

# Consequences of $t$ -channel unitarity for the interaction of real and virtual photons at high energies

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**Abstract.** We analyze the consequences of  $t$ -channel unitarity for photon cross sections and show what assumptions are necessary to allow for the existence of new singularities at  $Q^2 = 0$  for the  $\gamma p$  and  $\gamma\gamma$  total cross sections. For virtual photons, such singularities can in general be present, but we show that, apart from the perturbative singularity associated with  $\gamma^*\gamma^* \rightarrow q\bar{q}$ , no new ingredient is needed to reproduce the data from LEP and HERA, in the Regge region.

## 1. Introduction

It is well known [1] that due to unitarity one can relate the amplitudes describing three hadronic elastic processes  $aa \rightarrow aa, ab \rightarrow ab, bb \rightarrow bb$ . Namely, if a simple Regge pole at  $j = \alpha(t)$  contributes in the  $t$ -channel for each of the above-mentioned processes, the residues of the poles are factorized

$$\beta_{aa \rightarrow aa}(t)\beta_{bb \rightarrow bb}(t) = (\beta_{ab \rightarrow ab}(t))^2.$$

However it is difficult to check directly such a relation. Firstly, there are no experimental data for all three processes (for example,  $\pi\pi$  is missing, if one considers  $\pi\pi, \pi p$  and  $pp$  scattering). Secondly, the best fit to the hadronic cross section data is achieved in the models with multiple Regge poles rather than with simple ones [2]. Factorization properties of multiple poles are to be determined.

On the other hand, the DIS and total cross-section data [3] as well as the measurements of the  $\gamma\gamma$  total cross section and of the off-shell photon structure function  $F_2^\gamma$  [4] are available now. If factorization is valid in the case of the photon amplitudes then it can be checked for another set of related processes:  $pp, \gamma p, \gamma\gamma$  and, probably, for  $pp, \gamma^*p, \gamma^*\gamma^*$ .

In this talk, we show how to derive the generalized factorization for the partial amplitudes of the related processes at an arbitrary, but common for these amplitudes,  $t$ -channel Regge singularity. We also give arguments in favor of its validity in the photon case and apply the new factorization relations to describe  $\gamma\gamma$  and  $\gamma^*\gamma^*$  cross sections.

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## 2. $t$ -channel unitarity

It is an old result that one can relate the amplitudes describing three elastic processes  $aa \rightarrow aa$ ,  $ab \rightarrow ab$ ,  $bb \rightarrow bb$ . The trick is to continue these to the crossed channels  $a\bar{a} \rightarrow a\bar{a}$ ,  $a\bar{a} \rightarrow b\bar{b}$ ,  $b\bar{b} \rightarrow b\bar{b}$ , where they exhibit discontinuities because of the  $a$  and  $b$  thresholds. One then obtains a nonlinear system of equations, which can be solved. Working in the complex  $j$  plane above thresholds ( $t > 4m_a^2, 4m_b^2$ ), and defining the matrix

$$T_0 = \begin{pmatrix} A_{aa \rightarrow aa}(j, t) & A_{ba \rightarrow ba}(j, t) \\ A_{ab \rightarrow ab}(j, t) & A_{bb \rightarrow bb}(j, t) \end{pmatrix} \quad (1)$$

one obtains

$$T_0 = \frac{D}{\mathbb{I} - RD} \quad (2)$$

with  $R_{km} = 2i\sqrt{\frac{t-4m_k^2}{t}}\delta_{km}$  for the case of two thresholds and  $D = T_0^\dagger$ . The latter is made of the amplitudes on the other side of the cut. For any  $D$ , equation (2) is enough to derive factorization: the singularities of  $T_0$  can only come from the zeroes of

$$\Delta = \det(1 - RD). \quad (3)$$

Taking the determinant of both sides of eq. (2), we obtain in the vicinity of  $\Delta = 0$

$$A_{aa \rightarrow aa}(j, t)A_{bb \rightarrow bb}(j, t) - A_{ab \rightarrow ab}(j, t)A_{ba \rightarrow ba}(j, t) = \frac{C}{\Delta}, \quad (4)$$

where  $C$  is regular at the zeroes of  $\Delta$ . As the l.h.s. is of order  $1/\Delta^2$  we obtain the well-known factorization properties from eqs. (2) and (4):

- The elastic hadronic amplitudes have common singularities;
- At each singularity in the complex  $j$  plane, these amplitudes factorise.

