LIMITS ON THE TOP QUARK MASS
FROM \(N(W \rightarrow e\nu)/N(Z \rightarrow e^+e^-)\) AND \(B_0^\pm B_0^\mp\) MIXING.*

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Abstract

With a new UA1 lower limit of 41 GeV on the top quark mass \(m_t\), one might think that the best available bounds come from \(B_0^\pm B_0^\mp\) mixing. We show however that most analyses have been too optimistic and that the best lower limit (at the 90% C.L.) that one can extract from the ARGUS measurement is of the order of 30 GeV. We also discuss the status of the upper limit on \(m_t\) that one can derive from the p\(\bar{p}\) collider measurement of the ratio \(R = N(W \rightarrow e\nu)/N(Z \rightarrow e^+e^-)\) and combine the two to get \(25 \text{ GeV} \leq m_t \leq m_W\) at the 95% C.L.

I. Upper Limit on $m_t$ from the $R$ Ratio:

The $R$ ratio, experimentally defined by $R = N(W \rightarrow e\nu)/N(Z \rightarrow e^+e^-)$, is theoretically given by the following expression:

$$R = \left[ \frac{\Gamma(W \rightarrow e\nu)/\Gamma_W}{\Gamma(Z \rightarrow e^+e^-)/\Gamma_Z} \right] \times \left[ \frac{\sigma_W}{\sigma_Z} \equiv R_\sigma \right]$$  \hspace{1cm} (1)

The first factor, giving the ratio of the branching fractions, is entirely determined within the minimal standard model. Via the $W$ and $Z$ total widths $\Gamma_W$ and $\Gamma_Z$, it depends on the number of light neutrinos (which we set equal to 3 in the following) and on $m_t$. The second factor, representing the ratio of the $W$ and $Z$ production cross sections, contains all the theoretical uncertainties. At the parton level, $R_\sigma$ is well determined: the Feynman diagrams entering the calculation are identical to order $\alpha_s^2$, thus guaranteeing the absence of big corrections in the ratio, such as K factors. The order $\alpha_s$ corrections have been estimated\(^1\) to be at most 3% and have been included in the present analysis. The main uncertainty in the calculation comes from the embedding of the parton process into the hadronic structure. The first attempts\(^2\) used standard sets of structure functions. Figure 1 shows that such a procedure produces a bound $m_t \leq m_W$ at the 1.5 $\sigma$ level. It was then noticed\(^3\) that $R_\sigma$ depended effectively on the ratio $u/d$ ($x = m_W/\sqrt{s}$) only and that this ratio had a negligible $Q^2$ dependence. One can thus directly input experimental measurements of this ratio to evaluate $R_\sigma$. A compilation of the estimates from most of the available data sets is given in Fig. 2 and leads to the average:

$$< R_\sigma > = 3.42 \pm 0.01 \pm 0.02 \pm 0.04 \pm \epsilon$$  \hspace{1cm} (2)

The first uncertainty comes from averaging the results of Fig. 2, the other uncertainties originate respectively from the errors on $m_W$, $sin(\theta_W)$, and from the charm contribution to the process. As we are considering a variety of independent data sets, we have assumed that it is safe to neglect the systematic uncertainties. Comparing the resulting $R$ value with the combined measurements of UA1 and UA2 leads to an upper bound $m_t \leq m_W$ at the 96% C.L. In this approach, each individual data set has little weight of its own, so that the inclusion of the new BCDMS data\(^4\) in our analysis modifies $< R_\sigma >$ by less than 1%.

II. Lower Limit on $m_t$ from $B^0_d\bar{B}^0_d$ Mixing:

The ARGUS collaboration measurement of $B^0_d\bar{B}^0_d$ mixing\(^5\), combined with well-known box-diagram calculations\(^6\), gives:

$$\frac{|m_{B^0_d} - m_{\bar{B}^0_d}|}{\Gamma_B} \equiv |\delta m| = 0.73 \pm 0.18 = \left[ G_F \left| U_{td} \right|^2 \right] \left[ \left| f_B \right| m_B/\Gamma_B \right] \left| I_{\text{loop}} \eta_{\text{QCD}} \right| \left| \beta_B \right|.$$  \hspace{1cm} (3)

