Managed Futures	Macro	Multi-Strategy	Others
AUM	(in USD Mio.)	290.34	181.72	308.04	559.70	165.32	279.02	539.28	252.04
Subs. Freq.	(in days)	16.73	29.66	28.90	21.08	17.73	24.72	28.09	28.41
Red. Freq.	(in days)	25.48	55.82	89.62	44.81	17.00	36.83	57.05	60.00
Red. Not. Per.	(in days)	22.63	39.92	52.68	38.29	12.07	27.09	40.65	46.42
Man. Fee	(in %)	1.34	1.46	1.51	1.28	1.63	1.53	1.52	1.42
Perf. Fee	(in %)	11.67	18.53	18.54	12.77	18.66	16.10	16.76	16.54
Life	(in months)	97.17	98.21	102.61	88.93	100.50	93.40	205.47	99.15
Proportion by strategy									
HWM	No	0.35	0.07	0.09	0.33	0.17	0.16	0.17	0.15
	Yes	0.65	0.93	0.91	0.67	0.83	0.84	0.83	0.85
Lock-up	No	0.80	0.68	0.61	0.73	0.94	0.85	0.77	0.69
	Yes	0.20	0.32	0.39	0.27	0.06	0.15	0.23	0.31
Closed	No	0.94	0.91	0.91	0.95	0.93	0.94	0.88	0.89
	Yes	0.06	0.09	0.09	0.05	0.07	0.06	0.12	0.11
Dead	No	0.70	0.44	0.56	0.57	0.42	0.40	0.47	0.46
	Yes	0.30	0.56	0.44	0.43	0.58	0.60	0.53	0.54
HR	No	0.62	0.88	0.86	0.67	0.94	0.82	0.82	0.80
	Yes	0.38	0.12	0.14	0.33	0.06	0.18	0.18	0.21
Leverage	No	0.80	0.43	0.49	0.47	0.43	0.29	0.38	0.58
	Yes	0.20	0.57	0.51	0.53	0.57	0.72	0.62	0.42
Number of funds / Total		0.07	0.35	0.05	0.09	0.15	0.06	0.07	0.16

Table 1: Overview of the variables by strategies. For numeric values, the mean by strategies is given. For the categorical values, we provide the proportion of occurrences for each level.

Before applying our estimation framework, we model and filter the mean structure of hedge funds returns. To do so, we rely on the work of Patton and Ramadorai (2013) to model time-varying exposures to risk factors (details are given in Appendix C). We consider an extended model and select the risk factors and conditioning information based on a Lasso regularization.

Strategy	Mean	Std. Dev.	Skewness	Kurtosis
Bottom-Up	0.45	0.06	-0.62	5.97
Long Short Equities	0.44	0.04	-0.27	5.40
Event Driven	0.45	0.03	-0.78	9.00
Fixed Income	0.37	0.02	-1.56	14.54
CTA/Managed Futures	0.38	0.04	0.06	5.37
Macro	0.34	0.04	-0.11	5.38
Multi-Strategy	0.40	0.03	-0.53	7.80
Others	0.34	0.03	-1.12	12.16

Table 2: Average statistics estimated at the fund level grouped by strategy.

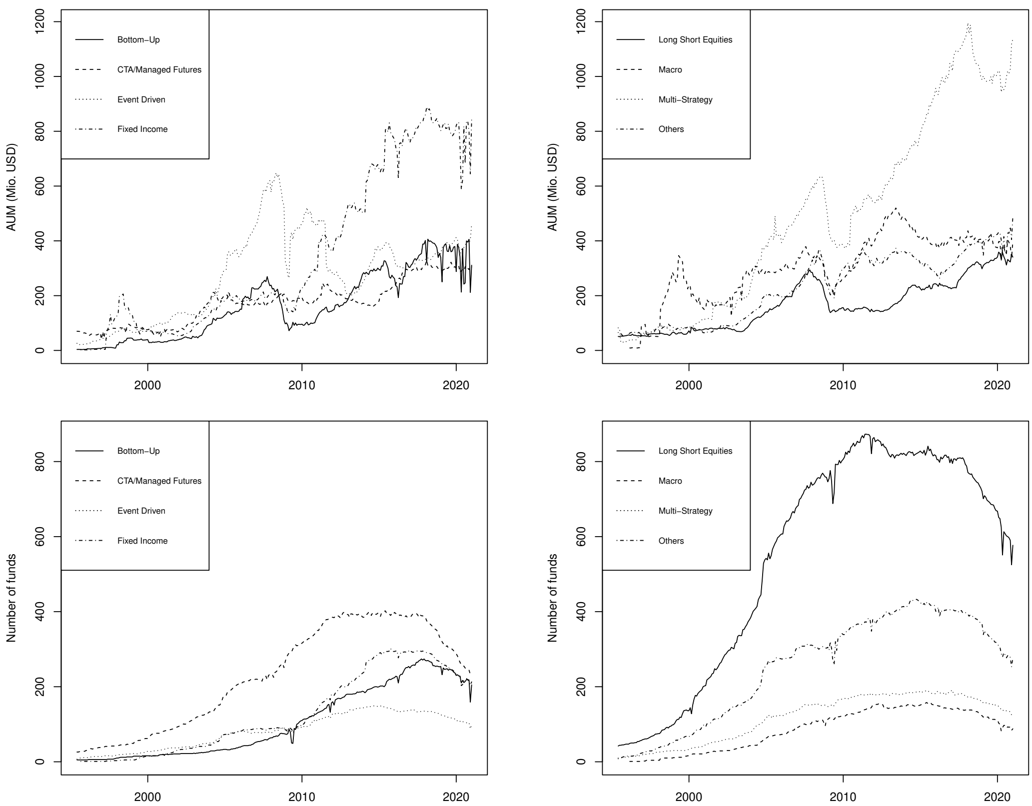

Figure 1: (Top) Evolution of the average funds size by strategy. (Bottom) Evolution of the number of hedge funds reporting Return and AUM, by strategy.

The resulting conditional mean model of hedge funds returns is of the form

$$R_{it} = \mu(f_t, Z_{(t-1)}, f_d^{\star}, Z_{(d-1)}^{\star}, \Theta_i^{\text{mean}}) + \epsilon_{it}, \tag{18}$$

where R_{it} is the monthly return of fund i at time t. The conditional mean of the returns is a function $\mu(\cdot)$ of f_t and $f_d^{\star}$, the risk factors measured on a monthly and daily basis, respectively, and of $Z_{(t-1)}$ and $Z_{(d-1)}^{\star}$, the corresponding lag conditioning variables measured on a monthly and daily basis. The set of parameters to estimate is Θ_i^{mean}, and is estimated through OLS for each fund i. The lagged absolute residuals of this model are also used as covariate in the estimation of the VaR of hedge funds (denoted $|\texttt{res}|_{t-1}$). To avoid numerical issues due to outliers, we remove the observations for which this covariate is higher than its empirical quantile 0.9998, corresponding to an approximate value of 0.45.

3.2. Banks data

Bank data originate from Eikon Refinitiv database. We use a panel of 22 banks (Table 3), selected by Mhalla et al. (2022) for their analysis of the tail dependence between banks and the different hedge funds strategies. For these entities, we retrieve the daily stock prices covering the time window of our hedge funds data and compute their log-returns. Two exceptions are Goldman Sachs and Mitsubishi UFJ, as they are quoted only from May 1999 and April 2001, respectively. Twenty of them are listed by the Financial Stability Board as systemically risky banking institutions. The others have been selected based on the importance of their assets under management. Although all selected entities are labeled as banks, AXA is also an insurer, while Goldman Sachs, Morgan Stanley, and Credit Suisse are also considered as broker-dealers or investment banks (Mhalla et al., 2022).

To derive VaR_{st} for a representative index of the banking sector, we first build the index using the approach of Adams et al. (2014) and derive the first component of a principal component analysis (PCA) using the log-returns of this set of 22 banks. The resulting index is comparable to the one used in the work of Adams et al. (2014). To compute daily VaR, we use a GARCH-EVT approach (McNeil and Frey, 2000). We then aggregate the daily VaR in a monthly measure.

Bank	Datastream code	Bank	Datastream code
Citigroup	U:C	Société Générale	F:SGE
JP Morgan	U:JPM	Deutsche Bank	D:DBK
Bank of America	U:BAC	AXA	F:MIDI
Bank of NY Melon	U:BK	ING	H:INGA
Goldman Sachs	U:GS	BNP Paribas Fortis	F:BNP
Morgan Stanley	U:MS	Intensa Sanpaolo	I:ISP
State Street	U:STT	Santander	E:SAN
Westpac Bank. Corp.	A:WBCX	BBVA	E:BBVA
Wells Fargo	U:WFC	Bank of Montreal	C:BMO
Standard Chartered	STAN	Bank of Nova Scotia	C:BNS
Barclays	BARC	Imperial Bank of Commerce	C:CM
HSBC, Lloyds	HSBA	Royal Bank of Canada	C:RY
Credit Suisse	S:CSGN	Toronto Dominion	C:TD
Commerzbank	D:CBK	Mitsubishi UFJ	J:MITF

Table 3: List of banks used in the analysis.

4. Results

To account for heterogeneity across investment strategies, we conduct our univariate and multivariate regression analyses separately for groups of funds sorted along their reported strategy. We use first strategies with at least 25.000 fund-month observations, thus creating 7 groups: *Bottom-Up* (`BU`), *Long Short Equity* (`LSE`), *Event Driven* (`ED`), *Fixed Income* (`FI`), *CTA/Managed Futures* (`CTAM`), *Macro* (`Macro`) and *Multi-strategy* (`MS`). Hedge funds that do not belong to one of the seven main strategies are classified in a group *Others* (`Others`). Sub-strategy heterogeneity inside this group is modeled though dummy variables.

