

# Consequences of $t$ -channel unitarity for $\gamma^{(*)}p$ and $\gamma^{(*)}\gamma^{(*)}$ amplitudes.

J.R. Cudell\*, E. Martynov<sup>†‡</sup> and G. Soyez<sup>§</sup>

Université de Liège, Bât. B5-a, Sart Tilman, B4000 Liège, Belgium

We show that  $t$ -channel unitarity constraints make it possible to obtain the photon-photon cross sections from the photon-proton and proton-proton cross sections, for  $Q^2 < 150 \text{ GeV}^2$ . In order to do so, one must postulate the existence of double-pole or triple-pole singularities in the complex  $j$  plane.

It is an old result [1] that one can relate the amplitudes describing three elastic processes  $aa \rightarrow aa$ ,  $ab \rightarrow ab$ ,  $bb \rightarrow bb$ . The trick is to continue these to the crossed channels  $a\bar{a} \rightarrow a\bar{a}$ ,  $a\bar{a} \rightarrow b\bar{b}$ ,  $b\bar{b} \rightarrow b\bar{b}$ , where they exhibit discontinuities because of the  $a$  and  $b$  thresholds. One then obtains a nonlinear system of equations, which can be solved. Working in the complex  $j$  plane above thresholds ( $t > 4m_a^2$ ,  $4m_b^2$ ), and defining the matrix

$$T_0 = \begin{pmatrix} A_{aa \rightarrow aa} & A_{ba \rightarrow ba} \\ A_{ab \rightarrow ab} & A_{bb \rightarrow bb} \end{pmatrix} \quad (1)$$

one obtains

$$T_0 = \frac{D}{1 - RD} \quad (2)$$

with  $R_{km} = 2i\sqrt{\frac{t-4m_k^2}{t}}\delta_{km}$  and  $D = T_0^\dagger$ . The latter is made of the amplitudes on the other side of the cut. For any  $D$ , equation (2) is enough to derive factorization: the singularities of  $T_0$  can only come from the zeroes of

$$\Delta = \det(1 - RD). \quad (3)$$

Taking the determinant of both sides of eq. (2), we obtain in the vicinity of  $\Delta = 0$

$$A_{aa \rightarrow aa}A_{bb \rightarrow bb} - A_{ab \rightarrow ab}A_{ba \rightarrow ba} = \frac{C}{\Delta}, \quad (4)$$

where  $C$  is regular at the zeroes of  $\Delta$ . As the l.h.s. is of order  $1/\Delta^2$  we obtain the well-known

\*J.R.Cudell@ulg.ac.be

†E.Martynov@guest.ulg.ac.be

‡on leave from Bogolyubov Institute for Theoretical Physics, Kiev.

§G.Soyez@ulg.ac.be

factorization properties from eqs. (2) and (4):

- The elastic hadronic amplitudes have common singularities;
- At each singularity in the complex  $j$  plane, these amplitudes factorise.

These equations are used to extract relations between the residues of the singularities, which can be continued back to the direct channel.

We have extended [2] the above argument including all possible thresholds, both elastic and inelastic. The net effect is to keep the structure (2), but with a matrix  $D$  that includes multi-particle thresholds. Furthermore, we have shown that one does not need to continue the amplitudes from one side of the cuts to the other, but that the existence of complex conjugation for the amplitudes is enough to derive (2).

Hence there is no doubt that the factorization of amplitudes in the complex  $j$  plane is correct, even when continued to the direct channel. For isolated simple poles one obtains the usual factorization relations for the residues.

