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Multi-actor optimization-based coordination of interacting power flow control devices or competing transaction schedulers in overlapping electricity markets

Dissertation présentée en vue
de l'obtention du grade de
Docteur en Sciences de l'Ingénieur

par

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Année académique 2009-2010

Acknowledgements

First of all, I would like to express my deepest gratitude to my supervisor, professor Thierry Van Cutsem for his constant support and material contribution in the realization of this work. Working with him has been a constructive and didactic experience for me, but also a pleasure, due to his quality as a scientist and as a human.

I also wish to dedicate very special thanks to professor Mevludin Glavic, professor Anastasios Bakirtzis and professor Bill Rosehart for sharing their knowledge with me in different parts of this work. I am thankful that this work gave me the opportunity to collaborate with them.

I should not omit thanking the members of the jury, who accepted to devote their time for assessing this report. Having each of them as referee for my PhD thesis is an honor for me.

A special word deserves to be said about professor Costas Vournas, thanks to whom this wonderful adventure began.

I could not forget my two colleagues and, most importantly, friends, Bogdan Otomega and Davide Fabozzi, with whom I shared a stimulative work environment and many unforgettable moments. Bogdan, you have been like an older brother for me at the beginning of my thesis. Davide, I wish you good luck and bon courage towards the accomplishment of your PhD thesis.

Apart from an academic experience, those years in Liège have also been a wonderful life experience. Admittedly, I owe this to all the friends, coming from around the world, that I made here and whom I hope from my heart to meet and share time with them again and again in the forthcoming years. I do not cite their names one by one, but I know that they can easily recognize themselves into these lines.

From the above-mentioned group of friends, I cannot resist thanking more explicitly Mohammed Boutaayamou, for hosting me in his home the last few (but more difficult and stressful) months of this work. Mohammed, I really appreciate your patience with me and wish you all the best.

A nice and fruitful change in my usual working environment has been the three-month interval I spent in Thessaloniki, hosted by professor Anastasios Bakirtzis. I wish to thank both him, as well as his then (and some still) PhD students, Athina Tellidou, Costas Baslis and Christos Simoglou for the wonderful environment they created for me.

Last but not least, I cannot but devoting a few lines to my parents and little brothers (who are less little than when I began this work, but who will always be the “little” brothers). I missed you so much! And, of course, my deepest thanks go to my beloved fiancee, Alexandra, for her patience and support until the end of this effort.

Abstract

This work deals with problems where multiple actors simultaneously take control decisions and implement the corresponding actions in large multi-area power systems. The fact that those actions take place in the same transmission grid introduces a coupling between the various decision-making problems. First, transmission constraints involving all actors' controls must be satisfied, while, second, the satisfaction of an actor's operational objective depends, in general, not only on its own actions but on the others' too.

Algorithms and/or operational procedures are, thus, developed seeking to reconcile the multiple actors' simultaneous decisions. The confidentiality and operational autonomy of the actors' decision-making procedures are preserved.

In particular, two specific problems leading to such a multi-actor situation have been treated.

The first is drawn from a recently emerging situation, at least in Europe, where several Transmission System Operators (TSOs) have installed and/or are planning to install Phase Shifting Transformers (PSTs) in such locations in their areas that, by properly adjusting the PST phase angle settings, they can significantly control the power flows entering and exiting their systems.

A general framework is proposed for the control of PSTs owned by several TSOs, taking into account their interactions. The proposed solution is the Nash equilibrium of a sequence of optimizations performed by the various TSOs, each of them taking into account the other TSOs' control settings as well as operating constraints relative to the whole system. The method is applied to a linearized network model and illustrated on the IEEE 118-bus system.

The second multi-actor situation dealt with in this work stems from the recently increasing amount of discussions and efforts made towards creating the right market structures and operational practices that would facilitate a seamless inter-area trade of electricity throughout large interconnections. In this respect, in accordance with European Union's goal of a fully functional Internal Electricity Market where ideally every consumer will be able to buy electric energy from every producer all across the interconnection, the possibility of every market participant to place its bid in whatever electricity market of an interconnection has been considered.

This results in overlapping markets, each with its own schedule of power injections and withdraws, comprising buses all around the interconnection, that are cleared simultaneously by Transaction Schedulers (TSs). An iterative procedure is proposed to reconcile the various TS schedules such that congestion is managed in a fair and efficient way. The procedure converges to such schedules that the various TS market clearings are in a Nash equilibrium. The method is then extended towards several directions: enabling market participants to place their bids simultaneously in more than one TS's market, incorporating $N-1$ security constraints, allowing for joint energy-reserve dispatch, and, accounting for transmission losses.

The corresponding iterative algorithms are thoroughly illustrated in detail on a 15-bus as well as the IEEE RTS-96 system.

Contents

List of symbols

To facilitate the reading, we have grouped hereafter the main abbreviations and symbols used throughout this report that appear in more than one (sub-)sections.

Abbreviations:

TSO	Transmission System Operator
PST	Phase Shifting Transformer
TS	Transaction Scheduler
PX	Power Exchange

Notation:

Bold lowercase letters denote vectors, while bold uppercase denote matrices. Symbols in non bold letters are scalars, irrespective of whether they are lowercase or uppercase. For a generic vector $\mathbf{x}$ the following notation is used:

$\mathbf{x}_i$	sub-vector of $\mathbf{x}$ (referring to the part of $\mathbf{x}$ that is under the ith actor's control)
x_j	jth element of vector $\mathbf{x}$
$(\mathbf{x})_j$	again, jth element of vector $\mathbf{x}$
$(\mathbf{x}_i)_j$	jth element of vector $\mathbf{x}_i$

Symbols:

$\mathbf{u}$	all actors' control variables vector
$\mathbf{u}_i$	ith actor's control variables vector
$\mathbf{u}_{i^-}$	all but the ith actors' control variables vector
$f_i(\mathbf{u})$	ith actor's objective function
$\mathbf{U}_i$	ith actor's feasible set
$\mathbf{g}_i(\mathbf{u}) \leq \mathbf{0}$	ith actor's feasible set expressed as inequality constraints
$\mathbf{U}$	all actors' feasible set
$\mathbf{g}(\mathbf{u}) \leq \mathbf{0}$	all actors' feasible set expressed as inequality constraints
$\mathbf{u}^\star$	Nash equilibrium of a game
$\mathbf{u}_i^\star$	ith actor's controls at the Nash equilibrium
$\mathbf{u}^o$	Pareto optimal solution of a multi-objective optimization problem
$\mathbf{p}$	branch flows vector
$\overline{\mathbf{p}}$	maximum branch capacities vector
$\boldsymbol{\varphi}$	PST angle settings vector
$\boldsymbol{\varphi}^k$	PST angle settings at the kth iteration
$\boldsymbol{\varphi}^{max}$	maximum PST angle settings vector
$\boldsymbol{\varphi}^{min}$	minimum PST angle settings vector
$\Delta\overline{\varphi}_i$	ith PST's maximum angle deviation between two consecutive iterations
$\mathbf{S}$	sensitivity matrix linking branch flow changes to PST setting changes
$\mathbf{p}^0$, $\boldsymbol{\varphi}^0$	operating point around which $\mathbf{S}$ has been computed

M	number of TSs
B	number of branches
N	number of buses

$\mathbf{g}$	generator productions vector
g_i	ith generator's energy production
$\mathbf{g}_m$	mth TS's schedule of generator productions
$\overline{\mathbf{g}}$	maximum generator productions vector
$\overline{\mathbf{g}}_m$	mth TS's maximum generator productions vector
$\widehat{\mathbf{g}}_m$	schedule of generator productions allocated to the mth TS at the last iteration of either the Energy or the Transmission allocation loop
$\widetilde{\mathbf{g}}_m$	schedule of generator productions demanded by the mth TS (communicated to the coordinator)
$\mathbf{r}$	reserve provisions vector
$\mathbf{r}_m$	mth TS's schedule of reserve provisions
$\overline{\mathbf{r}}$	maximum reserve provisions vector
$\overline{\mathbf{r}}_m$	mth TS's maximum reserve provisions vector
$\mathbf{d}$	load consumptions vector
$\mathbf{d}_m$	consumptions of loads served by the mth TS's schedule
$\overline{\mathbf{d}}$	maximum load consumptions vector
$\overline{\mathbf{d}}_m$	mth TS's maximum load consumptions vector
$\mathbf{n}$	bus power injections vector
$\mathbf{n}_m$	bus power injections assigned to the mth TS's schedule
$\widehat{\mathbf{n}}_m$	bus power injections assigned to the mth TS at the last transm. allocation iteration
$\mathbf{\Gamma}$	matrix with 0 and 1 elements, that links generator productions to bus injections
$\mathbf{\Delta}$	matrix with 0 and 1 elements, that links load consumptions to bus injections
$\mathbf{T}$	PTDF matrix linking branch power flows to bus injections
$\mathbf{t}_b$	bth row of the $\mathbf{T}$ matrix
$(\Delta\mathbf{p}_m^-)_b$	mth TS's contribution to the bth branch flow overload alleviation (flow decrease required)
$(\Delta\mathbf{p}_m^+)_b$	mth TS's contribution to the bth branch flow overload alleviation (flow increase required)
$\mathbf{c}_m$	vector of generator energy bids to the mth TS's market
$\boldsymbol{\varrho}_m$	vector of generator reserve bids to the mth TS's market
$\boldsymbol{\pi}_m$	vector of mth TS's offered prices for generation energy allocation
$\boldsymbol{\pi}'_m$	vector of mth TS's offered prices for generation reserves
$\boldsymbol{\pi}_m{}''$	vector of mth TS's offered prices for generation reserve allocation (prices modified so as to be comparable with the prices offered for energy)
R_s	reserve requirement in the sth area
$\mathbf{L}$	LODF matrix linking post- to pre-contingency branch flows
p_b^v	bth branch flow resulting from the outage of the vth branch
$(\Delta\widetilde{\mathbf{p}}_m^-)_b$	mth TS's contribution in the bth branch flow post-contingency overload alleviation (flow decrease required)
$(\Delta\widetilde{\mathbf{p}}_m^+)_b$	mth TS's contribution in the bth branch flow post-contingency overload alleviation (flow increase required)

Chapter 1

Introduction

1.1 Background

Historically, electric power systems have been planned and operated on an area basis. Each such area[1], was responsible for planning the installation of generation and transmission capacities so as to serve the demand efficiently. An entity, typically called the area's (country's) power system (or electric energy) company, was responsible for operating the bulk power system (i.e. generation and transmission) of the area. Transmission tie-lines have been built connecting those areas which, like that, have formed large synchronous interconnections (like the western continental European one). The main purpose for them being interconnected, however, has been the increased level of security that was provided to all involved parties.

During the 90's, this classical picture of a vertically integrated bulk power system started to change. Each area has been gradually transformed into an electric energy market, where electricity is now traded freely as any other commodity. Generators are now separate entities which compete with each other trying to sell their product to the area's electricity consumers. In this new liberalized, or deregulated as it has been also called, environment an entity called Transmission System Operator (TSO) was created for each area (in USA the terms RTO and ISO are mainly used, standing respectively for regional transmission organization and for independent system operator). The role of the TSO is purely to operate the transmission grid in a way that allows equal, non-discriminatory access to all market participants, assuring at the same time an adequate level of security. The TSO acts as a facilitator of the market, or even, in some countries, as a Market Operator (MO)[2].

Naturally, in the liberalized market-oriented and -driven framework, a demand for inter-area, or cross-border, trade of electricity has emerged. Transactions involving participants located in different areas in an interconnection are presently common practice in both the European and the North-American interconnections. Especially in Europe, facilitating such trading through

[1]In Europe, each area corresponds typically to a country.
[2]The term is used in this work to denote the entity that clears an area's electricity spot market.

the different countries is an important step towards arriving to a fully functional European Internal Energy Market (IEM), as dictated by the European Commission's goals [EUI, ETS09].

However, despite the tendency towards multi-national, interconnected, seamless electric markets, with electric transactions taking place according to the markets rules, the operation and control of each area's power system remains in the hands of the area's TSO. Expectedly, due to the presence of inter-area transactions, TSO control actions are in many cases affecting their neighbors, and, without proper coordination, this may result in far from optimal operation of the involved networks in the less severe case, while emergencies [ENTb] or even blackouts [ENTa] have also been reported as a result of such a lack of coordination.

At the same time, congestion often appears, most usually on the tie-lines connecting different control areas, due to increased demand to make commerce of electric energy from one part of the interconnection to another. The reason why tie-lines tend to get congested is the fact that, as previously commented, they have been initially built for security purposes, without provision for accommodating large transfer amounts, which now they do not have sufficient transfer capacity to support.

Let us recall here that, in power systems, the term "congestion" is used to describe a situation where the electric grid can no longer support a power transaction towards a certain direction (over a branch, or a set of branches) without this compromising its security of operation. Following this definition, "congestion management" can be defined as the actions taken to avoid or relieve congestion. More broadly, congestion management can be considered any systematic approach used in scheduling and matching generation and loads in order to manage congestion [KDMR02].

It is of interest to take a look at why cross-country transactions of electric energy are so present already and are expected to be even more pronounced as the inter-area transfer capabilities increase. Figure 1.1 shows the per country electricity production capacity by primary energy source in the European Union (EU) [EU08]. Clearly, the mix of production differs significantly among the EU countries. As a result, depending on variable parameters, like fuel prices (mainly oil and gas) in international markets or weather conditions (e.g. presence of wind), there may appear demand for transferring power towards different directions. In addition, considerable wind generation capacity is expected to be installed in Europe in the near future, mainly as offshore wind mills in the Nordic and Baltic seas, which should be absorbed by loads all over Europe making international transactions even more pronounced [Tra09]. Last but not least, the combustion plants emission limits set by the EU [EUG, EUP] could create price differences between areas, since power production should become more and more expensive for those plants that do not manage to follow the environmental criteria. All in all, cross-border power flows are expected to become more and more pronounced in Europe and more and more unpredictable.

In this context, is has been understood that the development of market structures and rules that would facilitate the inter-area power exchanges is of crucial importance. A major goal of the EU is to come up with a fully functional IEM, where ideally every consumer will be able to buy electric energy from every producer all across the interconnection.

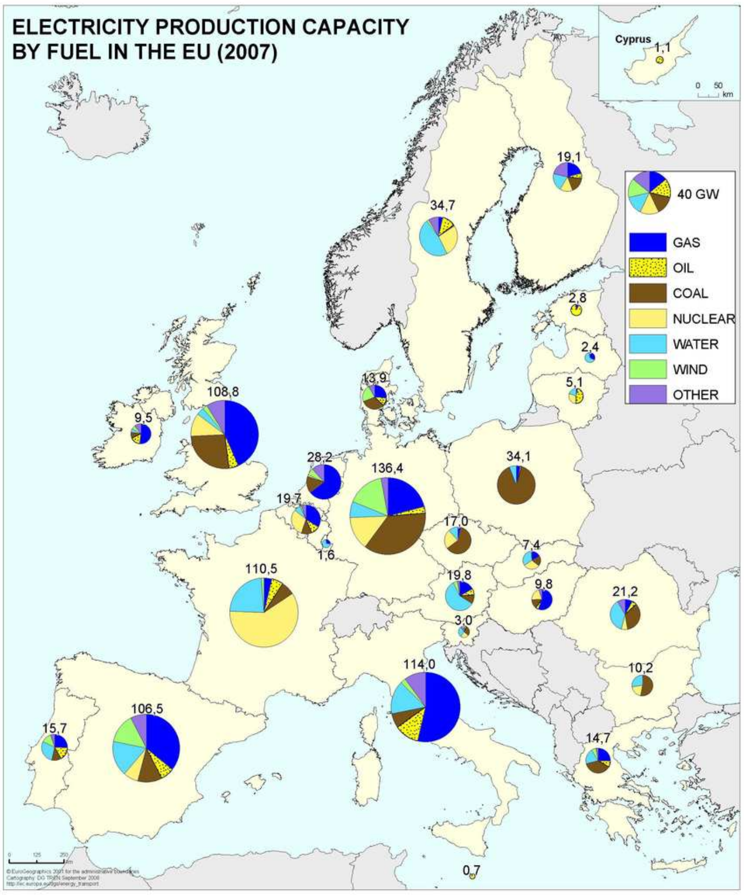

Figure 1.1: Electricity production in EU

1.2 Purpose and content of the thesis

This work, inspired by the new exciting situation, deals with two related emerging issues.

First, a multi-area control problem where the various TSOs of the interconnection simultaneously make control decisions and apply the corresponding control actions is investigated. Particular attention is given to the problem of the independent control of active power flows by the various TSOs using Phase Shifting Transformers (PST). Other typical multi-area control problems fitting the framework proposed in this thesis are the scheduling of an area's reactive power injections (Mvar scheduling) or the active generation re-dispatch problem.

Second, a market structure that allows free cross-border trade of electricity over an interconnection is developed. The main objective of the proposed algorithms is to allow market participants to trade electric energy using the transmission network in a coordinated way. Again, this corresponds to a situation where decisions are simultaneously made (i.e. the various markets are simultaneously cleared) in the same common transmission network.

1.2.1 Control of PSTs

A PST is a transformer that is installed in series with one or several transmission lines. It allows to introduce a phase angle shift of voltages across its ends. By so doing, one can control the active power that flows in a line (provided that there exist parallel paths, possibly consisting of several branches, that link the two ends of the line) or a set of lines. Figure 1.2 shows such a configuration. The two "boxes" at the ends of each figure represent the rest of the grid, which is assumed to provide paths connecting the three lines under question.

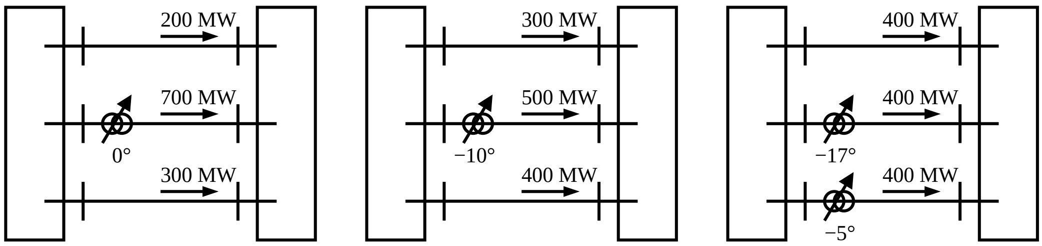

Figure 1.2: Example of PST operation

With PSTs properly located in series with some of the tie-lines of a control area, the TSO of this area is able to re-direct some power flowing through its area, sending it, unavoidably, through other areas of the interconnection. Although in normal operation the TSOs are not supposed to take such actions without prior coordination with their neighbors, it would be unrealistic, on the other hand, not to allow a TSO to use equipment it has installed if, in emergency situations, this will save its system from damaging events. In this respect, an algorithm has been developed aside the main line of this work that can be used by a TSO to control in real-time its PSTs in a way that it keeps its system secure with the least possible nuisance to the rest.

Clearly, if left uncoordinated, simultaneous or sequential adjustments of PST phase angles by different TSOs may end up in very undesirable power flows, endangering the security of the system. Furthermore, a phase adjustment made by one TSO resulting in change of power flows in another area may trigger a PST adjustment by the TSO of that area and so on, leading to very inefficient and dangerous "control fights" among the TSOs. Answering to this, a framework that allows the TSOs to simultaneously, and independently, control their respective PSTs while preserving the overall system security is also proposed in this work.

1.2.2 Overlapping markets

As mentioned in Section 1.1, an important aspect in multi-area electricity trade is to come up with market structures that facilitate such trade. In this respect, the possibility of every market participant to place its bid in whatever electricity market of an interconnection has been considered in this work. This supposition results in overlapping markets, each with its own schedule of power injections and withdraws, comprising buses all around the interconnection. Such a situation is visualized in Fig. 1.3. There, each contour corresponds to an area operated by a different TSO, while the different numbers next to the various generators and consumers suggest that they are dispatched in a different market. The lines linking the various TSO areas represent the fact that these areas are electrically connected with transmission lines. Clearly, this introduces a distinction between "TSO area" and "market". For the moment, in the remaining of the Introduction, we will call MO (Market Operator) the entity that clears a market. Further discussion is found in the related chapters of this thesis. So, in the example of Fig. 1.3 there are three MOs (named MO1, MO2 and MO3), each scheduling market participants from all around the interconnection, which justifies the use of the term "overlapping" to describe those markets.

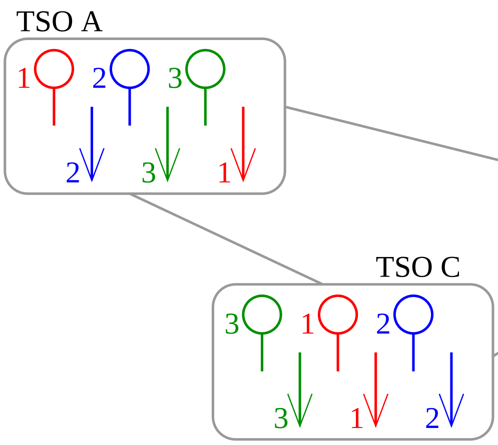

Figure 1.3: Visual example of overlapping markets

The main challenge of the proposed overlapping market structure is the management of congestion. All transactions, scheduled in the various markets, use the same interconnected grid. As a result, it is not obvious at first glance who should be responsible of approving the transaction schedules, allocating the scarce transmission capacity, and, more generally, coordinating the different acting entities. Neither is it obvious how this coordination should be performed. The algorithm developed in this work to enable cross-border trading based on free participation in multiple, generally overlapping, markets, deals with the above issues.

1.2.3 Unifying mathematical framework

Both problems dealt with in this work share some common characteristics that make it possible to treat them in a similar way. They both involve different entities (like TSOs in the PST case and MOs in the market case) taking decisions (PST phase angle adjustments and scheduling of generators and loads, respectively) in the same environment (the interconnected transmission grid) which makes those decisions interdependent.

Let us call "actor" each such entity. Every actor controls a set of variables; assigning values to these variables is a control action. The decision-making problems of choosing the control actions have been formulated in this thesis as optimization problems, where an objective function is minimized/maximized subject to a set of constraints involving the control variables. If $\mathbf{u}$ is the control vector of an actor, then its decision-making problem could be formalized as follows:

$$\min_{\mathbf{x},\mathbf{u}} f(\mathbf{x},\mathbf{u}) \tag{1.1a}$$
$$\text{s.t.} \quad \mathbf{h}(\mathbf{x},\mathbf{u}) = \mathbf{0} \tag{1.1b}$$
$$\mathbf{g}(\mathbf{x},\mathbf{u}) \leq \mathbf{0} \tag{1.1c}$$
$$\mathbf{u} \in \mathbf{U} \tag{1.1d}$$

where $\mathbf{x}$ represents the "state" of the environment defined by the whole set of actors' controls. Of course, when the actor solving the optimization problem modifies its controls, $\mathbf{x}$ is generally also affected. This is modeled by the equality constraints in (1.1b). So, $\mathbf{x}$ is a function of all the controls in the interconnection; $\mathbf{x} = \mathbf{x}(\mathbf{u}, \mathbf{u}^{-})$, with $\mathbf{u}^{-}$ being the vector containing all the other actors' control actions [3]. For example, in the PST control problem, $\mathbf{x}$ could contain the active power flow over a cutset inside the system of a TSO, which depends on the tap positions of all the PSTs of the interconnection (not only those controlled by the TSO solving the optimization problem). The inequality constraints in (1.1c) stand for the physical, security, operational, regulatory and other limits that should be respected, while $\mathbf{U}$ is the domain from where the controls $\mathbf{u}$ take their values.

From now on, for the sake of simplicity, $\mathbf{x}$ will be omitted from the presentation of the problems: $\mathbf{h}$ will be considered as implicitly expressed in $\mathbf{g}$, while the objective function $f(\cdot)$ and the inequality constraints $\mathbf{g}(\cdot) \leq \mathbf{0}$ will be directly expressed as functions of all the control actions. Like this, the optimization problem (1.1) of an actor is re-written as:

$$\min_{\mathbf{u}} f(\mathbf{u},\mathbf{u}^{-}) \tag{1.2a}$$
$$\text{s.t.} \quad \mathbf{g}(\mathbf{u},\mathbf{u}^{-}) \leq \mathbf{0} \tag{1.2b}$$
$$\mathbf{u} \in \mathbf{U} \tag{1.2c}$$

For instance, let us denote by $\boldsymbol{\varphi}$ the vector of all PST settings in the interconnection, $\mathbf{g}$ the vector of all generator injections, $\mathbf{d}$ the vector of all consumptions, $\mathbf{i}(\boldsymbol{\varphi}, \mathbf{g}, \mathbf{d})$ the vector of branch currents and $\mathbf{i}^{max}$ the thermal limits of the branches in the interconnection. Let

$$\mathbf{i}(\boldsymbol{\varphi}, \mathbf{g}, \mathbf{d}) \leq \mathbf{i}^{max} \tag{1.3}$$

be the set of constraints that should be respected at any operating point. In the PST control problem, the generations and loads are considered fixed and each TSO controls a part of vector $\boldsymbol{\varphi}$ (the PSTs of its area). In the overlapping market problem, the phase angles are considered fixed and each MO controls a part of vectors $\mathbf{g}$ and $\mathbf{d}$ (the generators and loads bidding in its market). In both cases, the overall control decisions should be such that the constraints (1.3) are satisfied.

[3] Generally $\mathbf{x}$ should also depend on its previous value, but in the problems dealt with in this thesis the system's state is defined by the actors' controls in a unique way, and the more general dynamic case is not considered.

Briefly stated, in each problem, the corresponding actors control their variables in order to satisfy their security or cost objectives. Clearly, it may not be always possible for an actor to modify its controls without this resulting in constraint violation, given the other actors' controls. The algorithms developed in this work seek to coordinate and reconcile the independent decision-making by the corresponding entities at the same time guaranteeing the feasibility of the overall solution.

In this work, basic notions stemming from Game Theory [FT91, Gib97] and Multi-Objective Optimization [Mie99] have been used in order to model the emerging situations and to evaluate the efficiency of the results. For instance, the above described situation can be viewed as a "game". The different actors take control decisions given the control decisions of the other actors trying to finally obtain the best possible satisfaction of their objective. The game ends at an operating point where no actor can further improve its objective given the control decisions of the others. A different approach to deal with the same situation is to put together the individual objectives of the various actors to form a single large, multi-objective, optimization problem. The solution of this problem yields a compromise between the satisfaction of the different (partially conflicting) objectives. The related material is briefly presented in Chapter 2 and referred to in the rest of the work.

1.2.4 Why not a single, centralized optimization?

A seemingly obvious solution to deal with the aforementioned problem would be to merge all individual objectives into a single one, thereby resorting to a single objective optimization involving all constraints.

Several reasons hamper this treatment.

- The objectives may differ significantly from one actor to another; the Mvar scheduling problem provides such an example where a TSO may want to increase its reactive reserves and another to decrease its active losses.
- The objectives may be somewhat contradictory to each other; for example, two TSOs may be using their respective PSTs each in order to decrease power flows over specified cutsets, but decreasing the flow over the cutset selected by the first TSO may result in a flow increase over the cutset selected by the second TSO and vice versa.
- Even when a single common objective would make sense, like in the case of overlapping markets where each MO's objective is to maximize its social welfare (so adding all the objectives together would maximize the total social welfare of the interconnection[4]), there remains the issue that this may imply favoring some objectives against the others. For example, let us assume that two MOs clear independently their markets and come up,

[4]The ith MO, when clearing its market, will typically be maximizing the total social welfare of all its participants, call it $sw_i(\mathbf{u}_i)$. Adding together all these MOs' social welfares makes up the total social welfare of the interconnection $SW(\mathbf{u}) = sw_1(\mathbf{u}_1) + sw_2(\mathbf{u}_2) + \ldots + sw_i(\mathbf{u}_i) + \ldots + sw_M(\mathbf{u}_M)$, with M the number of MOs.

respectively, with schedules $\mathbf{u}_1^\star$ and $\mathbf{u}_2^\star$ with corresponding social welfares $sw_1^\star$ and $sw_2^\star$. Let us also assume that if the total social welfare, $SW(\mathbf{u}) = sw_1(\mathbf{u}_1) + sw_2(\mathbf{u}_2)$, was minimized by a central entity, the resulting schedules would be $\widehat{\mathbf{u}}_1$ and $\widehat{\mathbf{u}}_2$, corresponding to individual social welfares $\widehat{sw}_1$ and $\widehat{sw}_2$. If for the ith MO $\widehat{sw}_i < sw_i^\star$ then this MO may consider that it is unfairly treated by the centralized entity. Considering a single common objective, even if it results in a higher total social welfare, would not be accepted by areas whose social welfare deteriorates due to the common optimization.

- The different actors may not be willing to leave their decision-making authority to a central entity. They may consider that this violates confidentiality issues, or simply that it does not serve their interests.

All in all, this thesis is built around the request that the different actors' objectives should be treated as fully private and undisclosed to the others. However, in selected places comparisons with a common objective optimization are made and commented.

1.3 How to (and not to) read this thesis

1.3.1 Structure of the thesis

The leitmotiv of this research work is the above described situation of multiple actors taking simultaneous actions in a large power system. The methodology and line of thinking that is used in this direction is presented in Chapter 2. Chapters 3 and 4 deal with the PST control and the overlapping market problems, respectively, in the spirit defined in Chapter 2. However, each of those problems deserves by itself special consideration, since it can be viewed as a self-standing research topic. For this reason, in both cases, the research work has been extended beyond the multi-actor framework, which is the backbone of the thesis, investigating in more detail the particular characteristics of each problem.

Namely, apart from the multi-TSO aspect of the PST control problem, the problem has been also considered from a single TSO viewpoint. In this respect, an algorithm addressed to a single TSO for operating its PSTs has been developed. This algorithm is presented in an Appendix because it is a deviation from the main line of thinking of the thesis. As regards the overlapping market problem, the developed coordination algorithm has been extended to treat a range of related issues. An additional chapter thus follows Chapter 4, where considerations about the treatment of N-1 security constraints, of losses as well as the scheduling of reserves are developed.

In fact, although it is not the main viewpoint adopted in this work, it is worth mentioning that the two problems can be viewed separately, dividing the work into two topics: one dealing with the control of PSTs in an interconnection, both from a single and a multiple TSOs perspective, and another dealing with the idea of allowing multiple overlapping markets to operate across a large power system, along with its implementation aspects.

Fig. 1.4 outlines the aforementioned considerations and serves as a guide to reading this thesis. The numbers in parenthesis give the chapter where the related material is located (A stands for Appendix A).

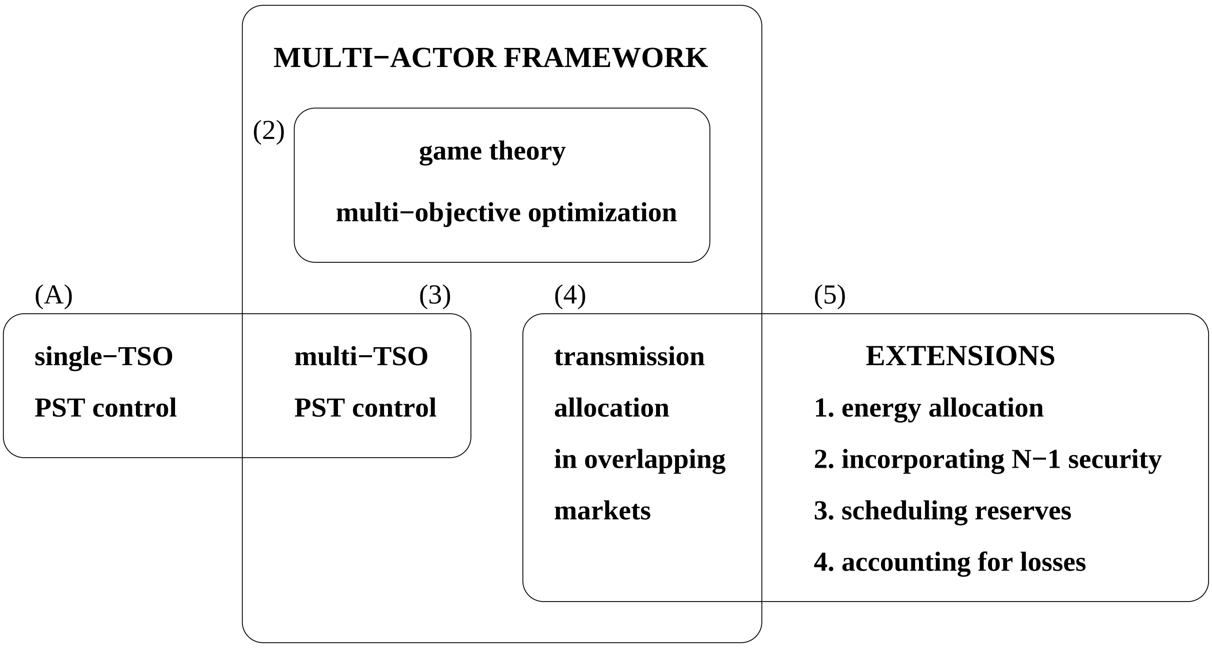

Figure 1.4: Structure of the thesis

1.3.2 Pathway to the thesis' content

Closing this Introduction, let us devote a few lines to the pathway of the presented research work. The initial inspiration has been a seemingly new emerging situation in Europe, with TSOs starting to equip their networks with PSTs in order them to gain controllability over power flows wheeling through their areas. Belgium and Switzerland are two typical such cases in Europe, which, due to their geographic location, are subject to significant power flows stemming from external transactions. The need and ways of coordinating the PST operation has, thus, been the first subject of this research work [Mar07].

This investigation drove consecutively our attention towards the operation and organization of electricity markets. Two reasons mainly stimulated this shift: first, the quest for market-oriented objectives for the TSOs to control their PST angle settings, and, second, the recognition that the PST control coordination problem can be classified as a special case of the, more general, congestion management problem that appears in the presence of multiple independent actors scheduling transactions in a common network.

As a matter of fact, a big discussion has been raised recently, at least in Europe, about the question of how several, up to now separated, electricity markets (coinciding with closed geographical areas) could be (re-)organized to give new structures that would eventually correspond to a large single market where electric energy is seamlessly traded. Again, Belgium is in the heart of these developments since it is, together with France and the Netherlands, one of the three countries whose Power Exchanges (PXs) set up the so-called Trilateral market Coupling (TLC) since 2006, coupling their day-ahead markets. The overlapping market approach proposed in this work contributes towards creating a big unified marketplace all across a large interconnection, like the European one. Its timely and practical interest justifies the relatively larger space devoted to this problem in this work compared to the PST coordination one.

1.4 Software implementation

Let us say two words about the software that has been used in order to implement and test the algorithms that have been developed in this work.

Both algorithms dealing with the control of PSTs (see Chapter 3 and Appendix A) have been implemented by modifying the source code of ARTERE, a power flow software developed at the University of Liège. In particular, the PSTs have been modeled as a Γ-equivalent in cascade with an ideal transformer with complex ratio, while linear sensitivities relating changes in PST angles with changes in branch power flows were derived from the Jacobian of the full AC network model. Optimization problems, stemming from the various TSO decisions, where solved by resorting to the corresponding solvers of the IMSL mathematical library.

For the implementation and testing of the algorithms presented in the context of overlapping markets (see Chapters 4 and 5), the mathematical programming environment of GAMS [GAM] has been used. This offers a variety of solvers, while its significant advantage in the context of this work (where, anyway, all optimization problems are linear) lies in the human-friendly way the various problems are formulated by the user and in the insight it offers regarding the results of its solved problem. Although not naturally designed for this purpose, we have used basic program flow control commands that exist in GAMS to implement the various iterative algorithms presented in Chapters 4 and 5.

1.5 Publications

The present work gave rise to the following publications (chapter(s) where the related material is presented are given in parenthesis):

- A. Marinakis, M. Glavic and T. Van Cutsem. Control of phase shifting transformers by multiple transmission system operators. In *Proc. of IEEE PowerTech Conference 2007*, Lausanne (Switzerland), pp. 119-124, 1-5 July 2007, Print ISBN: 978-1-4244-2189-3. (Chapter 3)
- A. Marinakis, A. G. Bakirtzis and T. Van Cutsem. Bidding and managing congestion across multiple electricity spot markets. In *Proc. of 6th International Conference on*

the European Energy Market 2009 (EEM09), Leuven (Belgium), 27-29 May 2009, Print ISBN: 978-1-4244-4455-7. (Chapter 4)

- A. Marinakis, W. D. Rosehart and T. Van Cutsem. A framework for the simultaneous clearing of multiple markets within a common transmission system. In *Proc. of IEEE PowerTech Conference 2009*, Bucharest (Romania), 28 June - 2 July 2009, Print ISBN: 978-1-4244-2234-0. (Chapter 4)

- A. Marinakis, M. Glavic and T. Van Cutsem. Minimal Reduction of Unscheduled Flows for Security Restoration: Application to Phase Shifter Control. *IEEE Transactions on Power Systems*, vol. 25, no. 1, pp. 506-515, February 2010. (Appendix A)

- A. Marinakis, A. G. Bakirtzis and T. Van Cutsem. Energy and Transmission Allocation in the Presence of Overlapping Electricity Markets. *IEEE Transactions on Power Systems*, paper accepted for publication in 2010. (Chapters 4 and 5)

- A. Marinakis and T. Van Cutsem. Energy and transmission allocation in overlapping electricity markets: incorporating N-1 security and accounting for losses. paper submitted to the *7th International Conference on the European Energy Market 2010 (EEM10)*, Madrid (Spain), June 23-25, 2010. (Chapter 5)

Besides the topics covered in this report, we were also involved in research work that led to the following publications:

- B. Otomega, A. Marinakis, M. Glavic and T. Van Cutsem. Model Predictive Control to Alleviate Thermal Overloads. *IEEE Transactions on Power Systems*, vol. 22, no. 3, pp. 1384-1385, August 2007.

- B. Otomega, A. Marinakis, M. Glavic and T. Van Cutsem. Emergency alleviation of thermal overloads using model predictive control. In *Proc. of IEEE PowerTech Conference 2007*, Lausanne (Switzerland), pp. 201-206, 1-5 July 2007, Print ISBN: 978-1-4244-2189-3.

Chapter 2

A framework for the optimization of multiple interacting objectives by multiple actors

In the Introduction the decision-making procedures employed by the different actors in both the PST control and the overlapping market problem were generally presented as optimization problems, described by equations like (1.2). It was stated that the interdependence of the actors' decisions in each problem allows to formulate the situation as a game or as a multi-objective optimization problem.

This chapter is devoted to presenting: (a) a basic background of these two fields of applied mathematics, (b) a discussion of how they are linked to each other, and (c) the relationship between them and the electric power systems problems treated in this thesis.

2.1 Different approaches to decision-making by multiple actors

For the sake of completeness and in order to put the viewpoints adopted in this work in perspective with existing practices and ideas, it is worth devoting some space to presenting a classification of the different organizational possibilities that could deal with the multi-actor problems under question.

These possibilities are differentiated in two ways. The first involves a separation between, on one hand, resorting to a large single-objective optimization encompassing all the various actors' problems and constraints, and, on the other hand, allowing concurrent optimizations of the various actors' objectives. The second differentiation involves a separation between, on one hand, a centralized solution of the various problems by one entity, and, on the other hand, a decentralized approach, where each actor solves its own optimization problem.

The above classification leads to four combinations as shown in Fig. 2.1. At one end there is a centralized, single-objective treatment of the problem, while, at the other end, we have the possibility of a decentralized, multiple-objectives approach.

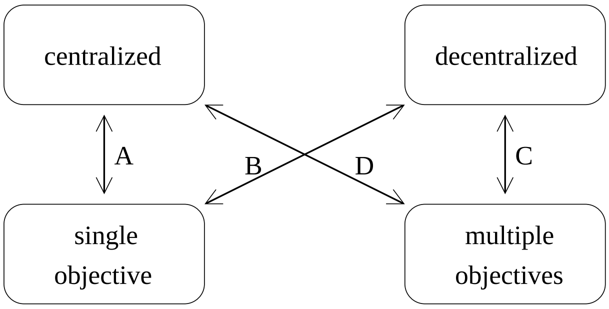

Figure 2.1: Different organizational possibilities for multi-actor framework.

Let us further examine what each of the four possibilities means.

Case A: This is a borderline case, where the existence of multiple actors is not acknowledged. It consists in operating the whole interconnection by a single central entity. This “super-TSO” would set all the available controls envisaging an objective which would probably be a combined one that seeks for maximizing the total social welfare while attaining at the same time a sufficient level of security. Such a super-TSO does not exist (neither it seems possible to exist at least in the near future). Section 1.2.4 provided a list of reasons explaining why such an approach is not very likely to be accepted and, thus, it is not the solution followed in this work.

It is of interest however to mention the recent creation of Coreso [Cor], an approach for a regional coordination center within the Central Western European region[1]. Coreso supplies the control centers of the relevant TSOs with forecasts about the security of the Central West European grid for the following day (‘D-1 activities’). The Coreso’s engineers base their analysis on data that are updated each day and submitted by the system operators, e.g. generation schedules, international electricity flows and unavailability of power stations and grid components. In this way, they assess the security of the grids, simulate various scenarios, such as the sudden unavailability of an interconnecting line, and then anticipate the measures that need to be taken to master the consequences. These analysis, together with the proposed measures, are submitted to the TSOs’ national control centers which assume operational responsibility for secure operation of their respective grids [Cor].

Clearly, Coreso is not a control center itself, but a center aiming at coordinating control centers. One cannot say whether, in some future, areas that are presently controlled separately will merge into larger control areas, but, for the moment, this does not seem to be envisaged. However, the exchange of information like generation schedules and line power flows between the TSO and the Coreso centers verifies what seems to be the present trend, at least in Europe: exchange non-market sensitive information in order to achieve a higher level of coordination in a pan-European level, ensuring the security of operation while enabling an integration of the various electricity markets.

[1]France, Luxembourg, Germany, Belgium and the Netherlands

Case B: This organizational possibility does not differ from the previous one (case A) in its potential result. In both cases a single objective, concerning the whole interconnection, is optimized. However, the implementation approach is completely different. Instead of requiring the setting up and operation of a unique control center, which would assemble all the areas' data and would send back to areas the computed controls, dedicated algorithms allow each area to treat its own, possibly confidential, data and set its controls such that it participates in the commonly optimized objective.

In this respect, considerable research results exist in the power system literature that deal with the solution of the so-called decentralized Optimal Power Flow (OPF) problem [KB97, CA98, NPC99, AQ01, BB03]. In their essence, they are decomposition methods which split an original single problem into several subproblems in such a way that little exchange of information is needed in order parallel iterative solutions of the subproblems by different agents to lead to the solution of the original problem. Section 2.2 contains a brief introduction to decentralized OPF.

The decentralized OPF algorithms allow a single optimization to be performed for the whole interconnection without the practical and maybe political complications that the creation of one single central control center would pose. However, the resort to a single common objective is anyway questionable as explained in Section 1.2.4. The remaining two organizational possibilities are those of interest in this work since they involve the explicit, separate optimization of the various objectives.

Case C: This possibility is the "natural" description of the situation under question: each actor optimizes itself its objective using its own controls. Due to the coupling of the various problems, however, those optimizations are not independent of each other. As explained in the Introduction, an actor's satisfaction from a control decision of its own, depends, generally, on the other actors' decisions as well. A game-theoretic framework provides the natural choice to describe such a case.

Clearly, ensuring the satisfaction of the coupled constraints requires a minimum level of coordination between the actors' control decisions. In this respect, in the algorithms presented in this work, the decentralized operation of multiple actors is in all cases complemented with rules that coordinate the concurrent optimizations.

Case D: Finally, an alternative worth considering, could be to step back from the decentralized operation viewpoint and have all the optimization problems solved by one entity, which, at the same time, would take care of all the constraints. This means that this central entity would solve a multi-objective optimization problem. The main difficulty in this possibility stems from the fact that it may not be obvious which of the, generally many, possible trade-offs should be finally chosen so that all involved actors are convinced to accept this solution.

However, the multi-objective approach can provide useful insight into the possible ways that the problems could be solved and, as will be commented later in this chapter, it could in some sense quantify the "quality" of the solutions resulting from a game-theoretic procedure. Thus, although the main line of this work goes with a coordinated game between the various actors, arguments and results stemming from a multi-objective consideration of the problems will be used when appropriate.

2.2 Decentralized Optimal Power Flow

This section is devoted to briefly presenting the decentralized OPF approach. As already said, this is not the direction followed in this work. The reader may as well skip this section and proceed with Section 2.3 without loss of information for the understanding of this work.

In [NPC99, CNP02], a modified Lagrangian relaxation procedure is used for the decomposition of the AC-OPF. The method results in a special treatment of complicating constraints (i.e. constraints invoking variables of two adjacent areas). Complicating constraints are the active and reactive power balance equations at the “from” and “to” buses of the tie-lines as well as the tie-line flow limits. The decentralized OPF solution is achieved by the iterative solution of modified area OPF subproblems. At each iteration, the modified OPF subproblem of a specific area differs from a standard OPF in the following: (a) the objective function is augmented by the Lagrangian terms corresponding to the complicating constraints of the adjacent area side of all tie-lines and (b) the variables as well as multipliers of adjacent areas, that appear in the complicating constraints, are held fixed to the values they attained during the previous iteration.

We outline hereafter the technique presented in [BB03] for the solution of the DC-OPF, building upon the ideas in [NPC99, CNP02].

Assume that there are only two areas, namely A and B, connected by a single tie-line ij (bus i located in area A and bus j in B). Each area's TSO controls a vector of variables, $\mathbf{u}_A$ for A and $\mathbf{u}_B$ for B, and minimizes a cost function, respectively $f_A(\mathbf{u}_A)$ and $f_B(\mathbf{u}_B)$. The vector of variables contains the bus active power injections, the bus voltage phase angles and the tie-line's active power flow. Thus, $\mathbf{u}_A = [\mathbf{P}_A \ \boldsymbol{\theta}_A \ T_A]$ and $\mathbf{u}_B = [\mathbf{P}_B \ \boldsymbol{\theta}_B \ T_B]$, where T_A is the tie-line flow from bus i to bus j as computed by TSO A, while T_B is the same flow as computed by TSO B. A set of “local” constraints has to be respected by each TSO, $\mathbf{g}_A(\mathbf{u}_A) \leq \mathbf{0}$ and $\mathbf{g}_B(\mathbf{u}_B) \leq \mathbf{0}$ respectively[2]. These constraints depend only on the local control variables.

The so-called “original” OPF problem, involving the entire interconnection is described as follows:

$$\min_{\mathbf{u}_A,\mathbf{u}_B} f_A(\mathbf{u}_A) + f_B(\mathbf{u}_B) \tag{2.1a}$$

$$\text{s.t.} \quad \mathbf{g}_A(\mathbf{u}_A) \leq \mathbf{0} \tag{2.1b}$$

$$\mathbf{g}_B(\mathbf{u}_B) \leq \mathbf{0} \tag{2.1c}$$

$$-T^{max} \leq T(\mathbf{u}_A, \mathbf{u}_B) \leq T^{max} \tag{2.1d}$$

where $T(\mathbf{u}_A, \mathbf{u}_B)$ gives the tie-line flow as a function of both areas variables: $T(\mathbf{u}_A, \mathbf{u}_B) = (\theta_i - \theta_j)/x_{ij}$, with x_{ij} the reactance of the tie-line.

[2]These constraints involve computation of phase angles and branch power flows using a DC model of the network and they ensure the power balance is respected and no branch gets overloaded.

Optimization problem (2.1) is decomposed into two separate ones, each involving the variables of only one TSO. The decentralized OPF algorithm consists in the two TSOs solving iteratively their own problems until converging to a solution of (2.1). The objective function (2.1a) is naturally decomposed into its two summands. Alike, constraints (2.1b) and (2.1c) are easily attributed to the corresponding TSOs. This cannot be done with (2.1d), since it involves variables from both sides. So, the tie-line flow T is duplicated into

$$T_A = T(\mathbf{u}_A, \widehat{\mathbf{u}}_B) \tag{2.2}$$

for TSO A, and

$$T_B = -T(\widehat{\mathbf{u}}_A, \mathbf{u}_B) \tag{2.3}$$

for TSO B, with $\widehat{\mathbf{u}}_A$ (resp. $\widehat{\mathbf{u}}_B$) the values of A's (resp. B's) variables communicated to TSO B (resp. A) stemming from the previous iteration. The constraint (2.1d) is now incorporated in (2.1b) as $-T^{max} \leq T_A \leq T^{max}$ and in (2.1c) as $-T^{max} \leq T_B \leq T^{max}$, while the two equality constraints (2.2) and (2.3) constitute the coupling constraints of the decomposed problem. Additionally, a term involving the other area's coupling constraint Lagrange multiplier is added to each subproblem's objective function, which leads to the following two subproblems:

$$\min_{\mathbf{u}_A} f_A(\mathbf{u}_A) + \widehat{\alpha}_B h_B(\mathbf{u}_A, \widehat{\mathbf{u}}_B) \tag{2.4a}$$

$$\text{s.t.} \quad \mathbf{g}_A(\mathbf{u}_A) \leq \mathbf{0} \tag{2.4b}$$

$$h_A(\mathbf{u}_A, \widehat{\mathbf{u}}_B) = T(\mathbf{u}_A, \widehat{\mathbf{u}}_B) - T_A = 0 \quad (\text{dual variable: } \alpha_A) \tag{2.4c}$$

and

$$\min_{\mathbf{u}_B} f_B(\mathbf{u}_B) + \widehat{\alpha}_A h_A(\widehat{\mathbf{u}}_A, \mathbf{u}_B) \tag{2.5a}$$

$$\text{s.t.} \quad \mathbf{g}_B(\mathbf{u}_B) \leq \mathbf{0} \tag{2.5b}$$

$$h_B(\widehat{\mathbf{u}}_A, \mathbf{u}_B) = T(\widehat{\mathbf{u}}_A, \mathbf{u}_B) - T_B = 0 \quad (\text{dual variable: } \alpha_B) \tag{2.5c}$$

where the hatted values are provided by the other TSO and correspond to its previous subproblem solution.

The algorithm stops when $T_A = T_B$ within some tolerance. At this optimal, the combined Karush-Kuhn-Tucker (KKT) first order optimality conditions of the areas' subproblems coincide with the KKT conditions of the original problem. This has been achieved thanks to the extra term added to each subproblem's objective function.

The main feature of these algorithms is that they allow each TSO to optimize its system individually, acting on its own controls only. Local constraints (i.e. referring only to a particular subsystem) are included by each TSO in the subproblem it solves. Coupled constraints are taken care by exchange of Lagrange multipliers of the optimization problems between the TSOs and incorporation of terms in their objective functions.

This approach is practically a distributed way to solve one single problem. The single objective that is finally optimized is the sum of all the TSOs individual objectives. Each TSO, if acting honestly (i.e. truly announcing the requested Lagrange multipliers at each iteration and truly incorporating those sent by the others in its OPF), is practically an agent participating in the solution of a global problem. The approach has been presented in problems where the objective of all TSOs was to minimize an operational cost (expressed in €/h). So, adding together all the objectives, to formulate the global problem, made sense; the overall objective could be viewed as the total operational cost of the interconnection. However, the algorithms equally work for objectives of different natures, provided that the different objectives are properly scaled prior to the execution of the algorithm.

It is worth stating explicitly the clear distinction between the techniques described in this section and the game-theoretic viewpoint considered in the remaining of this chapter. The decentralized OPF should not be viewed as a game where each TSO is a player. The reason is that the different actors are not self-interested; they do not seek to optimize their individual objective, but they participate in the optimization of a common one. A game-theoretic viewpoint would question the willingness of the TSOs to announce true values of their Lagrange multipliers and of properly incorporating the others' multipliers in their OPF problems. In a game-theoretic approach, the TSOs would aim at making the iterative algorithm converge to a solution where their individual objective function is optimized, even if the total cost does not. Clearly, this is not the spirit of the publications referred to in this section.

2.3 Game-theoretic framework

2.3.1 Dynamic non-cooperative game theory: a brief background

A short description of what the term "game theory" refers to is given hereafter based on [BO99].

In a nutshell, *game theory* involves multi-person decision making; it is *dynamic* if the order in which the decisions are made is important [3], and it is *non-cooperative* if each person involved pursues its own interests which are partly conflicting with others'.

It is relatively easy to delineate the main ingredients of a conflict situation: an individual has to make a decision and each possible decision leads to a different outcome or result, which are valued differently by that individual. This individual may not be the only one who decides about a particular outcome; a series of decisions by several individuals may be necessary. If all these individuals value the possible outcomes differently, the germs for a conflict situation are there.

The individuals involved are typically called *players* (the terms *decision-makers* or *actors* are also used).

The games played in the PST control and the overlapping market problems in this work are

[3] In other words, a game is dynamic if the individuals make sequential decisions by turn, while it is static if they simultaneously choose and implement their actions.

Table 2.1: The place of dynamic game theory

	One player	Many players
Static	Mathematical programming	(Static) game theory
Dynamic	Optimal control theory	Dynamic (and/or differential) game theory

dynamic because the different actors sequentially adjust their control actions after observing the result of the others' actions, while they are non-cooperative because each player seeks for its own interest, formulated as the minimization or maximization of its objective function.

Scientifically, *dynamic game theory* can be viewed as a child of the parents "game theory" and "optimal control theory" (see Table 2.1). Its character, however, is much more versatile than that of its parents, since it involves a dynamic decision process evolving in (discrete or continuous) time, with more than one decision maker, each with its own cost function and possibly having access to different information [BO99]. This view is the starting point behind the formulation of "games in extensive form", which started in the 1930s through the pioneering work of Von Neumann (culminated in his book with Morgenstern [NM47]), and was then made mathematically precise by Kuhn [Kuh53], all within the framework of "finite" games. The general idea in this formulation is that a game evolves according to a road or tree structure, where at every crossing or branching a decision has to be made as how to proceed.

In spite of this original set-up, the evolution of game theory has followed a rather different path. Most research in this field has been, and is being, concentrated on the normal or strategic form of a game. In this form all possible sequences of decisions of each player are set out against each other. In such a formulation dynamic aspects of a game are completely suppressed, and this is the reason why game theory is classified as "static" in Table 2.1. In this framework emphasis has been more on (mathematical) existence questions, rather than on the development of algorithms to obtain solutions.

Independently, control theory gradually evolved from Second World War servomechanisms, where questions of solution techniques and stability were studied. Then, Bellman's "dynamic programming" [Bel57] and Pontryagin's "maximum principle" [PBGM62] followed, which spurred the interest in a new field called optimal control theory. Here the concern has been on obtaining optimal solutions and developing numerical algorithms for one-person single-objective dynamic decision problems. The merging of the two fields, game theory and optimal control theory, leads to even more concepts and to actual computation schemes.

Heretofore, we have safely talked about "decisions" made by players, without being very explicit about what a decision really is. This will be made more precise now in terms of information available to each player. In particular, we shall distinguish between *actions* (also called controls) on the one hand and *strategies* (or, equivalently, decision rules) on the other.

If an individual has to decide about what to do the next day, and the options are fishing and going to work, then a strategy is: "if the weather report early tomorrow morning predicts dry weather, then I will go fishing, otherwise I will go to my office". This is a *strategy* or *decision rule*: what actually will be done depends on quantities not yet known and not controlled by the decision maker; the decision maker does not influence the course of the events further, once he has fixed his strategy. Any consequence of such a strategy, after the unknown quantities are realized, is called an *action*. In a sense, a constant strategy (such as an irrevocable decision to go fishing without any restrictions or reservations) coincides with the notion of action.

2.3.2 Nash games

Let us now recall what is considered to be a “game” in the context of game theory.

Definition. An N-person *game* is a formal representation or a mathematical model of a situation in which a number of players interact in a setting of strategic interdependence [CKK04]. This means that the welfare of a player depends upon its own actions and on the actions of the other participants in the game. An N-person game (in normal form) is defined as a three-tuple $\{\mathcal{N}, \mathbf{U}_i, f_i, i \in \mathcal{N}\}$, where:

- $\mathcal{N}$ is the set of players, $\mathcal{N} = \{1, 2, \ldots, N\}$;
- $\mathbf{U}_i$ is the set of possible actions (or strategy space, or feasible set) of the ith player ($\mathbf{U}_i \subset \mathbb{R}^{n_i}$);
- f_i is the payoff (or welfare, utility, profit, objective, etc.) function of the ith player that assigns a real number to each element of the Cartesian product of the strategy spaces $\mathbf{U} = \mathbf{U}_1 \times \mathbf{U}_2 \times \ldots \times \mathbf{U}_N$.

Players play a game through actions. The information that a player has about its own and other players’ past actions is the *information set* of that player. A *payoff function* expresses the utility that a player obtains given a strategy profile for all players.

Assume that there are $i = 1, \ldots, N$ players participating in a game. Each player can take an individual action represented by a vector $\mathbf{u}_i$. All players acting together makes up a collective action, which is a vector $\mathbf{u} = (\mathbf{u}_1, \ldots, \mathbf{u}_N)$. To emphasize the ith player’s variables within $\mathbf{u}$, we sometimes write $(\mathbf{u}_i, \mathbf{u}_{i-})$ instead of $\mathbf{u}$. Each player selects its action from its feasible set, i.e. $\mathbf{u}_i \in \mathbf{U}_i$, while a collective action belongs in the collective action set $\mathbf{U}$. If $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$ and $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_N)$ are elements of the collective action set, we define an element $(\mathbf{y}_i, \mathbf{x}_{i-}) \equiv (\mathbf{x}_1, \ldots, \mathbf{x}_{i-1}, \mathbf{y}_i, \mathbf{x}_{i+1}, \ldots, \mathbf{x}_N)$ of the collective action set as a set of actions where the ith player plays $\mathbf{y}_i$ while the remaining actors are playing $\mathbf{x}_j$, with $j = 1, \ldots, i-1, i+1, \ldots, N$.

Assuming, without loss of generality, that each player tries to minimize an objective function, the ith player’s decision-making problem can be formulated as:

$$\min_{\mathbf{u}_i} f_i(\mathbf{u}_i, \mathbf{u}_{i-}) \tag{2.6a}$$

$$\text{s.t.} \quad \mathbf{u}_i \in \mathbf{U}_i \tag{2.6b}$$

Definition. A point $\mathbf{u}^\star \in \mathbf{U}$ is called a *Nash equilibrium* of the game if, for all i, we have:

$$f_i(\mathbf{u}^\star) = \min_{\mathbf{u}_i \in \mathbf{U}_i} f_i(\mathbf{u}_i, \mathbf{u}^\star_{i^-}). \tag{2.7}$$

Notice that $\mathbf{u}^\star$ solves the game $\{\mathcal{N}, \mathbf{U}_i, f_i, i \in \mathcal{N}\}$ in the following sense: at $\mathbf{u}^\star$ no player can improve its individual payoff by a unilateral (i.e. its sole) action. For this reason, it is said that the Nash equilibrium is strategically stable or self-enforcing.

Depending on the problem, there may exist multiple, a unique, or even no Nash equilibrium at all[4]. The set of all Nash equilibria of a game is called the *Nash set*.

Problems that fit in the above described framework are termed as *Nash games*, or *Nash equilibrium problems* (NEPs) as a recognition to the mathematician John Nash who was the first to formally introduce them, in two papers [Nas50, Nas51] which are a landmark in the scientific history of the twentieth century.

2.3.3 Generalized Nash games

In Nash games, each player's payoff function depends on the actions of the other players, but each player's feasible set does not; the ith player chooses its action always from the same strategy space $\mathbf{U}_i$ irrespective of what the others' decisions are. However, there exist problems where the players interact also at the level of the strategy spaces. In other words, the players' feasible sets depend on the players' control actions, i.e.

$$\mathbf{U}_i = \mathbf{U}_i(\mathbf{u}_{i^-}).$$

Such problems arise quite naturally from standard Nash problems if the players share some common resource (a communication link, an electrical transmission line, a transportation link etc.) or limitations (for example a common limit on the total pollution in a certain area).

This class of problems has been named after a number of different terms in the literature, such as pseudo-games, social equilibrium problems, equilibrium programming problems and abstract economies. The two names that are used interchangeably in this thesis are: a) *generalized Nash equilibrium problems* (GNEPs) that seems to emerge as the favorite in Operations Research, and, b) *coupled constraints games*, because of its descriptive value for the problems treated in this work. An excellent survey on GNEPs can be found in [FK07], from where the material of this subsection has been borrowed.

Let us from now on assume that the sets $\mathbf{U}_i(\mathbf{u}_{i^-})$ are given by

$$\mathbf{U}_i(\mathbf{u}_{i^-}) = \{\mathbf{u}_i \in \Re^i : \mathbf{g}_i(\mathbf{u}_i, \mathbf{u}_{i^-}) \leq \mathbf{0}\}, \tag{2.8}$$

[4] In fact, no *pure* Nash equilibria; there will generally exist *mixed* Nash equilibria, but this is out of the scope of this work.

where $\mathbf{g}_i(\mathbf{u}_i, \mathbf{u}_{i^-}) : \mathbb{R}^{n_1 \times \ldots \times n_N} \rightarrow \mathbb{R}^{C_i}$, C_i being the number of constraints of the ith player. Equality constraints, as well as constraints of the ith player that depend only on its own variables, are included in (2.8) as particular cases.

Thus, the decision-making problem of the ith player in the coupled constraints games is formulated as:

$$\min_{\mathbf{u}_i} f_i(\mathbf{u}_i, \mathbf{u}_{i^-}) \tag{2.9a}$$

$$\text{s.t.} \quad \mathbf{g}_i(\mathbf{u}_i, \mathbf{u}_{i^-}) \leq \mathbf{0} \tag{2.9b}$$

Similarly to (2.7), a *generalized Nash equilibrium* is defined as a point $\mathbf{u}^\star \in \mathbf{U}$, where for all i, we have:

$$f_i(\mathbf{u}^\star) = \quad \min_{\mathbf{u}_i} f_i(\mathbf{u}_i, \mathbf{u}^\star_{i^-}) \tag{2.10a}$$

$$\text{s.t.} \; \mathbf{g}_i(\mathbf{u}_i, \mathbf{u}^\star_{i^-}) \leq \mathbf{0}. \tag{2.10b}$$

From here on, for simplicity, we will refer to such a point as a Nash equilibrium point, irrespective of whether there exist coupling constraints or not.

Simply stated, a Nash equilibrium of a GNEP is a point (i.e. a vector of collective actions) at which no player can, by a unilateral action, improve its objective without violating at least one of the coupled constraints.

One may argue that a generalized Nash game is "unnatural" since in a non-cooperative framework the players are expected to care only about their individual welfare and not about common limitations (constraints). However, this point of view appears to be rather limited, and severely undervalues [FK07]:

1. the descriptive and explanatory power of the GNEP model;

2. its normative value, i.e. the possibility to use GNEPs to design rules and protocols, set taxes and so forth, in order to achieve certain goals, a point of view that has been central to recent applications of GNEPs outside the economic field (see application below);

3. the fact that in any case different paradigms for games can and have been adopted, where, although in a noncooperative setting, there are mechanisms that make the satisfaction of the constraints possible.

As a matter of fact, the viewpoint expressed in the third point above has been adopted in this work when dealing with power system problems. The corresponding algorithms are presented in Chapter 3 for the PST control problem and Chapter 4 for the overlapping market problem.

Application (power allocation in a telecommunication system). This application, coming from the telecommunication field, is an example where the GNEP is used for its normative value. The considered problem is the power allocation in a Gaussian frequency-selective interference channel model [PSFW07]. In order to make the presentation self-contained and clear, we consider a simplified variant which, however, captures all the technical issues that are significant for our illustrative purposes.

Consider the Digital Subscriber Line (DSL) technology (a very common method for broadband internet access). DSL customers use a home modem to connect to a Central Office through a dedicated wire. In a standard setting, the wires are bundled together in a common telephone cable, at least in proximity of the Central Office. Due to electromagnetic couplings, the DSL signal in the wires can interfere with one another, causing a degradation of the quality of the service. The current standards prescribe the use of discrete multitone modulation which, in practice, divides the total available frequency band in each wire into a set of parallel subcarriers. In this setting, the parameter that can be controlled is, for each wire q and for each subcarrier k, the power p_k^q allocated for transmission. For each wire, the transmission quality is given by the maximum achievable transmission rate R_q.

