Creating NOON states with ultracold atoms via counterdiabatic driving

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Abstract

We theoretically investigate quantum control protocols for the creation of NOON states using ultracold bosonic atoms on two modes, corresponding to the coherent superposition $|N,0\rangle + |0,N\rangle$, for a small number N of bosons. One possible method to create this state is to consider a third mode where all bosons are initially placed, which is symmetrically coupled to the two other modes. Tuning the energy of this third mode across the energy level of the other two modes allows the adiabatic creation of the NOON state. We demonstrate that the use of a counterdiabatic protocol to accelerate the processus is feasible and effective for a single particle, and then discuss how to extend its application to a larger number of bosons.

Bose-Hubbard model

- We consider an ensemble of N interacting bosons trapped in a 3-site optical lattice :

$$H(t) = \frac{U}{2}\sum_{i=1}^{3}\hat{n}_i(\hat{n}_i - 1) - J\sum_{i=1}^{3}\left(\hat{a}^\dagger_{i+1}\hat{a}_i + \hat{a}^\dagger_i\hat{a}_{i+1}\right) + \varepsilon(t)\,\hat{a}^\dagger_2\hat{a}_2$$

with:

- on-site interaction U
- hopping between neighboring sites J
- time-dependant driving $\varepsilon(t)$ on the central well

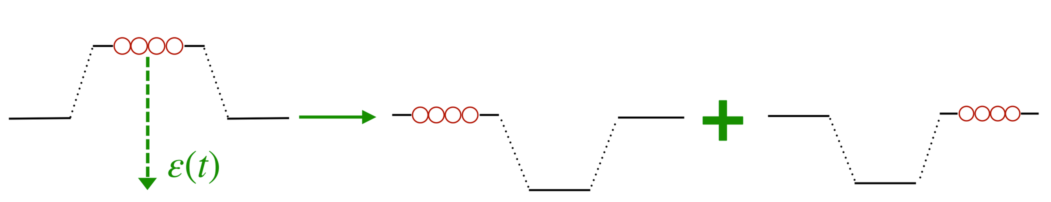
- Reduced 3-level system

$$H_{\text{red}}(t) = \begin{pmatrix} \mathcal{E} & -\mathcal{J} & 0 \\ -\mathcal{J} & \tilde{\mathcal{E}} + \mathcal{N}\varepsilon(t) & -\mathcal{J} \\ 0 & -\mathcal{J} & \mathcal{E} \end{pmatrix}$$

where $\mathcal{E}, \tilde{\mathcal{E}}, \mathcal{N}$ and $\mathcal{J}$ are fully determined by perturbation theory for $NU/J \gg 1$.

Adiabatic driving

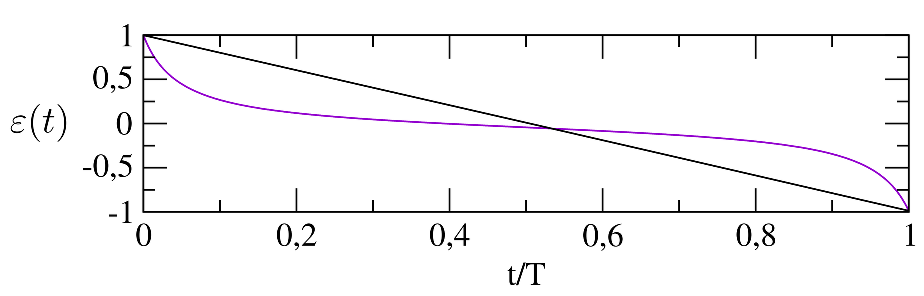
- Adiabatic theorem : a system that is slowly driven will remain in its eigenstates
- Consider a slowly driven third symmetrically coupled well → NOON state creation if driving $\delta(t)$ slow enough, $|NOON\rangle = (|N,0,0\rangle + |0,0,N\rangle)/\sqrt{2}$