For isolated simple poles one obtains the usual well-known factorization relations for the residues. However, it is appropriate to mention here that the relation

$$\lim_{j \rightarrow \alpha(t)} \left[ A_{aa \rightarrow aa}(j, t) - \frac{A_{ab \rightarrow ab}(j, t)A_{ba \rightarrow ba}(j, t)}{A_{bb \rightarrow bb}(j, t)} \right] = \text{finite terms}, \quad (5)$$

where  $\alpha(t)$  is the position of a zero of  $\Delta$ , is valid not only for a simple pole but also for any common  $j$ -singularity in the amplitudes. Moreover, it has a more general form than just a relation between residues. To avoid a misunderstanding we would like to note that ‘‘any  $j$ -singularity’’ means a singularity of the full unitarized amplitude rather than those partial singularities which are produced *e.g.* by  $n$ -pomeron exchange.

These equations are used to extract relations between the residues of the singularities, which can be continued back to the direct channel.

### 2.1. EXTENSION IN THE HADRONIC CASE

We have extended [5] the above argument including all possible thresholds, both elastic and inelastic. The net effect is to keep the structure (2), but with a matrix  $D$  that includes multi-particle thresholds. Furthermore, we have shown that one does not need to continue the amplitudes from one side of the cuts to the other, but that the existence of complex conjugation for the amplitudes is enough to derive (2) and consequently the factorization relations (4,5).

Hence there is no doubt that the factorization of amplitudes in the complex  $j$  plane is correct, even when continued to the direct channel.

If  $A_{pq}(j)$  has coinciding simple and double poles (at any  $t$  or *e.g.* colliding simple poles at  $t = 0$ ),

$$A_{pq} = \frac{S_{pq}}{j-z} + \frac{D_{pq}}{(j-z)^2}, \quad (6)$$

one obtains the new relations

$$\begin{aligned} D_{11}D_{22} &= (D_{12})^2, \\ D_{11}^2S_{22} &= D_{12}(2S_{12}D_{11} - S_{11}D_{12}). \end{aligned} \quad (7)$$

In the case of triple poles

$$A_{pq} = \frac{S_{pq}}{j-z} + \frac{D_{pq}}{(j-z)^2} + \frac{F_{pq}}{(j-z)^3}, \quad (8)$$

the relations become

$$\begin{aligned} F_{11}F_{22} &= (F_{12})^2, \\ F_{11}^2D_{22} &= F_{12}(2D_{12}F_{11} - D_{11}F_{12}), \\ F_{11}^3S_{22} &= F_{11}F_{12}(2S_{12}F_{11} - S_{11}F_{12}) + D_{12}F_{11}(D_{12}F_{11} - 2D_{11}F_{12}) \\ &\quad + D_{11}^2F_{12}^2. \end{aligned} \quad (9)$$

Although historically one has used  $t$ -channel unitarity to derive factorization relations in the case of simple poles, it is now clear [6] that a soft pomeron pole is not sufficient to reproduce the  $\gamma^*p$  data from HERA [3]. However, it is possible, using multiple poles, to account both for the soft cross sections and for the DIS data [7, 8]. We shall see later that relations (7, 9) enable us to account for the DIS photon-photon data from LEP.