This quantity depends both on the vector $\mathbf{p}^q = [p_1^q, \ldots, p_N^q]$ of power allocations across the N available subcarriers for wire q, and $\mathbf{p}^{q-}$, the vector representing the strategies of all the other wires (Q is the set of wires). Thus, $R_q = R_q(\mathbf{p}^q, \mathbf{p}^{q-})$.

In this setting, there is a single decision maker to decide the power allocation. This decision maker, loosely speaking, on the one hand wants to minimize the power employed while guaranteeing to each wire q a transmission rate of at least R_q^{min}. Telecommunication engineers have come to the conclusion that a desirable way to choose the power allocation is to take it as the equilibrium of a GNEP described below. Each wire q is a player of the game, whose objective function is to minimize the total power used in transmission, with the constraint that the maximum transmission rate is at least R_q^{min}, i.e. the problem of the generic player q is:

$$\begin{aligned} &\min_{\mathbf{p}^q} \sum_{k=1}^{N} p_k^q \\ \text{s.t.} \quad & R_q(\mathbf{p}^q, \mathbf{p}^{q-}) \geq R_q^{min} \\ & \mathbf{p}^q \geq \mathbf{0} \end{aligned}$$

We stress that here the GNEP is used in a normative way. No one is really playing a game; rather, a single decision maker has established that the outcome of the GNEP is desirable and therefore (calculates and) implements it. This perspective is rather common in many modern engineering applications of the GNEP.

It is interesting to note that the above technique fits in the context of Case D in Fig. 2.1. Instead of solving a multi-objective optimization problem, the central entity could simulate the execution of a game between the involved actors, taking into account the satisfaction of the coupled constraints. The difference, however, is that in the here-presented telecommunication example the players are "defined" by the central entity for the purposes of the allocation, while in the context considered in this work, the players, i.e. the various actors, are "self-existing" and may rather prefer to be left free to privately decide on and implement their strategies and behaviors.

2.3.4 Reaching a Nash equilibrium

In general, a game may have one or more Nash equilibria or even no equilibrium at all. Two classical topics in mathematical programming are, in fact, investigating the existence and uniqueness of Nash equilibria (e.g. [AD54]).

However, even if a unique Nash equilibrium is proven to exist, there is no guarantee that when the game is actually played by the players, it will finally converge to this equilibrium. Similarly, in a game with multiple equilibria, the game may not converge to any of them. Even when it does, one will not know in advance which equilibrium will be reached.

In its most general case, a game goes on as follows. The players, asynchronously with each other, based on their information sets, solve their decision-making problems and implement the corresponding actions. The actions taken by one player will generally modify the information sets thus triggering new actions by the other players and so on. At a Nash equilibrium, the information in each player's information set is such that the player has no motivation to take a new action (i.e. to modify its controls). Notice that, often, the information sets may contain observable quantities but not the others' control actions themselves. For example, in a PST control problem, a TSO may observe the active power flow in a line changing as a result of PST adjustments by the other TSOs without knowing what those actions are. Depending on their technology or strategy, different players may observe their environment and compute and take actions at different speeds.

Deviating from the above described general situation, one may easily imagine games where actions are taken in a synchronous manner at specific moments when all players compute and implement their actions. Additionally, a distinction can be made between games where the players act in parallel with each other and games where they act sequentially (one after another, each at its turn). Also, as a particular case, there may exist games where each player's information set coincides with all the players' already taken actions (i.e. at any moment, the players know the other players' control actions).

Figures 2.2 and 2.3 illustrate the difference between asynchronous and synchronous playing of a game, respectively, as well as between taking actions in parallel or sequentially.

All in all, a game may be played in different manners as regards the level and type of synchronization of the players' actions, as well as the information being available during its execution. Expectedly, the manner a game is played, in addition to its starting point will affect the equilibrium to which the game will converge. In addition, the speed at which a player is able to react is generally expected to affect the final outcome, since a change in that speed will change the whole "dynamics" of the game. However, it should be kept in mind that existence of equilibria does not guarantee convergence to one of them; the game may be trapped in a limit cycle situation (oscillating between two or more different collective control actions) or even progress endlessly without reaching a clearly observed pattern.

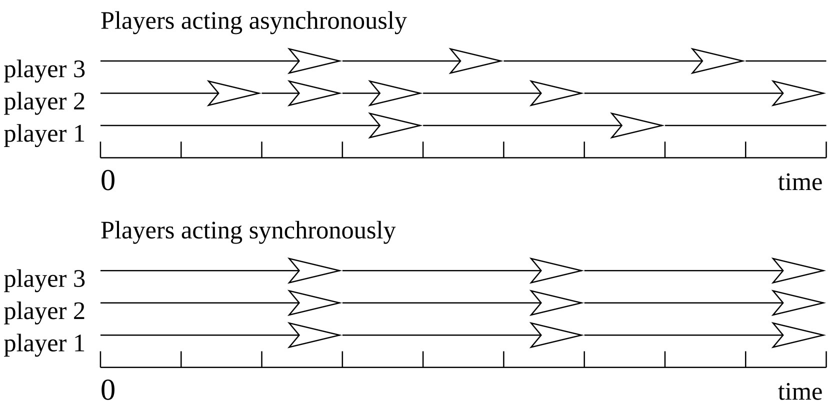

Figure 2.2: Asynchronous vs synchronous execution of a game.

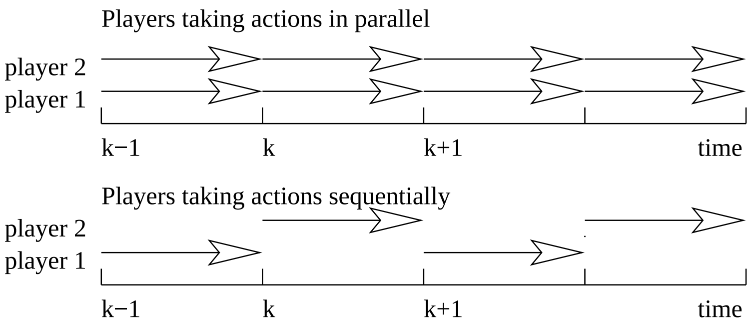

Figure 2.3: Parallel vs sequential execution of a game.

2.3.5 Illustrative example

A simple example, taken by [TC01], is used here to illustrate some of the above issues. Assume a two-person game, with respectively $p1$ and $p2$ the two players. They both control a scalar variable (u_1 and u_2) and minimize an objective function (f_1 and f_2). Both f_1 and f_2 depend on both control variables. The game is formed as follows:

$$p1: \quad \min_{u_1} f_1(u_1, u_2) \quad \text{s.t. } 0 \leq u_1 \leq 10 \qquad \Bigg| \qquad p2: \quad \min_{u_2} f_2(u_1, u_2) \quad \text{s.t. } 0 \leq u_2 \leq 10$$

where $f_1(u_1, u_2) = 44.76u_1^2 - 28.87u_1u_2 + 10.24u_2^2 - 150u_1 - 20u_2$
and $f_2(u_1, u_2) = 19.49u_1^2 - 34.48u_1u_2 + 25.51u_2^2 - 120u_1$.

Thanks to the simplicity of the example, one can compute what each player will play as a function of the other player's present action (we call this a player's reaction). For instance, when $p2$ plays $\widehat{u}_2$, $p1$ will choose $\widehat{u}_1 = \arg \min_{0 \leq u_1 \leq 10} f_1(u_1, \widehat{u}_2)$. Thus, $\widehat{u}_1$ is nothing but the point where the first derivative of f_1 with respect to u_1 becomes equal to zero, which gives:

$$\partial f_1(u_1, \widehat{u}_2)/\partial u_1|_{(u_1=\widehat{u}_1)} = 0 \Rightarrow 89.52\widehat{u}_1 - 28.87\widehat{u}_2 - 150 = 0 \Rightarrow \widehat{u}_1 = 0.32\widehat{u}_2 + 1.68. \quad (2.11)$$

In the same way, one can find that $p2$'s reaction is given by:

$$\widehat{u}_2 = 0.68\widehat{u}_1. \quad (2.12)$$

In Fig. 2.4, the two players' reactions as functions of the other player's action are shown graphically. The point $(u_1^\star, u_2^\star)$ where the two lines intersect is the (unique in this case) Nash equilibrium of the game. This collective action satisfies both (2.11) and (2.12).

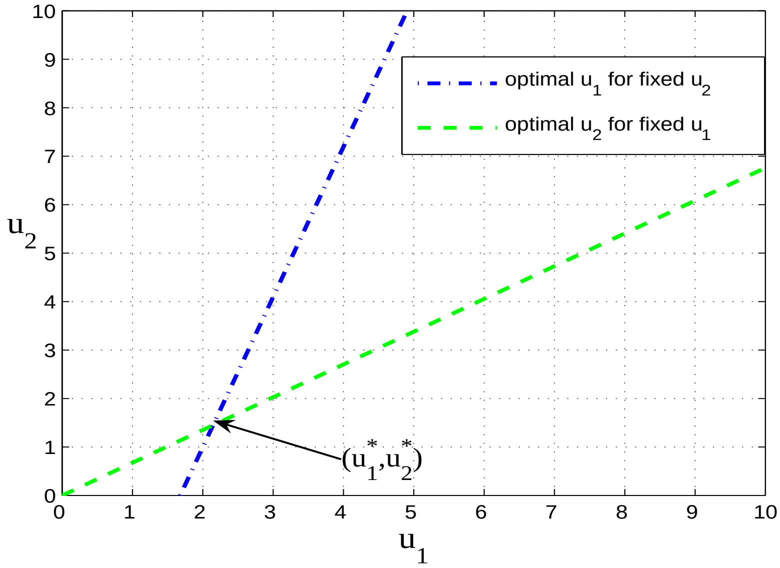

Figure 2.4: Players' reactions and Nash equilibrium of the game.

In Fig. 2.5, an execution of the game is shown. The two players set their control variable one after the other, as a response to the other player's action. These moves are illustrated with solid line. The end of each segment denotes an operating point, i.e. a collective action that is actually played. The rest of the segment does not correspond to any action, it just aims at visualizing the progress of the game by linking the different operating points. The starting point was $(\widehat{u}_1, \widehat{u}_2) = (9, 9)$ (right-up end of the solid line). The game is played sequentially and $p1$ is the first to act. It changes u_1 according to (2.11) bringing the operating point on the dash-dotted line (that gives the optimal u_1 for a given $\widehat{u}_2$). At its turn, $p2$ modifies u_2 according to (2.12) bringing the operating point on the dashed line (that gives the optimal u_2 for a given $\widehat{u}_1$). The game goes on like this, until the Nash equilibrium is reached. There, no player is motivated to change its control variable. This point can be easily computed in this simple example by solving (2.11) and (2.12). It is $(u_1^\star, u_2^\star) = (2.15, 1.46)$.

The same game can be played with the players acting in parallel; they both make and implement their decisions at the same time. Fig. 2.6 shows the first three steps of such an execution.

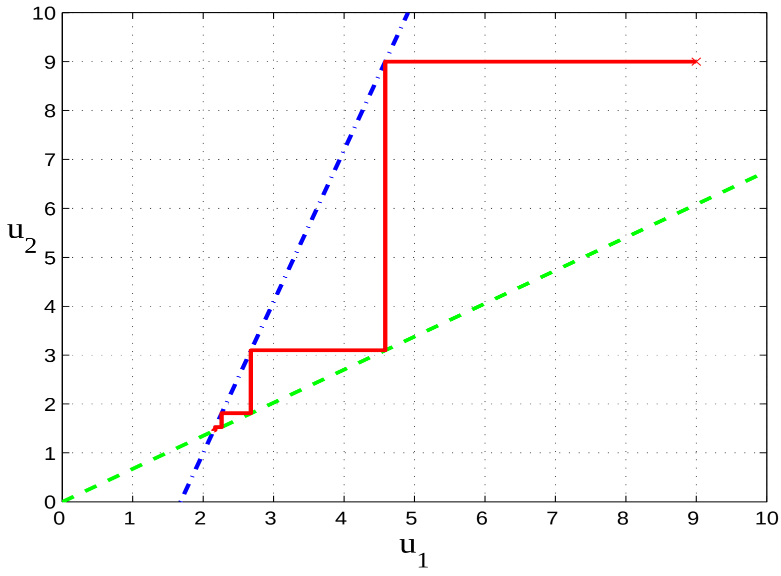

Figure 2.5: Playing sequentially until converging to the Nash equilibrium.

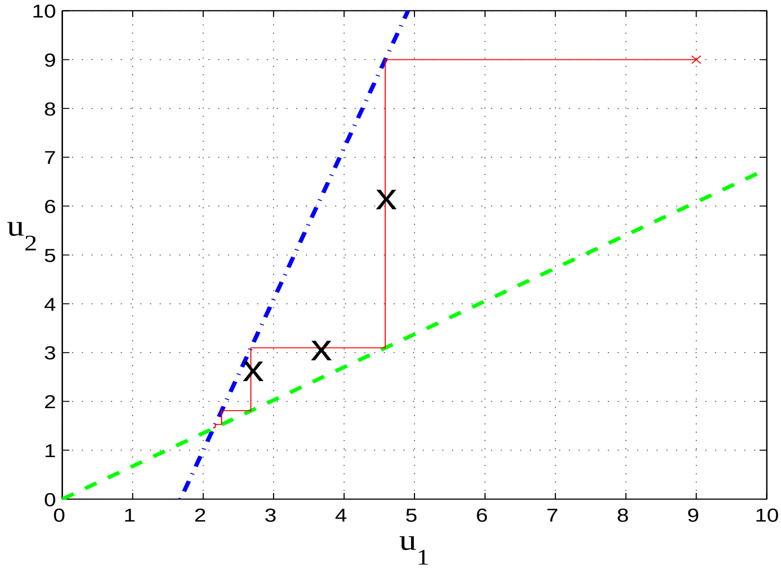

Figure 2.6: Playing in parallel until converging to the Nash equilibrium (the first three steps are shown with 'x's).

The 'x's correspond to operating points being actually implemented. For comparison, the solid line showing the progress of the sequential execution has been kept in the figure. Starting again from point $(9, 9)$ both players at the same time compute and implement their reactions according to (2.11) and (2.12). The collective action resulting from the two players' simultaneous actions corresponds to the operating point $(4.56, 6.12)$, shown with the upper-right 'x' in Fig. 2.6. Then, the players take new actions and so on up to convergence to the Nash equilibrium.

One can easily see that playing sequentially or in parallel changes the "dynamics" of the game; the operating point follows a different trajectory towards the Nash equilibrium.

It is worth noting that in this example the information set of each player consists of the other player's action.

Pursuing with the example in [TC01], let us now assume that there exist some constraints coupling the players' actions. Namely, let those constraints be: $u_1 + u_2 \geq 6$ and $u_1 + u_2 \leq 10$.

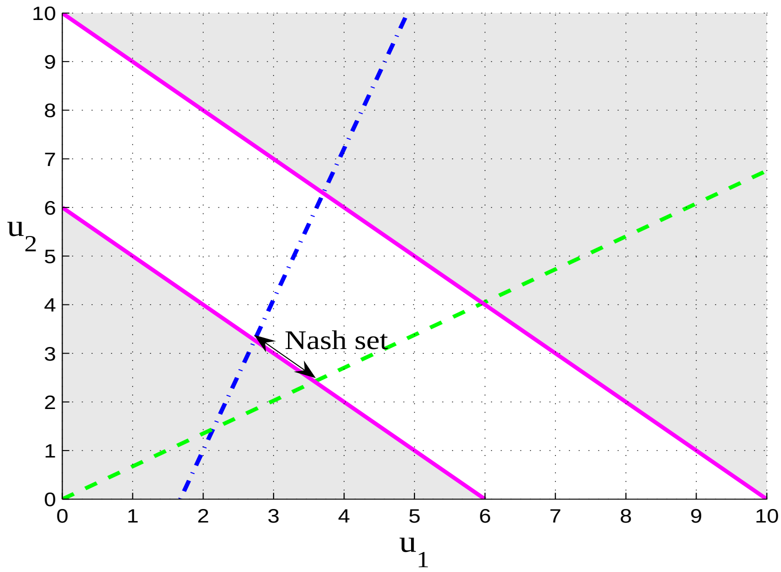

Figure 2.7: Players' reactions, feasible region and set of Nash equilibria of the game.

Fig. 2.7 is a reproduction of Fig. 2.4 where the feasible region, dictated by the two added inequality constraints, corresponds to the non-shaded area between the two sloping solid lines. In the new game, the intersection of the two players' reaction curves (i.e. the Nash equilibrium of the game without coupled constraints) is no longer acceptable since it violates one of the constraints. As a matter of fact, in this generalized Nash game there exist a whole set of collective actions that are all Nash equilibria of the game (making up the Nash set). This set is also illustrated in Fig. 2.7. The reader can easily check that at any point in the Nash set, no player can take an action that decreases the value of its payoff function without violating a constraint of the problem.

How the satisfaction of the two coupled constraints is guaranteed is an interesting issue. In this example, we assume that each player, when making a decision, limits itself not to violate any of those two constraints, given the actual value of the other player's control variable. In other words, the two players share these constraints, and they both include them in their decision-making problems, which become:

$$
\begin{array}{ll|ll}
p1: & \min_{u_1} f_1(u_1,u_2) & p2: & \min_{u_2} f_2(u_1,u_2) \\
\text{s.t.} & 0 \le u_1 \le 10 & \text{s.t.} & 0 \le u_2 \le 10 \\
 & u_1 + u_2 \ge 6 & & u_1 + u_2 \ge 6 \\
 & u_1 + u_2 \le 10 & & u_1 + u_2 \le 10
\end{array}
$$

One can notice that the new game is a generalized Nash game, since $\mathbf{U}_i = \mathbf{U}_i(u_j)$, $i, j = 1, 2$. Indeed $\mathbf{U}_i(u_j) = \{u_i : 0 \le u_i \le 10, u_i + u_j \ge 6, u_i + u_j \le 10\}$.

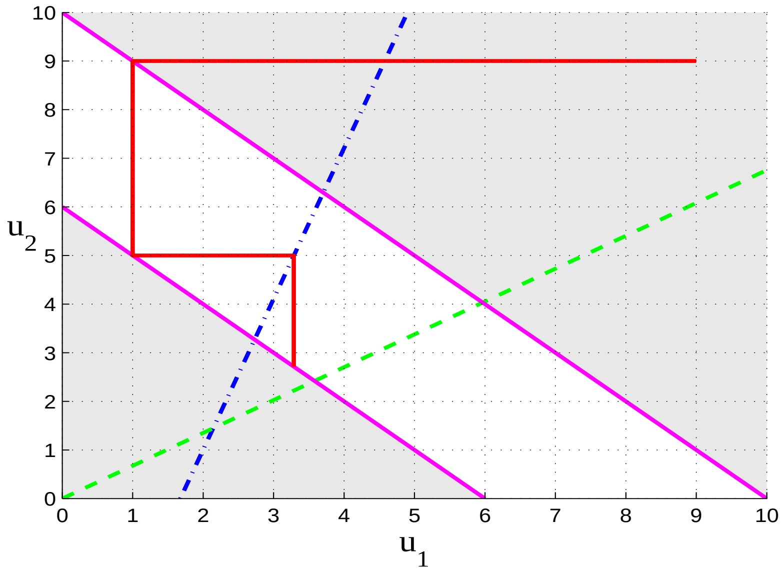

Figure 2.8: Playing sequentially until converging to one of the multiple Nash equilibria.

Playing the game sequentially under the above rule gives the sequence of moves presented in Fig. 2.8. The notation is as in Fig. 2.5 (players' actions) and Fig. 2.7 (feasible region). Notice that often the players are driven by the need to satisfy (1st move of $p1$) or not to violate (1st and 2nd move of $p2$) a constraint. The parts of dashed-dotted and dashed lines that are in the shaded area correspond to what $p1$ and respectively $p2$ would have played had it not been for the coupled constraints. Clearly, without a mechanism that guarantees the satisfaction of those constraints the game would have converged to the intersection of those reaction lines, which is not acceptable. On the contrary, the game shown in Fig. 2.8 converged to one of the points in the Nash set.

Figures 2.9 and 2.10 show another two executions of the same game, where, compared to Fig. 2.8, the order of moves has been inverted (see Fig. 2.9) or the starting point has changed (see Fig. 2.10). Again, one of the multiple Nash equilibria is attained, but different every time. This is due to the change in the "dynamics" of the game.

The results shown in Figs. 2.8, 2.9 and 2.10 are indicative of what was stated previously in this

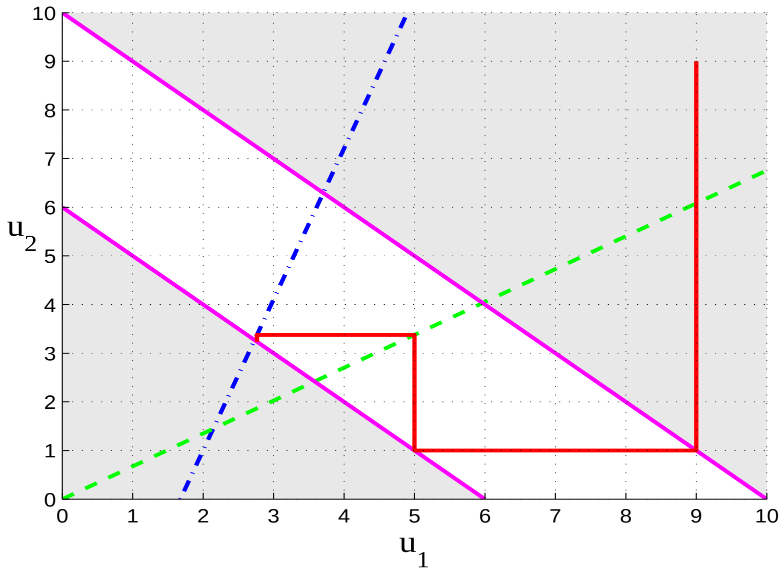

Figure 2.9: Playing sequentially, with $p2$ playing first, until converging to one of the multiple Nash equilibria.

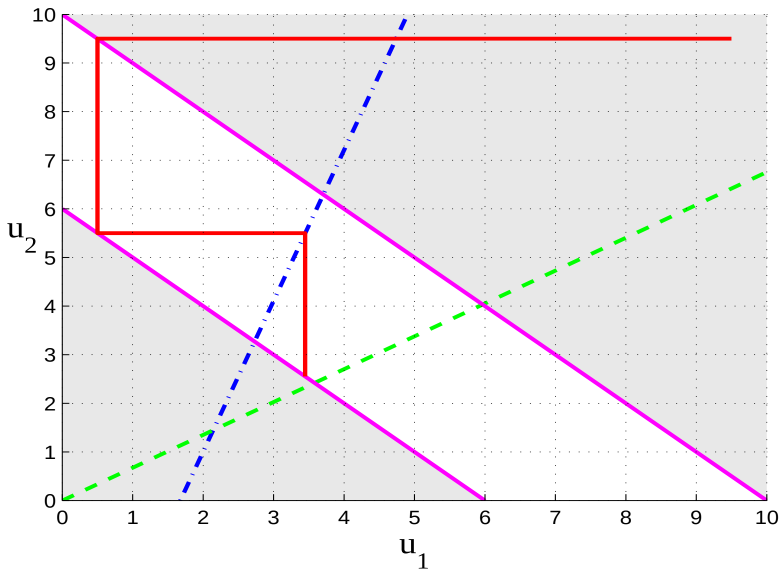

Figure 2.10: Playing sequentially, starting from (9.5,9.5), until converging to one of the multiple Nash equilibria.

section: "the manner a game is played, in addition to its starting point, affect the equilibrium to which the game will converge". The normal form of a game (see beginning of Section 2.3.2) is not enough to predict its final outcome.

Although simple, the example captures a very interesting situation. In the game played in

Fig. 2.9, $p2$, at its second move, plays $u_2 = 3.4$ minimizing f_2 given that $\widehat{u}_1 = 5$, as suggested by (2.12). However, just by looking at the figure, one can see that if $p2$ had played $\widehat{u}_2 = 2.5$ instead of 3.4 and then $p1$ had played trying to minimize its payoff function given the new $\widehat{u}_2$, then the game would have converged to a Nash equilibrium better for $p2$, i.e. where f_2 has a lower value. This illustrates a very common situation in games: it involves players trying to anticipate what the other players' strategies are (i.e. what actions they should be expected to take) and considering this when they choose their own actions. In other words, the players, each time they make a decision, instead of naively and myopically acting in a way that instantly minimizes their payoff functions, could *act strategically* envisaging the longer-term, more constant and stable, benefit, thus looking forward to driving the game towards the most profitable for them Nash equilibrium.

Let us now modify the example used throughout this section as follows:
$f_1(u_1, u_2) = 54.76u_1^2 - 98.87u_1u_2 + 10.24u_2^2 - 100u_1 - 20u_2$
and $f_2(u_1, u_2) = 19.49u_1^2 + 109.52u_1u_2 + 49.435u_2^2 - 120u_1 - 960u_2$.

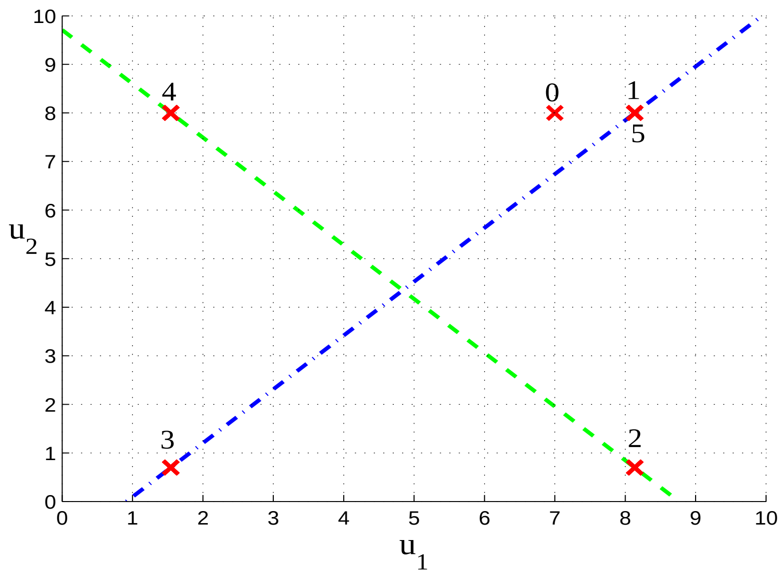

Figure 2.11: Limit cycle situation; the Nash equilibrium is never reached.

The resulting reactions, computed as in (2.11) and (2.12), are graphically shown in Fig. 2.11 (dash-dotted line for $p1$ and dashed line for $p2$). Coupled constraints are not considered in this example. The intersection of those two lines is the (unique) Nash equilibrium of this game.

In the same figure, an execution of the game, played sequentially with $p1$ making the first move, is also presented. The players' actions are shown with 'x's, where the numbers $0, 1, \ldots, 5$ refer to the sequence of action.

It is easily seen that the game does not converge to the Nash equilibrium. In fact, the players oscillate between 4 different operating points, making up a limit cycle. In particular, after the 4th move, $p1$ acts minimizing its payoff function and this brings the game at point '5' which coincides with the previously played point ‘1’. From then on, the game goes on repeating itself, i.e. point ‘6’ coincides with ‘2’, ‘7’ with ‘3’ and so on.

This example illustrates the fact that the existence of Nash equilibria does not guarantee convergence to one of them.

2.3.6 Computing Nash equilibria

For many applications, it is interesting or useful for an entity to compute beforehand the possible equilibria of a game (for example, the regulator of a market may want to check what the effect of a particular policy would be). Algorithms have been developed to this purpose. A good entry point to the related situation is [FK07]. In general, those algorithms take as input all players’ payoff functions and strategy spaces and, by solving sets of equations and/or optimization problems, they converge to a Nash equilibrium of the game. However, these algorithms rely on assumptions regarding the mathematical properties of the payoff functions (continuity, differentiability, etc.) and of the feasible sets (convexity, concavity, etc.). Thus they cannot be used for any game. Also, they generally do not make an exhaustive search for all possibilities; non convergence to an equilibrium does not mean that the latter does not exist. Finally, convergence to a Nash equilibrium does not provide information about the existence of others, while different starting points may lead to different Nash equilibria. It is probably safe to say that, as of writing this report, almost no algorithm can be shown to be globally convergent under clear or reasonable assumptions [FK07], in spite of a significant progress already made by researchers.

We outline hereafter some of the existing methods and cite some applications in the power system area.

We first consider *practitioners methods*, i.e. methods that are popular mostly among practitioners and whose rationale is easy to grasp. They are “natural” decomposition methods, be it of Jacobi- or Gauss-Seidel-type (with well-known counterparts in the case of systems of linear equations [SB02]).

Nonlinear Jacobi-type Method:

1. Choose a starting point $\mathbf{u}^0 = (\mathbf{u}_1^0, \ldots, \mathbf{u}_N^0)$ and set $k := 0$.
2. If $\mathbf{u}^k$ satisfies a suitable termination criterion, STOP.
3. For $i = 1, \ldots, N$: compute a solution $\mathbf{u}_i^{k+1}$ of
$$\min_{\mathbf{u}_i} f_i(\mathbf{u}_i, \mathbf{u}_{i^-}^k) \quad \text{s.t. } \mathbf{u}_i \in \mathbf{U}_i(\mathbf{u}_{i^-}^k).$$
4. Set $\mathbf{u}^{k+1} := (\mathbf{u}_1^{k+1}, \ldots, \mathbf{u}_N^{k+1})$, $k \leftarrow k + 1$ and go to step 2.

At each iteration k, N optimization problems have to be solved in step 3. The objective function $f_i(\mathbf{u}_1^k, \ldots, \mathbf{u}_{i-1}^k, \mathbf{u}_i, \mathbf{u}_{i+1}^k, \ldots, \mathbf{u}_N^k)$, $(i = 1, \ldots, N)$, has to be minimized over all $\mathbf{u}_i \in \mathbf{U}_i(\mathbf{u}_{i-}^k)$, whereas all block variables $\mathbf{u}_j^k$ of the other players $j \neq i$ are fixed. This algorithm does not use the newest information, since, when computing $\mathbf{u}_i$, we already have the new variables $\mathbf{u}_1^{k+1}, \ldots, \mathbf{u}_{i-1}^{k+1}$ and may use them instead of $\mathbf{u}_1^k, \ldots, \mathbf{u}_{i-1}^k$. Using these variables both in f_i and in the feasible sets, leads to the following Gauss-Seidel-type method.

<u>Nonlinear Gauss-Seidel-type Method:</u>

1. Choose a starting point $\mathbf{u}^0 = (\mathbf{u}_1^0, \ldots, \mathbf{u}_N^0)$ and set $k := 0$.
2. If $\mathbf{u}^k$ satisfies a suitable termination criterion, STOP.
3. For $i = 1, \ldots, N$: compute a solution $\mathbf{u}_i^{k+1}$ of

$$\begin{aligned} \min_{\mathbf{u}_i} \; & f_i(\mathbf{u}_1^{k+1}, \ldots, \mathbf{u}_{i-1}^{k+1}, \mathbf{u}_i, \mathbf{u}_{i+1}^k, \ldots, \mathbf{u}_N^k) \\ \text{s.t.} \; & \mathbf{u}_i \in \mathbf{U}_i(\mathbf{u}_1^{k+1}, \ldots, \mathbf{u}_{i-1}^{k+1}, \mathbf{u}_{i+1}^k, \ldots, \mathbf{u}_N^k). \end{aligned} \tag{2.13}$$

4. Set $\mathbf{u}^{k+1} := (\mathbf{u}_1^{k+1}, \ldots, \mathbf{u}_N^{k+1})$, $k \leftarrow k + 1$ and go to step 2.

With reference to the examples presented in Section 2.3.5, the iterations of the Jacobi-type method appear as the decisions taken by players acting in parallel, while those of the Gauss-Seidel-type method correspond to players acting sequentially.

In the case of a NEP (defined in Section 2.3.2), it is easy to prove [FK07] that if the entire sequence $\{\mathbf{u}^k\}$ generated by one of these methods converges to a point $\mathbf{u}^\star$, then $\mathbf{u}^\star$ is a Nash equilibrium of the NEP. Conditions which guarantee the convergence of the whole sequence $\{\mathbf{u}^k\}$, however, are typically not known or extremely restrictive. The situation becomes even more complicate for GNEPs where additional properties of the constraints are required in order to prove suitable convergence results.

The methods described above are straightforward and easy to implement, which explains their popularity among practitioners. However, they can be considered, at most, good and simple heuristics.

There exist more systematic methods to find Nash equilibria. For instance, a popular approach consists in writing down the Karush-Kuhn-Tucker first-order necessary optimality conditions [Kar39, KT51], for each player's optimization problem and then solving them altogether. The concatenation of all the players' KKT conditions gives the so-called *KKT conditions of the GNEP*.

Using the representation (2.9) and assuming that all functions involved are C^1 (differentiable with continuous derivatives), the KKT conditions for the ith player are:

$$\begin{aligned} \nabla_{\mathbf{u}_i} L_i(\mathbf{u}_i, \mathbf{u}_{i^-}, \boldsymbol{\lambda}_i) &= \mathbf{0} && (2.14a)\\ \mathbf{g}_i(\mathbf{u}_i, \mathbf{u}_{i^-}) &\leq \mathbf{0} && (2.14b)\\ \boldsymbol{\lambda}_i &\geq \mathbf{0} && (2.14c)\\ (\boldsymbol{\lambda}_i)_k \cdot (\mathbf{g}_i(\mathbf{u}_i, \mathbf{u}_{i^-}))_k &= 0 \qquad k = 1, \ldots, NC_i && (2.14d) \end{aligned}$$

where $L_i(\mathbf{u}, \boldsymbol{\lambda}_i) = f_i(\mathbf{u}) + \boldsymbol{\lambda}_i^T \mathbf{g}_i(\mathbf{u})$ is the Lagrangian associated with the ith player's optimization problem, $\nabla_{\mathbf{u}_i} L_i$ denotes the gradient of L_i with respect to $\mathbf{u}_i$, $(\boldsymbol{\lambda}_i)_k$ is the kth element of $\boldsymbol{\lambda}_i$, $(\mathbf{g}_i)_k$ is the kth element of $\mathbf{g}_i$ and NC_i is the number of constraints in the ith player's optimization problem.

Thus, the KKT conditions of the GNEP are given by:

$$\begin{aligned} \mathbf{L}(\mathbf{u}, \boldsymbol{\lambda}) &= \mathbf{0} && (2.15a)\\ \boldsymbol{\lambda} &\geq \mathbf{0} && (2.15b)\\ \mathbf{g}(\mathbf{u}) &\leq \mathbf{0} && (2.15c)\\ (\boldsymbol{\lambda})_k \cdot (\mathbf{g}(\mathbf{u}))_k &= 0 \qquad k = 1, \ldots, NC && (2.15d) \end{aligned}$$

where

$\boldsymbol{\lambda} = \left[\boldsymbol{\lambda}_1^T, \ldots, \boldsymbol{\lambda}_N^T\right]^T$,

$\mathbf{g}(\mathbf{u}) = \left[\mathbf{g}_1^T(\mathbf{u}), \ldots, \mathbf{g}_N^T(\mathbf{u})\right]^T$,

$\mathbf{L}(\mathbf{u}, \boldsymbol{\lambda}) = \left[\nabla_{\mathbf{u}_1}^T L_1(\mathbf{u}, \boldsymbol{\lambda}_1), \ldots, \nabla_{\mathbf{u}_N}^T L_N(\mathbf{u}, \boldsymbol{\lambda}_N)\right]^T$ and

NC is the total number of constraints in all the players' optimization problems.

System (2.15) can be regarded as a first order necessary condition for the GNEP. The following theorem [FK07] shows, provided some convexity assumptions hold true, that the $\mathbf{u}$-part of a solution of (2.15) is a solution of the GNEP, i.e. a Nash equilibrium of the game.

Theorem. Let a GNEP be defined by (2.9) and assume that all functions involved are continuously differentiable.
(a) Let $\mathbf{u}^\star$ be an equilibrium of the GNEP. Then, a $\boldsymbol{\lambda}^\star$ exists that together with $\mathbf{u}^\star$ solves (2.15).
(b) Assume that $(\mathbf{u}^\star, \boldsymbol{\lambda}^\star)$ solves (2.15) and that for every player i and every $\mathbf{u}_{i^-}$, $f_i(\mathbf{u}_i, \mathbf{u}_{i^-})$ is convex and $\mathbf{U}_i(\mathbf{u}_{i^-})$ is closed and convex. Then, $\mathbf{u}^\star$ is an equilibrium of the GNEP.

System (2.15) is a *Mixed Complementarity Problem* (MCP), i.e. a special variational inequality, for which efficient solvers are available [DF95, FP03, MFF$^+$01]. This paves the way to several approaches to the solution of GNEPs. Unfortunately this statement has to be immediately qualified, since the requirements that must be satisfied in order for those methods to convergence to a solution of the MCP are not easily applicable to the KKT system (2.15), and the conditions one obtains this way are rather unnatural in terms of the original GNEP and are not clear at all [FK07].

The above method of formulating and solving an MCP has been largely applied by the power system researchers trying to compute Nash equilibria of electricity markets. References [Hob01, Sme97] and several subsequent ones such as [BCG$^+$06, XYS04] formulate the market equilibrium conditions and solve the resulting MCP.

For instance, in [Hob01] two models of bilateral markets including a congestion management scheme for transmission are formulated as mixed Linear Complementarity Problems (LCP), a particular MCP formulation where the functions involved in the complementarity constraints are affine. Both models address a bilateral market in which imperfectly competitive generators purchase transmission services from an ISO who prices scarce transmission capacity in order to ration it efficiently. In terms of strategies, each generating company in both models plays a Nash game in quantities sold. This is equivalent to each generation company assuming that other firms will not alter their outputs, which is a case of Nash-Cournot game. In addition, each producer merely assumes that its outputs will not significantly affect transmission prices. In game-theoretic terms, this is a case of Bertrand game with respect to transmission. Given the above market and strategy assumptions, both models calculate a market equilibrium for generation and transmission. A market equilibrium is defined as a set of prices, generator outputs, transmission flows, and consumption that simultaneously satisfy each market participant's first order conditions for maximization of its profit while clearing the market (supply = demand). A solution satisfying those conditions possesses the property that no participant has incentive to alter its decisions unilaterally; it is a Nash equilibrium.

Reference [Sme97] concludes its survey of gas and electric market models by stating that explicit statement and solution of equilibrium conditions is a promising theoretical and computational approach to modeling strategic behavior. The models of this application are mixed LCPs as a result of using linear demand functions and marginal generation costs. Mixed LCPs involving thousands of variables and complementarity conditions can be solved using available LCP software, such as the MILES and PATH solvers within GAMS [GAM]. This permits application of strategic market models to large systems with thousands of power plants and hundreds of constrained transmission interfaces.

One may guess a relationship between the lower level problem of the MPECs mentioned in Section 2.3.5 and the here presented method of concatenating all the players problems' KKT conditions to formulate a MCP. In fact, if the players are solving MPECs instead of classical optimization problems to come up with their actions, then the problem of obtaining an equilibrium among such MPECs is called an *Equilibrium Problem with Equilibrium Constraints* (EPEC) [DS01]. Because the MPEC problem is generally non-convex, such an equilibrium might not exist or there might be multiple equilibria [HR04].

Other methods that are used to compute Nash equilibria include the "Nikaido-Isoda-function-type methods", the "penalty methods" and the "ODE-based methods" [FK07]. Some of them have been applied in the power system literature, like for example in [CKK04], where a Nikaido-Isoda-function method is presented for the calculation of Nash-Cournot equilibria in electricity markets.

2.4 Multi-objective optimization framework

The process of optimizing systematically and simultaneously a set of objective functions is called *multi-objective optimization* or *vector optimization* [MA04]. Typically, multi-objective optimization problems appear when a decision maker wants to optimize several objectives which are functions of its control variables. The case of interest is when those objectives are somewhat conflicting; setting the controls at optimal values for one objective deteriorates the others. Thus, the solution sought by the decision maker is an "optimal" compromise between its different objectives.

Several applications of multi-objective optimization appear in the power system literature and practice. Objectives that are typically combined and optimized include cost of active generation, social welfare, active losses, cost of reactive support, voltage profile, loading margin [BBIM01, BBM01, CL94, MCI03, RCQ03]. For example, in an electricity market environment, concerns of ensuring a fair market for participants and security of the system can lead to conflicting decisions for the system operator, since a reduced operating cost may not be achieved simultaneously with high security operating conditions [Rom06].

2.4.1 Pareto optimality

The general multi-objective optimization problem is posed as follows [MA04]:

$$\min_{\mathbf{u}} \mathbf{F}(\mathbf{u}) = [f_1(\mathbf{u}), f_2(\mathbf{u}), \ldots, f_N(\mathbf{u})]^T \tag{2.16a}$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{u}) \leq \mathbf{0} \tag{2.16b}$$

where N is the number of objective functions, $\mathbf{u}$ the vector of decision variables, $f_i(\cdot)$ the ith objective function, $\mathbf{g}(\cdot)$ the set of inequality constraints (possible equality constraints being implicitly taken care there) defining the *feasible set* or *feasible design space* $\mathbf{U} = \{\mathbf{u} : \mathbf{g}(\mathbf{u}) \leq \mathbf{0}\}$ and $\mathbf{F}$ is a vector containing all the objective functions. A *feasible* solution of the problem is a control vector $\mathbf{u} \in \mathbf{U}$. Finally, the set $\mathbf{Z}$ containing the values that $\mathbf{F}$ may take, $\{\mathbf{F}(\mathbf{u}) : \mathbf{u} \in \mathbf{U}\}$, is called the *attainable set*.

Notice that problem (2.16) is not well posed from the mathematical viewpoint (it is not a mathematical programming problem); it is rather an intuitive way to visualize with mathematical symbols the definition of the multi-objective optimization problem. However, a range of methods is available to convert the multi-objective formulation (2.16) into a substitute problem with a scalar objective (scalarization) that can be solved with the tools of single-objective optimization.

In contrast to single-objective optimization, a solution to a multi-objective problem [5] is more of a concept than a definition [MA04]. Typically, there is no single global solution and it is often necessary to determine a set of points that all fit a predetermined definition for an optimum.

[5] We use the terms multi-objective problem and multi-objective optimization problem interchangeably.

The predominant concept in defining an optimal point is that of *Pareto optimality* [Par06], which is defined as follows:

> **Definition.** A solution $\mathbf{u}^o$ of a multi-objective optimization problem is said to be *Pareto optimal* if it is feasible and is not dominated by any other feasible solution. This means that there is no other solution $\mathbf{u}'$ yielding at least one better objective function $f_i(\mathbf{u}')$ (i.e. $f_i(\mathbf{u}') < f_i(\mathbf{u}^o)$) without worsening any of the rest (i.e. satisfying that $f_{i^-}(\mathbf{u}') \leq f_{i^-}(\mathbf{u}^o)$, where index i^- denotes all objectives but the ith one).

For any given problem there may be an infinite number of Pareto optimal solutions, making up the *Pareto set*. When it comes to solving a multi-objective problem, one must distinguish between methods that provide the Pareto set or some portion of that set, and methods that actually seek a single final solution.

A simple method for determining whether a control vector $\mathbf{u}^o$ is Pareto optimal or not consists in solving the following single-objective problem [Ben78]:

$$\min_{\mathbf{u},\boldsymbol{\delta}} \sum_{i=1}^{N} \delta_i \tag{2.17a}$$

$$\text{s.t.} \quad f_i(\mathbf{u}) + \delta_i = f_i(\mathbf{u}^o) \qquad i = 1, \ldots, N \tag{2.17b}$$

$$\mathbf{g}(\mathbf{u}) \leq \mathbf{0} \tag{2.17c}$$

$$\boldsymbol{\delta} \geq \mathbf{0} \tag{2.17d}$$

If at the solution of problem (2.17) all δ_is are zero then $\mathbf{u}^o$ is Pareto optimal, otherwise it is not.

In terms of a global criterion F_g, [Sta88] presents the following sufficient condition for a Pareto optimal solution, which is useful for evaluating the effectiveness of a scalarization method:

> **Theorem.** Let $\mathbf{F} \in \mathbf{Z}$, $\mathbf{u}^o \in \mathbf{U}$, and $\mathbf{F}^o = \mathbf{F}(\mathbf{u}^o)$. Let a scalar global criterion $F_g(\mathbf{F}) : \mathbf{Z} \rightarrow \Re$ be differentiable with $\nabla_{\mathbf{F}} F_g(\mathbf{F}) > \mathbf{0}, \quad \forall\ \mathbf{F} \in \mathbf{Z}$. Assume $F_g(\mathbf{F}^o) = \min\{F_g(\mathbf{F}) : \mathbf{F} \in \mathbf{Z}\}$. Then, $\mathbf{u}^o$ is Pareto optimal.

The above theorem suggests that minimization of a global function $F_g(\mathbf{F})$ is sufficient for Pareto optimality if $F_g(\mathbf{F})$ increases monotonically with respect to each objective function. Furthermore, if $\mathbf{u}^o$ is Pareto optimal, then there exists a function $F_g(\mathbf{F})$ that satisfies the above theorem and captures $\mathbf{u}^o$ [MSM00]. If the Hessian of $F_g(\mathbf{F})$ with respect to $\mathbf{F}$ is negative definite, then the minimization of $F_g(\mathbf{F})$ is a necessary condition for Pareto optimality [AP96].

2.4.2 Utopia point

An alternative to the idea of Pareto optimality, which yields a single solution, is the idea of a *compromise solution* [Sal71a, Sal71b]. It entails minimizing the difference between the potential optimal solution and a *utopia point* (also called an *ideal point*)[6], which is defined as follows [VG81]:

Definition. A point $\mathbf{F}^{\triangle} = \left(f_1^{\triangle}, \ldots, f_i^{\triangle}, \ldots, f_N^{\triangle}\right)$ is a utopia point iff $\forall\ i = 1, \ldots, N$, it is $f_i^{\triangle} = \min\limits_{\mathbf{u} \in \mathbf{U}} f_i(\mathbf{u})$.

In general, $\mathbf{F}^{\triangle}$ is unattainable (if it was not, there would be no conflict between the objectives). The next best thing is a solution that is as close as possible to the utopia point. Such a solution is called a compromise solution and is Pareto optimal. A difficulty lies in the definition of "close". Usually it implies that one minimizes the Euclidean distance $D(\mathbf{u})$:

$$D(\mathbf{u}) = \left|\mathbf{F}(\mathbf{u}) - \mathbf{F}^{\triangle}\right| = \sqrt{\sum_{i=1}^{N} \left(f_i(\mathbf{u}) - f_i^{\triangle}\right)^2}$$

However, if the various objective functions have different units, they should be made dimensionless. A "robust" approach for doing this, regardless the original range of the objective functions, is called normalization and results in the following new objectives [KS87]:

$$f_i^{norm}(\mathbf{u}) = \frac{f_i(\mathbf{u}) - f_i^{\triangle}}{f_i^{max} - f_i^{\triangle}}$$

with $f_i^{max} = \max\limits_{1 \leq j \leq N} \left\{ f_i \left(\mathbf{u}_j^{\triangle} \right) \right\}$, where $\mathbf{u}_j^{\triangle}$ is the control vector that minimizes the jth objective function; $\mathbf{u}_j^{\triangle} = \arg \min\limits_{\mathbf{u} \in \mathbf{U}} f_j(\mathbf{u})$.

An interesting application of the above concept in the power system literature is [PBPE08]. There, the authors consider an interconnected power system where the various TSOs have agreed to transfer some of their controls to a centralized entity. The role of the latter is to come up with control decisions which are fair enough for every TSO. The resulting decision-making problem is formulated as a multi-objective problem, where each objective function corresponds to a TSO. An algorithm is proposed that selects the closest to the utopia point solution. To this purpose, the different objectives are normalized using some notions of fairness.

2.4.3 Methods for solving a multi-objective optimization problem

An excellent review of methods for solving multi-objective problems can be found in [MA04].

As a primary goal in multi-objective optimization is to model a decision maker's preferences (ordering of relative importance of objectives and goals), methods are categorized according to how the decision-maker articulates these preferences. This yields:

[6] The term "point" is used here referring to a specific set of values of the objective functions. Thus, a control vector $\mathbf{u}$ yields a point $(f_1(\mathbf{u}), \ldots, f_i(\mathbf{u}), \ldots, f_N(\mathbf{u}))$.

1. methods that involve *a priori articulation of preferences*, which implies that the user indicates the relative importance of the objective functions or desired goals before running the optimization algorithm;

2. methods with *a posteriori articulation of preferences*, which entail selecting a single solution from a set of mathematically equivalent solutions;

3. methods that require *no articulation of preferences*;

4. methods that involve a *progressive articulation of preferences*, in which the decision maker is continually providing input during the execution of the algorithm.

Most of the methods with a priori articulation of preferences (category 1 above) incorporate parameters (like coefficients, exponents, constraint limits, etc.) that can either be set to reflect the decision maker's preferences, or be continuously altered in an effort to represent the complete Pareto set. The best known method of this category is the *weighted sum method*, which consists in minimizing a positively weighted convex sum of all the objective functions:

$$\min_{\mathbf{u}} \sum_{i=1}^{N} w_i f_i(\mathbf{u}) \tag{2.18a}$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{u}) \leq \mathbf{0} \tag{2.18b}$$

where $w_i \geq 0$ are scalar-valued weights. They are typically given values in the interval $[0, 1]$.

The values of the weights define the importance given by the decision maker to each of the objectives and are selected in advance. The theorem in Section 2.4.1 suggests that any solution of (2.18) is Pareto optimal. Clearly, the relative values of the different w_i have a significant effect on the solution of (2.18), in terms of which element of the Pareto set will be chosen. Various systematic approaches have been developed to select weights [Hob80, HY81]. However, it should be kept in mind that an a priori selection of weights does not necessarily guarantee that the final solution will be acceptable (the decision maker may for example realize that some objectives were too much/little taken into account); sometimes, the problem has to be solved again with new weights.

The weighted sum problem may be also solved repeatedly with different sets of weights. In this manner the decision maker learns about available trade-offs between the satisfaction of its objectives and is offered a selection of candidate solutions. An approximation of the Pareto set can be obtained in this way. It should be noted however that this method succeeds in getting points from all parts of the Pareto set only when the Pareto set is convex [DD97] and that a uniform spread of weight parameters rarely produces a uniform spread of points in the Pareto set [DD97].

Other methods with a priori articulation of preferences include: the weighted global criterion method, the lexicographic method, the weighted min-max method, the exponential weighted criterion, the weighted product method, the goal programming methods, the bounded objective function method and, finally, the physical programming.

Methods with a posteriori articulation of preferences (category 2 above), on the other hand, allow the decision maker to choose from a palette of solutions. To this end, an algorithm is used to determine a representation of the Pareto set. Although an approximation of the Pareto set can be obtained by repeatedly solving a weighted formulation (like in the weighted sum method), this may be inefficient in providing points that uniformly sample and significantly represent the complete Pareto set. Methods dedicated to this task are the physical programming, the normal boundary intersection (NBI) method [DD98] and the normal constraint (NC) method [MIYM03]. These methods provide means for obtaining an even distribution of Pareto optimal points and, thus, allow the decision maker to view options before making a decision.

Clearly, when presenting solutions in tabular form, selecting a single solution can be an intimidating task in the presence of a relatively large number of objectives, variables, or solution points. Consequently, methods with a posteriori articulation of preferences are best suited to problems with a relatively small number of objectives.

2.5 Relationship between generalized Nash games and Multi-objective Optimization problems

Game theory models the interactions among different actors who are taking interdependent decisions in a common environment. The actors typically take their individual decisions by solving single-objective optimization problems. On the other hand, multi-objective optimization is a tool that allows one to set its control variables in a way that simultaneously optimizes a set of conflicting objectives.

At first glance, the two fields seem totally different from each other. And, in fact, they generally serve completely different purposes. However, it is noteworthy that they share the same basic situation: multiple, partially conflicting objectives are simultaneously optimized. To some extent, this allows a game to be treated as a multi-objective problem.

This section, built on the related analysis presented in [TC01], considers a game from a multi-objective viewpoint. This allows quantifying the “quality” of the game’s Nash equilibria.

In [TC01] the authors describe a situation where multiple actors make control decisions in a common environment. This typical game, after some iterations, settles down to a Nash equilibrium. On the other hand, supposing that the actors work together on the aggregate of their individual problems (they can do this either by placing themselves under centralized control, or by abandoning competition and self-interest in favor of cooperation and altruism), Pareto optimal solutions can be obtained.

More precisely, let us consider a generalized Nash game where each player’s decision problem is described by (2.9). Putting all the players’ objective functions and all constraints under a common umbrella, one can come up with a single decision maker multi-objective problem, generally described by (2.16). There, the elements of the vector objective function $\mathbf{F}$ are the individual objective functions f_i ($i = 1, \dots, N$), while the feasible set **U** is the aggregation of all players' feasible sets. From now on, we call this the *corresponding multi-objective problem of a game*.

Revisiting the example from reference [TC01] developed in Section 2.3.5, we formulate the corresponding multi-objective problem of the game as:

$$\min_{u_1,u_2} w_1 f_1(u_1,u_2) + w_2 f_2(u_1,u_2) \tag{2.19a}$$

$$\text{s.t.} \quad 0 \leq u_1 \leq 10 \tag{2.19b}$$

$$0 \leq u_2 \leq 10 \tag{2.19c}$$

where $f_1(u_1,u_2) = 44.76u_1^2 - 28.87u_1u_2 + 10.24u_2^2 - 150u_1 - 20u_2$
and $f_2(u_1,u_2) = 19.49u_1^2 - 34.48u_1u_2 + 25.51u_2^2 - 120u_1$.
A weighted-sum scalarization of the vector objective function is used, with $w_1 > 0$, $w_2 > 0$ and $w_1 + w_2 = 1$.

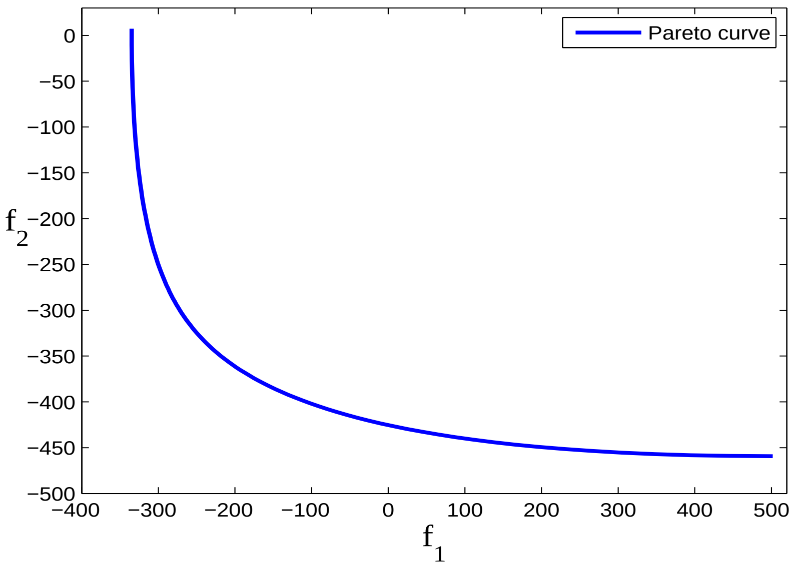

Figure 2.12: Pareto set of the corresponding multi-objective problem.

Solving problem (2.19) repeatedly for different values of w_1 and w_2 (sampling the entire range) the Pareto set of the multi-objective problem is sketched in Fig. 2.12. This graphical representation of the Pareto set is typically named as *Pareto curve*[7] or *Pareto front*. It shows the different Pareto optimal trade-offs that can be attained for the two objectives. Each point on the figure corresponds to a combination of (f_1, f_2) values, i.e. it belongs to the objective functions' space[8]. By definition of Pareto optimality, all the points that are located left and below of the Pareto curve do not belong to the attainable set **Z** defined in Section 2.4.1. Points that are located right and above the Pareto curve are generally attainable (this, of course, depends

[7] Pareto surface for more than two objective functions.

[8] The figures presented in Section 2.3.4 were all in the control variables' space.

on the values that u_1 and u_2 can take). Any attainable point in Fig. 2.12 (whether or not on the Pareto curve) results from a feasible collective action $(\widehat{u}_1, \widehat{u}_2)$[9].

A collective action (u_1^o, u_2^o) that yields a point on the Pareto curve in Fig. 2.12 is from now on called a Pareto optimal operating point of the game. If the outcome of the game was such a Pareto optimal point, by definition, this would mean that no other collective action exists at which at least one player's objective is decreased without increasing any of the other players' objectives. On the contrary, if a collective action yields a point at the right-upper side of the Pareto curve in Fig. 2.12, this means that another collective action could be played where at least one of the players' objectives is decreased without an increase in any of the remaining objectives.

It is straightforward to conclude that if the outcome of a game is not a Pareto optimal point, then the players can, by a coordinated collective action, modify altogether their controls and reach another operating point where they are all better-off than previously[10].

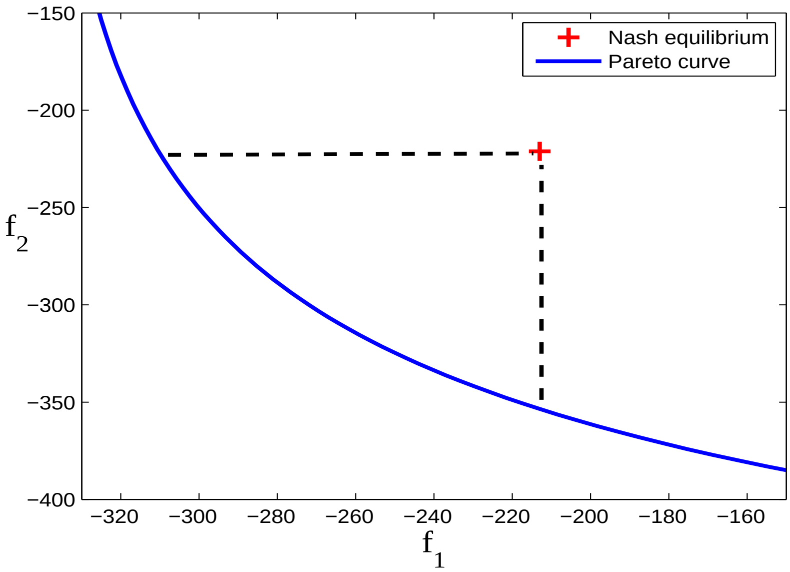

Figure 2.13: Nash equilibrium of the game with respect to the Pareto set of the corresponding multi-objective problem.

For instance, coming back to our example, Fig. 2.13 shows where the Nash equilibrium of the game (without the coupled constraints) is situated in the objective functions' space. A zoom on the Pareto curve (close to the Nash point) is also shown in the same figure. It can be seen that the Nash equilibrium of the game is not Pareto optimal. In fact, there exist a whole set of feasible points for which both actors obtain a better result (smaller objective function value) than if the collective action yielding the Nash equilibrium is played. In Fig. 2.13, these points are all those located between the Pareto curve and the two dashed lines. Both players would be willing to properly modify their control variables in order to reach one of those points.

[9]Note that the figure does not provide information on the action $(\widehat{u}_1, \widehat{u}_2)$ for each point.
[10]To be precise, there may be cases where one or more (but surely not all) players are equally well, not better.

In fact, it is not a coincidence of the example but rather a general property: the Nash equilibria of a game do not, in general, coincide with Pareto optimal solutions of the corresponding multi-objective problem. This means that, given a Nash equilibrium $\mathbf{u}^\star$, there exist collective actions yielding, for all actors, more satisfactory objective values than what $\mathbf{u}^\star$ does.

Unfortunately, the above mentioned Pareto optimal points are not self-enforcing, since they are not Nash equilibria of the original game. This means that if, at a given step of the game, the players' control vectors have values corresponding to a Pareto point, at least one of the players will be in position, by a unilateral action, to decrease the value of its objective, given the others' control vector values. Necessarily, this improvement of a player's objective will result in deteriorating at least another player's objective, otherwise the operating point under question would not have been a Pareto optimal one.

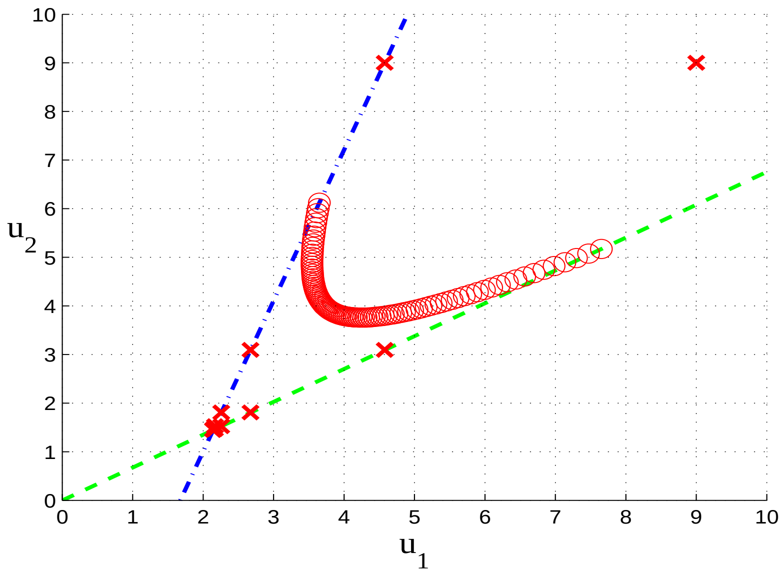

Figure 2.14: Collective actions giving the Pareto points of the corresponding multi-objective problem.

Returning to the control variables' space representation, Fig. 2.14 shows the values that give the Pareto curve of Figs. 2.12 and 2.13. Each 'o' in the figure corresponds to a point of the Pareto curve. The players' reaction curves (dashed-dotted and dashed lines) as well as the sequence of operating points played towards the Nash equilibrium ('x's) are also presented in the same figure. One can easily verify the statements made in previous paragraphs: 1. The (unique here) Nash equilibrium is not a Pareto optimal point. 2. None of the Pareto optimal points is a Nash equilibrium of the game; indeed, starting from whichever of those points the game will, after some moves, converge to the Nash equilibrium.

It is interesting to repeat the exercise for the second version of the example presented in Section 2.3.4, where some coupled constraints are added to the original Nash game. In the same way as in Fig. 2.14, Fig. 2.15 shows the collective actions that correspond to Pareto optimal points.

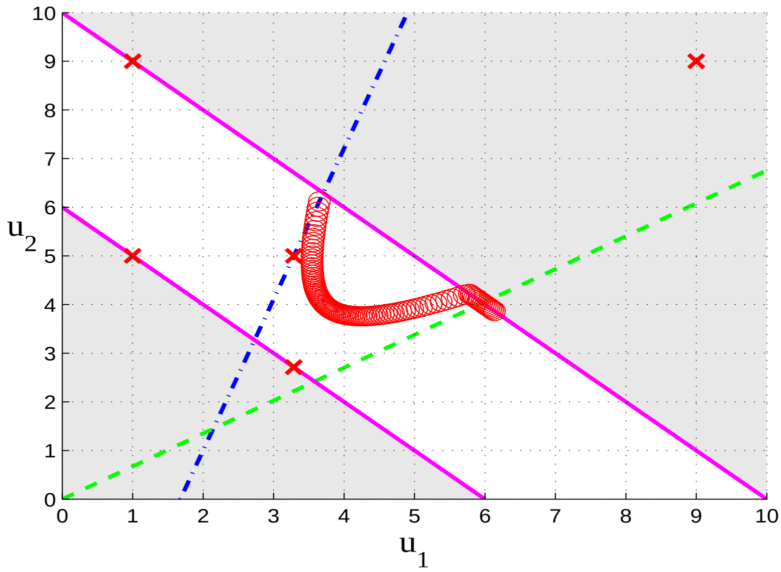

Figure 2.15: Collective actions giving the Pareto points of the corresponding multi-objective problem (example with coupled constraints).

A comparison of Figs. 2.14 and 2.15 suggests that one of the additional constraints causes the Nash set to move closer to the Pareto set. In other words, additional rules (regulations), expressed as constraints, if carefully designed, could move the Nash equilibria closer to the Pareto set [TC01]. This is in accordance to the economists' knowledge that the efficiency of a completely free market (their version of an unregulated game) can be improved by the addition of regulations. In [TC01], the authors suggest that in designing these regulations it is helpful to represent them as constraints in the actors' problems.