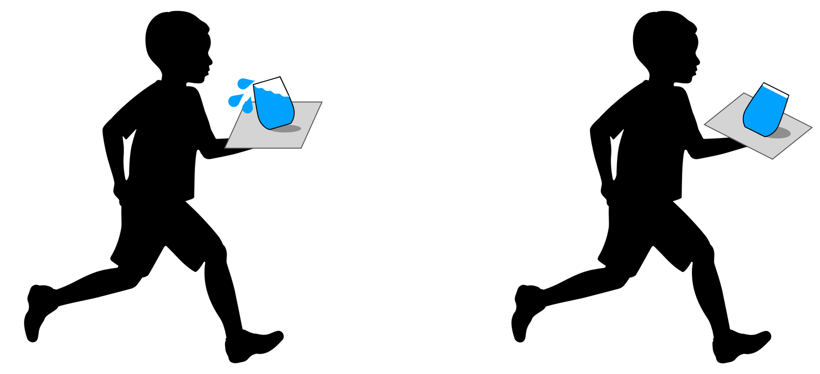
- Geodesic driving : shortest path between two points in time-dependant parameter manifold [1]

$$\mathcal{N}\varepsilon(t) = 2\sqrt{2}\mathcal{J}\tan\left(\pi/2 - \pi t/T\right) - (\tilde{\mathcal{E}} - \mathcal{E})$$

- For $N = 2$, **linear** and geodesic drivings



Counterdiabatic driving



- We add a term that cancel non adiabatic transitions: $H(t) \to H(t) + H_{\text{CD}}(t)$ [2]

$$H_{\text{CD}} = \dot{\varepsilon}\mathcal{A}_\varepsilon = \sum_n \sum_{m\neq n} \frac{\langle n|\dot{H}|m\rangle}{E_m - E_n}|m\rangle\langle n|$$

- Exact result for H_{red} at 1^{st} order [3] :

$$H_{\text{CD}}(t) = i\Omega(t)\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

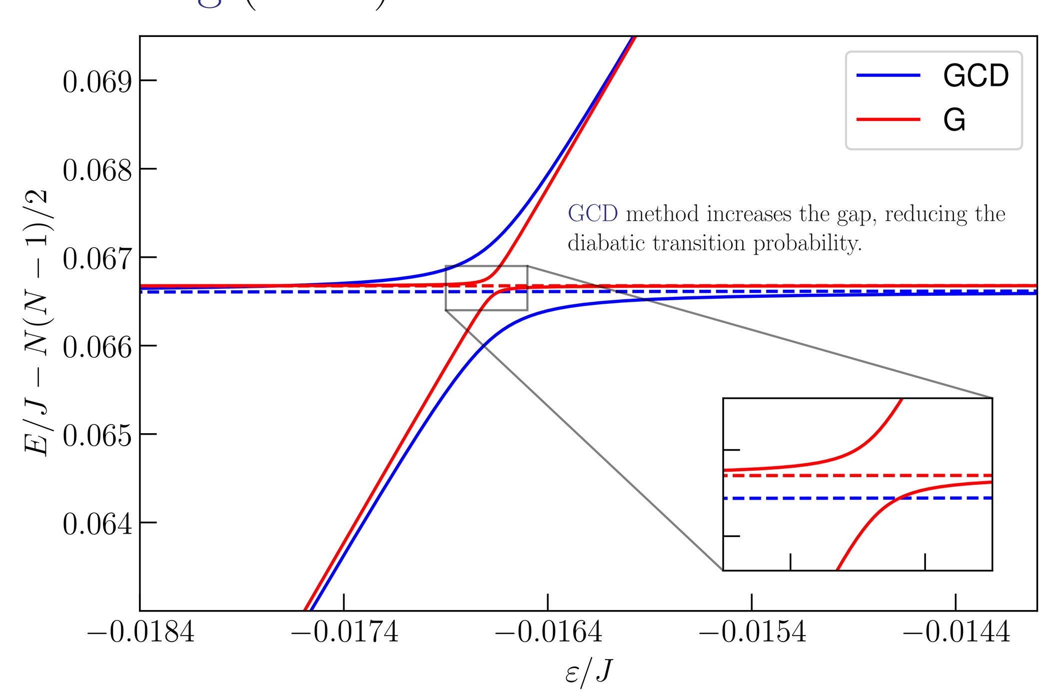
where $\Omega(t) = \frac{\mathcal{N}\mathcal{J}\dot{\varepsilon}(t)}{8\mathcal{J}^2 + \left((\tilde{\mathcal{E}} - \mathcal{E}) + \mathcal{N}\varepsilon(t)\right)^2}$

