PARTIALLY POLARIZED PHYSICAL OPTICS

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Operational Issues

Computation and Parametrization of Scattering Characteristics

Meaning of Mueller Elements

No explicit relationship between Mueller matrix elements and the physical object.

Analytical Characterization

No analytical characterization of the set of all Mueller matrices exists to this day.

Formulation

Different formalisms and notations for different scattering events.

Conservation Properties

No notion for energy conservation and conservation of the input polarization state.

Operational Issues

Parameter & Incompatibility

How can we achieve a unified parametrization if we don't know what physical properties the individual Mueller matrix elements represent?

Computationally Expensive

The tests for the physical realizability of the results (per iteration) and for checking the energy and polarization conservation are computationally very expensive.

Tests

What are the tests we can put in place to verify a Mueller matrix? In other words, what is the difference between a common 4x4 matrix and a Mueller matrix?

Experimental Errors

Experimental errors can violate many criteria that have to be fulfilled. Moreover, Mueller calculi do not conserve the Degree Of Polarization (DOP) of an arbitrarily polarized input state. This over complicates many modeling approaches.

All these issues affect not only the theoretical work. They also have an impact on operational work.

Methods

The Current and New Procedures

Coherency Matrix: (Born 99, p. 545)

Pauli Tensor

$$S^{\alpha\dot{\alpha}} = \frac{\sigma_M^{\alpha\dot{\alpha}}}{2} S^M = \frac{1}{2}\begin{bmatrix} S^0+S^3 & S^1+iS^2 \\ S^1-iS^2 & S^0-S^3 \end{bmatrix} \Rightarrow \begin{bmatrix} E_1E_1^* & E_1E_2^* \\ E_2E_1^* & E_2E_2^* \end{bmatrix}$$

(iff S^M fully polarized)

With $\alpha, \dot{\alpha} = [0, 1]$.

The Determinant of $S^{\alpha\dot{\alpha}}$ is relatet to the DOP like:

$$\det(S^{\alpha\dot{\alpha}}) = \frac{1}{4}\left[(S^0)^2 - (S^1)^2 - (S^2)^2 - (S^3)^2\right]$$

(Different way to see the DOP)

$\det(S^{\alpha\dot{\alpha}}) = 0 \Rightarrow$ Fully Polarized

$\det(S^{\alpha\dot{\alpha}}) = \frac{1}{4}(S^0)^2 \Rightarrow$ Unpolarized

$0 < \det(S^{\alpha\dot{\alpha}}) < \frac{1}{4}(S^0)^2 \Rightarrow$ Partially Polarized

Spinor Representation: (Causmaecker 81, Kleiss 85, ...)

Iff det $(S^{\alpha\dot{\alpha}}) = 0$ then

$$S^{\alpha\dot{\alpha}} = \pi^{\alpha} \otimes \tilde{\pi}^{\dot{\alpha}}$$ where π^{α} and $\tilde{\pi}^{\dot{\alpha}}$ are Spinors!

Relation between $S^{\alpha\dot{\alpha}}$ & $\vec{E}_n$ iff det$(S^{\alpha\dot{\alpha}}) = 0$:

$$S^{\alpha\dot{\alpha}} \longrightarrow \begin{bmatrix} E_1E_1^* & E_1E_2^* \\ E_2E_1^* & E_2E_2^* \end{bmatrix} \longrightarrow \pi^{\alpha}\tilde{\pi}^{\dot{\alpha}} = \underbrace{\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} [E_1^* \;\; E_2^*]}_{\tilde{\pi} = (\pi^{\alpha})^*}$$

=> Every $S^{\alpha\dot{\alpha}}$ defined with π & $\tilde{\pi}$ is manifestly fully Polarized!

↳ No quadratic constraints like:

$$(S^0)^2 = (S^1)^2 + (S^2)^2 + (S^3)^2 \, !$$

Redundancy:

If $\pi \to \boxed{z}\,\pi$ then $\tilde{\pi} \to z^{-1}\tilde{\pi}$

$\Rightarrow S^{\alpha\dot{\alpha}} \to z\pi^{\alpha}\tilde{\pi}^{\dot{\alpha}}z^{-1} = S^{\alpha\dot{\alpha}}$!