For instance, Table 2.2 compares the values of the objective functions for various collective actions. The first is the unique Nash equilibrium of the first example, while the remaining are three of the multiple Nash equilibria of the second example. Precisely, they are the equilibria reached after the executions of the game shown respectively in Figs. 2.8 (which is the same case as in Fig. 2.15), 2.9 and 2.10. It can be seen that in all three cases with coupled constraints both actors are better-off compared to the unconstrained Nash equilibrium. Worth mentioning is also the fact that at each of the three Nash equilibria of the same game (lines 3 to 5 in the table) a different trade-off is encountered between the two objectives; $p1$ would prefer the equilibrium $(2.77, 3.23)$ while $p2$ would favor $(3.45, 2.55)$.

The final outcome of a game, i.e. the reached Nash equilibrium, corresponds to a certain satisfaction for each actor. From a specific actor's perspective, different Nash equilibria can be easily compared. However, it is less obvious to classify Nash equilibria from a "neutral observer's" viewpoint.

Naming A, B and C the equilibria in the third, fourth and respectively fifth line of Table 2.2 and using the symbol $>$ to denote an order of preference, for $p1$ it is $B > A > C$ while for $p2$ it

Table 2.2: Objective functions' values for various Nash equilibria

Game played	$u_1^\star$	$u_2^\star$	$f_1(u_1^\star, u_2^\star)$	$f_2(u_1^\star, u_2^\star)$
without coupled constraints (Fig. 2.5)	2.15	1.45	-213.0	-221.1
with coupled constraints (Fig. 2.8)	3.29	2.71	-245.7	-303.7
with coupled constraints (Fig. 2.9)	2.77	3.23	-288.3	-224.3
with coupled constraints (Fig. 2.10)	3.45	2.55	-223.2	-319.4

is $C > A > B$. If a neutral observer had to make a choice among the three, it would not have a clear criterion to prefer one to another (we can see that for whichever combination of equilibria, the two actors would always disagree on what would satisfy them more). Maybe an instinctive choice would be A, as a compromise solution. But even this choice is not supported, while it does not even generalize well; if a fourth candidate solution D is such that $B > A > D > C$ for $p1$ and $C > D > A > B$ for $p2$, what should be the choice, A or D?

Borrowing the terminology from the multi-objective optimization literature, we say that a Nash equilibrium $\mathbf{u}^\star$ *strictly dominates* another Nash equilibrium $\mathbf{u}^{\star'}$ if for all the involved players $i = 1, \ldots, N$ we have $f_i(\mathbf{u}^\star) < f_i(\mathbf{u}^{\star'})$. We also say that $\mathbf{u}^\star$ *weakly dominates* (or just *dominates*) $\mathbf{u}^{\star'}$ if for at least one player i we have $f_i(\mathbf{u}^\star) < f_i(\mathbf{u}^{\star'})$ while for each of the remaining players j it is $f_j(\mathbf{u}^\star) \leq f_j(\mathbf{u}^{\star'})$. Clearly, dominated Nash equilibria can be classified as worse than those which dominate them. In the same line of reasoning, all players have interest to avoid converging to a strictly dominated equilibrium.

Since the outcome of a game (and hence the satisfaction of the players) depends, in part, on the rules of that game (if any), or, as it is called in [TC01], the players' *organization*, one should seek an organization such that convenient (at least non dominated) equilibria are reached. Even more, if a game converges to an equilibrium which is not Pareto optimal, the organization should be modified so that the new attracting equilibrium lies in the region of the objectives' space defined by the Nash equilibrium and the Pareto front as sketched in Fig. 2.13. The term organization is used in a general sense. It embeds the order and synchronization at which players take actions, the exchange and/or share of information between players, the existence of a coordinating entity or supervising authority, or even the level at which the players agree to import in their own problems, objectives and constraints from the problems of the other players (*altruism*) and the level at which they agree to cede some control of their decision variables to others (*deference*).

As explained in [TC01], the organizational possibilities may lie in a range from a completely unregulated game, where actors are autonomous and work asynchronously, unrestricted by rules or regulations and driven by self-interest, to a "totalitarian regime" (centralized operation), where actors have no autonomy; they only execute instructions stemming from a central planner. In between these two extremes, there is a continuum of increasingly regulated games. The role of regulations is to allow actors to compete along some dimensions while ensuring that they cooperate along others. In other words, they "require the actors to temper their self-interest with altruism" [TC01].

2.6 Final discussion

The problems dealt with in this work conceptually belong to the family of GNEPs; different actors optimize their controls while there exists a set of constraints, coupling all the actors' control decisions, that should be respected. These constraints stem from the operation and feasibility limits of the transmission grid.

Coordination is needed to, at least, make sure that the reached operating point is a feasible one. As stated in Sections 2.3.3 and 2.3.4, a mechanism is needed to ensure that the coupled constraints are satisfied. With reference to Fig. 2.8 for example, one can see that this mechanism should prevent the actors from converging to the intersection of the two reaction curves, which violates one of the coupled constraints. So, mechanisms for coordination between the different actors are sought. Of course, the coordination needs not necessarily be limited in ensuring the satisfaction of constraints, it could also embed some efficiency and fairness objectives, as is the case in the algorithm developed for the overlapping market problem.

Merging the various actors' decision problems into a large central one seems a tempting solution since it handles all constraints and interactions (it can be seen as the ultimate level of coordination between the actors). However, several practical disadvantages of such a centralization exist, as listed in the Introduction of this report.

More specifically, let us say two words about the choice of centralization from the particular viewpoint of each separate problem. In the case of the PST control problem , the single (common) objective would be to operate the grid in a secure way. The main inertia force against the benefits of centralization is the fact that each system is generally financially self-supported (example of countries in Europe), so TSOs do not trust ceding their control to a central decision-maker as each of them fears that this could result in itself eventually paying for the security of another TSO's system. In the case of the overlapping market problem it can be an objective by itself to allow different markets (thus, market structures) to co-exist. This issue is further developed and discussed in Chapter 4.

In the already cited reference [PBPE08], the authors use notions of efficiency, fairness and accountability to set up a single-objective optimization problem solved by a central decision-maker to solve the Mvar scheduling problems of several TSOs. There, the original TSO objectives are of different nature (maximizing reactive reserves vs. minimizing system losses). However, it seems still difficult to devise one or more metrics to judge whether a centralized scheme is acceptable by the various involved actors.

The direction followed in this work is that of allowing the various actors to, simultaneously and independently of each other, solve their decision-making problems and come up with their sought actions. A set of obligations related to the actors' decisions is imposed by a centrally operated coordinator to reconcile those independent decisions.

More precisely, coordination is achieved via the use of a common network model and via constraints that actors should be obliged to satisfy when solving their decision problems. The commonly shared models allow checking that each actor fulfils its obligations. On the contrary, *the decision-making problems themselves remain undisclosed. Furthermore, each actor may have its own objective and use its own operation procedures.*

Creating and sharing a model of the entire network has been considered acceptable by the involved TSOs. In fact, it is the present trend, at least in Europe, to come up with such large-scale models and use them for various security reasons [VP05]. In particular, the ongoing PEGASE project (Pan European grid advanced simulation and state estimation), supported by the 7th Framework Program of European Union and involving a group of TSOs, companies and research centers, is dealing with state estimation, optimal power flow and dynamic simulation at the European level [PEG, SKCW08].

Chapter 3

Control of phase shifting transformers by multiple transmission system operators

In this chapter, a general framework is proposed for the control of Phase Shifting Transformers (PSTs) owned by several TSOs, taking into account their interactions. The proposed solution is the Nash equilibrium of a sequence of optimizations performed by the various TSOs, each of them taking into account the other TSOs' control settings as well as operating constraints relative to the whole system. The method is applied to a linearized network model and illustrated on the IEEE 118-bus system.

3.1 The phase shifter and its use

The possibility of controlling power flows by PSTs (or "phase shifters" as they are also called), and thus increasing the utilization of the bulk power system, was recognized long ago [Lym30, Lym38, Blu51].

3.1.1 Description of PST operation

A PST allows to introduce a phase angle shift of the voltage (and current) at its ends. Phase shifting is implemented by a parallel connected three-phase transformer, which generates a quadrature component of voltage. This can be inserted into the line via a series connected boosting transformer. An on-load tap changer can be used to change the value of the quadrature component in order to obtain the corresponding variation of the voltage phase angle [Han82]. Fig. 3.1 illustrates such a set-up for one of the three phases of a transmission line. A fraction of the voltage taken between two phases is added to the voltage between the third phase and the neutral.

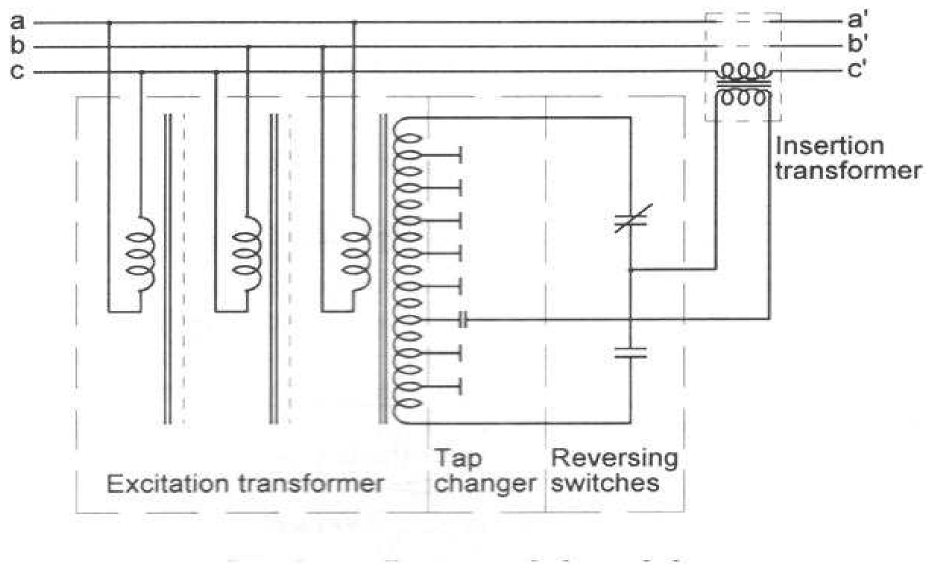

Figure 3.1: Phase Shifting Transformer

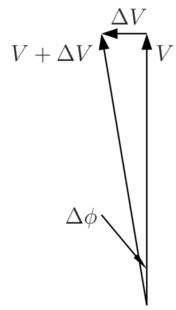

Figure 3.2: Phase angle change by a PST, one-phase diagram

Fig. 3.2 shows, in a one-phase diagram, the effect of a PST operation. One can see that a considerable phase angle shift ($\Delta\phi$ in the figure) can be obtained with a very small change of the voltage magnitude (denoted by the arrows' lengths in the figure). Note that there exist more elaborate schemes, allowing to introduce a voltage phase angle shift without affecting voltages magnitudes.

If a PST installed in a branch (connecting bus i with bus j) introduces a phase angle difference $\Delta\phi$ at its ends, this will result in a new bus angle difference $\theta'_j - \theta'_i = \theta'_j(\Delta\phi)$[1], depending on the system's electrical parameters.

Let us call an "ideal" PST a phase shifter for which two assumptions are made: 1. its reactance is zero, and 2. it can incur a phase angle change $\Delta\phi$ without introducing any change in the voltage magnitude. Inspired by the illustration in [Elg71], the effect of PST operation is presented hereafter by means of a simple example.

[1]We take bus i as the reference bus.

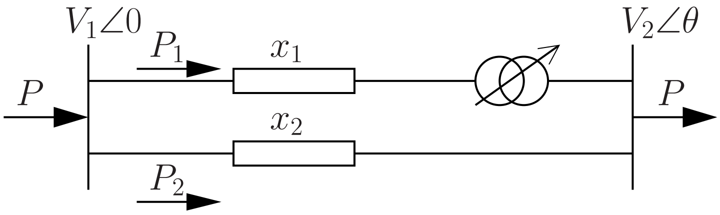

Figure 3.3: Example of PST operation: before the PST action.

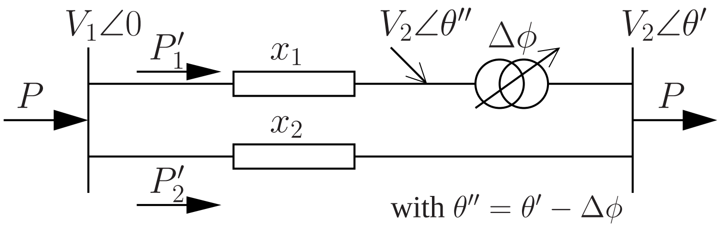

Figure 3.4: Example of PST operation: after the PST action.

Suppose an ideal PST installed in a transmission line 1 in Figs. 3.3 and 3.4. The line's resistance is neglected while its reactance is x_1. The parallel line 2 (see figures), with reactance x_2, is supposed to equivalently represent all the alternative paths (i.e. except line 1) in the network that connect the two buses. The network carries an active power P from bus 1 to bus 2.

Before the PST operation ($\Delta\phi = 0$, Fig. 3.3) P was divided between line 1 and the rest of the network (line 2) as follows:

$$P_1 = \frac{x_2}{x_1 + x_2}P \tag{3.1a}$$

$$P_2 = \frac{x_1}{x_1 + x_2}P \tag{3.1b}$$

After the introduction of $\Delta\phi$ (Fig. 3.4), the new line flows are given by the following equations (where, for simplicity, the DC approximation $\sin x \simeq x$ is made for all angles):

$$P_1' = \frac{V_1 V_2}{x_1}(\theta' - \Delta\phi) \tag{3.2a}$$

$$P_2' = \frac{V_1 V_2}{x_2}\theta' \tag{3.2b}$$

Since we assume an ideal PST, the voltage magnitude remains the same.

From (3.2), and using the fact that $P = P_1 + P_2$, the following line power flows result:

$$P_1' = \frac{x_2}{x_1 + x_2}P - \frac{V_1 V_2}{x_1 + x_2}\Delta\phi \tag{3.3a}$$

$$P_2' = \frac{x_1}{x_1 + x_2}P + \frac{V_1 V_2}{x_1 + x_2}\Delta\phi \tag{3.3b}$$

Comparing (3.1) with (3.3), one can see that a modification of the PST angle by $\Delta\phi$ caused a variation of power flow

$$\Delta P_1 = -\frac{V_1 V_2}{x_1 + x_2}\Delta\phi \tag{3.4}$$

in line 1. This power flow is redistributed through the remaining of the network. Clearly, if no alternative path exists for the power to flow from bus 1 to bus 2, then $x_2 \rightarrow \infty$ and $\Delta P_1 = 0$.

Also, from Eqs. (3.2) and (3.3), using the expression of the power flow before the PST action, $P_1 = \frac{V_1 V_2}{x_1}\theta$ or $P_2 = \frac{V_1 V_2}{x_2}\theta$, we end up with the following expression relating the phase angle difference $\theta' - \theta$ with the phase shift $\Delta\phi$:

$$\theta' - \theta = \frac{x_2}{x_1 + x_2}\Delta\phi \tag{3.5}$$

The above expression shows that, as expected, in the absence of a path parallel to line 1 ($x_2 \rightarrow \infty$) the entire PST shift $\Delta\phi$ is seen as a voltage phase angle change $\theta' - \theta$. On the contrary, the more parallel paths exist (i.e. $x_2 \rightarrow 0$), the more θ is held unchanged.

Although the effect of PST was illustrated using some approximations (ideal PST, $\sin x \simeq x$), the above presented results hold qualitatively true in general. If a PST installed in branch b introduces a phase angle difference $\Delta\phi$ at its ends, then the branch flow will be modified by an amount Δp_b approximately proportional to $\Delta\phi$, provided that there exist alternative paths connecting the ends of branch b. How sensitive Δp_b is to $\Delta\phi$ depends on the network topology, parameters and, in general, operating point. Noteworthy here is the fact that in this work as well as in other studies [Mar05, VHS+08], it has been observed that a linear approximation of the PST effects on active power flows is very accurate.

3.1.2 Scope of PST control: literature review

Due to the above-described flow redirection capability, phase shifters are often installed in branches that are deemed being at risk of getting overloaded and are operated according to the simple rule: if the branch gets (or tends to get) overloaded, adjust the PST phase angle so that the overload is cleared thanks to the resulting flow redistribution. This is a “local” control strategy, where the PST actions are independent of the system security and economic considerations. Another similar local control strategy consists in predefining the desired active power flows (typically called MW schedules) in the branches where PSTs are installed and adjust the PST settings such that the sought flows are imposed.

Alternatively, or sometimes in addition to a local control strategy, the PSTs that are installed in a system can be controlled altogether, in a coordinated way, as it has been (already) briefly suggested in Chapters 1 and 2, in order to enhance system security [CBC+02, MZBH01] or/and to facilitate economically beneficial transactions, in this way improving the economic follow-up for the market players [MC04].

Ref. [MZBH01] presents an integrated “OPF with PST” approach to enhance power system security by removing line overloads. The problem dealt with in that paper stems from the fact that, on one hand, the general OPF calculations are hourly based and the OPF control variables are continuous, while, on the other hand, the PST control calculations are daily based and the variables related to PSTs are discrete. To address this problem, the paper develops a scheme where PST control is incorporated into a rule-based OPF. In order to effectively alleviate the line overloads, the ranking of phase shifter locations is conducted based on contingency analysis and sensitivity analysis.

In the same spirit, Ref. [CBC+02] presents a methodology to include optimization of the PSTs MW schedules in security constrained scheduling applications. The approach advocated differs from conventional OPF in that it involves iterations between two modules: a schedule optimizer and a security monitor. The data passed from the security monitor to the schedule optimizer are critical security constraints, characterized by sensitivities of line flows to PST control actions and limits. The schedule optimizer calculates the minimum cost dispatch of generator resources and price-sensitive loads, while simultaneously satisfying both the control variable constraints and system constraints. Representative test results obtained from a security constrained unit commitment are shown to demonstrate the economic benefits and effectiveness of the developed methodology. It is shown that optimal setting of the PSTs MW schedules can significantly reduce the system operational cost.

The PST control problem, defined as the solution of an OPF-type optimization problem, incorporating proper modeling of the PST control effect, in order to come up with the selected PST tap settings, is formulated and solved for different objective functions in [Mar05]. A Model Predictive Control-inspired approach is developed in that work, implementing a control scheme where real-time PST adjustments keep the objective function at its optimal level despite normal (variation of load) or emergency (equipment outages) changes that modify the power system operating state.

An interesting coordinated PST control approach is presented in [MC04]. The proposed methodology consists in setting the phase shifter angles such that the overall transfer capacity is increased towards the most economically valuable directions. A mechanism is also presented so that the phase shifter owners are remunerated in proportion to the extra benefit created by the optimal setting of their phase shifters.

All in all, thanks to the introduced phase shifts, PSTs offer the opportunity to partially control the flows in a power system. They are one of the principal control devices used to direct power flows in specific parts of the transmission network. Although the operation of PSTs incurs maintenance costs and losses, it remains less costly than generation rescheduling and definitely preferred to load shedding [HMB+91]. Finally, it is one of the controls, together with topology changes, that fully remain in the hands of TSOs.

Within the just described perspective, several TSOs, in Europe noticeably, equip their networks with more and more PSTs. Most of them are located to remove congestions on important lines, typically tie-lines between countries, which are usual “bottlenecks” [BSA+04].

With reference to Fig. 3.5, the two PSTs can be controlled in a coordinated way to reduce the fraction of power flow passing through the network $\mathcal{N}$ as a result of the transaction from G to

D. More PSTs are likely to be installed for increased control of transit flows, as testified by the situation in Belgium, where three PSTs have been put in operation on the Northern border of the country [VHS+07, VHS+08].

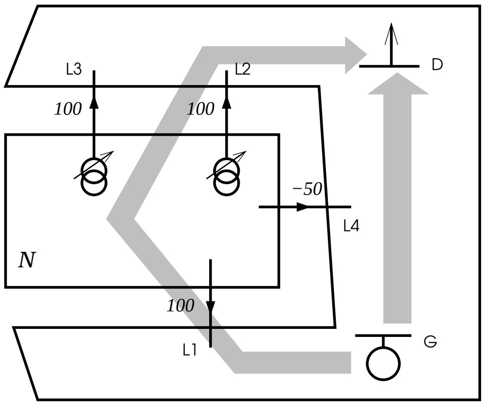

Figure 3.5: Transit flow due to external transaction

This situation is further analyzed in Appendix A, where an algorithm for real-time PST control is presented. The control scheme aims at using the PSTs of an area to restore security by decreasing as little as possible the transit flow passing through that area. The motivation behind the developed algorithm is that the control should balance between two somewhat conflicting objectives: be non intrusive to the rest of the interconnection on one hand, but protect the area under question from unscheduled flows stemming from transactions in other areas of the interconnection on the other hand.

3.1.3 Control of PSTs by multiple TSOs

In a large meshed interconnection, PSTs can impact active power flows in far-away distances. As a result, PST control actions taken by one TSO will generally affect the operation of the other TSOs' systems. In some future, these interactions might lead to dangerous conflicting situations, for instance if one TSO prevents transit power flows from passing through its system. Such "fights" are obviously undesirable, not only from market viewpoint [BK97], but above all for security of operation.

The optimal solution from a technical viewpoint would probably be a central "entity" coordinating the various PSTs so as to reach a global objective. However, TSOs may not be open to such a solution in which they would partly lose control on equipments they acquired to improve their own system. Thus, the viewpoint adopted here is more to allow each TSO to have its own objective, while avoiding conflicts that would endanger security.

To this purpose, in the remaining of this chapter, first a general framework is outlined, in which multiple objectives, each relative to a particular TSO, are optimized under a set of common security constraints. While it is assumed that information is shared by the partners in order to avoid violating those constraints, each TSO is supposed to keep its objective undisclosed. The formulation leads to solutions that altogether constitute Nash equilibria of the overall procedure. This general formulation is then particularized to the problem of PST control, for which linear programming is used by each TSO.

3.2 General multi-TSO optimization framework

3.2.1 Uncoordinated game between TSOs

We consider an environment in which each TSO uses its own controls to optimize an individual objective, all of them operating the same interconnected system. For simplicity, we refer to a case with two TSOs, named TSO1 and TSO2, with $\mathbf{u}_1$ and $\mathbf{u}_2$ their respective vectors of control variables. Let us also group together into a set of inequality constraints whatever the operation of the whole interconnection has to obey:

$$\mathbf{g}(\mathbf{u}_1, \mathbf{u}_2) \leq \mathbf{0} \tag{3.6}$$

This set of constraints is nothing but a very general way to express all security, operational, physical and other limits that the involved TSOs should respect when optimizing their systems.

The set $\mathbf{g}(\cdot) \leq \mathbf{0}$ is naturally decomposed according to the involved TSOs, into:

$$\mathbf{g}_1(\mathbf{u}_1, \mathbf{u}_2) \leq \mathbf{0} \tag{3.7a}$$

$$\mathbf{g}_2(\mathbf{u}_1, \mathbf{u}_2) \leq \mathbf{0} \tag{3.7b}$$

In other words, (3.6) is made up of all the involved TSOs individual set of constraints. Those sets shall be in the largest part distinct, except maybe some common constraints, typically involving tie-line flows, that can be duplicated in both $\mathbf{g}_1$ and $\mathbf{g}_2$ without loss of generality. Clearly, many constraints in (3.7a) and (3.7b) are expected not to depend on the other TSO's controls; they are all expressed in the same way though, to keep the presentation simpler and more homogenous.

Each TSO has its own objective function to be minimized. We denote them by $f_1(\mathbf{u}_1, \mathbf{u}_2)$ and $f_2(\mathbf{u}_1, \mathbf{u}_2)$, respectively. These objectives may be quite different, but we assume that each function is influenced by the whole set of controls, which is the expression of the already mentioned TSO interactions. Note that the decision-making procedure may be more complex than just solving a mathematical programming problem: it could be heuristic, or it could involve additional computations, dealing for instance with post-contingency security constraints. The latter, for instance, has been considered in the algorithm presented in Appendix A.

In the worst case, when there is no level of coordination among the TSOs, each TSO solves a problem including its own control variables only, the rest being explicitly or implicitly set to some constant value, and focusing on its own operating constraints only. In this perspective, TSO1 computes:

$$\widehat{\mathbf{u}}_1 = \arg\min_{\mathbf{u}_1} f_1\left(\mathbf{u}_1, \mathbf{u}_2^0\right) \tag{3.8a}$$

$$\text{subject to} \qquad \mathbf{g}_1\left(\mathbf{u}_1, \mathbf{u}_2^0\right) \leq \mathbf{0} \tag{3.8b}$$

where $\mathbf{u}_2^0$ is the value of $\mathbf{u}_2$ assumed by TSO1[2]. A similar set of equations holds for TSO2, which ends up with a solution $\widehat{\mathbf{u}}_2$. Since each TSO ignores the other TSO's control actions, the operating point resulting from these uncoordinated changes is likely to differ from what each TSO model predicts. More importantly, the solution may not be feasible, since $\mathbf{g}(\widehat{\mathbf{u}}_1, \widehat{\mathbf{u}}_2) \leq \mathbf{0}$ does not necessarily hold true.

As discussed in Chapter 2, each TSO i could be expected to modify its control action according to (3.8), updating $\mathbf{u}_{i-}^0$ based on its observations. If $\mathbf{u}_{i-}$ is observable, then the new $\mathbf{u}_{i-}^0$ will be the $\widehat{\mathbf{u}}_{i-}$ stemming from the other's solution of (3.8), otherwise $\mathbf{u}_{i-}^0$ will be estimated based on the observation of changed quantities, such as branch flows. In general, a TSO will modify its controls each time there is a modification triggered by the other TSOs. Nothing guarantees convergence of such iterations. Furthermore, even when they do converge to a Nash equilibrium, this is likely to happen together with some of the following disadvantages:

1. the system may be operated for some time, during the iterations, in an emergency state, i.e. with some of the constraints in (3.6) not respected;
2. too much control effort (with possible associated cost) may be wasted during the iterations, each TSO annulling the other's action.

3.2.2 Coordinated iterative procedure for multi-TSO optimization

A more "responsible and coordinated" scheme is considered in this work. It relies on the following rules:

1. each TSO provides information on its operating constraints;
2. each TSO takes into account the whole set of operating constraints;
3. each TSO communicates its current preferred control settings (i.e., $\widehat{\mathbf{u}}_i$ for the ith TSO), which are taken into account by the other TSOs;
4. they iterate until an equilibrium is reached.

The first item suggests that the TSOs collaborate and exchange the necessary information to jointly construct the set (3.6). This goes with the prerequisite considered in this thesis that the TSOs are willing to put their efforts together in this direction (see Section 2.6).

[2]This could also be denoted as $\mathbf{u}_{1-}^0$ according to the notation introduced in Chapter 2.

The second and third items aim at achieving a level of coordination among the TSOs. In the multi-TSO optimization presented in this chapter, this coordination is fully oriented towards guaranteeing the system's security, at any moment; thus the second item. The third item may look too strong a prerequisite at first glance: why should a TSO announce its control action? Its usefulness is justified by the fact that it allows the TSOs to correctly take into account the constraints (see second item), while it facilitates the verification, at any moment, that any TSO is respecting its obligations (i.e., it does not take an action that violates the common constraints). Not having the TSOs announcing their actions complicates the coordination, without, at least, really protecting some sense of confidentiality; the network model transparency, included in the construction of (3.6), is there to make sure that one TSO's control actions are "seen" by the others.

If, however, confidentiality issues are raised, a consensus has to be reached about the minimal amount of data to be communicated, withholding sensitive pieces of information, so as to render it commercially neutral for instance. On the other hand, a TSO should be able to justify the security constraints it announced (if requested to do so by a regulatory body, for instance).

Finally, the fourth item stems from another prerequisite followed in this work: the TSOs' decision-making procedures should remain undisclosed. This, as discussed in Chapter 2, excludes the solution of solving an overall single optimization for the whole interconnection and leaves the choice of having the TSOs iterate in order for them to altogether optimize their objectives.

It is important to point out that, for the moment, we make the assumption that each TSO has the controllability to satisfy all the constraints, whatever the action of the other is. This may not always be the case in practice. This issue of controllability is illustrated and commented later on in this chapter, as well as in the forthcoming Chapter 4.

Under the above assumptions, at the kth iteration of the procedure, TSO1 knows the current preferred value $\mathbf{u}_2^k$ of TSO2 controls[3]. Using this information, it updates its own preferred solution according to:

$$\begin{aligned} \mathbf{u}_1^{k+1} &= \arg\min_{\mathbf{u}_1} f_1\left(\mathbf{u}_1, \mathbf{u}_2^k\right) \\ \text{subject to} \quad & \mathbf{g}_1\left(\mathbf{u}_1, \mathbf{u}_2^k\right) \leq 0 \\ & \mathbf{g}_2\left(\mathbf{u}_1, \mathbf{u}_2^k\right) \leq 0 \end{aligned} \tag{3.9}$$

TSO2 carries out a similar computation, ending up with the updated solution $\mathbf{u}_2^{k+1}$. Both values are used at the next iteration.

If convergence is achieved, the final solution reached is:

$$\begin{aligned} \text{for TSO1:} \quad \mathbf{u}_1^\star &= \arg\min_{\mathbf{u}_1} f_1\left(\mathbf{u}_1, \mathbf{u}_2^\star\right) \\ \text{subject to} \quad & \mathbf{g}_1\left(\mathbf{u}_1, \mathbf{u}_2^\star\right) \leq 0 \\ & \mathbf{g}_2\left(\mathbf{u}_1, \mathbf{u}_2^\star\right) \leq 0 \end{aligned} \tag{3.10}$$

[3]The $\widehat{\ }$ symbol is omitted since the iteration index k is enough to show that the vector is the solution of a TSO's decision problem.

$$\text{and for TSO2:} \quad \mathbf{u}_2^\star = \arg\min_{\mathbf{u}_2} f_2\left(\mathbf{u}_1^\star, \mathbf{u}_2\right) \tag{3.11}$$
$$\text{subject to} \quad \mathbf{g}_1\left(\mathbf{u}_1^\star, \mathbf{u}_2\right) \leq 0$$
$$\mathbf{g}_2\left(\mathbf{u}_1^\star, \mathbf{u}_2\right) \leq 0$$

where all security constraints are satisfied.

One can recognize that the proposed scheme makes up a GNEP (see Section 2.3.3). In other words, the TSOs play a generalized Nash game. The solution $(\mathbf{u}_1^\star, \mathbf{u}_2^\star)$ is a Nash equilibrium of this game. The rule that is chosen to guarantee the satisfaction of the coupled constraints consists in all the players committing themselves to include those constraints in their decision-making problems. Each TSO may be viewed as a self-interested player acting towards optimizing its objective, all of them obeying the whole set of operating constraints. The information set of each player contains all the players' actions according to the third rule stated at the beginning of this section.

Of course, the convergence of the above procedure and the existence of several Nash equilibria remain questions of interest.

3.3 Application to PST control problem

As already mentioned, we consider an environment in which each TSO uses its PSTs to optimize an individual objective. To model the effect of PST control on the network we adopt the well-known DC approximation, which is acceptable for the problem of concern and leads to an insightful linear problem. The PSTs are considered as ideal; no voltage magnitude change with PST tap change is assumed. The impedance of a PST installed in series with a transmission line is considered fixed and is included into the line's impedance. Further efforts could be directed towards updating the operating constraints when large PST angle excursions take place as well as adjusting the PST impedances with the tap position.

Under the DC approximation, the active power flows in transmission lines can be linearized around a base case operating point, according to:

$$\mathbf{p} = \mathbf{p}^0 + \mathbf{S}\left(\boldsymbol{\varphi} - \boldsymbol{\varphi}^0\right) \tag{3.12}$$

where $\mathbf{p^0}$ is the base case value of active power flows $\mathbf{p}$, and similarly for $\boldsymbol{\varphi}^0$ with respect to the PST angles $\boldsymbol{\varphi}$. The sensitivity matrix $\mathbf{S}$ can be easily derived from the DC (or even AC) load flow equations using a well-known general sensitivity formula involving the inverse transposed Jacobian of the power flow equations [PPTT68]. The limits on branch power flows take on the form:

$$-\overline{\mathbf{p}} \leq \mathbf{S}\left(\boldsymbol{\varphi} - \boldsymbol{\varphi}^0\right) + \mathbf{p}^0 \leq \overline{\mathbf{p}} \tag{3.13}$$

where $\overline{\mathbf{p}}$ is a vector of maximum branch power flow.

Proceeding with the two-TSO example, these inequalities can be decomposed into:

$$-\overline{\mathbf{p}}_1 - \mathbf{p}_1^0 \leq \mathbf{S}_{11}\left(\boldsymbol{\varphi}_1 - \boldsymbol{\varphi}_1^0\right) + \mathbf{S}_{12}\left(\boldsymbol{\varphi}_2 - \boldsymbol{\varphi}_2^0\right) \leq \overline{\mathbf{p}}_1 - \mathbf{p}_1^0 \tag{3.14a}$$

$$-\overline{\mathbf{p}}_2 - \mathbf{p}_2^0 \leq \mathbf{S}_{21}\left(\boldsymbol{\varphi}_1 - \boldsymbol{\varphi}_1^0\right) + \mathbf{S}_{22}\left(\boldsymbol{\varphi}_2 - \boldsymbol{\varphi}_2^0\right) \leq \overline{\mathbf{p}}_2 - \mathbf{p}_2^0 \tag{3.14b}$$

where the notation is self-explanatory. To sketch out each TSO's participation in each set of constraints, the sensitivity matrix is decomposed into four parts (namely $\mathbf{S}_{11}$, $\mathbf{S}_{12}$, $\mathbf{S}_{21}$ and $\mathbf{S}_{22}$).

According to what was presented in Section 3.2, the two TSOs will compute a sequence of PST settings according to $(k = 1, 2, \ldots)$:

$$\text{for TSO1:} \qquad \boldsymbol{\varphi}_1^{k+1} = \arg\min_{\boldsymbol{\varphi}_1} f_1\left(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2^k\right) \tag{3.15}$$

$$\begin{aligned} \text{s.t.} \quad & -\overline{\mathbf{p}}_1 - \mathbf{p}_1^0 \leq \mathbf{S}_{11}\left(\boldsymbol{\varphi}_1 - \boldsymbol{\varphi}_1^0\right) + \mathbf{S}_{12}\left(\boldsymbol{\varphi}_2^k - \boldsymbol{\varphi}_2^0\right) \leq \overline{\mathbf{p}}_1 - \mathbf{p}_1^0 \\ & -\overline{\mathbf{p}}_2 - \mathbf{p}_2^0 \leq \mathbf{S}_{21}\left(\boldsymbol{\varphi}_1 - \boldsymbol{\varphi}_1^0\right) + \mathbf{S}_{22}\left(\boldsymbol{\varphi}_2^k - \boldsymbol{\varphi}_2^0\right) \leq \overline{\mathbf{p}}_2 - \mathbf{p}_2^0 \\ & \boldsymbol{\varphi}_1^{min} \leq \boldsymbol{\varphi}_1 \leq \boldsymbol{\varphi}_1^{max} \end{aligned}$$

$$\text{and for TSO2:} \qquad \boldsymbol{\varphi}_2^{k+1} = \arg\min_{\boldsymbol{\varphi}_2} f_2\left(\boldsymbol{\varphi}_1^k, \boldsymbol{\varphi}_2\right) \tag{3.16}$$

$$\begin{aligned} \text{s.t.} \quad & -\overline{\mathbf{p}}_1 - \mathbf{p}_1^0 \leq \mathbf{S}_{11}\left(\boldsymbol{\varphi}_1^k - \boldsymbol{\varphi}_1^0\right) + \mathbf{S}_{12}\left(\boldsymbol{\varphi}_2 - \boldsymbol{\varphi}_2^0\right) \leq \overline{\mathbf{p}}_1 - \mathbf{p}_1^0 \\ & -\overline{\mathbf{p}}_2 - \mathbf{p}_2^0 \leq \mathbf{S}_{21}\left(\boldsymbol{\varphi}_1^k - \boldsymbol{\varphi}_1^0\right) + \mathbf{S}_{22}\left(\boldsymbol{\varphi}_2 - \boldsymbol{\varphi}_2^0\right) \leq \overline{\mathbf{p}}_2 - \mathbf{p}_2^0 \\ & \boldsymbol{\varphi}_2^{min} \leq \boldsymbol{\varphi}_2 \leq \boldsymbol{\varphi}_2^{max} \end{aligned}$$

where bounds on control variables have been added to branch flow constraints. These PST angle bounds could correspond to physical limits, such as (a) maximum and minimum angles that can be enforced by the PST or (b) maximum and minimum angle changes that the PST can introduce within a defined time step. They could also correspond to "computational" limits stemming from coordination demands of the procedure, as clarified later in this chapter.

Several objective functions may be thought of, such as minimum deviation of controls from base case values, minimum active power losses (using an extension of the above DC model), minimum deviation from a desired value of power flowing through a set of branches, etc. An example is the algorithm for real-time PST control that has been developed aside of the main workline of this thesis and is presented in Appendix A. Thus, the optimization procedure may be more complex than shown above, the point being that each TSO takes into account the other TSO's controls and the whole set of operating constraints.

To implement the above ideas, information should be exchanged through a network of TSO computers, first to build the model, then to exchange PST setting values until convergence is reached. Before starting the iterations, the power flow Jacobian matrices of each system have to be sent to a central computer, in order to be assembled into a single Jacobian $\mathbf{J}$, subsequently factorized. The $\mathbf{S}$ matrix can be computed column by column; each column requires solving a sparse linear system with $\mathbf{J}$ as matrix of coefficients, and an independent term stemming from the individual TSO systems. Each TSO must also provide the value of its base case and maximum power flows. From there on, optimizations of the type (3.15,3.16) can be performed independently by the TSOs, with an exchange of the $\boldsymbol{\varphi}_1^k, \boldsymbol{\varphi}_2^k$ PST settings in between iterations.

3.4 The path to Nash equilibrium

As discussed in Section 2.3.4, the outcome of a game depends, in general, on the way it is played. In the previous two sections (3.2 and 3.3) the contour of the approach was presented (rules to be obeyed by the TSOs, exchange of information, PST and network models to be used). This section aims at presenting various possibilities that can be followed when translating the aforementioned principles into a precise procedure.

First, the iterative procedure suggested in the previous sections may take on form of:

- either a *(computer-to-computer) negotiation*, in which the iterations are performed until reaching an equilibrium, to be the control settings subsequently implemented on the system;
- or an actual *step-by-step implementation* of the control changes in the course of iterating.

Second, the communication between actors can be *synchronous* or *asynchronous*, as sketched in Fig. 3.6 for a three-TSO case. In asynchronous operation, each TSO announces[4] its control settings whenever it is ready for, while in a synchronous operation, each TSO is obliged to announce its settings at specific times. Clearly, the synchronous mode yields more ordered operation, in which each TSO calculation remain consistent with the present state of the system. On the contrary, in the asynchronous mode, each TSO performs its calculations based on data referring to different points in time, depending on the moments at which the other TSOs announced their settings. In synchronous operation, if the solution targeted by one TSO is not fully implemented at the time of communicating the settings, the part of it already implemented is communicated.

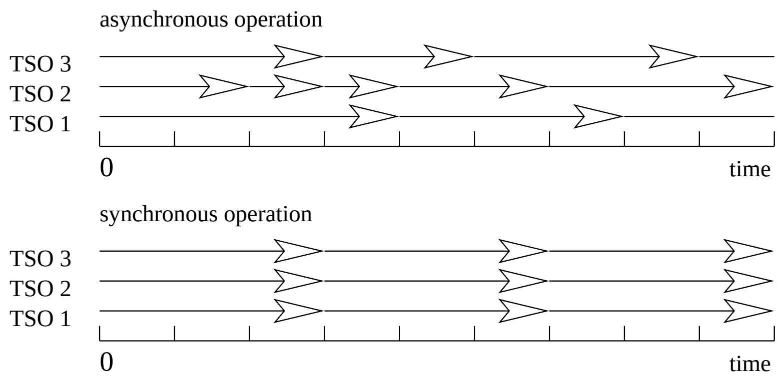

Figure 3.6: Asynchronous versus synchronous iterations

For a computer-to-computer negotiation, synchronization seems necessary since the procedure should converge to a final equilibrium, to be actually implemented, within a certain time. On the other hand, in a step-by-step implementation, synchronization is not a prerequisite; TSOs

[4]and maybe implements

could be free to adjust their PSTs whenever they want, as long as they respect the coupled constraints and they announce their control action. It should be pointed out, however, that introduction of synchronization in a step-by-step implementation procedure has some advantageous consequences, at the expense, of course, of a less "free" and "independent" operation by the TSOs. Those advantages are namely:

- in asynchronous operation, a TSO may not act against a constraint violation, waiting the others to take the curative action, while, if the operation is synchronous, the TSO obligations are better defined;
- in asynchronous operation, a TSO, when optimizing other parameters of its system, is exposed to the risk that, at any time, another TSO may significantly modify its PST settings, changing the first TSO's operating environment, while, if the operation is synchronous (and say executed on a hourly basis), the TSO can optimize its system knowing that the overall PST settings will remain at the reached Nash equilibrium.

The above points are further discussed after the presentation of the procedure via an example.

Let us further consider the synchronous approach. With reference to the two-TSO case illustrated in Fig. 3.7, the iterative procedure can be run:

- in a *parallel* way: TSO1 computes its new settings φ_1^{k+1} based on the previous setting φ_2^k of TSO2, while at the same time interval TSO2 computes φ_2^{k+1} based on φ_1^k;
- in a *sequential* way: each TSO waits for the other TSO to communicate its updated settings before performing its own optimization.

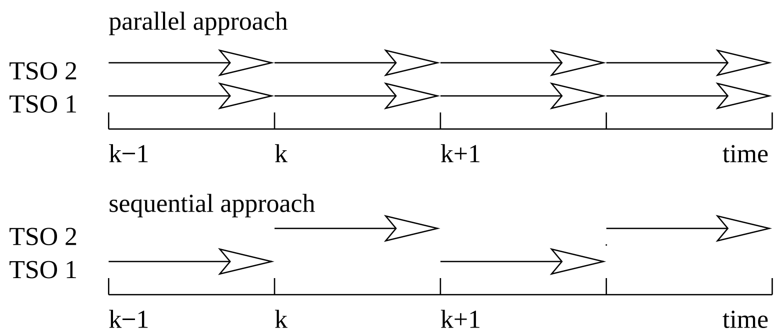

Figure 3.7: Parallel versus sequential iterations

The sequential approach has the disadvantage of being often slower than the parallel one. This becomes even more important when more than two TSOs are involved, which could be the case in practice. On the other hand, the parallel approach, if applied strictly, may not keep the system inside its feasible region at every moment. Indeed, although the solutions $(\varphi_1^{k+1}, \varphi_2^k)$ and $(\varphi_1^k, \varphi_2^{k+1})$ are both feasible, there is no guarantee that this holds true for the solution $(\varphi_1^{k+1}, \varphi_2^{k+1})$ to be implemented at the next time step. An additional level of coordination is needed to bring the solution back inside the feasible region. This must be designed to avoid oscillating from one side to the other of the feasible region boundary.

Finally, it may be useful to consider an additional degree of coordination between the TSOs that would consist in not allowing a TSO to modify its PST settings above a certain $\Delta\varphi^{max}$ every time it acts. In other words, the TSOs could agree to limit their rate of action. This could possibly damp big oscillations among actors as well as help moving towards an equilibrium more progressively. On the other hand, one could argue that such an additional restriction would be unfair for TSOs who have invested in technologically more advanced devices. Again, the question is further discussed after the illustration of an example.

3.5 Illustrative example

3.5.1 The test system

We illustrate the proposed method on the well-known IEEE 118-bus test system [IEE]. The latter has been decomposed into two sub-systems, named respectively "West" and "East" and assumed to be operated by two different TSOs. The overall structure of the so-decomposed system is shown in Fig. 3.8. Furthermore, a transaction of approximately 240 MW has been added from the Southern part of the East system (where most of its production is located) to the Northern part of the West system (where most of its load is located). The largest part of this transaction flows through the Northern part of the East system, thus passing through the "south-north cut" and "north interconnection" defined in Fig. 3.8. This makes the East system operate closer to its limits and with higher losses.

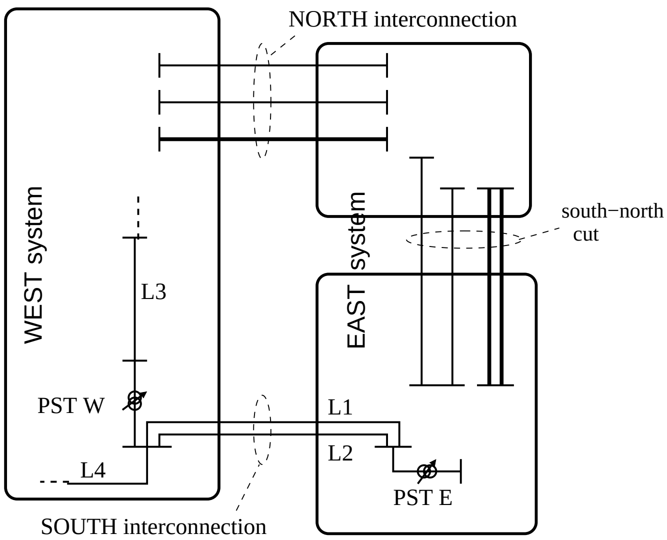

Figure 3.8: Overall structure of the decomposed IEEE 118-bus system

Under this perspective, we suppose that TSO East installed a PST in series with tie-lines L1 and L2 of the south interconnection. This allows East to control, up to a certain point (dictated by the PST limits), the share of the power flow between north and south interconnections. However, when a higher power flows in the south interconnection, line L3 (one of the links between Southern and Northern parts of West system) tends to be overloaded, due to its low thermal capability. Therefore, we assume that TSO West placed a PST in series with that line to protect it.

3.5.2 The objectives

For the above mentioned reasons, the objective of TSO East is to keep below a certain limit the power flowing in the south-north cut, which is equivalent to keeping above some value the flow in the south interconnection. On the other hand, TSO West wants to keep below a certain limit the power flow in line L3. These two objectives, though not directly connected to each other, turn out to be somewhat in contradiction, in the sense that improving one of them deteriorates the other. This will be shown graphically in the sequel.

In the examples presented hereafter, each TSO's decision-making problem has been expressed as a linear programming optimization problem, like (3.15), with the objective function being, for TSO East and respectively for TSO West, to make the power flow passing through the south-north cut and, respectively, line L3 equal to the TSO's maximum acceptable threshold value. The equivalence with the actual objectives described in the previous paragraph stems from the fact that in our examples we have always chosen the initial operating conditions to be such that the TSOs' power flow threshold values are violated and, thus, action is required by both East and West to bring the power flows down to their maximum sought values.

3.5.3 Examples in the context of step-by-step implementation

We first present results obtained in the context of a step-by-step implementation of controls by the two TSOs (see Section 3.4). Furthermore, we consider the synchronous and sequential schemes. As already discussed, this preserves feasibility of the solution during the iterations. Thus, we assume that each TSO has some time to calculate its next target PST setting, implement a part of it and communicate the resulting new setting to the other TSO. This can be expressed with the following constraint for the ith PST: $-\Delta\overline{\varphi}_i \leq \varphi_i^{k+1} - \varphi_i^k \leq \Delta\overline{\varphi}_i$, with $\Delta\overline{\varphi}_i \geq 0$.

Presentation in the control variables space: Figure 3.9 presents the evolution of the operating point in the control variables space. "phiE" denotes the phase angle of the PST in East and "phiW" the one in West. The two solid lines correspond to the thermal limits of lines L3 and L4, respectively. The shaded part of the diagram is the infeasible region. The two dashed dotted lines represent the TSO targets. East has the objective of keeping the active power flow in the south-north cut (see Fig. 3.8) at 210 MW. Points located to the right of that line correspond to higher (undesired) power flows. Similarly, West tries to keep the power flow in line L3 at 30

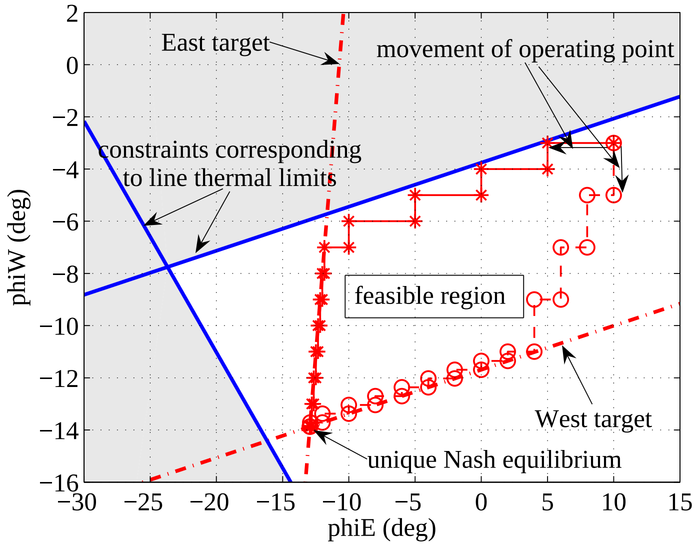

Figure 3.9: Convergence to a unique Nash equilibrium

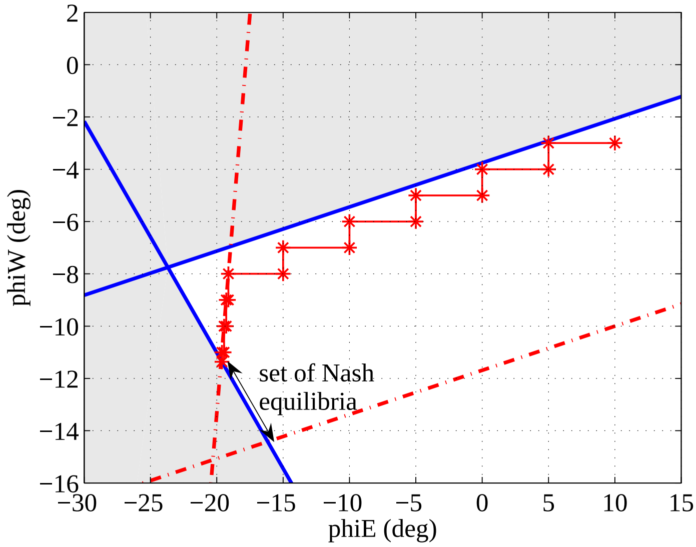

Figure 3.10: Target of East decreased to 190 MW; multiple Nash equilibria

MW. Clearly, the target line is parallel to the constraint line corresponding to the thermal limit of L3, which has been set to 50 MW.

The two trajectories in Fig. 3.9 correspond to different rates of change of the two PSTs. For the trajectory shown with solid line, it was assumed that, inside the time interval given to announce its new settings, East can change its phase angle by at most 5 degrees, and West by at most 1 degree. The dashed line, on the contrary, corresponds to faster moves by West.

As long as the system operates far enough from constraints, there is a single Nash equilibrium,

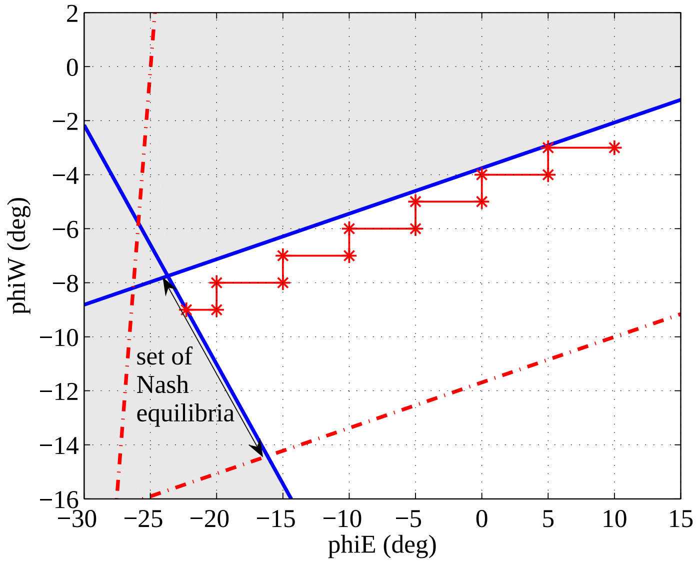

Figure 3.11: Target of East decreased to 170 MW; multiple Nash equilibria

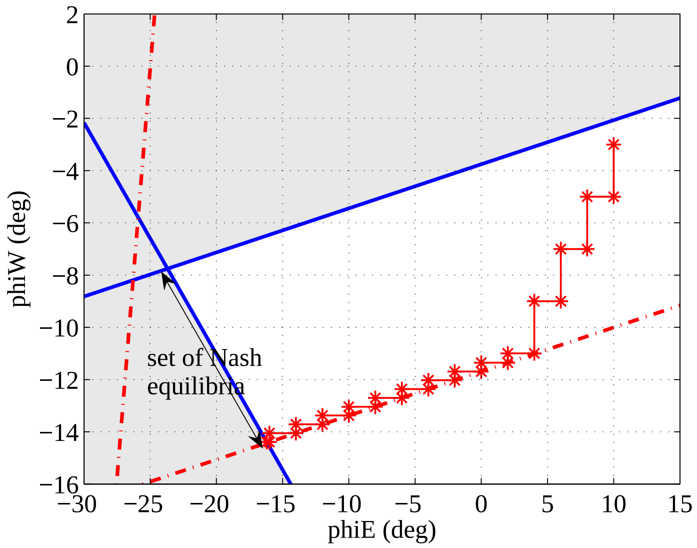

Figure 3.12: Same case as in Fig. 3.11 with different speeds of control changes

at the intersection of the target lines in Fig. 3.9. There, each TSO is satisfied with the solution so it has no motivation to proceed to any change. If the Nash equilibrium point lies inside the feasible region and if this region is convex, the procedure always converges to that point. Changing the relative speeds of the two PSTs does not influence the final point reached.

Remark. Lower limits on PST angle changes must be considered, to avoid moving by less than one step. This has been neglected in Fig. 3.9 and in subsequent ones, in order not to disturb the discussion with questions regarding discretization. Of course, in reality, the procedure will

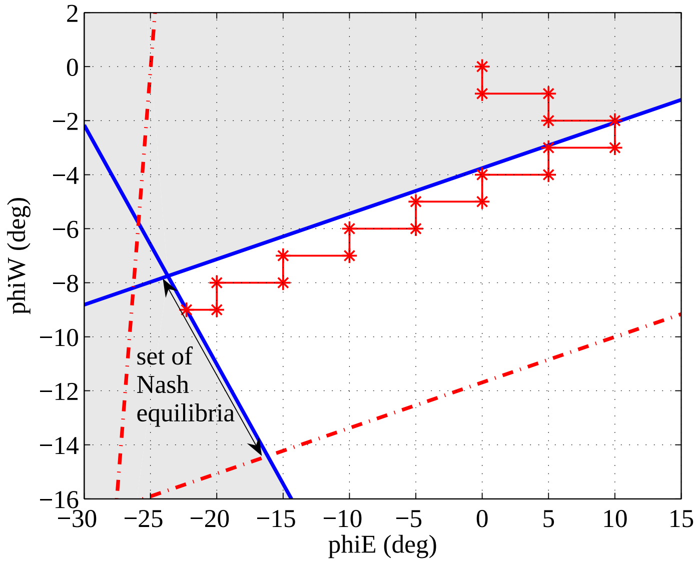

Figure 3.13: Sequence starting from an infeasible point

settle down somewhere very close to the aforementioned equilibrium.

In Fig. 3.10 the target power of East has been decreased to 190 MW. Due to the linearity of the model, this amounts to shifting the target line parallel to itself. As a result, the intersection point of the two target lines does no longer fall in the feasible region. Now the operating point moves along the East target line until it meets the constraint line corresponding to L4 overload. The point cannot move any further since this would either violate the constraint or increase the objective of East TSO. This final point is a Nash equilibrium. Furthermore, all points of the feasibility boundary pointed out in Fig. 3.10 have the same property and are all Nash equilibria.

A similar situation is shown in Fig. 3.11 corresponding to a 170 MW target power for East. The set of Nash equilibria is larger than in the previous case.

The final Nash equilibrium reached now depends on the system trajectory, and hence on the starting point and the relative speeds of action of TSOs. As an illustration, consider Fig. 3.12 which differs from Fig. 3.11 only by the speeds at which the TSOs change their PST angles (East five times faster than West in Fig. 3.11, both speeds identical in Fig. 3.12). A different Nash equilibrium is reached. Moreover, the faster the PST, the better the final value of the corresponding TSO objective.

Next, we consider in Fig. 3.13 a simulation starting from an infeasible point, which could result from a disturbance, for instance. According to the algorithm (3.15, 3.16), the first priority of TSOs is to restore feasibility. Hence, both start taking actions to remove the violation. Note that for TSO East, this action is in a direction opposite to the one dictated by its objective, while there is no such contradiction for TSO West.

Presentation in the objective functions space: Another view of the same simulation is pre-

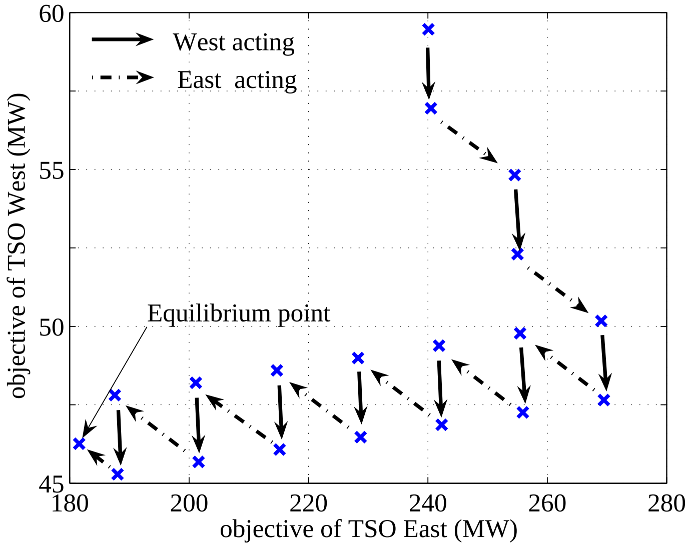

Figure 3.14: Case of Fig. 3.13 seen in the space of objective functions

sented in Fig. 3.14, showing the successive values of both objective functions. It is easily seen that TSO East has its objective deteriorated until feasibility is restored. From there on, at each iteration, one TSO ameliorates its objective while the other objective is deteriorated. Of course, this deterioration is just a side effect, since a TSO does even not know the other TSO's objective; it only knows its constraints.

Evaluation of the result in terms of its Pareto efficiency: An interesting property of the results is that the equilibria of the procedure happen to be Pareto optimal points of the corresponding multi-objective problem of the game (see Section 2.5). For all Nash equilibria in our examples, one cannot find another feasible operating point at which both objective functions would assume a better value.

To show this, let us come back to Fig. 3.11, which we reproduce in Fig. 3.15 without the operating point trajectory. In this diagram, we arbitrarily pick one of the Nash equilibria of the procedure, with the objective functions of TSOs East and West taking values $f_E^\star$ and $f_W^\star$ respectively. We then draw the two lines that correspond to points where the objectives have values $f_E^\star$ and $f_W^\star$, the solid line corresponding to TSO East and the dashed to TSO West. Because of the linear relationship between the objective functions and the PST angles, the two lines are parallel to the two target lines. For the objective of East to take a value better than $f_E^\star$, the operating point should be at the left of the solid line. In the same way, for the objective of West to take a value better than $f_W^\star$, the operating point should be below the dashed line. One can easily observe that no operating point in the feasible region falls at the same time left of the solid line and below the dashed one. Hence, the point is Pareto optimal. The same holds true for whichever point in the Nash set.

In the case where the (unique) Nash equilibrium is inside the feasible region (Fig. 3.9), the point is also Pareto optimal since both objectives have taken their best possible values. This

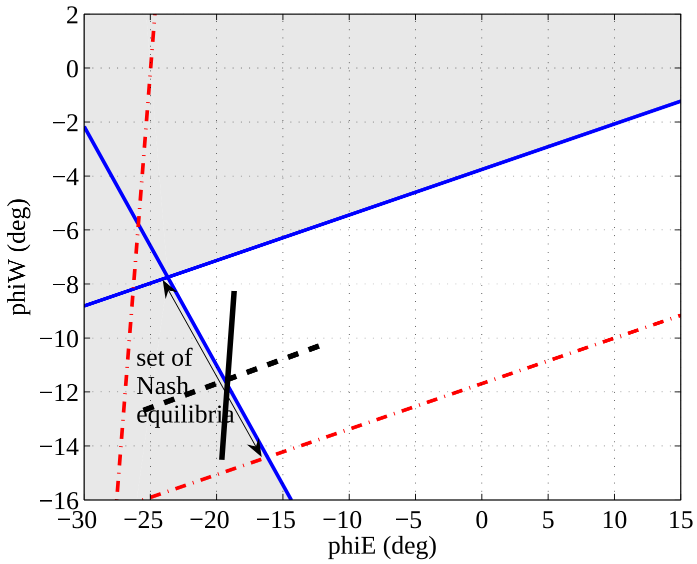

Figure 3.15: Showing that the Nash equilibrium is also Pareto optimal

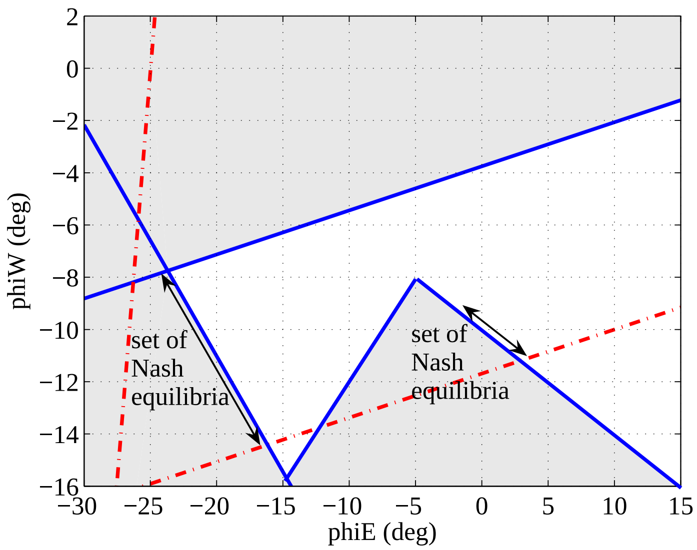

Figure 3.16: Non convex feasible region

holds true provided we assume that points corresponding to smaller line power flow than the target are equally good as those where line power flow is equal to the target.

The just described property of the Nash equilibria being Pareto optimal solutions of the corresponding multi-objective problem, is not a particularity of our examples. Even with different targets and different equilibrium points, the geometric properties depicted in Fig. 3.15 will always hold true. It is not the purpose of this work to give strict mathematical proofs, but we believe that what has been shown geometrically can also be proven in a more general algebraic way. An important assumption though is the convexity of the feasible region. If in is not convex, there may exist Nash equilibria that are not Pareto optimal. Figure 3.16 illustrates this situation. Here we have added artificial constraints to make the feasible region non convex.

The fact that the equilibria of our iterative procedure turn out to be Pareto optimal in the PST control problem, does not mean that this is a general property of the algorithm. The algorithm is designed to work for non linear objectives as well (the operational objectives need not even be formulated as mathematical programming problems), in which case there is no reason to believe that the Nash equilibria will constitute Pareto optimal points of the corresponding multi-objective problem.

Devising an additional coordination procedure so that the algorithm ends up in points as close to Pareto optima as possible remains an interesting challenge [TC01].

