



May 25<sup>th</sup> 2023



# Fatigue crack prediction in Multiphasic-Phasic Material

Anne Marie Habraken<sup>1,4</sup>, Chantal Bouffioux<sup>1</sup>, Olivier Dedry<sup>2</sup>, Seif Fetni<sup>1</sup>,  
Hector Sepulveda<sup>1,3</sup>, Laurent Duchêne<sup>1</sup>, Anne Mertens<sup>2</sup>

<sup>1</sup>: ArGEnCo department, University of Liège, Belgium

<sup>2</sup>: Department of Aerospace and Mechanical Engineering, University of Liège, Belgium

<sup>3</sup>: UCLouvain, Belgium

<sup>4</sup>: Fonds de la Recherche Scientifique –F.R.S. –F.N.R.S., Belgium



## Introduction

- Motivation
- Lemaitre and Chaboche model

## Welded structures

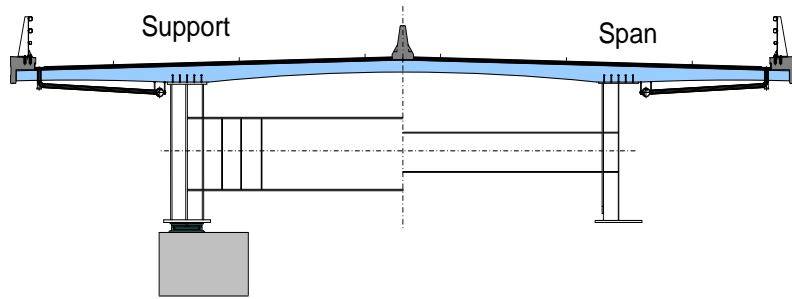
- Material parameter identification
- Results

## Bi-Phasic material

- RVE definition
- Results

## Concluding remarks

- Conclusions
- Future prospects



Road bridges  
RFCS OPTIBRI project

Replace standard S355 NL by HSS S690 QL in a welded steel bridge  
-save resources (material, welding meters)

Use lighter materials like AlSi10Mg and process like LPBF  
-save resources (fuel)



LongLifeAM RW project

Design of civil engineering structures, or transport vehicles  
without fatigue cracks



Damage modelling at different scales

Old fashion (?) but robust model

Many extensions...

Coupled or uncoupled...

For crack initiation: uncoupled model can do the job

Always: need of a carefull identification and validation

Lemaitre, J., Chaboche, J.-L., 1996. Mécanique des matériaux solides, Dunod

$$\frac{\partial D}{\partial N} = 0 \quad \text{if } f_D < 0$$

or

$$\frac{\partial D}{\partial N} = [1 - (1 - D)^{\beta+1}]^\alpha \left(\frac{\tilde{A}_{II}}{M}\right)^\beta \quad \text{if } f_D \geq 0$$

$$\tilde{f}_D = A_{II} - A_{II}^* \quad \hat{\sigma}_{ij} = \sigma_{ij} - \sum_k \frac{1}{3} \sigma_{kk}$$

$$A_{II} = \frac{1}{2} \sqrt{\frac{3}{2} (\hat{\sigma}_{ij\max} - \hat{\sigma}_{ij\min}) (\hat{\sigma}_{ij\max} - \hat{\sigma}_{ij\min})}$$

$$A_{II}^* = \sigma_{10} (1 - 3 \cdot b \cdot \sigma_{Hm}) \quad (\text{Sines' criterion})$$

$$\tilde{A}_{II} = \frac{A_{II}}{1 - D} \quad \sigma_{Hm} = \frac{1}{3} \left[ \frac{1}{T} \int_T \text{Tr}(\underline{\underline{\sigma}}(t)) dt \right]$$

$$\alpha = 1 - a \left( \frac{A_{II} - A_{II}^*}{\sigma_u - \sigma_{eq\max}} \right) \quad M = M_0 (1 - 3 \cdot b \cdot \sigma_{Hm})$$

D: damage value

N: number of cycles

$A_{II}$ : 2<sup>nd</sup> stress amplitude invariant

$A_{II}^*$ : fatigue limit

$f_D$ : damage yield locus

$\sigma_{Hm}$ : mean hydrostatic stress

$\sigma_{eq\max}$ : maximum VM stress / cycle

$\langle x \rangle = x$  if  $x > 0$  else = 0

$b = 1/\sigma_u$

$\sigma_u$ : ultimate tensile stress

$\sigma_{10}$ : fatigue limit if  $\sigma_{Hm} = 0$

$a, M_0, \beta$ : material parameters



## Introduction

- Motivation
- Lemaitre and Chaboche model

## Welded structures

- Material parameter identification
- Results

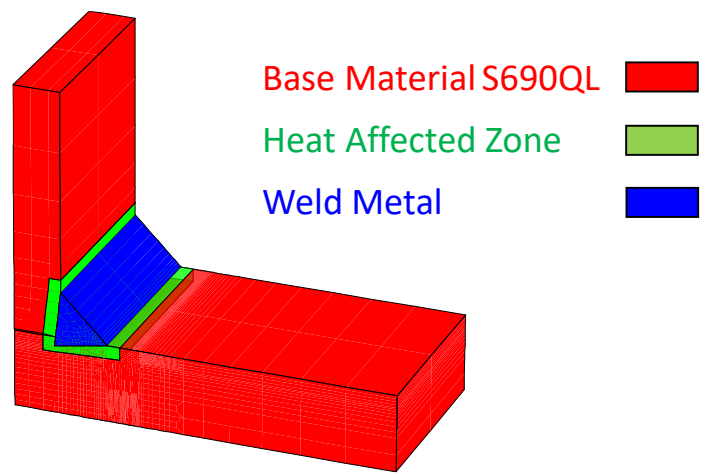
## Bi-Phasic material

- RVE definition
- Results

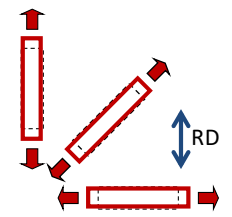
## Concluding remarks

- Conclusions
- Future prospects

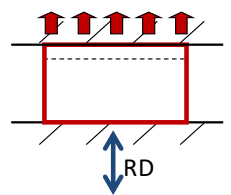
# Material parameter identification (STATIC)



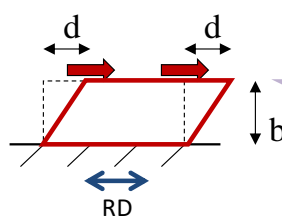
Tensile test



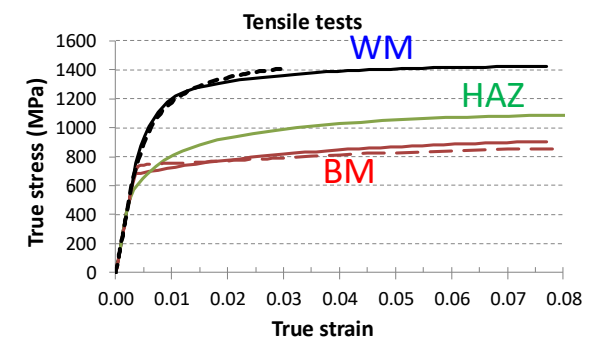
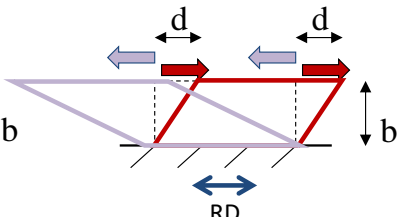
Large tensile test



Shear test



Bauschinger shear test

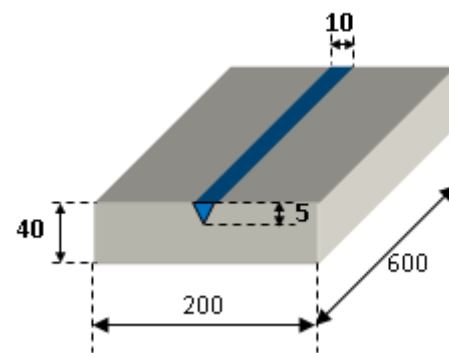


--- Experiment in RD  
 \_\_ Model

Material tests -- Small samples (

Data for Hooke, Hill, Voce and Armstrong-Frederick laws (units: MPa, s)													
Material	Ultimate tensile strength		Elastic data		Yield locus				Isotropic hardening			Kinematic hardening	
	$\sigma_{u,eng}$	$\sigma_{u,true}$	E	$\nu$	F	G	H	$N=L=M$	K	$\sigma_0$	n	$C_x$	$X_{sat}$
BM (S690QL)	838	905	210 116	0.3	1	1	1	3.9	0	674	0	31.9	167
HAZ	1338	1424	210 000	0.3	1	1	1	4.45	371	827	511	52.5	152
WM	1008	1101	210 000	0.3	1	1	1	3.2	241	531	285	42.6	218

BM bulk sample with WM



# Validation of Lemaitre and Chaboche (vgrad LC) model

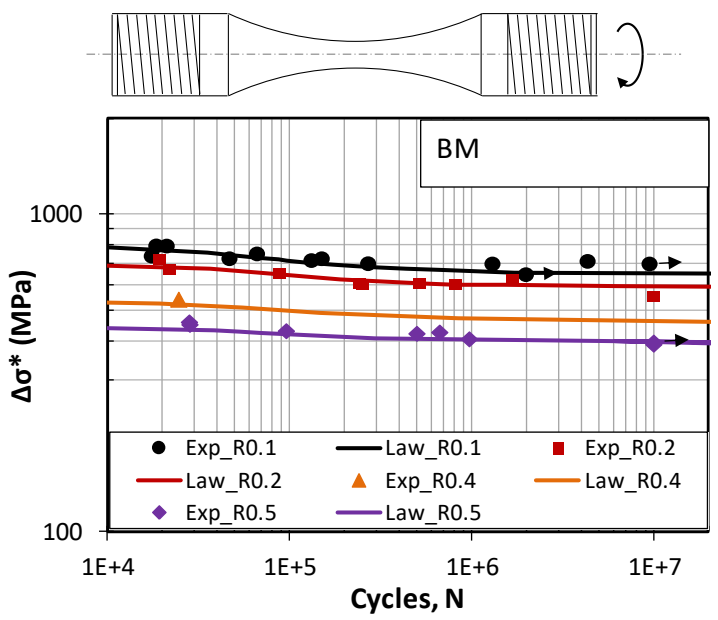
Small samples, Ra= 0.8 μm,  
Length: 96 mm  
Diameter: 5 mm

- Tests on a vibrophore
- Axial loading

Material	Smooth	Notch
BM	3 R	3 geom.
HAZ1	1 R	1 geom.
HAZ2	2 R	1 geom.
WM	2 R	1 geom.