$z = e^{i\varphi}$ (Phase)

(The action of the Hopf Fibration)

(Hopf 31, p. 654)

(Zee 16)

Partially Polarized Waves

$$\det\left(S^{\alpha\dot{\alpha}}\right) = \left(0, (S^0)^2\right]$$

↳ Rank 2 !

$$S^{\alpha\dot{\alpha}} = \boxed{\Pi_1^{\alpha}\tilde{\Pi}_1^{\dot{\alpha}} + \Pi_2^{\alpha}\hat{\Pi}_2^{\dot{\alpha}}}$$

$$Z_I = \begin{bmatrix} \Pi_1^{\alpha} \\ \hdashline \Pi_2^{\dot{\alpha}} \end{bmatrix} \qquad Z \in \mathbb{C}^4$$

$$Z^I = B^{IJ} Z'_J$$

$$B \in U(4)$$

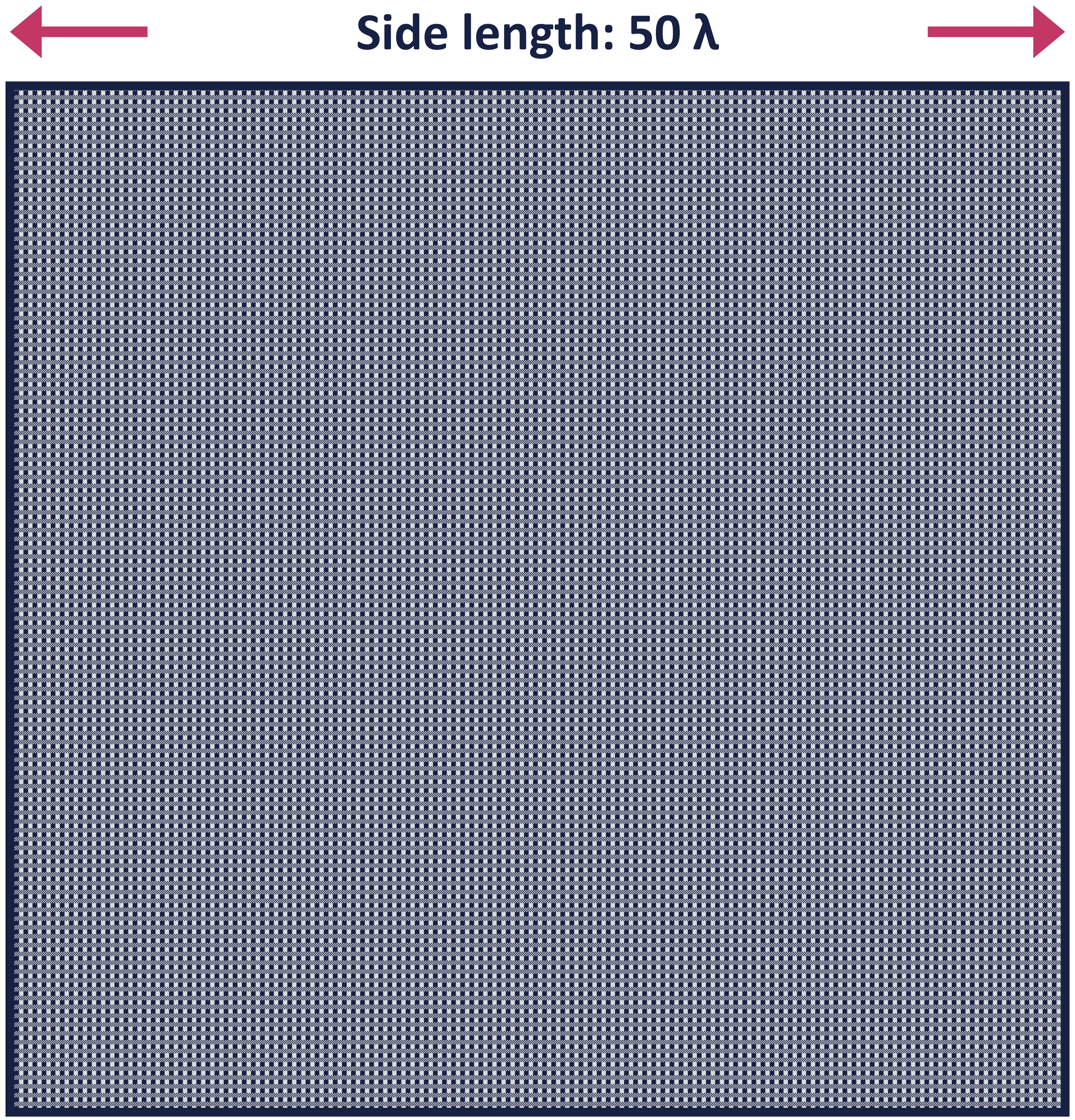
$$\det(\Pi_1 \tilde{\Pi}_2) = 0 \qquad \det(\Pi_2 \hat{\Pi}_2) = 0$$