We discuss first the results of the univariate regression analysis, before moving to the tail dependence. Finally, we discuss the obtained systemic risk measures.

4.1. Hedge funds downside risks

In Table 4, we report coefficient estimates of the univariate analysis for each selected covariate. Our results indicate that fund- and time-heterogeneity in the tail are mainly driven by variations in the scale parameter rather than in the shape. Using a QQ-plot of the pseudo-residuals (Figure E.1 in Appendix E), we show that the fit resulting from this estimation is of good quality. The estimated regression models are then used to infer on the values of the shape and scale parameters for each fund-month observation, which in turn allow us to compute the VaR of each fund-month observation at level $1 - p = 99.9\%$.

Looking at selected covariates across strategies, we find that the tail dynamic of hedge funds strongly differs from one strategy to the other. Nevertheless, one commonality across strategies is the selection of the lagged absolute residuals ($|\text{res}|_{t-1}$), which aims at modelling fund-specific heteroscedasticity. Moreover, the size of the fund (AUM) is also selected for all strategies, with the exception of *Multi-Strategy* funds. The negative regression coefficient indicates that a marginal increase in fund size is associated with a decrease in the tail risk of hedge funds, suggesting, e.g., the presence of an experience effect or economies of scale (Agarwal et al., 2017). For half of the strategies, we find a positive regression coefficient for management fees (Man. Fee), while the performance fees seem to matter for two strategies, namely *Macro* and *Others*. Thus, whereas Agarwal et al. (2009) show a link between managerial incentives and hedge funds performance, our results indicate that an increase in managerial incentives is associated with an increase in tail risks for some investment strategies. We also find a positive relationship between the use of lock-up periods (Lock-up) and the tail risk of hedge funds for *Long Short Equities* and *Macro* funds (representing 40% of our sample), a result consistent with Agarwal et al. (2017). Finally, we find evidence that Credit spread is positively associated with the tail risk of funds, for four out of the eight strategies. This effect suggests that the tail risk of hedge funds is partly explained by market funding conditions and default risks. It is also consistent with the shadow banking phenomenon, which saw major transfer of credit-risky activities from banks to lightly-regulated entities such as hedge funds.

Turning now to the time-series dimension, we observe that the average VaR across hedge funds has been slightly decreasing over time, from a level of 14.16% between 1997 and 2002 to 12.78% between 2014 and 2019. This decrease can be traced back to the strong negative relationship between fund size and tail risk, and to an increase in the average fund size over time (Figure 1). Nevertheless, we still observe large surges in the average VaR during periods of crisis such as 1998, 2008, or 2020, although their magnitude differs across strategies (see Figure F.1 in Appendix F). In particular, *Fixed Income* funds had their average VaR peaking at three different periods: in September 1998 (LTCM crisis), February 2000 (Dot-com bubble), and in March 2020 (COVID crisis), with the highest peak reaching 30%. On the contrary, *Bottom Up* and *Event Driven* funds did not experience peaks at the same periods: these funds were more affected during the 2008 crisis, with their average VaR peaking in October 2008, while we do not observe peaks over the period 1998-2000. This illustrates well the heterogeneity, in terms

	Covariates	BU	LSE	ED	FI	CTAM	Macro	MS	Others
	AUM	-0.10***	-0.04***	-0.10***	-0.05	-0.09***	-0.16***	-	-0.05***
	Subs. Freq.	-	-	-	0.13***	-	-	-0.07**	-
	Red. Freq.	-	-0.06***	0.10**	0.07*	-	-	0.04	-0.06**
	Red. Not. Per.	0.11***	-	-	-	0.07***	-	-	-0.05**
	Man. Fee	-	-	0.09**	0.06*	0.08***	0.091***	-	-
	Perf. Fee	-	-	-	-	-	0.04	-	0.12***
	$\|res\|_{t-1}$	0.26***	0.26***	0.21***	0.33***	0.22***	0.20***	0.31***	0.29***
	ACF	-	-	-0.08**	-0.01	0.04*	-0.11***	-	-0.03
	HR	-	-	-0.06**	0.05	-	0.07**	0.09***	0.05**
	HWM	-	-	0.07**	-	-	-	-	-0.10***
	Leverage	-	-	-	-	-	-	-0.07**	0.02
	Lock-up	-	0.07***	-	-	-	0.14***	-	-0.03
	SP500	-	-	-	-	-	-	0.04	-
	Size factor	-	-	-	-	-	-	-	-
	PTFSBD	-	-	-	0.11***	-	-	-	-
	PTFSFX	-	-	-0.06*	-	-	-	-	-
	PTFSCOM	-	-	0.05*	0.07**	-0.02	-	-	0.05**
σ	PTFSIR	-	-	-	-0.15***	-	-0.08**	-	-0.04**
	PTFSSTK	-	-	-	0.01	0.03	-	-	-
	Bond factor	-	-	-	-	-	-	-	-
	Credit spread	0.06**	-	0.10***	-	0.03	-	-	0.07***
	MSCI Em.	-	-	-	-	-	-	-	-
	Market illiquidity	-	-	-	-	-0.03	-	-	-
	MSCI W.	-0.02	-	-	-	-0.06***	-	-	-
	VIX	-	-	-	-	-	-	0.08**	-
	RV	-	-	0.04	-	-	-0.04	-	-0.02
	MIS - Dist. Debt								0.123***
	MIS - Div. Debt								0.09***
	MIS - DA								0.05
	MIS - Others								0.33***
	MIS - RV								0.06**
	MIS - TD								0.02
	MIS - V								0.17***
	$\bar{\sigma}$	0.02	0.02	0.02	0.01	0.03	0.02	0.02	0.02
	AUM	-2.03***	-	-	-0.67**	-	-	-2.76***	-
	Subs. Freq.	1.15**	-	-	-	-	-	-	-
	Red. Freq.	-1.92	-	-	-	-	-	-	-
	Red. Not. Per.	-	-	-	-0.37**	-	-	-	-
	Man. Fee	-	-	-	-	-	-	-	-
	Perf. Fee	-	-	-	-	-	-	-	-
	ACF	-0.66***	-0.20**	-	-	-	-	-	-
	HR	-	-	-	-	-	-	-	-
	HWM	-0.71***	-	1.91***	-	-	-	-	-
	Leverage	-	-	-	-	-	-	-	-
	Lock-up	-1.44**	-	5.52***	-	-	-	-	-
	SP500	-	-	-	-	-	-1.86	-	-
	Size factor	-	-	-	-	-	-	-	-
	PTFSBD	-	-	-	-	-	-	-	-
	PTFSFX	-	-	-	-	-	-	-	-
	PTFSCOM	-	-	-	-	-	-	-	-
	PTFSIR	-	-	-	-	-	-	-	-
γ	PTFSSTK	-	-	-	-	-	-	-	-
	Bond factor	-	-	-	-	-	-0.51*	-	-
	Credit spread	-	-	-	-	-	-	-	-
	MSCI Em.	-	-0.70***	-	-	-	-	-	-
	Market illiquidity	-	-	-	-	-	-	-	-
	MSCI W.	-	0.52***	0.14	-	-	1.36	-0.16	-
	VIX	-	-	-	-	-	-	-	-
	RV	-	-	-	-	-	-	-	-
	MIS - Dist. Debt								-
	MIS - Div. Debt								-
	MIS - DA								-1.45
	MIS - Others								-
	MIS - RV								-
	MIS - TD								-
	MIS - V								-
	$\bar{\gamma}$	0.08	0.11	0.08	0.24	0.05	0.08	0.18	0.07
	n^*	1636	8435	1268	1944	3544	1305	1826	3920
	$\overline{\text{VaR}}$	0.14	0.13	0.11	0.07	0.17	0.14	0.12	0.12
	$\overline{\text{ES}}$	0.18	0.17	0.13	0.10	0.20	0.17	0.16	0.15

Table 4: Adaptive Lasso estimation of α_l^σ (upper side) and α_l^γ (lower side) (equations 5 and 6). The covariates not selected by the model are reported as "-" while blank spaces indicate that the covariate was not candidate in the model. $\bar{\sigma}$ (resp. $\bar{\gamma}$) stands for the mean of the estimated σ (resp. γ) for all the observations of the group (i.e. not only data above the threshold). n^* stands for the number of observations used for the fit. $\overline{\text{VaR}}$ and $\overline{\text{ES}}$ correspond to the average VaR and ES at level $(1-p) = 99.9\%$ across all funds' observations belonging to the corresponding strategy. *, ** and *** indicate respectively significant coefficients at level 10%, 5% and 1%. A description of the variables is provided in Appendix D.

of tail behavior, between funds following different investment strategies.

Although informative about the risk profiles of a financial institution, the VaR might not be the best measure to investigate the systemic threat posed by a group of funds, since it does not reflect an expected loss in monetary units. From a policymaker standpoint, an approach of interest when considering a group of funds g at time t would consist in investigating the group Expected Shortfall (GES), defined as

$$\text{GES}_{gt} = \frac{1}{n_{gt}} \sum_{i \in g_t} \text{ES}_{it} \text{AUM}_{it},$$

where AUM_{it} is the AUM of fund i at time t, g_t is the set of indices for funds reporting at time t that belong to the group g and n_{gt} is the number of funds that report at time t in the group g. A natural choice for a group g is a given strategy. The intuition behind GES_{gt} is to measure the expected amount to be lost if one random fund among this group suffers from a loss larger than its 99.9% quantile. In Figure 2, we display the evolution, over time and across strategies, of this quantity. We observe an increase for all strategies, driven by the increase in average fund size over time (measured by the AUM) that more than compensates the decrease in VaR (expressed as a % of the AUM), and illustrates the growing threat hedge funds represent for financial stability.

