Dissipative Phase Transition in an Interacting Many-Particle System: From Qubits to Qudits

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EOS THE EXCELLENCE OF SCIENCE

- The Lipkin-Meshkov-Glick model is a model of N identical spins with all-to-all interaction. By adding individual and/or collective decay, a dissipative phase transition occurs as a function of interaction and decay rates.
- Previous results for two-level spins have shown a second-order phase transition for individual decay [1,2] and a first-order phase transition for collective decay [2].
- For multilevel spins: very similar behaviour if dissipator is a spin ladder operator, but richer phenomenology if decay rates are identical among all adjacent levels.
- Entanglement negativity is maximal at phase transition and depends on the nature of the dissipator.

[1] T. E. Lee, S. Gopalakrishnan, and M. D. Lukin *Phys. Rev. Lett.* **110**, 257204 (2013)

[2] T. E. Lee, C.-K. Chan, and S. F. Yelin, *Phys. Rev. A* **90**, 052109 (2014).

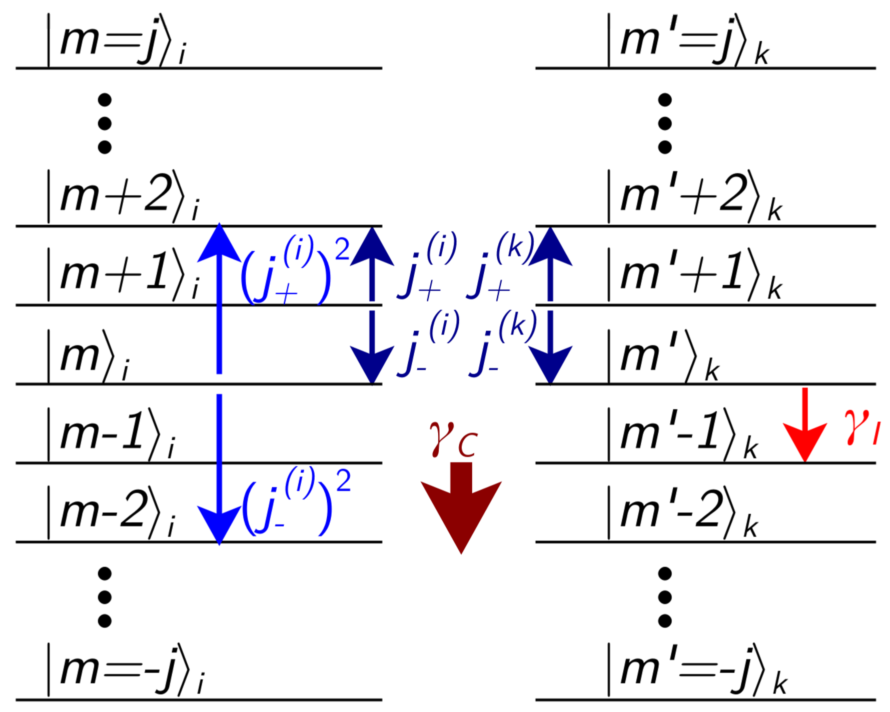
Dissipative Lipkin-Meshkov-Glick Model for Multilevel Systems

- N spins of length $\sqrt{j(j+1)}$, local spin operators $j_\alpha^{(i)}, \alpha=x,y,z,\pm, i=1,\dots,N$.
- Hamiltonian in terms of collective operators $J_\alpha=\sum_{i=1}^N j_\alpha^{(i)}$:

$$H=\frac{V}{Nj}\left(J_x^2-J_y^2\right)=\frac{V}{2Nj}\left(J_+^2+J_-^2\right)$$

- Dissipators:
  - Individual: $L_i=\sum_{m=-j}^{j-1}\ell_m|m\rangle_i\langle m+1|_i$,
  - Collective: $L_C=\sum_{i=1}^N L_i$.
- Lindblad master equation:

$$\frac{d\rho}{dt}=-i[H,\rho]+\frac{\gamma_C}{Nj}\left(L_C\rho L_C^\dagger-\frac{1}{2}\left\{L_C^\dagger L_C,\rho\right\}\right)+\frac{\gamma_I}{j}\sum_{i=1}^N\left(L_i\rho L_i^\dagger-\frac{1}{2}\left\{L_i^\dagger L_i,\rho\right\}\right).$$

