Annex 2
Discretization of the FdV variational principle for LEFM

Content

A2.1. Discretization of the FdV variational principle

The FdV variational equation writes:

$$\delta\Pi = \delta\Pi_1 + \delta\Pi_2 + \delta\Pi_3 + \delta\Pi_4 + \delta\Pi_5 + \delta\Pi_6 = 0 \qquad \text{(A2.1)}$$

$$\delta\Pi_1 = \sum_{I=1}^{N} \int_{A_I} \sigma_{ij}\,\delta\varepsilon_{ij}\, dA_I \qquad \text{(A2. 2)}$$

$$\delta\Pi_2 = \sum_{I=1}^{N} \int_{A_I} \Sigma_{ij} \left[\frac{1}{2}\left(\frac{\partial\delta u_i}{\partial X_j} + \frac{\partial\delta u_j}{\partial X_i}\right) - \delta\varepsilon_{ij} \right] dA_I \qquad \text{(A2. 3)}$$

$$\delta\Pi_3 = \sum_{I=1}^{N} \int_{A_I} \delta\Sigma_{ij} \left[\frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right) - \varepsilon_{ij} \right] dA_I \qquad \text{(A2. 4)}$$

$$\delta\Pi_4 = -\sum_{I=1}^{N} \int_{A_I} F_i\, \delta u_i\, dA_I \qquad \text{(A2. 5)}$$

$$\delta\Pi_5 = -\sum_{K=1}^{M_t} \int_{S_K} T_i\, \delta u_i\, dS_K \qquad \text{(A2. 6)}$$

$$\delta\Pi_6 = \sum_{K=1}^{M_u} \left[\int_{S_K} \delta r_i (\tilde{u}_i - u_i)\, dS_K - \int_{S_K} r_i\, \delta u_i\, dS_K \right] \qquad \text{(A2. 7)}$$

From the assumption on the stresses and strains in LFMVC, we get successively:

$$\{\delta P\}^J = \{\delta P^O\} + \delta K_{\Sigma 1}^J \{H^{\Sigma 1}\} + \delta K_{\Sigma 2}^J \{H^{\Sigma 2}\}, \quad J = 1, N_C \qquad \text{(A2.8)}$$

$$\{\delta\gamma\}^J = \{\delta\gamma^O\} + \delta K_{\Sigma 1}^J [D]^J \{H^{\Sigma 1}\} + \delta K_{\Sigma 2}^J [D]^J \{H^{\Sigma 2}\}, \quad J = 1, N_C \qquad \text{(A2.9)}$$

Development of $\delta\Pi_1$

Introducing (A2.8) and (A2.9) in (A2.2), we get:

$$\delta\Pi_1 = \sum_{I=1}^{N_C} \left\{ \int_{A_I} \langle\tau\rangle^I \{\delta\gamma^O\}^I dA_I + \delta K_{\Sigma 1}^I \int_{A_1} \langle\tau\rangle^I [D]^I \{H^{\Sigma 1}\} dA_I + \delta K_{\Sigma 2}^I \int_{A_1} \langle\tau\rangle^I [D]^I \{H^{\Sigma 2}\} dA_I \right\} + \sum_{I=N_C+1}^{N} A_I \langle\sigma\rangle^I \{\delta\varepsilon\}^I$$

$$= \sum_{I=1}^{N_C} \int_{A_I} \langle\tau\rangle^I \{\delta\gamma^O\}^I dA_I + \sum_{I=1}^{N_C} \langle\delta K_\Sigma\rangle^I \{e\}^I + \sum_{I=N_C+1}^{N} A_I \langle\sigma\rangle^I \{\delta\varepsilon\}^I \qquad \text{(A2.10)}$$

with

$$\{K_\Sigma\}^I = \begin{Bmatrix} K_{\Sigma 1} \\ K_{\Sigma 2} \end{Bmatrix}^I \;; \qquad \text{(A2.11)}$$

$$\{e\}^I = \begin{Bmatrix} e_1 \\ e_2 \end{Bmatrix}^I = \begin{Bmatrix} \int_{A_I} \langle \tau \rangle^I [D]^I \{H^{\Sigma 1}\} dA_I \\ \int_{A_I} \langle \tau \rangle^I [D]^I \{H^{\Sigma 2}\} dA_I \end{Bmatrix} = \begin{Bmatrix} \int_{A_I} \langle H^{\Sigma 1} \rangle dA_I \\ \int_{A_I} \langle H^{\Sigma 2} \rangle dA_I \end{Bmatrix} [D]^I \{\tau\}^I \qquad \text{(A2.12)}$$