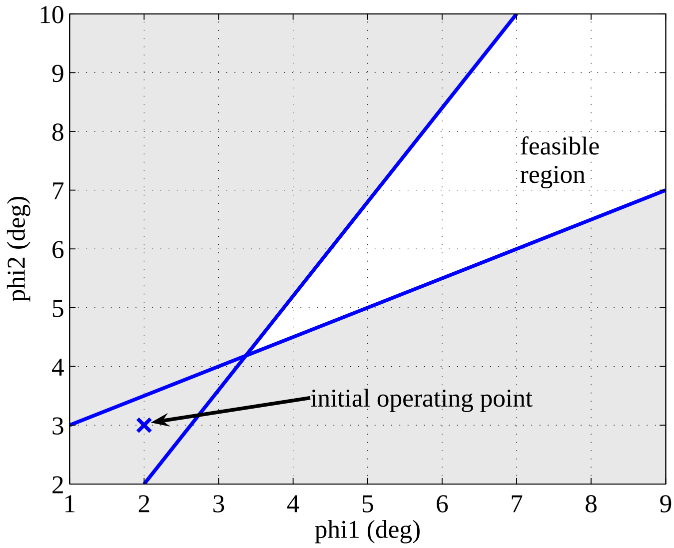

Figure 3.17: Initial infeasible point from which the procedure cannot start

Example where inability to restore feasibility is encountered: The sequential scheme considered in the previous examples guarantees that, once inside the feasible region the operating point will always remain inside. Furthermore, Fig. 3.13 has shown how the procedure can bring the system back inside its feasible region. However, the procedure may not succeed doing so in all cases. An example of difficult situation is depicted in Fig. 3.17, in which any *individual* change of the control variables fails bringing the operating point inside the feasible region.

This is, in fact, a general possible drawback of a scheme where coordination is achieved by having each actor satisfy all coupled constraints every time it takes an action. It may happen that none of the actors has enough controllability to restore feasibility. A more sophisticated type of coordination is needed in this case. In Chapter 4 another coordination scheme is proposed, dealing with the above issue, while some discussions comparing the two schemes can be found in Section 4.7.3.

3.5.4 Example in the context of negotiation

An example of (computer to computer) negotiation (see Section 3.4) is given in Fig. 3.18. Here, the intermediate points do not represent actual operating points, but are rather values announced by each TSO during the iterative procedure. What is sought is the Nash equilibrium, the real PST adjustment taking place later. The scheme is still sequential. The control changes are not restricted as in the previous figures, although in the example of Fig. 3.18, a maximum deviation of 25 degrees has still been imposed. As expected, the convergence to a Nash equilibrium is much faster. Due to the larger steps allowed, the order in which TSOs announce their settings makes an important difference on the final equilibrium.

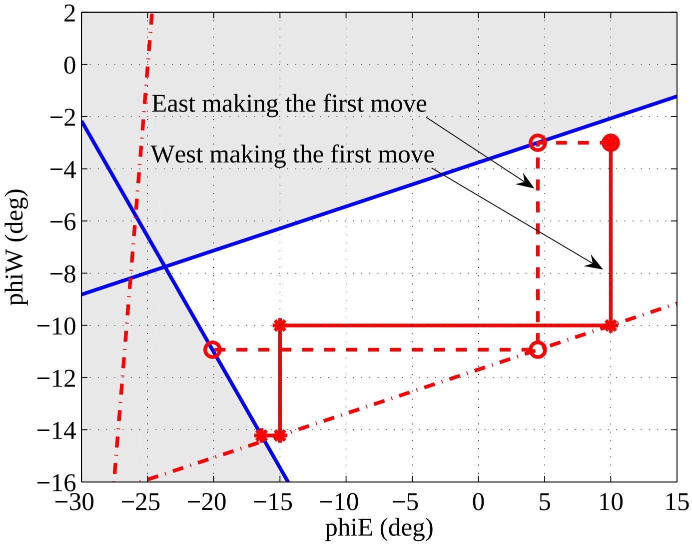

Figure 3.18: Example of negotiation

The example presented in Fig. 3.18 illustrates how vulnerable to gaming such a negotiation could be. In the negotiation of this example, TSO East would like the procedure to converge as close to the upper-left edge of the Nash equilibria segment as possible, while TSO West would like it to converge to the bottom-right edge of the segment. None of them has motivation to "myopically" solve its original decision-making problem at its turn, but rather to solve another problem which anticipates the evolution of the iterations (see Section 2.3.4). Clearly, such behavior is far from what the "willing-to-collaborate framework" among the TSOs intends to achieve.

3.6 Discussion

The coordination principles presented in Section 3.2.2 set the spirit of a possible application of the multi-TSO procedure. Let us repeat that the idea is to cut down the negative consequences of a completely uncoordinated operation (described in Section 3.2.1) preserving at the same time the independence and confidentiality of the TSOs objectives and operational procedures. Section 3.4 exposed different possible implementations of the framework delineated in Section 3.2.2. In light of the examples presented in the previous section, additional remarks can be made regarding those implementations.

3.6.1 Bounded vs. unbounded modification of PST settings

The different executions can be divided in two families:

- those where the TSOs' control modifications from one iteration to another are bounded, i.e. for PST i:

$$-\Delta\overline{\varphi}_i \leq \varphi_i^{k+1} - \varphi_i^k \leq \Delta\overline{\varphi}_i \qquad (3.17)$$

- those where the TSOs' control modifications are not bounded, i.e. a TSO's previous control action (or announcement) does not constrain its next one.

Indicatively, the first case corresponded to the step-by-step implementation examples presented in Section 3.5.3, while the second case to the negotiation examples of Section 3.5.4. This should not suggest that implementation necessarily goes with bounded update of controls and negotiation without: future electronically controlled PSTs will have their settings modified fast, while a rule in a computer to computer negotiation could be that TSOs must respect constraints like (3.17) every time they re-solve their decision problems. Thus, it is worth wonder, as a question by itself, whether it is preferable to operate under the first or the second of the above two cases.

Obviously, unbounded control modifications can make convergence extremely fast; in both examples shown in Fig. 3.18 two iterations have been enough for convergence. On the other hand, unbounded modifications could make the reached equilibrium too sensitive to the starting point and, in case of sequential operation, the order in which TSOs take actions. Furthermore, the TSOs may be more tempted to act strategically during the procedure, seeking for convergence to the most profitable equilibrium. Finally, if parallel execution is chosen, allowing large control modifications between iterations could create oscillations between feasible and infeasible operating points.

Clearly, if the control changes are actually implemented during the execution of the procedure, the actions of fast devices must be limited so that transients (or instabilities) are not caused by large PST angle excursions. However, for the reasons mentioned in the previous paragraph, resort to constraints of type (3.17) could be also made to improve the convergence properties of the procedure.

3.6.2 Further steps towards an implementation

An advantage of running the procedure only between computers, before actually implementing the computed and announced control actions, lies in the fact that this saves the time it takes to actually move the PST taps. As a result, the procedure could be run quickly, for example on an hourly-basis, and come up with the PST settings that the TSOs should implement and respect for the next hour. The TSOs could then optimize and operate their systems knowing all the PST settings in the interconnection. Clearly, a constraining time should be available to the TSOs to actually implement the computed solution.

The drawback of the above described approach is that it leaves the actual PST modifications uncoordinated; the trajectory that the operating point will follow from the PST present settings to the just computed new equilibrium could pass through the infeasible region since this is nowhere checked (see [Mar05] for a Model Predictive Control approach dealing with that issue). There are two ways to deal with this possibility. Either the TSOs estimate that their systems can tolerate some constraint violations for a limited time[5], or, they could impose such constraints to the negotiation procedure, which would then compute not only the final Nash equilibrium but also the trajectory towards it.

The latter would make the negotiation rather a simulation of the actual implementation; the intermediate steps announced by the TSOs during the negotiation should be those implemented step-by-step (and in the same order, of course) afterwards. The advantage of doing so is that the trajectory is computed faster (no need to reserve time between iterations for actual move of the PSTs) and it is made known to all involved TSOs, which then can further optimize their systems, while implementing their obligations (their previously announced PST modifications), taking the trajectory into account.

In the two-TSO examples of Section 3.5 sequential operation was considered, mainly for illustrative purposes. Thanks to the simplicity of the examples (two PSTs, linear objectives and models) the series of actions in case of parallel execution of the procedure (see Section 3.4) can be easily figured out. In the examples of Figs. 3.9-3.13 it would take half the time to arrive to practically the same Nash equilibrium. For more than two TSOs, the acceleration would be even larger. This is a motivation towards having the TSOs solve their decision problems in parallel with each other.

The basic drawback of parallel execution lies in the danger that some collective PST actions $(\boldsymbol{\varphi}_1^k, \boldsymbol{\varphi}_2^k, \ldots)$, computed during the execution, may not satisfy the security constraints, as explained in Section 3.4. However, if (a) some minor constraint violations are tolerated during the execution of the procedure (this would typically consist in the operating point slipping around a constraint), and (b) a mechanism exists that does not allow the operating point to finally end up oscillating inside and outside the feasible region, parallel execution seems the most reasonable choice. In the example of Fig. 3.19, starting from point 1, a parallel execution of the procedure would end up oscillating between points 2 and 3. The mechanism mentioned under item (b) should be able to capture such an oscillation and stop it at the feasible side operating point (i.e.

[5]This is acceptable for thermal overloads.

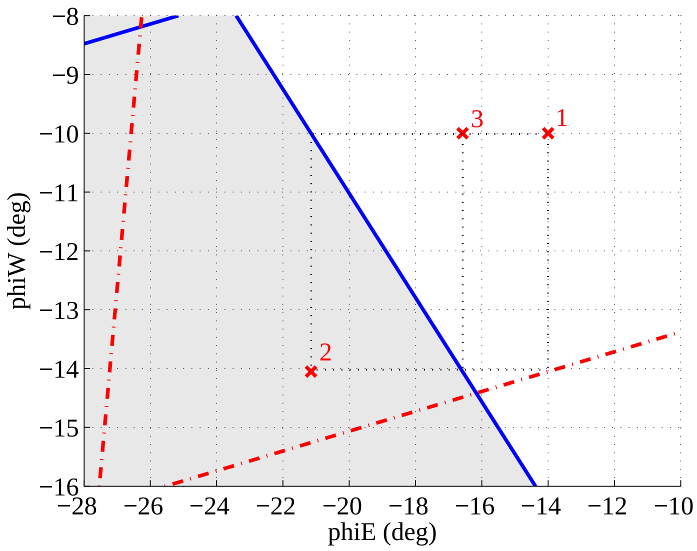

Figure 3.19: Oscillations in parallel execution

point 3 in Fig. 3.19). The TSOs could easily realize such behavior by observing the history of operating points, and each of them could fix its controls at the values corresponding to the feasible operating point.

If a step-by-step implementation is chosen, then asynchronous operation could be envisaged. This would practically mean that each TSO is allowed to act whenever it wants, but when it does, it must do it according to the principles described in Section 3.2.2. This is enough to keep the operating point always feasible (except if two or more TSOs happen to act exactly at the same time). Special care should be taken when for some reason the system is operating in the infeasible region.

A compromise should be found between security, speed, fairness and independence of TSOs. For example, parallel execution favors speed and fairness (there is no "privileged" who acts first), while sequential execution favors security. Asynchronous operation favors independence of the TSOs, but it slows down the procedure and it may endanger security.

To conclude the above discussion, we believe that the following procedure for coordinated PST control could be of practical interest and applicability:

1. On an hourly (or daily) basis the involved TSOs execute a (computer to computer) negotiation to come up with their PST settings of the next hour (or, respectively, day).

2. Results of this negotiation are not only the final PST settings (i.e., the Nash equilibrium reached) of the TSOs but also their trajectories towards those settings.

3. Thus, when the negotiation is over, the TSOs implement step-by-step the actions that had been announced previously, reaching in this way the Nash equilibrium.

4. A predefined time is available to the TSOs within which they must implement each of their PST moves.

5. In particular, within the negotiation procedure, the TSOs announce their actions in parallel to each other, i.e. each TSO computes its new action knowing the other TSOs previous actions.

6. A predefined time is available to each TSO to compute and announce its new settings. In case of failure to do so, it is considered not to make a move at the iteration under question. This will be mirrored to a corresponding no-action of the TSO in the implementation phase that follows the negotiation.

7. At each iteration of the negotiation, a TSO cannot modify its PST settings by more than a predefined amount $\Delta\overline{\varphi}_i$. This amount is in accordance with physical limits of the PST (the TSO must be able to implement the step change it announced) and security limits (i.e., no undesirable transient will be caused by such a step change). Furthermore, the $\Delta\overline{\varphi}_i$ values should be selected such that the procedure smoothly converges towards the equilibrium, without sudden changes of power flows that could disturb the operation of the network.

8. Finally, a higher level of coordination exists that detects if the procedure ends up in oscillating between a feasible and an infeasible operating point. In this case, it seems reasonable to stop and take the feasible point as the sought equilibrium.

After the negotiation phase is over, the TSOs know what will be the PST settings for the next hour (or day). They can include this information when they are dealing with their other security, operational and market issues.

3.6.3 Sharing a common objective in case of emergency

A slightly modified version of the iterative algorithm described in Section 3.2.2 could consist in changing the objectives in case of emergency. The idea is that if after an incident one or more constraints are violated, all TSO change their objective functions to a common one representing the least control effort (i.e. the fastest movement) and solve an optimization problem using all the control variables. In sequence, each one implements the part of the solution that involves its own controls. In the two-TSO case, for instance, this could be formulated as follows:

TSO 1 computes:

$$
\begin{aligned}
(\mathbf{u}_1^{k+1}, \mathbf{u}_2^{k+1}) = \arg \min_{(\mathbf{u}_1, \mathbf{u}_2)} & (\mathbf{u}_1 - \mathbf{u}_1^k)^2 + (\mathbf{u}_2 - \mathbf{u}_2^k)^2 \\
\text{subject to} \quad & \mathbf{g}\left(\mathbf{u}_1, \mathbf{u}_2\right) \leq 0
\end{aligned}
\tag{3.18}
$$

and implements: $\mathbf{u}_1^{k+1}$

at the same time TSO 2 computes:

$$(\mathbf{u}_1^{k+1}, \mathbf{u}_2^{k+1}) = \arg \min_{(\mathbf{u}_1,\mathbf{u}_2)} (\mathbf{u}_1 - \mathbf{u}_1^k)^2 + (\mathbf{u}_2 - \mathbf{u}_2^k)^2 \tag{3.19}$$

$$\text{subject to} \quad \mathbf{g}\left(\mathbf{u}_1, \mathbf{u}_2\right) \leq 0$$

and implements: $\mathbf{u}_2^{k+1}$

One can see that the two TSOs solve the same problem, and hence they will come up with the same solution. This results in globally acting in the most efficient way to alleviate the emergency problem. In the original method the alleviation of emergencies is done in a less coordinated way, where each one, forced by the constraints, moves its controls towards a direction that solves the emergency problem. No one guarantees, however, that the combination of TSO actions is the most efficient way to solve the problem. Changing the operating strategy in case of emergency to the one just described, ensures the treatment of the emergency to be most efficient. Furthermore, this approach solves the problem discussed in Fig. 3.17.

3.7 Conclusion

A multi-objective optimization framework has been proposed to deal with the operation of a system by multiple interacting TSOs. The essence of the algorithm is an iterative approach where TSOs successively compute control actions, taking into account the last actions of other TSOs and obeying the whole set of constraints. This involves information exchange between TSOs, although their individual objectives are kept undisclosed. This framework has been applied to the PST control problem with linearized constraints, and several schemes of potential implementation have been outlined.

Examples relative to a two-PST, two-TSO case have been presented. Several features of the procedure have been illustrated graphically: existence of one or multiple Nash equilibria, sensitivity to relative speeds of action, etc. In addition, some circumstances where the TSOs could switch to single objective were presented in Section 3.6.3.

Future research should address, among others, the questions of existence and convergence to Nash equilibria, as well as relationships with centralized control and Pareto optimum. In this respect, extensions to controls having a cost and, hence, to market-type objectives are of interest.

Chapter 4

Coordinated use of transmission resources by multiple transaction schedulers

The possibility for market participants to simultaneously place their bids in different markets across an interconnection is investigated in this and the next chapter. Transaction schedulers settle multilateral transactions among participants, while a single central entity coordinates the overall operation through interactions with the transaction schedulers. Two issues are dealt with in this context. First, the market participants are allowed to place their bids simultaneously in more than one transactions scheduler's market, and, second, the available transmission capacity is fairly shared among the transaction schedulers. Economically interesting transactions are favored, while confidentiality of market data and independence of transaction schedulers' clearing mechanisms are preserved. The corresponding iterative algorithm is illustrated in detail on a 15-bus as well as the IEEE-RTS system.

4.1 Introduction

4.1.1 Existing situation

In modern power systems, several areas, controlled by separate entities, form altogether larger interconnections inside which electricity is traded [KS04]. In Europe, for instance, the entities correspond to TSOs and, in most cases, the areas to countries[1]. While a lot of research effort has been devoted to improving electricity markets inside areas, comparatively less attention has been paid to organizational structures and algorithms allowing separate areas to be operated in a seamless way in terms of inter-area electricity trade.

Long-term forward contracts between different areas have been in practice even before the liberalization process. This work, however, focuses on the operation of spot markets, from

[1]There are four TSOs in Germany.

day-ahead up to real-time, and the development of algorithms to facilitate the inter-area trade.

A typical way to do so has been the posting by TSOs of Available Transfer Capacity (ATC) values for importing and/or exporting at each interconnection and the selling of consistent transmission rights to the market actors. This is referred to as explicit auction of transmission capacity, since the latter is auctioned and sold separately from energy. In such a framework, for instance, a broker purchases export and import rights from the areas where the generator and the consumer respectively are located, in this way settling an inter-area transaction. Explicit auction is currently the prevailing allocation mechanism of scarce interconnection resources in Europe. Although attractive in theory, this approach has been found in practice to yield some inefficient use of the network. The main reasons are: it is difficult for the participants to anticipate what the value of each transmission line will be for them, some participants tend to hoard capacity that they don't finally use, and pancaking of allocations appears when several borders are involved in a transaction [TLC06].

The alternative, increasingly used in the last years, is implicit auction for congestion management, where the use of the transmission system is allocated implicitly at the time the energy market is cleared [ETS09]. This is the main way intra-area congestion management is treated in some parts of North America, with the several pool-based Locational Marginal Pricing (LMP) approaches [SCTB88]. Another implicit auction approach, called market splitting, has been used for years in the Nordic market (Scandinavian countries) where in case of congestion the market is split in two or more price areas [CWW00].

It seems that implicit auction is the future (and already the present in some cases) way of managing cross-border transmission capacities in Europe. The prevailing mechanism for doing so is the so-called market coupling. Both the LMP and the market splitting approaches require a centralized market operator that combines the bids in a market clearing procedure. On the other hand, market coupling is an implicit auction similar to market splitting but performed in reverse order. First, each sub-market is cleared; then, these markets are coupled. It is thus a method performing coordination among different markets, each using its own rules inside its area [GBD+05].

The first implementation of this approach was the Trilateral Market Coupling (TLC) in operation since 2006 between France, Belgium and the Netherlands. It is organized as a decentralized, multilateral contractual arrangement between the participants [ETS09]. The Power Exchanges (PXs), namely APX, Belpex and Powernext, provide the IT systems and run the common coupling algorithm, while, the TSOs, namely RTE, Elia and TenneT, calculate cross border capacities, set up physical exchanges, share congestion revenues and pay the market coupling service fee that is determined locally. Regulatory oversight remains with the national regulators and/or is subject to national legislation.

A detailed description of the TLC algorithm can be found on the Web sites of the above PXs (e.g. [TLC06]). Basically, it consists of each market participant bidding in the day-ahead market of the area where it is physically located, using the rules and IT tools of the corresponding PX. These (sell or purchase) bids are used by the PXs to construct the net export curve of their markets, i.e. the difference between total sales and total purchases of this market as a function of the Market Clearing Price (MCP). These curves are assembled in the central coordination module so that markets with the highest MCPs import electricity from markets with the lowest MCPs. In the absence of congestion, the result is an import/export pattern between markets in such volumes that the three local MCPs become equal. Otherwise, import/export is settled up to the ATCs and the markets end up with different prices. This mechanism enables local markets participants to “see” a larger liquidity, not limited to their area, within the limits of the cross-border capacities of course.

Worth mentioning is the fact that, with the present TLC rules, this assemblement of PXs has a monopoly over the inter-area spot market trade. The only way that two (or more) market participants have in order to settle a bilateral (or multilateral) transaction in day-ahead is to pass through the TLC system. The producer will have to sell to its local PX at the local price, the consumer to buy from its local PXs at the local price and then they will have to pay one another the difference between the prices imposed by the PXs and the price they had privately agreed between themselves.

The extension of TLC to the five countries of the Central Western Europe (CWE) region has been announced for 2010. This involves Germany and Luxembourg in addition to the three TLC countries. A more sophisticated algorithm is envisaged [CWE08], although it retains the ideas that a market participant interacts only with the PX of its area, while some central calculations take care of energy being exported from low to high price areas, within the limits of transfer capacity. First, an ATC-based modeling of the network constraints will be used, but it is planned to switch soon to a more precise flow-based network model, in which critical branches (tie- and some internal lines) will be defined by the CWE TSOs. For the time being, the above market coupling mechanisms apply to day-ahead procedures only. Steps are also taken towards opening intra-day and real-time markets to foreign players [VMB].

The above outlined trilateral, and soon pentalateral, initiative couples the markets of the involved PXs. It should be noted, however, that these PXs do not involve but a fraction of the spot energy trade in Europe, where trading arrangements are mainly bilateral. Most of the wholesale trade is in the Over-The-Counter markets, often supplemented with day-ahead auction trade organized by the national PXs [MB07]. The advantage of having the PXs organizing these auctions is that they use simple rules to settle contracts at a point of time where it is not worth getting into time consuming negotiations. Power Exchanges are also counter-party for all transactions so that trade is anonymous and traders do not have to worry about counter-party risk. However, it could also be argued that PXs are not strictly necessary market components [MB07]. Still, most European countries have a PX often as a result of private initiatives. The PXs often do not have to take network constraints into account at all, or they do only partly.

It is worth noting here that usually there exists one PX (or none) per area, but in principle nothing prevents several PXs from co-existing and competing within an area. On the other hand, a PX can extend its activities over more than one area. This is going to happen in a near future with the merging of Powernext and EEX (French and German PXs, respectively).

Compared to Europe, the North American wholesale markets appear more weakly linked, if at all. As considered in [MB07, FER], it may be more difficult to couple these markets because they apply a different implementation of nodal pricing, making it practically difficult to harmonize the handling of network constraints; even more if the latter is already fine-tuned, which is less the case in Europe. There is, however, a common market initiative between MISO and PJM who are working towards the development of complementing system operations and one robust, non-discriminatory wholesale electricity market to meet the needs of all customers and stakeholders [MIS].

4.1.2 Approach proposed in this work

As there exist various electricity market implementations, and it is not the objective of this work to enter into the details of each one, let us call Transmission System Operator (TSO) the entity responsible for operating the transmission system of a particular area, while we call Transaction Scheduler (TS) every entity responsible for settling transactions between producers and consumers[2]. For instance, a PX is a TS, but other entities also fit the description, such as a broker who settles bilateral or multilateral transactions. The TSO is typically a TS when dealing with real-time operation (balancing market, generation re-dispatch, etc.).

This work investigates whether the constraint that a wholesale market participant should be part of a particular spot market, defined by its geographic location, could be relaxed. Thus, the presented approach assumes that every market participant is allowed to bid in whatever market (represented by a TS), irrespective of where it is located. More generally, a framework and an algorithm are proposed to let market actors use the grid in a coordinated way to perform commerce of electrical energy without them being constrained to do so via a TS covering only their geographic location.

Clearly, the idea that any market participant may place its bids in the market of any TS operating in the interconnection would result in the appearance of “overlapping markets” and would make inter-area congestion management even more important. The development of a coordinating framework is thus required. This framework should enable free spot trade of electricity. The TSs should be able to compete freely first to attract market participants interested in settling transactions and second to obtain transmission capacity in order to support these transactions.

Furthermore, as stated in the end of Section 2.6, this work is based on the assumption that the SOs of an interconnection are willing to co-operate in the setting up of a common model of the grid and to delegate part of the congestion management tasks to a central coordinating entity. These assumptions seem acceptable and go with the present trend, at least in Europe [Cor]. The objective of this coordination among TSOs will be to operate the grid in a way that electricity trade is maximized, with priority given to the most valuable transactions, without violating the security limits.

[2]The term “market operator” (MO) provisionally used in the Introduction is, thus, from now on abandoned and replaced by the more general term “transaction scheduler”. The term MO has been used in the Introduction, as possibly more familiar to the reader, just in order to avoid introducing a new term there.

All in all, the developed approach considers the following two prerequisites:

1. Transparency of the grid data: TSOs are responsible for constructing a common model of the grid and make it available to all participants. In this way everybody will be able to check that the coordinating computations made by the TSOs are fair.

2. Confidentiality of each market data and procedures: the TSs should not be asked to provide any intermediate information of their market clearing procedures. They should only announce their final schedules and prices.

4.1.3 Related work

The proposed approach offers a decentralized way of coordinating multilateral transactions. In this spirit, Ref. [WV99] proposed a new operating paradigm in which the decision mechanisms regarding economics and reliability (security) of system operation are separated. In this framework, economic decisions are carried out by private multilateral trades among generators and consumers. Reliability is ensured by the TSO who provides publicly accessible data, based upon which generators and consumers can determine profitable trades that meet the secure transmission loading limits.

In [Hao05], the author proposes two decentralized procedures in which each Regional Transmission Organization (RTO) administers its energy market and also acts as a transmission coordinator to achieve feasible and efficient use of congested transmission by all markets in the interconnection. Participants in any RTO market are allowed to schedule transactions into, out of, or across any RTO control areas. The resulting overlapping markets are modeled, while, since when transmission capacity is limited markets compete for the use of the limited transmission paths, two methods for allocating this capacity are proposed. In both methods, the author suggests that the TSs send to the coordinator the sensitivities of their cost functions to the branch available capacities. Using this information, for all congested branches, the coordinator, in what is referred to as “master problem”, shares their available capacities among TSs so that they have the same value for everyone of them.

Closely related is also the work in [LNWB07], which proposes a decentralized model for DC load flow based congestion management for the forward markets via optimal resource allocation.

4.2 Statement of the problem and outline of the approach

4.2.1 Market clearing and transmission system modeling

Let M be the number of TSs. Each TS clears the market it represents, using its own rules. The outcomes are scheduled generation and load quantities together with the corresponding prices offered to each generator or asked to each load.

Although the clearing may be implemented in various ways, it is convenient to formalize it as an optimization problem where the market's social cost is minimized (i.e. the social welfare is maximized) [KS04]. For the mth TS, this optimization takes on the form:

$$\min_{\mathbf{g}_m,\mathbf{d}_m} \{\mathbf{c}_m^T\mathbf{g}_m - \mathbf{b}_m^T\mathbf{d}_m\} \tag{4.1a}$$

$$\text{s. t.} \qquad \mathbf{1}^T\mathbf{g}_m = \mathbf{1}^T\mathbf{d}_m \tag{4.1b}$$

$$\mathbf{0} \leq \mathbf{g}_m \leq \overline{\mathbf{g}}_m \tag{4.1c}$$

$$\mathbf{0} \leq \mathbf{d}_m \leq \overline{\mathbf{d}}_m \tag{4.1d}$$

where $\mathbf{c}_m$ (respectively $\mathbf{b}_m$) is a vector containing the bids of all generators (consumers) bidding in market m, $\mathbf{g}_m$ ($\mathbf{d}_m$) contains the powers of generators (consumers) dispatched by the mth TS, $\mathbf{1}$ is a unit column vector, $\overline{\mathbf{g}}_m$ is the vector of maximum powers that generators are willing to produce for market m, while $\overline{\mathbf{d}}_m$ is the vector of maximum powers that loads are willing to consume. Equation (4.1b) expresses that each TS has a balanced schedule.

The net power injection at bus k scheduled by the mth TS is given by:

$$(\mathbf{n}_m)_k = \sum_{i\in k}(\mathbf{g}_m)_i - \sum_{j\in k}(\mathbf{d}_m)_j \tag{4.2}$$

where the expression $i \in k$ (resp. $j \in k$) is used to denote that the ith generator (resp. the jth load) is connected to the kth bus. Eq. (4.2) is written in vector form as

$$\mathbf{n}_m = \mathbf{\Gamma}\ \mathbf{g}_m - \mathbf{\Delta}\ \mathbf{d}_m$$

where the elements of matrices $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are zeros and ones so that they express whether a generator or, respectively load, is connected to a bus.

The vector of net bus power injections is obtained as the summation of all the TS schedules:

$$\mathbf{n} = \mathbf{\Gamma}\ \mathbf{g} - \mathbf{\Delta}\ \mathbf{d} = \sum_m \{\mathbf{\Gamma}\ \mathbf{g}_m - \mathbf{\Delta}\ \mathbf{d}_m\} \tag{4.3}$$

Once this vector is known, branch power flows can be computed using a model of the entire network. A DC model of the interconnection is used in this work. This is a commonly used model in market clearing problems and it is well suited to the linear computations presented in the remaining of the chapter. It is assumed that the various TSOs in the interconnection assemble and share such a network model, which they use to coordinate the overlapping markets simultaneous clearings.

Let B be the number of branches and N the number of buses in the system. In order to assess the impact of the power injection schedule on branch flows, we resort to well-known Power Transfer Distribution Factors (PTDF). Let T_b^{kl} be the fraction of a transaction from bus k to bus l that flows over branch b $(k, l = 1, \ldots, N; b = 1, \ldots, B)$. According to [CWW00]:

$$T_b^{kl} = \frac{X_{ik} - X_{jk} - X_{il} + X_{jl}}{x_b} \tag{4.4}$$

where i and j are the terminal buses of the branch, x_b is its reactance, X_{ik} is the entry in the ith row and kth column of the $N \times N$ bus reactance matrix $\mathbf{X}$, and similarly for the other entries. Assuming that bus N is the slack bus, the Nth row and the Nth column of $\mathbf{X}$ have all zero elements [WW96].

The effect of the power injection n_k at bus k on the power flow in branch b can be seen as the effect of a transaction n_k between bus k and the slack bus N. The power flowing through branch b is thus given by :

$$p_b = \sum_{k=1}^{N} T_b^{kN} \, n_k \tag{4.5}$$

This is easily written in matrix form as :

$$\mathbf{p} = \mathbf{T} \, \mathbf{n} \tag{4.6}$$

where $\mathbf{p}$ is the vector of branch power flows and $\mathbf{T}$ is the $B \times N$ matrix relating branch power flows to bus power injections, and defined by:

$$(\mathbf{T})_{bk} = T_b^{kN} \qquad b = 1, \ldots, B; \quad k = 1, \ldots, N \tag{4.7}$$

The choice of the slack bus influences the elements of $\mathbf{T}$. However, when assessing the contribution of the market schedules to branch flows, formula (4.6) will be applied to the injection vector $\mathbf{n}$ whose components sum up to zero, owing to (4.1b), (4.2) and (4.3). Therefore, the net power injection caused by the mth market at the slack bus is zero. Thus, the branch flows computed in (4.6) do not depend on the choice of the slack bus (losses being neglected in this derivation).

As long as there is enough reactive compensation to keep voltage magnitudes constant at all buses, PTDFs have been shown to remain practically unchanged as the pattern of injections changes the loading of branches [Bal07, BDO05, LG04].

4.2.2 Emerging issues

Clearing the above mentioned overlapping markets without any concern for the grid flows is very likely to end up in branch overloads. If $\widehat{\mathbf{n}}_m$ is the injection schedule of the mth TS, nothing guarantees that the resulting branch flows $\widehat{\mathbf{p}} = \mathbf{T} \sum_m \widehat{\mathbf{n}}_m$ respect the constraint

$$-\overline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}}, \tag{4.8}$$

with $\overline{\mathbf{p}}$ the vector of maximum branch power flows.

The congestion could be alleviated by the various TSOs by, ex post, rescheduling generation inside their areas. However, this has been shown to result in very inefficient use of the combined generation and transmission capacities (and, thus, it has prompted interest for implicit allocation of both energy and transmission). Clearly, in order for a market overlapping scheme to be put into practice, a mechanism is needed to coordinate the various TSs' simultaneous market clearings. The objective in this work is to deal with the congestion management problem implicitly at the same time when the TSs are clearing their markets.

Another issue has to do with the risk for the final schedule to be far from what could be reached by optimizing the whole system as a single market. The reason is that some attractive market participants (e.g. cheap generators), having placed their bids in a market, may be excluded when the latter is cleared, and thus remain inactive while they could still be used by another TS to reach a better schedule. One could argue that such a case should not persist in the long term, because market participants will "find their place". However, the problem will definitely appear in the short term. Hence, a mechanism should allow efficient shifting of participants between the various TS markets. This issue is covered in Section 5.1 of Chapter 5.

4.2.3 Outline of the proposed approach

Regarding the congestion management issue, the proposed approach consists in sharing between the TSs the capacity of the most used branches so that the mth TS, when clearing its market, would obey reduced flow limits $\overline{\mathbf{p}}_m^{ov}$, where the upperscript ov denotes the set of overloaded branches. The modified limits are such that $\sum_m \overline{\mathbf{p}}_m^{ov} = \overline{\mathbf{p}}^{ov}$, and are iteratively adjusted to the schedules announced by the TSs.

The treatment suggests the presence of a coordinating entity that will iteratively communicate to the TSs their corresponding reduced branch limits, which it will compute based on an agreed policy. This coordinator may result from the joint efforts of the involved TSOs. Its role is to facilitate electricity trading, while respecting the confidentiality of the TS data and the independence of their procedures. In this respect, the only information provided by the TSs to the central coordinator are their power injection schedules.

4.3 General framework for congestion management

From a game-theoretic viewpoint, the TSs make up a set of actors, each setting its control vector

$$\mathbf{u}_m = [\mathbf{g}_m^T \ \mathbf{d}_m^T]^T \ \in \mathbf{U}_m, \tag{4.9}$$

where $\mathbf{U}_m$ encompasses the constraints (4.1b), (4.1c) and (4.1d), in order to minimize an objective function

$$f_m(\mathbf{u}_m) = \mathbf{c}_m^T \mathbf{g}_m - \mathbf{b}_m^T \mathbf{d}_m. \tag{4.10}$$

At the same time, there is a set of constraints, coupling the various TSs' controls, that should be satisfied

$$-\overline{\mathbf{p}} \leq \mathbf{T}\ \mathbf{n} \leq \overline{\mathbf{p}}. \tag{4.11}$$

In other words, a generalized Nash game is played among the TSs, for which a mechanism is sought to ensure the satisfaction of (4.11).

4.3.1 Why not the solution proposed in Chapter 3 ?

One way to deal with the problem could be to resort to the iterative algorithm presented in Chapter 3. This would suggest that, at each iteration, the mth TS includes in its clearing problem (4.1) branch flow constraints of the type:

$$-\overline{\mathbf{p}} \leq \mathbf{T}(\mathbf{n}_m + \sum_{m^-} \mathbf{n}_{m^-}) \leq \overline{\mathbf{p}} \tag{4.12}$$

where m^- denotes all TS markets but the mth one. Indeed, this constraint means that a TS will come up with a schedule that does not cause branch limit violation, given the last schedule announced by the other TSs. Of course, since the other TSs are clearing their own markets at the same time, the combined schedule $\widehat{\mathbf{n}}$ may quite well lead to overloads.

The above idea was, in fact, tested. However, it turned out that the overlapping market problem is too complicated to be coordinated in such a way. Constraints (4.12) practically require that a TS clears any congestion by its own control means whenever it appears after an iteration. This may not be always possible; a TS, with its injection schedule, may have little participation in some overloads and thus little capability to unload them by changing its schedule. This issue is further discussed in Section 4.7.3.

The branch flow limits cannot be enforced by acting on each market irrespective of what the other markets are doing; instead, a *more* coordinated congestion management scheme is required.

4.3.2 Nash equilibrium and corresponding multi-objective problem of the game

In order to make the presentation more compact, let us refer to the mth TS's market clearing problem as follows:

$$\min_{\mathbf{u}_m \in \mathbf{U}_m} f_m(\mathbf{u}_m). \tag{4.13}$$

Furthermore, let us group the branch flow constraints in the following set of linear inequalities:

$$\mathbf{A}\mathbf{u} - \overline{\mathbf{p}} \leq \mathbf{0} \tag{4.14}$$

where $\mathbf{u}$ contains all TS injection schedules and $\mathbf{A}$ is a suitably adjusted matrix, constructed using the PTDFs in $\mathbf{T}$.

A solution defined by a control vector $\mathbf{u}^\star$, is a Nash equilibrium if all constraints are satisfied, and no TS can further improve its objective by modifying its own controls, given the control vectors of the other TSs. Thus, $\mathbf{u}^\star$ yields a Nash equilibrium if :

$$\begin{aligned} \forall m \in \{1, 2, \ldots, M\} : \quad & \mathbf{u}_m^\star = \arg \min_{\mathbf{u}_m \in \mathbf{U}_m} f_m(\mathbf{u}_m) \\ \text{subject to} \quad & \mathbf{A}_m \mathbf{u}_m + \mathbf{A}_{m^-} \ \mathbf{u}_{m^-}^\star - \overline{\mathbf{p}} \leq \mathbf{0} \end{aligned} \tag{4.15}$$

where $\mathbf{u}_{m^-}^\star$ denotes the sub-vector of $\mathbf{u}^\star$ containing the controls of all TSs but the mth one, and $\mathbf{A}_m$ and $\mathbf{A}_{m^-}$ are the corresponding sub-matrices of $\mathbf{A}$.

Following the discussion in Chapter 2, one can group the various TSs' objective functions into a single scalar one, $F(\mathbf{u}) = F(f_1(\mathbf{u}_1), \ldots, f_M(\mathbf{u}_M))$ and write down the game's corresponding multi-objective problem. Taking F as a linear combination of the individual objectives, as is typically done in such cases, yields the following optimization problem:

$$\begin{aligned} & \min_{\mathbf{u}} \sum_m w_m f_m(\mathbf{u}_m) \\ \text{subject to} \quad & \mathbf{A}\mathbf{u} - \overline{\mathbf{p}} \leq \mathbf{0} \end{aligned} \tag{4.16}$$

where the w_m's are weighting factors.

This optimization problem can be solved in the following two ways. Either in a centralized scheme, solved by a central entity applying some commonly agreed rules regarding the allocation of the common resources. Besides the high dimensionality issue, this approach has the drawback of not respecting possible confidentiality restrictions that each TS may want to preserve regarding individual data and strategy. Or, in a decentralized scheme, to deal with the above dimensionality and confidentiality issues, resorting to one of the decentralized algorithms that exist in the literature. There the interconnected system is decomposed into separate sub-systems, each controlled by a TS, the aim being to process the information of each subsystem locally, while at the same time solving the system-wide problem (4.16). To this purpose, a coordination entity is in charge of passing information between players and possibly performing some upper-level computation.

One practical issue when dealing with (4.16) is the choice of the weighting factors w_m. Indeed, the various TSs may question the priorities assigned to their respective objectives through these weighting factors. One option is to try different weighting factors, but this may become computationally intractable.

Normally, as far as market is of concern, all objectives correspond to costs (i.e. they are expressed in the same unit) and hence, a natural choice is to set all w_m's to 1, i.e. consider the objective:

$$F(\mathbf{u}) = \sum_m f_m(\mathbf{u}_m) \tag{4.17}$$

This leads to optimizing the total "social welfare of all participants" within the interconnection.

While this seems desirable from a global system perspective, a TS could argue that it would have better market opportunities (higher social welfare for the market it clears) if it was not incorporated into the overall optimization. Even more, it goes with the freedom and independence of each market to be cleared separately from the others, incorporating maybe its particular rules and operating strategies. The above justify our choice to consider several markets, instead of a single integrated one.

4.3.3 Independent optimizations with a Coordinator

In the previous multi-objective approaches, a central entity is in charge of either solving the system-wide multi-objective optimization or coordinating the decentralized computations aimed at solving that problem. Alternatively, a central entity may be responsible for monitoring and correcting multiple independent optimizations, performed by the M TSs, according to certain rules. These rules will reflect a pre-defined *policy* to share the available resources among the TSs.

Contrary to the single system-wide optimization approach previously considered, the idea is to preserve the operational independence of the TSs. The TSs are not constrained to adopt a common objective. On the contrary, they may formulate their operational strategies in different ways. Thus, the TSs' independence is preserved, but with additional rules applied by the coordinator to reconcile the TSs' decisions.

This approach is developed in the remaining of this chapter. The method consists in decoupling the TS optimization problems by dividing the constraints among them in such a way that each one respecting its part of the constraints will result in the whole, original set of constraints being satisfied. Formally, the mth TS will solve a modified optimization problem of the type:

$$\min_{\mathbf{u}_m} f_m(\mathbf{u}_m) \tag{4.18a}$$

$$\text{subject to} \quad \mathbf{A}_m\mathbf{u}_m - \overline{\mathbf{p}}_m \leq \mathbf{0} \tag{4.18b}$$

where new $\overline{\mathbf{p}}_m$ limits have to be found so that:

$$\mathbf{A}_m\mathbf{u}_m - \overline{\mathbf{p}}_m \leq \mathbf{0},\ \forall m \in \{1,\ldots,M\} \ \Rightarrow\ \mathbf{A}\mathbf{u} - \overline{\mathbf{p}} \leq \mathbf{0} \tag{4.19}$$

Furthermore, the vectors $\overline{\mathbf{p}}_m$ should be adjusted by the coordinator in such a way that a well defined and transparent policy is followed to share the available resources, allowing the TSs to check the coordinator decisions.

These vectors could be assigned *ex ante* by the coordinator, to have the TSs perform M completely independent optimizations. A better option, however, is to construct "dynamically" the vectors $\overline{\mathbf{p}}_m$ while observing the evolution of the successive optimizations performed by the TSs, allowing in some sense the coordinator to intervene in this evolution. This second option is selected here since it combines flexibility of the coordination policy with an as large as possible operational freedom for the TSs. In this spirit, a procedure is presented hereafter where after a number of iterations between the TSs and the coordinator, the whole original set of constraints is satisfied by the final solution of the individual optimization problems.

4.3.4 Constraint decomposition

For the sake of presentation simplicity, we refer here to a case with two TSs, denoted TS1 and TS2 respectively. The generalization to more TSs is straightforward.

After partitioning the control vector, (4.14) is rewritten as:

$$\mathbf{A}_1\mathbf{u}_1 + \mathbf{A}_2\mathbf{u}_2 - \overline{\mathbf{p}} \leq \mathbf{0} \tag{4.20}$$

It is easily seen that if the following constraints are satisfied:

$$\text{by TS1:} \quad \mathbf{A}_1\mathbf{u}_1 - \overline{\mathbf{p}}_1 \leq \mathbf{0} \tag{4.21a}$$

$$\text{by TS2:} \quad \mathbf{A}_2\mathbf{u}_2 - \overline{\mathbf{p}}_2 \leq \mathbf{0} \tag{4.21b}$$

$$\text{where:} \quad \overline{\mathbf{p}}_1 + \overline{\mathbf{p}}_2 = \overline{\mathbf{p}} \tag{4.21c}$$

then the overall constraints (4.20) are also satisfied. The constraints (4.21a) and (4.21b) are of the type (4.19).

Consider now the bth constraint in (4.20), with the corresponding components $(\overline{\mathbf{p}}_1)_b$, $(\overline{\mathbf{p}}_2)_b$ and $\overline{p}_b$ of the $\overline{\mathbf{p}}_1, \overline{\mathbf{p}}_2$ and $\overline{\mathbf{p}}$ vectors, respectively. Clearly, $(\overline{\mathbf{p}}_1)_b + (\overline{\mathbf{p}}_2)_b = \overline{p}_b$. It can be guessed that the values of $(\overline{\mathbf{p}}_1)_b$ and $(\overline{\mathbf{p}}_2)_b$ determine how much of the resource (branch capacity) is being allocated to TS1 and TS2 respectively. For instance, for a higher value of $(\overline{\mathbf{p}}_1)_b$, TS1 may be less constrained and a higher control effort will be put on TS2 to satisfy the bth constraint, and conversely. Thus, the coordinator may implement the agreed congestion management policy by suitably choosing the values $(\overline{\mathbf{p}}_m)_b$ for a congested branch b. Furthermore, the coordinator should share the limited resource in a transparent way, that is, its choice should be justified by information that can be made public to all involved TSs.

Note that a solution $(\mathbf{u}_1, \mathbf{u}_2)$ which satisfies (4.21) will satisfy the original constraints (4.20). However, the converse is not true: it is possible to find controls $\mathbf{u}_1$ and $\mathbf{u}_2$ satisfying (4.20) but not both (4.21a) and (4.21b). Thus the use of (4.21) somewhat reduces the feasible space of the original optimization problem. This is a price to pay for the convenience of the decomposition into independent optimizations.

This reduction of the feasible space, however, should be as low as possible. To this purpose, a procedure is proposed that iteratively adjusts the values of $\overline{\mathbf{p}}_1$ and $\overline{\mathbf{p}}_2$, while converging towards a solution satisfying (4.14).

4.3.5 Adjustment of constraints by the coordinator

Assume that, in a first step, the two TSs optimize their objective functions without taking care of the constraints; let $\widehat{\mathbf{u}}_1$ and $\widehat{\mathbf{u}}_2$ be the corresponding controls. Assume furthermore that the bth constraint in (4.14) is violated by these settings, i.e.

$$\mathbf{a}_{1b}\widehat{\mathbf{u}}_1 + \mathbf{a}_{2b}\widehat{\mathbf{u}}_2 - \overline{p}_b - \delta_b = 0 \tag{4.22}$$

where $\mathbf{a}_{1b}$ and $\mathbf{a}_{2b}$ are the bth rows of matrices $\mathbf{A}_1$ and $\mathbf{A}_2$, respectively, and $\delta_b > 0$ is the amount by which branch b is overloaded. New controls $\mathbf{u}_1$ and $\mathbf{u}_2$ are sought, such that:

$$\mathbf{a}_{1b}\mathbf{u}_1 + \mathbf{a}_{2b}\mathbf{u}_2 - \overline{p}_b \leq 0 \tag{4.23}$$

Subtracting (4.22) from (4.23) gives:

$$\mathbf{a}_{1b}(\mathbf{u}_1 - \widehat{\mathbf{u}}_1) + \mathbf{a}_{2b}(\mathbf{u}_2 - \widehat{\mathbf{u}}_2) + \delta_b \leq 0 \tag{4.24}$$

Let the amount δ_b be shared over the two TSs according to:

$$\delta_b = \alpha_1\delta_b + \alpha_2\delta_b \tag{4.25}$$

where the choice of the α_1 and α_2 coefficients reflects the coordinator's policy regarding the treatment of the constraint. Introducing (4.25) into (4.24) yields:

$$\mathbf{a}_{1b}\mathbf{u}_1 + \alpha_1\delta_b - \mathbf{a}_{1b}\widehat{\mathbf{u}}_1 + \mathbf{a}_{2b}\mathbf{u}_2 + \alpha_2\delta_b - \mathbf{a}_{2b}\widehat{\mathbf{u}}_2 \leq 0 \tag{4.26}$$

This inequality suggests the following decomposition of the bth constraint in accordance with (4.21):

$$\text{for TS1:} \quad \mathbf{a}_{1b}\mathbf{u}_1 + \alpha_1\delta_b - \mathbf{a}_{1b}\widehat{\mathbf{u}}_1 \leq 0 \tag{4.27a}$$

$$\text{for TS2:} \quad \mathbf{a}_{2b}\mathbf{u}_2 + \alpha_2\delta_b - \mathbf{a}_{2b}\widehat{\mathbf{u}}_2 \leq 0 \tag{4.27b}$$

This is equivalent to setting:

$$(\overline{\mathbf{p}}_1)_b = \mathbf{a}_{1b}\widehat{\mathbf{u}}_1 - \alpha_1\delta_b \tag{4.28a}$$

$$(\overline{\mathbf{p}}_2)_b = \mathbf{a}_{2b}\widehat{\mathbf{u}}_2 - \alpha_2\delta_b \tag{4.28b}$$

It is easily checked that $(\overline{\mathbf{p}}_1)_b + (\overline{\mathbf{p}}_2)_b = \overline{p}_b$.

Generalizing, irrespective of the number of TSs, for each overloaded branch corresponding to a constraint b, the coordinator should choose the coefficients α_m^b, with $\sum_m \alpha_m^b = 1$. As a result, the branch capacity will be shared among the TSs, the mth one receiving a modified bound $(\overline{\mathbf{p}}_m)_b$, with $\sum_m (\overline{\mathbf{p}}_m)_b = \overline{p}_b$.

If all TSs solve their market clearing problems (4.1), each of them with one additional constraint of the type:

$$\mathbf{a}_{mb}\mathbf{u}_m - (\overline{\mathbf{p}}_m)_b \leq 0 \tag{4.29}$$

then, the new overall solution $\widehat{\mathbf{u}}$ will be such that the bth constraint will be satisfied. Now, other constraints may be found violated by the new solution. If so, the coordinator will in the same way share their transmission capacities among the TSs which, in their turn, will clear their markets incorporating the new constraints. In order not to get violated again in the remaining of the procedure, each constraint b found violated once should remain in the set of constraints decomposed by the coordinator and incorporated into the TSs' clearings at subsequent iterations. If a constraint is no longer violated, δ_b will obviously be negative (or equal to zero) but this does not affect the validity of the formula used for sharing the transmission capacity.

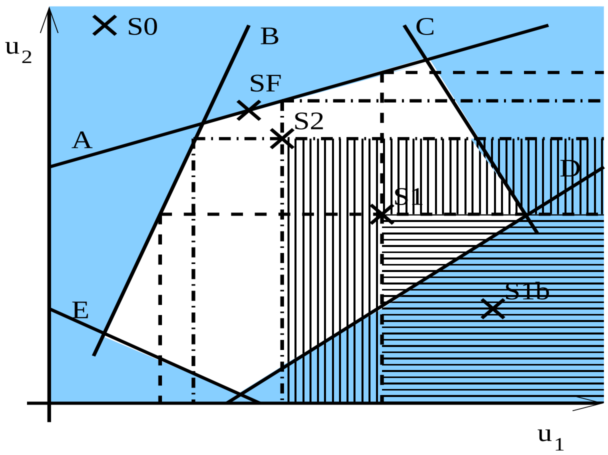

Figure 4.1: Graphical illustration

In summary, at every iteration k of the algorithm, the coordinator collects the TSs' control vectors $\mathbf{u}_m^{(k)}$, identifies the resulting branch overloads needing corrections $\delta_b^{(k)}$ and the remaining available capacities of branches that have been overloaded in a previous iteration, and, decides the coefficients $\alpha_m^{(k)}$ that define the next share of the branch capacities by the various TSs.

As long as a branch does not get overloaded, the coordinator does not impose any constraint to the TSs.

4.3.6 Graphical representation

The decomposition of the set of linear constraints, as well as the iterative adjustment of the decomposed constraints can be illustrated in a graphical way as follows. A two-TS case with one control variable per TS is assumed. Each constraint b is a linear combination of the two controls: $a_{1b}u_1 + a_{2b}u_2 + \beta_b \leq 0$.

In Fig. 4.1 the feasible region corresponding to five such constraints is presented (non colored area). Note that it is not possible to construct the same region by constraints that involve either u_1 only or u_2 only.

Let us assume that the solution resulting from the independent market clearings violates two of the constraints (point S0 and constraints A and B in Fig. 4.1). This infeasibility initiates the iterative algorithm and each of the two constraints is decomposed following the congestion management policy. This results into two constraints being communicated to each TS, one for each overloaded branch: $a_{1A}u_1 + \beta_{1A} \leq 0$ to TS1 and $a_{2A}u_2 + \beta_{2A} \leq 0$ to TS2 for branch A (vertical and horizontal dashed lines starting from a point on A), $a_{1B}u_1 + \beta_{1B} \leq 0$ to TS1 and $a_{2B}u_2 + \beta_{2B} \leq 0$ to TS2 for branch B (similarly, dashed lines starting on B). Each pair of these constraints guarantees that at the next iteration the original constraint will be satisfied while they share the corresponding available transmission capacity between the two TSs. Note that the non violated constraints remain "invisible" to the TSs; the searched space is not reduced unless a constraint violation is encountered. In fact, the searched space for the next solution is the intersection of the above decomposed constraints and is highlighted with horizontal lines in the figure.

Let the point S1 in Fig. 4.1 be the new solution that results from the next iteration. As this solution happens to be feasible, it could be chosen to be actually implemented and the procedure could stop here. However, in order to give the TSs the opportunity to improve their schedules, constraints A and B are once more decomposed, based on the present operating point (S1), again according to the congestion management policy. The dashed-dotted lines in Fig. 4.1 indicate this new decomposition. One can see that the searched space for the new solution has now been enlarged by the area shown with vertical lines in the figure. This results in a new solution (point S2). The procedure continues in the same, finally converging to the point SF where one of the two initially violated constraints is active.

It is noteworthy that the coordinator has not as objective to guarantee the feasibility of the next iteration's solution. It just checks for convergence and shares the capacity of the already overloaded lines. If at any step of the algorithm a new branch gets overloaded, the corresponding constraint will be also subsequently decomposed among the TSs. Coming back to the example of Fig. 4.1, if S1b had been the solution after iteration 1, then constraint D would have been also decomposed and communicated to the TSs, obliging them to provide solutions above (for TS2) and to the left (for TS1) of the two new decomposed constraints, making the searched space be a rectangle.

4.3.7 Nash equilibrium property of the solution

It is important for the algorithm to provide solutions that are Nash equilibria of the original uncoordinated problem, defined by each TS clearing independently its market as in (4.15). The reason is that this makes the final point acceptable by all TSs, since nobody has the power to modify it for its own profit by its sole means only.

This can be visualized in Fig. 4.1, where point SF denotes the final solution of the algorithm. No TS can, modifying its control, improve its objective (assuming that TS1 tries to decrease u_1 and TS2 to increase u_2 as suggested by the example) without violating the problem's original constraints (in particular constraint A). This makes SF a Nash equilibrium.

Let us show that this is, indeed, a general property of the algorithm.

Let us recall that even if no branch is overloaded at a given iteration (no $\delta_b > 0$) the procedure continues, sharing among the TSs the remaining capacities of the previously overloaded branches according to the congestion policy, until no change in flows is encountered between two subsequent iterations. Hence, at the solution, all branches fall into one of the three categories: 1. they have never been overloaded; 2. their capacity is totally used ($\delta_b = 0$); or 3. they have been overloaded but, finally, their capacity is not fully used ($\delta_b < 0$). The third case may happen if a line flow is limited as a side effect of the effort to unload another branch.

For the fully used branches, it can be shown using (4.22) and (4.28) that the corresponding inequality constraint in (4.15) is the same as the constraint (4.29) at the equilibrium of the proposed coordinated algorithm. The other constraints in (4.15) do not affect the solution obtained at the last iteration of the proposed algorithm, since they are not binding. So, they should not affect the solution of problem (4.15) either. As a result, the solution obtained by each TS when solving (4.15) with the other controls fixed to the solution of the algorithm, is to keep itself the same control settings. This by definition makes this solution a Nash equilibrium of the original uncoordinated problem.

4.4 Choosing a congestion management policy

4.4.1 Reduced transmission capacity allocation

The time has come to choose a policy for managing congestion. This policy essentially consists in, dynamically during the execution of the procedure, allocating transmission capacity to be used by the TSs to settle their transactions.

Assume that after the various market clearings the power flow $\widehat{p}_b$ in the bth branch $(b = 1, \ldots, B)$ exceeds its upper limit:

$$\widehat{p}_b > \overline{p}_b \tag{4.30}$$

Using Eqs. (4.3) and (4.6), this inequality can be rewritten as:

$$\sum_m \mathbf{t}_b\, \widehat{\mathbf{n}}_m > \overline{p}_b \tag{4.31}$$

where $\mathbf{t}_b$ is the bth row of the $\mathbf{T}$ matrix and $\widehat{\mathbf{n}}_m$ is the schedule of the mth TS, obtained as described in Section 4.2.

It turns out that $\mathbf{t}_b\widehat{\mathbf{n}}_m$ is the participation of the mth TS in the bth branch flow. Obviously, all TS participations add up to the actual branch flow p_b.

As mentioned in Section 4.2.3, only non commercially-sensitive information, such as the cleared schedules from TSs, should be communicated between involved parties. In this context, it is proposed to allocate transmission capacity to TSs *in proportion to their respective utilizations of the congested branches*.

Coming back to the overloaded branch b, let us call $\Delta p_m^- > 0$ the amount by which the mth TS is asked by the coordinator to decrease its contribution to the branch flow p_b by modifying its schedule from $\widehat{\mathbf{n}}_m$ to a new value $\mathbf{n}_m$. Following this notation, (4.27) takes on the form:

$$\mathbf{t}_b(\mathbf{n}_m - \widehat{\mathbf{n}}_m) \leq -\Delta p_m^- \tag{4.32}$$

with the sum of all Δp_m^- values being equal to the branch overload to be corrected:

$$\sum_m \Delta p_m^- = \delta_b = \sum_m \mathbf{t}_b\widehat{\mathbf{n}}_m \; - \; \overline{p}_b \tag{4.33}$$

Equation (4.32) can be equivalently written as:

$$\mathbf{t}_b\mathbf{n}_m \leq \mathbf{t}_b\widehat{\mathbf{n}}_m - \Delta p_m^- \tag{4.34}$$

where the left-hand side represents the new flow produced in branch b by the new schedule of the mth TS, and the right-hand side can be interpreted as a reduced capacity allocated to that TS.

The congestion management policy choice suggests that the constraint (4.34), reflecting the share of the transmission capacity among the TSs, should be:

$$\mathbf{t}_b\mathbf{n}_m \leq \frac{\mathbf{t}_b\widehat{\mathbf{n}}_m}{\sum_m \mathbf{t}_b\widehat{\mathbf{n}}_m}\overline{p}_b \tag{4.35}$$

The above equation is equivalent, as can be shown by using (4.33) and (4.34), to choosing:

$$\frac{\Delta p_m^-}{\sum_m \Delta p_m^-} = \frac{\mathbf{t}_b\widehat{\mathbf{n}}_m}{\sum_m \mathbf{t}_b\widehat{\mathbf{n}}_m} \tag{4.36}$$

Similarly, if the branch overload has the opposite sign, i.e.

$$\widehat{p}_b \leq -\overline{p}_b < 0 \tag{4.37}$$

the mth TS is required to change its schedule so that its contribution to the branch flow p_b is increased by at least a specified amount $\Delta p_m^+ > 0$ (with $\sum_m \Delta p_m^+ = -\overline{p}_b \; - \; \sum_m \mathbf{t}_b\widehat{\mathbf{n}}_m$):

$$\mathbf{t}_b(\mathbf{n}_m - \widehat{\mathbf{n}}_m) \geq \Delta p_m^+ \tag{4.38}$$

with Δp_m^+ taken as:

$$\frac{\Delta p_m^+}{\sum_m \Delta p_m^+} = \frac{\mathbf{t}_b\widehat{\mathbf{n}}_m}{\sum_m \mathbf{t}_b\widehat{\mathbf{n}}_m} \tag{4.39}$$

Equation (4.35) suggests that the more a TS is using a congested branch the more it has the right to keep on using it. This goes towards increasing efficiency: the more a TS uses a branch, the more this is likely to be valuable for its schedule.

On the other hand, (4.35) can be rewritten as

$$\mathbf{t}_b(\mathbf{n}_m - \widehat{\mathbf{n}}_m) \leq \frac{\mathbf{t}_b\widehat{\mathbf{n}}_m}{\sum_m \mathbf{t}_b\widehat{\mathbf{n}}_m}(\overline{p}_b - \mathbf{t}_b\widehat{\mathbf{n}})$$

which shows that the more a TS participates in a congestion, the more it has to participate in its alleviation. This meets the objective of fairness and practical acceptability of the policy: the larger the responsibility of a TS in a flow, the larger the correction requested from this TS.

These two interpretations of (4.35) may look contradictory at a first glance but are mathematically equivalent owing to the choice of proportionality. This and further aspects of the allocation rule are further discussed in Section 4.7.1.

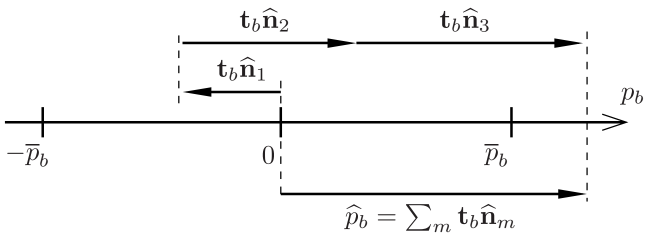

Figure 4.2: Example of counterflow situation

4.4.2 Counterflow situations

It may happen that the schedule of a TS creates a counterflow in an overloaded branch. This situation is depicted in Fig. 4.2, which refers to a case with three TSs. In the situation shown, the branch is overloaded but the contribution $\mathbf{t}_b\widehat{\mathbf{n}}_1$ of the first TS is in the opposite direction with respect to the power flow $\widehat{p}_b$. Clearly, this TS reduces the overload caused by the other two TSs.

It would not be fair to impose a congestion management constraint to a TS that contributes with such a counterflow, since the latter in fact reduces the overload created by the other TS schedules. On the contrary, the counterflow leaves more room for the transactions of the other TSs, which is good from the market viewpoint. Hence, when allocating the available capacities among TSs, it is reasonable to let unconstrained the TSs that cause counterflows and share the effort among the other TSs. To this purpose, for a branch with an upper limit violation (4.30) it suffices to use (4.36) with the sums extending only over the schedules with positive contributions $\mathbf{t}_b\widehat{\mathbf{n}}_m$. Similarly, for a branch with a lower limit violation (4.37), only the schedules with a negative contribution are considered when using (4.39).

As explained in Section 4.3.5, iterations are performed between market clearings by the TSs, on one hand, and Transmission allocation by the coordinator, on the other hand. If the TS producing the counterflow is not requested to change its schedule, there is no reason for that TS to depart from its optimum, and it will keep on contributing with the same counterflow. On the other hand, if the handling of another branch overload requires the TS to change its schedule, it may happen that its counterflow is decreased. In this case, at the next iteration, the branch will still be overloaded and through a new application of (4.32), (4.38) the other TSs will be requested to contribute more towards its alleviation. Obviously, if a TS stops counterflowing, it enters the congestion management procedure as the other TSs.

4.4.3 Handling multiple congestions

As explained in Section 4.3.5, after a branch overload has been handled it should be prevented from taking place again in subsequent iterations. To this purpose, the inequality constraints (4.32), (4.38) stemming from previous congestion managements remain in effect when dealing with new congestions. For the formerly congested branches, the constraints essentially share

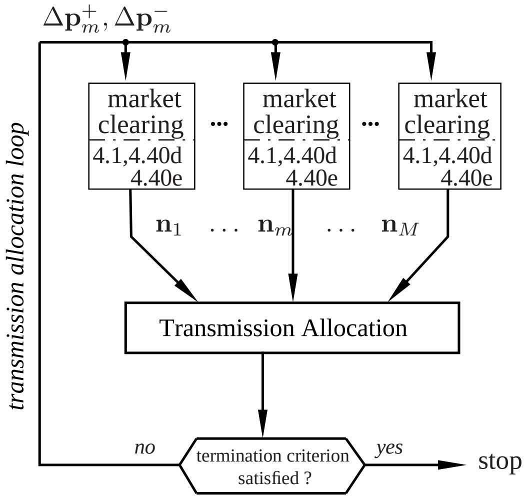

Figure 4.3: Flowchart of the iterative Transmission allocation

among the TSs the remaining part of available capacity (i.e. Δp_m^- and Δp_m^+ are negative for such branches).

4.5 Overview of the Transmission allocation procedure

In Fig. 4.3, the iterative procedure implored to manage congestion, from now referred to as "Transmission allocation loop", is illustrated in form of a flowchart. The criterion used to stop the iterations is explained hereafter.

In Section 4.3.6 it was suggested that the algorithm is executed until convergence to an equilibrium. In practice this is done by preforming a convergence test on all branches that have been involved in constraints (4.32, 4.38). If any power flow differs from the value at the previous iteration by more than a tolerance ϵ, the algorithm proceeds with a new Transmission allocation loop; otherwise the procedure is completed.