4.2. Tail dependence between banks and hedge funds

In this section, we study the tail dependence between the banking sector and the hedge funds. Our baseline regression model for τ_{sit} includes all the variables used in the univariate analysis. In addition, we include two-way interaction effects obtained from combining funds characteristics with either the VIX or the S&P500 monthly returns. The analysis is performed separately on each group of funds sorted along their investment strategies. Our final model is obtained by applying a stepwise selection procedure.[1]

We identify several covariates that are common across strategies. We first observe that, except for *Long Short Equity* and *Fixed Income* strategies, the tail dependence is significantly

[1]We perform two additional sets of analysis, one with all the existing covariates and no variable selection, and one with stepwise selection without the interaction variables. The results are provided in the Supplement.

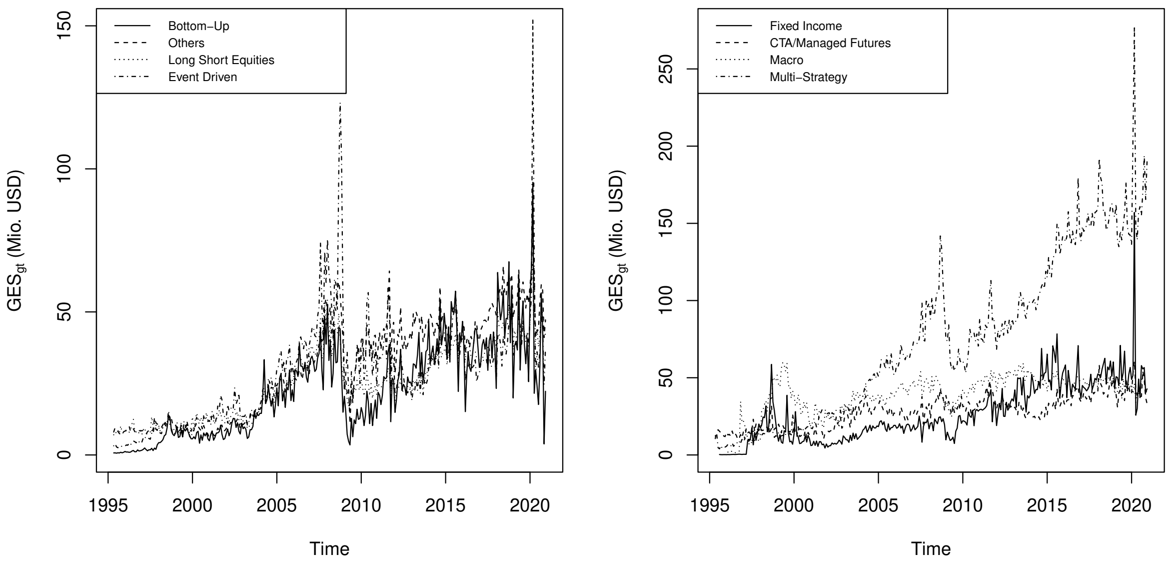

Figure 2: Evolution of GES_{gt} (Mio. USD), for g being the 8 different group of strategies considered in the analysis.

higher when funds are leveraged. Thus, besides the evidence reported in Dudley and Nimalendran (2011) that extremal linkage between funds is higher when they use leverage, our results suggest that the use of leverage increases also the linkage with the banking sector. From a theoretical standpoint, this result is consistent with the fact that the source of leveraged positions of hedge funds consists generally in collateralized borrowings financed by prime brokers. As such, the use of leverage should mechanically strengthen the linkage between funds and banks through brokerage activities.

Second, we find that an increase in market illiquidity (`Market illiquidity`) is associated with an increase of the tail dependence between hedge funds and banks, for all strategies except *Event Driven*. These results indicate that large negative shocks suffered by hedge funds are associated with a higher likelihood of simultaneous large losses among banks. In line with the effect of a prime broker channel highlighted above, this result is consistent with the documented high exposure of hedge funds to liquidity risk (Brunnermeier and Pedersen, 2009): a negative liquidity shock depresses the value of sophisticated illiquid products, which in turn increases margin requirements from banks, strengthening the linkages between funds and banks.

Then, focusing on uncertainty measures, we find that `VIX` is negatively related to the joint occurrence of large losses, indicating that a high uncertainty level is associated with less likely simultaneous large losses of banks. This drives the drop in tail dependence we observe for all the strategies around the 2008 crisis in Figure 3. In addition, we find a positive relationship between tail dependence and funds' size for four of the eight strategies (`AUM` in Table 5).

Finally, when looking at the time dimension of the estimates of tail dependence and systemic linkage, we observe a large increase of these measures after the 2008 crisis. When comparing the periods 2010-2020 and 1995-2007, the differences in average tail dependence are found to be significantly positive for all strategies, with the exception of the tail dependence of *Bottom-Up* funds (see Table G.1 in Appendix G). This increase in average tail dependence seems to be mainly driven by the increasing average size of the funds (positively associated with the tail dependence), and illustrates the raising concerns around the role played by hedge funds in the shadow banking phenomenon that appeared following the increase in capital constraints on banks after the crisis (Irani et al., 2021). Our results indicate that hedge funds with the *CTA/Managed Futures*, *Long Short Equities* and *Others* strategies have the highest average tail dependence and average systemic linkage, indicating a high propensity of these funds to participate to this phenomenon.

Thus, our findings suggest that a regulatory framework based on fund size and leverage would appear of interest to control for systemic linkages of hedge funds. Yet, direct regulation of hedge funds might not be feasible nor necessarily profitable (King and Maier, 2009).

	BU	LSE	ED	FI	CTAM	M	MS	Others
Intercept	0.44***	0.62***	0.24***	0.83***	0.92***	0.34***	0.62***	0.80***
AUM	0.06***	-	0.04***	0.03***	-	-	-	0.08***
Subs. Freq	-0.88***	-0.76***	-	-1.77***	-0.64***	-1.03***	-	-1.734***
Red. Freq.	-0.42***	-	-	-	-	-	-	-
Red. Not. Per.	-	-0.19***	-	0.45***	-	-	-	-
Man. Fee	0.14***	-	-	0.07***	-	-	-	0.17***
Perf. Fee	-0.01***	-	-	-0.01***	-	-	-	-
ACF	-0.35***	-	-	-0.48***	-0.32***	-	-	-0.22***
HR	0.11***	-	-	-	-	-	-	-0.07***
HWM	-	0.22***	-	-	-	-	-0.13***	-0.18***
Leverage	0.11***	-	0.10***	-	0.05***	0.24***	0.07***	0.14***
Lock-up	0.10***	0.08***	-	-0.05***	-0.18***	0.13***	-	-
SP500	1.14***	-2.40***	-	-	0.73***	-	-	-1.67***
Size factor	-0.78***	-	-	-0.74***	-0.70***	-0.89***	-	-0.98***
Bond factor	-	-	0.15***	-	0.10***	-	-	0.09***
Credit spread	-	-0.34***	0.17***	0.14***	-	-	-	-
Market illiquidity	0.84***	0.89***	-	0.70***	0.47***	0.537***	0.94***	0.67***
MSCI W.	-	2.60***	0.94***	-	-	-	-	2.44***
VIX	-0.01***	0.01***	-0.01***	-0.02***	-0.02***	-0.011***	-0.01***	-0.01***
RV	-	-8.22***	-	-	2.77***	-	-	-3.66***
MIS - Dist. Debt								0.06***
MIS - Div. Debt								0.46***
MIS - DA								0.19***
MIS - Others								0.15***
MIS - RV								0.14***
MIS - TD								0.03
MIS - V								0.22***
ACF × VIX	-	-	-	0.01***	-	-	-	-
Leverage × VIX	-	-	-	-	-	-0.01***	-	-
HWM × VIX	-	-0.01***	-	-	-	-	-	-
Subs. Freq. × SP500	-7.90***	-	-	-	-	-	-	-
Subs. Freq. × VIX	-	-	-	-	-	-	-	0.05***
Man. Fee. × VIX	-	-	-	-	-	-	-	-0.01***
n	3267	13413	2532	3888	7087	2609	3655	7845
BIC	-5209.24	-23234.26	-4685.27	-6079.25	-11175.40	-4684.11	-5750.93	-12779.54
$\bar{\tau}$	0.47	0.63	0.39	0.49	0.55	0.40	0.46	0.60
$\overline{SL}$	0.94	0.95	0.93	0.84	0.97	0.93	0.87	0.96
Min SL	0.71	0.80	0.77	0.47	0.89	0.81	0.73	0.85
Max SL	0.98	0.97	0.98	0.97	0.98	0.96	0.91	0.99

Table 5: Estimated regression coefficients for the tail dependence analysis (θ_p^{mult} in equation 11) applied for each group of strategy. n is the number of observations used in the corresponding regression fit, $\bar{\tau}$ and $\overline{SL}$ stand for the mean of the estimated tail dependence and SL and Min SL and Max SL are the minimum and maximum observed value of SL within each strategy group. *, ** and *** indicate respectively significant coefficients at level 10%, 5% and 1%. For the group Others, we add the variable related to the strategies remaining in this category (and for which the number of observed data was not sufficient for a full analysis). These dummy variables are all set to zero when the strategy is Arbitrage. A description of the variables is provided in Appendix D.