## BM: smooth samples

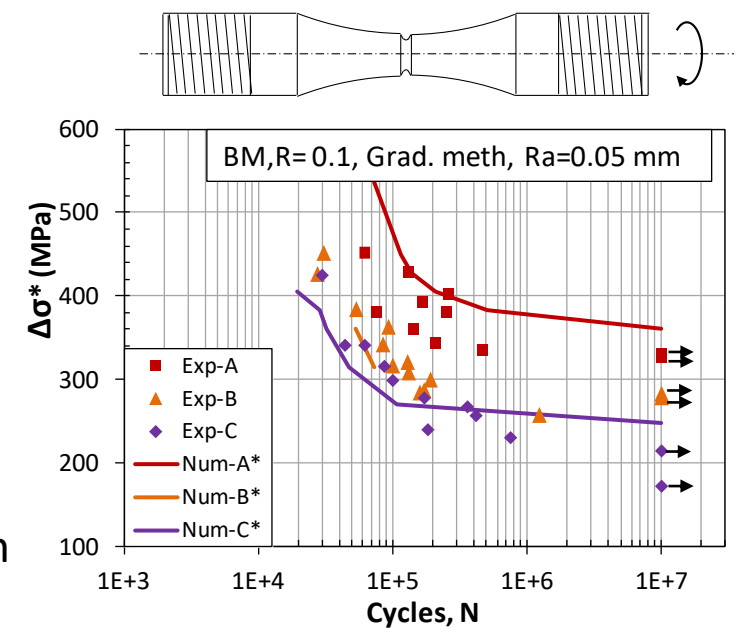


Notch radius:  
A: 1.2 mm  
B: 0.3 mm  
C: 0.15 mm

Vgrad LC  
↘ mesh dependency

Additional parameter  
Length scale Ra : 0.06 mm

## BM: notched samples



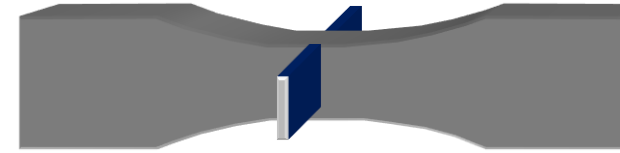


# Experimental campaign on representative parts

Plates



Welded plates + PIT or Tig rem. or nothing



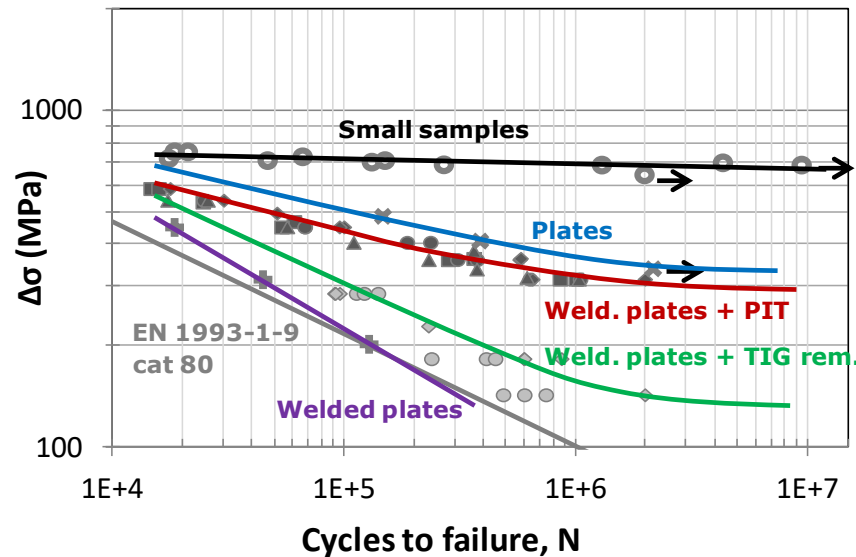
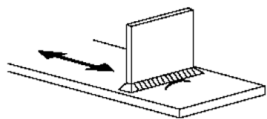
Effect of:

- size, surface roughness
- welding (HAZ,  $\sigma$  concentration,  $\sigma_{res}$ )
- post-treatment **Pneumatic Impact Treatment (PIT)** or **Tungsten Inert Gas remelting (TIG rem.)**
- Almost no geometrical effect (plate thickness 1.5- 4 cm, stiffener length 4-6 cm and thickness 6-15 mm)

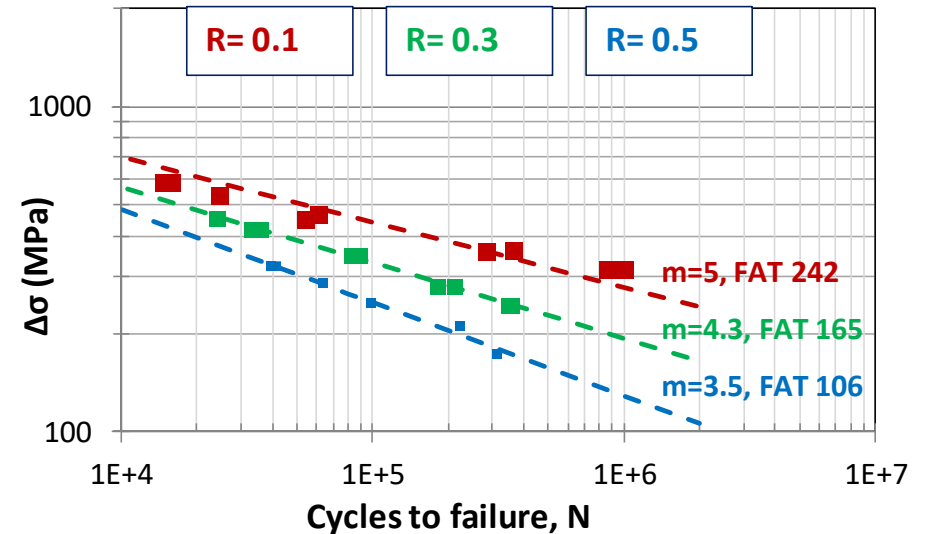
Comparison

with

EN 1993-1-9 cat 80



Effect of stress range on welded plates + PIT



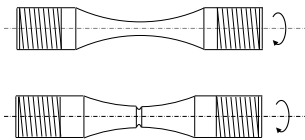
# Identified material parameter - Scale effect (FATIGUE)

$\sigma_u$ : ultimate tensile stress

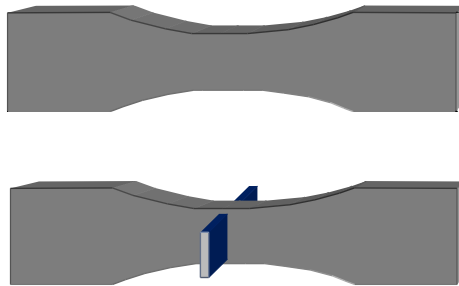
$\sigma_{10}$ : fatigue limit

$a, M_0, \beta$ : material data

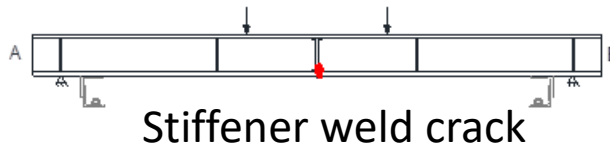
Length 96 mm  
Diameter 5 mm



Length 1 m  
Thickness 4 cm



Length 4 m



## Lemaître Chaboche multiaxial fatigue model improved by stress gradient method

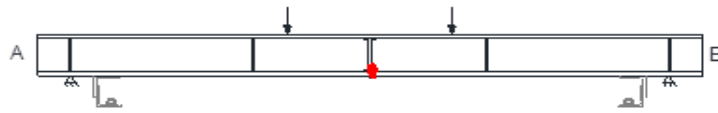
Material	$\sigma_U$ (Mpa)	$\sigma_{10}$ (Mpa)	b	$\beta$	a	$M_0$	Ra (mm)
BM - small	905.0	580.0	1.10 E-03	0.17	1	5.385 E+30	0.06
HAZ - small	1424.0	428.4	7.02E-04	2.094	1	4.410E+05	0.06
WM - small	1101.0	319.4	9.08E-04	0.161	1	7.245E+32	0.00



Material	$\sigma_U$ (Mpa)	$\sigma_{10}$ (Mpa)	b	$\beta$	a	$M_0$	Ra (mm)
BM - plate	905.0	203.0	1.10 E-03	0.17	1	5.385 E+30	0.06
HAZ - plate	1424.0	149.9	7.02E-04	2.094	1	4.410E+05	0.06



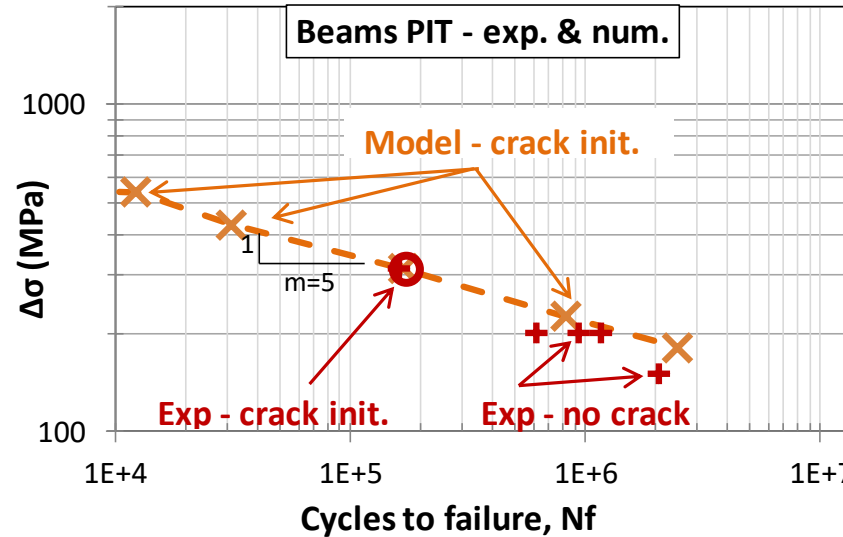
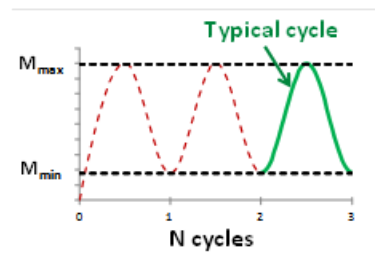
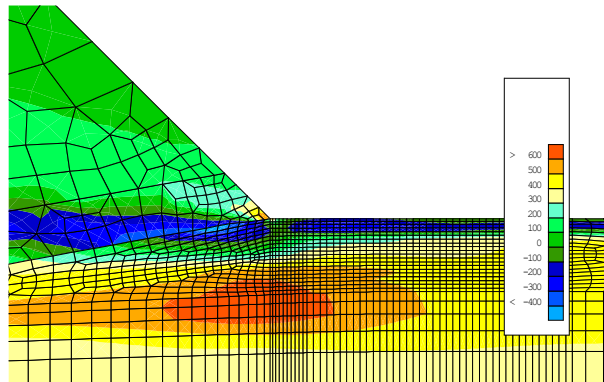
Material	$\sigma_U$ (Mpa)	$\sigma_{10}$ (Mpa)	b	$\beta$	a	$M_0$	Ra (mm)
BM - large plate	905.0	203.0	1.10 E-03	0.159	1	5.385 E+30	0.06
HAZ - large plate	1424.0	149.9	7.02E-04	1.965	1	4.410E+05	0.06