Let

$$[IH]^I = \begin{Bmatrix} IH_1 \\ IH_2 \end{Bmatrix}^I = \begin{Bmatrix} \int_{A_I} \langle H^{\Sigma 1} \rangle dA_I \\ \int_{A_I} \langle H^{\Sigma 2} \rangle dA_I \end{Bmatrix} \quad I = 1, N_C \qquad \text{(A2.13)}$$

Hence

$$\{e\}^I = [IH]^I [D]^I \{\tau\}^I \qquad \text{(A2.14)}$$

e_1^I and e_2^I can be seen as a generalized strains in LFMVC n° I conjugated to the generalized stresses (or stress parameters) $K_{\Sigma 1}^I$ and $K_{\Sigma 2}^I$

Development of $\delta\Pi_2$

$$\delta\Pi_2 = \sum_{I=1}^{N} \int_{A_I} \Sigma_{ij} \left[\frac{1}{2} \left(\frac{\partial \delta u_i}{\partial X_j} + \frac{\partial \delta u_j}{\partial X_i} \right) - \delta\varepsilon_{ij} \right] dA_I$$

For $I = 1, N_C$ (LFMVC's), we have:

$$\sum_{I=1}^{N_C} \left\{ \int_{A_I} P_{ij}^I \left[\frac{1}{2} \left(\frac{\partial \delta v_i}{\partial Y_j} + \frac{\partial \delta v_j}{\partial Y_i} \right) - \delta\gamma_{ij}^I \right] dA_I \right\}$$

$$= \sum_{I=1}^{N_C} \left\{ \int_{A_I} P_{ij}^I \left[\frac{1}{2} \left(\sum_{J=1}^{N} \frac{\partial \Phi_J}{\partial Y_j} \delta v_i^J + \sum_{J=1}^{N} \frac{\partial \Phi_J}{\partial Y_i} \delta v_j^J \right) - \delta\gamma_{ij}^I \right] dA_I \right\}$$

$$= \sum_{J=1}^{N} \left\{ \left[\sum_{I=1}^{N_C} \int_{A_I} \langle P \rangle^I [\partial\Phi]^J dA_I \right] \{\delta v\}^J \right\} - \sum_{I=1}^{N_C} \left[\int_{A_I} \langle P \rangle^I \{\delta\gamma\}^I dA_I \right]$$

$$= \sum_{J=1}^{N} \left\{ \sum_{I=1}^{N_C} \left[\int_{A_I} \langle P^0 \rangle^I [\partial\Phi]^J dA_I + K_{\Sigma 1}^I \int_{A_I} \langle H^{\Sigma 1} \rangle [\partial\Phi]^J dA_I + K_{\Sigma 2}^I \int_{A_I} \langle H^{\Sigma 2} \rangle [\partial\Phi]^J dA_I \right] \{\delta v\}^J \right\}$$

$$- \sum_{I=1}^{N_C} \int_{A_I} \left[\langle P^O \rangle^I + K_{\Sigma 1}^J \langle H^{\Sigma 1} \rangle + K_{\Sigma 2}^J \langle H^{\Sigma 2} \rangle \right] \left[\{\delta\gamma^O\}^I + \delta K_{\Sigma 1}^I [D]^I \{H^{\Sigma 1}\} + \delta K_{\Sigma 2}^I [D]^I \{H^{\Sigma 2}\} \right] dA_I$$