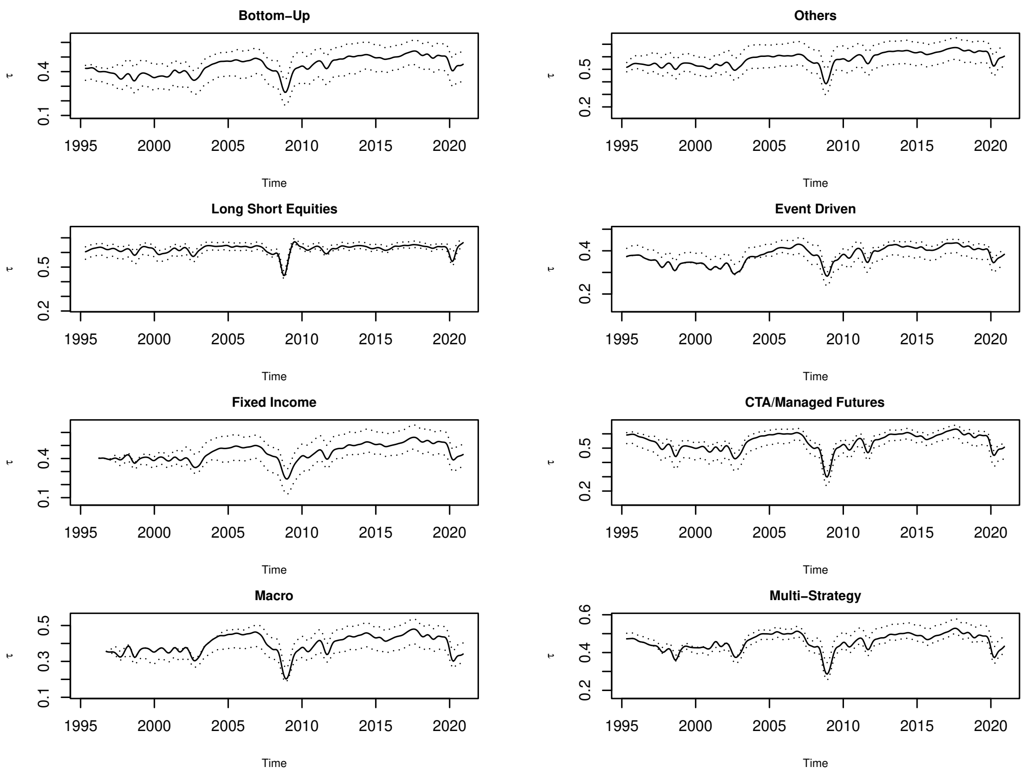

Figure 3: In continuous line, evolution of the median tail dependence, estimated at each point in time, for all hedge funds belonging to the corresponding strategy along with, in dashed, the estimated quantile at levels 5% and 95% of the same quantity. We use kernel smoothing for the ease of representation.

4.3. Systemic risks of hedge funds

When comparing funds with different investment strategies, *Fixed Income* funds display the highest average β_{sit} over the whole sample (Table 6), thus the highest systemic risk level. This quantity is a function of the VaR of the hedge funds, the VaR of the banks, and the tail dependence between the two (see equation (2)). Relying on this decomposition, we observe that the high β_{sit} is driven by a large ratio between the VaR of the banking sector and the VaR of the hedge funds, since the estimated SL is, on average, lower than the other strategies (see three first columns in Table 6). Nevertheless, the estimated systemic risk level of *Fixed Income* funds is not the highest at every point in time (Figure 4): when computing the average over all funds belonging to a given strategy, at each point in time, we observe that the levels of β_{sit} for *Long Short Equities* funds and the ones categorized as *Others* are the highest during the 2008 crisis. Hence, banks show a higher extremal sensitivity to these funds during this crisis period while *Fixed Income* funds stand out during more peaceful market contexts.

Focusing on the overall evolution of our systemic risk measures, we observe the largest average variation in β_{sit} between the pre- and post-crisis period for *Fixed Income* funds (Table 6). This result is also apparent when considering the full history (Figure 4). Using the decomposition provided by (2), we find that this increase is driven by the overall increase in tail dependence between banks and funds, and the relative decrease in the VaR of the hedge funds, with respect to the one of the banks. This evidence supports that the sensitivity of banks to the tail risk of hedge funds increased after the 2008 crisis, and points towards the increasing role played by hedge funds in the rise of shadow banking (Irani et al., 2021). Hence, our results highlight the growing importance of the hedge fund sector and the related concerns that need to be addressed (FSOC, 2021 and Abad et al., 2022).

To summarize these results, we show that *Fixed Income* funds appear to be the most important class of hedge funds, both in terms of their average β_{sit} level, and in their increase since the 2008 crisis. However, the dynamic and complex behavior of β_{sit} for the different strategies further highlights the necessity of a pro-active and dynamic monitoring. The case of *Long Short Equities* funds is a good example: these funds can have a larger impact on banks during periods of crisis while they are less important during peaceful periods.

	$\overline{\text{VaR}}$	$\overline{SL}$	$\overline{\beta}$	05/1995-12/2007	01/2010-01/2020	Δ
BU	0.14	0.94	1.83	-0.42	-0.16	0.27***
LSE	0.14	0.95	1.77	-0.33	-0.09	0.24***
ED	0.11	0.93	1.63	-0.09	0.14	0.23***
FI	0.07	0.84	2.39	0.08	0.48	0.40***
CTAM	0.17	0.97	0.99	-0.64	-0.28	0.36***
M	0.14	0.93	1.47	-0.42	-0.14	0.28***
MS	0.12	0.87	1.19	-0.29	-0.05	0.24***
Others	0.13	0.96	2.03	-0.11	0.00	0.11***

Table 6: $\overline{\text{VaR}}$, $\overline{SL}$, $\overline{\beta}$ are the empirical averages of the VaR, SL, and β for all observations of funds belonging to each strategy. Next to it, we report the empirical averages of log-β_{sit} for two sub-periods, before and after the 2008 crisis. The difference between the two means, Δ, is tested through a t-test. *, ** and *** indicate respectively significant coefficients at level 10%, 5% and 1%.

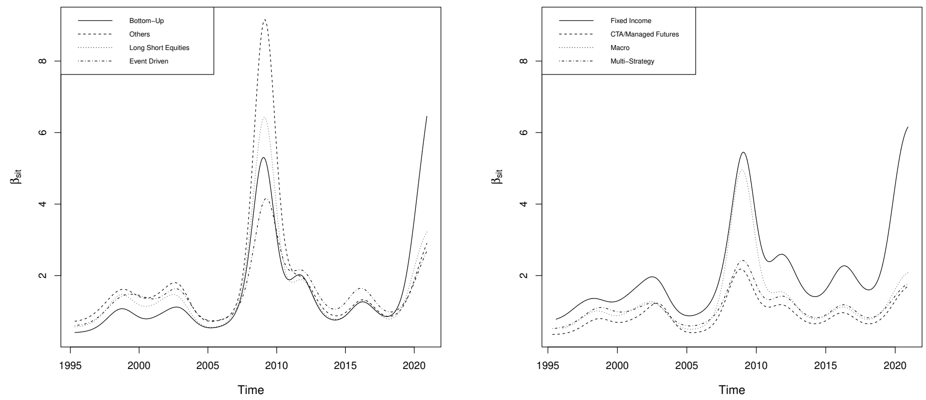

Figure 4: Evolution of β_{sit} by strategy using kernel smoothing with a bandwidth of 2.

To assess the importance of the factors driving the variations of β_{sit} over time, we follow van Oordt and Zhou (2019b) and regress the estimated log-β_{sit} against the covariates used in the estimation procedure[2]. Results are displayed in Table 7. We find that β_{sit} is significantly higher when funds are bigger, for all strategies (AUM). This result is consistent with the findings of Bellavite Pellegrini et al. (2022) for money market funds. In our framework, this relationship is driven by both the behavior of the VaR of hedge funds and the tail dependence with respect to the banking sector, since both are positively related to funds' size. Hence, the sensitivity of banks' extreme losses to funds' extreme losses is larger when funds' sizes increase. Moreover, the use of leverage is positively related to our systemic risk measure for five strategies, among which three also exhibit a link with market liquidity: β_{sit} increases for lower liquidity levels for *Multi Strategy*, *Fixed Income* and *Macro* funds. These two effects can be explained by the positive relationships between these variables and the tail dependence. Overall, these effects are consistent with the prime broker channel outlined in section 4.2.

To complement our main regression analysis, we graphically inspect the dependence between four covariates and β_{sit}. To do so, we sort fund-month observations in deciles along their estimated β_{sit}, and report the average value of the covariate of interest for each decile (Figure 5). The analysis is conducted separately for each investment strategy.

Starting with the individual size of funds (AUM, Figure 5, top left), we find a positive relationship with β_{sit} for all strategies, except *Multi-strategy* funds. This result is consistent with those reported in Table 7. Taking the example of *Fixed Income* funds, the highest β_{sit} decile

[2] We use Lasso estimators as well for this regression analysis.