One could think of stopping the iterations as soon as the schedules resulting from the M simultaneous market clearings do not lead to any new branch overload. The reason for not doing so can be seen from the following counterexample. Due to the flow it causes in branch b, the constraint $\mathbf{t}_b(\mathbf{n}_m - \widehat{\mathbf{n}}_m) \leq -\Delta p_m^-$ is imposed to the mth TS, and $\mathbf{t}_b(\mathbf{n}_k - \widehat{\mathbf{n}}_k) \leq -\Delta p_k^-$ to the kth TS. Assume furthermore that when clearing its market, the kth TS comes up with a schedule $\mathbf{n}_k^{new}$ such that its participation to the bth power flow is lower than expected, i.e. $\mathbf{t}_b(\mathbf{n}_k^{new} - \widehat{\mathbf{n}}_k) < -\Delta p_k^-$ (which may happen if this TS has to satisfy other constraints as well). Then, some transmission capacity is left unused. The procedure should not stop but leave the mth TS the opportunity to exploit this margin, for the sake of economic efficiency. However, if needed due to limited remaining time, the procedure could stop at an intermediate, already available, feasible schedule.

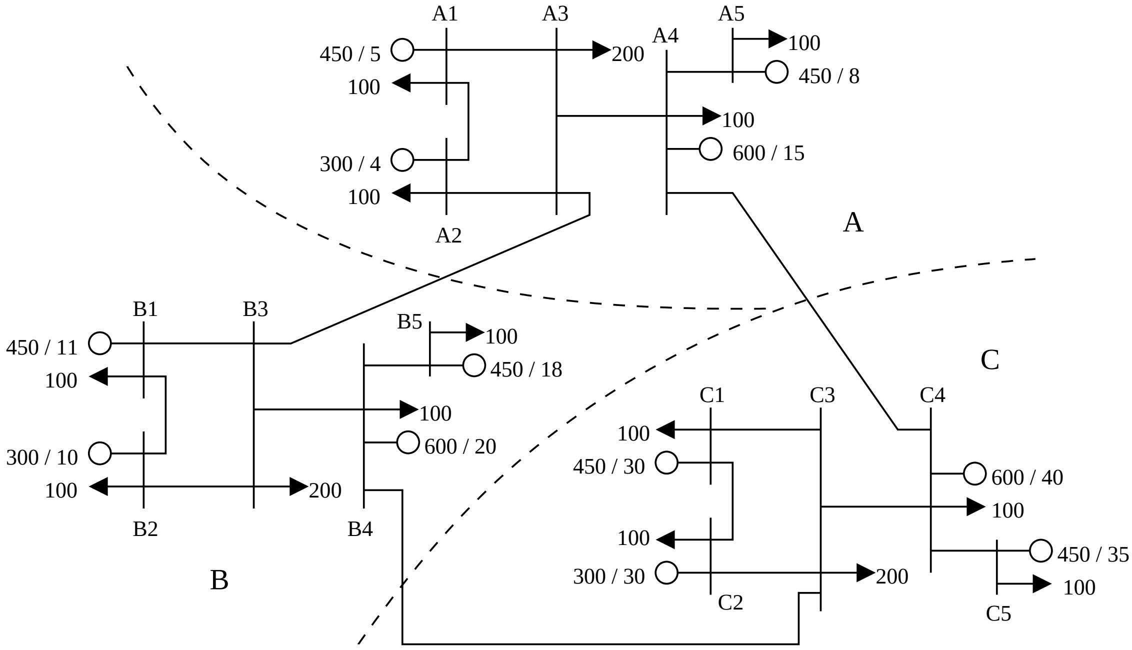

Figure 4.4: Three-area test system

4.6 Illustrative example

4.6.1 Test system

For clarity, we illustrate the features of the Transmission allocation algorithm on a problem where: (i) the loads are considered inelastic, i.e. only the generators are bidding, and (ii) each TS serves the load of an area. Note that the method is generally able to handle situations where each TS serves loads dispersed throughout the whole system, or some loads place bids to more than one TSs.

Thus, each TS dispatches generation, located anywhere in the interconnection, so as to satisfy the load located in its area. This leads to the simple market clearing for the mth TS:

$$\min_{\mathbf{g}_m} \mathbf{c}_m^T \mathbf{g}_m \tag{4.40a}$$

$$\text{s. t.} \quad \mathbf{1}^T \mathbf{g}_m = \mathbf{1}^T \mathbf{d}_m \tag{4.40b}$$

$$\mathbf{0} \le \mathbf{g}_m \le \overline{\mathbf{g}}_m \tag{4.40c}$$

$$\mathbf{t}_b(\mathbf{g}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m) \le -(\Delta \mathbf{p}_m^-)_b \quad b = 1, \ldots \tag{4.40d}$$

$$\mathbf{t}_b(\mathbf{g}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m) \ge (\Delta \mathbf{p}_m^+)_b \quad b = 1, \ldots \tag{4.40e}$$

where all symbols have been previously defined, and the last two constraints stem from the coordinator.

Consider the three-area 15-bus system shown in Fig. 4.4. It consists of three five-bus areas, each of them serving 600 MW of load, and denoted by a letter (A, B and C) also used to name the TS that serves the area ("TS A", "TS B" and "TS C"). The three areas are identical as regards the distribution of loads and the location and capacity of generators. However, they differ by the generator bids, which are the cheapest in area A and the most expensive in area C. Next to each generator, its maximum production capacity (in MW) as well as its bid (in €/MWh) are shown. In order to make the steps of the algorithm easier to follow in the provided example, each generator capacity has been divided by three, i.e. each generator bids one third of its capacity to every TS (for example, $(\overline{\mathbf{g}}_m)_{A1} = 150MW$ for all three TSs). For the same objective of clarity, the same bid per generator has been placed to all the TSs (i.e. $(\mathbf{c}_A)_i = (\mathbf{c}_B)_i = (\mathbf{c}_C)_i$ for all generators i). Generally, it is the choice of each generator how much of its capacity it offers to every market and at what price (a generator may bid differently to different markets). A table with the system's branch reactances is presented in Appendix B.

4.6.2 Insight into the Transmission allocation iterations

In order to provide insight on how the algorithm performs, we present hereafter the results obtained at the first three iterations of the procedure, followed by those of the final generation schedule.

At the initial point, all TSs are allowed to schedule the generators that have placed bids in their markets without any constraint other than (4.40b) and (4.40c). Obviously, this leads to all of them demanding the cheapest generations, i.e. all TSs schedule generation in ascending order of price until they reach the total load quantity. This yields the situation detailed in Table 4.1. For each generator, Columns 1 and 2 give its name and bid (€/MWh), Columns 3 to 5 show the power scheduled by each TS (MW), Column 6 gives its total dispatched generation (i.e. the sum of Columns 3 to 5), while Column 7 shows its maximum production capacity (dividing this quantity by three gives the maximum capacity that is offered to each TS).

At this stage, the coordinator can determine the resulting flows and check the corresponding limits. All the branch flows, computed by the coordinator, are given in Table 4.2 (in MW). Columns 2 to 4 show the participation of each TS to each branch flow, while Columns 5 and 6 give respectively the branch power flow and its limit. The last three columns of the table show by how much each TS will be requested to change each power flow in its next market clearing, according to (4.36). Adding together the various Δp_m^- values of a branch yields the overload $p_b - \overline{p}_b$ that has to be corrected. A dash (-) in this field means that the TS has no obligation regarding the corresponding branch flow when clearing its market at the next iteration.

It is noteworthy that TS A is obliged to decrease the flows in branches A1A3 and A2A3 by less than the other two TSs, even if all three have scheduled the same power from generators gA1 and gA2. This is due to the fact that TS A serves some loads on buses A1 and A2, which makes it less responsible for the flows in those two branches.

Finally, the dash in the last but one row of Table 4.2 stems from the fact that TS A is not requested to change its contribution to the branch flow A4C4 because it is counterflowing, as explained in Section 4.4.2. Indeed, TS A has a negative contribution of -41 MW to the final

Table 4.1: 1st iteration; generation scheduled by each TS

Gen	Bid	TS A	TS B	TS C	Total	Max
gA1	5	150	150	150	450	450
gA2	4	100	100	100	300	300
gA4	15	0	0	0	0	600
gA5	8	150	150	150	450	450
gB1	11	100	100	100	300	450
gB2	10	100	100	100	300	300
gB4	20	0	0	0	0	600
gB5	18	0	0	0	0	450
gC5	35	0	0	0	0	450

Table 4.2: 1st iteration; resulting flows

Branch	TS A	TS B	TS C	p_b	$\overline{p}_b$	Δp_A^-	Δp_B^-	Δp_C^-
A1A2	17	18	18	53	100	-	-	-
A1A3	32	133	133	**298**	150	16	66	66
A2A3	17	118	118	**253**	150	7	48	48
A3A4	8	-25	175	158	400	-	-	-
A4A5	-50	-150	-150	-350	400	-	-	-
B1B2	0	0	0	0	100	-	-	-
B1B3	100	0	100	**200**	150	25	0	25
B2B3	100	0	100	**200**	150	25	0	25
B3B4	41	75	275	391	400	-	-	-
B4B5	0	100	0	100	400	-	-	-
C1C2	0	0	0	0	100	-	-	-
C1C3	0	0	-100	-100	150	-	-	-
C2C3	0	0	-100	-100	150	-	-	-
C3C4	41	-125	-125	-209	400	-	-	-
C4C5	0	0	100	100	400	-	-	-
A3B3	-158	275	75	192	200	-	-	-
A4C4	-41	125	325	**409**	200	-	58	151
B4C3	41	-125	275	191	200	-	-	-

branch flow of 409 MW. The necessary power flow decrease by 409-200 = 209 MW is assigned to the other two TSs, in proportion to their participation.

This completes the first execution of the Transmission allocation loop. At this point the TSs perform new market clearings incorporating the constraints (4.40d) and (4.40e) (actually, in this example, all new constraints are of type (4.40d)). The corresponding demanded generations are shown in Columns 3 to 5 of Table 4.3.

What makes the TSs adjust their schedules with respect to the values in Table 4.1 is the addition of the constraints dealing with the overloaded branches. For instance, TS C is obliged to abandon most of the power it planned to obtain from generators located in system A, in order to decrease by 151 MW the flow it causes on the tie-line A4C4 (see Table 4.2). In the same way, TS A and B had to reschedule some generation in order to satisfy the additional constraints.

The new power flows are detailed in Table 4.4, which illustrates other features of the method.

Table 4.3: 2nd iteration; generation scheduled by each TS

Gen	Bid	TS A	TS B	TS C	Total	Max
gA1	5	125	63	0	188	450
gA2	4	100	73	100	273	300
gA4	15	75	0	0	75	600
gA5	8	150	110	0	260	450
gB1	11	75	100	75	250	450
gB2	10	75	100	75	250	300
gB4	20	0	4	149	153	600
gB5	18	0	150	150	300	450
gC5	35	0	0	51	51	450

Table 4.4: 2nd iteration; resulting flows

Branch	TS A	TS B	TS C	p_b	$\overline{p}_b$	Δp_A^-	Δp_B^-	Δp_C^-
A1A2	9	-3	-35	-29	100	-	-	-
A1A3	16	67	35	118	150	-4	-18	-10
A2A3	9	70	65	144	150	0	-3	-3
A3A4	-46	-43	175	86	400	-	-	-
A4A5	-50	-110	0	-160	400	-	-	-
B1B2	0	0	0	0	100	-	-	-
B1B3	75	0	75	150	150	0	0	0
B2B3	75	0	75	150	150	0	0	0
B3B4	21	-21	75	75	400	-	-	-
B4B5	0	-50	-150	-200	400	-	-	-
C1C2	0	0	0	0	100	-	-	-
C1C3	0	0	-100	-100	150	-	-	-
C2C3	0	0	-100	-100	150	-	-	-
C3C4	21	-67	-26	-72	400	-	-	-
C4C5	0	0	49	49	400	-	-	-
A3B3	-129	179	-75	-25	200	-	-	-
A4C4	-21	67	175	**221**	200	-	6	15
B4C3	21	-67	374	**328**	200	7	-	121

First, one can see that all the previously overloaded branches have been brought back within limits, except tie-line A4C4. The reason is that not all TSs have participated in alleviating the congestion of that branch. Indeed, after the first iteration, the necessary A4C4 flow decrease of 209 MW was assigned to TS B and C, while TS A was left unconstrained owing to the counterflow it was creating. As a matter of fact, TS B and TS C have decreased their contribution by the expected 209 MW amount, but the new market clearing of TS A contributes to the branch flow with -21 MW instead of the previous -41 MW. This change is driven by the new constraints imposed to TS A. Therefore, the line remains overloaded by -21 - (-41) = 20 MW[3], as shown in Table 4.4. Hence, new corrections are going to be imposed, in which, again, TS A will not participate since it continues to counterflow. In fact, when all TSs are assigned responsibility for an overload (i.e. no one counterflows), then, at the next step, the branch will certainly be unloaded, since (4.19) holds true. On the contrary, when at least one TS is coun-

[3]The 1 MW of difference with respect to the 200-221=-21 MW in the table is due to roundoff when presenting results without decimal digits.

Table 4.5: 3nd iteration; generation scheduled by each TS

Gen	Bid	TS A	TS B	TS C	Total	Max
gA1	5	134	99	17	250	450
gA2	4	96	59	95	250	300
gA4	15	93	0	0	93	600
gA5	8	150	84	0	234	450
gB1	11	27	100	75	202	450
gB2	10	100	100	75	275	300
gB4	20	0	8	0	8	600
gB5	18	0	150	150	300	450
gC1	30	0	0	58	58	450
gC2	30	0	0	100	100	300
gC4	40	0	0	0	0	600
gC5	35	0	0	30	30	450

Table 4.6: 3nd iteration; resulting flows

Branch	TS A	TS B	TS C	p_b	$\overline{p}_b$	Δp_A^-	Δp_B^-	Δp_C^-
A1A2	13	14	-27	0	100	-	-	-
A1A3	21	85	44	150	150	0	0	0
A2A3	9	73	68	150	150	0	0	0
A3A4	-57	-23	160	80	400	-	-	-
A4A5	-50	-84	0	-134	400	-	-	-
B1B2	-25	0	0	-25	100	-	-	-
B1B3	53	0	75	128	150	-9	0	-13
B2B3	75	0	75	150	150	0	0	0
B3B4	14	-19	103	98	400	-	-	-
B4B5	0	-50	-150	-200	400	-	-	-
C1C2	0	0	-15	-15	100	-	-	-
C1C3	0	0	-28	-28	150	-	-	-
C2C3	0	0	-15	-15	150	-	-	-
C3C4	15	-61	10	-36	400	-	-	-
C4C5	0	0	70	70	400	-	-	-
A3B3	-113	180	-47	20	200	-	-	-
A4C4	-14	61	160	**207**	200	-	2	5
B4C3	14	-61	253	**206**	200	1	-	5

terflowing an overloaded branch, then it is possible that the branch remains overloaded at the next step. However, this does not really cause a problem; these calculations are nothing but intermediate steps. At the end of the procedure no branch remains overloaded.

Next, it should be pointed out that for branches that were previously overloaded but are not anymore (namely, A1A3 and A2A3) the remaining capacity is now shared among the TSs in proportion to their contributions to the flows. This yields the negative values of Δp_m^- shown in the table. In fact, the reader can ascertain that, for each branch b that has been overloaded at least once, adding together the three Δp_m^- corresponding to the three TSs gives a total Δp_b which is exactly equal to the difference between the present flow and the maximum one: $\Delta p_b = p_b - \overline{p}_b$. This holds true irrespective of whether the branch is overloaded at this iteration or not. If the branch is overloaded, the constraint distributes among the TSs the effort to bring back

Table 4.7: Final point; generation scheduled by each TS

Gen	Bid	TS A	TS B	TS C	Total	Max
gA1	5	134	99	17	250	450
gA2	4	96	59	95	250	300
gA4	15	94	0	0	94	600
gA5	8	150	80	0	230	450
gB1	11	26	100	123	249	450
gB2	10	100	100	50	250	300
gB4	20	0	12	0	12	600
gB5	18	0	150	115	265	450
gC1	30	0	0	28	28	450
gC2	30	0	0	100	100	300
gC4	40	0	0	0	0	600
gC5	35	0	0	72	72	450

the branch flow within the feasible limits, while if it is not overloaded, the constraint shares the remaining branch capacity among the TSs. The same congestion management rule is used in both cases.

Finally, a new branch (B4C3) gets overloaded and hence enters the set of constraints (only for TSs A and C, since TS B is counterflowing in this branch).

A new round of market clearings with these updated branch flow constraints yields the generation schedules shown in Table 4.5 with the resulting flows of Table 4.6.

4.6.3 Features of the final generation schedules

The algorithm proceeds similarly until the congested branch flows differ by less than $\epsilon = 2$ MW from their values at the previous iteration. This takes place after 7 iterations and yields the final values presented in Table 4.7 (Columns 3 to 6). These are the generation productions to be actually implemented, i.e. they are the quantities that the TSs will ask from the generators to produce and for which they will have to pay them the corresponding prices (each TS according to its own pricing rules).

From a market participant's perspective, the results in Table 4.7 are the market clearing result(s) of the TS(s) where it placed its bid(s). The previously presented iterations (see Section 4.6.2) are computations executed between the TSs in order for them to "share" the use of the transmission network; these computations do not correspond to actual productions and consumptions by the there-scheduled market participants neither do they involve any action or decision from their (i.e. the market participants') part. The resulting branch flows are shown in Table 4.8. No branch is overloaded, while all previously congested branches are fully used. These are the branches that, from the first steps of the algorithm, turned out to be the most crucial for the satisfaction of the most economic generation schedules.

It is also noteworthy that TS A finally manages to allocate mainly the less expensive generators

Table 4.8: Final point; resulting flows

Branch	TS A	TS B	TS C	p_b	$\overline{p}_b$	Δp_A^-	Δp_B^-	Δp_C^-
A1A2	13	14	-27	0	100	-	-	-
A1A3	21	85	44	150	150	0	0	0
A2A3	9	73	68	150	150	0	0	0
A3A4	-57	-21	154	76	400	-	-	-
A4A5	-50	-80	0	-130	400	-	-	-
B1B2	-26	0	26	0	100	-	-	-
B1B3	52	0	98	150	150	0	0	0
B2B3	74	0	76	150	150	0	0	0
B3B4	14	-21	131	124	400	-	-	-
B4B5	0	-50	-114	-164	400	-	-	-
C1C2	0	0	-25	-25	100	-	-	-
C1C3	0	0	-47	-47	150	-	-	-
C2C3	0	0	-25	-25	150	-	-	-
C3C4	14	-59	-27	-72	400	-	-	-
C4C5	0	0	28	28	400	-	-	-
A3B3	-112	178	-42	24	200	-	-	-
A4C4	-14	60	154	200	200	-	0	0
B4C3	14	-60	246	200	200	0	-	0

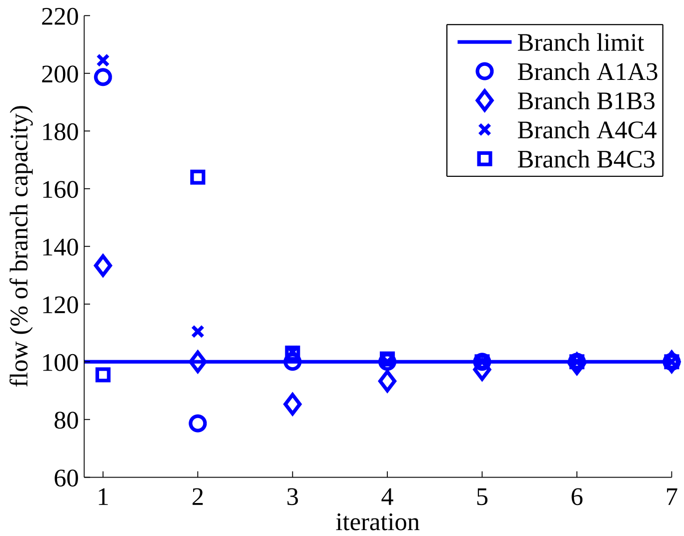

Figure 4.5: Evolution of power flows with iterations

(located geographically in its area), while on the other hand, TS C is mostly obliged to resort to some expensive generators (geographically located in its area). This makes sense since TS C is the main responsible for loading the tie-lines A4C4 and B4C3, and, consequently, it is the one who is mainly assigned the effort for unloading.

Figure 4.5 shows the evolution of four of the congested branch flows through the successive iterations. The horizontal line corresponds to the branch flow limit. Worth mentioning is the fact that already in four iterations the flows have almost converged to their final values. Full

Table 4.9: System-wide market clearing

Gen	Bid	TS A	TS B	TS C	Total
gA1	5	150	50	50	250
gA2	4	100	100	50	250
gA4	15	0	0	0	0
gA5	8	150	150	0	300
gB1	11	0	100	150	250
gB2	10	50	100	100	250
gB4	20	0	0	0	0
gB5	18	150	0	150	300
gC1	30	0	0	0	0
gC2	30	0	100	100	200
gC4	40	0	0	0	0
gC5	35	0	0	0	0

Table 4.10: System-wide market clearing; resulting flows

Branch	TS A	TS B	TS C	p_b	$\overline{p}_b$
A1A3	33	67	50	150	150
A2A3	17	83	50	150	150
B1B3	17	0	133	150	150
B2B3	33	0	117	150	150
A3B3	-133	225	-92	0	200
A4C4	67	-175	308	200	200
B4C3	-67	75	192	200	200

utilization of the branch capacities is finally achieved.

For comparison purposes, a system-wide market clearing has been considered. It consists in minimizing the total production cost throughout the interconnection, i.e. minimize (4.17), subject to all the TSs individual constraints as well as the branch flow coupling constraints. This yields the following optimization problem:

$$\min_{\mathbf{g}_A,\mathbf{g}_B,\mathbf{g}_C} \sum_m \mathbf{c}_m^T \mathbf{g}_m \tag{4.41a}$$

$$\text{s. t.} \quad \mathbf{1}^T \mathbf{g}_m = \mathbf{1}^T \mathbf{d}_m \qquad \forall\, m \in \{A, B, C\} \tag{4.41b}$$

$$\mathbf{0} \leq \mathbf{g}_m \leq \overline{\mathbf{g}}_m \qquad \forall\, m \in \{A, B, C\} \tag{4.41c}$$

$$-\overline{\mathbf{p}} \leq \mathbf{T} \sum_m (\mathbf{\Gamma}\, \mathbf{g}_m - \mathbf{\Delta}\, \mathbf{d}_m) \leq \overline{\mathbf{p}} \tag{4.41d}$$

The solution of (4.41) results in the schedules that correspond to the highest possible social welfare for the whole interconnection, given the generator decisions of where, how much and at what price they bid their available quantities (i.e. given the vectors $\mathbf{g}_m$ and $\mathbf{c}_m$).

The resulting generations are provided in Table 4.9, while the corresponding flows in the congested as well as the tie-branches are given in Table 4.10.

The congestion management policy is highlighted by comparing Tables 4.7, 4.8 with Tables 4.9, 4.10. As explained above, during the iterations TS C has been forced to reschedule gen-

Table 4.11: Costs comparison (in €/h)

Cost:	TS A	TS B	TS C	Total
one single system-wide clearing	5550	6950	8800	21300
transmission allocation iterative procedure	4950	6412	10740	22102
three independent clearings, one per area	3050	7050	18500	28600

eration and finally dispatch some more expensive one, located into its own area, in order to alleviate, proportionally to its responsibility, the congestion appearing in the tie-lines A4C4 and B4C3 (both importing into area C). On the contrary, when the problem is solved as a single optimization, the allocation of generators is made in such a way that, by properly creating some counterflows, the use of more expensive generators in area C is decreased.

The same observation can be made by looking at the resulting costs, shown in Table 4.11. Columns 2 to 4 show the cost of each TS, computed as $C_m = \mathbf{c}_m^T \widehat{\mathbf{g}}_m$, where $\widehat{\mathbf{g}}_m$ is the final generation schedule of the mth TS. In column 5 the total cost, $C^{tot} = \sum_m C_m$, is presented. The second row corresponds to the costs of the system-wide market clearing, and the third row to the costs of the proposed Transmission allocation procedure. Expectedly, the single system-wide optimization yields a set of TS schedules with lower total cost. This difference is due to the smaller cost of the generation that the system-wide clearing dispatched for TS C. On the other hand, the costs for TS A and TS B are larger. This confirms the comment made above, when comparing tables 4.7 and 4.9, regarding the effect of the chosen congestion management policy.

In fact, it is important to point out that the system-wide market clearing does not apply a congestion management policy. This qualitatively differentiates the results of the proposed approach from those of the system-wide clearing. The decomposition of the binding constraints by the coordinator is not just a trick to let TSs clear their markets independently from each other, it reflects a choice about how the use of the transmission network should be shared.

Clearly, the observed cost difference suggests that arrangements could be made between the TSs, economically profitable for all of them, such that more expensive generation is released in favor for some cheaper. It is not within the scope of this work to simulate such arrangements but it is not incompatible with the proposed approach to let the TSs communicate with each other and exchange allocated generation quantities while clearing their markets. Of course, these inter-TS arrangements should remain consistent with the congestion alleviation obligations as well as the already allocated quantities and prices resulting from the coordinator's computations.

The last row of Table 4.11 gives the costs of three individual market clearings performed without cross region bidding, i.e. with each TS considering only the generators located in its area. To do so, three optimization problems were solved, one for each area, each of them considering only generation, load and branch flows geographically located within the area. Thus, the generators of each area produce all together exactly the amount of the area's total load. By chance no line got overloaded. However this could happen in general, since no area considers the effect of its schedule on the other areas. This result is shown in order to confirm that there is indeed high inter-area trade potential benefit in our test system.

The total generation cost that resulted from the proposed method is only 3.76% higher than the minimum cost that can be obtained (21300 €/h) and significantly lower (29.4%) than the cost resulting from the independent market clearings (28600 €/h). This shows that the proposed method and congestion management policy go with the objective of dispatching as much as possible the cheapest generators, while in the same time preserving the independency of the different markets. Let us emphasize, however, that the proposed algorithm is not aimed at minimizing the total operating cost; it should not be confused with algorithms for optimizing a single objective in a distributed manner [AQ01, BB03]. However, the fact that it yields an overall cost very close to the one obtained when handling the whole system as a single market (i.e. perform the system-wide market clearing) appears to be an attractive feature. This issue is further discussed in Section 4.7.2.

Finally, let us recall from Section 4.3.7 that the final dispatches consist a Nash equilibrium of the procedure, as well as a Nash equilibrium of the original game itself. At the final schedules no TS can further decrease its cost, by rescheduling its already dispatched generation or replacing some of it with some of the remaining available one, without causing the violation of one or more constraints.

This Nash equilibrium feature of the final solution explains why some cheaper generation remains not fully exploited. For instance, TS C cannot resort to gC1 or gC2 instead of gC5 because shifting some generation from gC5 to gC1, for example, would cause the overload of one or more branches. More generally, there is no other combination involving all the generators' available quantities that results in a cost for TS C lower than 10740 €/h. There is no concern, though; TS C requested gC5 instead of gC1 or gC2 during the execution of the algorithm, since this allowed to schedule more interesting cheap generation outside area C.

4.6.4 Assessing the final solution in multi-objective optimization terms

In order for the participants to adhere to a coordination framework like the proposed one, they have to be convinced that the final result will be fair and will exploit in the best possible way the transfer capacity of the electric network.

To this purpose, the Pareto efficiency of the final point has been checked. Given an operating point defined by the generation schedules ($\widehat{\mathbf{g}}_A, \widehat{\mathbf{g}}_B, \widehat{\mathbf{g}}_C$), with resulting costs (C_A, C_B, C_C), a way to check whether this is Pareto optimal is to solve the system-wide market clearing problem (4.41) described in the previous subsection, with the following three additional constraints:

$$\mathbf{c}_m^T \mathbf{g}_m \leq C_m \; , \quad m \in \{A, B, C\} \tag{4.42}$$

Let us call this the *Pareto Efficiency Optimization Problem* (PEOP)[4].

[4] One can easily observe that this problem is equivalent to (2.17).

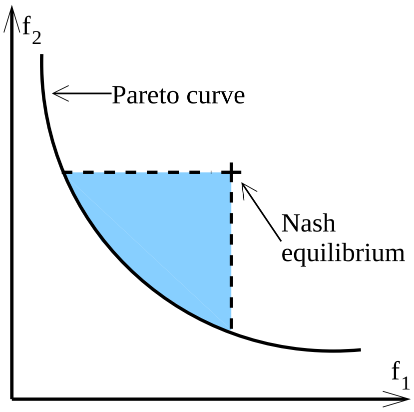

Figure 4.6: Nash equilibrium compared to the Pareto set

Table 4.12: Cost comparison with a Pareto point

Cost:	TS A	TS B	TS C	Total
proposed Transmission allocation	4950	6412	10740	22102
PEOP	4900	6348	10052	21300

The proposed method provided a feasible solution, where:

$$\mathbf{c}_m^T \widehat{\mathbf{g}}_m = C_m \ , \ \forall \ m \in \{A, B, C\} \tag{4.43}$$

So, if the outcome of PEOP satisfies (4.43) (this may happen even with a schedule different than $(\widehat{\mathbf{g}}_A, \widehat{\mathbf{g}}_B, \widehat{\mathbf{g}}_C)$), then the equilibrium point of the proposed algorithm is a Pareto optimal one. Otherwise, at the solution $\mathbf{g}^o = \sum_m \mathbf{g}_m^o$ of PEOP, at least one of the inequalities in (4.42) is a strict one ($\mathbf{c}_m^T \mathbf{g}_m^o < C_m$), which means that there exists (at least) one solution that decreases at least one of the cost functions without increasing any of the others; so the equilibrium point is not a Pareto optimal one.

Figure 4.6 illustrates this discussion, in a two-dimensional example. A solution inside the colored area dominates the Nash equilibrium, since both objectives are better off there. On the contrary, a solution outside that area cannot be considered “better” than the Nash equilibrium, since there one of the involved TSs is worse off than at the Nash solution.

It turned out that the final solution of the iterative procedure is not a Pareto optimal point. In Table 4.12 the resulting costs are compared. Obviously, if that PEOP solution could be implemented, it would be for the profit of all TSs, since it dominates the solution of the proposed algorithm. However, finding this point has been made possible only after assembling together, into a single problem, all the private information of the TSs, which would not preserve the independence of the different markets.

The system-wide market clearing solution (see Table 4.11) is also Pareto optimal. However, it cannot be judged “better” than the outcome of the proposed algorithm because it is not a simultaneous improvement of all the TSs’ costs.

Finally, using the objective in (4.16) instead of (4.17) and varying the factors w_i, gave more

Table 4.13: Costs of different Pareto points

w_A	w_B	w_C	$\mathbf{c}_A^T\mathbf{g}_A$	$\mathbf{c}_B^T\mathbf{g}_B$	$\mathbf{c}_C^T\mathbf{g}_C$	Total Cost
1.0	0.0	0.0	4450	6348	10633	21431
0.4	0.3	0.3	4450	6348	10502	21300
0.3	0.4	0.3	4900	5767	10633	21300
0.0	1.0	0.0	4900	5767	10633	21300
0.3	0.3	0.4	4900	6348	10052	21300
0.0	0.0	1.0	4900	6348	10052	21300

points dominating the equilibrium solution. However, the one presented in Table 4.12 turned out to be the only one where all three TS costs are simultaneously decreased. In order to find more generation schedules that improve all three objectives, (4.42) has been modified to the following:

$$\mathbf{c}_m^T\mathbf{g}_m \leq \alpha C_m \ , \quad \text{with } \ \alpha < 1 \tag{4.44}$$

For $\alpha < 0.99$ the optimization problem turned out to be infeasible. This shows how close to the Pareto set is the solution of the proposed algorithm. In Table 4.13 some results for $\alpha = 0.99$ are presented for different weighting factors w_i. A minimum reduction of 1% is guaranteed for all costs in all cases, while, depending on the relative values of the weighting factors, some costs may be further decreased.

Interestingly, the simultaneous market clearing problem treated here belongs to a family of games where all Pareto optimal points of the corresponding multi-objective problem consist at the same time Nash equilibria of the game. This is due to the fact that each actor's (i.e. TS's) objective depends only on its own control variables.

For instance, let us assume a Pareto optimal collective action $\mathbf{u}^o = (\mathbf{u}_1^o, \ldots, \mathbf{u}_i^o, \ldots)$. If this action was not a Nash equilibrium this would suggest that at least one of the actors, say the ith one, could improve its objective function by modifying its action $\mathbf{u}_i$. However, since the others' objectives do not depend on this actor's control values, the result of the ith's action would be to improve the ith objective while keeping the remaining constant at their previous values. But this would negate the Pareto optimality assumption that was made regarding $\mathbf{u}^o$. Hence, every Pareto optimal $\mathbf{u}^o$ makes up as well a Nash equilibrium of the game.

4.7 Discussion

4.7.1 On the choice of the congestion management policy

A possibly controversial choice in the proposed algorithm, is the way the coordinator shares the use of the branches that tend to get overloaded. Economic theory would suggest that, in order to optimize the use of the whole system, each branch capacity should be shared according to the economic value it has for each TS. More precisely, it was shown in [LNWB07] that at the operating point where total social welfare is maximum, all TSs equally value the use of any congested branch. Indeed, if at least one branch b had a larger value for TS A than for TS B, then the total social welfare could be further maximized by decreasing the share of the branch capacity allocated to B and increasing correspondingly the share allocated to A. This in turn requires computing the sensitivity of the individual TS social welfare (4.1a) to the branch capacity assigned to that TS. Clearly, in order the above sensitivities to be compared, they must be communicated to a coordinator [Hao05, LNWB07].

First, it must be recalled that the method proposed in this work does not aim at maximizing the above total social welfare but instead focuses on simultaneously optimizing multiple overlapping markets (while making the best possible use of the transmission system). Next, the proposed algorithm has been built on the premise that no TS should be asked to provide sensitive private information. In this respect, the choice of relying on the TS participation in branch flows preserves confidentiality, while it sounds reasonable, fair, and according to the test results, economically efficient. Even more, due to its simplicity, it is more transparent and could be more easily accepted by market participants and TSs.

Even if this sensitivity information was asked from the TSs, it might not be possible for the coordinator to check its validity. A mechanism should be thought to motivate the TSs to announce true sensitivity values. This can be done through TSs bidding (in explicit auctions) for individual branch transmission capacity. This would be a step back towards separate transmission and energy markets. Moreover, it may not be easy for a TS to value the use of each branch individually, in the presence of several congested branches, especially in meshed systems. Indeed, these values are much interdependent; the value of a branch for a TS would vary depending on the TS expectation to allocate the use of other branches. This passes the complications of the overlapping markets approach to the responsibility of the TS.

Clearly, the best way for allocating transmission capacity according to its real economic value for each TS (instead of doing this according to the TS intention of use) would be to have them revealing the bids that the market participants have placed to them in order for the coordinator to run an optimization problem and figure out the transmission branches economic value per TS. This would be a step towards centralization of the markets, while the proposed approach aims at allowing co-existence of separate decentralized markets.

4.7.2 Comparison with centralized, fully integrated approach

The direction followed in this work is that of a decentralized approach for merging separate interconnected markets into a single large one. An alternative is that the involved entities (market participants, SOs, regulators and others) in the separate areas-markets agree to overcome the administrative and maybe political difficulties to merge into a single centrally operated system. In this case, the new central authority could clear the entire interconnection using an algorithm that collects bids from all market participants and maximizes the social welfare of the entire interconnection. Two objections may be raised at this point.

First, the willingness of all involved parties to adhere to such a central common operation may be argued. Indeed, an individual area may not want to participate into an overall social welfare maximization because this may lead to a lower social welfare locally inside this area. A set of market participants would not agree to be part of a central solution if making an arrangement between themselves is more profitable for them.

Second, whether it is preferable to operate the market in a centralized manner or coordinate multilateral trades, has been extensively discussed. It is not the intention of this work to come up with a choice between the two, but it is worth pointing out some pros and cons of each. In principle, centralized operation mimics the old vertical organization, with the market participants' bids replacing their marginal costs and benefits. Major advantages of this approach are: (a) transmission network constraints are taken care of implicitly when clearing the energy market and (b) experience shows that it is less exposed to "gaming" by market participants. Centralized market clearing results in nodal LMPs, that is all market participants connected on the same bus pay or get paid the same price.

The choice/need for centralization stems from the difficulty to efficiently coordinate multilateral trades being simultaneously scheduled; it is not an objective by itself. On the contrary, it goes with the principle of free trading to let market participants the option to buy and sell electric energy in the terms they agree between themselves. However, given the transmission network constraints that couple the different transactions, it is more challenging to coordinate them in a decentralized way.

The proposed decentralized scheme allows the participants to directly trade electricity in the terms they wish. Different markets could operate with different individual rules, while competition should encourage the evolution of the TSs market designs, products, software interfaces, efficiency of market clearing algorithms etc.

The co-existence of different markets allows for different ways of sharing the social welfare and for different pricing mechanisms. A generator could sell part of its production at a high price to consumers that value it a lot and another part at a lower price to consumers who are not willing to pay this much. With this price discrimination [VHV05], neither low-paying consumers are excluded from the market, nor are cheap generators obliged to obtain low profit for energy sold to consumers that value it a lot.

The above reasoning is better illustrated through the simple example sketched in Fig. 4.7, where a high price area is connected to a single-bus low price area through a 300-MW transmission link. All generators of the high price area are assumed to have a marginal cost (mc) greater than 10€/MWh and all loads a marginal benefit (mb) greater than 10€/MWh as well. There is cheap generation available (mc=4€/MWh) in the low price area, but it cannot be fully utilized owing to the transmission constraint. Additionally, there is some low-value load (mb=6€/MWh) located in the low price area.

Let us first consider the case of a central market clearing resulting in nodal LMPs. If the cheap generator bids its marginal cost, it will be scheduled for a 400 MW production at a price of 4€/MWh, which will result in a revenue of 1600€/h and zero profit (it will be the marginal

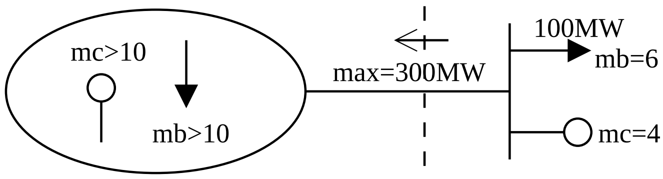

Figure 4.7: Example illustrating the different shares of social welfare

generator within the low price area). The generator could anticipate that the load located in the low price area is willing to pay more for energy and thus it could bid a price of 6€/MWh. In this case the generator will be again scheduled for a 400 MW production, but now at the price of 6€/MWh resulting in a revenue of 2400€/h and a profit of 800€/h. Furthermore, the cheap generator could anticipate the costs and willingness to pay of the participants located in the high price area and, thus, it could submit a bid of 10€/MWh. In this case the low-value load will not be served and the cheap generator will be scheduled to produce 300 MW at a price of 10€/MWh, resulting in a revenue of 3000€/h and a profit of 1400€/h. This behavior maximizes the generator's profit under the centralized LMP-based market clearing. However, there remains some unserved load in the low price area, that is willing to pay more for energy than the marginal cost of a generator who is able to provide this energy. Social welfare of (6-4)x100=200€/h is lost.

In the decentralized approach proposed in this work, different TSs could serve the high-value and the low-value load of the example. The cheap generator can again bid its capacity at 10€/MWh to the high-value load and make a revenue of 3000€/h. However, in this scheme, the generator can also place a bid in the market of the TS that serves the low-value load. The value of the generator's bid price, between 4 and 6 €/MWh, will define how the extra welfare of 200€/h will be shared between the generator and the load of the low price area. For instance, the cheap generator could be scheduled a 100 MW production at 6€/MWh to serve the low-value load, resulting in some extra 600€/h revenue.

As suggested, there is a welfare equal to $\sum_i (prg_i - mc_i) + \sum_j (mb_j - prl_j)$ (with prg_i and prl_j the price paid to the i-th generator and paid by the j-th load) that, depending on the market clearing mechanism, is to be shared between the participants. A part of this money should be withheld by the TS who clears the market in order to cover its operational costs. One can see that letting market participants the choice of TS, introduces competition among TSs to clear their markets as efficiently as possible.

To close the centralized vs. decentralized discussion, it must be emphasized that a decentralized approach like the one presented in this work does not aim at maximizing total social welfare in the short-term, unlike what typically a centralized approach does. The former rather allows for free electricity trade according to the market participants' preferences. It is in the longer-term that a decentralized approach may be more beneficial than the centralized one, due to the market openness and the innovation it promotes. In this respect, if the short-term results of a decentralized approach are far worse than those of the centralized one, this suggests that it is not worth being considered, since its possible longer-term benefits will not be expected to compensate for the short-term inefficient use of the energy and transmission resources. On the other hand, if a decentralized approach results in schedules with total economic value close to the optimum obtained by the centralized solution, this is a good indication that the approach under examination may be a worthy one.

4.7.3 Satisfying set of common constraints vs. sharing control effort for feasibility restoration

Two different ways for coordinating the various actors' control decisions have been used in this work:

1. The coordination method presented in Chapter 3, where each actor is constrained to satisfy all the system's coupled constraints given the other actors' last action, and,

2. the coordination method presented in this chapter, where the various actors solve their decision problems without considering coupled constraints and, in case of constraint violation, a coordinator shares the feasibility restoration effort among actors according to an agreed rule.

In Sections 3.5.3 and 4.3.1, a potential weakness of the first approach has been outlined. If, at a moment during the iterations, the collective controls correspond to an infeasible operating point, restoring feasibility by their sole actions may turn out to be an impossible task for some actors; they may not have enough controllability over the operating point. On the contrary, the second approach requires from each actor to do less than the whole effort needed to restore feasibility. In this respect, the rule of decomposing the effort in proportion to each actor's responsibility goes towards the direction of not asking from an actor to do more than it is capable of. It should be noted, however, that even this approach is not theoretically protected from the here-discussed problem: it could happen, in case of multiple constraints getting violated, that the way the coordinator shares the correction effort for each constraint individually ends up in an overall impossible task for some actors. It should be noted however that we have not been able to "create" a case that would encounter this problem, which may suggest that it is not very probable to happen with the proposed congestion management policy.

Why is the first approach suitable for the PST problem (but not for the market problem)? The answer stems from the fact that most likely the coupled constraints checked when coordinating the operation of PSTs are those that are mainly affected by the PST actions. Given the locations of the involved PSTs, off-line sensitivity studies can easily provide the information of which constraints should be considered in the scheme. In addition, it can be checked up to which point each TSO can affect every constraint and this controllability information may be used as a maximum limit of required action for each TSO-constraint pair.

Applying a rule for sharing the feasibility restoration effort is not just a "computational trick"; it applies a policy for managing congestion. On the other hand, in the absence of such a reasonable policy, it may be difficult to convince the various actors to adhere to a scheme like the one presented in this chapter, even if it actually results in better coordination of their actions.

4.8 Conclusion

This chapter has investigated the possibility of allowing external actors to bid in whatever market of an interconnection, thereby leading to co-existence of several overlapping markets. The procedure is based on the following premises:

- the TSOs put efforts together in order to come up with and share a common network model as well as jointly operate a central coordinator;
- the various TSs can resort to different market clearing mechanisms;
- the coordination does not require the TSs to provide information that is either economically sensitive or difficult to validate (such as Lagrange multipliers).

An iterative method, named Transmission allocation procedure, has been proposed to deal with the resulting congestion management problem. Its essence consists in checking, at each iteration, for branch overloads and sharing among TSs the effort of alleviation. For this purpose, a specific congestion management policy has been implemented, according to which the involved TSs are asked to participate in the overload alleviation in proportion to their participation in the branch loading.

The approach has been thoroughly illustrated on a small-scale example. The resulting solution has been assessed in two ways. First, its property of being a Nash equilibrium has been shown, and, second, its proximity to the set of Pareto optimal solutions has been checked with satisfactory results, since it turned out that, even by collecting all the supposedly private information and solving a single optimization problem, the TSs' social costs can be improved simultaneously by only 1%.

The following chapter builds on the here-presented Transmission allocation loop to extend the overlapping market proposal, dealing with additional issues, namely: (a) allow market participants to place their bids simultaneously into more than one TS markets, (b) incorporate $N-1$ security constraints, (c) jointly schedule reserves, and, (d) account for losses.

Chapter 5

Extensions towards a marketplace encompassing transmission, energy and security

A procedure that allows market participants to place their bids across multiple markets has been proposed in Chapter 4. The developed Transmission allocation loop manages the resulting congestion, coordinating the use of the transmission network by the various markets' schedules.

In Section 4.2.2, another issue related to the overlapping market scheme was raised: attractive market participants, having placed their bids in one TS, may be left inactive at the end of the Transmission allocation procedure while they could be scheduled in another TS market. This chapter starts with proposing an additional loop in the procedure, which we call Energy allocation loop, to deal with that issue. The proposed solution consists in allowing market participants to place their bids in more than one market simultaneously. After the market clearings, a participant should be allocated to the TS from which it received the best offer (the highest price to be paid if it is a generator, or the lowest price to pay if it is a consumer).

After the development of such an integrated Energy and Transmission allocation procedure, extensions dealing with various additional issues, are presented in this chapter. First, security constraints related to equipment outages are incorporated in the mechanism of transmission allocation. Second, one possible way is proposed for clearing, in the same procedure, not only the energy but the reserve market as well. Third, transmission losses are accounted for during the iterations, by having every TS scheduling some additional generation.

The last two sections of the chapter deal with two topics that could be further investigated. First, the rule for transmission allocation is somewhat criticized, and then, it is briefly exposed how a TS could try to anticipate the outcome of the Energy and Transmission allocation procedure when clearing its market within the iterations.

5.1 Energy and Transmission allocation procedure

5.1.1 Proposed Energy allocation loop

As explained in the beginning of this chapter, with the market participants deciding and firmly placing their bids to one or more TSs and then having the Transmission allocation procedure executed, cheap generators (or high bidding elastic loads) may be finally left unused. Table 4.7 suggests, for example, that TS A could decrease its cost if it used some of the remaining capacity of generator gA5[1]. For this reason the previously presented Transmission allocation procedure has been enhanced with an additional feature, allowing market participants to bid their entire capacities to all (or some of) the TSs at the same time, as explained hereafter.

An iterative procedure, referred to as "Energy allocation loop", is implemented by the coordinator to allow this simultaneous dispatching of the market participants by all the TSs.

The procedure starts with the market participants placing their bids, each consisting of a maximum quantity (corresponding to available generation or to load asking to be served) and one price per TS. Why market participants could bid differently to different TSs will be discussed in the sequel. Let $\overline{\mathbf{g}}$ be the vector containing all generators' capacities and $\overline{\mathbf{d}}$ the vector containing the powers of all loads asking to be served. Let also $(\mathbf{c}_m)_i$ be the bid of the ith generator submitted to the mth TS and $(\mathbf{b}_m)_j$ the bid of the jth load to the mth TS. Those bids are private, in the sense that they are announced directly to the TS under question and are not revealed during or after the execution of the Energy allocation loop.

The TSs compete with each other trying to allocate in their final dispatch the most interesting participants. Thus, after having cleared its market, the mth TS communicates to the coordinator its demanded bus generation vector $\widetilde{\mathbf{g}}_m$ and consumption vector $\widetilde{\mathbf{d}}_m$, together with the corresponding offered price vectors $\boldsymbol{\pi}^g_m$ and $\boldsymbol{\pi}^d_m$.

For a given generator i, if the total power demanded by the various TSs is below (or equal to) its capacity, i.e. $\sum_m (\widetilde{\mathbf{g}}_m)_i \leq \overline{g}_i$, that power is simply allocated to the various TSs as they requested. Otherwise, there is a conflict, and the role of the coordinator is to take care that the generator is finally dispatched at the most profitable possible prices. To this purpose, the coordinator allocates the power to one or several of the involved TSs by decreasing order of offered price. In case several TSs compete for the same generator with equal offered prices, the available power is shared in proportion with the requested quantities.

Hence, generally, some TSs will be left with power imbalances, and the markets have to be cleared again. In order the power just allocated to a TS not to be available to the others, the coordinator communicates reduced bounds $(\overline{\mathbf{g}}_m)$ and $(\overline{\mathbf{d}}_m)$ to the latter TSs.

Thus, the TSs come up with new demanded quantities and offered prices. At this stage, the coordinator repeats the above procedure, with the following two additional rules:

[1]Capacity that it cannot dispatch since it was not initially bidden into its market.

1. what was previously allocated to a TS and is still requested remains with that TS;

2. what was previously allocated to a TS and is not requested any longer is made right away available to the other TSs.

These iterative adjustments lead to a gradual allocation of all demanded generations. Loads are handled in a similar way, but with the allocation performed by increasing order of prices requested by the TSs in order to serve them.

The procedure terminates when each market is balanced, no TS has incentive to further improve its schedule by dispatching available generation or load, and no conflict is left for any resource.

Note that no market participant is obliged to participate in the Energy allocation procedure. Indeed, a market participant may prefer to place its bid directly in a TS market because of a beneficial arrangement made with this TS or because it believes the announcement of the clearing price by the TS would unveil its bid. Furthermore, no TS is obliged to accept such bids. However, a TS may be willing to receive bids from the above described Energy allocation procedure owing to the risk of being left without enough participants interested in placing their bids in its market. Thus, what has been described refers to participants and TSs who choose to take advantage of the higher liquidity offered by the proposed mechanism.

Note also that different markets may impose different obligations or offer different benefits to their participants, which can make the prices that a participant receives from the various TSs for the same amount of energy not directly comparable with each other. This will be generally reflected on the individual price a market participant offers to each TS in its bid. Additionally, a predefined correction term can be applied when prices are compared by the central coordinator. This is easily incorporated in the presented procedure. Further discussion of this issue can be found in [GAK99].

At the end of the Energy allocation procedure described above, the bus injection vector defined in (4.3) is available. Note that in general this vector also includes power injections that result from a bilateral (or multilateral) agreement between parties, and hence have not been determined iteratively as described in this section.

5.1.2 Overall procedure for Energy and Transmission allocation

In the general case, iterations need to be performed between the Energy and Transmission allocation procedures. The overall procedure is outlined in Fig. 5.1.

The procedure starts with each TS clearing its market according to its own procedures and rules. The resulting demanded (not approved yet) schedules and corresponding offered prices are communicated to the coordinator.

The latter first deals with Energy allocation. When the received schedules are in conflict, resources are allocated as explained in Section 5.1 and new constraints regarding the availability

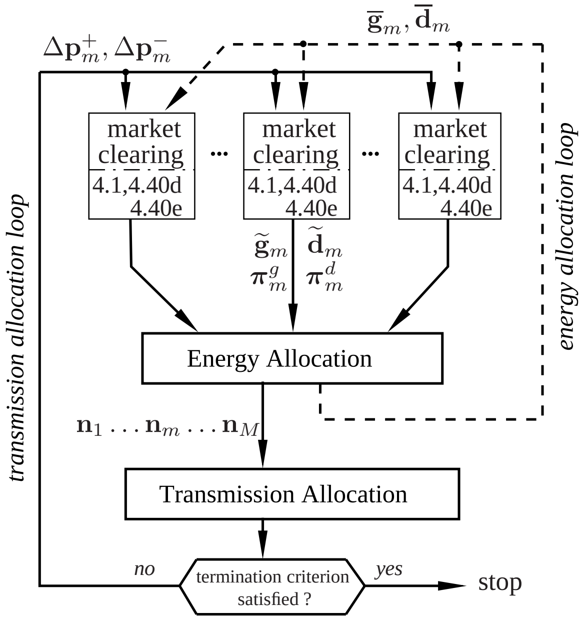

Figure 5.1: Flowchart of the iterative Energy and Transmission allocations

of these resources are communicated to the TSs, which clear again their markets. The procedure, depicted with dashed line in Fig. 5.1 is repeated until the coordinator eventually receives schedules with no availability conflict; the latter are used in the Transmission allocation block.

This block performs the computations presented in Section 4.4 and, in case of congestions, sends back the constraints (4.32, 4.38) to the TSs for inclusion in their market clearing. This makes up an outer loop, shown with heavy line in Fig. 5.1.

Before doing so, the convergence test is performed on all branches that have been involved in constraints (4.32, 4.38). If any power flow differs from the value at the previous iteration by more than a tolerance ϵ, the algorithm proceeds with a new Energy allocation loop; otherwise the procedure is completed.

5.1.3 Information flow during the execution of the algorithm

It is appropriate to summarize the information disclosed and communicated between parties.

Each market participant places its bid to a number of TSs (generally, different per TS). This information is given only to the TS receiving the bid. At no point of the procedure it is revealed to any other entity.

Every time the TSs simultaneously clear their markets, they announce to the coordinator their preferred schedules and the prices they offer to the market participants. This information is made available only to the coordinator during the procedure, but it could be disclosed at the

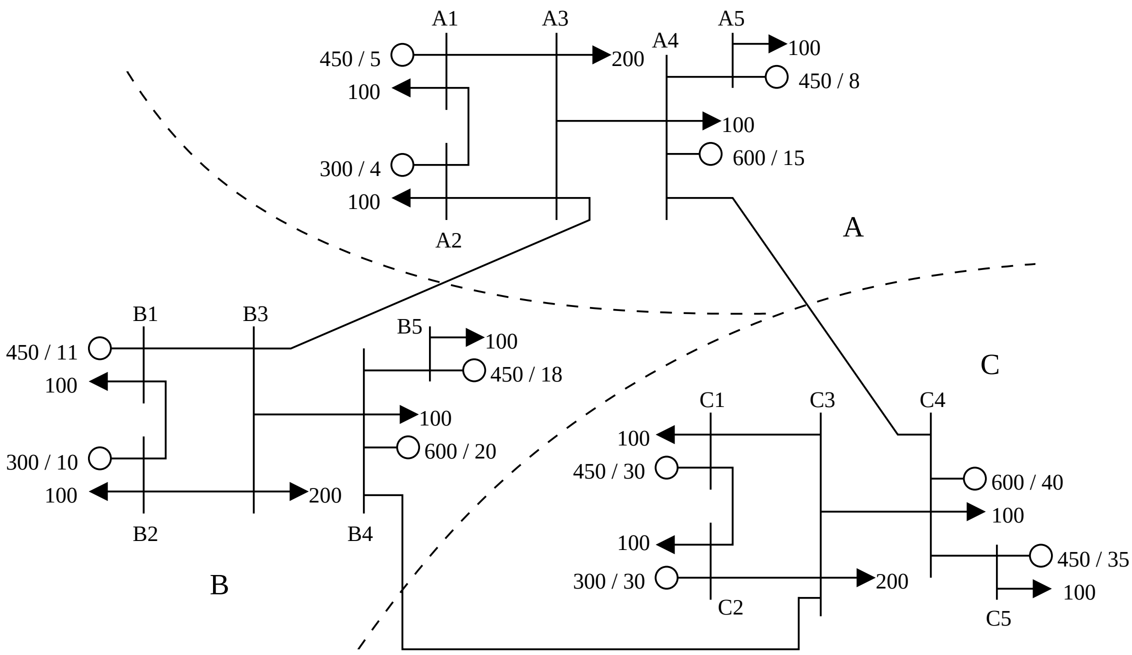

Figure 5.2: Three-area test system

end so that interested parties can check that the coordinator has acted according to the rules.

The coordinator communicates to the TSs linear constraints relating their net bus power injections with sought changes in branch flows. The model used by the coordinator to compute those flows is in principle available to all market participants, allowing them to check that they have been properly treated during the execution of the algorithm.

5.2 Illustrative examples

5.2.1 Simulation results on a 15-bus test system

The three-area 15-bus system presented in Section 4.6.1 is re-used to illustrate the combined energy and transmission allocation. For the reader's convenience, the test system is reproduced in Fig. 5.2. As in Section 4.6.1, for the sake of clarity, each TS serves the inelastic loads of an area, while each generator bids the same price to all TSs. A marginal clearing price mechanism has been assumed for all three TSs. Hence, the price offered by each TS, irrespective of the generator, is the bid of the most expensive generator in its dispatch.

In order to provide insight on how the algorithm performs, we present hereafter the results obtained at the first three iterations of the procedure, followed by those of the final generation schedule.

Table 5.1: Iteration 1: Generation allocated to each TS (in MW)

Gen	Bid	TS A	TS B	TS C	Total	Max
gA1	5	150	150	150	450	450
gA2	4	100	100	100	300	300
gA4	15	0	0	0	0	600
gA5	8	150	150	150	450	450
gB1	11	100	100	100	300	450
gB2	10	100	100	100	300	300
gB4	20	0	0	0	0	600
gB5	18	0	0	0	0	450
gC1	30	0	0	0	0	450
gC2	30	0	0	0	0	300
gC4	40	0	0	0	0	600
gC5	35	0	0	0	0	450

Table 5.2: Iteration 1: Power flows and requested corrections (in MW)

Line	TS A	TS B	TS C	p_b	$\overline{p}_b$	Δp_A^-	Δp_B^-	Δp_C^-
A1A3	32	133	133	298	150	16	66	66
A2A3	18	117	117	252	150	8	47	47
B1B3	100	0	100	200	150	25	0	25
B2B3	100	0	100	200	150	25	0	25
A4C4	-42	125	325	408	200	-	58	150

5.2.2 Examples of iterations

At the initial point, all TSs are allowed to compete for all generators without any other constraint than (4.40b) and (4.40c), with $(\overline{\mathbf{g}}_m)_i = \overline{\mathbf{g}}_i, \ \forall m$. Obviously, this leads to all of them simultaneously demanding the cheapest generations, namely all TSs ask for 300 MW from gA2 and 300 MW from gA1. Hence, the Energy allocation procedure merely divides the available generation in equal parts[2], and these constraints are sent back to the TSs for them to perform new market clearings. This step is repeated, as shown by the dashed line in Fig. 5.1, until no two TSs compete for the same power generation. This yields the situation detailed in Table 5.1. Columns 3 to 5 show the power allocated to each TS.

At this stage, the coordinator can determine the resulting flows and check the corresponding limits. The results for the overloaded branches are given in Table 5.2. As already explained in Section 4.6.2, a Δp_m^- is computed per overloaded branch for each TS m according to the congestion management rule. Again, the dash in the last row of Table 5.2 means that TS A is not requested to change its contribution to the branch flow A4C4 because it is counterflowing. The new constraints computed by the coordinator are communicated to the TSs.

This completes the first execution of the Transmission allocation loop shown with solid line in Fig. 5.1. At this point the TSs perform new market clearings incorporating the constraints (4.40d, 4.40e). The corresponding demanded generations are shown in Columns 3 to 5 of

[2]Due to the fact that in this example all TSs serve the same amount of load and use the same pricing rule, in the absence of branch flow constraints they all offer the same price to generators.

Table 5.3: Iteration 2: Generation schedule (in MW) after first iteration of the Energy allocation loop

Gen	Bid	demanded by			allocated to			Total
		TS A	TS B	TS C	TS A	TS B	TS C	
gA1	5	125	63	0	125	63	0	188
gA2	4	100	73	74	100	73	74	247
gA4	15	75	0	0	75	0	0	75
gA5	8	150	110	0	150	110	0	260
gB1	11	75	100	0	75	100	0	175
gB2	10	75	100	100	75	100	100	275
gB4	20	0	0	0	0	0	0	0
gB5	18	0	**154**	**426**	0	119	331	450
gC1	30	0	0	0	0	0	0	0
gC2	30	0	0	0	0	0	0	0
gC4	40	0	0	0	0	0	0	0
gC5	35	0	0	0	0	0	0	0

Table 5.3.

What makes the TSs adjust their schedules with respect to the values in Table 5.1 is the addition of the constraints dealing with the overloaded branches. For instance, TS C is obliged to abandon most of the power it planned to obtain from generators located in system A, in order to decrease by 150 MW the flow it causes on the tie-line A4C4 (see Table 5.2).

When the second iteration starts, no TS can use the power allocated to another TS at the first iteration. For example, TS A can only resort to 150 MW from generator gA5 since the remaining 300 MW were already allocated to TSs B and C (see Table 5.1). More precisely, TS A can either keep from gA5 those 150 MW already allocated to it or make it partly or fully available to the other TSs, depending upon the outcome of its new market clearing. Indeed, Table 5.3 shows that TS A is obliged to release part of the powers allocated to it from gA1, gB1 and gB2, in order to meet the constraints stemming from branches A1A3, A2A3, B1B3 and B2B3. It should be noted how the constraint on the tie-line A4C4 has affected the market clearing solutions of TS B and even more TS C, both obliged to replace cheap generation in area A by more expensive in area B.

For generator gB5, the total demanded generation exceeds its capacity (see bold values in the table). According to the rule discussed in Section 5.1, the TS making the best bid has priority. In this particular case, it happens that both TS B and TS C (TS A does not ask any power from gB5) offer the same price of 18 €/MWh. Hence, according to the default rule suggested in Section 5.1, the remaining capacity (in this case the whole 450 MW available) is allocated to each TS proportionally to what it asks. Columns 6 to 8 in Table 5.3 show the quantities allocated as a result of the above decisions.

Since there was one generator with demand higher than capacity, another execution of the Energy allocation loop is performed, involving new market clearings. In the latter, the congestion management constraints remain unchanged, but the $(\overline{\mathrm{g}}_m)_i$ bounds in (4.40c) have been up-

Table 5.4: Iteration 2: Generation allocated to each TS (in MW)

Gen	Bid	TS A	TS B	TS C	Total	Max
gA1	5	123	63	0	186	450
gA2	4	103	73	107	283	300
gA4	15	0	0	0	0	600
gA5	8	340	110	0	450	450
gB1	11	0	100	7	107	450
gB2	10	33	100	111	244	300
gB4	20	0	34	0	34	600
gB5	18	0	119	331	450	450
gC1	30	0	0	0	0	450
gC2	30	0	0	0	0	300
gC4	40	0	0	0	0	600
gC5	35	0	0	43	43	450

Table 5.5: Iteration 2: Power flows and requested corrections (in MW)

Line	TS A	TS B	TS C	p_b	$\overline{p}_b$	Δp_A^-	Δp_B^-	Δp_C^-
A1A3	16	67	38	121	150	-4	-16	-9
A2A3	10	70	70	150	150	0	0	0
B1B3	11	0	44	55	150	-20	0	-75
B2B3	21	0	75	96	150	-12	0	-42
A4C4	18	67	175	260	200	5	15	40
B4C3	-18	-67	382	297	200	-	-	97

dated. For instance, TS A now sees 450 - 110 = 340 MW available from gA5, and 450 - 331 - 119 = 0 MW available from gB5. From the latter, TS B and TS C see 119 MW and 331 MW respectively.

The resulting generation schedule is given in Table 5.4. As can be seen, TS A has released most of the generation it had in area B in order to dispatch the less expensive that is now available in area A (gA5). As there is no conflict between demanded and available quantities, the algorithm proceeds with the Transmission allocation.

The new power flow corrections are detailed in Table 5.5.

A new market clearing with these updated branch flow constraints yield the generation schedule shown in Table 5.6.

5.2.3 Features of the final generation schedule

The algorithm proceeds similarly until the congested branch flows differ by less than ϵ =2 MW from their values at the previous iteration. This takes place after 5 iterations and yields the final values presented in Table 5.7 (Columns 3 to 6).

Figure 5.3 shows the evolution of four of the congested branch flows through the successive

Table 5.6: Iteration 3: Generation allocated to each TS (in MW)

Gen	Bid	TS A	TS B	TS C	Total	Max
gA1	5	132	98	0	230	450
gA2	4	99	54	89	242	300
gA4	15	0	0	0	0	600
gA5	8	326	68	0	394	450
gB1	11	10	100	23	133	450
gB2	10	33	100	167	300	300
gB4	20	0	0	0	0	600
gB5	18	0	179	141	320	450
gC1	30	0	0	0	0	450
gC2	30	0	0	0	0	300
gC4	40	0	0	0	0	600
gC5	35	0	0	181	181	450

Table 5.7: Final generation allocation (in MW)

Gen	Bid	TS A	TS B	TS C	Total	Single	Max
gA1	5	136	113	0	249	250	450
gA2	4	98	56	96	250	250	300
gA4	15	0	0	0	0	0	600
gA5	8	324	58	0	382	300	450
gB1	11	9	100	48	157	250	450
gB2	10	33	100	167	300	250	300
gB4	20	0	0	0	0	0	600
gB5	18	0	173	89	262	300	450
gC1	30	0	0	0	0	0	450
gC2	30	0	0	0	0	200	300
gC4	40	0	0	0	0	0	600
gC5	35	0	0	200	200	0	450

iterations. The horizontal line corresponds to the branch flow limit. The branch flows almost converge to their final values already from the 4th iteration.