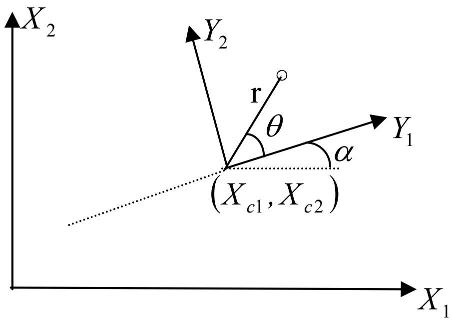
$$= \sum_{J=1}^{N} \left\{ \sum_{I=1}^{N_C} \left[K_{\Sigma 1}^I \langle w_1 \rangle^{IJ} + K_{\Sigma 2}^I \langle w_2 \rangle^{IJ} + \int_{A_I} \langle P^0 \rangle^I [\partial\phi]^J dA_I \right] \{\delta v\}^J \right\}$$

$$
\begin{aligned}
&-\sum_{I=1}^{N_C}\left\{\langle\delta\gamma^0\rangle^I\{P^0\}^I A_I+\langle\delta\gamma^0\rangle^I\int_{A_I}\left[K_{\Sigma1}^I\{H^{\Sigma1}\}+K_{\Sigma2}^I\{H^{\Sigma2}\}\right]dA_I\right\}\\
&-\sum_{I=1}^{N_C}\left\{\langle\delta K_\Sigma\rangle^I\int_{A_I}\{e^0\}^I dA_I+\int_{A_I}\left[\langle K_\Sigma\rangle^I[V]^I\{\delta K_\Sigma\}^I\right]dA_I\right\}\\
&=\sum_{J=1}^{N}\left\{\sum_{I=1}^{N_C}\left\{\langle K_\Sigma\rangle^I[W]^{IJ,T}[R]^I+\int_{A_I}\langle P^0\rangle^I[\partial\phi]^J[R]^I dA_I\right\}\{\delta u\}^J\right\}\\
&-\sum_{I=1}^{N_C}\left\{\langle\delta\gamma^0\rangle^I\{P^0\}^I A_I+\langle\delta\gamma^0\rangle^I\int_{A_I}\left[K_{\Sigma1}^I\{H^{\Sigma1}\}+K_{\Sigma2}^I\{H^{\Sigma2}\}\right]dA_I\right\}\\
&-\sum_{I=1}^{N_C}\left\{\langle\delta K_\Sigma\rangle^I\int_{A_I}\{e^0\}^I dA_I+\int_{A_I}\left[\langle K_\Sigma\rangle^I[V]^I\{\delta K_\Sigma\}^I\right]dA_I\right\}
\end{aligned}
$$

(A2.15)

with

$\{v\}^J=\begin{Bmatrix}v_1\\v_2\end{Bmatrix}^J=[R]^I\{u\}^J$ the displacements of node J expressed in the local reference frame attached to the LFMVC n° I



$$\{\delta v\}^J=\begin{Bmatrix}\delta v_1\\ \delta v_2\end{Bmatrix}^J=[R]^I\{\delta u\}^J$$

$$[\partial\Phi]^J=\begin{bmatrix}\dfrac{\partial\Phi_J}{\partial Y_1} & 0\\ 0 & \dfrac{\partial\Phi_J}{\partial Y_2}\\ \dfrac{\partial\Phi_J}{\partial Y_2} & \dfrac{\partial\Phi_J}{\partial Y_1}\end{bmatrix};\ [R]^I=\begin{bmatrix}\cos\alpha_I & \sin\alpha_I\\ -\sin\alpha_I & \cos\alpha_I\end{bmatrix}$$

$$\{w_1\}^{IJ}=\int_{A_I}[\partial\Phi]^{J,T}\{H^{\Sigma1}\}dA_I\ ;\qquad \{w_2\}^{IJ}=\int_{A_I}[\partial\Phi]^{J,T}\{H^{\Sigma2}\}dA_I \qquad \text{(A2.16)}$$

$$[W]^{IJ}=\begin{bmatrix}\langle w_1\rangle^{IJ}\\ \langle w_2\rangle^{IJ}\end{bmatrix} \qquad \text{(A2.17)}$$