	BU	MS	LSE	ED	FI	CTAM	M	Others
Intercept	-1.14***	-1.11***	-0.34***	-0.06***	-1.09***	-0.94***	-0.95***	-0.91***
AUM	0.18***	0.03***	0.14***	0.06***	0.09***	0.24***	0.01***	0.07***
Subs. Freq	-	-0.37***	-	-3.23***	-0.29***	-0.48***	1.54***	-
Red. Freq.	-0.21***	0.29***	-0.21***	-0.27***	-0.06*	-	-0.19***	0.21***
Red. Not. Per.	-0.96***	-	-	-	-1.32***	-	-	0.42***
Man. Fee	5.06***	4.21***	-11.07***	-1.86***	-5.33***	-8.34***	2.20***	-
Perf. Fee	-	-0.10***	-0.15*	-0.27***	0.23***	-0.36***	-	-1.40***
$\lvert\text{res}\rvert_{t-1}$	-5.83***	-4.87***	-4.15***	-13.34***	-3.64***	-3.82***	-7.14***	-5.32***
ACF	0.13***	-	0.54***	-	-0.37***	0.64***	-0.10***	0.11***
HR	0.03***	-	0.21***	-0.07***	0.02***	-0.19***	-0.15***	-0.07***
HWM	0.18***	-0.03***	-0.32***	-	-0.05***	-0.05***	-0.08***	0.20***
Leverage	-	0.03***	0.05***	-	0.02***	0.06***	0.15***	-
Lock-up	-	-0.09***	-0.26***	-	-	-0.15***	0.03***	0.042***
MIS - Dist. Debt								-0.23***
MIS - Div. Debt								-0.85***
MIS - DA								0.11***
MIS - Others								-0.59***
MIS - RV								-0.08***
MIS - TD								0.08***
MIS - V								-0.16***
SP500	2.37***	2.56***	3.01***	3.13***	2.82***	2.58***	1.95***	2.56***
Size factor	1.21***	1.35***	1.42***	-	-	0.25***	0.61***	0.60***
Bond factor	-0.18***	-0.12***	-	-0.17***	-0.14***	-0.15***	-0.12***	-0.13***
Credit spread	-0.37***	-0.19***	-0.54***	-0.20***	-0.24***	-0.18***	-0.17***	-0.47***
Market illiquidity	-	0.33***	-	-	0.33***	-	0.27***	-
MSCI Em.	2.89***	0.55***	0.47***	0.63***	-	-	0.22***	0.72***
MSCI W.	3.55***	1.96***	1.78***	1.41***	0.98***	1.16***	1.34***	1.40***
VIX	0.05***	0.05***	0.05***	0.05***	0.05***	0.05***	0.04***	0.05***
RV	22.32***	16.26***	6.39***	11.41***	17.96***	14.359***	14.65***	15.60***
PTFSBD	-	-	-	-0.25***	0.20***	0.17***	-	-
PTFSFX	-0.14***	-0.14***	-	-0.30***	-0.17***	-0.16***	-0.16***	-0.18***
PTFSCOM	-	-	-0.16***	-0.32***	0.26***	-	0.10***	-0.18***
PTFSIR	-0.12***	-0.05***	-0.07***	0.45***	-0.03***	0.21***	-0.05***	0.10***
PTFSSTK	-0.41***	-0.43***	-0.43***	-0.56***	-0.65***	-0.48***	-0.46***	-0.50***
Man. Fee. $\times$ VIX	-	-	-	-	-	-	-	0.00***
ACF $\times$ VIX	-	-	-	0.01***	-	-	-	-
adj. R^2	0.60	0.61	0.62	0.59	0.61	0.64	0.58	0.65

Table 7: Regression coefficient for the Lasso-OLS regression of all the covariates previously selected by our models on the estimated log-β_{sit}. *, ** and *** indicate respectively significant coefficients at level 10%, 5% and 1%. A description of the variables is provided in Appendix D.

is associated with an average AUM above 1,000 Mio. USD. Among all the *Fixed Income* funds reporting over the studied period, 20% of the funds reached a size at least as large during their lifetime. At the last reporting date in our dataset, 2.94% of the reporting *Fixed Income* funds reported a size larger than this threshold.

Focusing on the VIX (Figure 5, bottom left corner), we observe an exponential increase across deciles for all strategies. In particular, for *Fixed Income* funds, the last decile in β_{sit} is associated with the largest average VIX, with a level of around 20.

Third, looking at the relationship with Credit spread (Figure 5, top right corner), we find particularly high levels of average credit spreads for the two last deciles of β_{sit}. On the contrary, for lower deciles, we observe much lower levels and fewer variations across deciles. Hence, we show that our systemic risk measure takes the highest values when we observe a high level of credit risk. These results suggest that an important increase of credit risk might thus put pressure on the risk levels of banks. Since banks are strongly regulated for their risk-taking behavior, they might find an incentive to hedge costly risks, for example by transferring them to lightly regulated entities such as hedge funds. Our results support the hypothesis that such transfers would increase systemic risks of hedge funds only when particularly high levels of credit risk are observed.

Finally, we cannot find a meaningful relationship between the leverage and β_{sit} (Figure 5, bottom right corner).

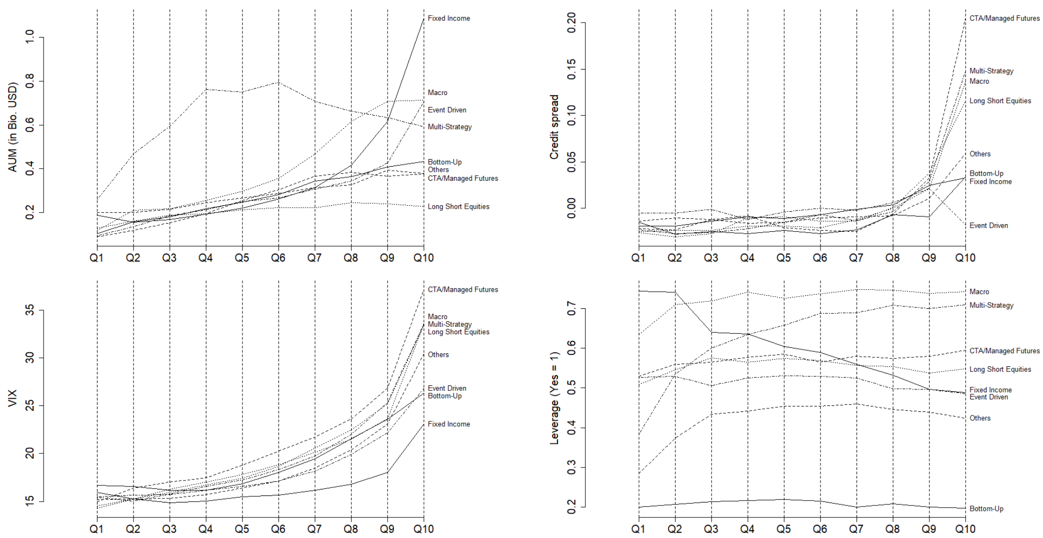

Figure 5: Evolution of the mean of key variables (AUM, credit spread, VIX and Leverage) across the ten deciles of β_{sit}, for each strategy.

We now derive $\Delta\text{CoVaR}_{sit}(\bar{p})$ through equations 16 and 17. To aggregate this information across funds, we define

$$\Delta\text{CoVaR}_t^{HF}(\bar{p}) = \text{AUM}_{st}\,\frac{1}{n_t}\sum_{i\in I_t}\Delta\text{CoVaR}_{sit}(\bar{p})$$

where I_t and n_t are the set of hedge funds indices for which we have an estimation for β_{sit} at time t and the corresponding number of funds at this time, respectively. AUM_{st} is the AUM of the banking index we build, whose weights are the loadings of the first principal component of our PCA (see section 3.2). We estimate this quantity for a level $1 - p = 99.9\%$. Recall that $\Delta\text{CoVaR}_{sit}(\bar{p})$ measures the change in VaR of the banking index due to a shock on fund i. Hence, the average of the $\Delta\text{CoVaR}_{sit}(\bar{p})$ over all funds can be considered as a proxy for the additional VaR the banking index is facing due to a shock materialized on a randomly-picked fund at a given time. We multiply this value by the AUM of the banks' portfolio to translate this exposure into monetary units (USD).

On Figure 6, we display the evolution of $\Delta\text{CoVaR}_t^{HF}(\bar{p})$ and its value expressed as a percentage of AUM_{st}. Over the whole sample, this latter measure reaches an average of 12.48%, with a maximum of 63.54% in October 2008, showing that crisis periods tend to correspond to higher exposure to hedge funds tail risk. We also observe a shift in the average level of this measure when we compare the periods before and after the crisis, consistent with previous findings. Between the two periods considered in Table 6, we find that the average $\Delta\text{CoVaR}_t^{HF}(\bar{p})$ roughly doubled, from USD 4.820 Bio. to USD 9.058 Bio. This shift is driven by an increase in AUM_{st}, from USD 50.398 to 78.048 Bio., but also by β_{sit}. Hence, the average $\Delta\text{CoVaR}_t^{HF}(\bar{p})$ as a percentage of AUM_{st} increases from 9.93% to 12.48%, and is solely driven by an increase in β_{sit}, since we had normalized by the size of the funds.

In summary, our results first suggest that the systemic risk of hedge funds, measured by β_{sit} and $\Delta\text{CoVaR}_t^{HF}(\bar{p})$, increased after the 2008 crisis. Secondly, we find that *Fixed Income* funds represent one of the biggest systemic threats among the different strategies, for two reasons: the average systemic risk level over the considered time window is the highest among all strategies, and we observe a large increase in the average of the systemic risk measure β_{sit} after 2008.

