Generating Dynamical Decoupling Sequences for Qudits using Parallel Tempering

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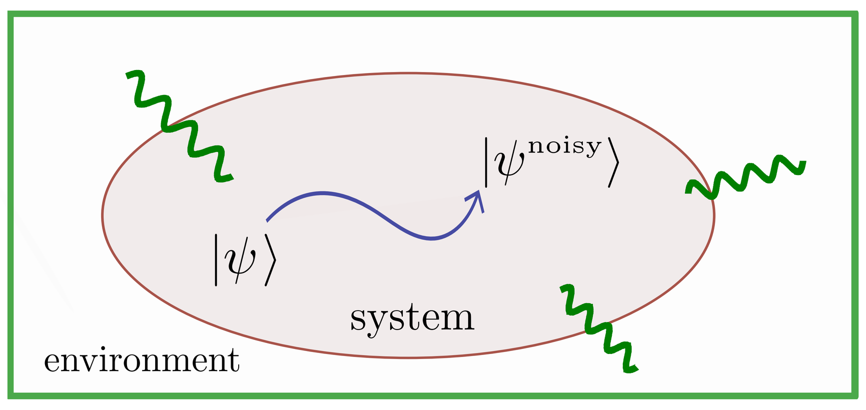
Motivation

- Current quantum technologies are limited by their noisy hardware.
- Dynamical decoupling can help mitigating decoherence and extending the lifetime of useful quantum states. Constructing optimized decoupling sequences is thus of great importance.
- While linear programming can be used for first-order decoupling [1], heuristic algorithms are needed to find higher-order, optimized sequences [2].
- Here, we consider DD sequences as optimal paths on a graph (as in Ref. [1]) and generate sequences using a search algorithm.

Dynamical decoupling

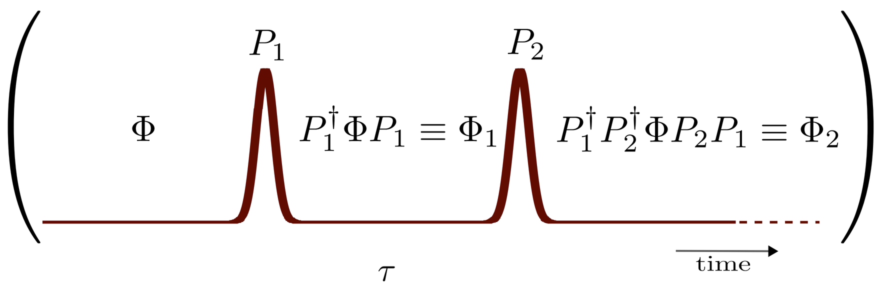
Consider a Hamiltonian H_{err} responsible for decoherence.



$|\psi^{\text{noisy}}(t)\rangle = e^{-i\Phi}(|\psi(t_0)\rangle \otimes |e\rangle)$

error phase
$\Phi = (t - t_0)H_{\text{err}}$

By applying a sequence of N equidistant pulses, i.e. a *dynamical decoupling* (DD) sequence, we can design the error phase on each time step. An "average" error phase can then be calculated using a Magnus expansion.



in average →

$\Phi_{\text{DD}} = \sum_{n=1}^{\infty} \Phi_{\text{DD}}^{[n]}$ with
$\Phi^{[1]} = \sum_k \Phi_k$
$\Phi^{[2]} = \frac{-i}{2}\sum_k \sum_{k'<k} [\Phi_k, \Phi_{k'}]$
$\vdots$

nth-order error phase $\|\Phi_{\text{DD}}^{[n]}\| \propto (\tau \|H_{\text{err}}\|)^n$

If decoherence is small enough ($\tau_c \|H_{\text{err}}\| < 1$), we can design a nth-order DD sequence by choosing the pulses such that $\Phi_{\text{DD}}^{[r]} = 0$ ($r \leq n$), leaving an error scaling as $\|\Phi_{\text{DD}}\| \propto (\tau_c \|H_{\text{err}}\|)^{n+1}$.