$$[V]^I=\begin{bmatrix}\int_{A_I}\langle H^{\Sigma1}\rangle[D]^I\{H^{\Sigma1}\}dA_I & \int_{A_I}\langle H^{\Sigma1}\rangle[D]^I\{H^{\Sigma2}\}dA_I\\ \int_{A_I}\langle H^{\Sigma2}\rangle[D]^I\{H^{\Sigma1}\}dA_I & \int_{A_I}\langle H^{\Sigma2}\rangle[D]^I\{H^{\Sigma2}\}dA_I\end{bmatrix} \qquad \text{(A2.18)}$$

$$\{e^0\}^I = \begin{Bmatrix} e_1^0 \\ e_2^0 \end{Bmatrix}^I = \begin{Bmatrix} \int_{A_I} \langle P^0 \rangle^I [D]^I \{H^{\Sigma 1}\} dA_I \\ \int_{A_I} \langle P^0 \rangle^I [D]^I \{H^{\Sigma 2}\} dA_I \end{Bmatrix} = \begin{Bmatrix} \int_{A_I} \langle H^{\Sigma 1} \rangle dA_I \\ \int_{A_I} \langle H^{\Sigma 2} \rangle dA_I \end{Bmatrix} [D]^I \{P^0\}^I \tag{A2.19}$$

For $I = N_C + 1, N$ (OVC's), we have:

$$\sum_{I=N_C+1}^{N} \int_{A_I} \Sigma_{ij} \left[\frac{1}{2} \left(\frac{\partial \delta u_i}{\partial X_j} + \frac{\partial \delta u_j}{\partial X_i} \right) - \delta\varepsilon_{ij} \right] dA_I$$

$$= \sum_{I=N_C+1}^{N} \Sigma_{ij}^I \oint_{C_I} \left[\frac{1}{2} (N_j^I \delta u_i + N_i^I \delta u_j) \right] dC_I - \sum_{I=N_C+1}^{N} \Sigma_{ij}^I \delta\varepsilon_{ij}^I A_I \tag{A2.20}$$

Taking account of $\delta u_i = \sum_{J=1}^{N} \Phi_J \delta u_i^J$ as well as of the symmetry of the Σ_{ij}, this becomes :

$$\sum_{I=N_C+1}^{N} \int_{A_I} \Sigma_{ij} \left[\frac{1}{2} \left(\frac{\partial \delta u_i}{\partial X_j} + \frac{\partial \delta u_j}{\partial X_i} \right) - \delta\varepsilon_{ij} \right] dA_I$$

$$= \sum_{I=N_C+1}^{N} \Sigma_{ij}^I \left[\sum_{J=1}^{N} A_i^{IJ} \delta u_j^J \right] - \sum_{I=N_C+1}^{N} \Sigma_{ij}^I \delta\varepsilon_{ij}^I A_I = \sum_{I=N_C+1}^{N} \langle \Sigma \rangle^I \left[\sum_{J=1}^{N} [A]^{IJ,T} \{\delta u\}^J \right] - \sum_{I=N_C+1}^{N} \langle \Sigma \rangle^I \{\delta\varepsilon\}^I A_I$$

(A2.21)

with

$$\{\delta u\}^J = \begin{Bmatrix} \delta u_1 \\ \delta u_2 \end{Bmatrix}^J \quad ; \quad \{\delta\varepsilon\}^I = \begin{Bmatrix} \delta\varepsilon_{11} \\ \delta\varepsilon_{22} \\ 2\delta\varepsilon_{12} \end{Bmatrix}^I \quad ; \quad \{\Sigma\}^I = \begin{Bmatrix} \Sigma_{11} \\ \Sigma_{22} \\ \Sigma_{12} \end{Bmatrix}^I$$