- Geodesic driving makes Ω constant :
  - $\dot{\varepsilon}(t) = \frac{2\sqrt{2}\mathcal{J}(\alpha_f - \alpha_i)}{NT\,\cos(\alpha_i + (\alpha_f - \alpha_i)\frac{t}{T})^2} \to \Omega = \frac{\pi\sqrt{2}}{4T}$
- $H_{\text{red}}(U, J, \varepsilon(t)) + H_{\text{CD}} = H_{\text{red}}\left(U_{\text{eff}}, J_{\text{eff}}, \varepsilon_{\text{eff}}(t)\right)$
- Effective parameters that incorporate the action of H_{CD}:
  - $U_{\text{eff}} = U + 2\delta\mathcal{E}/[N(N-1)]$
  - $J_{\text{eff}} = U_{\text{eff}}^{(N-1)/N}\left[\frac{J^N}{U^{N-1}} + i\hbar\frac{\Omega(N-1)!}{N}\right]^{1/N}$
  - $\varepsilon_{\text{eff}}(t) = \varepsilon(t) + (N-1)\left(U - U_{\text{eff}}\right)/2 + 2\delta\tilde{\mathcal{E}}/\mathcal{N}$

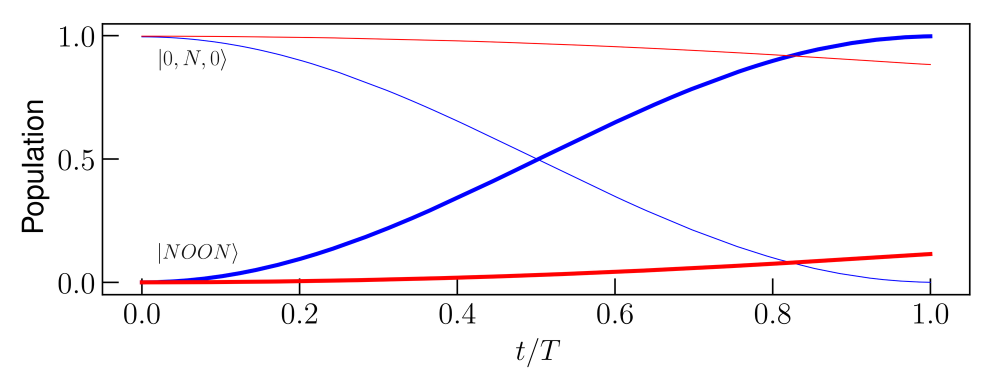
  with $\delta\mathcal{E} = \mathcal{E}(U,J) - \mathcal{E}(U_{\text{eff}}, J_{\text{eff}}) - N(N-1)(U + U_{\text{eff}})/2$

Four particles : spectrum

- Energy levels as a function of driving ε/J. Focus on the 3 relevant levels for Geodesic driving (G) and Geodesic + Counterdiabatic driving (GCD)