Generating DD sequences numerically

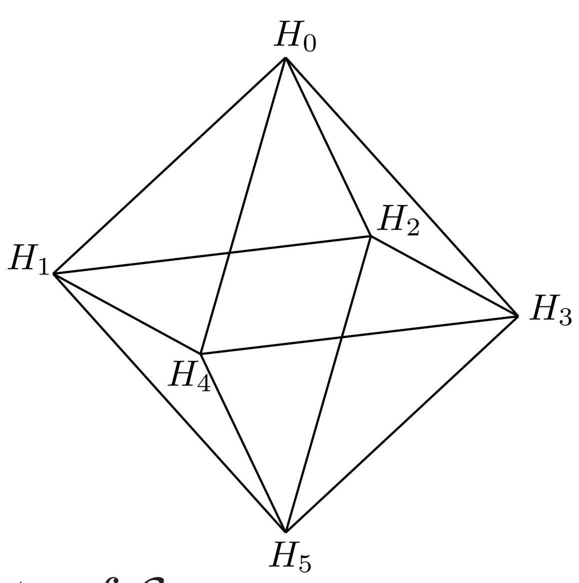
Objective : designing a pulse sequence that mitigates decoherence issued from some error Hamiltonian H_{err}.

General method :

step 1. Specify H_{err} and a set of accessible pulses $\mathcal{P} = \{P_k\}$.

step 2. Find all accessible error phases,

$$\mathcal{E} = \left\{ H_k \middle| \exists U_k = \prod_q P_q : U_k^\dagger H_{\text{err}} U_k = H_k \right\}.$$



step 3. Construct a graph:

- Vertices → elements of $\mathcal{E}$.
- Edges → pulses that connect two elements of $\mathcal{E}$.

Remark. The existence of a pulse of $\mathcal{P}$ that connects H_k to H_{k+1} should not depend on the unitary that generates H_k. A sufficient condition on the pulse set is that $\forall P, Q \in \mathcal{P}$, we have $P^\dagger \in \mathcal{P}$ and $P^\dagger Q P \in \mathcal{P}$.

step 4. Find the optimal cyclic path:

- Algorithm → *parallel tempering* (PT) [3].
- Cost function → environment-invariant Hilbert-Schmidt norm distance D [4]: measures the distance between the system+environment propagator and a target system-only propagator (here the identity).

Remark. If nth-order decoupling is achieved, the distance scales as $D \propto (\tau_c \|H_{\text{err}}\|)^{n+1}$ with τ_c the duration of the sequence.

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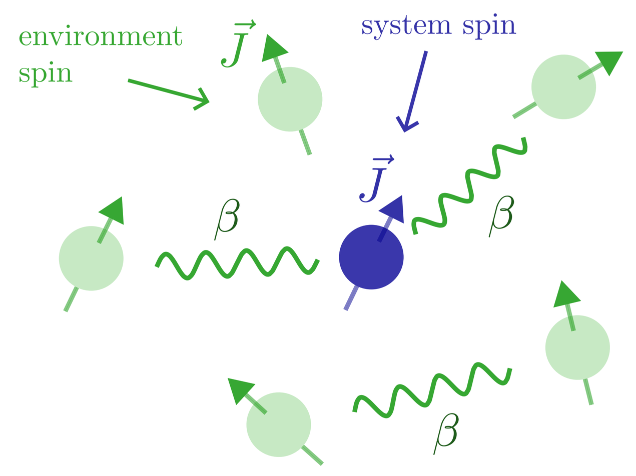
Benchmarking : an ensemble of interacting spin-1



Error Hamiltonian.

$$H_{\text{err}} = \beta \sum_{i,j} \left(3J_z^i J_z^j - \vec{J}^i \cdot \vec{J}^j\right) + \Gamma \sum_i \left(J_z^i + \epsilon \hat{n} \cdot \vec{J}^i\right)$$

strength of the dipole-dipole interaction; strength of unwanted rotations around z; additional rotations around a random axis

Set of accessible pulses.

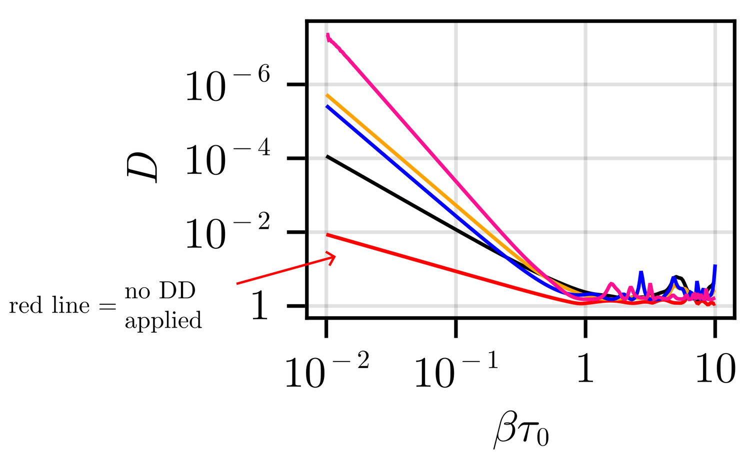
$\mathcal{P} = \{\mathbb{1}, \pm\frac{\pi_x}{2}, \pm\frac{\pi_y}{2}\}$