## 2.2. THE PHOTON CASE

For photons, due to the fact that an undetermined number of soft photons can be emitted, two theoretical possibilities exist:

i) The photon cross sections are zero for any fixed number of incoming or outgoing photons [9]. In this case, it is impossible to define an S matrix, and one can only use unitarity relations for the hadronic part of the photon wave function. Because of this, photon states do not contribute to the threshold singularities, and the system of equations does not close. The net effect is that the singularity structure of the photon amplitudes is less constrained.  $\gamma p$  and  $\gamma\gamma$  amplitudes must have the same singularities as the hadronic amplitudes, but extra singularities are possible: in the  $\gamma p$  case, these may be of perturbative origin, but must have non perturbative residues. In the  $\gamma\gamma$  case, these singularities have their order doubled. It is also possible for  $\gamma\gamma$  to have purely perturbative additional singularities.

ii) It may be possible to define collective states in QED for which an S matrix would exist [10]. In this case, we obtain the same situation for on-shell photons as for hadrons. However, in the case of DIS, virtual photons come only as external states. Because they are virtual, they do not contribute to the  $t$ -channel discontinuities, and hence the singularity structure for off-shell photons is as described in i).

Let us consider the second possibility in more details and define virtuality of photons as shown in Fig.1.

$$T = \begin{pmatrix} \text{Diagram 1} & \text{Diagram 2} \\ \text{Diagram 3} & \text{Diagram 4} \end{pmatrix}$$

Figure 1. Graphic representation of the matrix T in the case of photons with virtualities  $Q^2$  and  $P^2$ .

In the case of real photons ( $Q^2 = P^2 = 0$ ) we have obtained for the matrix  $T$  ( $a \equiv p, b \equiv \gamma$ ) (see [5]) the same expression as in the hadron case (eq.(2)). It means that there are no extra singularities in the photon amplitudes besides those contributing to  $pp$  amplitude.

In the DIS case ( $Q^2 \neq 0, P^2 = 0$ ) we have

$$T(Q^2, 0)(\mathbb{1} - RD(0, 0)) = D(Q^2, 0), \quad (10)$$

where  $R$  is a diagonal matrix with elements  $R_{11} = 2i\sqrt{(t - 4m_p)^2/t}$  and  $R_{22} = 2i$ ,  $D(Q^2, P^2)$  is expressed through  $T^\dagger(Q^2, P^2)$ . Hence we see that all the on-shell singularities must be present in the off-shell case, but we can have new ones coming from the singularities of  $D(Q^2, 0)$ . These singularities can be of perturbative origin (*e.g.* the singularities generated by the DGLAP evolution) but their coupling will depend on the threshold matrix  $R$ , and hence they must know about hadronic masses, or in other words they are not directly accessible by perturbation theory.

In the case of  $\gamma^*\gamma^*$  scattering, we take  $Q^2 \neq 0$  and  $P^2 \neq 0$ , and obtain [5]

$$T(Q^2, P^2) = D(Q^2, P^2) + \frac{D(Q^2, 0)RD(0, P^2)}{\mathbb{1} - RD(0, 0)}. \quad (11)$$

This shows that the DIS singularities will again be present, either through  $\Delta = \det(\mathbb{1} - RD(0, 0))$ , or through extra singularities present in DIS (in which case their order will be different in  $\gamma\gamma$  scattering, at least for  $Q^2 = P^2$ ).

It is also possible to have extra singularities purely from  $D(Q^2, P^2)$ . *A priori* these could be independent from the threshold matrix, and hence be of purely perturbative origin (*e.g.*  $\gamma^*\gamma^* \rightarrow \bar{q}q$  or the BFKL pomeron coupled to photons through a perturbative impact factor).

We also want to point out that the intercepts of these new singularities can depend on  $Q^2$ , and as the off-shell states do not enter unitarity equations, these singularities can be fixed in  $t$ . However, their residues must vanish as  $Q^2 \rightarrow 0$ .

In the following, we shall explore the possibility that no new singularity is present for on-shell photon amplitudes, and show that it is in fact possible to reproduce present data using pomerons with double or triple poles at  $j = 1$ .