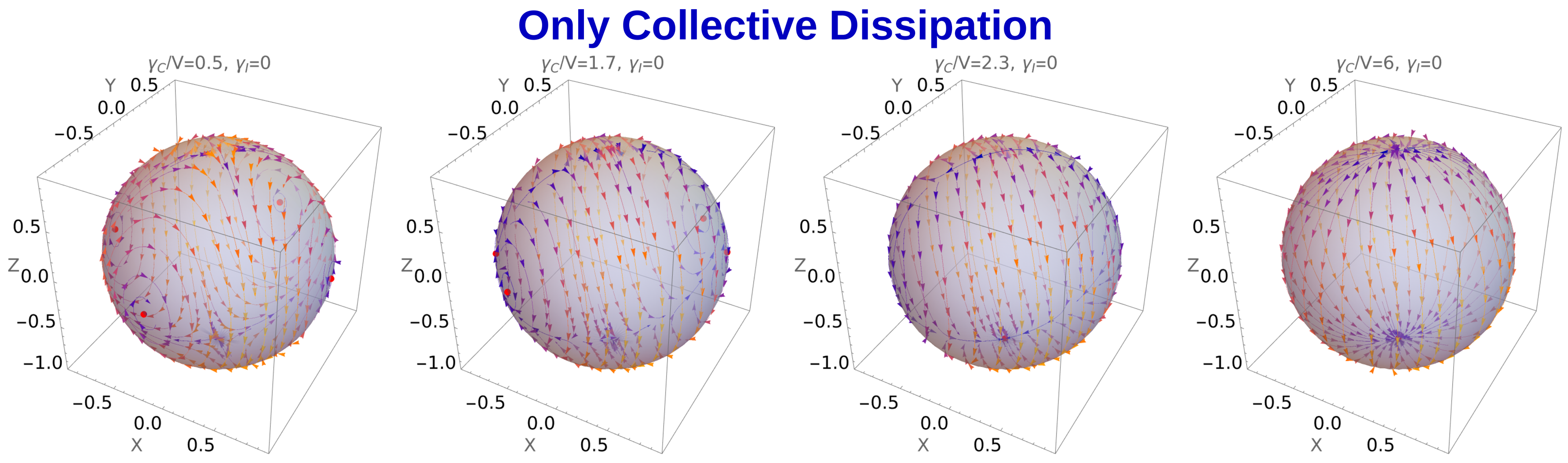


Known Properties for Two-Level Systems

Mean-field equations for $X=\langle J_x\rangle/Nj, Y=\langle J_y\rangle/Nj, Z=\langle J_z\rangle/Nj$