Crack initiation on PIT treated welded joints

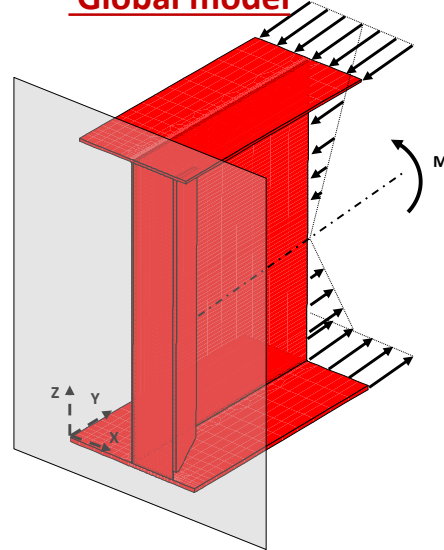


Large scale reduction factor on fatigue data

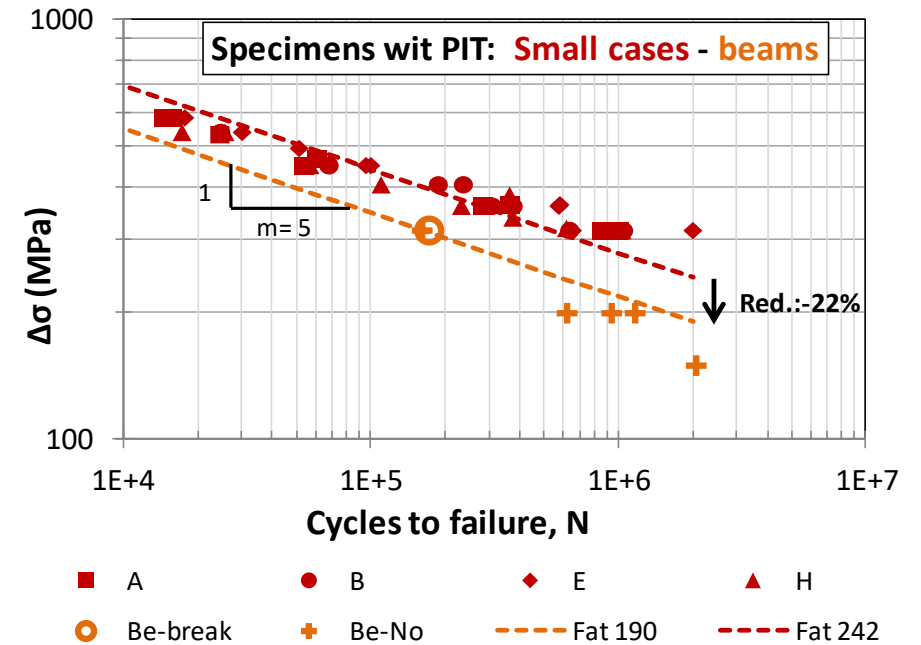
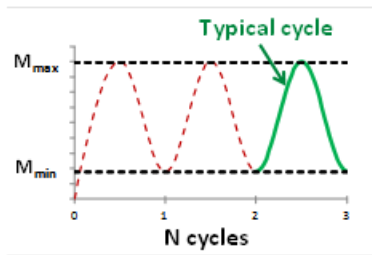
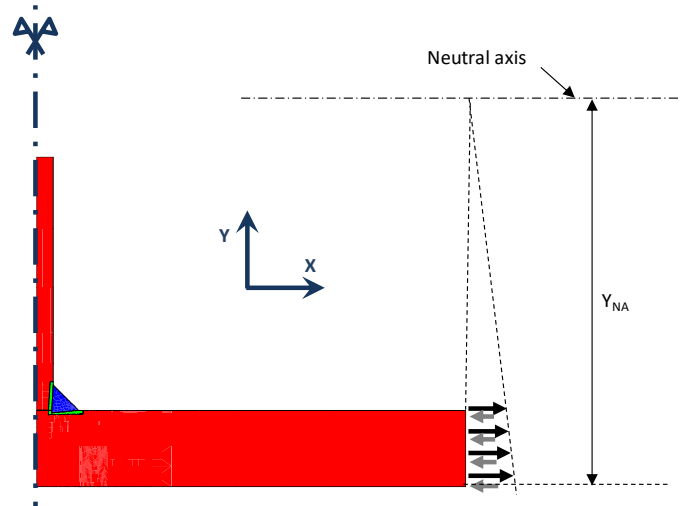




**Global model**



**Simplified sub-model**



Clear scale effect between laboratory samples and civil engineer structure

Predictions of Vgrad Lemaitre and Chaboche model validated,  
**however tuning parameter is required when scale varies**  
**very heavy experimental campaign**

*‘User guided multi-scale simulations’:*

- *FE simulations identify stress cycles in hot points or mandatory check point from civil engineer Standard*
- *Modeling of weld joint*

OPTIBRI project proves interest to replace S355 NL by HSS S690 QL from  
LCA analysis + cost analysis  
(impact of decrease of material but mostly weld multipass process thanks to plate reduction)

Optimal use of high strength steel grades within bridge (**OPTIBRI**) Habraken A.M., Duchêne L., Bouffioux C. 2019 (EUR 29546 EN) doi: 10.2777/93807



## Introduction

- Motivation
- Lemaitre and Chaboche model

## Welded structures

- Material parameter identification
- Results

## Bi-Phasic material

- RVE definition
- Results

## Concluding remarks

- Conclusions
- Future prospects

## Assumptions about AlSi10Mg

post processing of LPBF samples to suppress the porosity

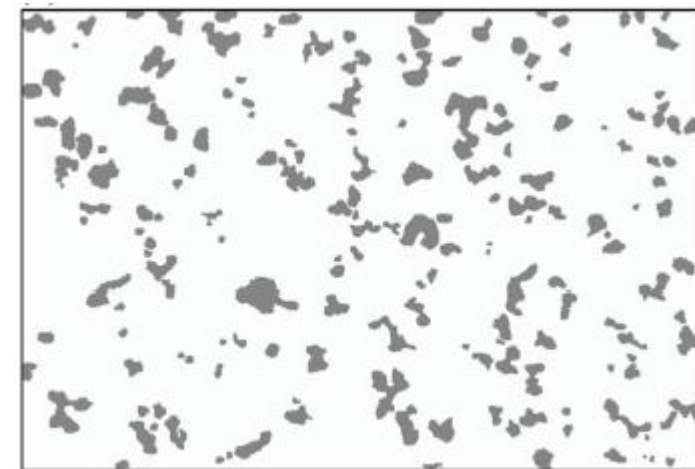
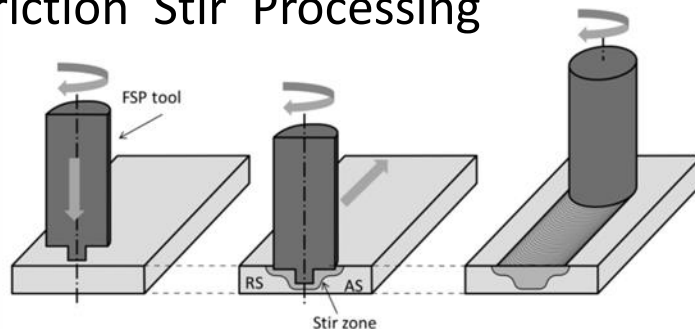
→ the microstructure itself defines the mechanical behavior

→ definition of a representative volume element: Al  $\alpha$  phase matrix  
+ Si precipitates

→ RVE validation 1 : static behaviour

→ fatigue ?

post processing Friction Stir Processing



1000 nm

Zhao et al. MSEA 2019

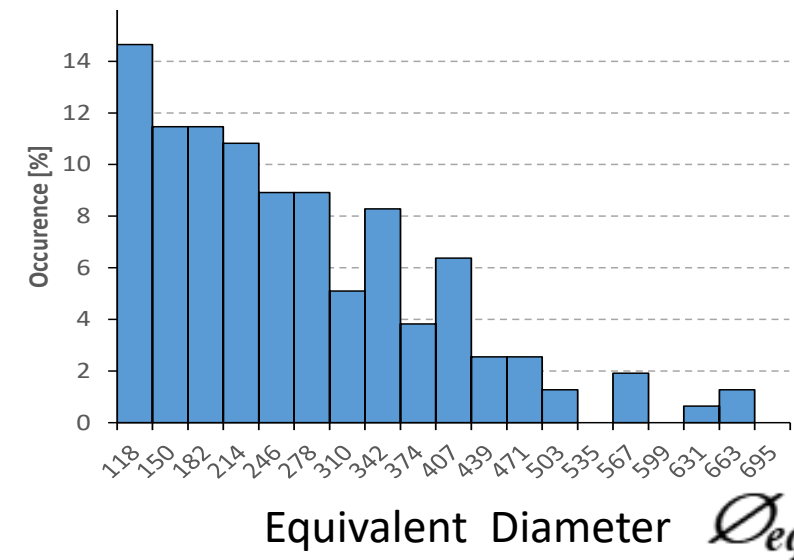
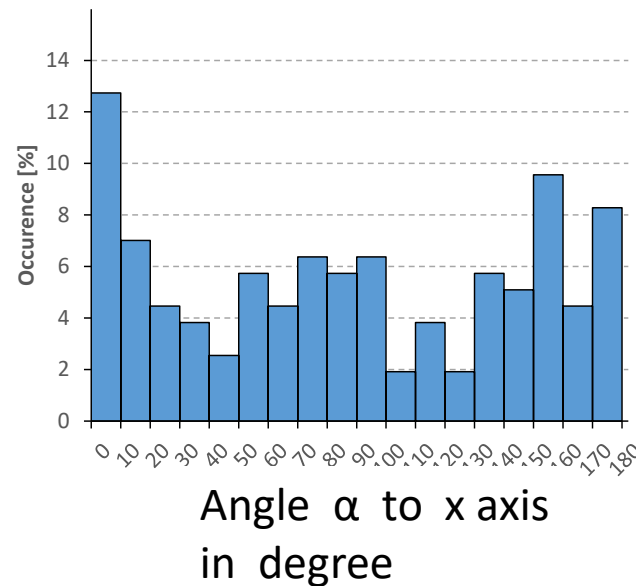
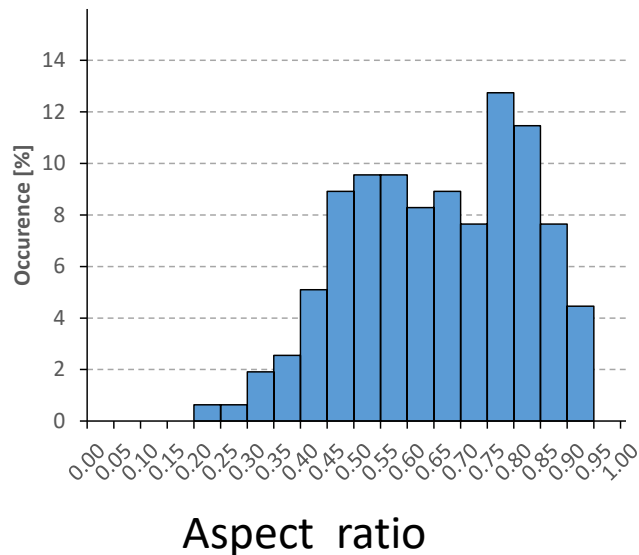
Analysis of precipitate distribution  $r_{mn} = \frac{cov(m,n)}{\sigma_m \cdot \sigma_n}$

$AR$ vs. $\varnothing_{eq}$	$\alpha$ vs. $\varnothing_{eq}$	$\alpha$ vs. $AR$
-0.336	0.042	0.066

Similar in RVE and measured microstructure

No correlation  $r_{mn}=0$  strong |1|

Pearson correlation coefficient



Si volume meets Si composition in Al  $\alpha$  phase matrix + Si precipitate  $\rightarrow$  statistical representativity of images and RVE



Si : literature + nano-indentation of big particles

(Young modulus, mostly elastic behavior, fracture stress very high)

$\alpha$ -Al matrix:

1. nano-indentation of  $\alpha$ -Al matrix with 3 indenters  $\rightarrow$  info on hardening curve  
( $\sigma_0$  still multiple solutions)<sup>1</sup>

2. inverse modelling of Berkovich nano-indentation<sup>2</sup>

$\rightarrow$  tensile curve = validation)

3. analytical formulas (many parameters, microscopic experiment validations)<sup>3,4</sup>

+ links with data driven approach

$\swarrow$   
In the idea to go to material design  
no dependence on experiment  
on large sample or any sample is searched

[1] Dedry et al. *Proc. ESAFORM 2021 (PoPuPs)*

[2] Tran Song Hoang et. al *Int.Journal of Mec.Sciences 2022*

[3] PhD J. Delahaye ULiege 2022

[4] ongoing Master thesis V. Borlaf and PhD H. Spulveda

Experimental method with 3 indenters

a representative point  $(\epsilon_{r,\theta}, \sigma_{r,\theta})$  by indenter characterized by its angle  $\alpha$

• **K.L. Johnson (J. Mech. Phys. Solids, 1970)**  $\rightarrow \epsilon_{r,\theta} = K \cdot \cotan(\theta)$

$\theta$ : angle of the equivalent cone,  $K = 0,105$  (Bucaille, Acta Mat. 51, 2003) with  $\alpha$ : angle of indenter

$\rightarrow$  the characteristic strain  $\epsilon_{r,\theta}$

$$\theta = \arctan \left( \sqrt{\frac{3 \cdot \sqrt{3}}{\pi}} \cdot \tan(\alpha) \right)$$

• **Dao's method (Acta Mat. 49, 2001) + Bucaille method (Acta Mat. 51, 2003)**

$$\Pi_{1\theta} = \frac{C_\theta}{\sigma_{r,\theta}} = \tan^2(\theta) \cdot \left\{ P1 \cdot \left[ \ln \left( \frac{E^*}{\sigma_{r,\theta}} \right) \right]^3 + P2 \cdot \left[ \ln \left( \frac{E^*}{\sigma_{r,\theta}} \right) \right]^2 + P3 \cdot \left[ \ln \left( \frac{E^*}{\sigma_{r,\theta}} \right) \right] + P4 \right\}$$