### 3. Application to HERA and LEP

For a given singularity structure, a fit to the  $C = +1$  part of proton cross sections, and to  $\gamma^{(*)}p$  data enables one, via relations (2), to predict the  $\gamma^{(*)}\gamma^{(*)}$  cross sections. Hence we have fitted [5]  $pp$  and  $\bar{p}p$  cross sections and  $\rho$  parameters, as well as DIS data from HERA [3].

The general form of the parametrizations which we used is given, for total cross sections of  $a$  on  $b$ , by the generic formula  $\sigma_{ab}^{tot} = (R_{ab} + H_{ab})$ . The first term, from the highest meson trajectories ( $\rho$ ,  $\omega$ ,  $a$  and  $f$ ), is parametrized via Regge theory as

$$R_{ab} = Y_{ab}^+(\tilde{s})^{\alpha_+ - 1} \pm Y_{ab}^-(\tilde{s})^{\alpha_- - 1} \quad (12)$$

with  $\tilde{s} = 2\nu/(1 \text{ GeV}^2)$ . Here the residues  $Y_+$  factorize. The second term, from the pomeron, is parametrized either as a double pole [7, 11]

$$H_{ab} = D_{ab}(Q^2) \Re \left[ \log \left( 1 + \Lambda_{ab}(Q^2) \tilde{s}^\delta \right) \right] + C_{ab}(Q^2) + (\tilde{s} \rightarrow -\tilde{s}) \quad (13)$$

or as a triple pole [8]

$$H_{ab} = t_{ab}(Q^2) \left[ \log^2 \left( \frac{\tilde{s}}{d_{ab}(Q^2)} \right) + c_{ab}(Q^2) \right]. \quad (14)$$

It may be noted, in the double-pole case, that the parameter  $\delta$  is close to the hard pomeron intercept of [6]. At high  $Q^2$ , because the form factor  $\Lambda$  falls off, the logarithm starts looking like a power of  $\tilde{s}$ , and somehow mimics a simple pole. It may thus be thought of as a unitarized version of the hard pomeron, which would in fact apply to hard and soft scatterings.

In the triple-pole case, this is accomplished by a different mechanism: the scale of the logarithm is a rapidly falling function of  $Q^2$ , and hence the  $\log^2$  term becomes relatively more important at high  $Q^2$ .

### 3.1. RESULTS

The details of the form factors entering (13, 14) can be found in [5]. Such parametrizations give  $\chi^2/dof$  values less than 1.05 in the region  $\cos(\vartheta_t) \geq \frac{49}{2m_p^2}$ ,  $\sqrt{2\nu} \geq 7 \text{ GeV}$ ,  $x \leq 0.3$ ,  $Q^2 \leq 150 \text{ GeV}^2$ . What is really new is that these forms can be extended to photon-photon scattering, using relations (7, 9). The total  $\gamma\gamma$  cross section is well reproduced (see Fig. 2) and the de-convolution using PHOJET is preferred.

The fit to  $F_2$  has quite a good  $\chi^2$  as well. We have checked that one can easily extend it to  $Q^2 \approx 400 \text{ GeV}^2$  for the triple pole, and to  $Q^2 \approx 800 \text{ GeV}^2$  in the double-pole case. It is interesting that one cannot go as high as in ref. [8]. This can be attributed either to too simple a choice for the form factors, or more probably to the onset of perturbative evolution.