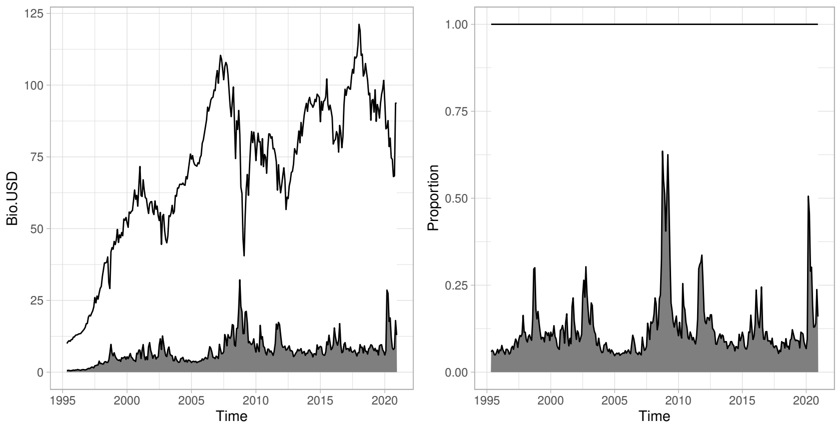

Figure 6: (Left) Evolution of $\Delta\text{CoVaR}_t^{HF}$ represented by the grey area, along with the evolution of AUM_{st} as the upper line. (Right) Evolution of the proportion of $\Delta\text{CoVaR}_t^{HF}$ with respect to AUM_{st}

Finally, our different regression results suggest that an increase in the size of hedge funds and the use of leverage are signals among the funds characteristics for higher systemic risk, and that market conditions corresponding to higher uncertainty, lower market liquidity and especially high credit risk levels are associated with a higher level of β_{sit}.

5. Conclusion

In this paper, we investigate the systemic risk contribution of the hedge funds industry. To do so, we use the model of van Oordt and Zhou (2019a) to measure the sensitivity of a banking index to shocks experienced by individual hedge funds, which can also be interpreted as a sensitivity to the tail risk of these funds. We also examine the systemic linkage and the tail dependence between banks and hedge funds, a measure of extremal dependence highlighted in the systemic risk literature. An innovation of the present paper is that we overcome data scarcity issues related to the low frequency of hedge funds data and short reporting history with the help of extreme value regression methods (Chavez-Demoulin and Davison, 2005; Mhalla et al., 2019). Hence, we are able to estimate time-varying and fund-specific systemic risk measures driven by covariates through a regression structure. Relevant covariates are selected with Lasso-type estimation methods.

Among our key findings, we evidence that banks are more sensitive to the tail distribution of hedge funds returns during crisis and uncertainty periods, although the tail dependence between these two entities concomitantly drops. We also find that the main drivers of the systemic risk contribution are the individual size of the funds and their use of leverage. When analyzing funds based on their investment strategies, *Fixed Income* hedge funds seem to represent the highest threat as of today, due to both their systemic risk level and its speed of increase over the last decade. Overall, we observe an increase of the sensivity of banks to hedge funds shocks over time. This increase is mostly explained by an increase in funds' sizes over time, and is consistent with the development of the shadow banking phenomenon over the past 15 years.

Regarding our study of the systemic linkage, we find that an increase in market illiquidity is associated with an increase in tail dependence. This result suggests therefore that an important risk channel between banks and hedge funds might be related to the trade of illiquid financial products. As such, from the perspective of a bank, it could indicate that hedge funds exhibit a counterparty risk that is highly sensitive to liquidity shocks.

From a policy standpoint, our findings highlight the need for regulators to monitor the systemic risks posed by hedge funds, in light of the large amounts of economic capital at risk. In particular, when highly uncertain or illiquid market conditions are expected, a thorough assessment of the counterparty risks posed by large and leveraged funds could help to avoid adverse effects on the financial sectors.

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Appendices

A. Threshold estimation

To estimate the threshold u_{it} in equation (4), we follow the methodology applied by Mhalla et al. (2022) and use quantile regression with time as a covariate. We write

$$u_{it} = \arg \max_{\beta_0, \beta_1 \in \mathbb{R}} \sum_{j=1}^{N} \rho_q \{R^{\star}_{it} - (\theta^q_0 + \theta^q_1 t_j)\},$$

where $\rho_q(u) = (q - \mathbb{1}_{q<0})q$ and where the time index is given by

$$t_j = \left(\underbrace{1, \ldots, 1}_{\times n(1)}, \ldots, \underbrace{T, \ldots, T}_{\times n(T)} \right),$$

where $n(t)$ is the number of observations at time t such that $N = \sum_{t=1}^{T} n(t)$, T being the last observation date index in the dataset. This threshold selection ensures that the number of observations across time is well balanced across time. In our empirical analysis, we set $q = .95$.

B. Hedge funds data cleaning procedure

Hedge funds data were retrieved from Eurekahedge on 28th November 2021. The dataset provides information on 26,855 funds with 127 variables. For each fund, the database provides the time series of returns and Assets Under Management (AUM). Among all the funds, we only consider the ones whose reporting currency is USD and with the following available information: inception date, currency, hurdle rate, high water mark, minimum investment size, leverage, redemption notification period, lock-up, management fee, and performance Fee. Among the resulting 6693 funds, we then look at the proportion of data available for each remaining variable. Based on these proportions, we require the funds to report their data for variables that are available in more than 99% of the cases among the remaining population. This allows us to also have information on redemption frequency or the subscription frequency and results in the exclusion of 93 additional funds. We further restrict ourselves to funds reporting more than 48 months, taking into account that we remove the 12 first months returns. For the main systemic risk analysis, we further exclude observations where the `AUM` is not reported, the covariate $|\texttt{res}|_{t-1}$ is higher than its empirical quantile 0.9998 and the estimated VaR is negative. The resulting set of funds and their observations are the ones described in the data section and the ones for which we are able to compute all our risk measures.

C. Mean model for hedge funds returns

In our analysis, we filter out first the mean structure of the hedge funds, before applying our EVR methodology on the residuals (equation (3)). To do so, we rely on the work of Patton and Ramadorai (2013) who analyze the intra-month exposure of the hedge funds returns. As a starting point, we assume

$$\begin{aligned} R_{id} &= \alpha_i + \theta_{id}^{\text{mean}} f_d^{\star} + \epsilon_{id}^{\star}, \\ \theta_{id}^{\text{mean}} &= g_i(\mathbf{Z}_d), \end{aligned} \tag{C.1}$$

where R_{id} is the daily return of fund i at time d, $f_d^{\star}$ is a daily risk factor and $\theta_{id}^{\text{mean}}$ describes the time-varying exposure to this factor. The time-varying nature of the coefficient is made possible by conditioning it to relevant daily information $\mathbf{Z}_d$, the information set available at time d. The dependence between this set of information and $\theta_{id}^{\text{mean}}$ is described by the function $g_i(\cdot)$. The rationale of this model is that managers change their exposure depending on the additional information they have each new day (Mitchell and Pulvino, 2001). We consider here one single conditioning information on a monthly basis Z_{d-1} and on a daily basis $Z_{d-1}^{\star}$, where Z_{d-1} is constant within a month. For $g_i(\cdot)$, we use a linear relationship described by

$$g_i(\mathbf{Z}_d) = \theta_i^{\text{mean}} + \phi_i^{\text{mean}} Z_{d-1} + \delta_i^{\text{mean}} Z_{d-1}^{\star}. \tag{C.2}$$

Then if $\mathcal{M}(t)$ is the space of day belonging to month t, we can write

$$R_{it} = \alpha_i m_t + \theta_i^{\text{mean}} f_t + \phi_i^{\text{mean}} f_t Z_{t-1} + \delta_i^{\text{mean}} \sum_{d \in \mathcal{M}(t)} f_d^{\star} Z_{d-1}^{\star} + \epsilon_{it}, \tag{C.3}$$

$$\text{where} \qquad \begin{aligned} R_{it} &= \sum_{d \in \mathcal{M}(t)} R_{id}, \\ f_t &= \sum_{d \in \mathcal{M}(t)} f_d^{\star}, \\ \epsilon_{it} &= \sum_{d \in \mathcal{M}(t)} \epsilon_{id}^{\star}, \end{aligned}$$

with m_t being the number of days within month t and Z_{t-1} being the conditioning information measured in monthly frequency.

Like Patton and Ramadorai (2013), we use four variables as conditioning information $\mathbf{Z}_d$:

1. The liquidity of the market, measured by the TED spread, i.e. 3-months LIBOR rate minus the 3-months T-Bill rate (TED).

2. The funding and leverage cost, measured by the cost of borrowing captured by the constant-maturity three-month U.S. T-bill rate (dLevel).

3. The volatility of the market, capturing the risk of the current environment, through the VIX index (VIX).

4. The performance of a common benchmark to which hedge funds would refer, here the S&P500 returns (SP500).

Regarding the risk exposures (f_t), we use four out of the seven factors provided by Fung and Hsieh (2004) that are available on a daily basis, namely:

1. The equity risk factor (SP500), measured by the excess return of the S&P500.

2. The size factor (SMB), measured by the difference between the Russell 2000 index and the S&P500 index returns (note that this one is different from the one used by Fung and Hsieh (2001)).

3. The Bond factor (TCM10Y), measured by the change in the 10-year treasury constant maturity yield.

4. The credit spread factor (BAAMTSY) measured by the change in the Moody's Baa yield less the 10-year treasury constant maturity yield.