For comparison purposes, a single market clearing has been considered. It consists in solving a single optimization for the whole system, with the objective of minimizing the total cost (i.e. maximizing total social welfare) while respecting branch flow limits. The resulting generations are provided in Column 7 of Table 5.7, while the corresponding cost is given in Table 5.8. As regards the proposed method, Columns 2 to 4 in the same table show the generation costs relative to the three TS final schedules, and Column 5 the sum of the latter costs which corresponds to the social welfare of the entire system, obtained by the proposed method.

One can notice that with the proposed method TS A managed to allocate the cheapest schedule while TS C ended up with the most expensive one. This is due to the limited capacities of the three tie-lines A4C4, A3B3 and B4C3 and to the fact that, during the execution of the procedure, the TSs have been obliged to reschedule their generations in order to unload congested branches. TS C has been assigned most of the effort to alleviate the overloads of these tie-lines during the execution of the algorithm (see Tables 5.2 and 5.5).

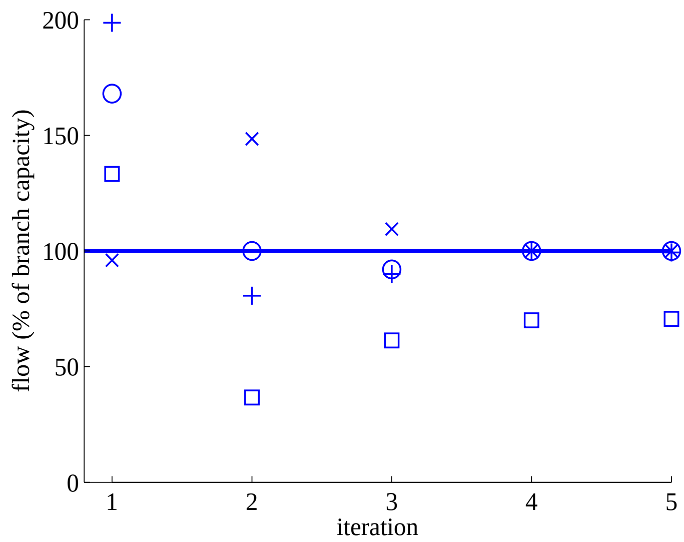

Figure 5.3: Evolution of power flows with iterations: branch A1A3 is shown with $+$, A2A3 with $\circ$, B1B3 with $\square$, B4C3 with $\times$

Table 5.8: Final generation costs (in €/h)

Single	TS A	TS B	TS C	Total
21300	4093	6467	11184	21743

At the final allocation no TS can further decrease its cost, by rescheduling its already allocated generation or replacing some of it with some of the remaining available one, without causing the violation of one or more constraints. This is why some cheaper generation remains not fully exploited. For instance, TS C cannot resort to gC1 or gC2 instead of gC5 because shifting some generation from gC5 to gC1, for example, would cause the overload of one or more branches. More generally, there is no other combination involving all the generators' available quantities (i.e. not already allocated to TSs A and B) that results in a cost for TS C lower than 11184 €/h. There is no concern, though; TS C requested gC5 instead of gC1 or gC2 during the execution of the algorithm, since this allowed to allocate more interesting cheap generation outside area C.

Expectedly, the single system-wide optimization yields a schedule with lower total cost than the proposed algorithm.

The cost of the system-wide optimal solution (21300 €/h) is 2 % lower than the total cost obtained by the proposed algorithm (21743 €/h). Let us emphasize, however, that the proposed algorithm is not aimed at minimizing the total operating cost; it should not be confused with algorithms for optimizing a single objective in a distributed manner [AQ01, BB03]. However, the fact that it yields an overall cost very close to the one obtained when handling the whole system as a single market appears to be an attractive feature, as already discussed in Section 4.7.2.

Table 5.9: Final point; generation scheduled by each TS

Gen	Bid	only Transmission allocation				Energy & Transm. allocation			
		TS A	TS B	TS C	Total	TS A	TS B	TS C	Total
gA1	5	134	99	17	250	136	113	0	249
gA2	4	96	59	95	250	98	56	96	250
gA4	15	94	0	0	94	0	0	0	0
gA5	8	150	80	0	230	324	58	0	382
gB1	11	26	100	123	249	9	100	48	157
gB2	10	100	100	50	250	33	100	167	300
gB4	20	0	12	0	12	0	0	0	0
gB5	18	0	150	115	265	0	173	89	262
gC1	30	0	0	28	28	0	0	0	0
gC2	30	0	0	100	100	0	0	0	0
gC4	40	0	0	0	0	0	0	0	0
gC5	35	0	0	72	72	0	0	200	200
Costs: (€/h)		4950	6412	10740	22102	4093	6467	11184	21743

It is of interest to compare the schedules that resulted when the generators had already shared their available capacities among the TSs prior to the execution of the Transmission allocation loop (see Table 4.7) with those that resulted with the generators making their capacities at the same time available to all TSs and then having the combined Energy and Transmission allocation method executed (see Table 5.7). The information contained in those tables, as well as the related costs taken from Tables 4.11 and 5.8, have been grouped into Table 5.9. Each of rows 2 to 13 in this table corresponds to a generator, whose production per TS and its total production are shown in columns 3 to 6 and 7 to 10 for the execution of, respectively, the sole Transmission allocation loop and both loops. The last row of the table contains the resulting costs, per TS and total, for the two executions.

The generation allocation of TS A in the full method (column 7 in Table 5.9) is indicative of the benefit of the Energy allocation loop. One can see that some capacity of gA5 that was left unused without the Energy allocation loop is dispatched by TS A in the full method, driving down TS A's as well as the overall generation cost. Furthermore, gA5 is alleviated from the, maybe difficult, decision of choosing how much of its capacity it should offer to each TS; it just announces its whole and the procedure takes care that, if it is economically interesting, the generation is dispatched.

5.2.4 Simulation results on IEEE RTS-96 test system

The algorithm was also tested on the IEEE Three-Area Reliability Test System - 1996 documented in [RTS99]. This somewhat larger system was obtained by triplicating the One Area RTS-96 system, and consists of three topologically identical 24-bus systems connected with five tie-lines. Fig. 5.4 provides a one-line diagram of this three-area system. Area 1 is at the left, area 2 in the middle and area 3 at the right.

In order to create different price areas, the marginal costs of generators have been modified with

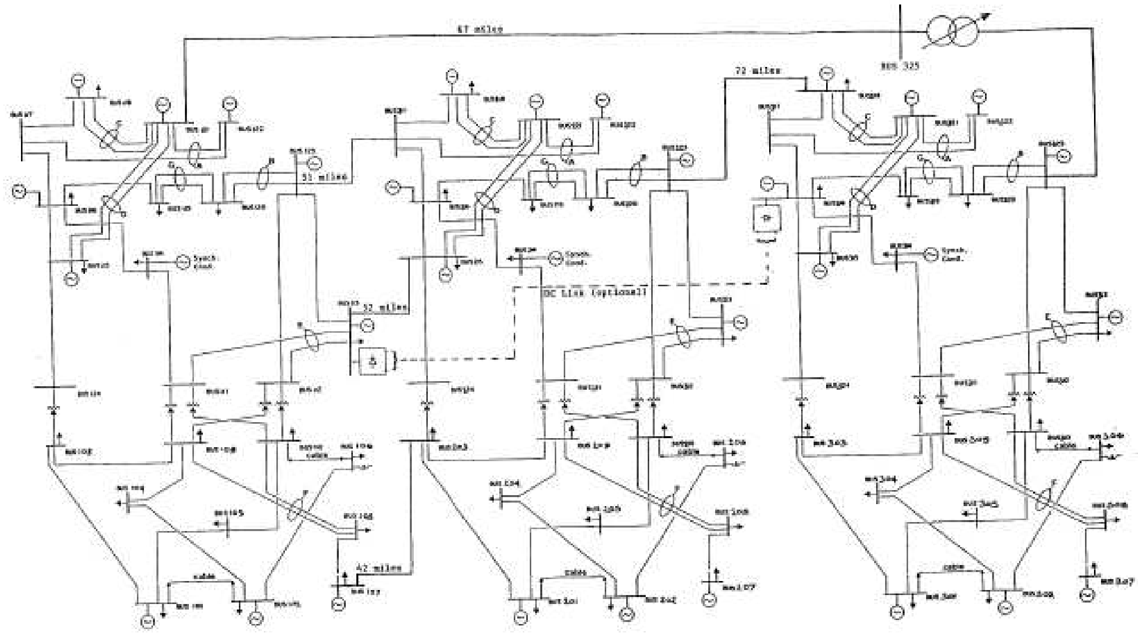

Figure 5.4: Three-area RTS-96 test system

Table 5.10: IEEE RTS-96 system; intermediate results

outer loop	inner loop	cost (in €/h)			
iter. count	iterations	of TS 1	of TS 2	of TS 3	total
1	11	10457.5	10457.5	10457.5	31372.5
2	1	10457.5	10457.5	10589.0	31504.0
3	3	10374.7	10374.7	10587.4	31336.8
4	1	10374.7	10374.7	10814.4	31563.8
5	4	10280.4	10280.4	10811.7	31372.5
6	1	10280.4	10280.4	11058.6	31619.4
7	4	10158.2	10158.2	11056.2	31372.6
8	5	9994.0	10120.0	11297.3	31411.3
9	1	9995.3	10120.9	11410.1	31526.3
10	2	9957.4	10091.3	11402.9	31451.6
11	1	9957.5	10091.6	11417.5	31466.6

respect to [RTS99] so that every generator in area 2 is twice as expensive as its counterpart in area 1, while the generators in area 3 are made three times as expensive as those in area 1. The generator data are presented in Appendix C. Note that in spite of these price increases, area 3 still includes attractive generators compared to the other areas. Again, it was assumed that load demand is inelastic, each TS serves the loads of one area resorting to any generator, and a marginal clearing pricing mechanism is used by every TS. The resulting scenario is interesting owing to the involved generation (re-)allocation, as shown hereafter.

It took 11 iterations for the procedure to converge with a tolerance $\epsilon = 2$ MW. Intermediate results are presented in Table 5.10. Each row refers to results obtained after executing the outer (Transmission allocation) loop, while the second column gives the number of inner (Energy allocation) loop executions. Columns 3 to 5 present the individual TS costs, while Column 6 shows the sum of those three individual costs.

The overall procedure can be summarized as follows. At the first iteration, network congestions are not handled yet and, since equal loads have to be served by all TSs, the cheapest generations are allocated in equal parts to each of them. This explains the identical costs shown in the table. As a result, the tie-lines of Area 3 are congested. Only TS 3 is responsible for these overloads since the other two TSs contribute with counterflows. Hence, TS 3 has to de-allocate generation in Areas 1 and 2 and replace it by more expensive in Area 3. This explains why only the cost of TS 3 increases at iteration 2. The so released capacity is used by TS 1 and 2 at iteration 3, which explains the corresponding cost decreases. This goes with a decrease in the generation allocated to TS 1 and 2 in Area 3. Therefore, the counterflows in the above mentioned tie-lines somewhat decrease, which causes overload again. Hence, at iteration 4, TS 3 has to further correct its schedule to keep the tie-line power flows within limits. The situation is unchanged until iteration 7 when TS 1 and 2 stop counterflowing, and hence have to participate in the congestion alleviation. Note that, in case of limiting time, the algorithm could even stop at this stage, as suggested at the end of Section 4.5. From there on, no further line is congested and no further power flow contribution changes sign; the last iterations are devoted to satisfying the convergence criterion, i.e. small adjustments of generation schedules are made and the power flows progressively converge to their final values (as is the case in the last 2 and, respectively, 4 iterations in the examples shown in Figs. 5.3 and 4.5).

As for the 15-bus system, a comparison was carried out with a single market clearing for the whole system. The corresponding cost was found to be 31456.8 €/h, which is to be compared with the final total cost of 31466.6 €/h obtained with the proposed procedure (see Table 5.10). Again, it is noteworthy that the two costs are quite close to each other; they differ by 0.031 % only.

5.3 Discussion on Energy allocation

5.3.1 Incorporating bilateral trades

It should be noted that the Energy allocation loop is optional in the proposed procedure; it is the Transmission allocation that enables the simultaneous use of the network for multiple trades. For instance, the procedure can easily accommodate bilateral trades scheduled in the spot markets[3].

A bilateral trade is nothing but a schedule submitted to the coordinator by one of the sides of the trade (i.e. either the producer or the consumer plays the role of the TS). Clearly, in the Energy allocation loop the bilateral trades are always allocated as they are announced. When the feasibility of the overall schedule is checked in the Transmission allocation loop, however, it

[3]Bilateral trades that have been scheduled in forward markets are not involved in the proposed approach (although they are taken into account when estimating the available transmission capacities).

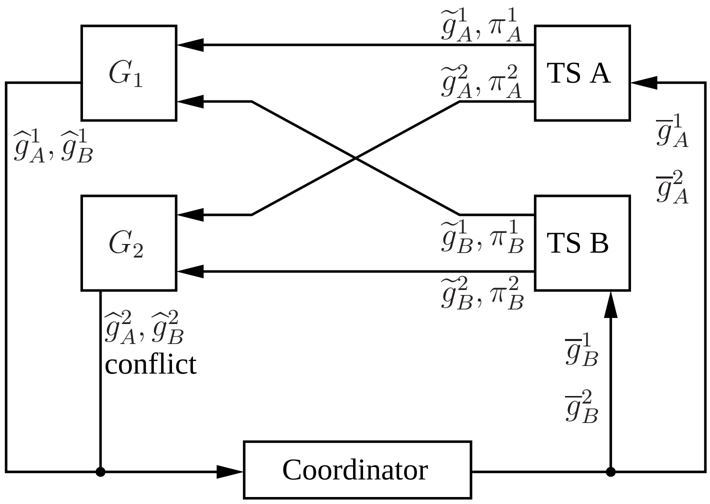

Figure 5.5: Energy allocation without revealing to the coordinator the prices offered by the TSs to market participants

may be possible that a TS scheduling a bilateral trade is asked to decrease its flow contribution to one or several overloaded lines. In this case, it will have to cut a part of the trade. Of course, if later on during the execution of the algorithm some of this transmission capacity is made available, the TS could use it to satisfy as much of the intended trade as possible.

5.3.2 Non-disclosure of offered prices

In the proposed Energy allocation procedure, the TSs announce to the coordinator the prices they offer to the market participants. Since this information is to become public after the execution of the procedure (so that anyone can check that it has been properly treated), some market participants may argue that this disclosure of offered prices violates the rule of confidentiality. Two arguments can be said against this. First, the TSs do not announce the bids they received by the market participants but only the prices resulting from their market clearings (unavoidably, any market clearing mechanism could, to a smaller or larger extent, reveal some of the participants bids). Second, the Energy allocation procedure is anyway optional; as already explained, any market participant can place its bid in one TS market only, thereby avoiding the need for announcing the price offered to it.

However, if required, the procedure could become "price-proof", at the expense of additional communication effort. The energy allocation can be achieved with information exchange between the different TSs and the market participants individually. At the end of a set of market clearings, the various TSs can communicate their demanded quantities and corresponding prices individually to each of the various market participants. Each market participant can then decide on its own on the quantities to offer in the next iteration to each TS (without conflict in its capacity), without the need for central coordination. In this case, at the end of each iteration, each market participant has to merely send to the coordinator: (i) an indication of a conflict and (ii) the quantities allocated to the TSs. If the coordinator does not receive any conflict notification, then the Energy allocation loop is complete and the resulting injection schedules should be announced to the coordinator in order to proceed with the Transmission allocation loop.

Figure 5.5 illustrates the flow of information at one step of the Energy allocation loop in an example with two participating generators (G_1 and G_2) and two TSs (TS A and TS B) where G_2 encounters a conflict.

5.3.3 Which prices are finally paid by/to the TSs ?

A question that deserves some discussion is: at the end of the procedure (when the various TSs have dispatched a set of market participants each) what actual price will each market participant pay to (in case of load) or be paid by (in case of generator) the TS that has scheduled (a part of) its available capacity?

As explained in Section 5.1, each time there is a conflict in the quantities that the various TSs wish to dispatch in their markets, the coordinator resorts to the offered prices in order to allocate market participants to TSs. Those allocations, made during the iterations, affect the final outcome of the procedure. In this respect, it appears that if the TSs just pay to (or are paid by) the market participants the prices that resulted from the last set of market clearings, then the price signals and announced schedules used during the iterations to allocate energy (and transmission) resources have no "actual cost" for the TSs.

For simplicity, let us refer to generators only. Consumers should be considered in an equivalent way.

Let us assume that, at an energy allocation step, the mth TS is allocated by the coordinator a power $(\mathbf{g}_m)_i$ from the ith generator based on an offered price $(\boldsymbol{\pi}_m)_i$. Thus, this power is no longer available to the other TSs, provided that the mth TS continues to dispatch at least $(\mathbf{g}_m)_i$ in future iterations. Clearly, if in future clearings of its market the pricing rule used by this TS suggested a new price $(\boldsymbol{\pi}_m)'_i < (\boldsymbol{\pi}_m)_i$, it would not be fair that the ith generator is paid $(\boldsymbol{\pi}_m)'_i$ instead of $(\boldsymbol{\pi}_m)_i$. For this reason, it seems reasonable to apply the following rule:

> A TS that, during the iterations, has been allocated a power $\widehat{g}_i$ at an offered price π_i, is obliged to pay at least this price to the ith generator for a quantity $g_i \leq \widehat{g}_i$ finally allocated to that TS at the end of the Energy allocation procedure.

Note that if the TS under question finally dispatches more than $\widehat{g}_i$, the above rule of paying at least π_i should apply for only $\widehat{g}_i$. With reference to the above rule, the remaining power $g_i - \widehat{g}_i$ can be paid at a price maybe lower than π_i.

Tables 5.3 and 5.4 provide a suitable illustration of the above rule. At the end of the first energy allocation iteration (Table 5.3), TS A is allocated 150 MW of gA5 at 11 €/MWh (price of the marginal generator gB1). But, in the same time, TS B and TS C had to release capacity from gA5 due to branch flow constraints. Thus, at the end of the second (and final in this case) energy allocation iteration (Table 5.4), TS A resorts to additional power from gA5, releasing some more expensive power from gA4, gB1 and gB2. The price offered by TS A to all dispatched generators is 10 €/MWh (the marginal generator is now gB2). Applying the above-explained pricing rule, TS A has to pay 150 $\times$ 11 + (340 - 150) $\times$ 10 €/h to gA5 and 33 $\times$ 11 €/h to gB2. It pays nothing to gA4 and gB1 which it did not finally dispatch.

With the above pricing rule, a TS is prevented from offering artificially very good prices during the iterations just to be allocated the most interesting participants, intending to decrease those prices later on during the procedure.

5.4 Incorporating security constraints in the Transmission allocation procedure

A basic security requirement in power system operation is that the system should be able to withstand the loss of any single element (i.e. $N-1$ contingency) without entering into an emergency situation. Generally, it is within the duties of each area's TSO to check and make sure that the system it operates can safely withstand any $N-1$ contingency, both to what regards the existence of a feasible post-contingency operating point, as well as the stability of the dynamic behavior towards the post-contingency operating point. System security can make up a special market by itself (e.g. market for ancillary services [RKTR07b]). Often, security is checked and restored after the energy markets have been cleared, where, typically, simplified considerations about security are made, if at all.

To what regards the proposed structure for clearing overlapping markets, it is reasonable to assume that, after the final TS schedules are available, each TSO will take proper actions, if necessary, to guarantee that its area of responsibility is in a secure state. Clearly, those actions could involve rescheduling some generation, whose cost is to be finally paid by the area's local participants. It would be unrealistic to pass the whole security assessment complexity to the market clearing procedure. However, as for the branch flow limits that are implicitly treated by the proposed procedure, it would be a step towards improving security if the flows resulting from a branch or a generator outage (i.e. a $N-1$ contingency) were also limited.

To this purpose, the congestion management problem has been extended incorporating the additional constraint that the overall injection schedule, $\mathbf{n}$, should be such that the power flows resulting from the loss of any branch or generator do not overload any of the remaining branches.

5.4.1 Line Outage Distribution Factors

Following the choice of a linear network model, we resort to well-known Line Outage Distribution Factors (LODF) [CWW00]. For each branch, these factors result from the PTDFs of the system configuration with and without the branch under question [GFLS09, GGL07]. The LODFs are linear sensitivities, each of them giving the fraction of the power flowing in a branch v before its outage, that is flowing in branch b after the outage. Let $\mathbf{L}$ the $B \times B$ matrix of LODFs and p_b^v the flow in branch b that results from the outage of branch v. We have:

$$\Delta p_b^v = p_b^v - p_b = (\mathbf{L})_{bv} p_v \tag{5.1}$$

where p_b and p_v are the bth and vth branch flows before any outage. By definition of $\mathbf{L}$, we have $(\mathbf{L})_{bb} = -1$.

In (5.1) the pre-outage flows can be replaced by (4.6), which yields the post-outage flow as a linear function of the injection schedule:

$$p_b^v = (\mathbf{L})_{bv} \mathbf{t}_v \mathbf{n} + \mathbf{t}_b \mathbf{n} = \left((\mathbf{L})_{bv} \mathbf{t}_v + \mathbf{t}_b \right) \mathbf{n} \tag{5.2}$$

where the row vectors $\mathbf{t}$ have been defined in Section 4.4.

Leaving aside contingency selection, the $N-1$ security criterion requires to check, for each of the B branches, the $B-1$ power flows that take place after the outage of another branch. Thus, for each pair (b, v) we check a security constraint of the type:

$$-\alpha\, \overline{p}_b \leq p_b^v \leq \alpha\, \overline{p}_b \tag{5.3}$$

where $\alpha \geq 1$ accounts for possible overload allowed in post-contingency situation (typically $1.05 \leq \alpha \leq 1.1$).

Using (5.2) for every post-outage flow p_b^v yields a linear relationship between the post-contingency flows and the pre-contingency bus power injections.

The satisfaction of the B constraints of the type (4.8) as well as the $B \times (B-1)$ constraints of the type (5.3) makes up the congestion management problem dealt with in this section.

5.4.2 LODF-based constraints in the Transmission allocation loop

Let us assume that, after the M TSs have cleared their markets, an $N-1$ constraint is violated, i.e. for the given injections $\widehat{\mathbf{n}}_m$ ($m = 1, \ldots, M$) we have:

$$\sum_{m=1}^{M} \left((\mathbf{L})_{bv} \mathbf{t}_v + \mathbf{t}_b \right)\, \widehat{\mathbf{n}}_m > \alpha\, \overline{p}_b \tag{5.4}$$

for a pair (b, v).

One can see that this constraint violation depends on the values of two branch flows, namely $\widehat{p}_b = \sum_m \mathbf{t}_b\widehat{\mathbf{n}}_m$ and $\widehat{p}_v = \sum_m \mathbf{t}_v\widehat{\mathbf{n}}_m$. The post-outage overload can be managed by decreasing the pre-outage flow in either of the two involved branches.

In the same way as we previously defined the participation of the mth TS in the bth branch's flow as $\mathbf{t}_b\widehat{\mathbf{n}}_m$, we can now define the participation of the mth TS in the overload of the bth branch after the outage of the vth one as $((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\ \widehat{\mathbf{n}}_m$. Again, all TS participations add up to the post-outage overload (5.4).

The effort to alleviate the congestion is again shared among the TSs, with the coordinator communicating to every TS a constraint involving only its own injection schedule in a way that if all TSs satisfy their constraints, then the initial overload is cleared, as was the case with (4.34). Let us call $\Delta\widetilde{p}_m^-$ the amount by which the mth TS is requested to contribute to the congestion alleviation. Note that this change refers to a post-outage flow, while the TS is requested to modify its pre-outage schedule. This means that $\Delta\widetilde{p}_m^-$ can be obtained from a $\Delta\widetilde{p}_m^-$ change of the TS's participation in the bth branch flow, or by a $\Delta\widetilde{p}_m^-/(\mathbf{L})_{bv}$ change of its participation in the vth branch flow, or by a combination involving both flows.

The policy we advocate remains that of contributing proportionally to the participation in the (now post-outage) overload, i.e. $\Delta\widetilde{p}_m^-$ is such that:

$$\frac{\Delta\widetilde{p}_m^-}{\sum_m((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\widehat{\mathbf{n}}_m - \alpha\overline{p}_b} = \frac{((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\widehat{\mathbf{n}}_m}{\sum_m((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\widehat{\mathbf{n}}_m} \tag{5.5}$$

and the mth TS will have to clear its market with the additional constraint:

$$((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\ (\mathbf{n}_m - \widehat{\mathbf{n}}_m) \leq -\Delta\widetilde{p}_m^- \tag{5.6}$$

A similar approach is followed for branches with $\sum_m ((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\ \ \widehat{\mathbf{n}}_m < -\alpha\ \overline{p}_b$. The mth TS is requested to change its participation in the post-outage overload by $\Delta\widetilde{p}_m^+$, with $\sum_m \Delta\widetilde{p}_m^+ = -\alpha\overline{p}_b - \sum_m((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\widehat{\mathbf{n}}_m$. This gives the following constraint for the mth TS:

$$((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\ (\mathbf{n}_m - \widehat{\mathbf{n}}_m) \geq \Delta\widetilde{p}_m^+ \tag{5.7}$$

with

$$\frac{\Delta\widetilde{p}_m^+}{-\alpha\overline{p}_b - \sum_m((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\widehat{\mathbf{n}}_m} = \frac{((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\widehat{\mathbf{n}}_m}{\sum_m((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\widehat{\mathbf{n}}_m} \tag{5.8}$$

Following the same reasoning as with the pre-contingency overloads (see Section 4.4.2), "counterflowing" TSs are assigned no constraint for a post-contingency overload. The term "counterflowing" is used, maybe in a little abusing manner, to refer to any TS whose participation $((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\ \ \widehat{\mathbf{n}}_m$ to the post-outage overload has a sign opposite to the overloaded branch flow. Thus, when using (5.5) the sums extend only over the schedules with positive contributions $((\mathbf{L})_{bv}\mathbf{t}_v + \mathbf{t}_b)\,\widehat{\mathbf{n}}_m$. Similarly, when using (5.8) the sums extend only over the schedules with negative contributions.

All in all, the Transmission allocation loop presented in Chapter 4 has be extended to incorporate some $N-1$ contingencies when managing congestion. The procedure remains otherwise

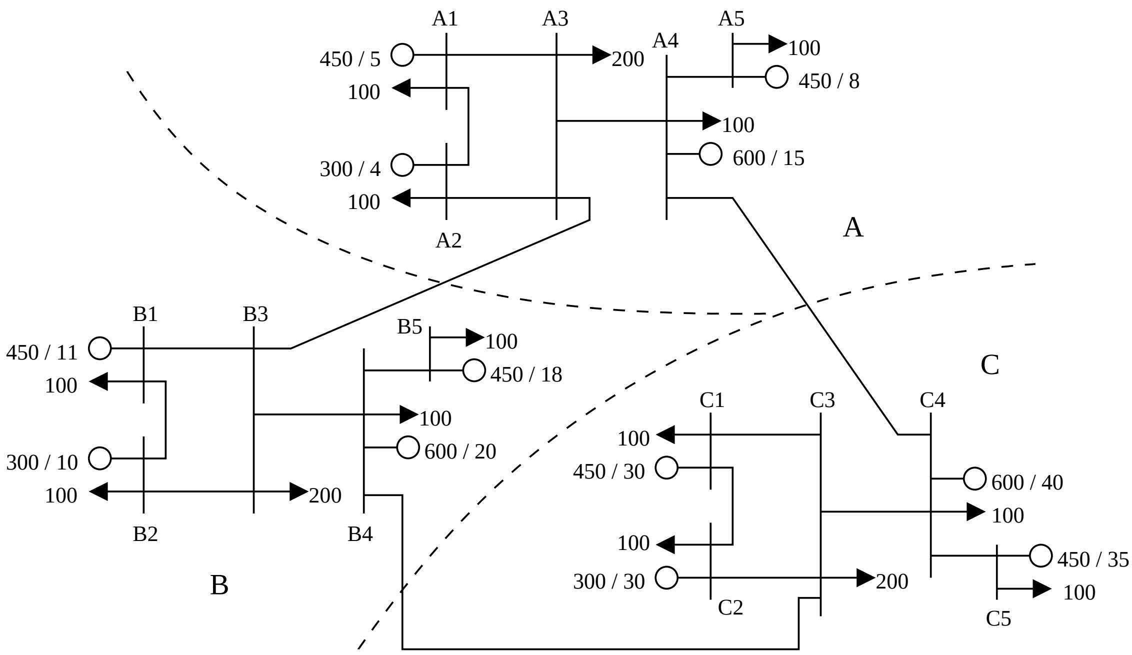

Figure 5.6: Three-area test system

unchanged. Iterations are performed between TSs and the coordinator until the termination criterion (see Section 4.5) is satisfied. The latter test also includes the post-outage overloaded branches. As for pre-outage branch overloads, post-outage overloads that have been solved during the iterations are prevented from taking place again by applying for those, formerly congested branches, constraints of type (5.6) and/or (5.7), which now eventually share the possible remaining capacity among TSs.

Note that the Energy allocation procedure remains compatible with this extended Transmission allocation one.

5.4.3 Illustrative example

The same example (with the test system's diagram reproduced in Fig. 5.6) as in Section 5.2.1 is used to illustrate the additional $N-1$ security feature of the Transmission allocation loop. In the post-outage limits (5.3) a parameter $\alpha = 1.1$ has been chosen. Table 5.11 shows, in columns 3 to 6, the resulting final generation schedules (per TS and total). All branches have been tripped, except A4A5, B4B5 and C4C5 whose tripping would island the system. For comparison, a single market clearing for the whole interconnection has been performed. The outcome of this clearing minimizes the total generation cost, while respecting all pre and post-outage power flow limits. The resulting generation schedules are presented in column 7 of Table 5.11. Finally, the last two columns of the table contain, respectively, the outcome of the proposed procedure and of a single system-wide market clearing when $N-1$ security constraints are not considered.

Table 5.11: Final generation allocation (in MW); incorporating $N-1$ security constraints

Gen	Bid	with $N-1$ constraints					without $N-1$	
		TS A	TS B	TS C	Total	Single	Total	Single
gA1	5	105	25	25	155	155	249	250
gA2	4	110	50	50	210	210	250	250
gA4	15	0	0	0	0	0	0	0
gA5	8	232	115	74	421	420	382	300
gB1	11	43	100	0	143	155	157	250
gB2	10	110	100	0	210	210	300	250
gB4	20	0	0	0	0	0	0	0
gB5	18	0	210	71	281	270	262	300
gC1	30	0	0	210	210	210	0	0
gC2	30	0	0	155	155	155	0	200
gC4	40	0	0	0	0	0	0	0
gC5	35	0	0	15	15	15	200	0

Table 5.12: Final point; resulting flows (in MW)

Branch	p_b	$\overline{p}_b$	Branch	p_b	$\overline{p}_b$	Branch	p_b	$\overline{p}_b$
A1A2	-19	100	B1B2	-23	100	C1C2	19	100
A1A3	74	150	B1B3	66	150	C1C3	91	150
A2A3	91	150	B2B3	87	150	C2C3	74	150
A3A4	-62	400	B3B4	-20	400	C3C4	27	400
A4A5	-320	400	B4B5	-182	400	C4C5	85	400
A3B3	27	200	A4C4	158	200	B4C3	62	200

Table 5.13: Final generation costs (in €/h)

	TS A	TS B	TS C	Total	Single
with N-1 security	4395	7127	13675	25197	25115
without N-1 security	4093	6467	11184	21743	21300
% of cost increase	7.38	10.21	22.27	15.89	17.91

Table 5.12 shows the branch power flows that result from the final generation schedules. The information contained in this table together with the information in Table 5.11 illustrate the effect of considering branch-outage constraints as well. For instance, with one of branches A1A3 and A2A3 out, the maximum power that can flow from buses A1 and A2 towards bus A3 equals 165 (= 1.1 $\times$ 150) MW. This is reflected to the total power injection schedules from those two buses (155 + 210 - 100 - 100 = 165 MW). Similarly, the maximum power that can flow inside area C equals 1.1 $\times$ 200 = 220 MW. Thus, the remaining 600 - 220 = 380 MW of the local load, served by TS C, must be produced inside the area. Indeed, one can notice that when branch outages are considered, less generation is scheduled, compared to the case without branch outages, from (a) generators gA1 and gA2, and, (b) generators gB1 and gB2. More generation is scheduled from generators inside area C.

The effect in terms of costs can be seen in Table 5.13. Expectedly, all TS costs are higher compared to the case where $N-1$ constraints were not considered. TS A is less affected, while TS C is the most. This is due to the fact that the post-outage branch flow limits have decreased the inter-area transfer capacities and, thus, each TS is obliged to resort more to generation from inside the area where its load is located. It is noteworthy that the total generation cost that resulted from the execution of the proposed algorithm is only 0.33% higher than the minimum total that can be attained (system-wide single clearing).

5.4.4 Incorporating generator outage security constraints

Another typical $N-1$ security constraint is the ability of the system to withstand the outage of a generator. In case of such an outage, other generators make up for the lacking active power through frequency control typically. It is acceptable to assume that they will increase their generation in proportion to a predefined participation factor. Assuming that the generators that participate in the "correction" are anyway dispatched to produce (we do not consider here how this is ensured), participation factors can be used to compute the bus power injections that would result after a generator outage and, thus, the resulting branch power flows.

Let as assume that after the outage of the jth generator, the ith one takes on some additional power according to:

$$\Delta g_i = h_{ij} \times g_j$$

where g_j is the power that was produced by the jth generator before the contingency and h_{ij} is the above mentioned participation factor. In general, the amount of extra power that the ith generator will produce after a generator outage does not depend on which is the lost generator, but only on the amount of lost power. So, in most cases, the participation factors h_{ij} will be equal for the various j. An exception stems from the fact that if the lost generator j is itself participating in frequency control, then the various factors h_{ij} are somewhat larger in order to account for the participation that was originally assigned to the jth generator.

In fact, before saying that the ith generator will augment its power production by $\Delta g_i = h_{ij} \times g_j$, it should be checked that, given the pre-contingency production g_i, there is enough remaining capacity available, i.e. generator i takes on Δg_i additional power only if $\Delta g_i \leq (\overline{g_i} - g_i)$, otherwise it increases its production up to $\overline{g_i}$ and the remaining $\Delta g_i - (\overline{g_i} - g_i)$ is shared among the other participating generators according to corresponding updated participation factors.

To summarize, after the outage of a generator j, we may assume that the making up of lacking power g_j will be distributed among some other generators in a way that depends on the predefined participation factors and the operating point of the participating generators. To simplify the reading, let us call $\Delta \mathbf{g}^j$ the vector containing the change in power production of *all* generators. By definition, we have $(\Delta \mathbf{g}^j)_j = -g_j$ and $(\Delta \mathbf{g}^j)_i = 0$ for a generator i that does not participate in frequency control, independently of j.

Thus, the post-contingency injection schedule, $\mathbf{n}'$, is given by:

$$\mathbf{n}' = \mathbf{n} + \mathbf{\Gamma} \, \Delta \mathbf{g}^j \tag{5.9}$$

where matrix $\mathbf{\Gamma}$, defined in Section 4.2.1, accounts for whether a generator is connected to a bus.

One can easily see that the branch flows $\mathbf{p}^j$ that result from the outage of the jth generator are given by:

$$\mathbf{p}^j = \mathbf{T}\,\mathbf{n}' = \mathbf{p} + \mathbf{T}\,\mathbf{\Gamma}\,\Delta\mathbf{g}^j \tag{5.10}$$

where $\mathbf{p}$ is the vector of pre-outage branch flows and matrix $\mathbf{T}$, defined in Section 4.2.1, contains the PTDFs, linking branch power flows with bus power injections.

The $N-1$ security criterion requires that all the flows resulting from a generator outage are within some limits:

$$-\alpha\,\overline{\mathbf{p}} \leq \mathbf{p}^j \leq \alpha\,\overline{\mathbf{p}} \tag{5.11}$$

where $\alpha \geq 1$ accounts for possible overload allowed in post-contingency situation (typically $1.05 \leq \alpha \leq 1.1$).

Coming back to our multi-TS problem, it is reasonable to consider that the generation changes that result from the outage of the jth generator are assigned to each TS schedule in proportion to how much of the lost generator's power it had dispatched, i.e. the mth TS's new generation schedule should be $\mathbf{g}'_m = \mathbf{g}_m + \Delta\mathbf{g}^j_m$, where $\mathbf{g}_m$ is its pre-contingency generation schedule. In other words, the outage of the jth generator results in a change $\Delta\mathbf{g}^j_m$ to the mth TS's allocated generator schedule.

Equation (5.10) shows that the post-outage flows $\mathbf{p}^j$ can be expressed as a linear function of all the TS schedules. Thus, for a branch overload resulting from a generator outage, the overload alleviation effort could be assigned to the various TSs in proportion to their participation in the post-contingency branch flow, exactly in the same way that (4.36) and (5.5) were built.

Another, maybe more reasonable (because it is based on a TS's involvement in the cause of the overload), possibility is to assign responsibility to TSs, not in proportion to their participation in the post-outage branch flow but, in proportion to how much of the lost generation's production they were dispatching. For instance, let us assume that after the outage of the jth generator, branch b gets overloaded with $p^j_b > \alpha\,\overline{p}$. We suggest that the alleviation

$$\Delta p = p^j_b - \alpha\,\overline{p}$$

is shared among the TSs which have allocated some capacity of the jth generator as follows:

$$\frac{\Delta p_m}{\Delta p} = \frac{(\mathbf{g}_m)_j}{\sum_m (\mathbf{g}_m)_j} \tag{5.12}$$

Again, this results in linear constraints being assigned by the coordinator to the various TSs and is easily incorporated into the proposed Transmission allocation loop.

5.5 Scheduling of reserves

5.5.1 Motivation

In electricity networks, the supply of power must equal the demand at all times in every location, or the system could experience disturbances including load shedding and cascading blackouts. The failure of a generator results in an imbalance between supply and demand that needs to be corrected. To prevent involuntary load shedding as a result of potential equipment failure, or contingency, system operators schedule operating reserves [FOH].

In a broad sense, the term “operating reserves” covers a wide range of applications related with the availability of generators and controllable loads to increase or decrease their production or consumption within a short timeframe. In [RKTR07a], they are classified as primary, secondary and tertiary frequency control reserves, while a variety of power systems around the world is considered, demonstrating the sometimes very different approaches followed by different TSOs in the used terminology and classification of their reserve services.

In [HK03], operating reserves are grouped into regulation and contingency-replacement reserves. Regulation, or load following, is an increase or decrease in production or consumption in response to unscheduled fluctuations. Contingency reserves are procured to guard against cascading outages in the wake of contingencies. There are various types of contingency reserves (spinning, non-spinning, up- and down-reserves etc.) that are used at different times after a disturbance has occurred. In principle, the idea is that, after a contingency, the system may no longer be in a secure state (i.e. it may be unable to withstand a second contingency). Thus, it is important for the TSO to have in its hands available corrective actions, such as generation and/or load re-dispatch, so that it can bring the new post-contingency system configuration into a secure state of operation.

To avoid technicalities which could be different from one system to another, in this section we group, for simplicity, different types of reserves, used to control frequency and maintain system security, into one single type of ancillary service which we simply refer to as “reserves”. What follows describes the treatment of this generic ancillary service in its essence. It is not suggested that each specific type of operating reserve fits this concept.

Typically, reserves are obtained by the system operator through a market process [NE, PJM, ERC, IES], where generators and certain loads bid (part of) their capacities, making them available to the system operator to use them if needed. For a generator this means that it does not sell its whole energy production potential, keeping some capacity available if asked to increase its output, while for a load this means that it offers the possibility to be partially shed. For simplicity and without loss of generality, let us consider only generators as available reserve units.

Obviously, a quantity that is scheduled as reserve cannot at the same time be dispatched in the energy market, i.e. the generator has an opportunity cost for offering its production capacity as reserve. In separate energy and reserve markets, generators have to anticipate that cost in order to include it when assessing their reserve offer price as well as when deciding how much of their capacity they will offer as reserve and how much they will keep to offer in the energy market (or vice versa, depending on which order the two markets are cleared). This is a rather complex problem for the generators and may result in inefficient use of generation capacities. For this reason, *energy-reserve co-optimization* (also referred to as *joint dispatch*), i.e. a simultaneous market clearing for energy and reserves, provides the most efficient way for allocating resources [ZL08]. Experience in Singapore, New Zealand and Australia, suggests that the co-optimization approach is successful in ensuring adequate provision of reserve and in lowering the overall cost of providing a secure supply of electrical energy [TK06]. This co-optimization simultaneously determines a price for energy and a price for reserve.

In fact, an intrinsic property of the joint energy-reserve approach, with a marginal pricing rule for both energy and reserves, is that it makes up for the opportunity costs incurred by generators which are competitive for producing energy but are called to participate in the provision of reserves [CC07]. The energy price reflects the marginal cost of supplying an increment in load and is equal to the cost of generating the additional energy while respecting the reserve requirement. On the other hand, the price of reserve reflects the marginal producer's offer to provide one more unit of reserve and the opportunity cost that this producer incurs when decreasing generation to provide reserve.

In the Energy and Transmission allocation method proposed in this work, it could be possible to let the various TSOs clear their local reserve markets independently, before or after the execution of the energy markets procedure. However, due to the above-mentioned higher efficiency of a joint clearing of those resources, it would be of interest to extend the proposed procedure to allow, during the scheduling of energy transactions, the implicit scheduling of reserves as well.

In this section a solution track is presented regarding the issue of incorporating the scheduling of reserves in the proposed procedure.

5.5.2 Statement of the energy and reserve co-optimization problem

Ideally, energy and reserve offers should be cleared in such a way that the overall cost is minimized while all pre and post-contingency constraints are satisfied.

In this respect, Ref. [AG05] examines the short-term operation and pricing of the various products traded in a joint energy-reserve market while accounting for transmission network flow limits and security constraints. In this market, besides submitting offers to sell and bids to buy energy, the participants can also contribute toward system security by offering to sell both up and down-spinning reserves at different rates. The system operator clears such a market by scheduling all the energy and reserve offers and bids so as to maximize the system social welfare while satisfying all operational constraints including those imposed by the need to survive the set of credible contingencies. Corrective security actions are explicitly accounted for in the market-clearing process by ensuring that all operational constraints are satisfied under all credible contingencies. These corrective actions define the required levels of two distinct types of reserve, namely, up and down-spinning reserves.

Recognizing the convenience for trading, and despite its theoretical suboptimality compared to the explicit consideration of every single contingency approach, a zonal reserve model is used in [ZL08, CC07, MSC99, AGM$^+$98]. In this model, a reserve zone is established for each import-constrained area based on historical studies. A zonal reserve requirement is then determined based on the simulation of an $N-2$ contingency event inside the local reserve zone. To satisfy local reserve requirements, resources, both inside and outside of the reserve zone, are utilized. Even simpler, Refs. [CSS$^+$00, WRAP04, MQ00, GL03, WWW05] consider a system-wide reserve requirement only.

For the purposes of our presentation, each TSO is assumed to have performed off-line studies and have come up with the adequate reserve requirement for its area. This translates into a total MW generation capacity, R_s, that needs to be scheduled as reserve inside the sth TSO area. Assuming that the sth TSO resorts to a pool-based joint (energy and reserves) dispatch, with transmission constraints also considered, yields the following optimization problem solved by the TSO:

$$\min_{\mathbf{g}_s,\mathbf{r}_s} \left\{ \mathbf{c}^T \mathbf{g}_s + \boldsymbol{\varrho}^T \mathbf{r}_s \right\} \tag{5.13a}$$

$$\text{s. t.} \quad \mathbf{1}^T \mathbf{g}_s = \mathbf{1}^T \mathbf{d}_s \tag{5.13b}$$

$$\mathbf{0} \le \mathbf{g}_s \le \overline{\mathbf{g}}_s \tag{5.13c}$$

$$\mathbf{0} \le \mathbf{r}_s \le \overline{\mathbf{r}}_s \tag{5.13d}$$

$$\mathbf{0} \le \mathbf{g}_s + \mathbf{r}_s \le \overline{\mathbf{g}}_s \tag{5.13e}$$

$$\mathbf{1}^T \mathbf{r}_s = R_s \tag{5.13f}$$

$$-\overline{\mathbf{p}}_s \le \mathbf{T} \left(\boldsymbol{\Gamma} \mathbf{g}_s - \boldsymbol{\Delta} \mathbf{d}_s \right) \le \overline{\mathbf{p}}_s \tag{5.13g}$$

where generator i bids its maximum production and reserve capacity, respectively $(\overline{\mathbf{g}}_s)_i$ and $(\overline{\mathbf{r}}_s)_i$ (with obviously $(\overline{\mathbf{r}}_s)_i \le (\overline{\mathbf{g}}_s)_i$) at the corresponding (marginal) costs c_i and ϱ_i. The various reserve bids, ϱ_i, do not refer to the cost of lost opportunity to sell energy (this is taken care implicitly by the dispatch's pricing rule), but are, instead, related to the expected cost of providing reserves, which might include some fixed administrative costs and some variable operating costs associated with providing reserve (e.g. a generator may operate at a higher heat rate and thus less efficiently when it produces less than its optimal output power) [TK06]. Load is assumed to be inelastic. Equation (5.13f) stands for the area's total reserve requirement, while (5.13g) expresses the requirement that the energy schedules should satisfy the transmission limits. Tables $\mathbf{T}$, $\boldsymbol{\Gamma}$ and $\boldsymbol{\Delta}$ are as defined in Section 4.2.1 and used throughout this report.

The outcome of the above optimization is the vector of scheduled generation powers $\mathbf{g}_s$ with offered (by the sth TSO) prices $\boldsymbol{\pi}$ and the vector of scheduled reserves $\mathbf{r}_s$ with offered prices $\boldsymbol{\pi}'$. Let us recall that the pricing rule is necessary such that the price $(\boldsymbol{\pi}')_i$ offered to the ith generator for providing reserves covers both its bid for reserve and its opportunity cost if some of its capacity has not been scheduled for energy production because it scheduled as reserve. A typical such reserve pricing mechanism is to offer to all generators a price equal to the Lagrange multiplier of constraint (5.13f).

The above formulation may be easily extended to allow a TSO define zones in its area, each of them having a different reserve requirement. To this purpose, (5.13f) would be substituted by as many such equations as the defined zones, with each import-constrained zone involving only generators that are located inside that zone.

5.5.3 Approach for scheduling reserves jointly with Energy allocation

Coming back to the overlapping market structure that has been proposed in this work, the "high-level" objective would be to have the proper amount of reserves, R_s, scheduled in each area s at the end of the iterations. Furthermore, those reserves should be scheduled according to market principles, i.e. most economic generators should be favored on one hand, while, on the other hand, generators' interests should be preserved (they should not undergo opportunity costs).

A natural choice seems to have the TSOs clearing reserves, while letting the TSs clear their energy markets. At first glance, it seems that this can be incorporated into the procedure, making use of the Energy allocation loop. At every iteration of the loop, the coordinator would receive prices offered to the generators for energy from the TSs and for reserve from the TSOs. That is, the TSOs would be solving a reduced version of problem (5.13), with the objective function containing the second term only and with the constraints (5.13d) and (5.13f) only. The coordinator would have to solve possible conflicts stemming from the fact that the total energy production asked by the various TSs for a generator i, together with the reserve quantity asked by the TSO s where the generator is located, may overpass the generator's maximum capacity, i.e. $\sum_m (\widetilde{\mathbf{g}}_m)_i + (\widetilde{\mathbf{r}}_s)_i > \overline{g}_i$, where we recall that $(\widetilde{\mathbf{g}}_m)_i$ is the ith generator's energy demanded by the mth TS and we similarly define $(\widetilde{\mathbf{r}}_s)_i$ as the ith generator's reserve demanded by the sth TSO.

However, the coordinator cannot take, based on offered prices, the dispatch decision that maximizes the generator's profit, as is the case when only energy is in question. The reason is that in order to compare an energy offer with a reserve offer for a given generator, one needs to know the generator's bid denoting its marginal operating cost. For instance, in case of a conflict between the mth TS's demand for energy from the ith generator and the sth TSO's demand for reserve from the same generator, with corresponding offered prices, respectively, $(\boldsymbol{\pi}_m)_i$ and $(\boldsymbol{\pi}'_s)_i$, the coordinator should give priority to the mth TS if

$$(\boldsymbol{\pi}_m)_i - c_i > (\boldsymbol{\pi}'_s)_i - \varrho_i \ ,$$

where we recall that c_i and ϱ_i are the generator's bids for energy and reserve, respectively, while each side of the inequality gives the generator's profit (for producing energy and for providing reserve). But the coordinator does not know the generators' bids, which are assumed to be confidential and, as a result, it cannot allocate generation capacity to TSs and TSOs using the rule described in Section 5.1.

The above difficulty could be circumvented by resorting to the "price-proof" approach discussed in Section 5.3.2, where generators enter the Energy allocation loop to take the allocation decisions on their own. However, two issues would remain unresolved:

1. It may not be appropriate to treat TSOs as competitors to TSs in the allocation of generators. Even more, considering the fact that the prices $\boldsymbol{\pi}'$ offered for reserves by the TSOs do not result from a energy-reserve co-optimization, it may be very difficult for a TSO to guess what price it should offer in order to allocate a generator as reserve.

2. For every area s, the satisfaction of the following inequality should be ensured at the end of the procedure: $\sum_i (\mathbf{r}_s)_i = R_s$. It is not acceptable to have all (or most of) the generators allocating their capacities for energy production, without enough being left for reserve. This suggests that a mechanism would be needed to tell which of the generators should produce less energy in order to provide reserve and at what price. In other words, scheduling reserves jointly with energy, requires to compare, not only energy with reserve offers for the same generator, but, also, offers made to different generators.

Let us further elaborate on point 2 presented above. If the generators are making the energy and reserve allocation on their own, the coordinator would still have to check that the following constraints are satisfied at each iteration for all TSOs:

$$\sum_{i \in s} \{(\overline{\mathbf{g}})_i - \textstyle\sum_m (\mathbf{g}_m)_i\} \geq R_s \qquad s = 1 \ldots \tag{5.14}$$

where $i \in s$ denotes that the ith generator is located inside the sth TSO area. The difficulty with implementing the above is that the coordinator needs a rule in order to compare different generator offers and decide which TS demand should be left unserved if needed. Let us clarify this with an example.

We consider a very simple case, with two generators (1 and 2) located in an area with a reserve requirement R and two TSs (A and B) dispatching those generators. Suppose that TS A asks for a $(\widetilde{\mathbf{g}}_A)_1$ production from generator 1 and a $(\widetilde{\mathbf{g}}_A)_2$ from generator 2. At the same time, TS B asks, respectively, $(\widetilde{\mathbf{g}}_B)_1$ and $(\widetilde{\mathbf{g}}_B)_2$. If $(\widetilde{\mathbf{g}}_A)_1 + (\widetilde{\mathbf{g}}_A)_2 + (\widetilde{\mathbf{g}}_B)_1 + (\widetilde{\mathbf{g}}_B)_2 > \overline{g}_1 + \overline{g}_2 - R$ then the TS demands cannot be fully satisfied because they do not leave enough capacity for reserve. Some of the TSs requested quantities should not be allocated. Assuming that $(\boldsymbol{\pi}_A)_1 > (\boldsymbol{\pi}_B)_1$ and $(\boldsymbol{\pi}_B)_2 > (\boldsymbol{\pi}_A)_2$, one can easily say that the demand of TS A for generator 1 should be favored against the demand of TS B for the same generator, while the demand of TS B should be favored against the demand of TS A for generator 2. However, with the above information only, one cannot say whether the coordinator should first satisfy the demand of TS A for generator 2 or the demand of TS B for generator 1.

Note that the Energy allocation procedure of Section 5.1 was free of the above problem because all comparisons were made for the same generator (or load in the general case); there had been no need to compare offers for different generators.

All in all, it does not seem possible to have the TSOs involved into the Energy allocation procedure as actors scheduling reserves: in order to achieve a joint dispatch, energy and reserve offers should be revealed to and treated by the same entity. Thus, we propose to incorporate the scheduling of reserves into the Energy allocation procedure by requiring *from the TSs* to take care of scheduling the required reserves jointly with their energy dispatches.

For this purpose, it is assumed that the various TSOs can express their area reserve requirement R_s as a percentage α_s of the total load dispatched (by the various TSs) in that area:

$$R_s = \alpha_s \sum_m D_m^s \ ,$$

where D_m^s is the total load dispatched by the mth TS in the sth area, given by $D_m^s = \sum_{j\in s}(\mathbf{d}_m)_j$. In this way, each TS could be asked to schedule a certain amount of reserves in each area where it dispatches some energy consumption. If the mth TS dispatches a total load D_m^s in area s, then it should schedule reserves $\mathbf{r}_m$ inside that area such that

$$\sum_{i\in s}(\mathbf{r}_m)_i = \alpha_s D_m^s \ .$$

The approach could be viewed as demanding from the TSs to schedule enough reserves to "support" their energy transactions, in the same spirit as they are asked to participate in congestion alleviation and in covering of losses (as will be discussed in the next section).

The above idea could work as follows.

- The generators submit to TSs bids for energy production ($\mathbf{c}_m$ being the vector of such bids for the mth TS) and for reserve provision ($\boldsymbol{\varrho}_m$ being the vector of such bids for the mth TS). They also submit to the coordinator their maximum energy and reserve capacities $\overline{\mathbf{g}}$ and $\overline{\mathbf{r}}$, respectively.

- Each TS clears a joint energy-reserve market, for instance solving a problem like (5.13), with constraint (5.13f) being replaced by $\mathbf{1}^T\mathbf{r}_s = \alpha_s\mathbf{1}^T\mathbf{d}_s$ for each area s where the TS dispatches some load. This gives the following joint market clearing for the mth TS:

$$\min_{\mathbf{g}_m,\mathbf{r}_m} \left\{\mathbf{c}_m^T\mathbf{g}_m + \boldsymbol{\varrho}_m^T\mathbf{r}_m\right\} \tag{5.15a}$$
$$\text{s. t.} \quad \mathbf{1}^T\mathbf{g}_m = \mathbf{1}^T\mathbf{d}_m \tag{5.15b}$$
$$\mathbf{0} \le \mathbf{g}_m \le \overline{\mathbf{g}}_m \tag{5.15c}$$
$$\mathbf{0} \le \mathbf{r}_m \le \overline{\mathbf{r}}_m \tag{5.15d}$$
$$\mathbf{0} \le \mathbf{g}_m + \mathbf{r}_m \le \overline{\mathbf{g}}_m \tag{5.15e}$$
$$\textstyle\sum_{i\in s}(\mathbf{r}_m)_i = \alpha_s \sum_{j\in s}(\mathbf{d}_m)_j \qquad s = 1,\ldots \tag{5.15f}$$
$$\mathbf{t}_b(\mathbf{g}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m) \le -(\Delta\mathbf{p}_m^-)_b \quad b = 1,\ldots \tag{5.15g}$$
$$\mathbf{t}_b(\mathbf{g}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m) \ge (\Delta\mathbf{p}_m^+)_b \quad b = 1,\ldots \tag{5.15h}$$

  where the last two constraints stem from the last transmission allocation iteration. Vectors $\overline{\mathbf{g}}_m$ and $\overline{\mathbf{r}}_m$ contain the maximum available capacities for energy and, respectively, reserve, communicated to the mth TS by the coordinator at the previous energy allocation iteration.

- Thus, the mth TS comes up with a set of demanded energy quantities $\widetilde{\mathbf{g}}_m$ with corresponding offered prices $\boldsymbol{\pi}_m$ and a set of demanded reserve quantities $\widetilde{\mathbf{r}}_m$ with corresponding offered prices $\boldsymbol{\pi}'_m$.

- This information cannot be treated by the coordinator to allocate quantities to TSs, since, as explained, it does not have enough information to compare energy with reserve offers (as already explained, bids $\mathbf{c}_m$ and $\boldsymbol{\varrho}_m$ are not disclosed to the coordinator for confidentiality reasons). To this reason, each TS "transforms" its offers for reserves into equivalent (in terms of the generators' profits) energy offers. This can be easily done by the mth TS if instead of offering a set of prices $\boldsymbol{\pi}'_m$ for demanded reserve capacities, it offers $\boldsymbol{\pi}_m'' = \boldsymbol{\pi}'_m - \boldsymbol{\varrho}_m + \mathbf{c}_m$. Like this, the modified price for reserve provision from the ith generator $(\boldsymbol{\pi}_m'')_i$ offered by the mth TS, can be compared with the offered price for energy production from the same generator $(\boldsymbol{\pi}_k)_i$ offered by another TS k, using the allocation mechanism that is used in the sole Energy allocation loop presented in Section 5.1.

- The Energy allocation loop is executed by the coordinator as explained in Section 5.1. Energy and reserve per generation are allocated in decreasing order of offered prices, up to the generators' available capacities $\overline{\mathbf{g}}$ and $\overline{\mathbf{r}}$. Again, what has been allocated to a TS in the previous iteration and is still asked by that TS, remains to the TS.

Similarly, the procedure could be applied with the TSs being requested to schedule reserves in proportion with the generation they dispatch in an area, instead of the load.

5.5.4 Illustrative example

Let us resort to the three-area 15-bus system used throughout this work (and recalled in Fig. 5.7) to illustrate how the above joint energy and reserve allocation method works. We assume that the amount of reserves in each area should equal 30% of the area's total load. Since in our example the loads are inelastic and each TS serves the load of an area, the reserve requirement translates into each TS having the obligation to schedule 180 MW of reserves from generators that are located inside its area.

Again, each generator is assumed to place the same marginal cost bid to all TSs for energy production, while, to what regards reserves, each generator's bid is taken as 0.3 times its energy bid, i.e. for every generator i we have: $(\mathbf{c}_A)_i = (\mathbf{c}_B)_i = (\mathbf{c}_C)_i = c_i$ and $\varrho_i = 0.3 \times c_i$. In addition, generators are assumed to make their whole capacities available for reserve provision, i.e. $\overline{\mathbf{r}} = \overline{\mathbf{g}}$. In this example, each generator bids for reserve provision only to the TS that serves the load of the area where the generator is located (since each area's load is served by a single TS). In general, of course, more than one TS may be dispatching load in an area, and, thus, each generator is expected to place a reserve bid to all the TSs serving load in the area where it is located. For simplicity, $N-1$ security constraints are not considered. All TSs come up with energy prices using a marginal clearing price rule, while, for reserves, their offered price

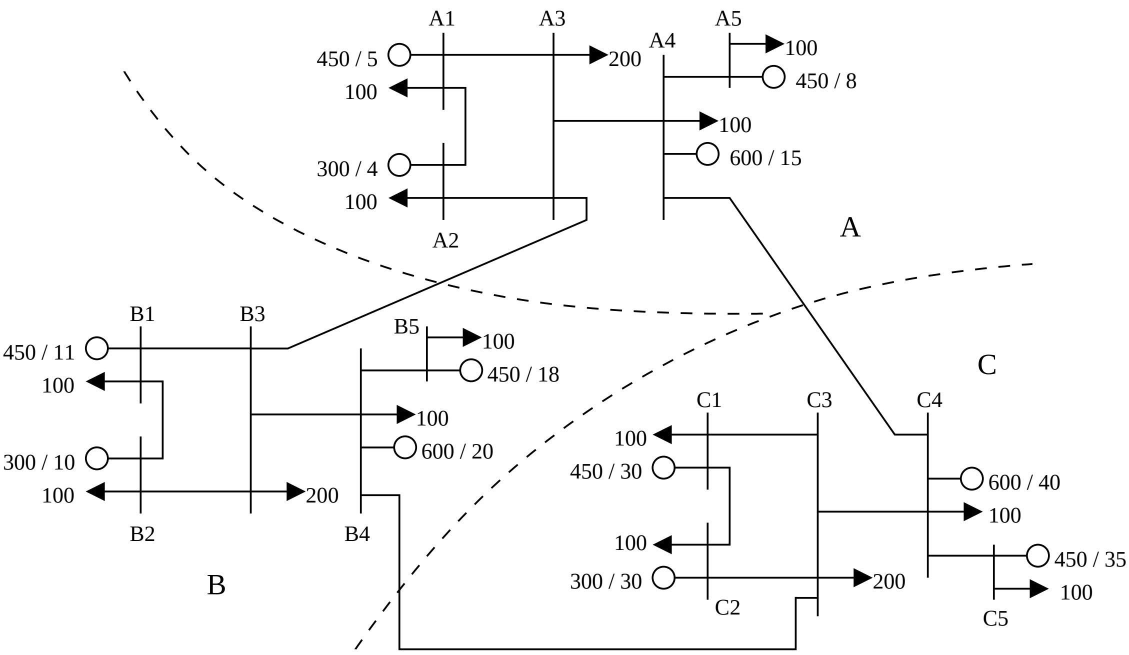

Figure 5.7: Three-area test system

Table 5.14: Iteration 1: 1st Energy allocation step

Gen	Energy Bid	Reserve Bid	Energy			Reserves			Max
			TS A	TS B	TS C	TS A	TS B	TS C	
	(in €/MWh)		Demanded Quantities (in MW)						
gA1	5	1.5	300	300	300	150	-	-	450
gA2	4	1.2	300	300	300	0	-	-	300
gA5	8	2.4	0	0	0	30	-	-	450
gB2	10	3.0	0	0	0	-	180	-	300
gC1	30	9.0	0	0	0	-	-	180	450
Clearing Prices (in €/MWh)			5	5	5	2.4	3	9	
			Allocated Quantities (in MW)						
gA1	5	1.5	100	100	100	150	-	-	450
gA2	4	1.2	100	100	100	0	-	-	300
gA5	8	2.4	0	0	0	30	-	-	450
gB2	10	3.0	0	0	0	-	180	-	300
gC1	30	9.0	0	0	0	-	-	180	450

is the Lagrange multiplier of constraint (5.15f) (note that each TS has one such constraint in its market clearing problem, corresponding to the area where its load is located).

The first two Energy allocation loop iterations (where reserves are now also allocated) are detailed as an illustration of how the method works. Note that those are inner loop executions, inside the first transmission allocation iteration of the procedure, and hence no branch flow constraints are yet considered by the TSs. In Table 5.14 results from the first energy allocation step are shown for the generators that have been demanded by the TSs for energy production or reserve provision. The first three columns of this table show the name of the generation, its

Table 5.15: Iteration 1: 1st Energy allocation step; prices related to TS A (in €/MWh)

Gen	Energy				Reserves			
	Bid $\mathbf{c}_A$	Clearing Price $(\boldsymbol{\pi}_A)$	Offered Price $(\boldsymbol{\pi}_A)$	Profit $(\boldsymbol{\pi}_A) - \mathbf{c}_A$	Bid ϱ_A	Clearing Price $(\boldsymbol{\pi}'_A)$	Offered Price $(\boldsymbol{\pi}_A'')$	Profit $(\boldsymbol{\pi}'_A) - \varrho_A$
gA1	5	5	5	0	1.5	2.4	5.9	0.9
gA2	4	5	5	1	1.2	/	/	/
gA5	8	/	/	/	2.4	2.4	8	0

energy bid and its reserve bid, respectively, while in the last column their respective capacities are shown. The remaining of Table 5.14 (columns 4-9) is divided into three blocks. Rows 4-8 contain the demanded energy and reserve quantities per TS (a dash in some reserve fields means that the corresponding TS does not schedule the corresponding generator as a reserve). Row 9 contains the energy and reserve prices resulting from the market clearing that gave the aforementioned demanded quantities. Finally, rows 11-15 contain the energy and reserve quantities that have been allocated by the coordinator to the various TSs.