Fig. 3 shows the  $F_2^p$  fit for some selected  $Q^2$  bins (figures for other  $Q^2$  bins can be found in [5]). As pointed out before, our fits do reproduce

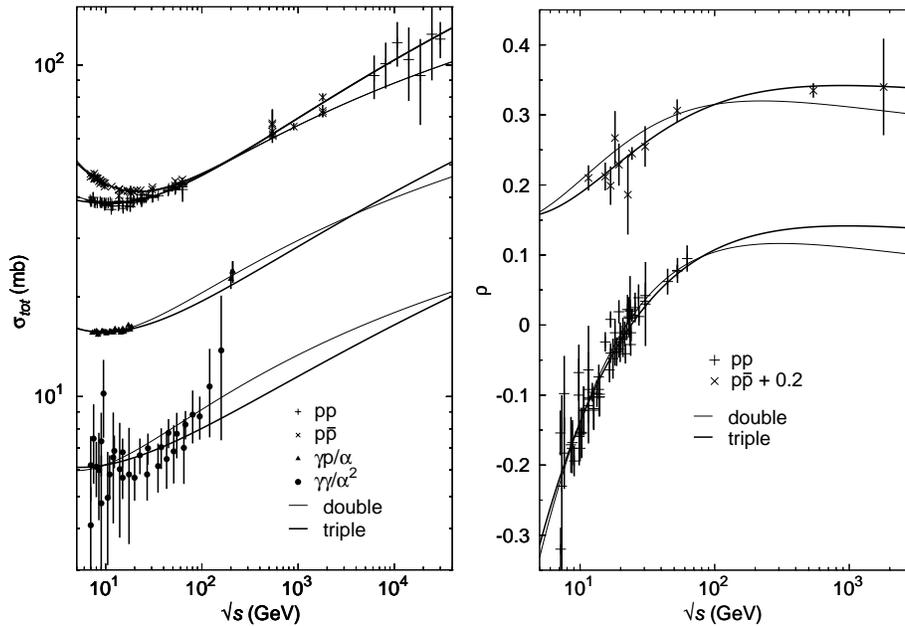


Figure 2. Fits to the total cross-sections and to the  $\rho$  parameter. The thick and thin curves correspond respectively to the triple-pole and to double pole cases.

the low- $Q^2$  region quite well, but predict total cross sections on the lower side of the error bands. Hence the extrapolation to  $Q^2 = 0$  of DIS data does not require a hard pomeron.

For photon structure functions, one needs to add one singularity at  $j = 0$  corresponding to the box diagram [12], but otherwise the  $\gamma\gamma$  amplitude is fully specified by the factorization relations. One can see from Fig. 4 that the data on photon structure are well reproduced by both parametrizations.

Even more surprisingly, it is possible to reproduce the  $\gamma^*\gamma^*$  cross sections when both photons are off-shell, as shown in Fig. 5. This is the place where BFKL singularities may manifest themselves, but as can be seen such singularities are not needed.

In conclusion, we have shown that  $t$ -channel unitarity can be used to map the regions where new singularities can occur, be they of perturbative or non-perturbative origin. Indeed, we have seen that although hadronic singularities must be universal, this is not the case for  $F_2^p$  and  $F_2^\gamma$ , as DIS involves off-shell particles. Nevertheless, up to  $Q^2 = 150 \text{ GeV}^2$ , the data do not call for the existence of new singularities, except perhaps the box diagram. In the case of total cross sections, this suggests that it is indeed possible to define an  $S$  matrix for QED.

For off-shell photons, our fits are rather surprising as the standard claim is that the perturbative evolution sets in quite early. This evolution is indeed allowed by  $t$ -channel unitarity constraints: it is possible to have extra singularities in off-shell photon cross sections, which are built on top of the non-perturbative singularities. But it seems that Regge parametrisations can be extended quite high in  $Q^2$  without the need for these new singularities.

Thus it is possible to reproduce soft data (*e.g.* total cross sections) and hard data (*e.g.*  $F_2$  at large  $Q^2$ ) using a common  $j$ -plane singularity structure, provided the latter is more complicated than simple poles. Furthermore, we have shown that it is then possible to predict  $\gamma\gamma$  data using  $t$ -channel unitarity. How to reconcile such a simple description with DGLAP evolution, or BFKL results, remains a challenge.