The main theoretical uncertainties are as follows: the first factor contains the K.M: matrix element $|U_{td}|^2$ which can be accessed only via the measurements of the other K.M: matrix elements and the imposition of unitarity; the second factor introduces the $B$ decay constant $f_B$ (coming from the vacuum insertion) and the $B$ bag parameter $B_B$ (from the insertion of higher order intermediate states), both of which can only be approximately estimated via lattice QCD, QCD sum rules or large N expansions. Furthermore, the other factors (i.e. the loop integral $I_{\text{loop}} \approx m_t^2$ and the QCD radiative corrections $\eta_{\text{QCD}}$) depend on quark masses and $\Delta_{\text{QCD}}$. To establish a real bound on $m_t$, one needs to let all the parameters of the problem vary in a reasonable range and to fit all the data constraining $|U_{td}|$. The present
analysis\(^7\) thus simultaneously fits i) the K.M matrix elements \(|U_{ud}|, |U_{uc}|, |U_{us}|, |U_{ub}|; ii) the \(c\) parameter of the \(K^0\)\(\bar{K}^0\) system; iii) the B lifetime \(\tau_B\); iv) \(|\delta m|/\tau_B\). These quantities depend on the following parameters, which we vary: i) the K.M. angles and phase; ii) the quark masses \(m_c\) and \(m_b\); iii) the bag parameters \(B_K\) and \(B_B\), as well as the decay constant \(f_B\); iv) the QCD parameter \(\Delta_{QCD}\). We further assume that \(|U_{ub}|/|U_{cb}| \leq 0.2\) and use a QCD model for the B semileptonic width, from which we extract \(\tau_B\) using the measured semileptonic branching fraction. Using a procedure close to the maximum likelihood method, we fit all the above data at the 1.5 \(\sigma\) level for \(m_t \geq 25\) GeV (if \(f_B^2 B_B \leq 0.04\) GeV\(^2\)) and for \(m_t \geq 30\) GeV (if \(f_B^2 B_B \leq 0.03\) GeV\(^2\)).

All the parameters are fitted to conventional values, except the matrix element \(|U_{cb}|\) and the ratio \(r_u \equiv |U_{ub}|/|U_{cb}|\) which respectively take the values 0.07 and 0.2 for \(m_t \approx 30\) GeV. This might seem at first sight to contradict the CLEO analysis\(^8\) which gives bounds \(|U_{cb}| \leq 0.05\) and \(r_u \leq 0.09\) for QCD-based models. We thus reconsider their analysis in the explicit case of the Altarelli model\(^9\) of B semileptonic decay, which we extend somewhat. The original model differed from previous ones by the inclusion of first-order QCD corrections and more importantly by the elimination of the \(m_b^2\) dependence of the rate. To achieve this, it assumed that the produced light spectator quark had a momentum distribution \(\Phi(p)\) (taken to be gaussian) and that the b quark was off-shell by an amount determined by exact kinematics. We keep exactly the same assumptions, except that we allow the distribution \(\Phi(p)\) to be undetermined. We show in Fig. 3 the spectra resulting from rectangular spikes in \(p^2 \Phi(p)\) of width 100 MeV centred around \(p_0\). We clearly fit both the semi-leptonic width and the endpoint spectrum if we assume a momentum distribution peaked around 0.7 GeV. This would mean that the b quark would have an effective mass close to 4.6 GeV which is the best value of \(m_t\) determined by our simultaneous fit. Finally, the bounds that one got from considering the \(D^*\) polarization in \(B \to D^*\ell\nu\) exclusive decay have to be reconsidered in view of the new ARGUS analysis\(^10\), which contradicts the original CLEO results and favours QCD-based models for B decay.

III. Conclusion:

Although large theoretical uncertainties affect the bounds one can get on \(m_t\), we conclude that from \(\mathcal{R}\) and \(|\delta m|/\tau_B\), one can extract: 30 GeV \(\leq m_t \leq 63\) GeV (90\% C.L.) and 25 GeV \(\leq m_t \leq m_W\) (95\% C.L.).

References:
Fig. 1: Dependence of $R$ on $m_t$ assuming 3 generations of quarks and leptons. The calculation\textsuperscript{2}, using standard sets of structure functions, is compared with the measured value obtained by combining UA1 and UA2 results.

Fig. 2: Values of $R_\sigma$ are calculated\textsuperscript{3}) by extracting the ratio $u/d$ from a series of experiments. We assumed $m_W = 80.1$ GeV and fixed $m_2$ from $\sin^2 \theta_W = 0.232$. After including the errors on these quantities we obtain $< R_\sigma > = 3.42 \pm 0.05$.

Fig. 3: The values of $|\delta m|/\Gamma_B$ resulting from our fit are compared with the ARGUS measurement for $f_B^2 B_B \leq 0.03$ (1), 0.04 (2) and 0.06 (3) GeV$^2$.

Fig. 4: Simultaneous fit to the endpoint spectrum of B semileptonic decay\textsuperscript{8}) (A) and to the B semileptonic width (B) for $|U_{ub}| = 0.07$, $|U_{ub}| = 0.014$, $m_b = 4.6$ GeV, $m_c = 1.8$ GeV and $0 \leq m_3 \leq 300$ MeV. $p_0$ is the momentum of the light spectator quark (see text).