In addition, we add the Emerging Market index factor (MSEMKF) suggested by the authors as an eighth factor (Fung and Hsieh, 2001).
In their work, Patton and Ramadorai (2013) suggest to use a more sparse set of risk factors including only two of them based on the BIC, testing separately for the the other four variables. Instead, we propose to estimate the full model including all available information conditioning $\theta_{it}^{\text{mean}}$ and all the risk factors mentioned above.As the estimation of such a model is possible with OLS, we can use a regularization technique such as Lasso penalization. The model becomes

$$
\begin{aligned}
R_{it} = &\ \alpha_i n_t + \sum_{j=1}^{5} \theta_{ij}^{\text{mean}} f_{jt} + \sum_{j=1}^{5}\sum_{k=1}^{4} \phi_{ijk}^{\text{mean}} f_{jt} Z_{k(t-1)} + \sum_{j=1}^{5}\sum_{k=1}^{4} \delta_{ijk}^{\text{mean}} \sum_{d \in \mathcal{M}(t)} f_{j,d}^{\star} Z_{k(d-1)}^{\star} + \epsilon_{it} \\
= &\ \mu(f_t, Z_{(t-1)}, f_d^{\star}, Z_{(d-1)}^{\star}, \Theta^{\text{mean}}) + \epsilon_{it},
\end{aligned}
$$

where f_{jt} and $Z_{k(t-1)}$ respectively denote the j^{th} risk factor and the k^{th} conditioning information at time t and $t-1$, respectively. $\Theta_i^{\text{mean}} = \{\theta_{ij}^{\text{mean}}, \phi_{ijk}^{\text{mean}}, \delta_{ijk}^{\text{mean}} | \forall j, k\}$ are the parameters to be estimated for each fund i. We first implement a simple Lasso-approach where we select the best penalization coefficient through the AIC. We first run the Lasso and then use the non-zero coefficients to determine the covariates that must be selected in a new fit of a simple OLS regression. This is what we call the post-Lasso coefficients, i.e. the ones obtained by taking into account the selection provided by the Lasso. To gauge the performance of this approach, we compare it with the distinctive features of the outcomes provided by the 8-factors model of Fung and Hsieh (2004) and the 6 factors model of Patton and Ramadorai (2013) (Figure C.1). The median number of coefficients selected by the Lasso is 6, which is comparable to the pre-determined number with the two other models. We confirm the overall over-performance of our model compared to the ones of Patton and Ramadorai (2013) and Fung and Hsieh (2004), both in terms of adjusted R^2 and AIC (Table C.1 and Figure C.1). Consequently, we use our Lasso approach to estimate the conditional mean of the hedge funds returns, denoted $\hat{\mu}_{it}$.

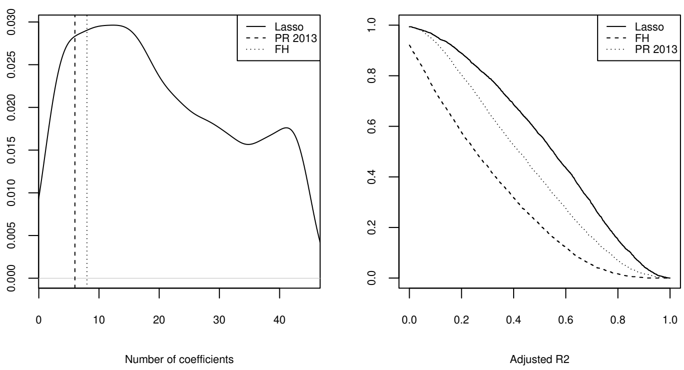

Figure C.1: (Left) Empirical density of the number of coefficients selected by the three models: Fung and Hsieh (FH), Patton and Ramadorai 6 factors model (PR 2013), Lasso (Lasso). The intercept is excluded. (Right) Survival curve of the adjusted R^2 for these three models. The bigger the surface below the curve of a model, the better it is in terms of adjusted R^2. We can make a parallel with Figure 5 in the work of Patton and Ramadorai (2013) where they plot instead the empirical cdf of the adj. R^2.

	Mean		Quantile 10%		Quantile 90%	
	Adjusted R^2	Δ AIC	Adjusted R^2	Δ AIC	Adjusted R^2	Δ AIC
Lasso	0.53	0.00	0.19	0.00	0.85	0.00
PR 2013	0.43	-19.95	0.13	-42.62	0.77	-4.62
FH	0.29	-47.98	0.01	-98.33	0.63	-12.23

Table C.1: Mean, quantile at level 10% and 90% of the Δ AIC and adjusted R^2 for the three models: Fung and Hsieh (FH), Patton and Ramadorai 6 factors model (PR 2013), Lasso (Lasso). Δ AIC is the average of the differences between the AIC of the Lasso model (Lasso) and the AIC of the corresponding model. A positive value is associated with an improvement of AIC compared to the Lasso.

D. Variables used in the analysis

In Table D.1, we provide a more detailed description of the variables we use in the analysis.

Variable Code	Description	Source
`AUM`	Asset Under Management	Eurekahedge
`Subs. Freq.`	Subscription frequency	Eurekahedge
`Red. Freq.`	Redemption frequency	Eurekahedge
`Red. Not. Per.`	Redemption Notification Period	Eurekahedge
`Man. Fee`	Management Fee expressed in %	Eurekahedge
`Perf. Fee`	Performance Fee expressed in %	Eurekahedge
`ACF`	Auto-correlation coefficient lag 1 of the funds returns	Eurekahedge
`MIS - BU`	Dummy equal to 1 when Main Investment Strategy is Bottom-Up	Eurekahedge
`MIS - CTAM`	Dummy equal to 1 when Main Investment Strategy is CTA/Managed Futures	Eurekahedge
`MIS - Dis. Debt`	Dummy equal to 1 when Main Investment Strategy is Distressed Debt	Eurekahedge
`MIS - Div.` Debt	Dummy equal to 1 when Main Investment Strategy is Diversified debt	Eurekahedge
`MIS - DA`	Dummy equal to 1 when Main Investment Strategy is Dual Approach	Eurekahedge
`MIS - ED`	Dummy equal to 1 when Main Investment Strategy is Event-Driven	Eurekahedge
`MIS - FI`	Dummy equal to 1 when Main Investment Strategy is Fixed income	Eurekahedge
`MIS - LSE`	Dummy equal to 1 when Main Investment Strategy is Long Short Equity	Eurekahedge
`MIS - Macro`	Dummy equal to 1 when Main Investment Strategy is Macro	Eurekahedge
`MIS - MS`	Dummy equal to 1 when Main Investment Strategy is Multi-Strategy	Eurekahedge
`MIS - Others`	Dummy equal to 1 when Main Investment Strategy is Others	Eurekahedge
`MIS - RV`	Dummy equal to 1 when Main Investment Strategy is Relative Value	Eurekahedge
`MIS - TD`	Dummy equal to 1 when Main Investment Strategy is Top-Down	Eurekahedge
`MIS - V`	Dummy equal to 1 when Main Investment Strategy is Value	Eurekahedge
`HR`	Dummy equal to 1 if the fund uses hurdle rate	Eurekahedge
`HWM`	Dummy equal to 1 if the fund uses High Water Mark	Eurekahedge
`Leverage`	Dummy equal to 1 if the fund uses Leverage	Eurekahedge
`Lock-up`	Dummy equal to 1 if the fund uses Lock-up periods	Eurekahedge
`SP500`	S&P500 returns	Eikon Datastream
`Size factor`	Size factor measured by Fung and Hsieh (2004)	Eikon Datastream
`PTFSBD`	Trend following factor on Bonds	Data from Fung and Hsieh (2004)[3]
`PTFSFX`	Trend following factor on Foreign Exchange	Data from Fung and Hsieh (2004)[3]
`PTFSCOM`	Trend following factor on Commodities	Data from Fung and Hsieh (2004)[3]
`PTFSIR`	Trend following factor on Interest Rates	Data from Fung and Hsieh (2004)[3]
`PTFSSTK`	Trend following factor on Stocks	Data from Fung and Hsieh (2004)[3]
`Bond factor`	Bond factor, derived by the change in 10-year constant maturity yield	Data from Federal Reserve Bank of St. Louis[4]
`Credit spread factor`	Credit spread factor measured by Fung and Hsieh (2004)	Data from Federal Reserve Bank of St. Louis[4] [5]
`MSCI Em.`	MSCI emerging markets index returns	Eikon Datastream
`Market illquidity`	Liquidity factor of Pástor and F. Stambaugh (2015)	Pástor and F. Stambaugh (2015)[6]
`MSCI W.`	MSCI World index returns	Eikon Datastream
`VIX`	VIX	Data from Federal Reserve Bank of St. Louis[7]
`RV`	Monthly realized volatility on the S&P500	Derived from S&P500 data

Table D.1: List of the different variables used in the analysis. For the dummy variable `MIS`, all are set to zero when MIS is Arbitrage.

[3] `http://people.duke.edu/~dah7/DataLibrary/TF-Fac.xls`
[4] `https://fred.stlouisfed.org/series/DGS10`
[5] `https://fred.stlouisfed.org/series/DBAA`
[6] `https://finance.wharton.upenn.edu/~stambaug/liq\_data\_1962\_2021.txt`
[7] `https://fred.stlouisfed.org/series/VIXCLS`

E. Goodness-of-fit for the marginal distribution of hedge funds and banks

In this section, we illustrate the goodness-of-fit of our approach with QQ-plots. For the hedge funds, we focus on the fit of the univariate EVR approach on the corresponding data (Figure E.1). For the banks, we show the QQ-plot for the data on which we apply the univariate EVR approach (left) but also on the whole distribution of $R^{\star}_{kd}$. The use of the empirical cdf leads to a poor fit in the very left tail of $R^{\star}_{kd}$, which does not impact the analysis because these estimates are not used for the multivariate EVR.

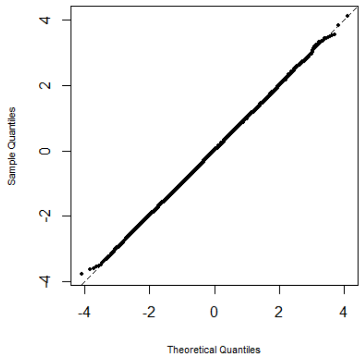

Figure E.1: QQ-plot of the pseudo-residuals computed from the estimated GPD distribution on the exceedances. The further we go in the right-up corner, the further we are in the left tail of the hedge funds returns distribution.