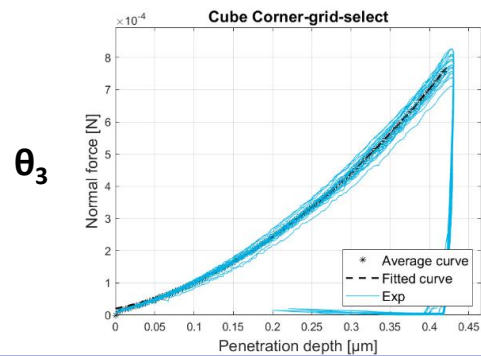
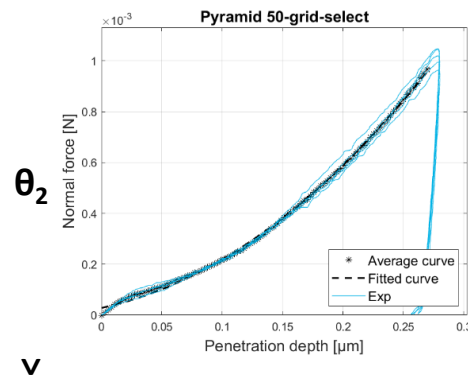
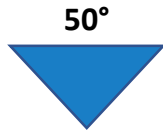
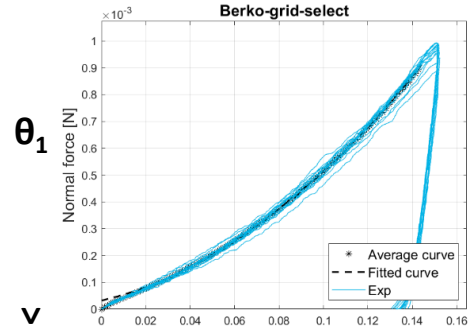
$E^*$ : reduced modulus of the nano-indentation curve

$\rightarrow$  the characteristic stress  $\sigma_{r,\theta}$

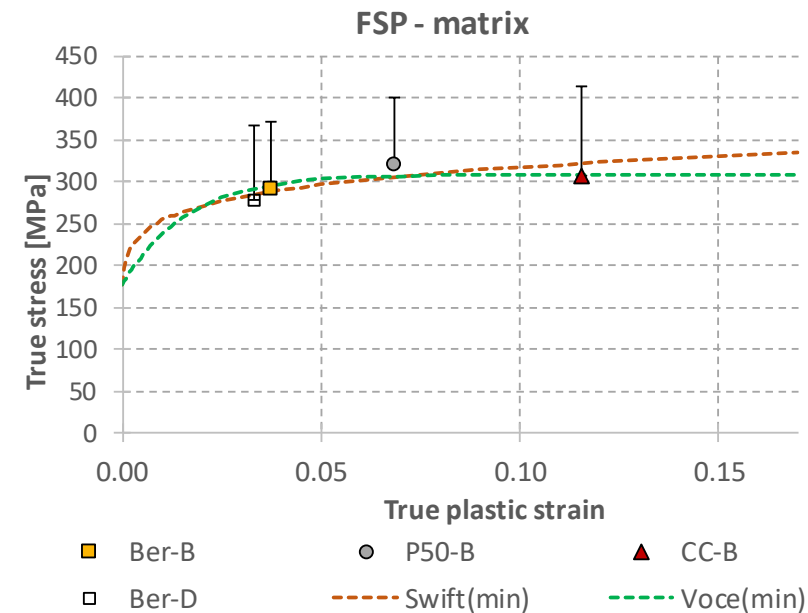
P1	P2	P3	P4
0.02552	-0.72526	6.34493	-6.47458



Indentations on grids with 3 different indenters: lowest curves for matrix = **assumption**

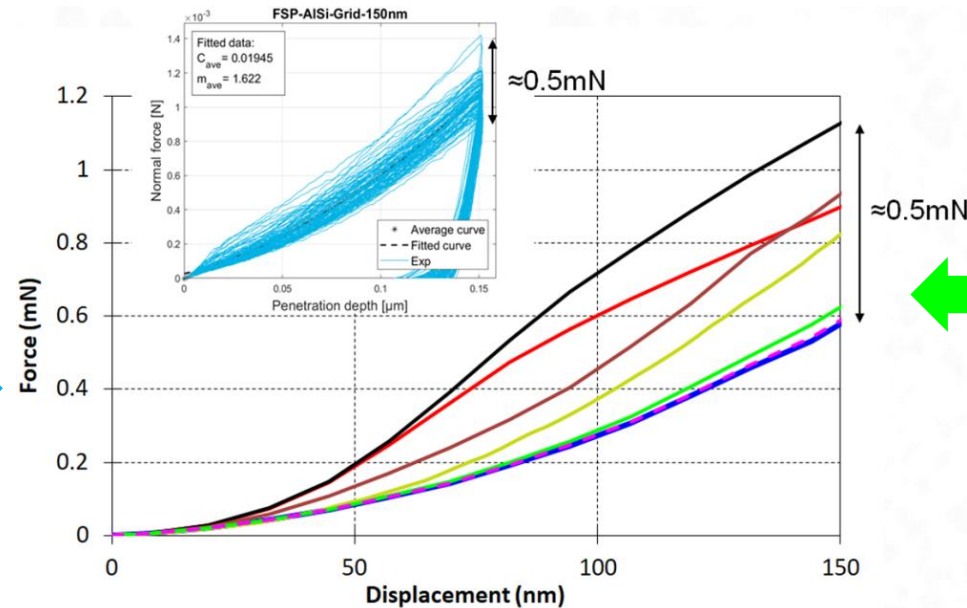
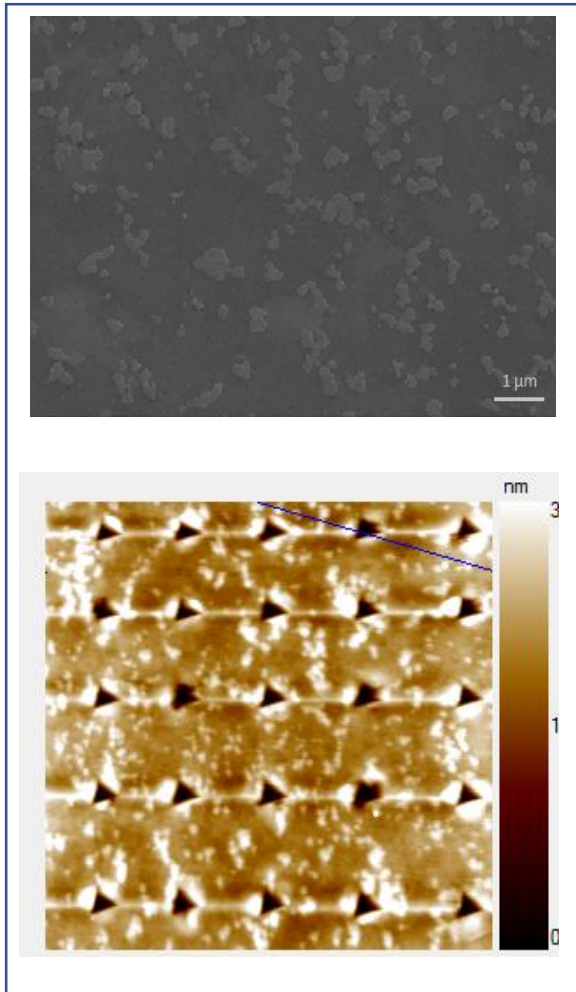


- Swift or Voce hardening law fitting infinite solution if  $\sigma_y$  not defined
- $\sigma_y$  from macro tensile macro



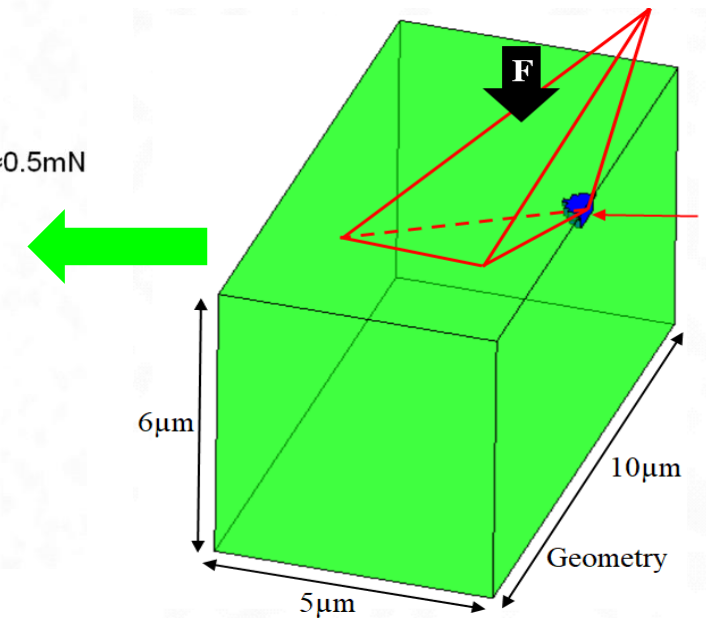
O. Dedry, et al., (2021) ESAFORM proceedings (PoPuPs)

Berkovich nanoindentation + Finite Element Method to identify plastic behavior of the  $\alpha$ -Al matrix in FSP- LPBF AISi10M