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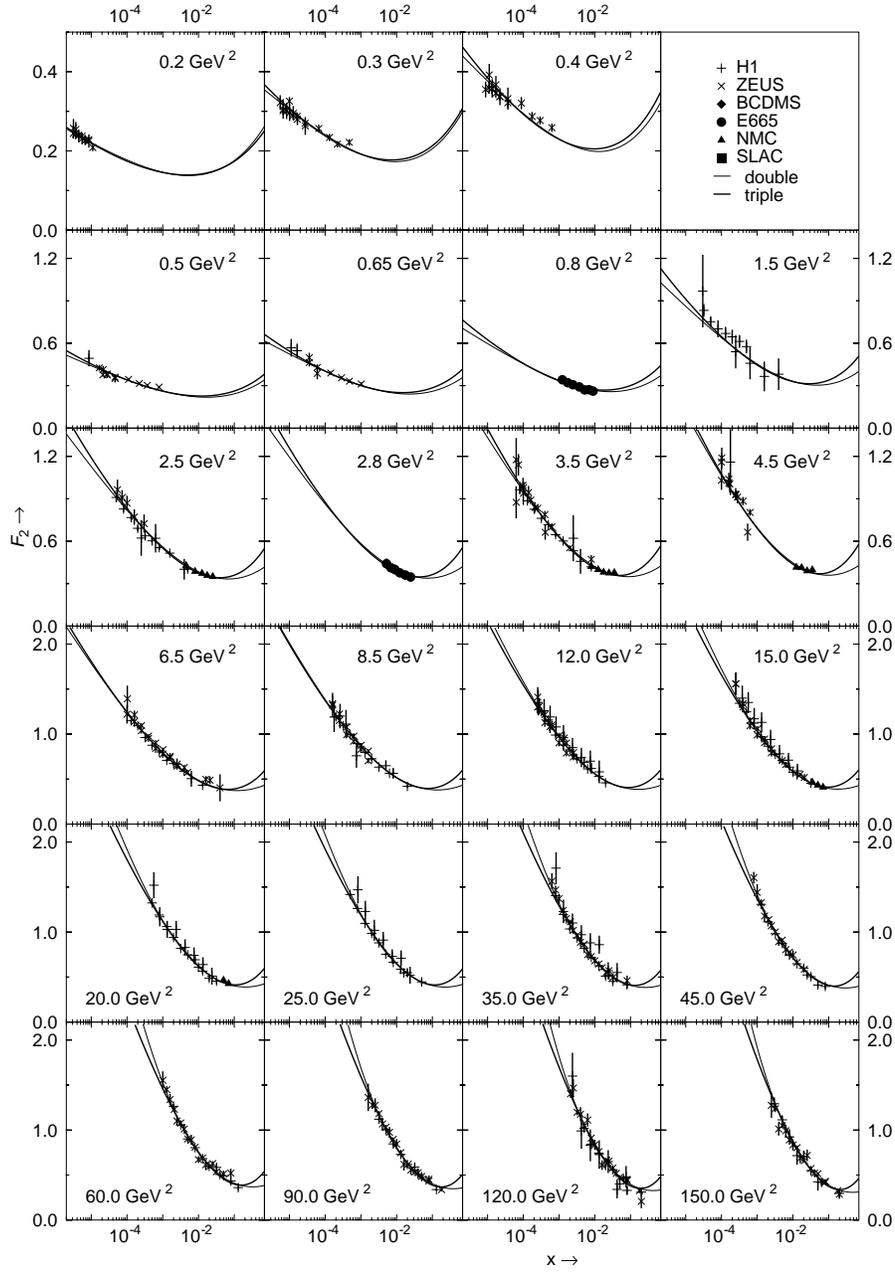


Figure 3. Fits to  $F_2^p$ . We show only graphs for which there are more than 6 experimental points, as well as the lowest  $Q^2$  ones. The curves are as in Fig. 2