The sum of two Coherency Matrices with Determinant 0 can create a Partially Polarized Coherency Matrix

$$\xrightarrow{Z!} W^I{}_J = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} \end{pmatrix}$$

Simulation Setup

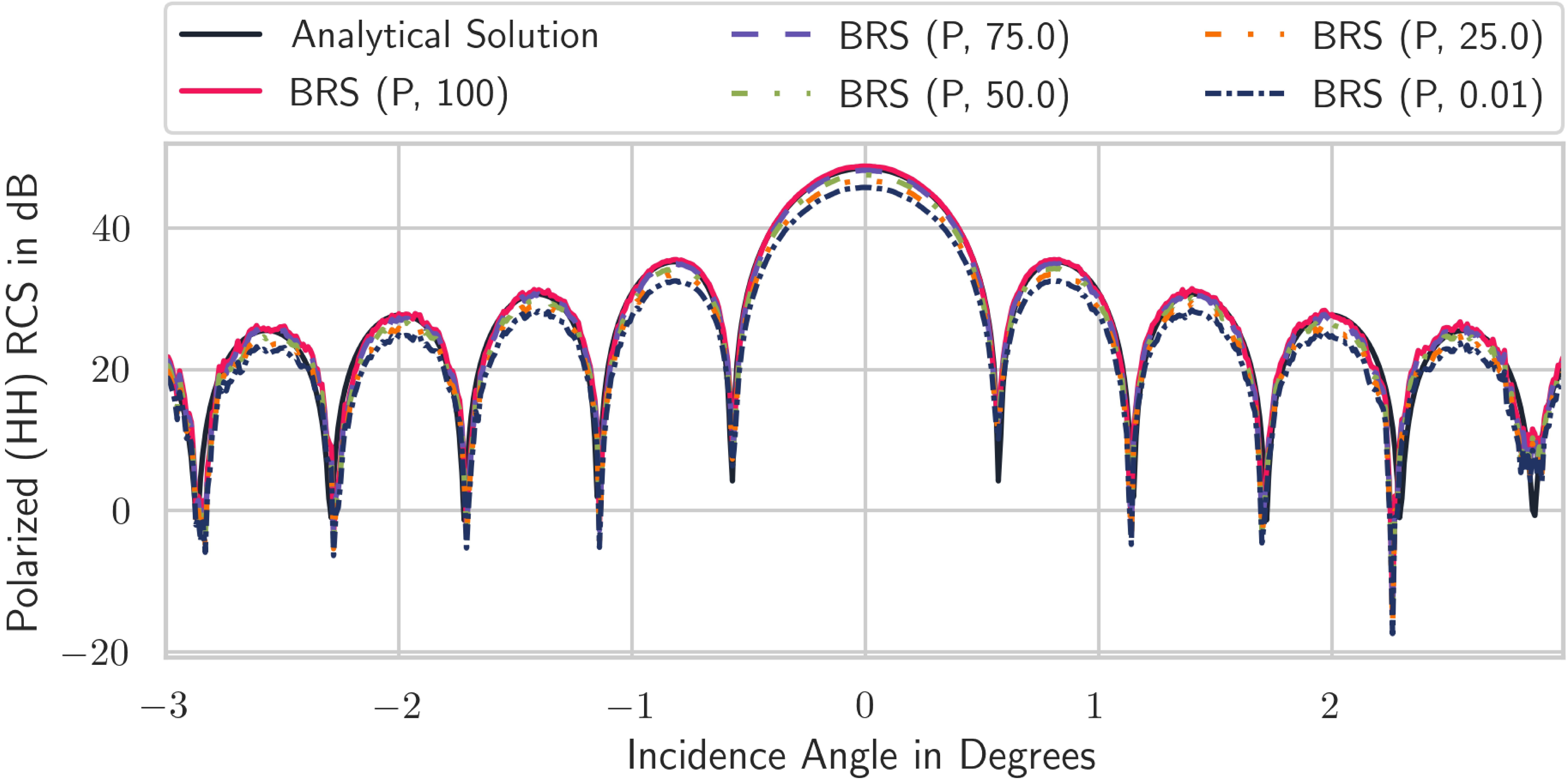
- Frequency: 10 GHz.
- Computation: Monostatic RCS.
- Plate dimension: 50 λ (1.5 Meter).
- Sampling rate: λ / 10.
- Roughness: [0, λ / 4].
- The correlation length will be 15 times the RMS height.
- We consider only the coherent part of the wave.
- Even if it is not realistic, we model the surface with the assumption that it consists of very complex materials capable of changing the polarization degree of an incident wave.