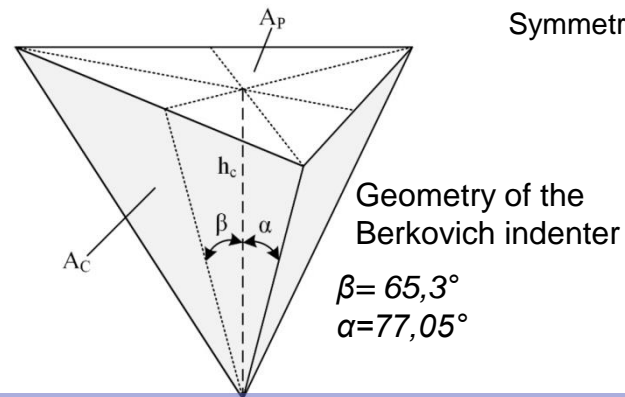
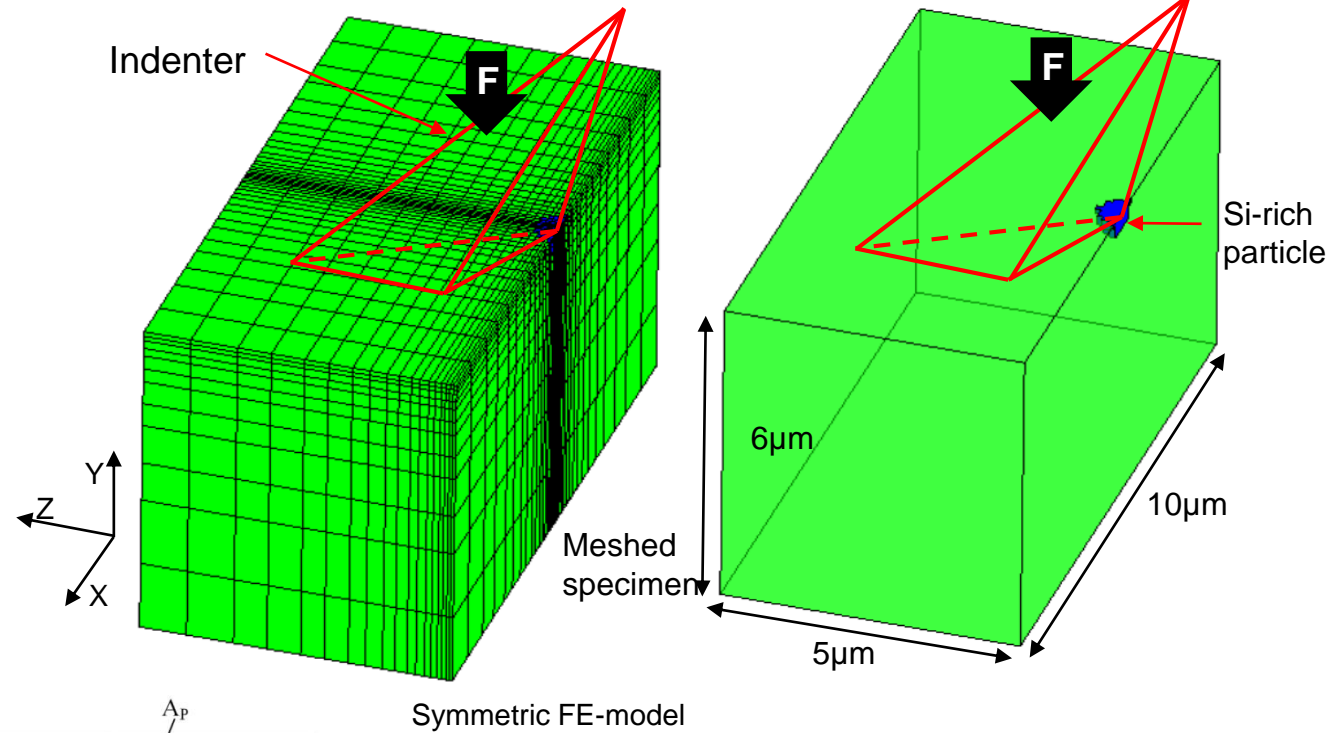
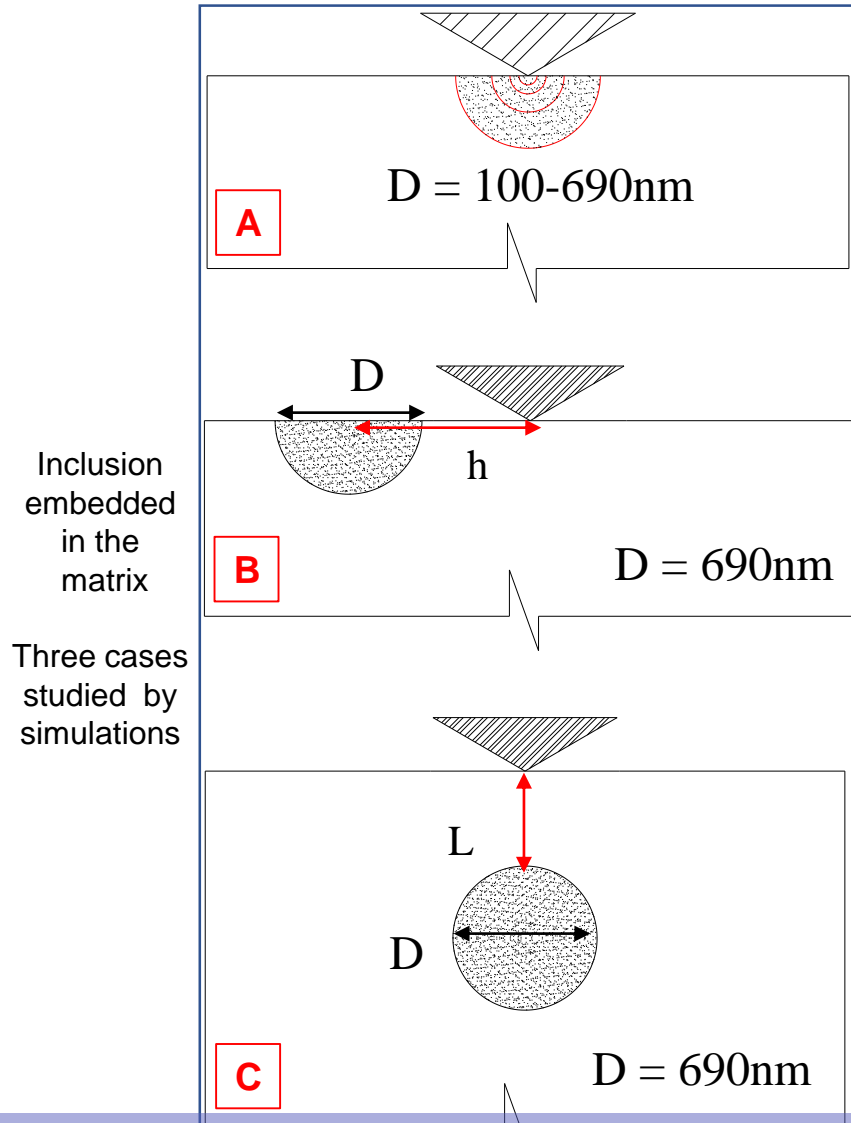


Large scattering  
 $\alpha$ -Al and Si

Identification the mechanical  
behavior of  $\alpha$ -Al ?



3D finite element model



Voce law for Matrix

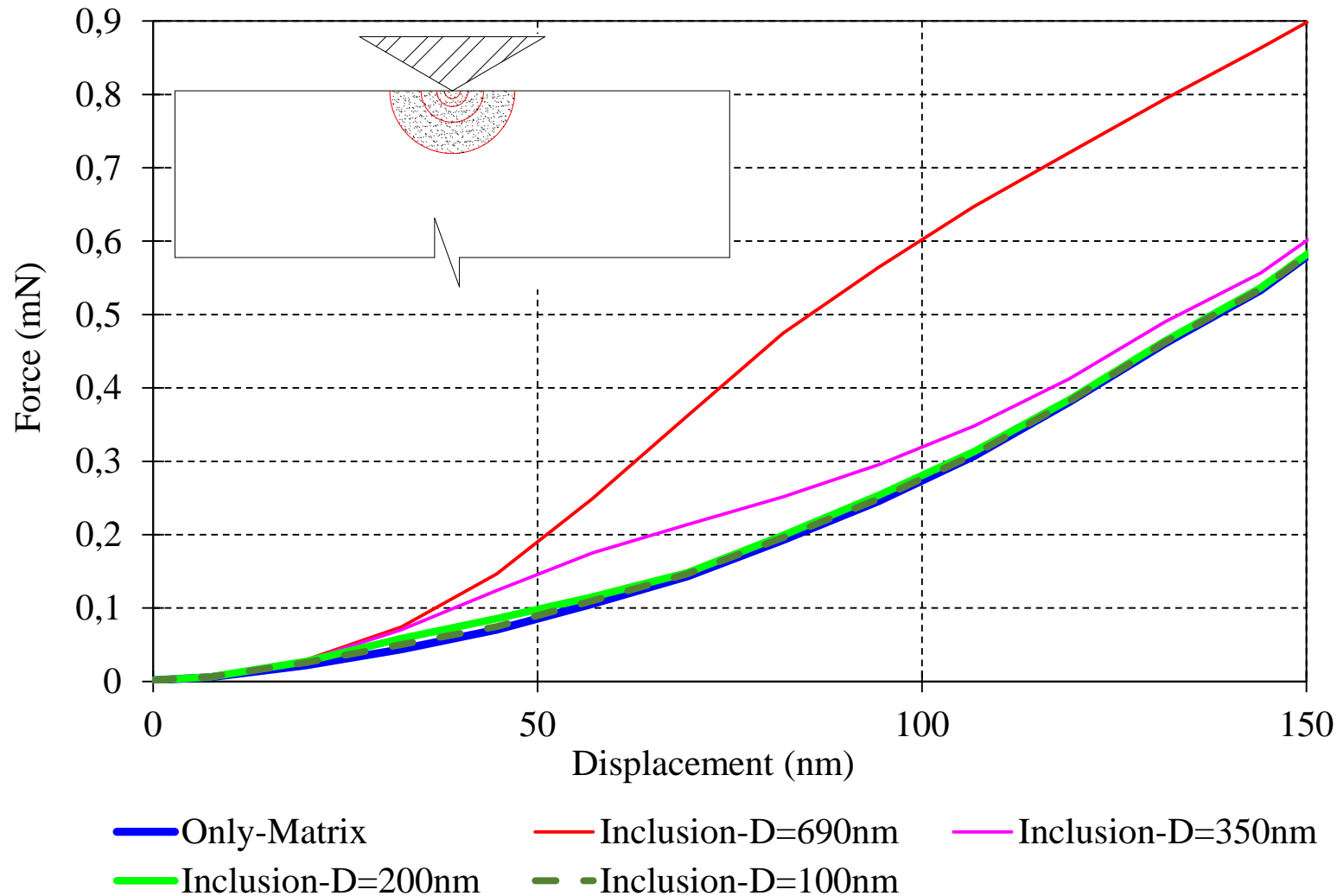
$$\sigma_F = \sigma_0 + K(1 - \exp(-n \cdot \epsilon^{pl}))$$

	Inclusion	Matrix
E (Gpa)	138	83.74
$\mu$	0.3	0.3

**Typical geometry case A**

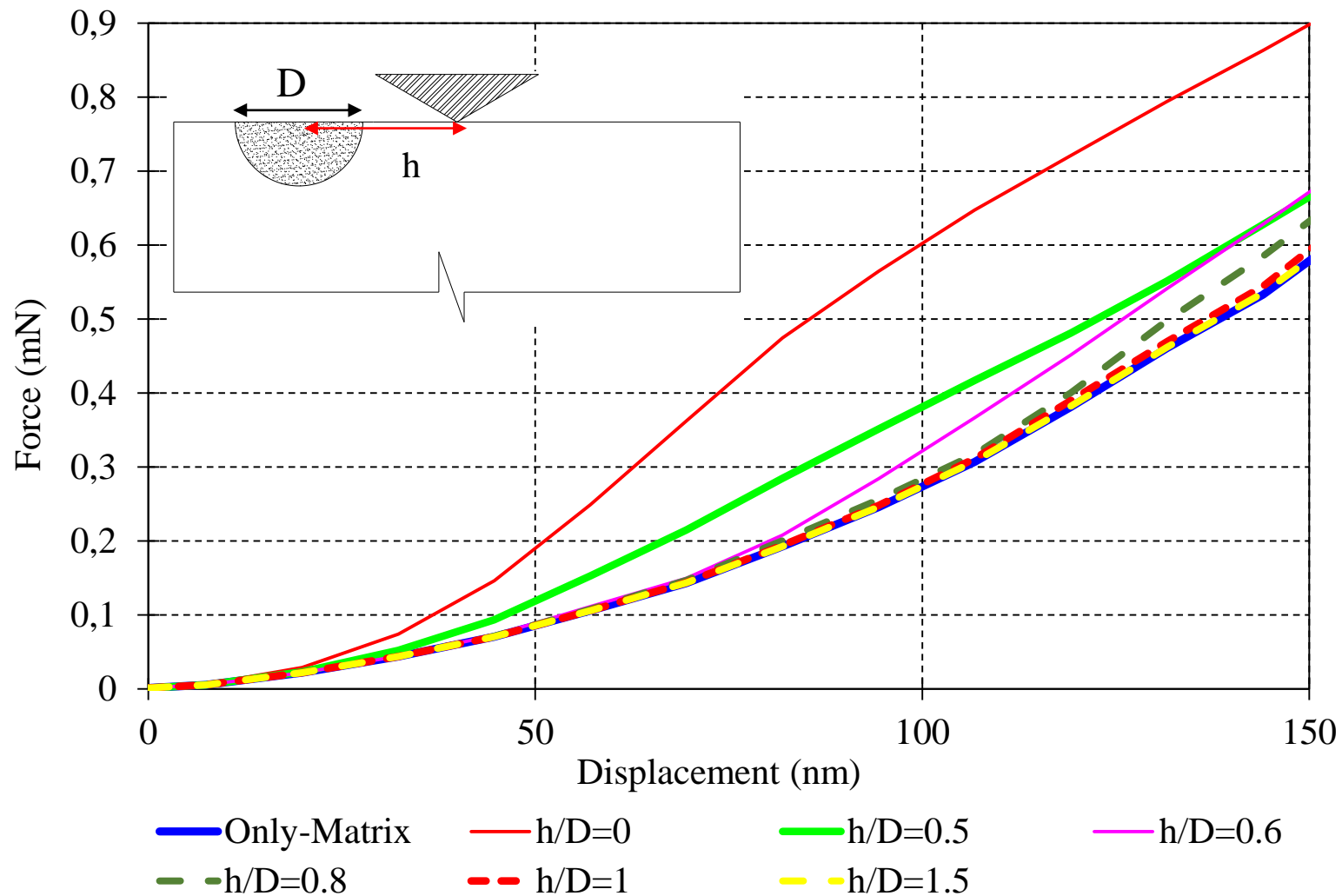
Simulation assumption:  
Matrix law based on lowest measured Force Displ. Curve+ inverse model