Development of $\delta\Pi_3$

For $I = 1, N_C$ (LFMVC's), we have:

$$\sum_{I=1}^{N_C} \left\{ \int_{A_I} \delta P_{ij}^I \left[\frac{1}{2} \left(\frac{\partial v_i}{\partial Y_j} + \frac{\partial v_j}{\partial Y_i} \right) - \gamma_{ij}^I \right] dA_I \right\}$$

$$= \sum_{J=1}^{N} \left\{ \sum_{I=1}^{N_C} \left\{ \langle \delta K_\Sigma \rangle^I [W]^{IJ,T} [R]^I \{u\}^J + \int_{A_I} \langle \delta P^0 \rangle^I [\partial\phi]^J [R]^I \{u\}^J dA_I \right\} \right\}$$

$$- \sum_{I=1}^{N_C} \left\{ \langle \delta P^0 \rangle^I \{\gamma^0\}^I A_I + \langle \delta P^0 \rangle^I \int_{A_I} \left[K_{\Sigma 1}^I [D]^I \{H^{\Sigma 1}\} + K_{\Sigma 2}^I [D]^I \{H^{\Sigma 2}\} \right] dA_I \right\}$$

$$- \sum_{I=1}^{N_C} \left\{ \langle \delta K_\Sigma \rangle^I \{e^\gamma\}^I + \int_{A_I} \left[\langle \delta K_\Sigma \rangle^I [V]^I \{K_\Sigma\}^I \right] dA_I \right\} \tag{A2.22}$$

with

$$\left\{e^{\gamma}\right\}^{I}=\begin{Bmatrix} e_1^{\gamma} \\ e_2^{\gamma} \end{Bmatrix}^{I}=\begin{Bmatrix} \int_{A_I}\langle H_1\rangle\left\{\gamma^0\right\}dA_I \\ \int_{A_I}\langle H_2\rangle\left\{\gamma^0\right\}dA_I \end{Bmatrix}=\left[IH\right]^{I}\left\{\gamma^0\right\}^{I} \tag{A2.23}$$

For $I = N_C + 1, N$ (OVC's), we have:

$$\sum_{I=N_c+1}^{N}\int_{A_I}\delta\Sigma_{ij}\left[\frac{1}{2}(\frac{\partial u_i}{\partial X_j}+\frac{\partial u_j}{\partial X_i})-\varepsilon_{ij}\right]dA_I=\sum_{I=N_c+1}^{N}\delta\Sigma_{ij}^{I}\left[\sum_{J=1}^{N}A_i^{IJ}u_j^J\right]-\sum_{I=N_c+1}^{N}\delta\Sigma_{ij}^{I}\varepsilon_{ij}^{I}A_I$$

$$=\sum_{I=N_C+1}^{N}\langle\delta\Sigma\rangle^{I}\left[\sum_{J=1}^{N}[A]^{IJ,T}\{u\}^{J}\right]-\sum_{I=N_C+1}^{N}\langle\delta\Sigma\rangle^{I}\{\varepsilon\}^{I}A_I \tag{A2.24}$$

with

$$\{\delta\Sigma\}^{I}=\begin{Bmatrix}\delta\Sigma_{11}\\ \delta\Sigma_{22}\\ \delta\Sigma_{12}\end{Bmatrix}^{I}$$

Development of $\delta\Pi_4$

$$\delta\Pi_4=-\sum_{I=1}^{N}\int_{A_I}F_i\,\delta u_i\,dA_I=-\sum_{I=1}^{N}\sum_{J=1}^{N}\delta u_i^J\int_{A_I}F_i\,\Phi_J\,dA_I=-\sum_{J=1}^{N}\langle\delta u\rangle^{J}\left\{\widetilde{F}\right\}^{J} \tag{A2.25}$$

Development of $\delta\Pi_5$

$$\delta\Pi_5=-\oint_{S_t}T_i\,\delta u_i\,dS=-\sum_{K=1}^{M_t}\sum_{J=1}^{N}\delta u_i^J\int_{S_K}T_i\,\Phi_J\,dS_K=-\sum_{J=1}^{N}\langle\delta u\rangle^{J}\left\{\widetilde{T}\right\}^{J} \tag{A2.26}$$

Development of $\delta\Pi_6$

$$\delta\Pi_6=\sum_{K=1}^{M_u}\left[\delta t_i^K\int_{S_K}(\widetilde{u}_i-u_i)dS_K-t_i^K\int_{S_K}\delta u_i\,dS_K\right]$$

We logically assume that none of the edges of the LFMVC's is subjected to imposed displacements. So, they do not belong to S_u.