Table 5.15 serves as a complement to Table 5.14, helping to illustrate how the TSs are transforming their reserve clearing prices to equivalent energy offered prices. It refers only to TS A. For both energy (columns 2-5) and reserves (columns 6-9), each row refers to one generator and contains, respectively: the generator's bid, the price that resulted from the market clearing for that generator, the offered price announced to the coordinator by the TS that is to be used for energy or reserve allocation, and, finally, the generator's profit that corresponds to the clearing price. The latter is the clearing price minus the generator's bid, both for energy as for reserves as well. In case of energy, the offered price, announced to the coordinator, is the energy clearing price of the market. To what regards the reserves, one can check that, for each generator, this price stems from the "transformation" (explained in the previous subsection) allowing the coordinator to compare reserve with energy prices without any knowledge of the bids. For instance, the reserve clearing price of 2.4 €/MWh that TS A is offering to gA1, is equivalent, in terms of gA1's profit, to offering 5.9 €/MWh for energy production. The / in some fields means that no energy or reserve is requested from this generator by TS A.

Back to Table 5.14, one can see that, as already explained in Section 5.1, due to the absolute "symmetry" of the TSs (equal served load, same marginal clearing pricing rule, absence of branch flow constraints) they all three request the same energy quantities with equal offered energy prices. As for reserves, each TS selects the cheapest solution from each area (for TS A, this means that it schedules the remaining 150 MW from gA1 and an additional 30 MW from the more expensive gA5). The reserve clearing price of each TS (row 9) equals the reserve bid of the most expensive generator. This reflects the fact that no generator encounters an opportunity cost for being scheduled as reserve.

The quantities are allocated according to the already explained rule. Only in the case of gA1 and gA2 there is a conflict. Note that the 150 MW that TS A asked as reserve from gA1 have been allocated to it, while the remaining 300 MW of gA1 have been equally shared among the (equally demanding) TSs for energy production. The reason why TS A got the whole reserve it asked from this generator, is that its offered *reserve* price for gA1 is higher than the offered

Table 5.16: Iteration 1: 2nd Energy allocation step

Gen	Energy Bid	Reserve Bid	Energy TS A	Energy TS B	Energy TS C	Reserves TS A	Reserves TS B	Reserves TS C	Max
	(in €/MWh)		Demanded Quantities (in MW)						
gA1	5	1.5	250	100	100	0	-	-	450
gA2	4	1.2	100	100	100	0	-	-	300
gA5	8	2.4	250	400	400	180	-	-	450
gB2	10	3.0	0	0	0	-	180	-	300
gC1	30	9.0	0	0	0	-	-	180	450
Clearing Prices (in €/MWh)			8	8	8	2.4	3	9	
			Allocated Quantities (in MW)						
gA1	5	1.5	250	100	100	0	-	-	450
gA2	4	1.2	100	100	100	0	-	-	300
gA5	8	2.4	87.5	140	140	82.5	-	-	450
gB2	10	3.0	0	0	0	-	180	-	300
gC1	30	9.0	0	0	0	-	-	180	450

Table 5.17: Final energy and reserve generation schedules

Gen	Energy Bid	Reserve Bid	Energy TS A	Energy TS B	Energy TS C	Energy Total	Energy Single	Reserves TS A	Reserves TS B	Reserves TS C	Reserves Single	Max
gA1	5	1.5	173	78	1	252	250	128	-	-	130	450
gA2	4	1.2	173	70	4	247	250	52	-	-	50	300
gA5	8	2.4	254	47	0	301	300	0	-	-	0	450
gB1	11	3.3	0	145	105	250	250	-	130	-	130	450
gB2	10	3.0	0	122	128	250	250	-	50	-	50	300
gB5	18	5.4	0	138	162	300	300	-	0	-	0	450
gC1	30	9.0	0	0	0	0	0	-	-	78	180	450
gC2	30	9.0	0	0	0	198	200	-	-	102	0	300
gC5	35	10.5	0	0	2	2	0	-	-	0	0	450

energy prices of all TSs (see Table 5.15 and row 9 of Table 5.14). The latter is due to the fact that TS A had to resort to (the more expensive) gA5 to cover all the reserves it needed, which increased the marginal reserve price above the gA1's reserve bid.

Table 5.16 shows the demanded, and allocated, quantities after the second step of the Energy allocation loop. An interesting feature of the approach appears at this step and deserves some comment. One can notice that TS A used the whole capacity of gA1 allocated to it by the coordinator, for energy production, even if 150 MW of this capacity was initially meant to be allocated to TS A as reserves. This is not an issue, but it should be stressed that 150 MW of the now scheduled 250 MW of gA1's energy production had been allocated to TS A at a price of 5.9 €/MWh and, thus, as explained in Section 5.3.3, TS A should pay this price (while for the remaining 150 MW, it has to pay 5 €/MWh, at which it was allocated this power).

The procedure goes on iterating between energy and transmission allocations, and, after 8 transmission allocation iterations, it converges to the final generation schedules shown in Table 5.17. One can see that, since the use of gA1, gA2, gB1 and gB2 for energy production has been limited due to transmission constraints, those generators, being less expensive, have been used for reserve provision. It is noteworthy that the total schedules that resulted from the proposed procedure are practically identical to the schedules that result from a single system-wide energy-reserve co-optimization (note that the reserve schedules in area C resulting from the proposed method and those resulting from the system-wide clearing are equivalent in terms of cost).

It seems that the proposed way to jointly schedule reserves in the Energy allocation loop works as expected, without compromising the already presented feature of the approach. Admittedly, more tests are needed to draw more definite conclusions.

Finally, let us recall that the proposed method has been built on the assumption that it is acceptable to have TSs scheduling reserves and that the areas' TSOs can express their reserve requirements as proportional to their respective areas' total dispatched load (or generation). In case the above conditions are not met, other possible schemes that would achieve efficient energy and reserve co-optimization should be thought of. Clearly, the intrinsic difficulty of finding such schemes lies on the fact that generators' energy and reserve bids need to be assembled by one entity. Given the market participants' confidentiality restrictions, this may not be an easy task.

5.6 Accounting for losses in Transmission allocation

In the Energy and Transmission allocation procedure presented down to here, the transmission system has been assumed lossless. However, losses correspond to a non negligible percentage of the energy production (for instance a figure of approximately 4% is cited in [GT00, dSdCC03]). Thus, it is appropriate, when scheduling generation and allocating transmission capacity, to also account for losses. The viewpoint adopted here is that each TS should be assigned the responsibility for the losses it "creates" due to its schedule.

5.6.1 Estimating transmission losses in DC models

In [SJA09], a review of DC models used in the power system literature and applications is presented. DC models are classified as *hot-start* or *cold-start* models. The former are constructed at a solved AC power flow base case, while the latter are typically resorted to when a reliable reference AC power flow solution is unavailable (usually due to lack of good voltage/var data). PTDF models belong to the general category of incremental DC models, i.e. sensitivities for changes around a base case. The base case can be an existing AC or DC solution. Thus, in general, PTDFs may correspond to a hot-start or to a cold-start model. The PTDF model used in this work is a cold-start DC model, as it is derived directly from the network's branch reactance matrix (see Section 4.2.1).

Not accounting for transmission losses when using a DC model may result in significant errors, which tend to accumulate close to the slack bus (or buses) chosen to compensate those losses.

In order to avoid this, the losses of each branch can be modeled as additional power withdraws at the two end buses of the branch [SJA09]. In case of hot-start models, those withdraws can be evaluated from the already known branch losses. On the contrary, in cold-start models the net transmission losses have to be estimated and then dispersed among the various buses in the system.

For the simultaneous market clearings proposed in this work, in the absence of an accurate estimate of losses, we resort to an approach proposed in [SJA09, LB07], where the estimation of losses and their distribution among the system buses are performed iteratively while clearing the market.

Initially the branch flows are computed according to the lossless model, as in (4.6). Then, the losses l_b in each branch b are calculated using the approximation $l_b = r_b p_b^2$, where r_b is the branch series resistance. Those branch losses are translated into bus power withdraws, to be treated as loads at the next iteration. To this purpose, an additional power withdraw $l_b/2$ is assigned to each end bus of the branch. New generation schedules are then computed in order to compensate for the additional withdraws and branch flows are again computed using (4.6). The branch losses can then be updated based on the new flows, and so on. The procedure is fast, it usually converges in at most three iterations [SJA09, LB07].

The above technique can be easily applied to the overlapping market problem, taking advantage of the iterative nature of the market clearing procedure to update the power withdraws accounting for losses. This is easily added to the Transmission allocation loop; the coordinator, after computing the branch flows, calculates the corresponding losses as well. But, since each of the M markets is power balanced, a mechanism is needed to share among the various TS the additional generation needed to cover the additional power withdraws.

5.6.2 Allocating losses to TSs

Allocating responsibility for transmission losses to the various market participants is a topic that has attracted a lot of attention in the power system literature. The basic motivation lies in the need to allocate the cost of those losses. Some methods allocate branch losses to individual generators and loads [Bia97, KAS97, CGK01, DA06], or to bilateral or multilateral transactions [A. 00, DA04, LG04]. Furthermore, there exist methods where transmission losses are computed from the full AC network model [Bia97, KAS97, CGK01, DA04, DA06] and others where a DC model is applied [A. 00, LG04].

Different loss allocation techniques have been proposed, such as *pro rata* techniques [IGF98], *marginal* techniques [Elg71, GCK02] (based on incremental transmission loss coefficients) and *flow tracing* techniques [Bia97, KAS97] (based on the neither provable nor disprovable assumption that the inflows are proportionally shared among the outflows at each network node). We resort to *loss formula* methods [A. 00, LG04, CGK01, DA04], which are more appropriate in terms of expressing losses with individual nodal injections or transactions [DA06]. These methods express the losses in each branch according to the power flow equations, either in AC

[CGK01, DA04], where the expression $l_b = r_b|i_b|^2$ is used, $|i_b|$ being the magnitude of the current branch b, or in DC [A. 00, LG04], with resort to the simplified expression $l_b = r_b p_b^2$.

The main challenge stems from the fact that system losses are a nonseparable, nonlinear function of the bus power injections, which makes it impossible to divide the system losses into a sum of terms, each one uniquely attributable to a generation/load or a transaction. Thus, the final allocation contains always a degree of arbitrariness. This issue of fairness will probably never be fully resolved [CGK01].

The nonseparable nature of losses is easily seen by using (4.3) and (4.6) in the DC approximation of the branch losses:

$$l_b = r_b \left(\sum_m \mathbf{t}_b \mathbf{n}_m \right)^2 = r_b \left(\sum_m (\mathbf{t}_b \mathbf{n}_m)^2 + \sum_m \sum_{k \neq m} (\mathbf{t}_b \mathbf{n}_m)(\mathbf{t}_b \mathbf{n}_k) \right) \qquad (5.16)$$

where each bilinear term involves the participation of two TSs in the branch flow.

When allocating the losses to the various TSs, it seems straightforward to allocate each term $r_b(\mathbf{t}_b\mathbf{n}_m)^2$ to the mth TS. On the other hand, terms involving two TSs, i.e. $r_b(\mathbf{t}_b\mathbf{n}_m)(\mathbf{t}_b\mathbf{n}_k)$, need to be shared among them. In [A. 00], the authors argue that it may be unfair to equally divide each such term between the two TSs (as is done for example in [LG04, CGK01, DA04]), i.e. to allocate to the mth TS $\frac{r_b}{2}(\mathbf{t}_b\mathbf{n}_m)(\mathbf{t}_b\mathbf{n}_k)$ for the term it shares with the kth TS.

To illustrate the authors' reasoning in [A. 00], let us consider a two-TS case where sharing the bth branch flow $p_b = 1.1$p.u. among the TSs gives participations $p_b^A = 1.0$p.u. for TS A and $p_b^B = 0.1$p.u. for TS B. Assuming that $r_b = 0.1$p.u., the branch losses equal $l_b = r_b(p_b^A + p_b^B)^2 = 0.121$p.u., with $r_b(p_b^A)^2 = 0.1$p.u., $r_b(p_b^B)^2 = 0.001$MW and $2r_b(p_b^A)(p_b^B) = 0.02$MW. It may be argued that simply diving the bilinear term by two gives a disproportional responsibility to TS B for the losses.

Different ways for allocating the bilinear terms are thus proposed in [A. 00]. In the present work, we followed the idea of allocating the bilinear term in proportion to the square of each TS participation in the branch flow. The motivation for this choice is the quadratic relationship between power flows and losses and the will to be consistent with the chosen policy for congestion management (see Section 4.4). Hence, every bilinear term is assigned as follows:

$$\text{to the } m\text{th TS: } \frac{(\mathbf{t}_b\mathbf{n}_m)^2}{(\mathbf{t}_b\mathbf{n}_m)^2 + (\mathbf{t}_b\mathbf{n}_k)^2} \, r_b(\mathbf{t}_b\mathbf{n}_m)(\mathbf{t}_b\mathbf{n}_k)$$

$$\text{to the } k\text{th TS: } \frac{(\mathbf{t}_b\mathbf{n}_k)^2}{(\mathbf{t}_b\mathbf{n}_m)^2 + (\mathbf{t}_b\mathbf{n}_k)^2} \, r_b(\mathbf{t}_b\mathbf{n}_m)(\mathbf{t}_b\mathbf{n}_k)$$

Thus, coming back to the loss allocation mechanism performed in the Transmission allocation loop, the coordinator, after computing the branch flows, allocates the branch losses to the various TS and, together with the congestion management constraints, it communicates to the TSs the corresponding bus withdraws to cover in their new market clearings. For example, if the

ith bus is connected with branches b and v, then the following injection will be communicated to the mth TS for this bus:

$$-\frac{r_b}{2}\left((\mathbf{t}_b\mathbf{n}_m)^2+\sum_{k\neq m}\frac{(\mathbf{t}_b\mathbf{n}_m)^2}{(\mathbf{t}_b\mathbf{n}_m)^2+(\mathbf{t}_b\mathbf{n}_k)^2}(\mathbf{t}_b\mathbf{n}_m)(\mathbf{t}_b\mathbf{n}_k)\right)$$
$$-\frac{r_v}{2}\left((\mathbf{t}_v\mathbf{n}_m)^2+\sum_{k\neq m}\frac{(\mathbf{t}_v\mathbf{n}_m)^2}{(\mathbf{t}_v\mathbf{n}_m)^2+(\mathbf{t}_v\mathbf{n}_k)^2}(\mathbf{t}_v\mathbf{n}_m)(\mathbf{t}_v\mathbf{n}_k)\right)$$

5.6.3 Illustrative example

Again, we resort to the 15-bus three-area system to illustrate the operation of the enhanced procedure. Each branch series resistance has been taken equal to 1/10 of the branch reactance. Table 5.18 shows the final generation schedules that resulted from the execution of the Energy and Transmission allocation procedure incorporating $N-1$ constraints (columns 3-6), as well as ignoring the $N-1$ constraints (columns 8-11). Columns 7 and 12 present, with and without $N-1$ constraints respectively, the generation schedules that result from a single system-wide market clearing. Iterations of market clearings, as described in Section 5.6.1, have been performed in order to account for losses.

For comparison, Table 5.19 shows the generation schedules that have resulted from executing the proposed procedure and a single system-wide clearing, without accounting for losses, both with (columns 3-7) and without (columns 8-12) $N-1$ constraints. Expectedly, some additional generation had to be dispatched to cover losses. In the case of Transmission allocation considering only pre-outage branch flow limits, one can notice that the final schedules are similar in the two cases (with and without losses), with only some additional productions from some generators in the case where losses are accounted for. This "similarity" holds in fact during the whole sequence of iterations; whether losses are accounted for or not, modifies the demanded and allocated TS generations by only a few MW for all generators and TSs.

On the contrary, the reader can notice that, when post-outage branch flow limits are considered in the Transmission allocation, accounting for losses resulted in a "qualitative" difference in some TS generation schedules. Namely, "thanks to" the loss allocation mechanism, TS C has been able to allocate some production from generators gB1 and, mostly, gB2, which allowed it to resort by a less amount to the, more expensive compared to gB1 and gB2, generator gB5 (see numbers in bold in Tables 5.18 and 5.19). This affected TS A, who, in the case with losses, did not allocated as much of gB2 as in the lossless case (again, see bold numbers in the aforementioned tables).

The above result reveals, in fact, a general property of the proposed procedure, worth receiving a comment. There exist two types of discrete "decisions", taken by the coordinator in respectively the Energy and the Transmission allocation loop, the outcome of which may significantly change the remaining iterations.

1. When market participants are allocated to TSs in the Energy allocation loop, in case of

Table 5.18: Final generation allocation (in MW); accounting for losses

Gen	Bid	incorporating $N-1$ constraints					without $N-1$ constraints				
		TS A	TS B	TS C	Total	Single	TS A	TS B	TS C	Total	Single
gA1	5	105	25	25	155	155	135	111	4	250	250
gA2	4	110	50	50	210	210	97	50	103	250	250
gA4	15	0	0	0	0	0	0	0	0	0	0
gA5	8	229	116	83	428	428	326	65	0	391	305
gB1	11	**48**	100	**7**	155	155	8	100	49	157	250
gB2	10	**65**	100	**45**	210	210	40	100	160	300	250
gB4	20	0	0	0	0	0	0	0	0	0	0
gB5	18	**46**	214	**12**	272	273	0	178	89	267	306
gC1	30	0	0	210	210	210	0	0	0	0	0
gC2	30	0	0	155	155	155	0	0	0	0	204
gC4	40	0	0	0	0	0	0	0	0	0	0
gC5	35	0	0	17	17	17	0	0	207	207	0

Table 5.19: Final generation allocation (in MW); no account for losses

Gen	Bid	incorporating $N-1$ constraints					without $N-1$ constraints				
		TS A	TS B	TS C	Total	Single	TS A	TS B	TS C	Total	Single
gA1	5	105	25	25	155	155	136	113	0	249	250
gA2	4	110	50	50	210	210	98	56	96	250	250
gA4	15	0	0	0	0	0	0	0	0	0	0
gA5	8	232	115	74	421	420	324	58	0	382	300
gB1	11	**43**	100	**0**	143	155	9	100	48	157	250
gB2	10	**110**	100	**0**	210	210	33	100	167	300	250
gB4	20	0	0	0	0	0	0	0	0	0	0
gB5	18	**0**	210	**71**	281	270	0	173	89	262	300
gC1	30	0	0	210	210	210	0	0	0	0	0
gC2	30	0	0	155	155	155	0	0	0	0	200
gC4	40	0	0	0	0	0	0	0	0	0	0
gC5	35	0	0	15	15	15	0	0	200	200	0

conflict between two (or more) TSs, whether a TS will get or not the right to dispatch a participant depends on its offered price π in a switch-like manner. There is a threshold value, defined by the other TSs offered prices for the market participant under question, above/below[4] which the TS will get all the quantity it asked for, and below/above which it will get nothing[5].

2. In the Transmission allocation loop, the switch-type rule stems from the treatment of counterflowing TSs. For any congested branch, a flow participation of just below/above[6] zero MW makes the difference between the TS under question receiving no constraint for that branch or receiving a constraint that does not allow the TS to increase/decrease its branch flow contribution.

[4]Depending on whether the participant is a generator or a load.
[5]A third case is when π equals the threshold value, which results in the TS getting a part of what it asked for.
[6]Depending on the direction of the main flow.

Table 5.20: Final generation costs (in €/h)

$N-1$	losses	TS A	TS B	TS C	Total	Single
yes	no	4395	7127	13675	25197	25115
	yes	4817	7210	13271	25298	25300
no	no	4093	6467	11184	21743	21300
	yes	4155	6572	11416	22142	21568

Coming back to our results, the counterflow-related switch-type behavior is the reason why incorporating $N-1$ constraints ended up with the above mentioned difference between the cases with and without losses accounted for, respectively. After the first iteration of the Transmission allocation loop, in both cases (losses accounted or not), branches B1B3 and B2B3 get overloaded and the alleviation effort is shared among the three TSs (in both cases a power flow decrease of 25 MW is assigned to TS C for each of the two branches). After the second Transmission allocation iteration, both branch flows are below their limits and according to the congestion management policy the remaining capacity is shared among the TSs proportionally to their participations in the flows. In the lossless case, it happens that TS C is creating zero flow in branches B1B3 and B2B3 (due to the various constraints it received, it had to deallocate all the generation it had dispatched from gB1 and gB2) and thus, according to the rule, it receives zero from the remaining MW capacities of those branches. This results in TS C not being able to schedule generation from gB1 and gB2 at the next iteration (and, more generally, in the remaining of the procedure). On the contrary, in the case with losses, again TS C is obliged to deallocate all generation it had from gB1 and gB2, but now, due to the loss allocation mechanism, power withdraws at buses B1 and B2 have been assigned to it, stemming from its participation in the previous iteration's losses in branches B1B3, B1B2 and B2B3. Thus, it happens that, after the second Transmission allocation iteration, TS C's participation in the B1B3 and B2B3 branch flows is a -0.242 MW flow for each branch. In other words, TS C is now counterflowing in those branches and, according to the rule, no constraint related to them is assigned to it for the next iteration. This results in TS C scheduling, and keeping until the end of the procedure, some generation from gB1 and gB2.

A possible way for dealing with this switch-type issue is mentioned in the section that follows.

Finally, Table 5.20 collects the resulting generation costs, for all four cases presented in Tables 5.18 and 5.19. One can see that accounting for transmission losses leads to a small augmentation of generation costs. Where post-contingency constraints are incorporated into the congestion management problem, the final total generation cost is practically the same as what would have resulted from a single system-wide optimization (the fact that it turns out to be 2 €/h less than the single optimization cost is just due to some rounding effects). Where only pre-contingency flow limits are considered in the congestion management problem, the cost from the system-wide optimization is 2.6% lower than the cost obtained with the proposed approach.

5.7 A comment on the energy and the transmission allocation rule

In the previous section, it was shown with an example that a small change in a TS schedule could result in a significant change in the final outcome, due to the two switch-type behaviors of the Energy and Transmission allocation procedure.

To what regards energy allocation, this behavior could be easily overcome by having the coordinator allocate capacities *in proportion* to the TSs' price offers. Like this, an infinitesimal difference in a TS's price offer would always result in an infinitesimal difference in the quantity it will be allocated. However, such a policy would go against the generators' profits and, more generally, against economic efficiency. Furthermore, it would be difficult to have it accepted (why should a generator be obliged to sell part of its capacity to somebody who values it less than a competitor?). In fact, this switch-type behavior already exists and is accepted in the various pool-based electricity markets, where, typically, a generator whose bid is even infinitesimally higher than the bid of the marginal generator, is not dispatched for energy production.

On the contrary, to what regards transmission allocation, the differentiation made between counterflowing and non counterflowing TSs might not be considered acceptable. It would seem more reasonable that a TS with a small positive contribution to an overload is not treated significantly different than a TS with a small negative contribution (i.e. a counterflow) to the same overload. In this respect, Fig. 5.8 outlines how low contributions to an overloaded branch flow could be treated. In both diagrams, the horizontal axis shows the participation of a TS in the branch flow, while the vertical axis shows the part of the branch's capacity that is allocated to this TS as a result of the congestion management policy. The left diagram corresponds to the policy that has been used in this work and the right diagram to a "smoother" policy. For simplicity it has been assumed that a counterflowing TS can use up to the whole capacity of a branch.

The left diagram shows, as already explained in Chapter 4, that a TS counterflowing in a congested branch (i.e. a branch that at some point during the iterations has been overloaded) will be unlimited regarding the flow it creates in this branch at the next iteration, while a TS which is not counterflowing will be allowed to use, at most, such part of the branch's capacity as it is presently using. One can easily understand that a TS with a small contribution to an overloaded branch, will practically be obliged to keep this small contribution for the remaining of the iterations (except if another TS, with high contribution, decreases significantly its participation in the flow).

The principle of the solution that is qualitatively shown in the right diagram of Fig. 5.8, consists in allowing: (a) TSs with low flow contributions to somewhat increase their utilization of the branch, and (b) counterflowing TSs to remain unconstrained *progressively*, such that a TS with an infinitesimal low counterflow is treated in almost the same way as a TS with an infinitesimal low positive contribution.

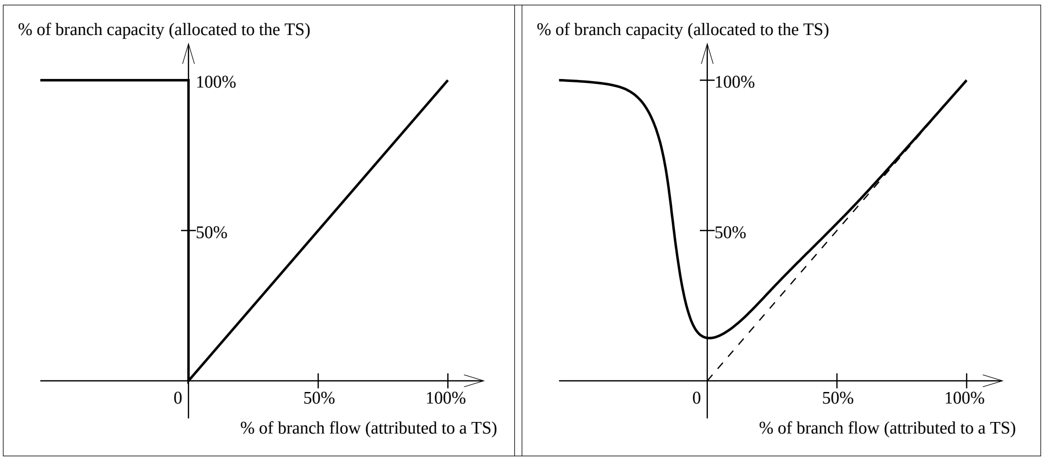

Figure 5.8: Making the transmission allocation rule "smoother"

The rationale behind the "smoother" approach is twofold: first, a TS should not be treated significantly different from another if their flow contributions are relatively close to each other, and second, a TS should not be left trapped into having the right to use only a very small percentage of a branch's capacity. Note that for larger positive or counter- flows, the "smoother" approach goes asymptotically to the rule that has been used in this work. However, it may not be obvious to choose the appropriate smooth function, while convergence difficulties may be experienced (since the decomposed constraints assigned to the TSs contributing to the counterflow do not altogether alleviate the overload). Future implementation and testing of this approach could be of interest.

5.8 Will TSs try to act strategically ?

5.8.1 Motivation

It is natural that the players in a game try to act strategically. To this purpose, a player may anticipate what the others' actions will be and include this information into its decision-making problem. For instance, in the proposed method for Energy and Transmission allocation, the various TSs, when clearing their markets during the successive iterations of the algorithm, could aim at dispatching their participants in a way that, given the other TSs' market clearings and the coordinator's decisions, they obtain an as small as possible cost at the equilibrium of the procedure.

The problem of an electricity producer's optimal bidding strategy in a pool electricity market provides an example of strategic behavior from the power system literature [BZTB07, BCG$^+$06, HMP00, WO02, RC09]. In this problem, the producer chooses the bid it should submit to the market operator such that the market clearing's outcome (i.e. the scheduled quantity to be actually produced and the corresponding price to be received by the producer) maximizes the producer's profit. The producer knows that the market clearing depends on all the producers' submitted bids. For this reason, in the above publications, the producer's bid selection is formulated as a bi-level optimization problem with the variable being the producer's bid. In the "lower" (internal) level, the producer solves the operator's market clearing problem, using an estimate of the other producers' bids. This optimization problem is a constraint in turn, embedded into the "upper" (external) level optimization problem, whose objective is to maximize the producer's profit (given the market clearing resulting from the lower level) subject to additional technical constraints.

Since the lower-level problem is assumed to be continuous and convex, it can be replaced by its Karush-Kuhn-Tucker first-order optimality conditions [Kar39, KT51], which yields a *Mathematical Problem with Equilibrium Constraints* (MPEC) [RC09]. MPECs are inherently non-convex, nonlinear optimization problems. The methods used to solve them cannot, thus, guarantee that the global optimal solution has been found [BZTB07]. A short description of what a bi-level optimization problem is, as well as its connection with MPECs, is given in Appendix D.

Clearly, solving the lower-level market clearing problem requires one to guess the behavior and data of both "rival" producers and consumers. Hence, the better will a producer anticipate the other producers' submitted bids, the most profitable choice will it make regarding its own bid. In this respect, in [RC09] the authors incorporate the uncertainty associated with demand and generating offers into the producer's model by considering multiple lower-level market clearing problems, each of them representing a possible realization of the uncertain parameters.

Alternatively, the producer can exploit the fact that the same market clearing is performed on daily basis and resort to an automatic learning algorithm in order to model its competitors' behavior. In [BO01] the authors present a learning algorithm for generators that shares the same essence with reactive learning, while in [YLT07, KBC$^+$06] generation companies are modeled as Q-learning agents. The analysis of electricity markets considering strategic bidding market players with learning capabilities is called agent-based simulation [YLP10]. For example, in [TB07] the authors employ agent-based simulation to study energy market performance and, in particular, capacity withholding and the emergence of tacit collusion among the market participants. In this respect, generators are modeled as adaptive agents capable of learning through the interaction with their environment, following a reinforcement learning algorithm (the SA-Q- learning algorithm).

Acting strategically is, in general, a difficult task to achieve. In the particular problem dealt with in the last two chapters, what makes it even more complicate are, first, the iterative nature of the method, and, second, the existence of two steps of coordination. On the other hand, the same (or, at least, a similar) game between the TSs is expected to be played on a regular basis (say, every day) which could unveil some statistically repeated patterns that could be exploited by the TSs when strategically clearing their markets.

At this point, let us see what the fact that a TS clears its market in a strategic way would mean, i.e. what could a TS make else than just solving problem (4.40). The outcome of the latter consists of a set of (demanded or allocated) generation quantities and a set of corresponding offered prices. Clearly, the TS strategically formulating its market clearing would announce generations that do not, at this moment, minimize its cost and/or prices that do not stem from its official pricing mechanism.

Clearly, before putting any market clearing procedure into practice, it should be carefully examined that it is not vulnerable to gaming by players who act strategically, in a way that the procedure's outcome no longer meets the objectives of openness and fairness that were originally envisaged by the designers of the procedure. In this section, very preliminary considerations are given, mainly focusing on how a TS could formulate its strategic behavior. The issue of gaming remains an interesting research direction towards along the here-presented work could be continued.

Before going on with the more involved case of the combined Energy and Transmission allocation procedure, it is helpful to first consider how a TS could include into its clearing problem information regarding a single iteration of one of the two loops.

5.8.2 Strategic behavior of a TS inside the Energy allocation loop

At a step of the Energy allocation loop, a TS announces to the coordinator demanded quantities and corresponding prices and receives some generation allocation. This allocation, determined by the coordinator's computations, depends on the market clearings of all the involved TSs. If the TS under question cleared its market considering what would be the outcome of the remaining TSs' market clearings as well as the outcome of the coordinator's computations, it could announce demanded quantities and prices yielding more profitable allocation.

Let us assume that the mth TS has been able to derive analytical models approximating in a satisfactory manner the other TSs' market clearings as well as the coordinator's energy allocation. This would mean that the mth TS could express the kth TS's market clearing outcome, i.e. the demanded generation $\widetilde{\mathbf{g}}_k$ and the corresponding offered prices $\boldsymbol{\pi}_k$ as functions of other variables involved in the procedure. For instance as:

$$\widetilde{\mathbf{g}}_k = \widetilde{\mathbf{g}}_k\left(\overline{\mathbf{g}}_k, \Delta\mathbf{p}_k^-, \Delta\mathbf{p}_k^+\right) \tag{5.17a}$$

$$\boldsymbol{\pi}_k = \boldsymbol{\pi}_k\left(\overline{\mathbf{g}}_k, \Delta\mathbf{p}_k^-, \Delta\mathbf{p}_k^+\right) \tag{5.17b}$$

where $\overline{\mathbf{g}}_k$, $\Delta\mathbf{p}_k^-$ and $\Delta\mathbf{p}_k^+$ stem from previous computations of the coordinator. They have been defined in Chapters 4 and 5 as, respectively, the new generation capacities available to the kth TS (resulting from the last energy allocation computation) and the branch flow decremental and incremental changes that should be provoked by the kth TS's new injection schedule (resulting from the last transmission allocation computation).

Note that the functions in (5.17) can, in general, be constructed to depend on more than the three shown variables (those communicated to the kth TS by the coordinator). For instance, if the mth TS wants to consider that the kth TS may be also modeling the others, it could express the kth TS's market clearing approximation as a function of *more* variables appearing in the procedure. For example an approximation could be a function of the type

$\widetilde{\mathbf{g}}_k = \widetilde{\mathbf{g}}_k\left(\overline{\mathbf{g}}_1, \ldots, \overline{\mathbf{g}}_M, \Delta\mathbf{p}_1^-, \ldots, \Delta\mathbf{p}_M^-, \Delta\mathbf{p}_1^+, \ldots, \Delta\mathbf{p}_M^+, \boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_M, \ldots\right)$, with the vectors in the parenthesis being already defined. The same can be said for $\boldsymbol{\pi}_k$ as well. Note also that, in general, the expression modeling the kth TS's market clearing implicitly incorporates some estimation to what regards private information of others, like the various market participants' bids to the kth TS.

Similarly, we assume that the mth TS expresses the coordinator's energy allocation computations with the following function:

$$\mathbf{g} = \mathbf{g}\left(\widetilde{\mathbf{g}}, \boldsymbol{\pi}, \widehat{\mathbf{g}}\right) \tag{5.18}$$

where g includes the generation quantities allocated to TSs. As defined in previous sections, vectors $\widetilde{\mathbf{g}}$ and $\boldsymbol{\pi}$ include all the TSs' announced offers at this step of the Energy allocation loop, while vector $\widehat{\mathbf{g}}$ contains the generation productions that where allocated to each TS in the previous iteration of the Energy allocation loop. The new generation capacities available to each TS are trivially computed from (5.18). For instance, the vector of generation limits communicated to the mth TS is $\overline{\mathbf{g}}_m = \overline{\mathbf{g}} - \sum_{k \neq m} \mathbf{g}_k$.

Thus, the mth TS can make use of the $M-1$ models of type (5.17) and the model (5.18) so that, by properly modifying its original problem (i.e. the one given in (4.40)), it clears its market in a way that yields some desirable generation allocation. For instance, if the mth TS wishes to clear its market so that after the coordinator's allocation of generators it gets, at minimum cost, generation quantities that cover the demand it serves, it could: first, compute $\widetilde{\mathbf{g}}_k$ and $\boldsymbol{\pi}_k$ $\forall\, k \neq m$, using (5.17), and then, perform the following modified market clearing:

$$\min_{\mathbf{g}_m, \widetilde{\mathbf{g}}_m, \boldsymbol{\pi}_m} \left\{\boldsymbol{\pi}_m^T \mathbf{g}_m\right\} \tag{5.19a}$$

$$\text{s.t.} \quad \boldsymbol{\pi}_m \geq \mathbf{c}_m \tag{5.19b}$$

$$\mathbf{1}^T \mathbf{g}_m = \mathbf{1}^T \mathbf{d}_m \tag{5.19c}$$

$$\mathbf{1}^T \widetilde{\mathbf{g}}_m = \mathbf{1}^T \mathbf{d}_m \tag{5.19d}$$

$$\mathbf{0} \leq \widetilde{\mathbf{g}}_m \leq \overline{\mathbf{g}}_m \tag{5.19e}$$

$$\mathbf{t}_b(\widetilde{\mathbf{g}}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m) \leq -(\Delta\mathbf{p}_m^-)_b \qquad b = 1, \ldots \tag{5.19f}$$

$$\mathbf{t}_b(\widetilde{\mathbf{g}}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m) \geq (\Delta\mathbf{p}_m^+)_b \qquad b = 1, \ldots \tag{5.19g}$$

$$\mathbf{g}_m = \mathbf{g}_m\left(\widetilde{\mathbf{g}}, \boldsymbol{\pi}, \widehat{\mathbf{g}}\right) \tag{5.19h}$$

At this point, it may be helpful to recall what is the market clearing problem, used in this work, for a TS that does not act strategically:

$$\min_{\widetilde{\mathbf{g}}_m} \left\{\mathbf{c}_m^T \widetilde{\mathbf{g}}_m\right\} \tag{5.20a}$$

$$\text{s.t.} \quad \mathbf{1}^T \widetilde{\mathbf{g}}_m = \mathbf{1}^T \mathbf{d}_m \tag{5.20b}$$

$$\mathbf{0} \leq \widetilde{\mathbf{g}}_m \leq \overline{\mathbf{g}}_m \tag{5.20c}$$

$$\mathbf{t}_b(\widetilde{\mathbf{g}}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m) \leq -(\Delta\mathbf{p}_m^-)_b \quad b = 1, \ldots \tag{5.20d}$$

$$\mathbf{t}_b(\widetilde{\mathbf{g}}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m) \geq (\Delta\mathbf{p}_m^+)_b \quad b = 1, \ldots \tag{5.20e}$$

In the above two optimization problems, $\widehat{\mathbf{n}}_m$ is the mth TS's injection schedule based on which the requested branch flow corrections $\Delta\mathbf{p}_m^-$ and $\Delta\mathbf{p}_m^+$ were computed at the last transmission allocation iteration. As already explained, $\widehat{\mathbf{g}}$ is the vector containing the generation productions that were allocated to each TS in the previous energy allocation iteration. If problem (5.19) is solved at the first energy allocation iteration, i.e. the one before which transmission allocation was performed, then $\widehat{\mathbf{g}}_m$ (i.e. the generator powers allocated to the mth TS) is involved in $\widehat{\mathbf{n}}_m$ (let us recall that $\mathbf{n}_m = \boldsymbol{\Gamma}\ \mathbf{g}_m - \boldsymbol{\Delta}\ \mathbf{d}_m$). However, if the problem is solved at a next iteration of the Energy allocation loop (this could happen if, for instance, the mth TS, due to inaccurate anticipation, did not get, after the first energy allocation iteration, all what it was aiming at), then $\widehat{\mathbf{g}}$, which in (5.19h) refers to the last energy allocation iteration, corresponds to different schedules than $\widehat{\mathbf{n}}_m$. Finally, let us recall that $\mathbf{c}_m$ is the vector containing all generators' bids placed in the mth TS's market at the beginning of the procedure.

The outcome of both (5.19) and (5.20) is a vector of demanded generation quantities $\widetilde{\mathbf{g}}_m$ and a vector of offered prices $\boldsymbol{\pi}_m$ to be announced to the coordinator in order to proceed with the Energy allocation. It should be noted, however, that, while in (5.20) $\boldsymbol{\pi}_m$ is computed as a side-effect of the optimization problem (according to the pricing rule used by the mth TS, for instance a marginal clearing price rule), in (5.19) $\boldsymbol{\pi}_m$ is explicitly treated as a problem's variable.

Let us, indeed, have a deeper look in problem (5.19). In this problem, apart from $\widetilde{\mathbf{g}}_m$ and $\boldsymbol{\pi}_m$, the generation quantities $\mathbf{g}_m$ that the coordinator is expected to allocate to the mth TS are also modeled as variables. To compute them, the part of model (5.18) giving $\mathbf{g}_m$ is incorporated into the optimization problem as equality constraints, namely (5.19h). These constraints relate $\mathbf{g}_m$ with the other variables of the optimization, $\widetilde{\mathbf{g}}_m$ and $\boldsymbol{\pi}_m$. The estimate of the other TSs' demanded quantities and offered prices are fixed parameters in (5.19), which have been computed before performing the optimization. The objective of this strategic market clearing is to minimize the actual cost of the TS, i.e. the amount $\boldsymbol{\pi}_m^T\mathbf{g}_m$ that it will have to pay to satisfy its demand. Constraint (5.19c) ensures that the TS will get exactly the amount of generation is needs. The offered prices $\boldsymbol{\pi}_m$ do not stem from a pre-defined pricing rule but are part of the TS's strategic behavior: they result from the optimization such that, on one hand, they minimize the cost to be paid (see the objective function), while, on the other hand, they allow to the TS to get the generators of its choice (see constraints (5.19h)). The role of constraint (5.19b) is to make sure that those prices are not lower than the bids submitted by the generators. Constraints (5.19e) to (5.19g), which are the same as in the non-strategic market clearing (5.20), stem from the rules of the procedure. Finally, it is interesting to note that, in principle, constraint (5.19d) could have been omitted since (5.19c) anyway ensures that the TS will get what it needs. However, the announcement of a balanced schedule (i.e. $\mathbf{1}^T\widetilde{\mathbf{g}}_m = \mathbf{1}^T\mathbf{d}_m$) is necessary in order for the change in branch flows in constraints (5.19f) and (5.19g) to be correctly computed.

One can see that problem (5.19) has no longer the form of a typical market clearing. Generators are not necessarily dispatched in ascending order of price, neither are the offered prices computed according to a pre-defined, clear to the participants, rule (such as a common marginal clearing price or locational marginal prices). This may not be acceptable if the TS is a PX or a TSO, since such entities have the obligation to dispatch the market participants applying a publicly announced algorithm. On the other hand, in the proposed framework the TS is a general entity settling multilateral transactions, it does not necessarily coincide with a PX, and as a result it could use undisclosed algorithms to clear its market. The role of a generic TS is to settle transactions that are economically profitable for the involved participants and, as long as it offers prices larger or equal to the generators' bids and smaller or equal to the loads' bids, then this role is fulfilled. In this respect, it would be appropriate to view the Energy allocation loop as a common marketplace where any market participant can place its bid(s) knowing that the coordinator will take care that the participant is dispatched to its highest profit.

Problem (5.19) tackles one single iteration of the Energy allocation loop. However, what really counts for a TS is which generators will be allocated to it and at what price, at the end of the loop. Clearing its market as in (5.19), at every step of the Energy allocation loop, could already be enough for a TS to end up with a satisfactory generation allocation[7]. The reason is that, along the iterations, the most interesting generators will tend to be allocated first; hence it makes sense for a TS to have as a strategy to be allocated, at each step of the loop, the cheapest generators. The anticipation will be more complete, though, if the TS models in its market clearing problem several of (in theory, even all) the remaining iterations up to the point when the method proceeds with the Transmission allocation.

It may be profitable for a TS to "refrain from rushing" to get all the generation capacity it needs in one energy allocation step, because some other TSs are expected to release interesting generators, previously allocated to them (and thus not presently available to the TS under question), due to constraints of type (5.19f) and (5.19g) related to alleviation of branch overloads. Tables 5.3 and 5.4 provide such an example, where at the first energy allocation iteration TS A cannot ask for more than 150 MW from gA5 (see Table 5.3), but, at the same time, TS B and TS C are obliged to release some or all the capacity of gA5 they had been allocated and, thus, at the next iteration of the same Energy allocation loop TS A has access to an increased available capacity of gA5. The pricing rule introduced in Section 5.3.3 suggests that a TS may be motivated to wait for some cheap generation to be released. Dispatching expensive generation and then releasing it for some cheaper one may end up in the TS paying more for what it could have dispatched at lower price.

For example, if the mth TS wishes to anticipate two Energy allocation iterations, with the objective of being after those iterations allocated, at minimum cost, a total generation equal to the total load it serves, it could clear its market by solving an optimization problem that is presented step-by-step as follows.

Let us call $\mathbf{g}_m^{(1)}$ and $\mathbf{g}_m^{(2)}$ the generator powers that the mth TS expects to be allocated after the first and the second energy allocation iteration, respectively. Thus, the total expected allocated power is $\mathbf{g}_m = \mathbf{g}_m^{(1)} + \mathbf{g}_m^{(2)}$. Let us also denote by $\left(\widetilde{\mathbf{g}}_m^{(1)}, \boldsymbol{\pi}_m^{(1)}\right)$ and $\left(\widetilde{\mathbf{g}}_m^{(2)}, \boldsymbol{\pi}_m^{(2)}\right)$ the pairs of demanded quantities and corresponding offered prices communicated by the mth TS to the coordinator at the first and second energy allocation iteration, respectively. More precisely, since

[7] It should be kept in mind that the models used by the anticipating TS are estimations only; in reality the TS may not get all what it had predicted.

the market clearing is supposed to take place at the first energy allocation iteration, $\left(\widetilde{\mathbf{g}}_m^{(1)}, \boldsymbol{\pi}_m^{(1)}\right)$ are the values that will be actually communicated to the coordinator, while $\left(\widetilde{\mathbf{g}}_m^{(2)}, \boldsymbol{\pi}_m^{(2)}\right)$ are what the mth TS anticipates to communicate to the coordinator at the mext energy allocation iteration, provided that its predictions for the first energy allocation turns out to be correct.

The wish of the mth TS to allocate the whole generation it needs to satisfy the demand it serves at minimum cost, suggests using the following objective in its market clearing:

$$\min\left\{\boldsymbol{\pi}_m^{(1)T}\mathbf{g}_m^{(1)} + \boldsymbol{\pi}_m^{(2)T}\mathbf{g}_m^{(2)}\right\} \tag{5.21}$$

with the constraints:

$$\boldsymbol{\pi}_m^{(1)} \geq \mathbf{c}_m \tag{5.22a}$$
$$\boldsymbol{\pi}_m^{(2)} \geq \mathbf{c}_m \tag{5.22b}$$
$$\mathbf{1}^T\mathbf{g}_m^{(1)} + \mathbf{1}^T\mathbf{g}_m^{(2)} = \mathbf{1}^T\mathbf{d}_m \tag{5.22c}$$

For the reasons already explained, the TS demanded generation quantities are at each step such that they cover the (inelastic in our example) demand, and are bounded by the capacities available to the mth TS. This yields the constraints:

$$\mathbf{1}^T\widetilde{\mathbf{g}}_m^{(1)} = \mathbf{1}^T\mathbf{d}_m \tag{5.23a}$$
$$\mathbf{1}^T\widetilde{\mathbf{g}}_m^{(2)} = \mathbf{1}^T\mathbf{d}_m \tag{5.23b}$$
$$\mathbf{0} \leq \widetilde{\mathbf{g}}_m^{(1)} \leq \overline{\mathbf{g}}_m \tag{5.23c}$$
$$\mathbf{0} \leq \widetilde{\mathbf{g}}_m^{(2)} \leq \overline{\mathbf{g}}_m^{(1)} \tag{5.23d}$$

where $\overline{\mathbf{g}}_m$ stems from the previous energy allocation and is a fixed parameter in the optimization problem. On the contrary, vector $\overline{\mathbf{g}}_m^{(1)}$ is a variable in the problem. It contains the generation availability limits that are expected to be communicated to the mth TS at the end of the first energy allocation iteration. The way $\overline{\mathbf{g}}_m^{(1)}$ is computed within the optimization is explained in the sequel.

Clearly, the schedule of the mth TS should satisfy the branch flow constraints stemming from the last transmission allocation iteration:

$$\mathbf{t}_b\left(\widetilde{\mathbf{g}}_m^{(1)} - \mathbf{d}_m - \widehat{\mathbf{n}}_m\right) \leq -\left(\Delta\mathbf{p}_m^-\right)_b \qquad b = 1, \ldots \tag{5.24a}$$
$$\mathbf{t}_b\left(\widetilde{\mathbf{g}}_m^{(1)} - \mathbf{d}_m - \widehat{\mathbf{n}}_m\right) \geq \left(\Delta\mathbf{p}_m^+\right)_b \qquad b = 1, \ldots \tag{5.24b}$$
$$\mathbf{t}_b\left(\widetilde{\mathbf{g}}_m^{(2)} - \mathbf{d}_m - \widehat{\mathbf{n}}_m\right) \leq -\left(\Delta\mathbf{p}_m^-\right)_b \qquad b = 1, \ldots \tag{5.24c}$$
$$\mathbf{t}_b\left(\widetilde{\mathbf{g}}_m^{(2)} - \mathbf{d}_m - \widehat{\mathbf{n}}_m\right) \geq \left(\Delta\mathbf{p}_m^+\right)_b \qquad b = 1, \ldots \tag{5.24d}$$

Note that all branch flow constraints refer to the schedule $\widehat{\mathbf{n}}_m$ considered by the coordinator in the last transmission allocation.

In order to anticipate the outcome of the coordinator's computations, at both the first and second energy allocation iteration, the mth TS resorts to the coordinator behavior model it uses, i.e. to

Eq. (5.18). This gives the following constraints to be incorporated in the optimization problem:

$$\mathbf{g}^{(1)} = \mathbf{g}\left(\widetilde{\mathbf{g}}^{(1)}, \boldsymbol{\pi}^{(1)}, \widehat{\mathbf{g}}\right) \tag{5.25a}$$
$$\mathbf{g}_m^{(2)} = \mathbf{g}_m\left(\widetilde{\mathbf{g}}^{(2)}, \boldsymbol{\pi}^{(2)}, \mathbf{g}^{(1)}\right) - \mathbf{g}_m^{(1)} \tag{5.25b}$$
$$\overline{\mathbf{g}}_k^{(1)} = \overline{\mathbf{g}} - \sum_{n \neq k} \mathbf{g}_n^{(1)} \qquad \forall\, k \tag{5.25c}$$

where $\left(\widetilde{\mathrm{g}}^{(1)}, \boldsymbol{\pi}^{(1)}\right)$ and $\left(\widetilde{\mathrm{g}}^{(2)}, \boldsymbol{\pi}^{(2)}\right)$ group all TSs' demanded quantities and corresponding offered prices submitted to the coordinator at the first and the second energy allocation iteration, respectively. Vector $\mathrm{g}^{(1)}$ contains the generation powers allocated to *all* the TSs by the coordinator at the end of the first energy allocation iteration. To what regards the second iteration, only the generation powers allocated to the mth TS itself are modeled ($\mathrm{g}_m^{(2)}$ in (5.25b), similar to (5.19h)), since the others are not used in the mth TS's market clearing. Finally, (5.25c) gives the expected generation limits needed in (5.23d). Let us recall that $\overline{\mathrm{g}}$ includes the generator limits while $\widehat{\mathrm{g}}$ includes the allocated generations stemming from the previous energy allocation.

It is noteworthy that, for any TS $k \neq m$, $\widetilde{\mathbf{g}}_k^{(1)}$ and $\boldsymbol{\pi}_k^{(1)}$ are fixed parameters in the optimization problem. They are estimated by the mth TS, before clearing its market, using its models (5.17). On the contrary, all $\widetilde{\mathrm{g}}_k^{(2)}$ and $\boldsymbol{\pi}_k^{(2)}$ are *variables* in the mth TS's market clearing problem. They cannot be estimated in advance because they depend on other variables of the optimization. So, to compute them, the mth TS, resorting to its models (5.17) of the other TSs' market clearings, adds the following constraints to the optimization:

$$\widetilde{\mathbf{g}}_k^{(2)} = \widetilde{\mathbf{g}}_k\left(\overline{\mathbf{g}}_k^{(1)}, \Delta\mathbf{p}_k^-, \Delta\mathbf{p}_k^+\right) \quad \forall\, k \neq m \tag{5.26a}$$
$$\boldsymbol{\pi}_k^{(2)} = \boldsymbol{\pi}_k\left(\overline{\mathbf{g}}_k^{(1)}, \Delta\mathbf{p}_k^-, \Delta\mathbf{p}_k^+\right) \quad \forall\, k \neq m \tag{5.26b}$$

Note that $\overline{\mathrm{g}}_k^{(1)}$ is computed in (5.25c) for all k (i.e. including m).

All in all, at the first energy allocation iteration, in order to come up with $\widetilde{\mathrm{g}}_m^{(1)}$ and $\boldsymbol{\pi}_m^{(1)}$ to be announced to the coordinator, the mth TS clears its market by solving the optimization problem consisting of (5.21), (5.22), (5.23), (5.24), (5.25) and (5.26). The variables of this problem are: $\widetilde{\mathrm{g}}_m^{(1)}$, $\boldsymbol{\pi}_m^{(1)}$, $\overline{\mathrm{g}}^{(1)}$,$\mathbf{g}^{(1)}$, $\widetilde{\mathrm{g}}^{(2)}$, $\boldsymbol{\pi}^{(2)}$ and $\mathrm{g}_m^{(2)}$.

5.8.3 Strategic behavior of a TS inside the Transmission allocation loop

In a similar way, a TS could anticipate what the outcome of the Transmission allocation loop will be and properly dispatch its market participants to obtain the most profitable use of the transmission network.

We now consider the only Transmission allocation procedure, without iterations for energy allocation (this corresponds to the algorithm presented in Chapter 4). We assume that the mth

TS has constructed analytical models of the other TSs' market clearings, expressed as functions of the coordinator's requests for branch flow decreases or increases and of the modeled TS's previous injection schedule:

$$\mathbf{g}_k = \mathbf{g}_k\left(\Delta\mathbf{p}_k^-, \Delta\mathbf{p}_k^+, \widehat{\mathbf{n}}_k\right) \quad \forall\, k \neq m \tag{5.27}$$

where $\mathbf{g}_k$ denotes the kth TS's new generation schedule (load is again assumed inelastic).

Let us assume that, at a given step of the Transmission allocation loop, the mth TS, solely by using the $\Delta\mathbf{p}_m^+$ and $\Delta\mathbf{p}_m^-$ corrections received from the coordinator, as well as the $M-1$ models (5.27), is able to predict the sets $\mathcal{O}'_-$ and $\mathcal{O}'_+$ of overloaded branches for which it will be requested to modify its flow (downwards and upwards, respectively). Let us finally assume that the mth TS is able to identify for each branch $b^- \in \mathcal{O}'_-$ and $b^+ \in \mathcal{O}'_+$ the sets $\mathcal{K}(b^-)$ and, respectively, $\mathcal{K}(b^+)$ of TSs that will also be requested to modify their flows (i.e. those that are not counterflowing). Note that $\mathcal{O}'_-$ and $\mathcal{O}'_+$ may also contain branches that have been overloaded in previous iterations. Let us call $\mathcal{A}_-$ and $\mathcal{A}_+$ the sets of branches for which the mth TS has already received decremental and, respectively, incremental flow constraints from the coordinator. Clearly, it is $\mathcal{A}_- \subseteq \mathcal{O}'_-$ and $\mathcal{A}_+ \subseteq \mathcal{O}'_+$.

The mth TS can then modify its market clearing, having as an objective to minimize its generation cost at the next transmission allocation iteration, i.e. after having adjusted its schedule in order to meet the coordinator's constraints. To this purpose, the mth TS has to solve the following optimization problem:

$$\min_{\mathbf{g}_m, \mathbf{g}'_m, \Delta\mathbf{p}_m^{-\prime}, \Delta\mathbf{p}_m^{+\prime}} \left\{\mathbf{c}_m^T \mathbf{g}'_m\right\} \tag{5.28a}$$

$$\text{s.t.} \quad \mathbf{1}^T\mathbf{g}'_m = \mathbf{1}^T\mathbf{d}_m \tag{5.28b}$$

$$\mathbf{0} \le \mathbf{g}'_m \le \overline{\mathbf{g}}_m \tag{5.28c}$$

$$\mathbf{t}_b\left(\mathbf{g}'_m - \mathbf{g}_m\right) \le -\left(\Delta\mathbf{p}_m^{-\prime}\right)_b \qquad b \in \mathcal{O}'_- \tag{5.28d}$$

$$\mathbf{t}_b\left(\mathbf{g}'_m - \mathbf{g}_m\right) \ge \left(\Delta\mathbf{p}_m^{+\prime}\right)_b \qquad b \in \mathcal{O}'_+ \tag{5.28e}$$

$$\mathbf{1}^T\mathbf{g}_m = \mathbf{1}^T\mathbf{d}_m \tag{5.28f}$$

$$\mathbf{0} \le \mathbf{g}_m \le \overline{\mathbf{g}}_m \tag{5.28g}$$

$$\mathbf{t}_b\left(\mathbf{g}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m\right) \le -\left(\Delta\mathbf{p}_m^-\right)_b \qquad b \in \mathcal{A}_- \tag{5.28h}$$

$$\mathbf{t}_b\left(\mathbf{g}_m - \mathbf{d}_m - \widehat{\mathbf{n}}_m\right) \ge \left(\Delta\mathbf{p}_m^+\right)_b \qquad b \in \mathcal{A}_+ \tag{5.28i}$$

$$\left(\Delta\mathbf{p}_m^{-\prime}\right)_b = \frac{\mathbf{t}_b(\mathbf{g}_m - \mathbf{d}_m)}{\sum_{k\in\mathcal{K}(b)} \mathbf{t}_b(\mathbf{g}_k - \mathbf{d}_k)} \left(\sum_{k\in\mathcal{K}(b)} \mathbf{t}_b(\mathbf{g}_k - \mathbf{d}_k) - \overline{p}_b\right) \qquad b \in \mathcal{O}'_- \tag{5.28j}$$

$$\left(\Delta\mathbf{p}_m^{-\prime}\right)_b = \frac{\mathbf{t}_b(\mathbf{g}_m - \mathbf{d}_m)}{\sum_{k\in\mathcal{K}(b)} \mathbf{t}_b(\mathbf{g}_k - \mathbf{d}_k)} \left(-\sum_{k\in\mathcal{K}(b)} \mathbf{t}_b(\mathbf{g}_k - \mathbf{d}_k) - \overline{p}_b\right) \qquad b \in \mathcal{O}'_+ \tag{5.28k}$$

In the above formulation, variables that refer to the second (i.e. anticipated) market clearing are denoted by a $'$ symbol. Thus, $\mathbf{g}_m$ is the actual outcome of the strategic market clearing (i.e. the one resulting in the schedule $\mathbf{n}_m = \mathbf{\Gamma}\ \mathbf{g}_m - \mathbf{\Delta}\ \mathbf{d}_m$ to be communicated to the coordinator), while $\mathbf{g}'_m$ is the generation schedule that the mth TS expects to set up at the next transmission allocation iteration.

Constraints (5.28b) and (5.28c) stand for the fact that the next generation schedule should cover the demand and be within the generators' capacities. The similar constraints (5.28f) and (5.28g) relate to the present generation schedule, while (5.28h) and (5.28i) are the branch flow limits stemming from the previous transmission allocation iteration.

The coordinator's computations are anticipated with (5.28j) and (5.28k), giving the next branch flow corrections ($\Delta\mathbf{p}_m^{-'}$ and $\Delta\mathbf{p}_m^{+'}$) expected to be requested from the mth TS. Those new limits depend on all the TSs schedules. The mth TS generation schedule is a variable in the optimization, while all the others' have been computed according to the models (5.27) prior to solving (5.28). Finally, the constraints in (5.28d) and (5.28e) are the estimated new branch flow-related inequality constraints that the mth TS will have to incorporate into its market clearing at the next transmission allocation iteration.

As for the energy allocation iterations, a TS acting strategically could anticipate more than one transmission allocation iterations by incorporating the corresponding models and variables into its market clearing problem.

5.8.4 Strategic behavior in both Energy and Transmission allocation loops

One can envisage, at least in theory, that a TS combines the market clearing formulations that were presented in the previous two subsections for, separately, energy and transmission allocation, and makes up an optimization problem where future energy *and* transmission iterations are anticipated. This would offer similarities with Model Predictive Control approaches (e.g. [Mac02, OMGC07b, OMGC07a]), in so far as a TS would optimize its sequence of actions over multiple future steps, and each time implement only those actions that correspond to the first step. The remaining computed actions (referring to future steps) would not be used, since at the next iteration of the procedure the TS under question would once again solve its optimization problem.

All in all, some strategic formulations of a TS's market clearing have been sketched in this section. To keep the presentation as simple as possible, those formulations have been based on the assumption that the anticipating TS has been able to construct analytical models of the other TSs' market clearings and of the coordinator's computations. In practice, however, this would be a very difficult task to achieve, if at all possible. The various TS market clearings take on, in fact, the form of optimization problems (like (5.20)). The same holds true for energy allocation, where the coordinator's objective is to maximize each generator's profit. In addition, in both energy and transmission allocation, the coordinator takes some if-then decisions which would have to be modeled as complementarity constraints. Namely, those decisions stem from: (a) in energy allocation, the rule that a TS should keep whatever has been allocated to it in previous iteration and it continues to ask, as well as the rule that, in case of equal offered prices, allocation should be made proportionally to demanded quantities; and (b) in transmission allocation, the rule that no alleviation constraints should be assigned to the TSs that are counterflowing in an overloaded branch.

Thus, the modeling of other TSs and the coordinator by a TS acting strategically, would yield multilevel optimization problems, i.e. optimization problems which involve as constraints other optimization problems and/or equilibrium constraints (see Appendix D for a formulation of such a problem). For instance, the models (5.17) would, in fact, be optimization problems of the type (5.20). Reference [CMS07] provides a good initial point to the literature of algorithms for solving bi-level (i.e. a particular case of multilevel) optimization problems. It should be noted, however, that these problems are intrinsically nonlinear and non convex and, thus, difficult to solve (even the simplest bi-level problem, i.e. with all involved constraints and objective functions being linear, has been shown to be $\mathcal{NP}$-hard in [Jer85]). In the beginning of this section, some approaches, taken from the power system literature, that involve solving bi-level problems have been cited. Solving the bi- (or higher) level optimization problems that stem from strategic behaviors by the TSs, in the approach proposed in this work, seems to be an interesting and exciting topic for future research.

Alternatively, the strategic behavior of a TS could involve resorting to automatic learning algorithms, such as reinforcement learning [EGCW09, YLT07, KBC$^+$06], in order a TS to avoid explicitly modeling the other TSs but, instead, use its experience from previous executions of the procedure in order to, gradually, formulate an appropriate strategy for its market clearings. This is another exciting topic worth of further investigation.

5.9 Conclusion

Starting from the Transmission allocation procedure developed in Chapter 4, this chapter built upon covering a variety of issues, overall resulting in an enhanced Energy and Transmission allocation scheme that contributes another step towards creating a common electric energy marketplace in an interconnection, where congestion is implicitly managed in an efficient way, from both a social welfare and an engineering viewpoint.

As regards the common marketplace, the proposed Energy allocation procedure allows different electricity markets to be coupled, thus offering more options to participants and more liquidity to TSs. As for congestion management, the proposed Transmission allocation procedure, complemented with $N-1$ security constraints, offers a mechanism that is fair and easy to implement, while leading to efficient and secure use of the transmission network. Security can be enhanced in an efficient way by allowing for joint energy and reserve scheduling. Finally, the issue of transmission losses, which could be significant in case of long-distance transactions, has been dealt with. In fact, embedding in the procedure the issues of $N-1$ security constraints, reserves and losses, helps avoiding the occurrence of a situation where the TSOs of the involved areas would have, ex-post (i.e. after the outcome of the iterations), to make important corrective adjustments.

The approach has been thoroughly illustrated on small-scale examples. Although they refer, for clarity, to a simplified situation (inelastic load, all TSs using the same pricing mechanism, etc.), the approach can encompass more involved situations. Admittedly, more testings are needed before considering the proposed method for practical application. Future work should deal with several issues such as: (a) incorporating complex bid structures; (b) vulnerability to participants or TSs trying to "game" the procedure; (c) link to existing transmission pricing mechanisms and (d) possibility of reducing the number of iterations, if prohibitive.

Regarding (c), it was assumed in this work that one or several transmission pricing schemes are in effect throughout the system. The latter are expected to be reflected in the prices offered by the TSs or/and the market participants.

Regarding (d), it is recognized that with the current state of the art the proposed iterative clearing methodology would pose an important burden in the bidding-settling process and would increase the transaction costs. However, as electricity markets mature, the bidding process is expected to become routine for generators and the motive for profit will drive them to bid across multiple markets, given the relevant framework. In addition, advances in online negotiation and electronic trade using intelligent agents [VCJ08, NPT01] are likely to wipe out the increased time requirements and transactions costs of the proposed iterative scheme.

Chapter 6

Conclusion

6.1 Brief summary of the work

Operating large power systems in a decentralized manner is sometimes a challenging task, which requires proper coordination of the different involved actors' control decisions. With reference to two specific, self-standing, power system problems, some algorithms and/or operational procedures have been developed in this work seeking to reconcile the multiple actors' simultaneous decisions, while a unifying mathematical framework, borrowed from the fields of Game Theory (basically) and of Multi-Objective Optimization (as a complement), has been used as the main conceptual tool to formulate the proposed ideas.

Precisely, the situation where various TSOs, whose respective control areas are within the same interconnection, simultaneously modify the angle settings of their respective PSTs, has been our first field of application of a coordinated decentralized framework. The second application has stemmed from the development of a decentralized, transaction-based, market structure where TSs settle multilateral power transactions throughout an interconnection. An improved extension of the coordination framework that was used for the PST control problem has been considered in order to properly manage the congestion resulting from those overlapping markets.