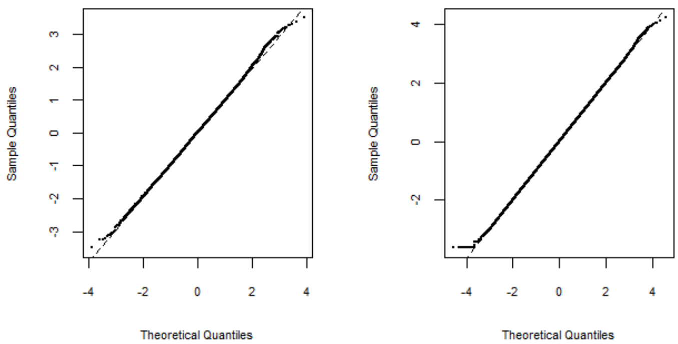

Figure E.2: (Left) QQ-plot of the negative pseudo-residuals from the mean-GARCH model, computed from the estimated GPD distribution on the exceedances. (Right) Same as left, but we add also the data that do not exceed the threshold and we apply the empirical cdf on the residuals that are not exceedances.

F. Marginal regression analysis

At each reporting time, for each strategy, we compute the average estimated VaR at level 99.9% for all funds belonging to the given strategy. We illustrate the evolution of this quantity for each strategy in Figure F.1. For most of them, we can observe the peaks around the different crisis periods (LTCM, bubble crisis, 2008 crisis, and COVID), in addition to a general decreasing trend over time.

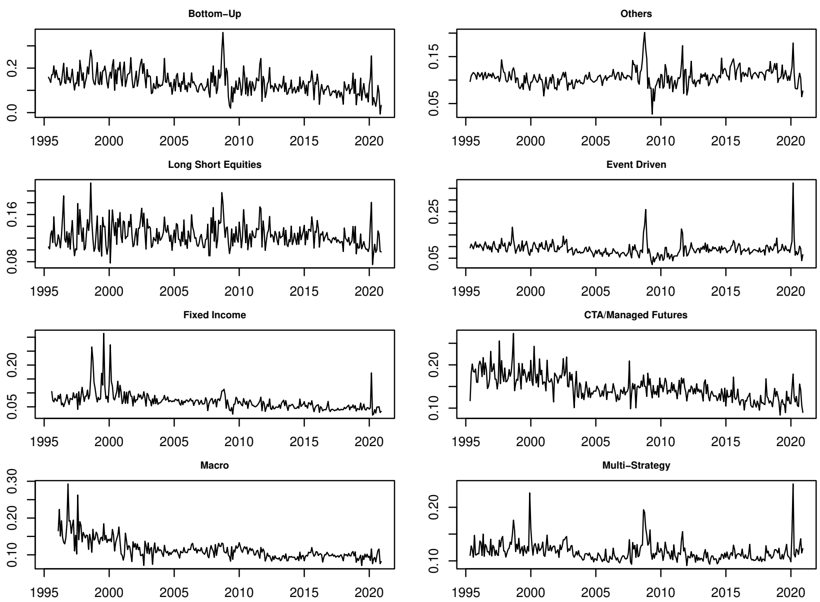

Figure F.1: Evolution of the average VaR over all funds belonging to the corresponding strategy at each point in time.

G. Tail dependence and systemic linkage before/after the 2008 crisis

We show in more detail the comparison of hedge funds linkage with respect to the banking sector through the tail dependence and systemic linkage (Table G.1). We compare the estimated quantities before and after the crisis and conclude for both measures, for all hedge funds, that there has been an increase in the extremal dependence between hedge funds and banks. The only exception is *Bottom-Up* hedge funds for which we have a decreasing systemic linkage, on the contrary of the tail dependence, due to the increasing shape parameter of these hedge funds. For the two periods and each strategy, we illustrate the distribution of the estimates using kernel density (Figures G.1 and G.2).

	$\overline{SL}_{1995/05\text{-}2007/12}$	$\overline{SL}_{2010/01\text{-}2020/01}$	$\Delta\bar{SL}$
BU	0.945	0.942	-0.003***
LSE	0.950	0.951	0.001***
ED	0.916	0.935	0.018***
FI	0.824	0.850	0.026***
CTAM	0.971	0.973	0.003***
M	0.929	0.935	0.006***
MS	0.869	0.878	0.009***
Others	0.961	0.967	0.006***
	$\bar{\tau}_{1995/05\text{-}2007/12}$	$\bar{\tau}_{2010/01\text{-}2020/01}$	$\Delta\bar{\tau}$
BU	0.422	0.494	0.072***
LSE	0.632	0.639	0.007***
ED	0.381	0.398	0.017***
FI	0.450	0.511	0.062***
CTAM	0.548	0.576	0.028***
M	0.397	0.427	0.030***
MS	0.460	0.485	0.025***
Others	0.577	0.629	0.052***

Table G.1: $\overline{SL}$ and $\bar{\tau}$ denote the average estimated SL and τ, respectively, grouped by strategies and for two sub-periods, before and after the 2008 crisis. The difference between the two-period means is tested for each group through a t-test. *, ** and *** indicate respectively significant coefficients at level 10%, 5% and 1%. Numbers are displayed with 3 decimal places for clarity.

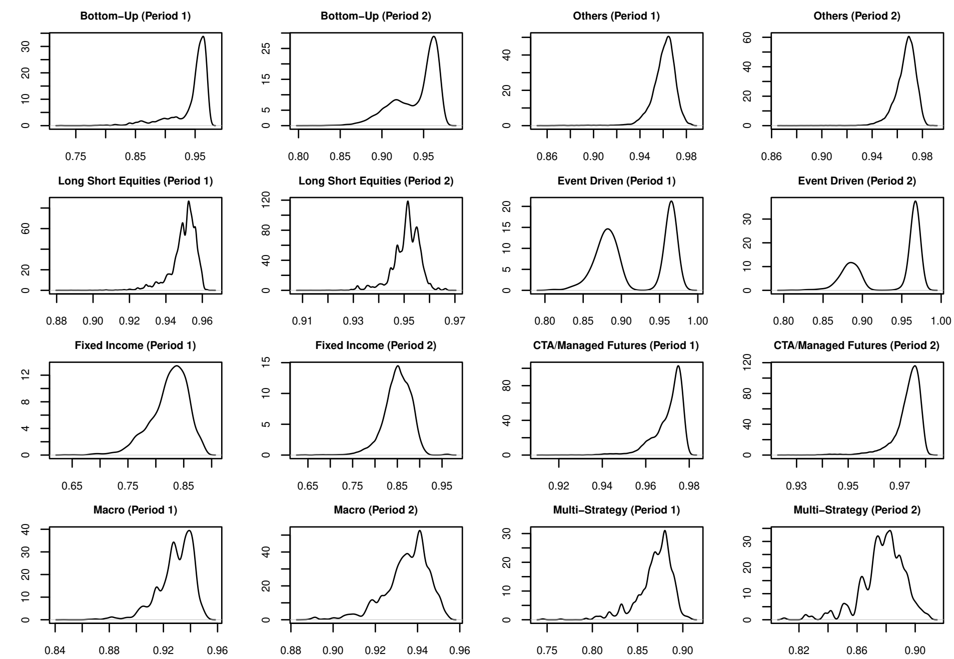

Figure G.1: Kernel density of the estimated SL within each of the two periods considered in Table G.1, grouped by strategies.

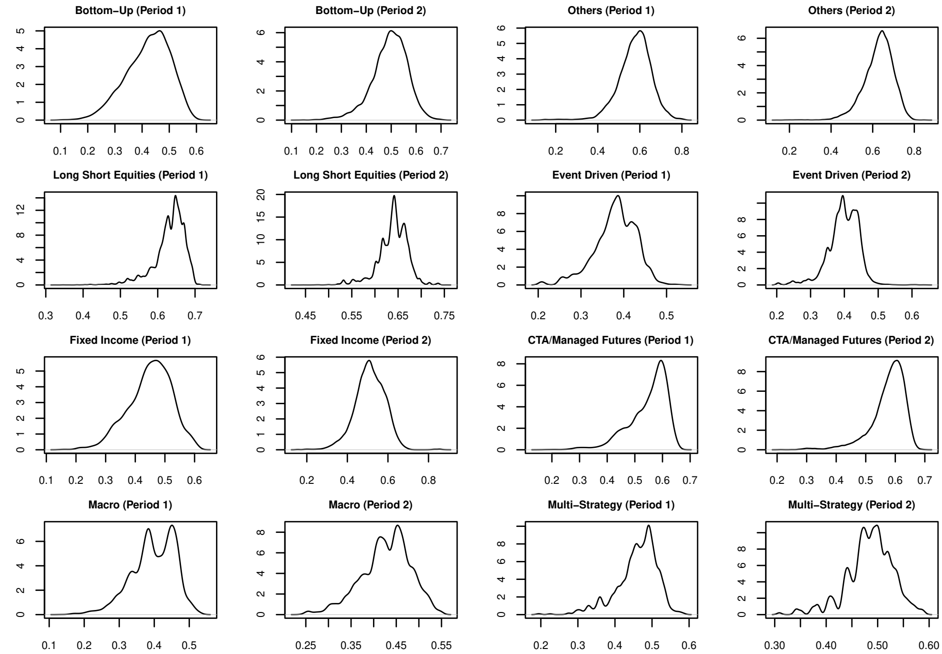

Figure G.2: Kernel density of the estimated τ_{sit} within each of the two periods considered in Table G.1, grouped by strategies.