If  $A_{pq}(j)$  has coinciding simple and double poles (*e.g.* colliding simple poles at  $t = 0$ ),

$$A_{pq} = \frac{S_{pq}}{j - z} + \frac{D_{pq}}{(j - z)^2}, \quad (5)$$

one obtains the new relations

$$\begin{aligned} D_{11}D_{22} &= (D_{12})^2, \\ D_{11}^2S_{22} &= D_{12}(2S_{12}D_{11} - S_{11}D_{12}). \end{aligned} \quad (6)$$

In the case of triple poles

$$A_{pq} = \frac{S_{pq}}{j - z} + \frac{D_{pq}}{(j - z)^2} + \frac{F_{pq}}{(j - z)^3}, \quad (7)$$

the relations become

$$F_{11}F_{22} = (F_{12})^2,$$

$$\begin{aligned}
F_{11}^2 D_{22} &= F_{12}(2D_{12}F_{11} - D_{11}F_{12}), \\
F_{11}^3 S_{22} &= F_{11}F_{12}(2S_{12}F_{11} - S_{11}F_{12}) \\
&+ D_{12}F_{11}(D_{12}F_{11} - 2D_{11}F_{12}) \\
&+ D_{11}^2 F_{12}^2.
\end{aligned} \tag{8}$$

Although historically one has used  $t$ -channel unitarity to derive factorization relations in the case of simple poles, it is now clear [3] that a soft pomeron pole is not sufficient to reproduce the  $\gamma^*$  data from HERA [4]. However, it is possible, using multiple poles, to account both for the soft cross sections and for the DIS data [5]. We shall see later that relations (6, 8) enable us to account for the DIS photon-photon data from LEP.

For photons, two theoretical possibilities exist: i) The photon cross sections are zero for any fixed number of incoming or outgoing photons [7]. In this case, it is impossible to define an S matrix, and one can only use unitarity relations for the hadronic part of the photon wave function. Because of this, photon states do not contribute to the threshold singularities, and the system of equations does not close. The net effect is that the singularity structure of the photon amplitudes is less constrained.  $\gamma p$  and  $\gamma\gamma$  amplitudes must have the same singularities as the hadronic amplitudes, but extra singularities are possible: in the  $\gamma p$  case, these may be of perturbative origin, but must have non perturbative residues. In the  $\gamma\gamma$  case, these singularities have their order doubled. It is also possible for  $\gamma\gamma$  to have purely perturbative additional singularities.

ii) It may be possible to define collective states in QED for which an S matrix would exist. In this case, we obtain the same situation for on-shell photons as for hadrons. However, in the case of DIS, virtual photons come as external states. Because they are virtual, they do not contribute to the  $t$ -channel discontinuities, and hence the singularity structure for off-shell photons is as described in i).

In the following, we shall explore the possibility that no new singularity is present for on-shell photon amplitudes, and show that it is in fact possible to reproduce present data using pomerons with double or triple poles at  $j = 1$ .

For a given singularity structure, a fit to the

$C = +1$  part of proton cross sections, and to  $\gamma^{(*)}p$  data enables one, via relations (2), to predict the  $\gamma^{(*)}\gamma^{(*)}$  cross sections. Hence we have fitted [2]  $pp$  and  $\bar{p}p$  cross sections and  $\rho$  parameters, as well as DIS data from HERA [4].

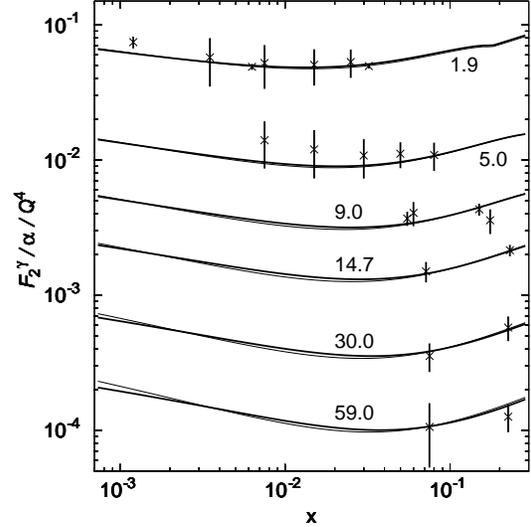


Figure 1. Fits to  $F_2^\gamma$ . The thick and thin curves correspond respectively to the triple-pole and to the double-pole cases. The data are from [6].