Sampling Rate: λ / 10

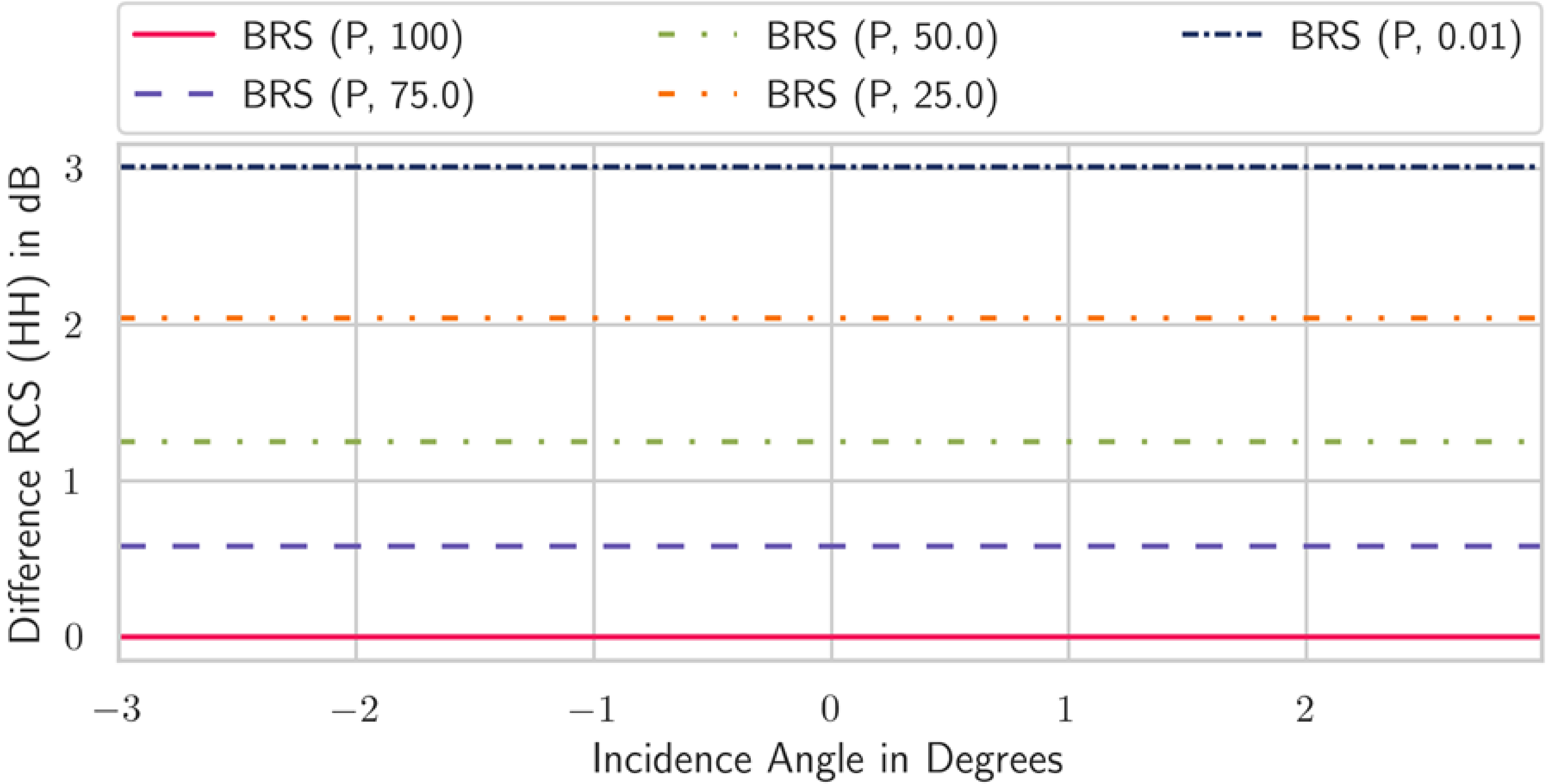
First Results (No Roughness)