**Influence of inclusion size**



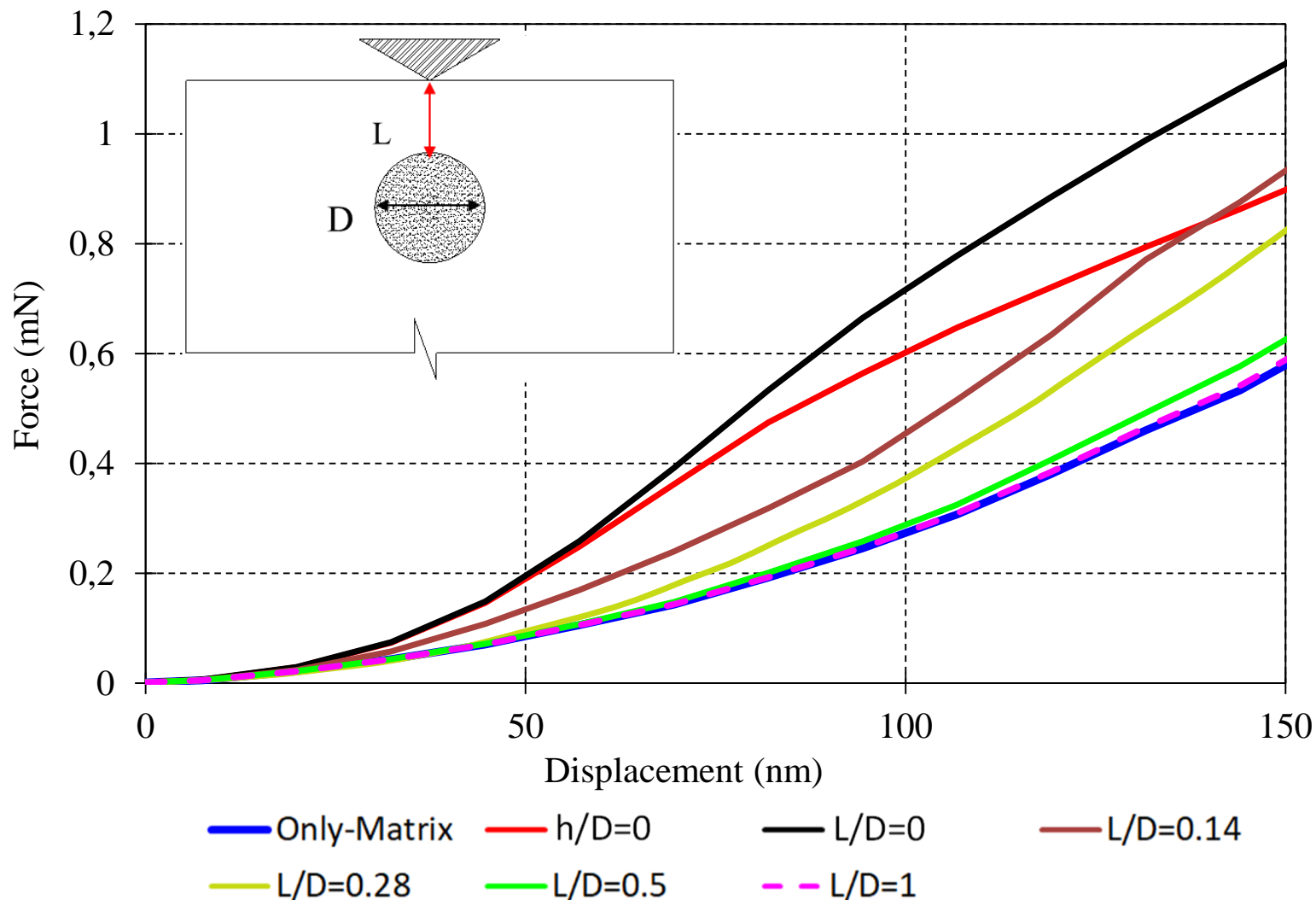
Typical geometry case B

Influence of distance/max inclusion size ratio



Typical geometry case C

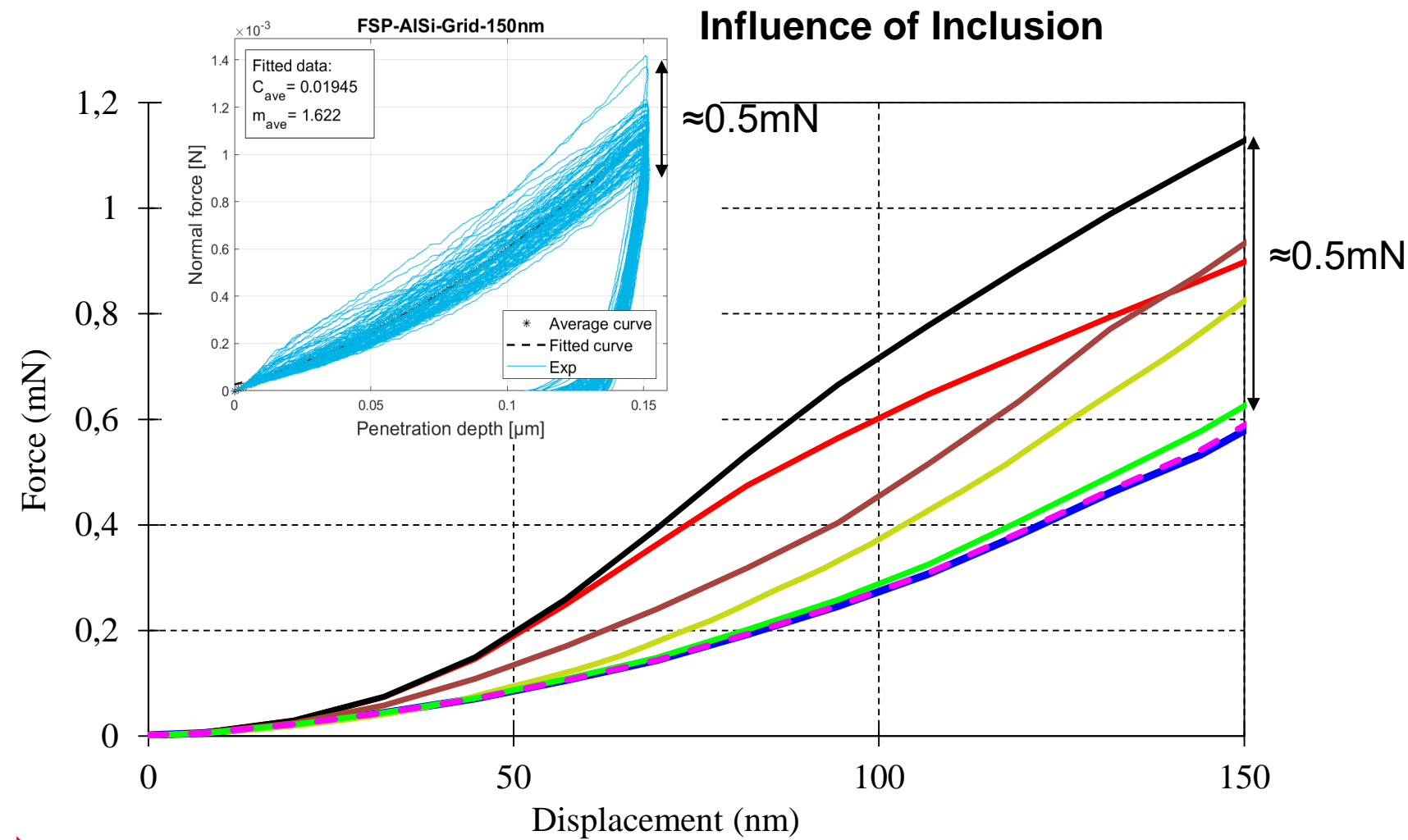
### Influence of Inclusion







**Cases A, B, C recover Experm. scattering range**

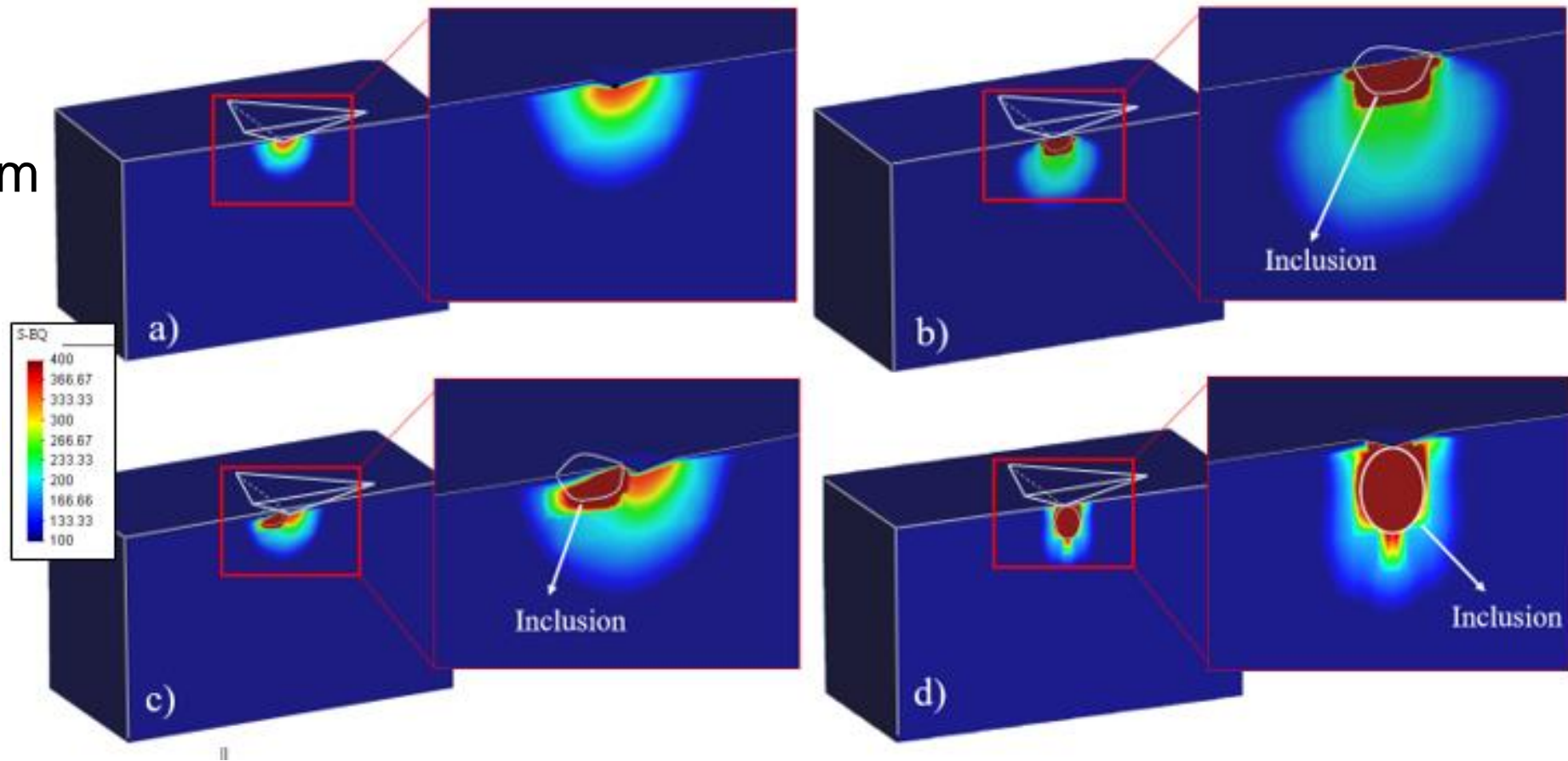


**➔ Possibility to find the  $\alpha$ -Al matrix properties : lower curves**

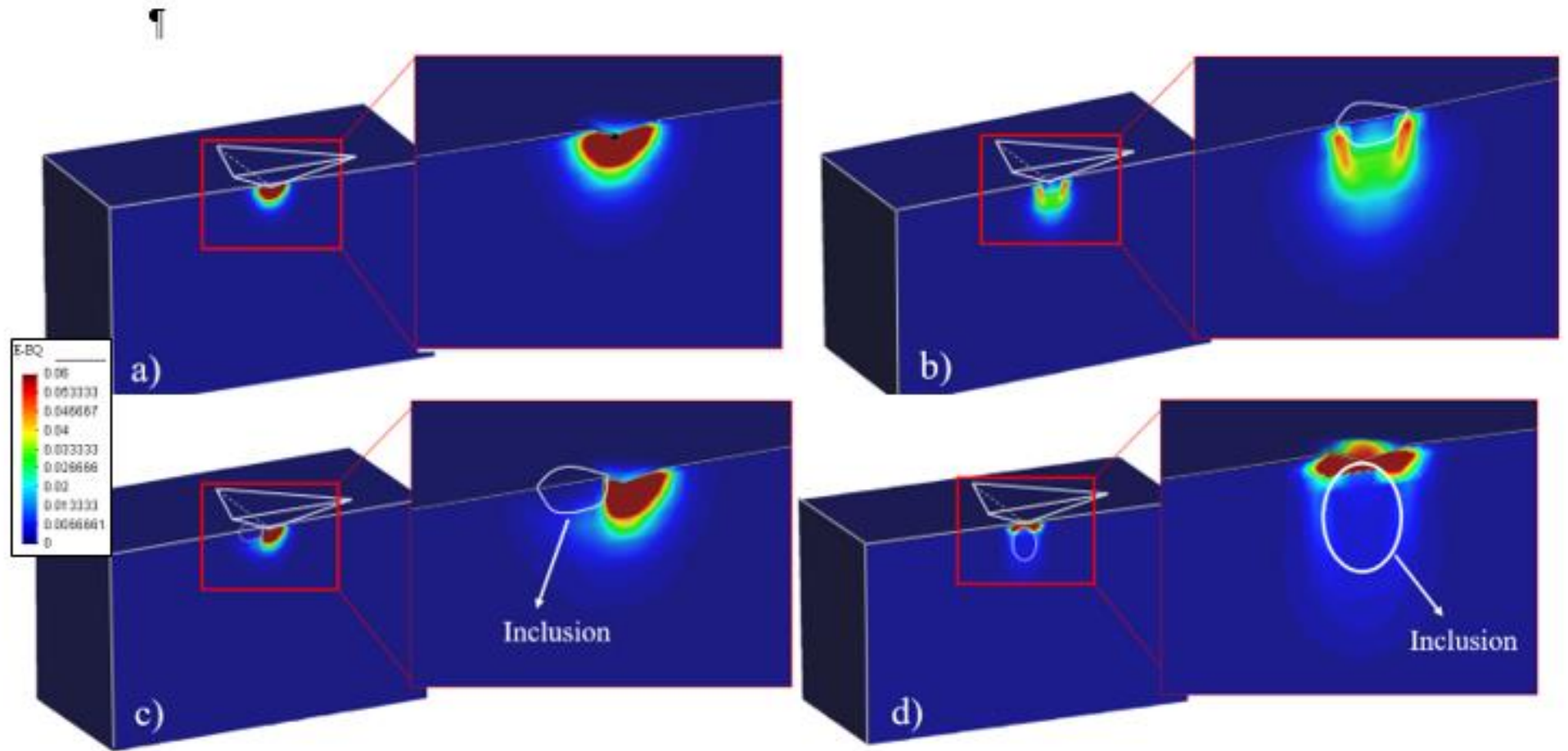


## Von Mises field

Displacement  
 $U=100$  nm,  
 $D_{inclusion}=690$  nm



Strain stress  
field  
Displacement  
 $U=100\text{ nm}$ ,  
 $D_{\text{inclusion}}=690\text{ nm}$

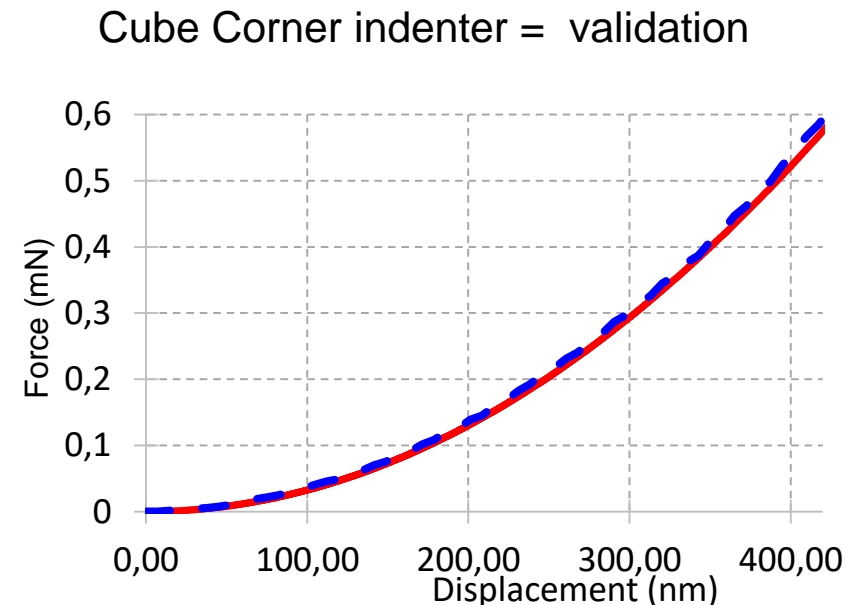
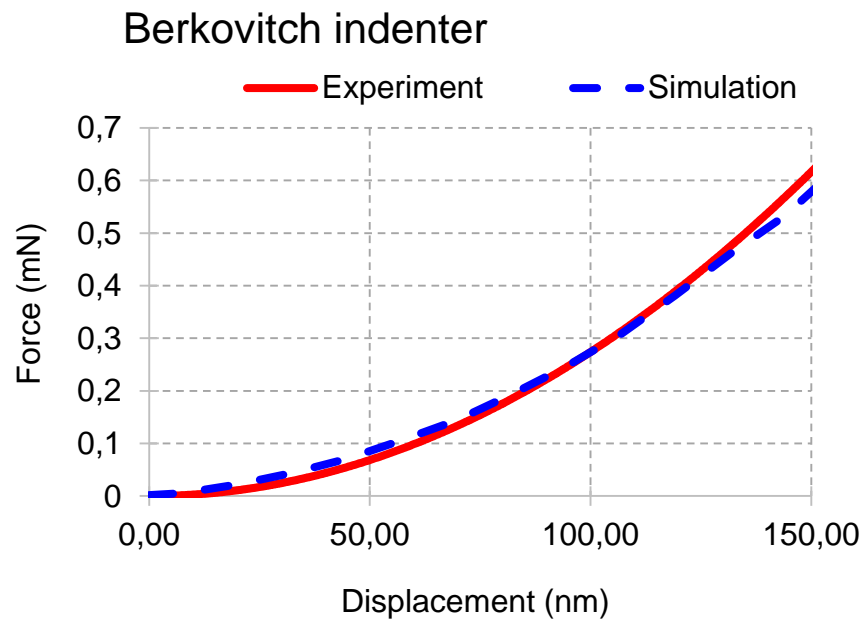


- Influence of individual inclusion on the indenter loading response
- A critical distance between the inclusion and indentation location annihilates the inclusion effect.

$$\frac{\sigma_{y,matrix}}{\sigma_{y,inclusion}} = 0.036 \text{ \& Si size [100-690] nm Berkovich curve not affected if :}$$

Si size < 200nm or distance from particule > 345 nm (proved for largest Si size)

➔ Efficient tool for curve selection process and post treatment to identify the mechanical properties of matrix phase.



## Analytical Yield stress calculation from microstructure ?

$$\text{Hall-Petch's law} = \sigma_0 + \frac{k}{\sqrt{d}}$$