The general form of the parametrisations which we used is given, for total cross sections of  $a$  on  $b$ , by the generic formula  $\sigma_{ab}^{tot} = (R_{ab} + H_{ab})$ . The first term, from the highest meson trajectories ( $\rho$ ,  $\omega$ ,  $a$  and  $f$ ), is parametrized via Regge theory as

$$R_{ab} = Y_{ab}^+(\tilde{s})^{\alpha+1} \pm Y_{ab}^-(\tilde{s})^{\alpha-1} \tag{9}$$

with  $\tilde{s} = 2\nu/(1 \text{ GeV}^2)$ . Here the residues  $Y_+$  factorise. The second term, from the pomeron, is parametrized either as a double pole [8]

$$\begin{aligned}
H_{ab} &= D_{ab}(Q^2)\Re e [\log(1 + \Lambda_{ab}(Q^2)\tilde{s}^\delta)] \\
&+ C_{ab}(Q^2) + (\tilde{s} \rightarrow -\tilde{s})
\end{aligned} \tag{10}$$

or as a triple pole

$$H_{ab} = t_{ab}(Q^2) \left[ \log^2 \frac{\tilde{s}}{d_{ab}(Q^2)} + c_{ab}(Q^2) \right]. \tag{11}$$

The details of the form factors entering (10, 11) can be found in [2]. Such parametrisations give  $\chi^2/dof$  values less than 1.05 in the region  $\cos(\vartheta_t) \geq \frac{49}{2m_p^2}$ ,  $\sqrt{2\nu} \geq 7$  GeV,  $x \leq 0.3$ ,  $Q^2 \leq 150$  GeV<sup>2</sup>.

What is really new is that these forms can be extended to photon-photon scattering, using relations (6, 8). The total  $\gamma\gamma$  cross section is well reproduced and the de-convolution using PHOJET is preferred. For photon structure functions, one needs to add one singularity at  $j = 0$  corresponding to the box diagram [9], but otherwise the  $\gamma\gamma$  amplitude is fully specified by the factorization relations. We see in Fig. 1 that DIS data are well reproduced by both parametrisations.

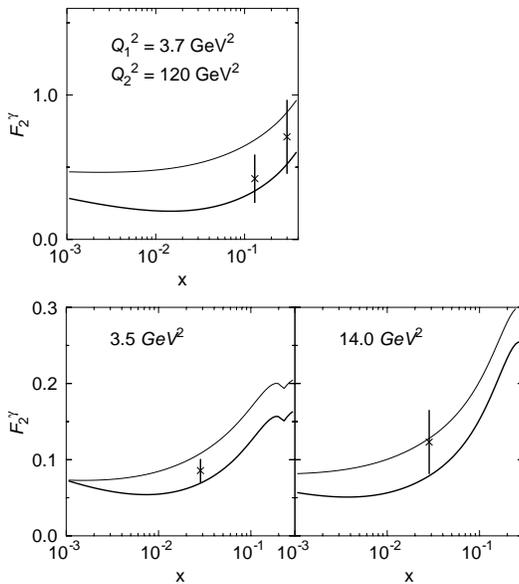


Figure 2. Fits to  $F_2^\gamma$  for nonzero asymmetric values of  $P^2$  and  $Q^2$  and for  $P^2 = Q^2$ . The curves are as in Fig. 1. The data are from [6].

Even more surprisingly, it is possible to reproduce the  $\gamma^*\gamma^*$  cross sections when both photons are off-shell, as shown in Fig. 2. This is the place where BFKL singularities may manifest them-

selves, but as can be seen such singularities are not needed.

In conclusion, we have shown that it is possible to reproduce soft data (*e.g.* total cross sections) and hard data (*e.g.*  $F_2$  at large  $Q^2$ ) using a common  $j$ -plane singularity structure, provided the latter is more complicated than simple poles. Furthermore, we have shown that it is then possible to predict  $\gamma\gamma$  data using  $t$ -channel unitarity. How to reconcile such a simple description with DGLAP evolution, or BFKL results, remains a challenge.

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