Consequently, the result of the calculation of $\delta\Pi_6$ is the same as in [5]:

$$\delta\Pi_6=\sum_{K=1}^{M_u}\delta t_i^K\left\{\widetilde{U}_i^K-\sum_{J=1}^{N}u_i^J B^{KJ}\right\}-\sum_{J=1}^{N}\left\langle\delta u^J\right\rangle\left\{\sum_{K=1}^{M_u}B^{KJ}\{t\}^K\right\}$$

$$=\sum_{K=1}^{M_u}\{\delta t\}^K\left\{\left\{\widetilde{U}\right\}^K-\sum_{J=1}^{N}B^{KJ}\{u\}^J\right\}-\sum_{J=1}^{N}\left\langle\delta u^J\right\rangle\left\{\sum_{K=1}^{M_u}B^{KJ}\{t\}^K\right\} \tag{A2.27}$$

A2.2. Recasting in matrix form

In matrix form, these results can be summarized as follows:

$$\delta\Pi_1 = \sum_{I=1}^{N_C} \int_{A_I} \langle\tau\rangle^I \{\delta\gamma^O\}^I dA_I + \sum_{I=1}^{N_C} \langle\delta K_\Sigma\rangle^I \{e\}^I + \sum_{I=N_C+1}^{N} A_I \langle\sigma\rangle^I \{\delta\varepsilon\}^I \tag{A2.28}$$

$$\delta\Pi_2 = \sum_{J=1}^{N} \left\{ \sum_{I=1}^{N_C} \left\{ \langle K_\Sigma\rangle^I [W]^{IJ,T} [R]^I + \int_{A_I} \langle P^0\rangle^I [\partial\phi]^J [R]^I dA_I \right\} \{\delta u\}^J \right\} \tag{A2.29}$$

$$\begin{aligned} &- \sum_{I=1}^{N_C} \left\{ \langle\delta\gamma^0\rangle^I \{P^0\}^I A_I + \langle\delta\gamma^0\rangle^I \int_{A_I} \left[K_{\Sigma 1}^I \{H^{\Sigma 1}\} + K_{\Sigma 2}^I \{H^{\Sigma 2}\} \right] dA_I \right\} \\ &- \sum_{I=1}^{N_C} \left\{ \langle\delta K_\Sigma\rangle^I \{e^0\}^I + \int_{A_I} \left[\langle K_\Sigma\rangle^I [V]^I \{\delta K_\Sigma\}^I \right] dA_I \right\} \\ &+ \sum_{I=N_C+1}^{N} \langle\Sigma\rangle^I \left[\sum_{J=1}^{N} [A]^{IJ,T} \{\delta u\}^J \right] - \sum_{I=N_C+1}^{N} \langle\Sigma\rangle^I \{\delta\varepsilon\}^I A_I \end{aligned} \tag{A2.30}$$

$$\delta\Pi_3 = \sum_{J=1}^{N} \left\{ \sum_{I=1}^{N_C} \left\{ \langle\delta K_\Sigma\rangle^I [W]^{IJ,T} [R]^I \{u\}^J + \int_{A_I} \langle\delta P^0\rangle^I [\partial\phi]^J [R]^I \{u\}^J dA_I \right\} \right\}$$

$$\begin{aligned} &- \sum_{I=1}^{N_C} \left\{ \langle\delta P^0\rangle^I \{\gamma^0\}^I A_I + \langle\delta P^0\rangle^I \int_{A_I} \left[K_{\Sigma 1}^I [D]^I \{H^{\Sigma 1}\} + K_{\Sigma 2}^I [D]^I \{H^{\Sigma 2}\} \right] dA_I \right\} \\ &- \sum_{I=1}^{N_C} \left\{ \langle\delta K_\Sigma\rangle^I \{e^\gamma\}^I + \int_{A_I} \left[\langle\delta K_\Sigma\rangle^I [V]^I \{K_\Sigma\}^I \right] dA_I \right\} \\ &+ \sum_{I=N_C+1}^{N} \langle\delta\Sigma\rangle^I \left[\sum_{J=1}^{N} [A]^{IJ,T} \{u\}^J \right] - \sum_{I=N_C+1}^{N} \langle\delta\Sigma\rangle^I \{\varepsilon\}^I A_I \end{aligned} \tag{A2.31}$$