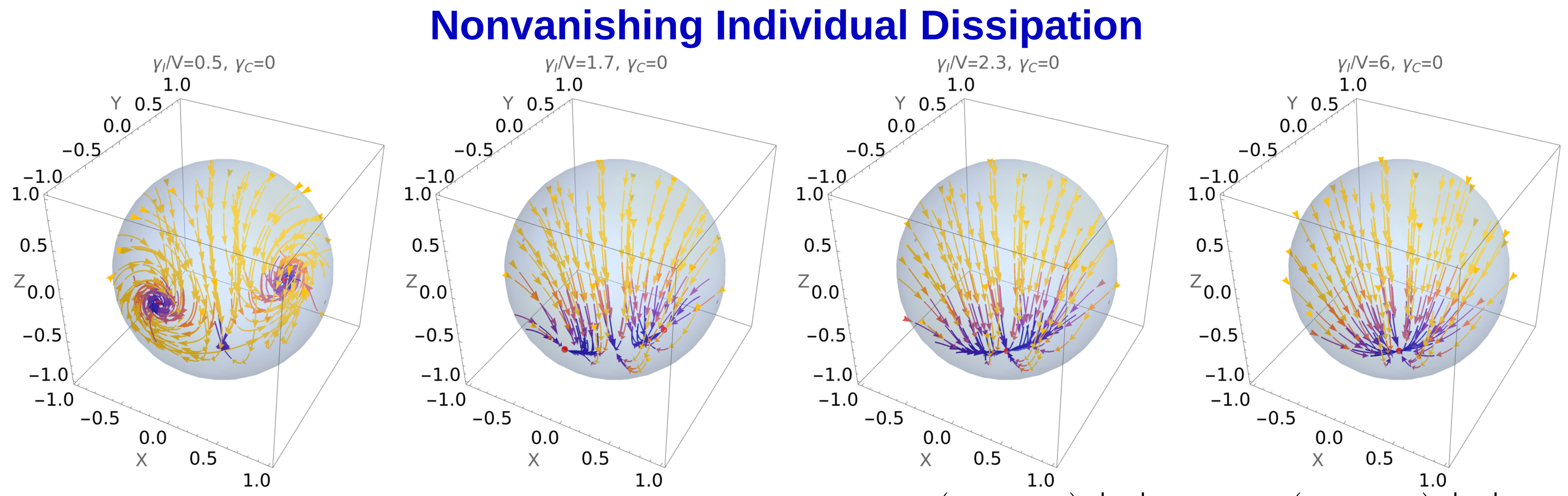
$$\frac{dX}{dt}=-2VYZ-\gamma_I X+\gamma_C XZ,\qquad \frac{dY}{dt}=-2VXZ-\gamma_I Y+\gamma_C YZ,\qquad \frac{dZ}{dt}=4VXY-2\gamma_I(Z+1)-\gamma_C(X^2+Y^2).$$

Only Collective Dissipation



- Oscillatory phase with $\langle X\rangle_t\neq 0, \langle Y\rangle_t\neq 0, \langle Z\rangle_t=0$ for $\gamma_C/|V|\leq 2$, spin-polarized phase $X=Y=0, Z=-1$ for $\gamma_C/|V|\geq 2$.
- First-order phase transition: Discontinuity in $\langle J_z\rangle$.

Nonvanishing Individual Dissipation



- Two phases of well-defined steady-state spin for $(\gamma_I+\gamma_C)/|V|\leq 2$ and $(\gamma_I+\gamma_C)/|V|\geq 2$:

$$X=\text{sgn}(V)Y=\pm\frac{\sqrt{\gamma_I(2|V|-\gamma_I-\gamma_C)}}{2|V|-\gamma_C},\ Z=-\frac{\gamma_I}{2|V|-\gamma_C}\quad\text{and}\quad X=Y=0, Z=-1.$$