➤ NaMo model [1,2]:

$$\sigma_y = \sigma_0 + \sigma_{disl} + \sigma_{sol} + \sigma_{bp}$$

- $\sigma_0$  : Pure aluminium yield strength
- $\sigma_{disl}$  : Dislocations contribution
- $\sigma_{sol}$  : Solid solution contribution
- $\sigma_{gb}$  : Grain boundaries contribution
- $\sigma_{sh}$  : Sheared precipitates contribution
- $\sigma_{bp}$  : By-passed precipitates contribution

$M$  → Taylor factor

$k_B$  → Boltzmann constant

$\dot{\epsilon}_0$  → Reference strain rate

$\dot{\epsilon}$  → Nominal strain rate

$\tau_{y0}$  → Zero-temperature stress

$\Delta E_b$  → The energy barrier.

➤ Quadratic model [3]:

$$\sigma_y = \sigma_0 + \sigma_{gb} + \sigma_{sol} + \sqrt{\sigma_{sh}^2 + \sigma_{bp}^2 + \sigma_{disl}^2}$$

For by-passed precipitates

$$\sigma_{bp} = \frac{M \cdot F_{bp}}{b \cdot l} = \frac{M}{br_p} \cdot \sqrt{\frac{3}{4\pi} \cdot \frac{f_p}{\alpha G b^2}} \cdot F^{\frac{3}{2}}$$

$b$  → Burger's vector

$G$  → shear modulus

$\alpha$  → geometric constant close to 0.5

$r_p$  → particle mean radius

$G$  → shear modulus

$T$  → Line tension generated by a dislocation inked between two obstacles

For sheared precipitates

$$\sigma_{sh} = \frac{M \cdot F_{sh}}{b \cdot l} = \frac{M F_{sh}^{3/2}}{b} \cdot \sqrt{\frac{N_a}{2T}}$$