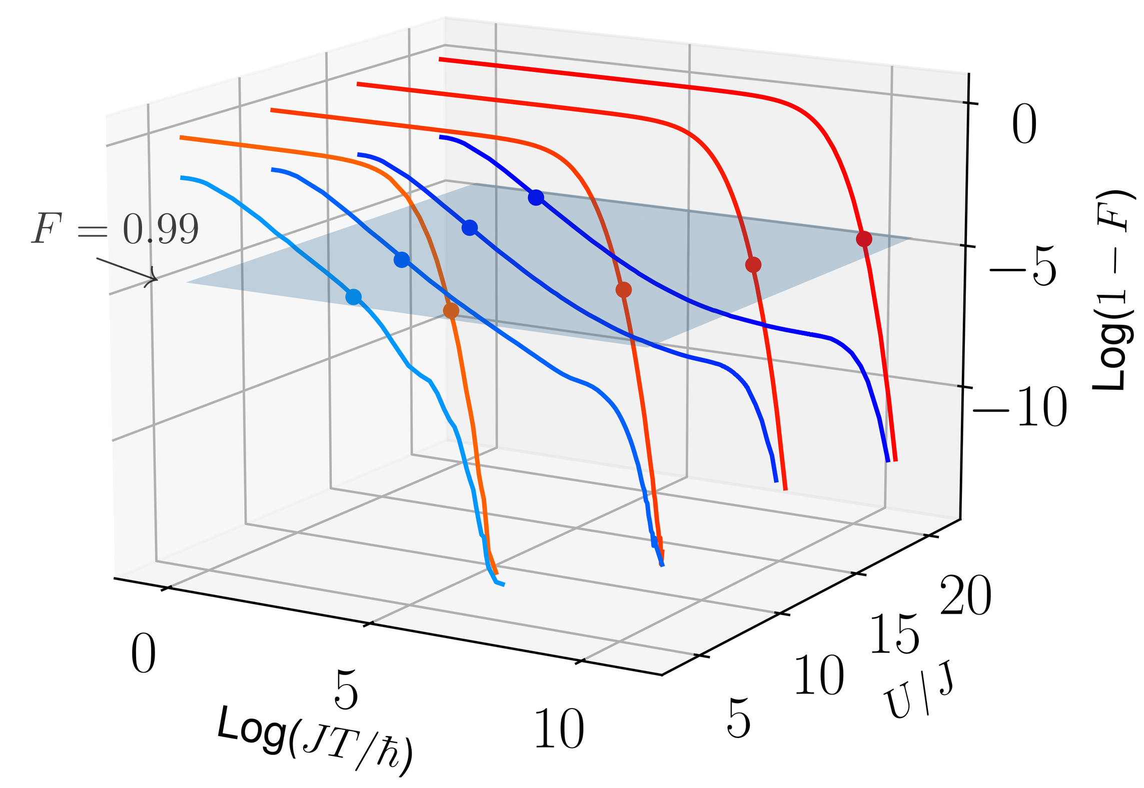
- Population as a function of time over total protocol time t/T for G and GCD methods in basis $\mathcal{B} = \{|0,N,0\rangle, |NOON\rangle\}$



- For $T = 3\times10^3\hbar/J$, purity of NOON state obtained ratio : $F_{GCD}/F_G = 0.98/0.1$