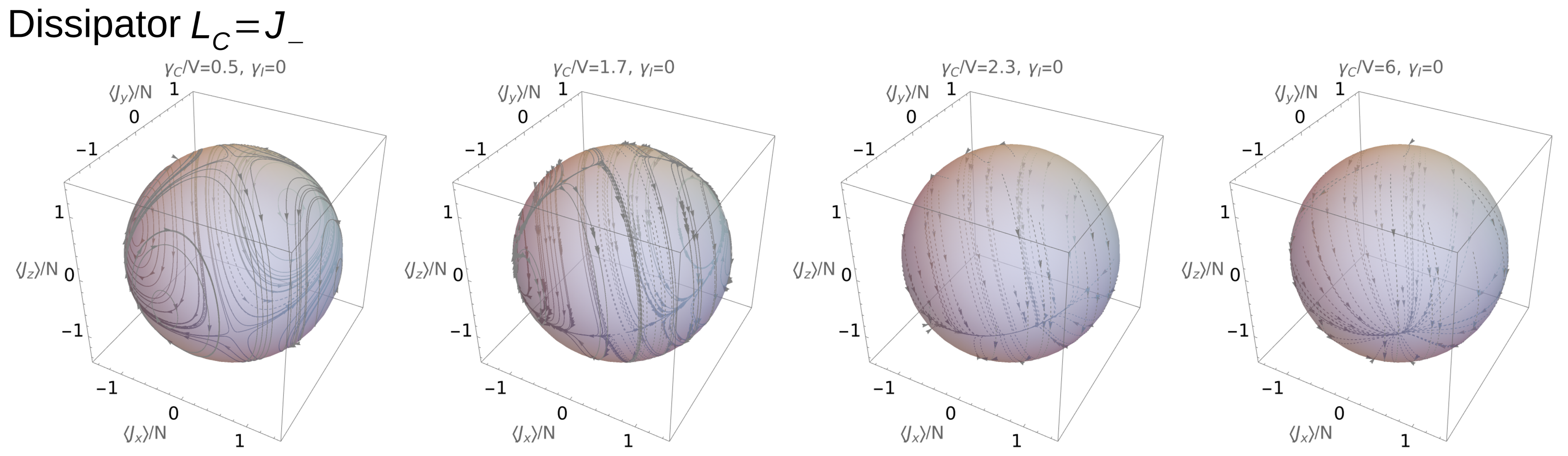
- Second-order phase transition: No discontinuity in $\langle J_{x,y,z}\rangle$, but in its derivative.

Compare T. E. Lee, C.-K. Chan, and S. F. Yelin, *Phys. Rev. A* **90**, 052109 (2014).

Mean-Field Dynamics for Multilevel Systems

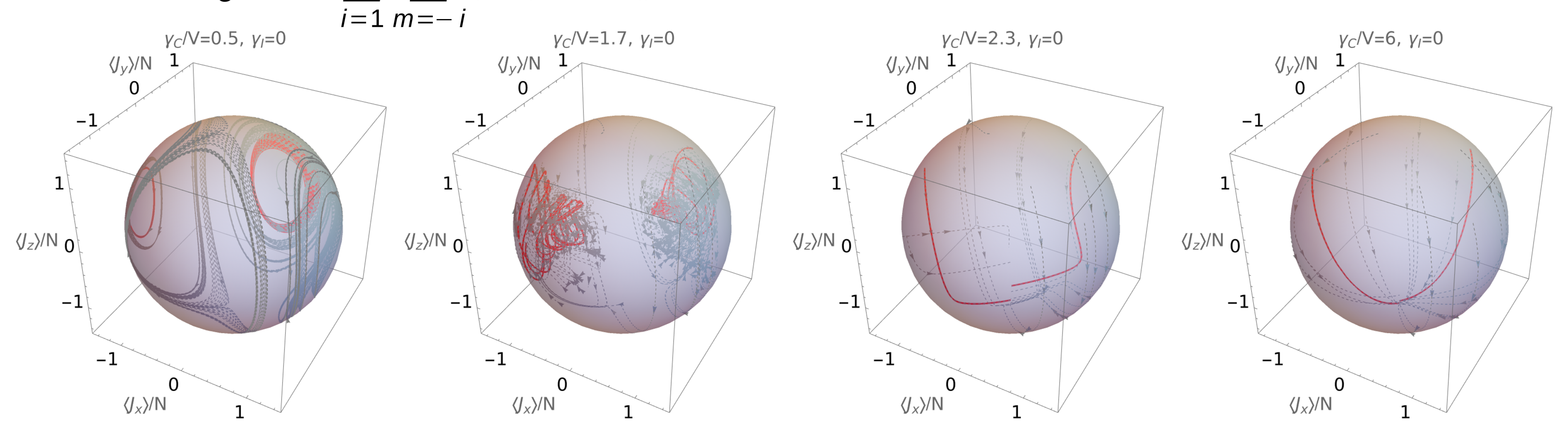
Dynamics for Only Collective Dissipation