$$\delta\Pi_4 = -\sum_{J=1}^{N} \langle\delta u\rangle^J \{\tilde{F}\}^J \tag{A2.32}$$

$$\delta\Pi_5 = -\sum_{J=1}^{N} \langle\delta u\rangle^J \{\tilde{T}\}^J \tag{A2.33}$$

$$\delta\Pi_6 = \sum_{K=1}^{M_u} \{\delta t\}^K \left\{ \{\tilde{U}\}^K - \sum_{J=1}^{N} B^{KJ} \{u\}^J \right\} - \sum_{J=1}^{N} \langle\delta u^J\rangle \left\{ \sum_{K=1}^{M_u} B^{KJ} \{t\}^K \right\} \tag{A2.34}$$

Substitution in $\delta\Pi = 0$ gives:

$$\delta\Pi = \sum_{I=1}^{N_C} \langle\delta K_\Sigma\rangle^I \left\{ \{e\}^I - \{e^0\}^I - \{e^\gamma\}^I - ([V]^I + [V]^{I,T})\{K_\Sigma\}^I + \sum_{J=1}^{N} [W]^{IJ,T} [R]^I \{u\}^J \right\}$$

$$
\begin{aligned}
&+ \sum_{I=N_C+1}^{N} \langle \delta\varepsilon \rangle^I \left\{ A_I \{\sigma\}^I - A_I \{\Sigma\}^I \right\} \\
&+ \sum_{I=N_C+1}^{N} \langle \delta\Sigma \rangle^I \left\{ \sum_{J=1}^{N} [A]^{IJ,T} \{u\}^J - \{\varepsilon\}^I \; A_I \right\} + \sum_{K=1}^{M_u} \{\delta t\}^K \left\{ \{\tilde{U}\}^K - \sum_{J=1}^{N} B^{KJ} \{u\}^J \right\} \\
&+ \sum_{J=1}^{N} \langle \delta u \rangle^J \left\{ \sum_{I=1}^{N_C} \left\{ [R]^{I,T} [W]^{IJ} \{K_\Sigma\}^I + \int_{A_I} [R]^{I,T} [\partial\phi]^J \langle P^0 \rangle^I dA_I \right\} \right. \\
&\left. + \sum_{I=N_C+1}^{N} [A]^{IJ} \{\Sigma\}^I - \{\tilde{F}\}^J - \{\tilde{T}\}^J - \sum_{K=1}^{M_u} B^{KJ} \{t\}^K \right\} \\
&+ \sum_{I=1}^{N_C} \langle \delta\gamma^0 \rangle^I \left\{ \int_{A_I} \left[\{\tau\}^I - \{P^0\}^I - K_{\Sigma 1}^I \{H^{\Sigma 1}\} - K_{\Sigma 2}^I \{H^{\Sigma 1}\} \right] dA_I \right\} \\
&+ \sum_{I=1}^{N_C} \langle \delta P^0 \rangle^I \left\{ \sum_{J=1}^{N} \int_{A_I} [\partial\phi]^J [R]^I \{u\}^J dA_I - A_I \{\gamma^0\}^I - \int_{A_I} \left[K_{\Sigma 1}^I [D]^I \{H^{\Sigma 1}\} + K_{\Sigma 2}^I [D]^I \{H^{\Sigma 1}\} \right] dA_I \right\} \\
&= 0 \qquad \text{(A2.35)}
\end{aligned}
$$

with $[V]^I + [V]^{I,T} = 2\,[V]^I$ since $[V]^I$ is symmetric.

From (A2.35), all the equations (V.27) to (V.33) in chapter V can be deduced.