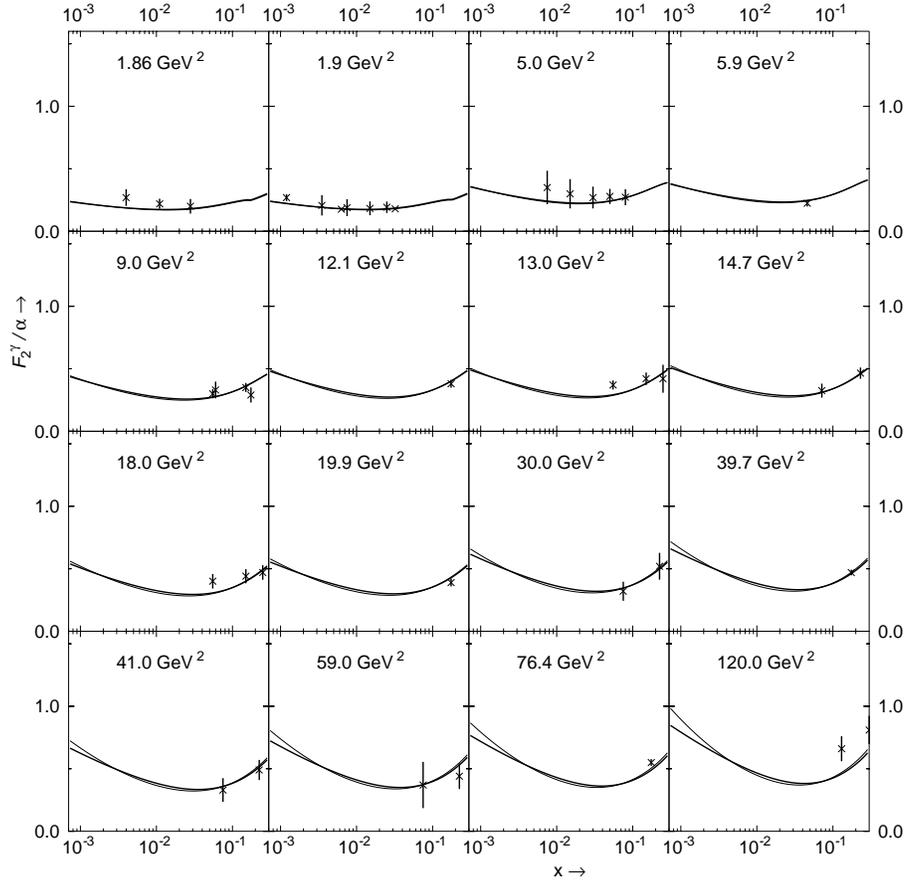


Figure 4. Fits to  $F_2^\gamma$ . The thick and thin curves correspond respectively to the triple-pole and to the double-pole cases. The data are from [4].

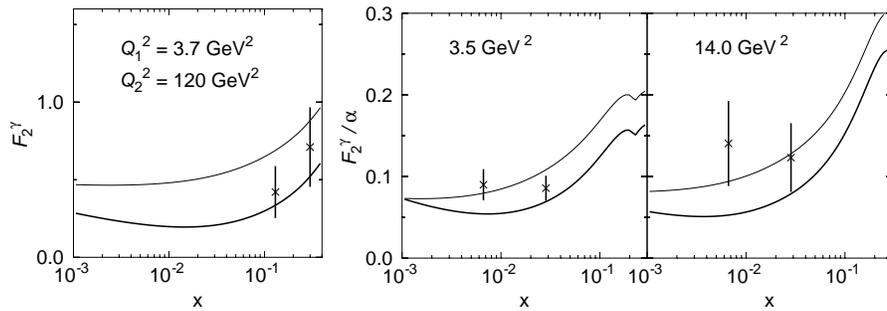


Figure 5. Fits to  $F_2^\gamma$  for nonzero asymmetric values of  $P^2$  and  $Q^2$  and for  $P^2 = Q^2$ . The curves are as in Fig. 4. The data are from [4].