Dissipator $L_C=J_-$



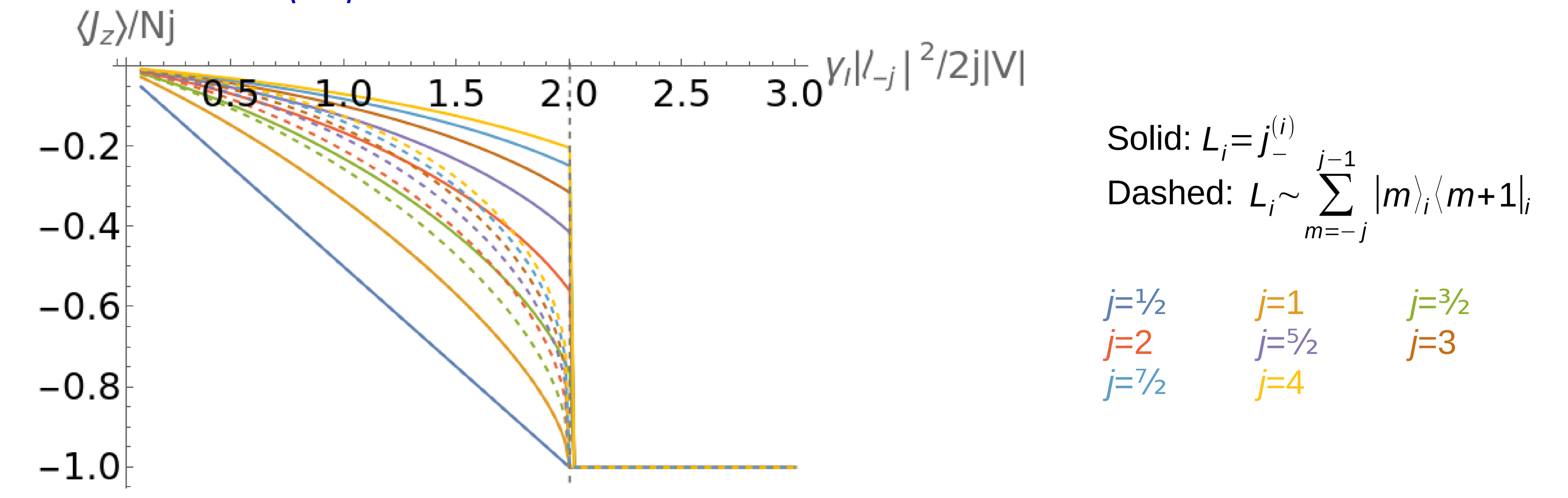
50 random initial states of maximal $\langle \mathbf{J}^2\rangle$, for $j=3/2$.