Polarized part of the wave



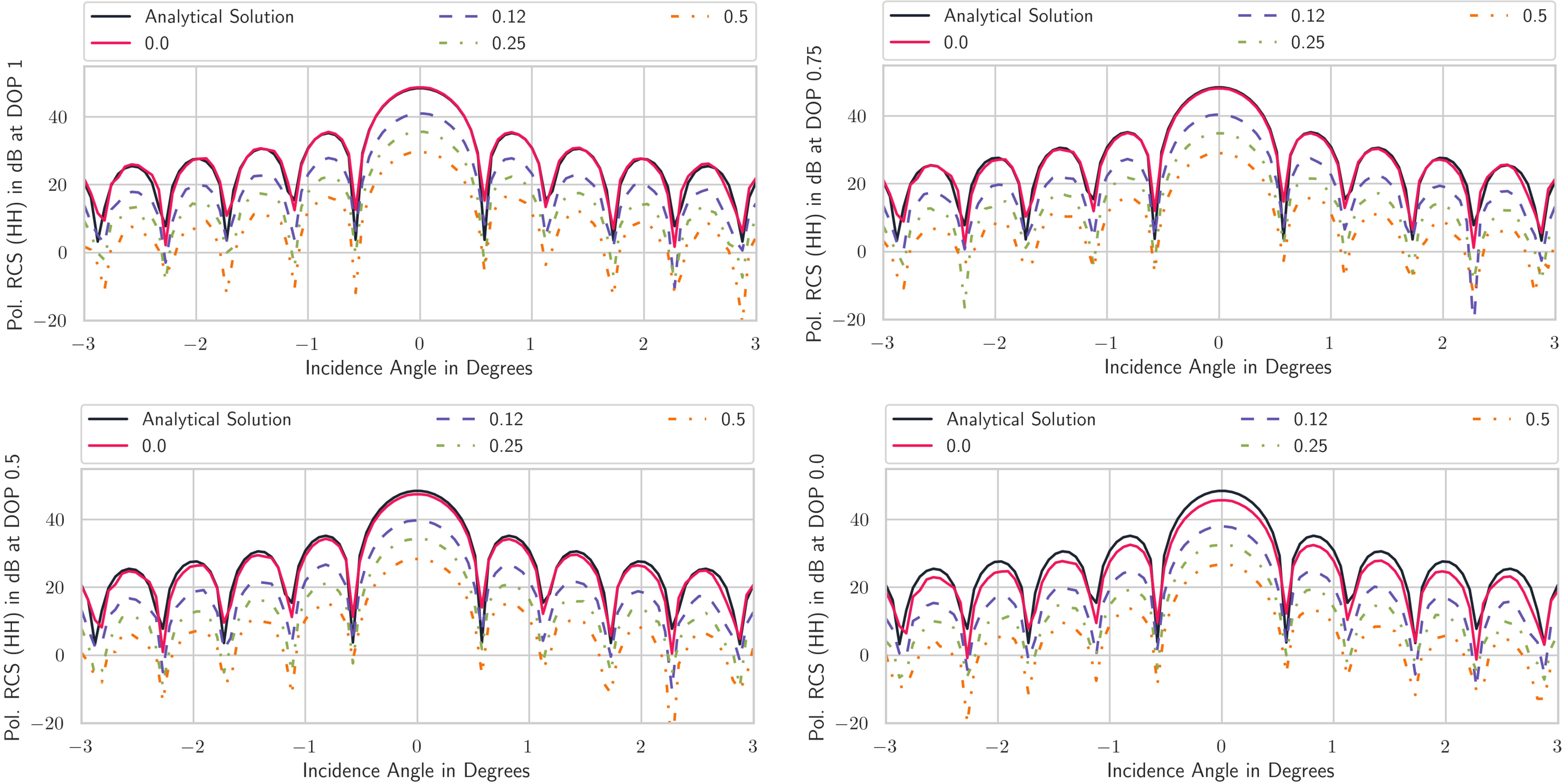
First Results (With Roughness)