The choice of allowing the simultaneous optimization of multiple actor objectives has been preferred against resorting to the optimization of a single objective that would be a combination of the individual objectives. This choice stemmed both from practical reasons dealing with the acceptability of a single-objective approach (for example, the various TSOs may not agree conceding the control of their investments to a "super-TSO") and from a viewpoint of promoting openness and innovation by seeking coordination rather than centralization (in this respect, market participants should, in principle, be let free to settle energy transactions between themselves). The confidentiality and operational autonomy of the actors' procedures has been also respected.

In fact, both multi-actor problems that have been dealt with can be classified as generalized Nash games. The proposed algorithms have been shown to lead to Nash equilibria of those games. Furthermore, the corresponding multi-objective problem of such a game has been defined as an optimization problem that seeks to optimize (a trade-off of) all actors' individual objectives. Like this, the aforementioned Nash equilibria have been assessed in terms of how close they are to Pareto efficiency.

Besides the main theme of this work, it has been considered appropriate to enlarge the scope of investigation in each of the two problems. To what regards the PST control problem, an algorithm has been developed for a single TSO to control the several PSTs of its area in a way that, by minimally reducing the transit flow passing through its system through preventive PST actions, makes its system correctively secure vis-à-vis a selected set of contingencies[1]. To what regards the proposed overlapping market structure, the developed Energy and Transmission allocation procedure has been enhanced with, namely, incorporation of $N-1$ security constraints, account for transmission losses and some considerations regarding the scheduling of reserves, in an effort to make it, overall, a practically implementable proposal for a decentralized power market.

It is worth noting that the extended Energy and Transmission allocation procedures presented in Chapter 5 remain a multi-actor game. However, in that chapter's presentation it has been preferred to focus on the development of a practically operational overlapping market structure viewpoint, rather than repeating multi-actor issues that have already been covered in Chapters 3 and 4.

6.2 Main contributions of the work

The following can be stated as the main contributions of this work:

1. *The framework which has been applied in the PST control problem allowing the optimization of multiple objectives while coordinating the operation of a system by multiple interacting TSOs.*

   The algorithm requires that, before its execution, the involved TSOs exchange information in order to construct and share a common model of the network that links phase angle modifications to resulting branch flow changes. In addition, each TSO communicates to the others a set of linear feasibility constraints representing branch flow limits. The essence of the algorithm is an iterative approach where the TSOs successively compute control actions taking into account the last actions of other TSOs and obeying the whole set of constraints.

2. *The extension of the above approach to deal with the congestion management issue that*

[1]Since it is not needed for the understanding of the remaining of this work, the single-TSO PST control algorithm has been presented separately in Appendix A.

arises from multiple overlapping markets simultaneously cleared in a common interconnection, yielding the so-named "Transmission allocation loop".

Again, a common model of the interconnection is constructed, and linear constraints are expressed via this model. However, contrary to the previous approach, those constraints are not incorporated into the various TS market clearings, but they are treated by a coordinator. The essence of the iterative algorithm is that the coordinator checks for constraint violations and, by applying a predefined congestion management policy, shares the alleviation effort among the TSs.

3. *The extension of the Transmission allocation procedure with an "Energy allocation loop" that couples the previously separate markets, allowing market participants to bid their whole capacities to more than one TS market simultaneously.*

   This feature increases the economic efficiency of the final dispatch because it permits to internalize (i.e. make implicit in the algorithm) the, possibly difficult, choice of how a market participant should place its bid between the various TSs.

4. *The enhancement of the Transmission allocation procedure with the additional features of: incorporating $N-1$ security constraints, allowing joint scheduling of reserves, and accounting for transmission losses.*

   At every transmission allocation iteration, the coordinator checks also for constraint violations that would result from a branch or a generator outage and shares the alleviation effort to TSs according to the same rule of proportional participation. Regarding reserves, the TSs have been assumed as been assigned the responsibility to schedule some reserves together with their energy transactions. Finally, as for transmission losses, at every transmission allocation iteration the coordinator computes an estimate of transmission losses and allocates them as additional bus withdraws to the various TSs according to their schedules.

5. *The assessment of the two proposed coordination schemes (see items 1 and 2 above) in terms of resulting in Nash equilibria of the game and of their Pareto optimality.*

   In all cases, the converged final control settings of the various actors are Nash equilibria of the generalized game; no actor can further improve its objective by its sole actions without violating the coupled constraints. In the PST control problem, those equilibria are also Pareto optimal solutions, while in the case of Transmission allocation they have be found to be very close to Pareto optimality.

6. *The algorithm, to be used by a TSO to control its PSTs, for security restoration via minimal reduction of unscheduled flows.*

   The algorithm is presented in detail in Appendix A.

The operation of all presented algorithms have been thoroughly illustrated with properly set, small-scale comprehensive examples. The test cases are complex not by the size but by the conflict between actors they involve.

6.3 Directions for future work

This work can be improved, complemented and extended towards several directions. We quote some that seem as most natural:

- The switch-type behavior of the energy and transmission allocation rules could be made smoother, as discussed in Section 5.6.3.

- The effects of possible strategic behavior of the various TSs and market participants should be thoroughly investigated before putting such a scheme into practice.

  Some preliminary reflections about the issue have been presented in Section 5.8. A complete investigation of the topic should, among others, involve: (a) consideration of the problem from an individual TS's or market participant's perspective. Strategies that maximize the TS's or, respectively, the market participant's profit at the end of an execution of the Energy and Transmission allocation procedure should be developed; (b) consideration of the problem from a market designer perspective. The result of the various TSs and/or market participants strategic behavior should be evaluated. Techniques (in the form of additional rules and modifications of the procedure) to mitigate gaming should be envisaged.

- The Energy allocation procedure could be extended to incorporate complex bidding structures (including start-up generation costs and spanning over several periods of time), placed in the TS markets by the various generators and loads.

  Clearly, this would make energy allocation a more sophisticated task. On the other hand, if one manages to coordinate the procedure, it seems that it would be another step towards economic efficiency. For instance, the so-extended energy allocation procedure, if properly designed, could, under the responsibility of the coordinator, allow a generator to be dispatched in one time-period by a TS and in the next by another, sharing the generator's start-up cost among the two involved TSs.

- Although the use of a DC network model seems justified by the nature of the problems treated in this work, extensions towards using a full AC model could be envisaged.

- The problem of jointly scheduling reserves with energy should be investigated in more detail.

  The difficulty in embedding energy and reserve co-optimization into the proposed Energy allocation procedure, stems from the fact that reserve offers and energy offers should be somehow revealed to and treated by the same entity. In Section 5.5, it has been proposed to overpass this difficulty by having the TSs responsible for scheduling reserves. However, this may not be the best option, and it deviates from present practice. In addition, in order for the latter idea to be put into practice, more research should be devoted into how each TSO could "divide" among TSs the amount of reserves it needs to be provisioned in its area.

- The method should be applied to and tested with large-scale systems.

  If made possible, it would be very interesting to collect real data from an existing interconnection, like the continental European one, in order to test what the proposed Energy and Transmission allocation procedure would give. A comparison with an existing centralized scheme would also be of interest.

- The convergence speed of all the iterative schemes presented remains an issue requiring further investigation.

- Coupling the problem of optimizing the settings of PSTs (and, more generally, of FACTS devices) by the TSOs with that of clearing overlapping markets by various TSs could be of interest.

  The PST angles could be dynamically set during the iterations of the procedure such that they alleviate congestion and increase the network's transfer capacity towards the most interesting directions. A remuneration mechanism should be developed for the PST owners.

- Finally, a very interesting, but somewhat vague, research direction could consist in envisaging more sophisticated coordination schemes, that would systematically lead to Pareto efficient solutions.

Appendix A

Minimal reduction of unscheduled flows for security restoration: Application to phase shifter control

More and more TSOs, noticeably in Europe, equip their systems with PSTs to counteract transit flows that take place in a large meshed interconnection. In Chapter 3, a framework for coordinating the interactions of the various TSO control actions has been developed. The work presented here, as an appendix to the main body of the thesis, triggered from the investigation of the multi-TSO PST control problem, consists in proposing an algorithm for the coordinated control, by one TSO, of several PSTs located in its system, with the objective of reducing the unscheduled flow through its system. Minimum reduction of unscheduled flow and minimum deviation with respect to present operating point are sought in order to minimize the trouble caused to other TSOs, while ensuring secure operation. Attention is paid to combining pre- and post-contingency controls. The resulting algorithm, simple and compatible with real-time applications, is illustrated on a realistic test system.

A.1 Introduction

A.1.1 Transit flows: causes and consequences

Loop flows, parallel path flows, inadvertent flows, and circulating flows are synonymous terms that basically refer to the fact that power can flow through several paths in a meshed network [HMB+91]. The term transit flow is used by ETSO (European Transmission System Operators) [DS05] and is adopted throughout the appendix.

This share of flow between parallel paths has been observed in large interconnections since the early '60s. In USA, parallel flows have been reported in the PJM interconnection as well

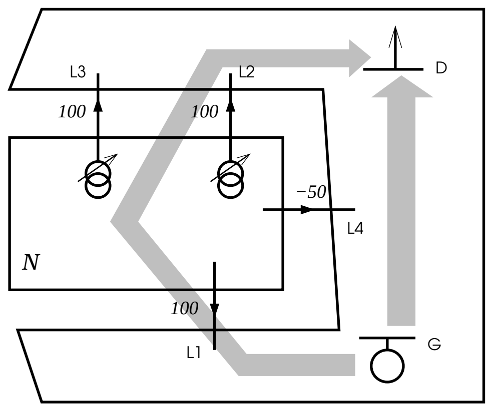

Figure A.1: Transit flow due to external transaction

as in the WECC system [HMB+91]. Transit flows are also common in Europe, where the borders of some countries are crossed, at least partially, by power exchanges involving other countries [GMZ+06, BSA+04]. This situation is symbolically depicted in Fig. A.1 where a fraction of the power due to external transaction passes through the network $\mathcal{N}$ not involved in the transaction. In recent times, transit flows have played an important role in the 2003 North American blackout [Liv05] and in cross-border trading in European markets [Bow02] thus necessitating proper management.

In large interconnections, consisting of several areas operated by different Transmission System Operators (TSO), the common practice is to plan inter-area transactions in advance, in forward, day-ahead or even intra-day markets. For the sake of coordination, Available Transfer Capacities (ATC) are computed between the different areas, taking into account security criteria. The final transactions settlements should respect these ATCs.

In real-time operation, however, actual power flows may differ significantly from what has been scheduled in ahead. This may originate from:

- unknown or uncoordinated transactions involving other partners in the interconnection, for instance if transactions are scheduled according to the contract path logic without making use of a flow-based model of the whole interconnection;
- changes in external generation pattern, e.g. due to wind generation variability;
- outage of external equipments.

The Unscheduled Flow (UF), i.e. the discrepancy between actual and expected flows, becomes a concern when it adds to the loading of inner and interconnection transmission lines and endangers security, moving the system to insecure state (when some credible contingencies could not be stood) or even emergency state (when thermal limits are overstepped even in the current operating conditions) [BSA+04, UCT].

A.1.2 Accommodating vs. controlling unscheduled flows

Several procedures are in place to deal with UFs [SFHC04, SH08, TLF06, WEC]. As long as it does not endanger security, a certain level of UF can be accommodated and priced. On the other hand, curtailment of transactions, such as in the transmission loading relief procedure used in the USA, or re-dispatch of generation may be required in severe situations.

Additionally, power flows can be controlled by Phase Shifting Transformers (PSTs) or possibly the faster, but more expensive FACTS devices [HMB+91, BSA+04, WEC, MZBH01, CBC+02]. PSTs are among the few controls, together with topology changes, that fully remain in the hands of TSOs. With reference to Fig. A.1, the two PSTs can be controlled in a coordinated way to reduce the fraction of power flow passing through $\mathcal{N}$ as a result of the transaction from G to D. More PSTs are likely to be installed for increased control of transit flows, as testified by the situation in Belgium, where three PSTs are going to be put in operation on the Northern border of the country [VHS+07, VHS+08].

In the European interconnection, an *ex post* inter-TSO payment has been put into practice since 2002. Countries receive a compensation for the use made by external agents of their networks. At the same time, they are charged for their use of the other partners' networks. The net outcome of the compensation and charges for one country must be used to modify the annual regulated transmission cost from which the transmission tariffs are computed. This results in a system of entry/exit tariffs whereby an agent who pays the modified local access tariff gains access to the entire European grid. Losses are compensated, while for infrastructure the compensation is based on the cost of hosting cross-border flows [ITC07, ITC05]. However, no real-time inter-TSO coordination procedure exists in Europe yet to mitigate UFs.

A.1.3 Objective of this work

This work deals with the real-time restoration of security when the appearance of some UF causes the system to operate in insecure or even emergency mode (i.e. the system would be in normal and secure state without the UF). Ahead scheduling through an ATC-type procedure is assumed to be in operation, as well as a real-time or *ex post* UF accommodation and compensation scheme.

A real-time control tool is proposed enabling a TSO to quickly restore security in its system through actions on its own controls. At the same time, this control is aimed at being as unintrusive as possible for the rest of the interconnection [RW01]. The first motivation for not acting more than needed (and not acting at all when not required) is to facilitate overall system operation and not to create congestions elsewhere. A second motivation may come from the above-mentioned *ex post* financial scheme which compensates the TSO for accommodating the UF.

In this context, the possibility is considered to let the system operate without satisfying the strict N-1 security criterion, but take advantage of post-disturbance corrective actions. Since equipment outages are relatively rare events, it is cost-effective to operate the system at the economic (or market) optimum that corresponds to its present (intact) configuration, and wait for the disturbance occurrence to take corrective action. However, post-contingency adjustments may be limited, given the time left by thermal overloads, because the operator is unavailable or not trained to react or because of constraints related to the functioning of the available controls (generator ramps, change of PST settings etc). This suggests that a compromise should be found between preventive and corrective control actions.

This fits the general problem of operating the system in the optimal, correctively secure manner [MPG87, SAM87, CW08]. The general approach to this problem is the Corrective Security Constrained Optimal Power Flow (CSCOPF).

However, as UFs are to be handled in real time, resorting to a standard CSCOPF may prove inappropriate, owing to the complexity of this approach. Instead, through the introduction of an inequality constraint on the UF and the use of a specific decomposition procedure, the proposed algorithm avoids the above complexity and yields a procedure more compatible with real-time application.

The remaining of this appendix is organized as follows. In Section II, the above simplification of the CSCOPF problem is exposed. The mathematical expression of the UF used to this purpose is presented in Section III. After this general presentation, the approach is applied specifically to the coordinated PST control in Section IV, considering a simplified optimization. An illustrative example is detailed in Section V, while various additional aspects are discussed in Section VI. The Conclusion in Section VII summarizes the main features of the approach.

A.2 Outline of the proposed procedure

A.2.1 Security constrained optimal power flow

Security constrained optimal power flow is the framework that has been advocated for a long time to support security control activities in power systems. This problem itself has been formulated under two modes: preventive (PSCOPF) and corrective (CSCOPF). In the former, the adjustment of control variables in post-contingency states is not allowed, except if stemming from automatic response to contingencies. The underlying assumption of CSCOPF is that operational limits violation can be generally tolerated for some time without equipment damages, thereby allowing post-contingency corrective actions to be implemented.

The CSCOPF approach of interest in this work can be compactly formulated as follows:

$$\min_{\mathbf{x},\mathbf{x}_1,\ldots,\mathbf{x}_c,\mathbf{u},\mathbf{u}_1,\ldots,\mathbf{u}_c} f(\mathbf{x},\mathbf{u}) \tag{A.1}$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0} \tag{A.2}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) \leq \mathbf{0} \tag{A.3}$$

$$\mathbf{g}_k(\mathbf{x}_k,\mathbf{u}_k) = \mathbf{0} \qquad k = 1,\ldots,c \tag{A.4}$$

$$\mathbf{h}_k(\mathbf{x}_k,\mathbf{u}_k) \leq \mathbf{0} \qquad k = 1,\ldots,c \tag{A.5}$$

$$|\mathbf{u}_k - \mathbf{u}| \leq \Delta\mathbf{u}_k^{max} \qquad k = 1,\ldots,c \tag{A.6}$$

The objective f may be either economical (e.g. maximize social welfare) or technical (e.g. minimize deviations with respect to a reference stemming from market). $\mathbf{x}$ (respectively $\mathbf{u}$) denotes the vector of state (resp. control) variables in the pre-contingency configuration, (A.2) are the pre-contingency power flow equations and (A.3) the corresponding operating constraints, c is the number of contingencies, $\mathbf{x}_k$ and $\mathbf{u}_k$ are the state and control variables in the k-th post-contingency configuration, with the corresponding power flow equations (A.4) and operating constraints (A.5). Finally, $\Delta\mathbf{u}_k^{max}$ is the vector of bounds on the variation of control variables between the base case and the k-th post-contingency state.

For some problems, the above general formulation may not be the most appropriate. The obvious issue is the high dimensionality of the problem, resulting in prohibitive computing times and complexity of computations. To mitigate these drawbacks, the usual approach is to consider a subset of potentially active contingencies, identified by means of (steady-state) security analysis and contingency filtering techniques [SAM87]. Benders decomposition has been also proposed [MPG87, SR96], as will be discussed in Section A.6.3 . Even with these mitigating approaches, designing a CSCOPF compatible with real-time requirements remains a challenge for large systems and/or when many contingencies are considered. For the specific situation of UFs threatening security, the simplification explained hereafter makes the problem much more compatible with real-time requirements.

A.2.2 Simplifying the optimization problem

We consider the impact of contingencies such as branch or generator outages. We assume that the system has entered an insecure (or even emergency) state with respect to some contingencies owing to an excessive transit flow[1]. Exploiting this correlation between excessive transit flow and severity of contingencies, the idea is to force the transit flow to decrease up to the point where the system is correctively secure.

[1] In fact, the unscheduled part of the transit flow is expected to be responsible for insecurity. For the scheduled part, the system should have been already checked and made secure.

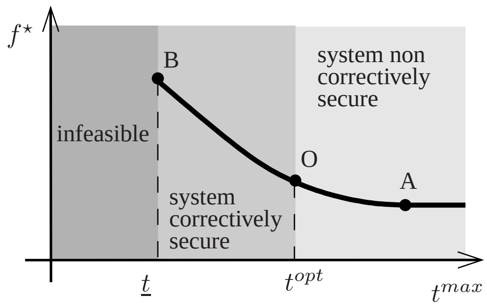

Figure A.2: Variation of objective function with t^{max}

To this purpose, consider the simpler OPF problem including pre-contingency constraints only:

$$\min_{\mathbf{x},\mathbf{u}} f(\mathbf{x},\mathbf{u}) \tag{A.7}$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0} \tag{A.8}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) \leq \mathbf{0} \tag{A.9}$$

$$t(\mathbf{x},\mathbf{u}) \leq t^{max} \tag{A.10}$$

where t represents the transit flow and t^{max} a bound on the latter. The $t(.)$ function is defined more precisely in the next section. Let $f^\star$ be the value of the objective at the optimum.

A variation of $f^\star$ with t^{max} is sketched in Fig. A.2. Consider a progressive decrease of t^{max}, starting from a large value for which the constraint (A.10) is not binding. At point A, this constraint becomes active and starts impacting the value $f^\star$. From there on, the smaller t^{max}, the larger $f^\star$. At the same time, smaller and smaller values of the transit flow t are forced and, hence, the impact of contingencies becomes less severe. Therefore, we assume that there exists a point O, where the system becomes correctively secure and remains so for even smaller values of t^{max}. The curve stops at point B, where (A.7-A.10) becomes infeasible if t^{max} is further decreased.

Point O is the sought operating point in the proposed method.Operating at this point is interesting because security is restored but the transit flow is decreased to the least extent, thereby disturbing the external system as little as possible.

Point O can be determined by searching iteratively for t^{opt}, the largest value of t^{max} such that the system is correctively secure. This single-dimensional search is simple. For a given t^{max}, the corresponding OPF (A.7-A.10) is solved to obtain the pre-contingency operating state $\mathbf{x}^\star$ and controls $\mathbf{u}^\star$. The next step is to determine if this operating state is correctively secure.

For the k-th contingency ($k = 1, \ldots, c$), we check whether there exists (at least) one $\mathbf{u}_k$ with $|\mathbf{u}_k - \mathbf{u}^\star| \leq \Delta\mathbf{u}_k^{max}$, such that the post-contingency state given by (A.4) satisfies the operating constraints (A.5). This could be done by solving the following optimization problem:

$$\min_{\mathbf{x}_k,\mathbf{u}_k,\mathbf{e}_k} \mathbf{1}^T\mathbf{e}_k \tag{A.11}$$

$$\text{s.t.} \quad \mathbf{g}_k\left(\mathbf{x}_k,\mathbf{u}_k\right)=\mathbf{0} \tag{A.12}$$

$$\mathbf{h}_k\left(\mathbf{x}_k,\mathbf{u}_k\right)\leq\mathbf{0} \tag{A.13}$$

$$|\mathbf{u}_k-\mathbf{u}^\star|\leq\mathbf{\Delta u}_k^{max}+\mathbf{e}_k \tag{A.14}$$

$$\mathbf{e}_k\geq\mathbf{0} \tag{A.15}$$

where $\mathbf{1}$ denotes a column vector with all components equal to 1. If the solution of this problem is such that $\mathbf{e}_k=\mathbf{0}$, then the post-contingency operating point is correctively secure.

An alternative way to check for the existence of $\mathbf{u}_k$, chosen in this work, consists in solving the following post-contingency OPF problem:

$$\min_{\mathbf{x}_k,\mathbf{u}_k} F(\mathbf{x}_k,\mathbf{u}_k) \tag{A.16}$$

$$\text{s.t.} \quad \mathbf{g}_k\left(\mathbf{x}_k,\mathbf{u}_k\right)=\mathbf{0} \tag{A.17}$$

$$\mathbf{h}_k\left(\mathbf{x}_k,\mathbf{u}_k\right)\leq\mathbf{0} \tag{A.18}$$

$$|\mathbf{u}_k-\mathbf{u}^\star|\leq\mathbf{\Delta u}_k^{max} \tag{A.19}$$

If this turns out to be infeasible, it can be concluded that the post-contingency operating point is not correctively secure. The advantage of this approach is that, if the optimization is feasible, its solution provides the operator with a set of post-contingency control actions that can be stored and implemented directly if the contingency ever actually occurs. Typically, the objective F deals with control adjustments; alternatively, the objective f of the pre-contingency OPF problem could be re-used.

The operating point is not correctively secure if there is at least one contingency making (A.16-A.19) infeasible.

A.2.3 Proposed decomposed CSCOPF approach

Figure A.3 shows the various steps of the proposed approach. First, contingencies are simulated. If none of them creates a limit violation, the procedure stops; otherwise, the possibility to correct the violations in post-contingency conditions is checked by solving the OPF problem (A.16-A.19) for each contingency (block 1). If all problems are feasible, the system is correctively secure and the procedure terminates. Otherwise, insecurity being attributed to an excessive transit flow, t^{max} (initialized to the observed transit flow) is set to a lower value (block 2), and the corresponding pre-contingency states $\mathbf{x}^\star$ and controls $\mathbf{u}^\star$ are obtained by solving the OPF problem (A.7 - A.10) (block 3). Based on the latter, corrective security is checked again by block 1. If some contingencies still cannot be corrected, the value of t^{max} is further decreased by block 2, while if all contingencies can be corrected, a higher value of t^{max} is tried. The procedure continues refining the value of t^{max} until t^{opt} is known up to some tolerance.

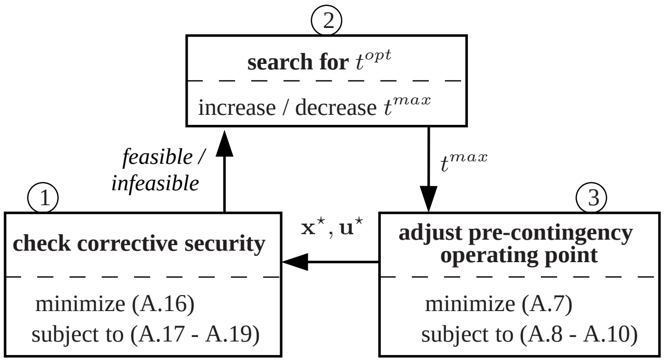

Figure A.3: Proposed decomposed CSCOPF approach

The above description clearly shows that by introducing (A.10) and iterating on t^{max}, the original large problem (A.1-A.6) has been decomposed into $c + 1$ much simpler sub-problems: the problem (A.7-A.10) relative to pre-contingency conditions and the c problems (A.16-A.19) relative to post-contingency.

Of course, adding the constraint (A.10) yields a sub-optimal solution, but this may be quite acceptable in a real-time environment. Further discussion of this aspect is provided in the results.

A.3 Formulation of the transit flow

There is no unique definition of a transit flow, and there is some degree of arbitrariness in its definition. We introduce hereafter the notion used throughout this work, with the objective of using it in the inequality constraint (A.10).

Consider a system exchanging power with the remaining of the interconnection through l tie-lines, in which the active power flows p_i are counted positively when exiting the system. Intuitively, there is a transit flow if some lines are bringing power in and some others are taking it out. This means that not all p_i's have the same sign. We thus define the transit flow as:

$$t = \frac{1}{2}\left(\sum_{i=1}^{l} |p_i| - |\sum_{i=1}^{l} p_i|\right) \tag{A.20}$$

In this expression, $\sum_i p_i$ is the net power interchange (typically controlled by AGC if the system coincides with a control area), a positive value indicating a net power export. Clearly, if all p_i's have the same sign, then $\sum_{i=1}^{l} |p_i| = |\sum_{i=1}^{l} p_i|$ and $t = 0$. If not all flows have the same sign, $t > 0$ whatever the net power interchange[2].

[2]Compared to the definition given in [DS05], the above formula gives the same transit flow values but allows an analytical treatment.

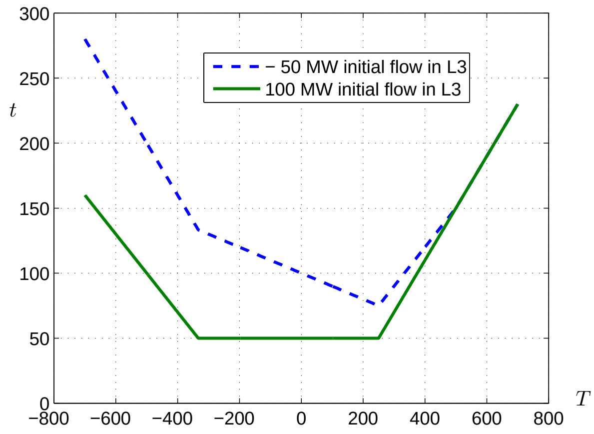

Figure A.4: Transit flow as a function of external transaction

The effect of an external transaction is easily shown in the following example. Consider Fig. A.1, with the base case power flows shown next to the tie-lines. The transit flow computed from Eq. (A.20) is 50 MW. Assume now that a transaction T takes place from G to D, with 40 % of the additional power passing through $\mathcal{N}$. Assume furthermore the following flow distribution (the variation of losses being neglected): -0.4 T in line L1, 0.3T in L2, 0.1T in L3 and nothing in L4. Thus, the power flow is 100-0.4 T in line L1, 100 + 0.3 T in L2, 100+0.1 T in L3, and -50 in L4. The variation of t with T is shown with solid line in Fig. A.4. The transit flow does not change as long as T remains below 250 MW. Indeed, no line flow changes sign; instead, a mere redistribution of flows is taking place. For T larger than 250 MW, the flow in L1 reverses and the transit flow starts increasing as expected. A similar observation is made for a reverse transactions ($T < 0$). The dotted line in Fig. A.4 refers to a base case with an initial flow of -50 MW in L3. In this case, the transaction creates a counterflow in both L1 and L3 and makes the transit flow decrease until T exceeds 250 MW.

Note that (A.20) includes both scheduled and unscheduled parts of the transit flow. As indicated earlier, it is likely that system security has been checked for the scheduled part and insecurity stems from the unscheduled part.

A.4 Application to phase shifter control

A.4.1 Modeling simplifications

In the remaining of this appendix, the decomposition method presented in the previous section is applied to security restoration through PST control. Since the emphasis is on coordinated control of PSTs instead of OPF algorithms, the following simplifying assumptions are made:

1. a linear model is considered, for simplicity and computational efficiency. Although it

might be obtained right away from the well-known DC approximation, a linearization of the AC power flow equations has been considered in this work. This assumption is justified by the almost linear variation of active power flows with PST angles;

2. control variables are assumed to be the PST angles only. We seek here for dedicated algorithms that can quickly help operators in the specific task of adjusting PSTs, or in some future even adjust the PSTs automatically;

3. the objective function f is of technical (instead of economical) nature. A minimum change of PST angles is considered. The motivation may be to minimize the increase in power losses that generally accompanies such changes, or to deviate as few as possible from the operating point set by the market, especially when PSTs are used to increase transactions [MC04].

Under assumption 1, the branch active power flows $\mathbf{p}$ vary with PST angles $\boldsymbol{\phi}$ according to:

$$\mathbf{p} - \mathbf{p}^0 = \mathbf{S} \left(\boldsymbol{\phi} - \boldsymbol{\phi}^0 \right) \tag{A.21}$$

where $\mathbf{p}^0$ and $\boldsymbol{\phi}^0$ are the base case values of the power flows and phase angles, respectively, and $\mathbf{S}$ is a $b \times n$ sensitivity matrix, where b is the number of branches and n the number of PSTs.

The PSTs have no influence on the net power interchange, under the approximation that the power losses remain unchanged. Thus the expression:

$$|\sum_{i=1}^{l} p_i| = d \tag{A.22}$$

does not vary with the PST angles. Using (A.20) and (A.22), the transit flow constraint can be rewritten as:

$$\frac{\sum_{i=1}^{l} |p_i| - |\sum_{i=1}^{l} p_i|}{2} \leq t^{max} \quad \Leftrightarrow \quad \sum_{i=1}^{l} |p_i| \leq d + 2t^{max} \tag{A.23}$$

A.4.2 Controllability of transit flow by PSTs

We assume that the available PSTs are able to control the transit flow t up to a certain point. To this purpose, there must be an adequate number of PSTs, they must be properly located so that the terms of the $\mathbf{S}$ matrix relating tie-line power flows to phase angles are large enough, and the range of PST angles should be wide enough. These important aspects, to be decided at the planning stage, are out of scope of this work [GMZ$^+$06, PVBY99].

In practice, the number, location and range of PSTs may not make it possible to decrease the transit flow below some value. The smallest transit flow $\underline{t}$ that can be enforced with the available PSTs can be computed as:

$$\begin{aligned} \underline{t} = & \min_{\mathbf{p},\boldsymbol{\phi}} \left\{ \frac{1}{2} \sum_{i=1}^{l} |p_i| - \frac{d}{2} \right\} \\ \text{subject to} \quad & \mathbf{p} - \mathbf{p}^0 - \mathbf{S}\left(\boldsymbol{\phi} - \boldsymbol{\phi}^0\right) = \mathbf{0} \\ & -\mathbf{p}^{max} \leq \mathbf{p} \leq \mathbf{p}^{max} \\ & \boldsymbol{\phi}^{min} \leq \boldsymbol{\phi} \leq \boldsymbol{\phi}^{max} \end{aligned}$$

$\underline{t}$ is also the smallest value of t^{max} such that the optimization problem (A.24-A.28) is feasible. It corresponds to point B in Fig. A.2.

A.4.3 The pre-contingency OPF

With a minimum deviation objective, the linear model (A.21) and the transit flow constraint (A.23), the pre-contingency OPF (A.7-A.10) may take on the form:

$$\min_{\mathbf{p},\boldsymbol{\phi}} \sum_{i=1}^{n} (\phi_i - \phi_i^0)^2 \tag{A.24}$$

$$\text{subject to} \quad \mathbf{p} - \mathbf{p}^0 - \mathbf{S}\left(\boldsymbol{\phi} - \boldsymbol{\phi}^0\right) = \mathbf{0} \tag{A.25}$$

$$-\mathbf{p}^{max} \leq \mathbf{p} \leq \mathbf{p}^{max} \tag{A.26}$$

$$\boldsymbol{\phi}^{min} \leq \boldsymbol{\phi} \leq \boldsymbol{\phi}^{max} \tag{A.27}$$

$$\sum_{i=1}^{l} |p_i| \leq d + 2t^{max} \tag{A.28}$$

where (A.26) accounts for the thermal limits of the branches, and (A.27) for the available range of PST angles. For a low enough t^{max}, the constraint (A.28) will be active at the optimum, unless an active constraint (A.26) forces a lower transit flow.

An L_1-norm objective $\sum_{i=1}^{n} |\phi_i - \phi_i^0|$ can be also considered but has been found to cause undesirable distortion of power flows, as it tends to make full use of controls with higher sensitivities. The L_2 norm (A.24) distributes the control effort more evenly over the PSTs.

Since the PSTs are discrete devices, each ϕ has to be rounded to the value corresponding to the nearest tap position.

To deal with the absolute value in (A.28) it is convenient to define two new variables, respectively p_i^+ and p_i^-, such that $p_i = p_i^+ - p_i^-$ with $p_i^+, p_i^- \geq 0$. The constraint (A.28) is then rewritten as:

$$\sum_{i=1}^{l} (p_i^+ + p_i^-) \leq d + 2t^{max} \,, \qquad \text{with } \; p_i^+, p_i^- \geq 0$$

A.4.4 The post-contingency OPF

Let $\boldsymbol{\phi}^{\star}$ be the solution of the pre-contingency OPF (A.24-A.28).

The post-contingency OPF problem (A.16-A.19), aimed at checking if the system is correctively secure with respect to the k-th contingency $(k = 1, \ldots, c)$ takes on the form:

$$\min_{\mathbf{p},\boldsymbol{\phi}} \sum_{i=1}^{n} |\phi_i - \phi_i^{\star}| \tag{A.29}$$

$$\text{subject to} \quad \mathbf{p} = \mathbf{p}^{(k)} + \mathbf{S}^{(k)} (\boldsymbol{\phi} - \boldsymbol{\phi}^{\star}) \tag{A.30}$$

$$-\mathbf{p}^{max} \leq \mathbf{p} \leq \mathbf{p}^{max} \tag{A.31}$$

$$\boldsymbol{\phi}^{min} \leq \boldsymbol{\phi} \leq \boldsymbol{\phi}^{max} \tag{A.32}$$

$$-\Delta\boldsymbol{\phi}^{max} \leq \boldsymbol{\phi} - \boldsymbol{\phi}^{\star} \leq \Delta\boldsymbol{\phi}^{max} \tag{A.33}$$

where $\mathbf{S}^{(k)}$ is the post-contingency sensitivity matrix and $\mathbf{p}^{(k)}$ the vector of post-contingency branch flows, provided by a preliminary contingency analysis. The constraint (A.33) expresses that in post-contingency conditions, PST angles cannot be changed from the pre-contingency values $\boldsymbol{\phi}^{\star}$ by more than $\Delta\boldsymbol{\phi}^{max}$, which is supposed to reflect the limited rate of change of PSTs and/or the initial response delay of operators. The choice of the objective has been discussed in Section A.2.2.

The following items are noteworthy:

1. The above optimization has to be performed for each contingency endangering the system. Obviously, the correction $\boldsymbol{\phi} - \boldsymbol{\phi}^{\star}$ is expected to vary with the contingency;
2. in the above procedure, it is implicitly assumed that the available PSTs have controllability over the overload problem. Thus, the contingencies of concern here are those that can be corrected by the PSTs. To check this, the above optimization can be performed with the constraints (A.33) removed. If the problem remains infeasible, the PSTs cannot help, and the corresponding contingencies should be treated by other means;
3. $\Delta\boldsymbol{\phi}^{max}$ may change with the contingency severity: a higher overload must be corrected in a smaller time and hence a smaller $\Delta\boldsymbol{\phi}^{max}$ should be imposed.

A.5 Illustrative example

A.5.1 Test system

The results have been obtained on a test system,loosely inspired of a small portion of the UCTE system. Its overall structure is shown in Fig. A.5. It is made up of four sub-systems, corresponding to different countries and different TSOs. The figure provides the number of

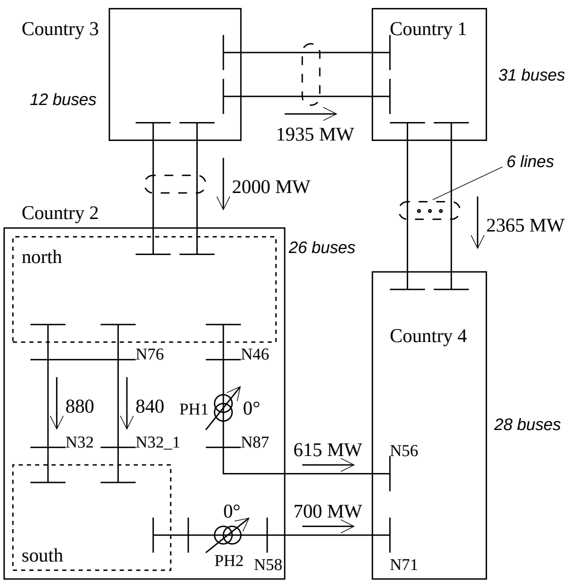

Figure A.5: Test system structure and base case operating point

buses in each sub-system. The subsystem of Country 2 is equipped with two PSTs, identified by PH1 and PH2.

The active power flows that exist in the base case situation, with both PST angles equal to zero are shown in Fig. A.5. The transit flow through Country 2 is $t = \frac{3315-685}{2} = 1315$ MW.

A deeper look at the diagram reveals the presence of a "major" and a "minor" loop. The major loop includes the tie-lines connecting the four systems. Inside this loop, Countries 1 and 3 are exporting power while Countries 2 and 4 are importing. The two PSTs of Country 2 are placed cutting the loop, in parallel to each other. Moving their angles in the same direction, the TSO of Country 2 can redirect some power flow from path 3-2-4 towards path 3-1-4. The minor loop includes two paths from north to south of Country 2, one through the internal lines N76-N32 and N76-N32_1 and the other through the tie-lines N87-N56 and N58-N71. The two PSTs are placed in series with each other inside this minor loop, and moving their angles in opposite directions redistributes the power between the two above-mentioned paths.

A.5.2 Security analysis

We consider security analysis in Country 2. Out of all N-1 contingencies, two of them end up in line overloads: the loss of lines N76-N32 and N76-N32_1. Figure A.6 shows the distribution of power flows after the tripping of N76-N32_1: line N76-N32 is significantly loaded above its capacity of 1215 MW (taken as 90 % of its MVA capacity to account for reactive power and leave a security margin).

As for the security analysis of any system nested inside an interconnection, a correct representation of the external system (Countries 1, 3 and 4 in this case) is essential to assess the effect

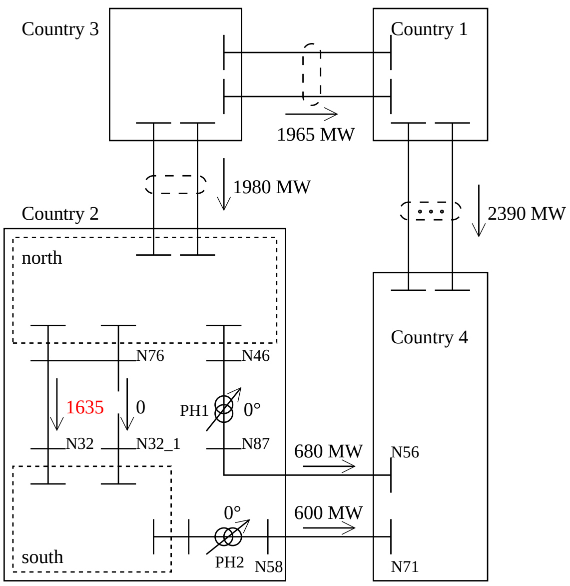

Figure A.6: Power flows after tripping of line N76-N32_1

of both contingencies and PST adjustments. The tests have been performed assuming that the whole system model is available to the TSO of Country 2, but an equivalent, or a combination of unreduced and equivalent models could be also used to account for the system external to Country 2.

A.5.3 Linearization

The model is obtained by linearizing the AC power flow equations as follows.

We start from a base case situation with PST angles $\boldsymbol{\phi}^0$ and power flows $\mathbf{p}^0$. The sparse power flow Jacobian is computed at this operating point and LU-decomposed. Using a well-known sensitivity formula [PPTT68], each column of the $\mathbf{S}$ matrix is obtained by solving one sparse linear system involving the available factors of the transposed Jacobian. $\boldsymbol{\phi}^0$, $\mathbf{p}^0$ and $\mathbf{S}$ are re-used each time the pre-contingency problem (A.24-A.28) is solved (block 3 in Fig. A.3) to obtain an updated $\boldsymbol{\phi}^\star$.

Before solving the post-contingency problem (A.29-A.33) (block 1 in Fig. A.3), and given the PST angles $\boldsymbol{\phi}^\star$, a full AC power flow is solved to obtain the flows $\mathbf{p}^{(k)}$ that result from both the k-th contingency and the pre-contingency PST adjustments. The corresponding Jacobian is LU-decomposed and used to determine the $\mathbf{S}^{(k)}$ matrix, using the above mentioned formula.

The power flow model used to compute the $\mathbf{S}$ and $\mathbf{S}^{(k)}$ matrices involves the external system, unreduced and/or equivalenced, according to what is available to the TSO of concern. The former option has been considered in this work.

Thanks to the very close to linear relationship between branch power flows and PST angles, as

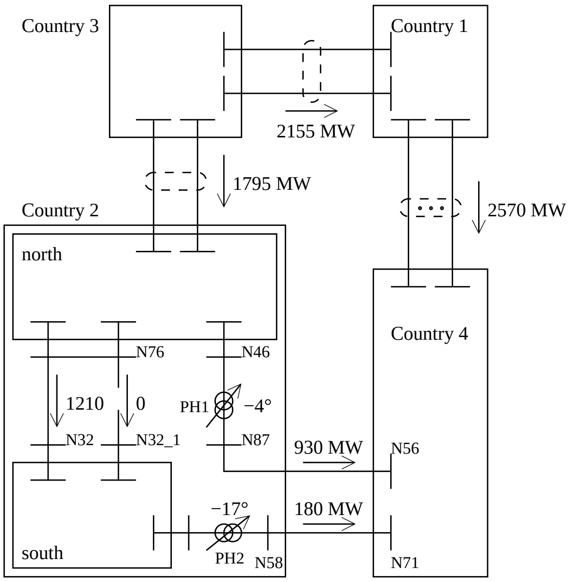

Figure A.7: Power flows after tripping of line N76-N32_1 and corrective control by PSTs

well as the sensitivity matrix updates, the linearized model was found to be extremely accurate. A comparison of power flows obtained from respectively the linearized and the full AC power flow models, revealed discrepancies no larger than 0.2 MW on the branch flows.

A.5.4 Corrective control of line overloads by PSTs

Before the application of the algorithm, we demonstrate the effectiveness of the PSTs in alleviating the overload caused by the tripping of line N76-N32_1, which is the contingency requiring the largest control effort.

We first consider the PST angles that correctively clear the overload without any limit of the type (A.33). We thus solve the optimization problem (A.29-A.32) with $\boldsymbol{\phi}^{\star}$ equal to the base case values $\boldsymbol{\phi}^0 = (0^o, 0^o)$. The angles and the resulting power flows are shown in Fig. A.7. The line flow is reduced below its limit thanks to: (i) a common decrease of PST angles that redistributes the flows in the major loop, decreasing the transit flow though Country 2 from 1280 to 1175 MW; (ii) a more pronounced action of PH2 that redistributes the flows inside the minor loop.

If the post-contingency change of ϕ_2 from 0^o to -17^o (see Fig. A.7) is deemed too large and limited to a lower value, ϕ_1 cannot compensate and the optimization problem (A.29-A.32) becomes infeasible, indicating that the system is not correctively secure.

Table A.1: Iterations to restore corrective security

iter. No	t^{max} (in MW)	block 3 ϕ_1^*	block 3 ϕ_2^*	block 1 outcome
0	1315	0^o	0^o	not correctively secure
1	658	infeasible		
2	986	-19^o	-20^o	correctively secure
3	1151	-10^o	-11^o	correctively secure
4	1233	-5^o	-6^o	not correctively secure
5	1192	-7^o	-8^o	correctively secure
6	1213	-6^o	-7^o	correctively secure

A.5.5 Preventive restoration of corrective security

We now illustrate the method presented in Section A.2 (see also Fig. A.3) to make the system secure with respect to both contingencies previously mentioned.

We assume a maximum post-contingency angle change $\Delta\phi^{max}$ of 10 degrees. Hence, for the initial operating point shown in Fig. A.5, the system is not correctively secure (as shown in Section A.5.4 for the loss of line N76-N32_1) and the PST angles have to be adjusted in the pre-contingency configuration.

A binary search (also known as dichotomic search, or bisection method) is used in block 2 of Fig. A.3 to determine the highest value of t^{max} such that the system is correctively secure. This consists in building a smaller and smaller interval $[t_l\ t_u]$ such that for $t^{max} = t_l$ the system is correctively secure while for $t^{max} = t_u$, it is not. At each step the value $t^{max} = \frac{t_u+t_l}{2}$ is tested and taken as the new t_l (resp. t_u) if the system is found correctively secure (resp. insecure). The procedure is repeated until $|t_u - t_l|$ becomes smaller than a tolerance ϵ. The best initial value for t_l is $\underline{t}$ (discussed in Section A.4.2) but a 0 MW value has been taken in the tests, saving the computation of $\underline{t}$ at the expense of an additional iteration of the binary search. t_u has been initialized at the base transit flow (1315 MW).

The main results are listed in Table A.1. At the first iteration, with t^{max} set to $\frac{1315+0}{2} = 658$ MW, the optimization of block 3 is infeasible, meaning that the PSTs cannot force such a low transit flow. Obviously, block 1 cannot be executed. Thus, after setting t_l to 658 MW, we proceed with the second iteration, corresponding to $t^{max} = \frac{1315+658}{2} = 986$ MW.

The third and fourth column of Table A.1 give the pre-contingency settings determined by block 3, while the last column indicates whether this new operating point is found correctively secure by block 1. The tolerance ϵ being set to 25 MW, the procedure stops after six iterations.

The settings to be finally actually implemented, in a preventive mode, are $\phi_1^\star = -6^o$ and $\phi_2^\star = -7^o$, which decrease the transit flow to 1213 MW.

Table A.2 presents the results obtained by repeating the procedure for various values of $\Delta\phi_1^{max} = \Delta\phi_2^{max} = \Delta\phi^{max}$. The second and third columns give the pre-contingency PST angles, leading to the transit flow value shown in the fourth column. The last two columns provide the final values that should be given to PST angles, in the post-contingency configuration, to clear the

Table A.2: Preventive and corrective PST settings for various $\Delta\phi^{max}$

$\Delta\phi^{max}$	ϕ_1^*	ϕ_2^*	t (in MW)	ϕ_1^{post}	ϕ_2^{post}
5^o	-14^o	-16^o	1065	-10^o	-21^o
10^o	-6^o	-7^o	1213	-5^o	-17^o
15^o	-2^o	-2^o	1280	-4^o	-17^o
20^o	0^o	0^o	1315	-4^o	-17^o

line overload caused by the tripping of N76-N32_1. As expected, the more one resorts to corrective control actions (i.e. the larger $\Delta\phi^{max}$), the less the pre-contingency operating point is changed (and, hence, the less intrusive the change in transit flow).

The variations observed in the table can be explained as follows. First, the post-contingency angles are the closest to the pre-contingency ones ($\boldsymbol{\phi}^\star$) that alleviate the post-contingency overloads. Second, for some pre-contingency PST angle settings, $\Delta\boldsymbol{\phi}^{max}$ may be not large enough to allow for post-contingency correction. In this case, the pre-contingency angles are modified in the direction that reduces the transit flow, resulting into new values of $\boldsymbol{\phi}^\star$. As a result, when seeking for post-contingency corrections, starting from the new $\boldsymbol{\phi}^\star$, different post-contingency settings will be found (still closest to this new $\boldsymbol{\phi}^\star$). This is why the post-contingency settings vary so much with $\Delta\boldsymbol{\phi}^{max}$.

A.6 Discussions

A.6.1 Requirements of the method

The following conditions have to be fulfilled for the proposed procedure to be successful. First, the available PSTs must have controllability over the transit flow. Second, the contingency should be secured by decreasing the transit flow. A typical situation is when a corridor is loaded by the transit flow and the outage of a line in this corridor causes overload of parallel lines. If the transit flow reduction cannot help, the contingency will remain harmful at the minimum transit flow $\underline{t}$. This point is further illustrated hereafter.

If these conditions are not met, another objective and/or additional (probably more expensive) controls should be considered to address the security problem.

A.6.2 Optimality of the method

Figure A.8 shows a characterization of the pre-contingency operating points corresponding to various values of (ϕ_1, ϕ_2). This diagram was obtained by repeatedly solving the optimization problem (A.29-A.33) with $(\phi_1^\star, \phi_2^\star)$ set to each pair of integer values in the shown range. The maximum post-contingency correction $\Delta\phi^{max}$ was set to 10^o, as in Table A.1. At the points

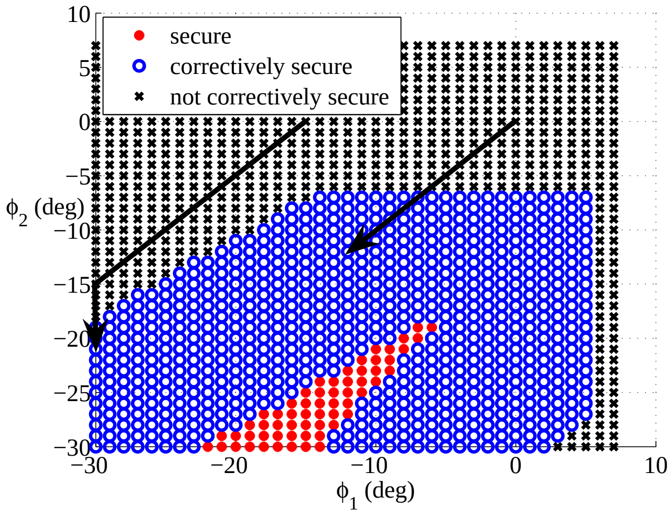

Figure A.8: Characterization of pre-contingency operating points

shown with crosses the optimization problem was infeasible; hence, the system is not correctively secure. At the points shown with circles, the problem had a solution, indicating that the system is correctively secure. Finally, at the points shown with disks, the contingencies were harmless and the system secure; there was thus no need for PST adjustments.

Assume that the system is operating initially at $(\phi_1^o = 0^o, \phi_2^o = 0^o)$. The arrow that starts from this point in Fig. A.8 is the path of (pre-contingency) PST angles obtained by solving (A.24-A.28) for decreasing values of t^{max} in (A.28), i.e. smaller and smaller transit flow. The points generated by block 3 of the proposed procedure (see Fig. A.3) lie on this path. The binary search converges to the point $(\phi_1 = -6^o, \phi_2 = -7^o)$, where the arrow enters the correctively secure region.

The variations of the post-contingency angle settings shown in Table A.2 can be further explained in the light of Fig. A.8. For smaller $\Delta\phi^{max}$, the correctively secure region shrinks closer to the secure area. Hence, when moving along the arrow in Fig. A.8 (which decreases the transit flow), the operating point enters the correctively secure region for different angle settings. In particular, with $\Delta\phi^{max} = 5^o$, this happens for $\boldsymbol{\phi}^\star = (-14^o, -16^o)$, from which the closest secure angle settings are $(-10^o, -21^o)$.

In fact there are many ways to enter the correctively secure region. For instance, minimizing the Euclidian distance to the initial point $(\phi_1^o = 0, \phi_2^o = 0)$ would lead to the solution $(\phi_1 = 0^o, \phi_2 = -7^o)$. This operating point is closer to the initial point but at this point the operation of system 2 is more disturbed due to a significant redistribution of power flows inside the minor loop. The proposed algorithm does not yield this solution because the pre-contingency changes are constrained to obey (A.28). In fact, having attributed the security problem to a certain cause (an excessive transit flow), the algorithm tries to find the closest correctively secure operating point towards the direction that mitigates this cause.

Assume now that the initial operating point is $(\phi_1^o = -14^o, \phi_2^o = 0^o)$. The search direction is parallel to the previously discussed path, until ϕ_1 hits its minimum of -30^o, causing the path to change direction. In this case, the binary search will converge to $(\phi_1 = -30^o, \phi_2 = -19^o)$.

The fact that the search is limited towards the direction that mitigates t may lead to not finding a solution. This happens when $\Delta\phi^{max}$ is small, shrinking the correctively secure region and causing the path to pass around it. In addition, if the requirements listed in section A.6.1 are not met, then the correctively secure region will not be reached by applying the method, since either the search direction will not be towards this region, or the PSTs will not be able to affect the transit flow and hence move the operating point towards the sought direction.

A.6.3 Analogy with Benders decomposition

The proposed problem decomposition offers some similarities with the Benders decomposition method [MPG87, CW08, SR96, LM09, SV07] from which it differs, however, as discussed hereafter.

In the context of PSCOPF and CSCOPF, the most appealing application of Benders decomposition consists of splitting the original problem into:

- one master problem, in which a solution is found to the pre-contingency sub-problem (A.1 - A.3), and
- several smaller slave problems, each dealing with one contingency and checking if there exists a control $\mathbf{u}_k$ satisfying (A.4 - A.6).

Each infeasible slave sub-problem generates the so-called feasibility cut constraint to be added at the next iteration to the master problem. Iterations between the master and the slave sub-problems continue, with the cuts updated at each iteration, until the original problem (A.1-A.6) is solved to some tolerance.

In the proposed approach the problem is also split into a master problem dealing with the pre-contingency situation (block 3 in Fig. A.3) and slave problems, each relative to a post-contingency situation (block 1 in the same figure). The information passed from slave to master problems is used to adjust the pre-contingency operating point.

However, the main differences with respect to Benders method lie in both the nature and the handling of the information returned to the master problem. The latter consists of a synthetic two-valued variable per contingency. The values stemming from the various contingencies are easily combined into a single infeasible/feasible information. Instead of adding mathematical constraints to the master optimization, the engineering knowledge of the problem (insecurity attributed to transit flow) drives the pre-contingency adjustments. While being less general (the situation of Fig. A.2 must apply) and sub-optimal (to the extent discussed in the previous section), the proposed scheme guarantees fast convergence to the solution, as a binary search is used to find point O in Fig. A.2. This may not be the case with Benders decomposition, as quoted in some papers reporting on the non-monotonic decrease of the objective function [CW08] or the slow final convergence (known as "tailing-off effect") [SV07]. Finally, with Benders decomposition, the size and the structure of the master problem vary from one iteration to the other, depending on the cut constraints added. This is not the case in the proposed method.

A.6.4 Computational efficiency

Several features contribute to making the overall procedure suitable for real-time applications.

First, the decomposition presented in Section II (and applicable to nonlinear CSCOPF) succeeds replacing the highly-dimensional problem (A.1-A.6) with smaller sub-problems. The binary search leads to a low, predictable number of iterations, which could even be decreased by extrapolating/interpolating the next value of the transit flow from past iterations.

As regards the particular application to PST control considered in Section IV:

- the linearized formulation allows resorting to proven, efficient optimization solvers;
- by focusing on the PSTs, the optimization involves a reduced number of control variables;
- the computation of a sensitivity matrix $\mathbf{S}$ involves factorizing the sparse power flow Jacobian and substituting one sparse vector per column of the matrix, i.e. per PST. Efficient sparsity programming solvers are available to this purpose. Furthermore, the optimal ordering step can be performed once for all in the pre-contingency topology.

A.7 Conclusion

The coordinated control of multiple PSTs to decrease unscheduled flow experienced by a TSO inside an interconnection has been considered.

First, a definition of the transit flow has been proposed, linked to tie-line power flows in opposite directions.

Next, a simplification to the general corrective security-constrained optimal power flow problem has been proposed, which allows decomposing this large-scale problem into simpler sub-problems. Based on the assumption that the security problem can be attributed to an excessive transit flow, the algorithm investigates a sequence of pre-contingency operating points towards the direction that decreases this flow. It converges to the correctively secure operating point with the transit flow reduced to the lowest extent possible. By so doing, the control is aimed at being as few intrusive as possible for other TSOs in the interconnection.

Finally, this approach has been applied to the reduction of an excessive transit flow by PSTs in order to deal with insecure situations. The algorithm determines the best possible combination of pre- and post-contingency PST adjustments, with limits specified on the post-contingency angle changes.

The features and limitations of this procedure have been illustrated on a test system.

The embedded optimization problems are simple and suitable to real-time operation. The method could assist the operator in quickly checking if transit flow control by PSTs can restore security or if more expensive actions are needed. The algorithm could be at the heart of a controller coordinating the PSTs, and allowing faster post-contingency adjustments.

Appendix B

Branch data of the three-area 15-bus test system used in this work

The branch reactances, series resistances, as well as maximum MW limits of the three-area 15-bus test system that has been used throughout this report are presented in Table B.1. A 100-MVA base has been used.

Table B.1: Three-area 15-bus system

Branch	Reactance (in p.u.)	Resistance (in p.u.)	Limit $\overline{p}$ (in MW)
A1A2	0.020851	0.0020851	100
A1A3	0.024241	0.0024241	150
A2A3	0.024241	0.0024241	150
A3A4	0.069502	0.0069502	400
A4A5	0.069502	0.0069502	400
B1B2	0.020851	0.0020851	100
B1B3	0.024241	0.0024241	150
B2B3	0.024241	0.0024241	150
B3B4	0.069502	0.0069502	400
B4B5	0.069502	0.0069502	400
C1C2	0.020851	0.0020851	100
C1C3	0.024241	0.0024241	150
C2C3	0.024241	0.0024241	150
C3C4	0.069502	0.0069502	400
C4C5	0.069502	0.0069502	400
A3B3	0.069502	0.0069502	200
A4C4	0.069502	0.0069502	200
B4C3	0.069502	0.0069502	200

Appendix C

Generator data of the IEEE RTS-96 test system used in this work

The generator maximum capacities $\overline{\mathbf{g}}$ and marginal cost bids $\mathbf{c}$ that have been used in this work are presented in Table C.1 . All other data of the IEEE RTS-96 system are as in Ref [RTS99]. Each three-column block corresponds to a TS area (we recall that each TS serves the inelastic load of an area, dispatching generators from all areas). The first column of each such block, gives the name of the bus where the generator is connected.

Table C.1: Generator data of the three-area IEEE RTS-96 test system

Area 1			Area 2			Area 3		
Bus	$\overline{g}$ (in MW)	**c** (in €/h)	Bus	$\overline{g}$ (in MW)	**c** (in €/h)	Bus	$\overline{g}$ (in MW)	**c** (in €/h)
101	20	3.121	201	20	6.242	301	20	9.363
101	20	3.121	201	20	6.242	301	20	9.363
101	76	2.693	201	76	5.386	301	76	8.079
101	76	2.693	201	76	5.386	301	76	8.079
102	20	3.121	202	20	6.242	302	20	9.363
102	20	3.121	202	20	6.242	302	20	9.363
102	76	2.693	202	76	5.386	302	76	8.079
102	76	2.693	202	76	5.386	302	76	8.079
107	100	2.268	207	100	4.536	307	100	6.804
107	100	2.268	207	100	4.536	307	100	6.804
107	100	2.268	207	100	4.536	307	100	6.804
113	197	2.263	213	197	4.526	313	197	6.789
113	197	2.263	213	197	4.526	313	197	6.789
113	197	2.263	213	197	4.526	313	197	6.789
115	12	2.762	215	12	5.524	315	12	8.286
115	12	2.762	215	12	5.524	315	12	8.286
115	12	2.762	215	12	5.524	315	12	8.286
115	12	2.762	215	12	5.524	315	12	8.286
115	12	2.762	215	12	5.524	315	12	8.286
115	155	2.195	215	155	4.390	315	155	6.585
116	155	2.195	216	155	4.390	316	155	6.585
118	400	2.288	218	400	4.576	318	400	6.864
121	400	2.288	221	400	4.576	321	400	6.864
122	50	0.0	222	50	0.0	322	50	0.0
122	50	0.0	222	50	0.0	322	50	0.0
122	50	0.0	222	50	0.0	322	50	0.0
122	50	0.0	222	50	0.0	322	50	0.0
122	50	0.0	222	50	0.0	322	50	0.0
122	50	0.0	222	50	0.0	322	50	0.0
123	155	2.195	223	155	4.390	323	155	6.585
123	155	2.195	223	155	4.390	323	155	6.585
123	350	2.276	223	350	4.552	323	350	6.828

Appendix D

Multilevel optimization

Multilevel optimization represents a hierarchy of optimization problems, where the outer optimization problem is subject to the outcome of a set of enclosed optimization problems. Partly motivated by the practical complexity of the multilevel optimization, most work in the recent past has addressed the special case of bi-level optimization, i.e. with one enclosed optimization problem only. The material in this appendix is largely borrowed from Ref. [CMS07], which is a recent review on bi-level optimization.

The general formulation of a bi-level programming problem is:

$$\min_{\mathbf{x}\in\mathbf{X},\mathbf{y}} \quad F(\mathbf{x},\mathbf{y}) \tag{D.1a}$$

$$\text{s.t.} \quad \mathbf{G}(\mathbf{x},\mathbf{y}) \leq \mathbf{0} \tag{D.1b}$$

$$\min_{\mathbf{y}} f(\mathbf{x},\mathbf{y}) \tag{D.1c}$$

$$\text{s.t.} \ \ \mathbf{g}(\mathbf{x},\mathbf{y}) \leq \mathbf{0} \tag{D.1d}$$

where $\mathbf{x} \in \mathbb{R}^{n_1}$ and $\mathbf{y} \in \mathbb{R}^{n_2}$. The variables of problem (D.1) are divided into two classes, namely the *upper-level variables* $\mathbf{x}$ and the *lower-level variables* $\mathbf{y}$. Similarly, the functions $F : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$ and $f : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$ are the *upper-level* and *lower-level objective functions*, respectively, while the vector-valued functions $\mathbf{G} : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}^{m_1}$ and $\mathbf{g} : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}^{m_2}$ are called the *upper-level* and *lower-level constraints*, respectively. In view of the hierarchical relationship, Eqs. (D.1a) and (D.1b) make up the *upper-level problem*, while Eqs. (D.1c) and (D.1d) the *lower-level problem*.

Two decision-makers are involved in (D.1), the upper- and the lower-level one. The upper-level decision-maker sets the upper-level variables $\mathbf{x}$ and, similarly, the lower-level decision-maker sets $\mathbf{y}$. In some applications, the upper-level decision-maker is called the *leader* and is supposed to issue directives to the lower-level decision-maker, called the *follower*. In this respect, the leader, anticipating the follower's reaction, solves problem (D.1) in order to choose its best (optimal) strategy accordingly. Like this, it comes up with its sought action $\mathbf{x} \in \mathbf{X} \subseteq \mathbb{R}^{n_1}$. Upper-level constraints involve variables from both levels (in contrast to the constraints specified by the set $\mathbf{X}$) and play a very specific role. Indeed, they must be enforced indirectly, as they do not bind the lower-level decision-maker.

It is not unusual to have more than one followers, in which case the bi-level problem expands to a multilevel optimization problem, where every follower is represented by an optimization problem like the lower-level problem in (D.1). This gives for every follower i a reaction set $\mathbf{Y}_i(\mathbf{x})$ corresponding to each action $\mathbf{x} \in \mathbf{X}$ of the leader. The latter wishes to optimize its objective function subject to all the followers' anticipated reactions.

As indicated in [CMS07], solving bi-level problems is a difficult task due to intrinsic nonlinearity and non convexity. Clearly, solving multilevel problems is even more difficult.

Finally, it is worth pointing out a connection between bi-level optimization problems and *Mathematical Problems with Equilibrium Constraints* (MPECs), i.e. optimization problems with mixed complementarity problems (see Section 2.3.6). In fact, if the bi-level involves a lower-level problem that is convex and differentiable, then its KKT necessary optimality conditions can be derived and introduced as constraints to the upper-level problem. Thus, the resulting MPEC would consist of minimizing (D.1a) subject to (D.1b) and to the KKT conditions of (D.1c)-(D.1d).

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