Dissipator $L_C=\sqrt{2j}\sum_{i=1}^N\sum_{m=-j}^{j-1}|m\rangle_i\langle m+1|_i$

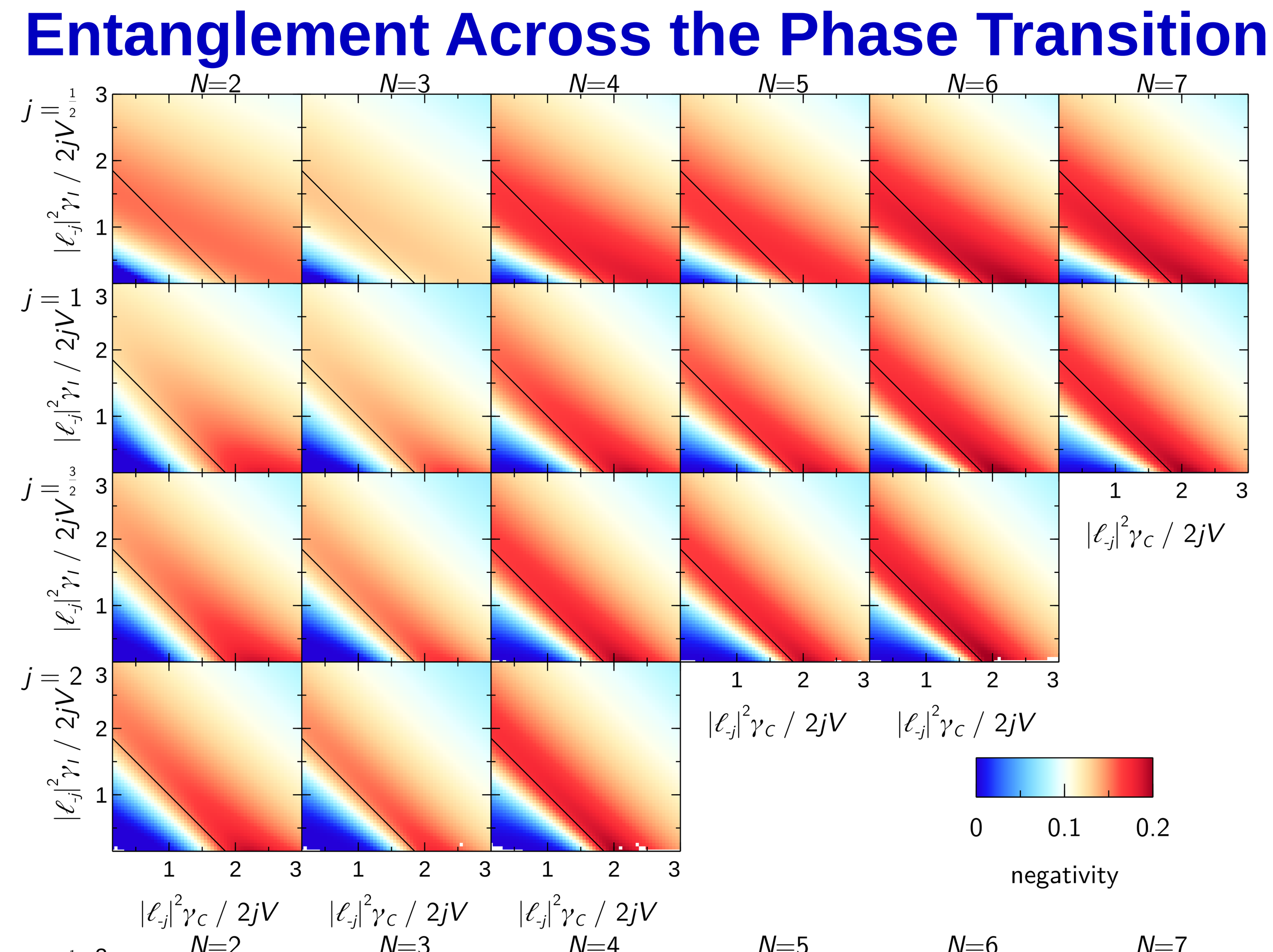


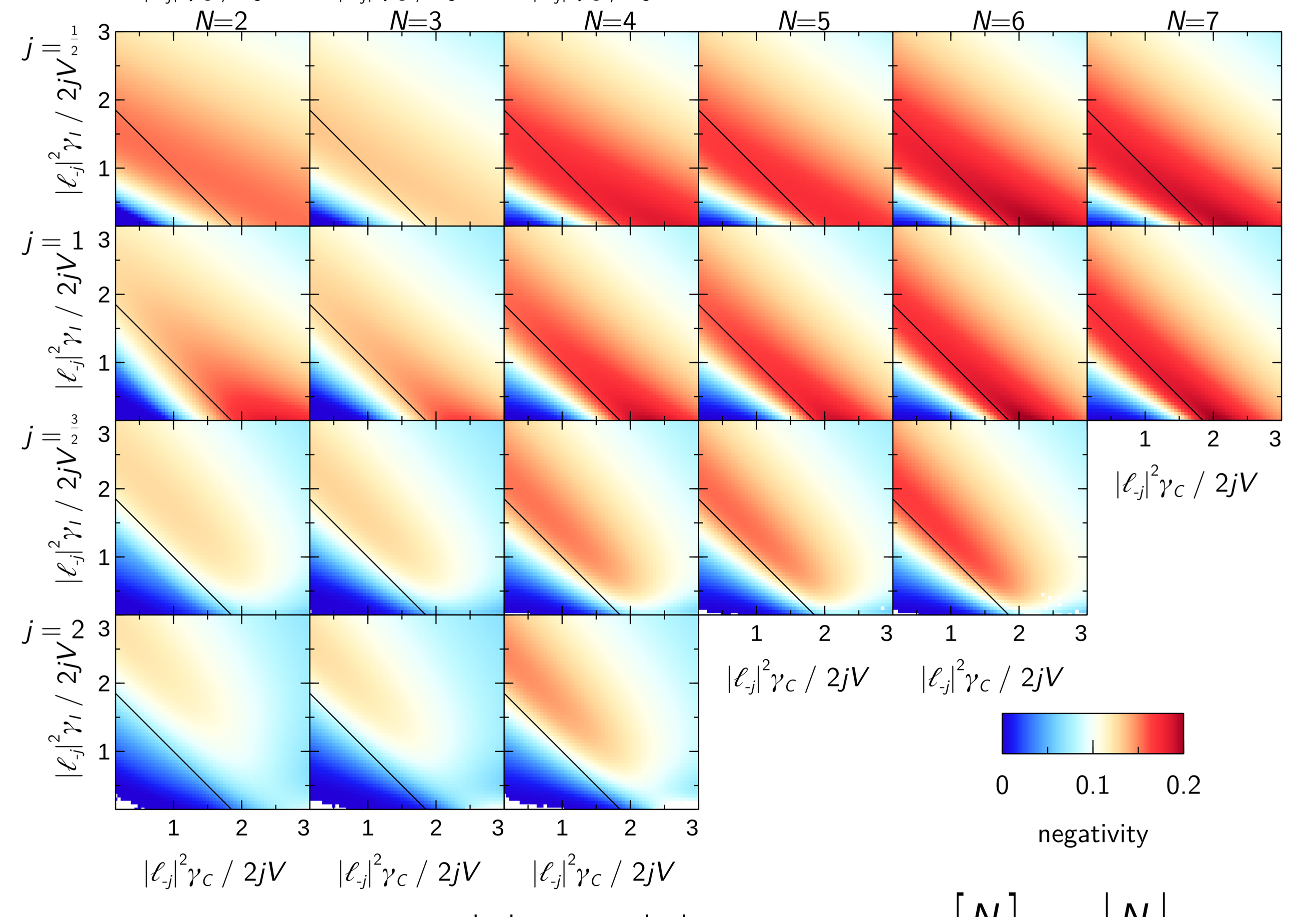
20 random initial states of maximal $\langle \mathbf{J}^2\rangle$, for $j=3/2$.

Steady-State $\langle J_z\rangle$ for Only Individual Dissipation



Entanglement Across the Phase Transition





Steady-state negativity versus $\gamma_C/|V|$ and $\gamma_I/|V|$, bipartition into $\left[\frac{N}{2}\right]$ and $\left[\frac{N}{2}\right]$ particles.

Top: $L_i=j_-^{(i)}$, bottom: $L_i\sim\sum_{m=-j}^{j-1}|m\rangle_i\langle m+1|_i$, line: mean-field phase transition $\frac{|\ell_{-j}|^2(\gamma_I+\gamma_C)}{2j|V|}=2$.