Difference between analytical solution and the polarized part of the wave



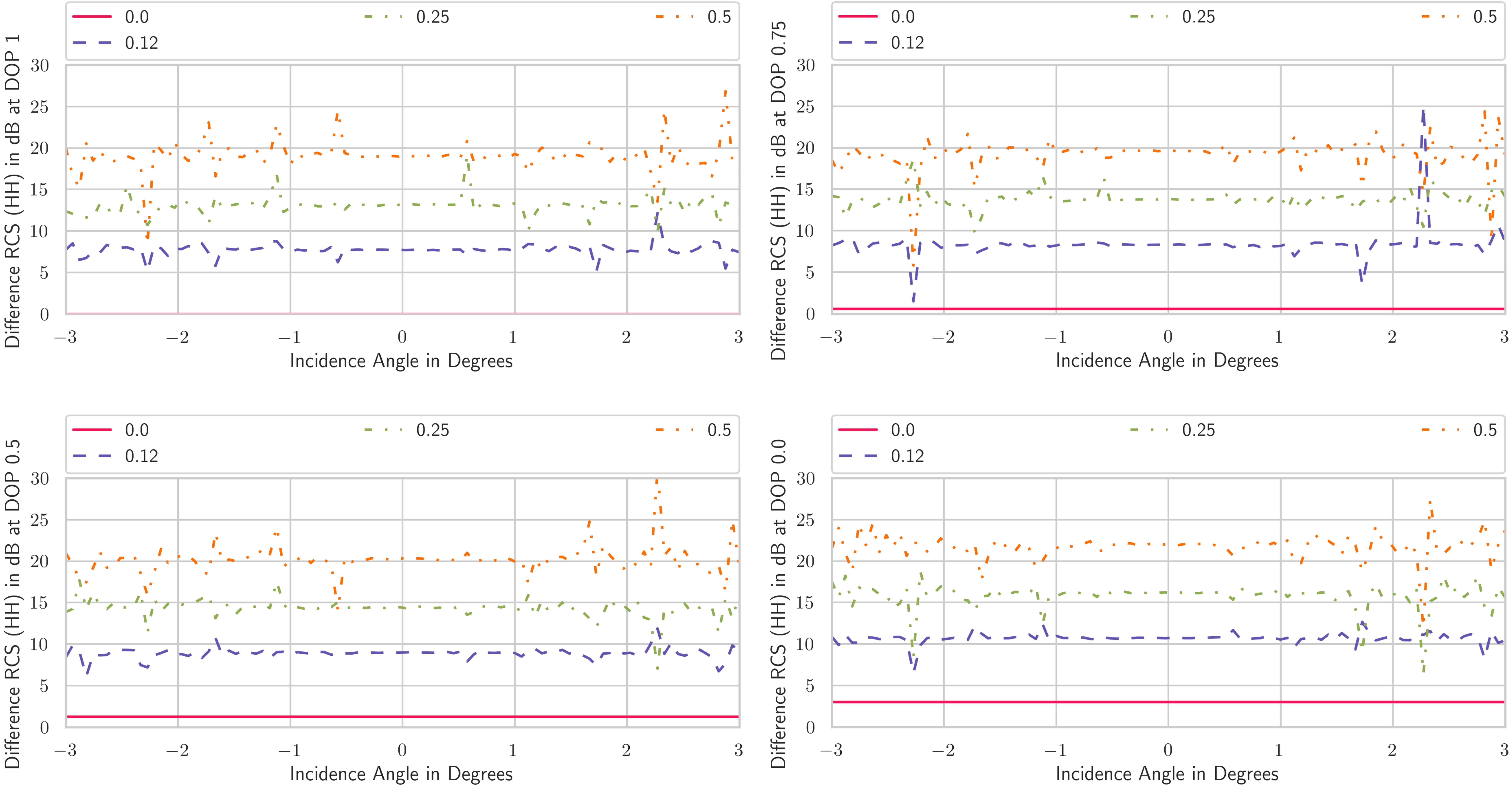
First Results (With Roughness)

Polarized part of the wave at different λ/RMS Height



First Results (With Roughness)

Difference between analytical solution and the polarized part of the wave at different λ/RMS Height