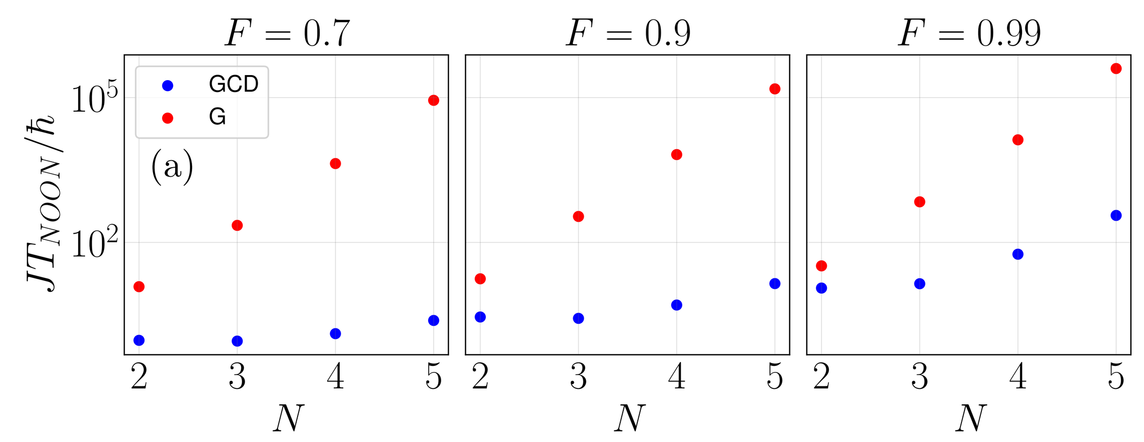
Four particles : dynamics

- Infidelities $1 - F = 1 - |\langle\psi(T)|m(T)\rangle|^2$ as a function of total protocol time T, for different values of U/J

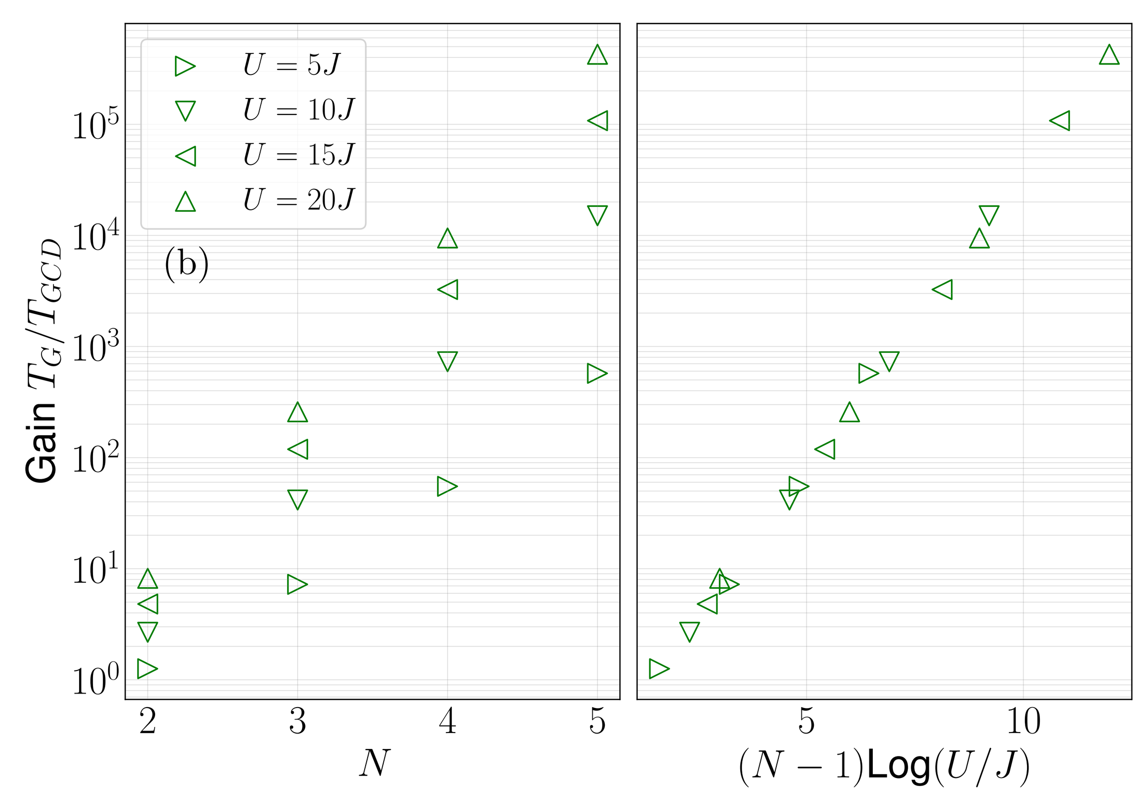


Results

- NOON time as a function of N for different fidelity thresholds at fixed $NU/J = 60$



- Gain as a function of N and U/J



- **Exponential** law w.r.t. particles number N
- **Power** law w.r.t. interaction strength U/J

→ Gain scales as $Ga(N, U/J) \sim (U/J)^{N-1}$

References

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