- $\pi/2$ rotations of $\vec{J}$ around the x and y axis
- Gobal pulses on the whole ensemble

Pulse sequences found using PT.

decoupling order	$\Gamma = 0$, $\epsilon = 0$	$\Gamma \neq 0$, $\epsilon = 0$	$\Gamma \neq 0$, $\epsilon \neq 0$
1	$\mathbf{PT}_3 = \frac{\pi_x}{2}\frac{\pi_y}{2}\frac{\pi_x}{2}$	$\mathbf{PT}_{6a}$ $\mathbf{PT}_{6b} = \left[\frac{\pi_x}{2}\frac{\pi_y}{2}\right]^3$	$\mathbf{PT}_{12c}$
2	$\mathbf{PT}_{6a} = \left[\frac{\pi_x}{2}\frac{\pi_x}{2}\frac{\pi_y}{2}\right]^2$ $\mathbf{PT}_{6c} = (\text{PT}_3)(\text{PT}_3)^\dagger$	$\mathbf{PT}_{12a} = (\text{PT}_{6a})(\text{PT}_{6a})^\dagger$ $\mathbf{PT}_{12b} = (\text{PT}_{6b})(\text{PT}_{6b})^\dagger$ $\mathbf{PT}_{12c} = \left[\frac{\pi_y}{2}\frac{\pi_x}{2}\frac{\pi_y}{2}\frac{\pi_y}{2}\frac{\pi_x}{2}\frac{\pi_y}{2}\right]^2$	
3	$\mathbf{PT}_{18a}$ $\mathbf{PT}_{18b}$		

verification of the decouling order; τ_0 = interval between pulses



scaling of the error ?

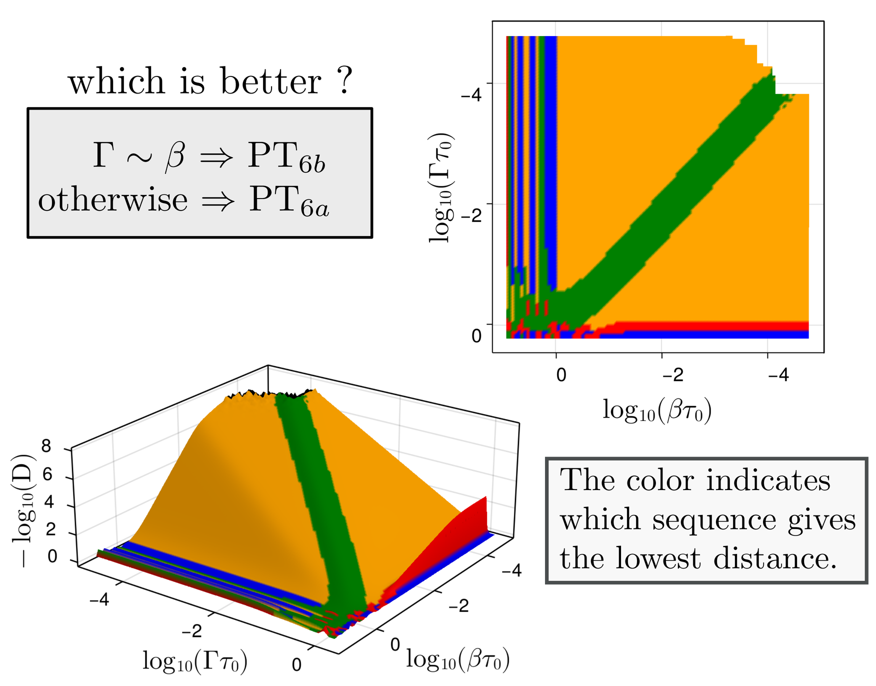
	$(\beta\tau_0 \ll 1)$ scaling of D	
no DD	$D \propto (\beta\tau_0)$	
PT_3	$D \propto (\beta\tau_0)^2$	→ 1st-order decoupled
$\text{PT}_{6a,c}$	$D \propto (\beta\tau_0)^3$	→ 2nd-order decoupled
$\text{PT}_{18a,b}$	$D \propto (\beta\tau_0)^4$	→ 3rd-order decoupled

The slope indicates the decoupling order.