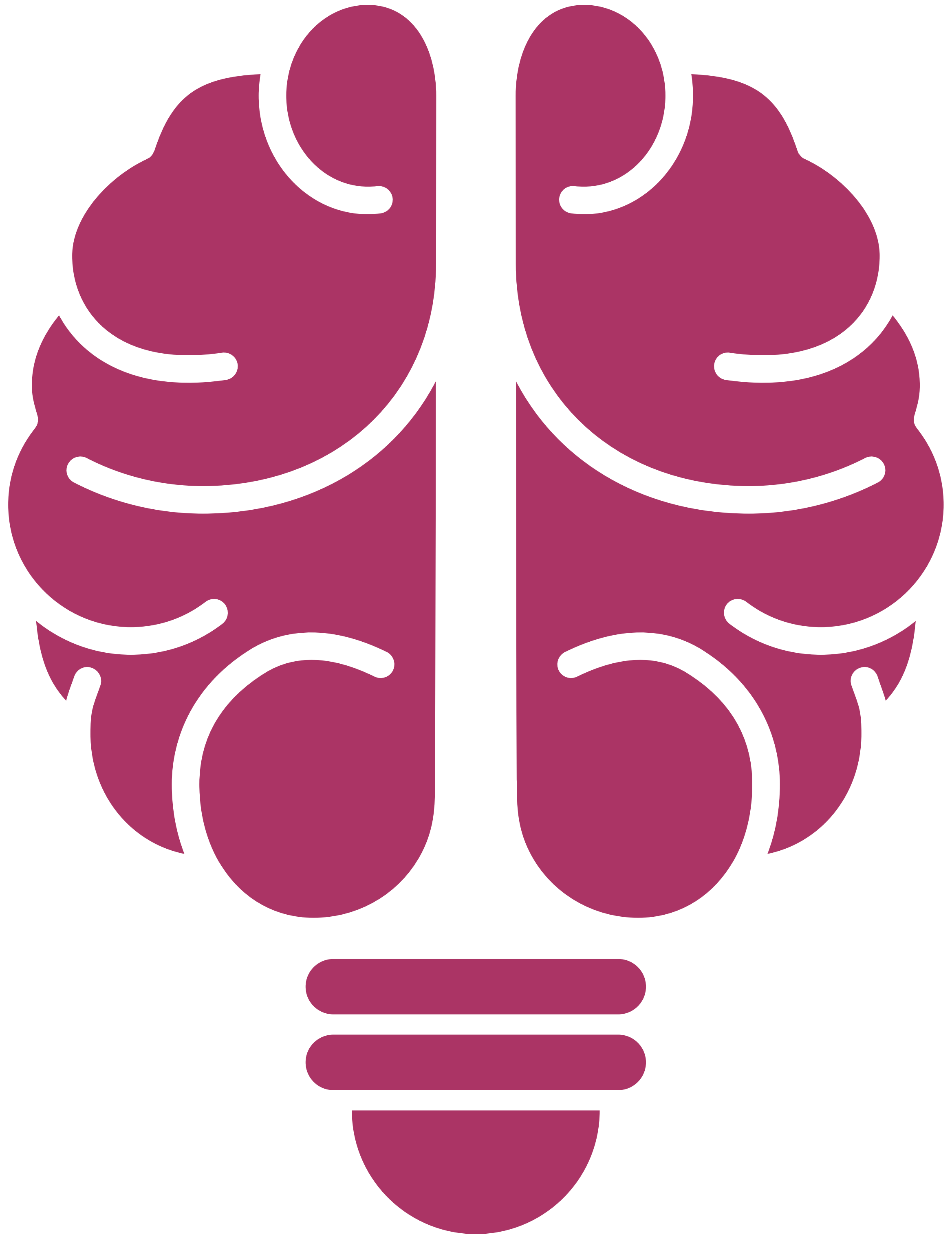
Summary and Outlook

Incoherent Fields

Modelling of incoherent fields within the simulations.

SAR-Simulation

Modelling of entire SAR images and signals.



Integrate into other Theories

Integrate the model in other Theories like SSA.

Real-Time Simulations

Due to its efficiency, it could be capable of performing real-time simulations.

Our work here today doesn't just represent another step in advancing scientific knowledge. Rather, it is a stride towards a future where we can more accurately perceive, interpret and respond to the world around us.

> Look up at the stars,
> not down at your feet.
>
> Stephen Hawking

Thank you for your Attention

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