Performance comparison in the $(\Gamma\tau_0, \beta\tau_0)$ parameter space, with $\epsilon = 0$.

6-pulses sequence PT_6

which is better ?
$\Gamma \sim \beta \Rightarrow \text{PT}_{6b}$
otherwise $\Rightarrow \text{PT}_{6a}$

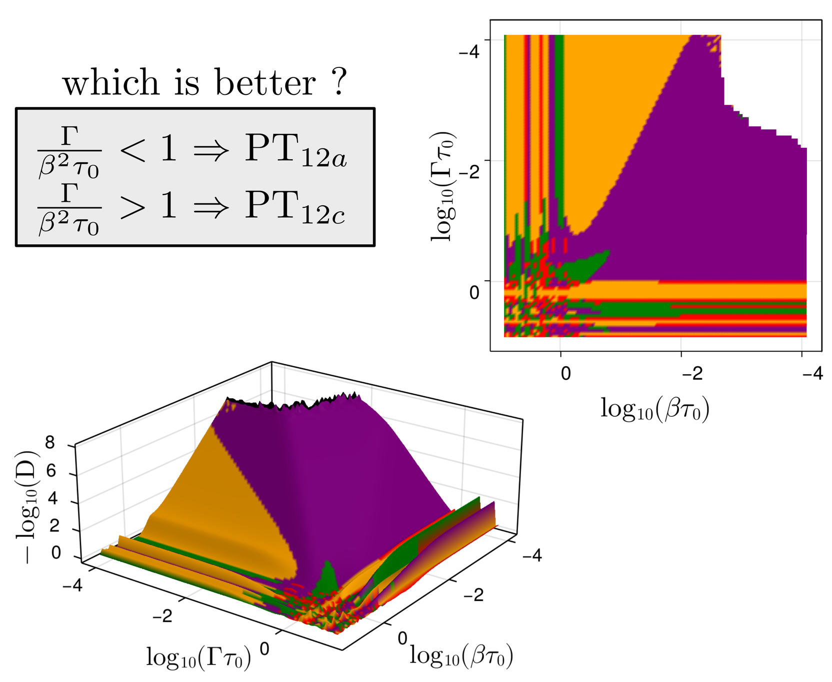


The color indicates which sequence gives the lowest distance.

	$\beta \to 0$	$\Gamma \to 0$
PT_{6a}	$D \propto (\Gamma\tau_0)^2$	$D \propto (\beta\tau_0)^3$
PT_{6b}	$D \propto (\Gamma\tau_0)^2$	$D \propto (\beta\tau_0)^2$
PT_{6c}	$D \propto \Gamma\tau_0$	$D \propto (\beta\tau_0)^3$

12-pulses sequence PT_{12}

which is better ?
$\frac{\Gamma}{\beta^2\tau_0} < 1 \Rightarrow \text{PT}_{12a}$
$\frac{\Gamma}{\beta^2\tau_0} > 1 \Rightarrow \text{PT}_{12c}$



	$\beta \to 0$	$\Gamma \to 0$
$\text{PT}_{12a,b}$	$D \propto (\Gamma\tau_0)^3$	$D \propto (\beta\tau_0)^3$
PT_{12c}	$D \propto (\Gamma\tau_0)^4$	$D \propto (\beta\tau_0)^3$

Conclusion and Outlook

We presented a simple method for generating DD sequences that suppress the first few orders of the MS for an arbitrary Hamiltonian and used it to construct short sequences of different decoupling properties for a spin-1.

Outlook :

- Find patterns and building blocks for the construction of higher-order sequences for the suppression of dipole-dipole interactions.
- Add robustness requirements in the numerical search.

References

[1] H. Zhou, H. Gao, N.T. Leitao, O. Makarova, I. Cong, A.M. Douglas, L.S. Martin and M.D. Lukin, arXiv:2305.09757
[2] G. Quiroz and D.A. Lidar, Phys. Rev. A **88**, 052306 (2013)
[3] M. Sambridge, Geophys. J. Int. **196**, 357 (2013)
[4] M.D. Grace *et al*, New J. Phys. **12**, 015001 (2010)