$N_a$  → Density of precipitates by surface unit

$f_p$  → Volume fraction of the precipitates

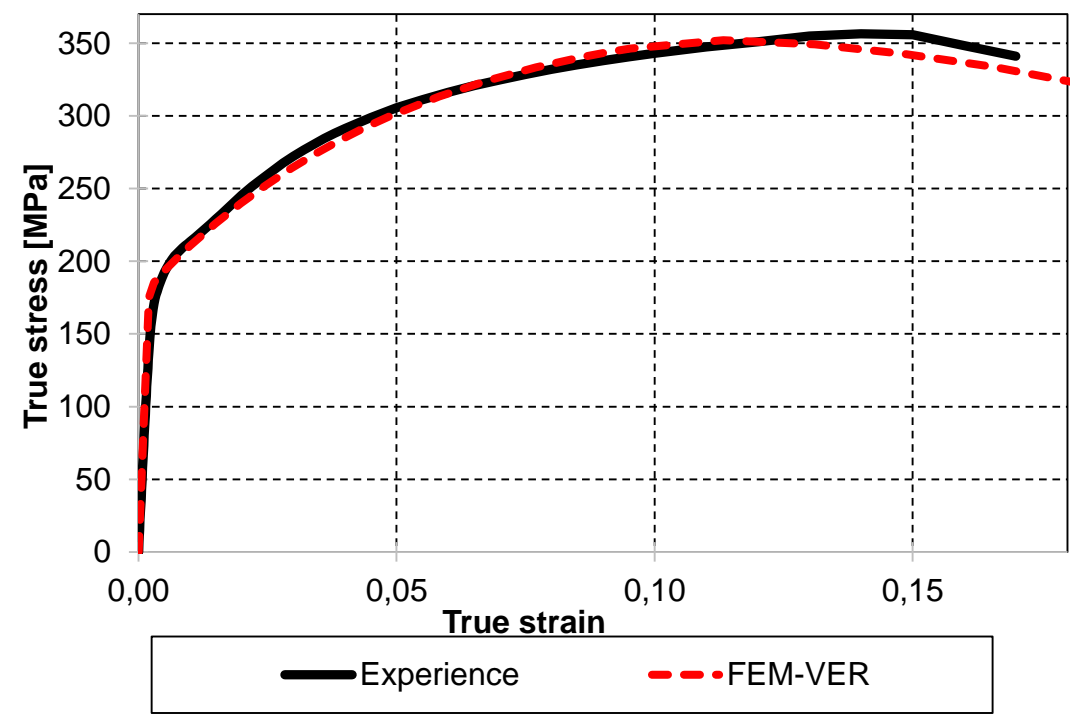
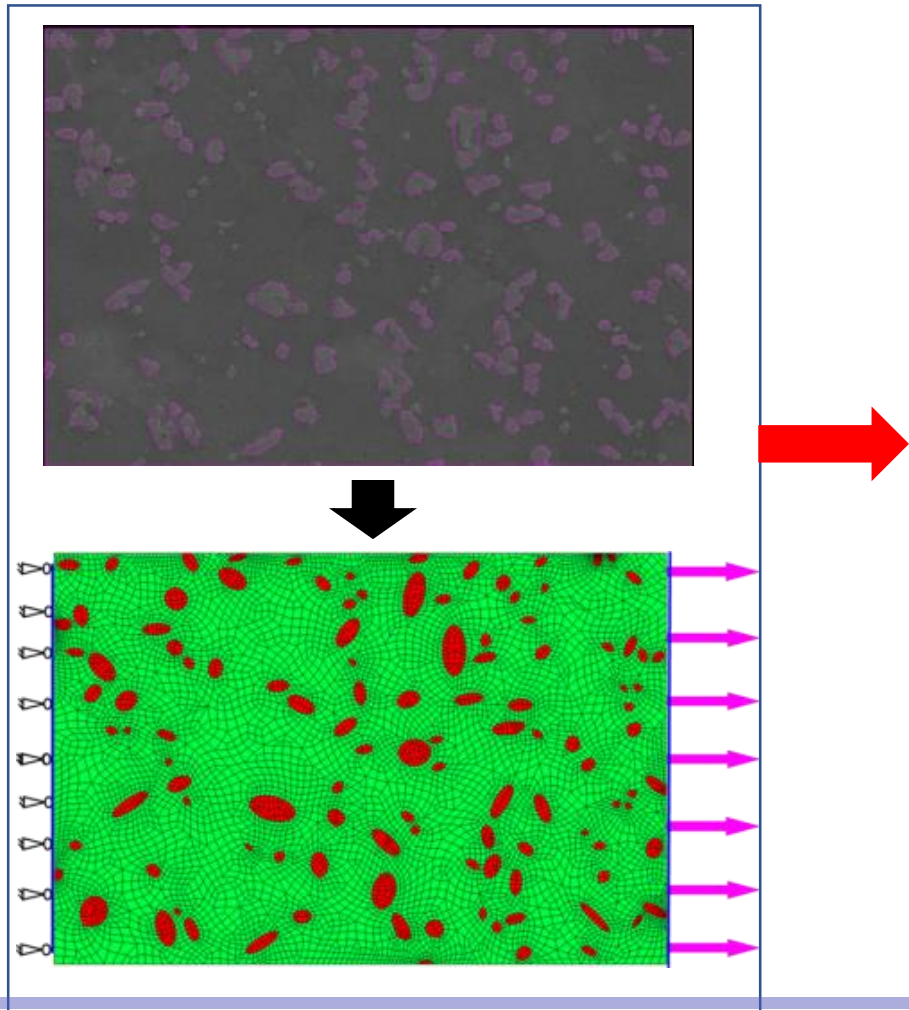
$l$  → Mean distance between precipitates

$F_{bp}, F_{sh}$  → Force required to overcome the pinning of the dislocation

[1] O. Engler et al., Materials Science & Engineering A, 2019

[2] J. Delahaye, PhD thesis, ULiège, 2022

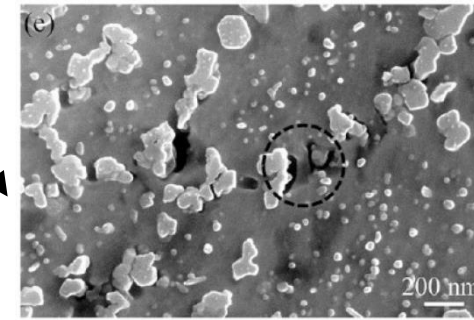
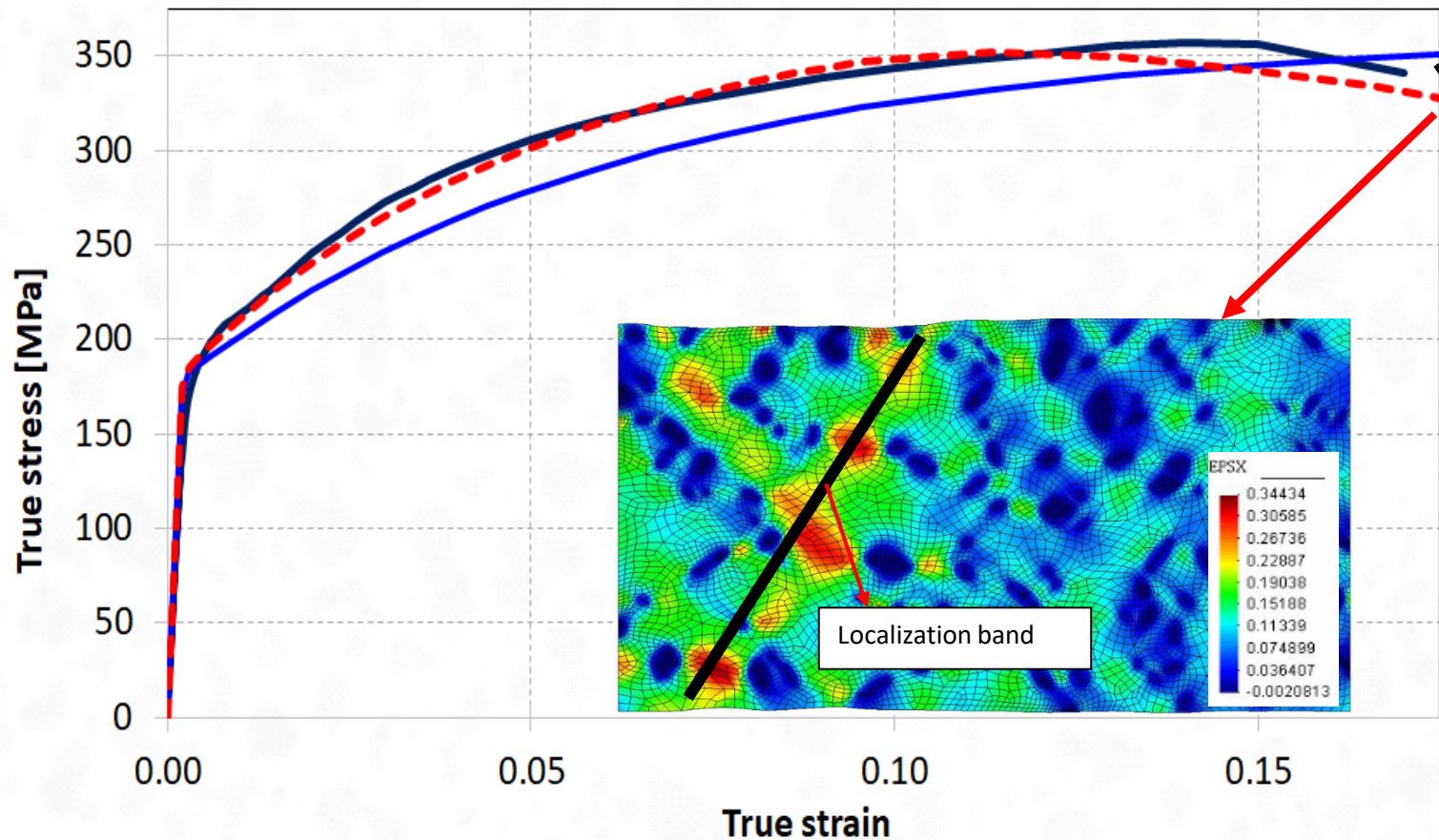
[3] J. Taurines, PhD thesis, UDLyon, 2023



Validation of large VER by tensile test model



# Large RVE Application - no cohesive element



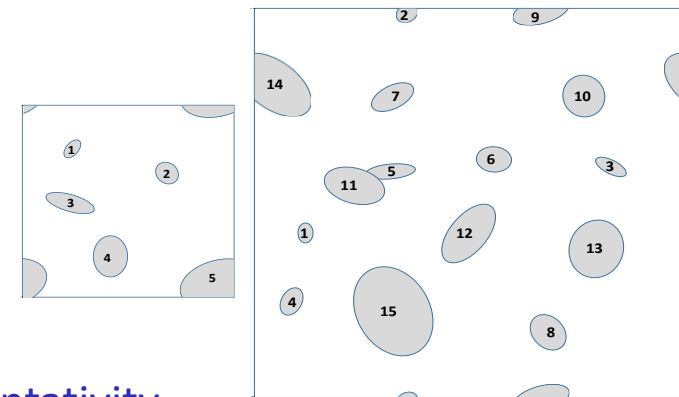
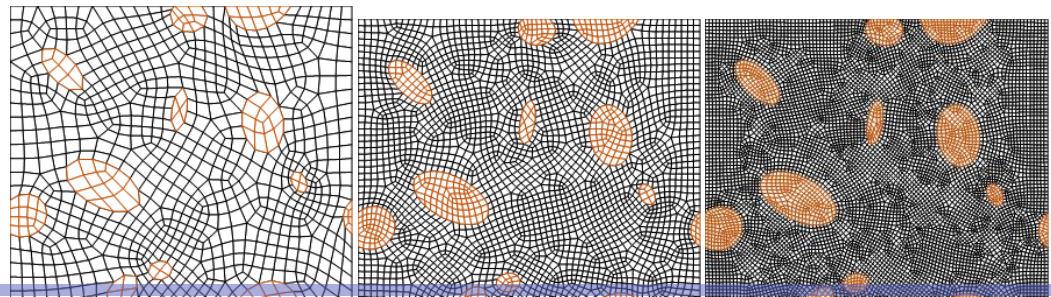
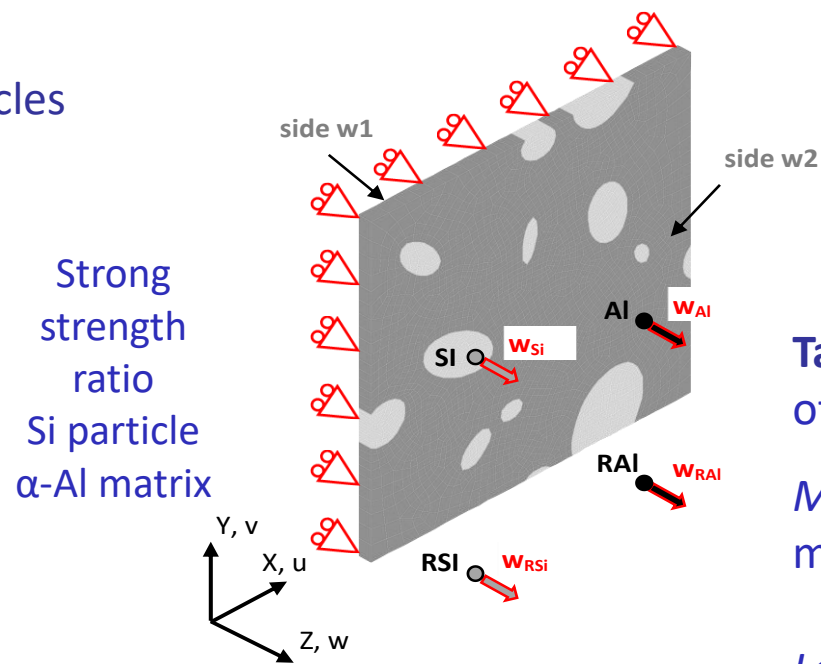
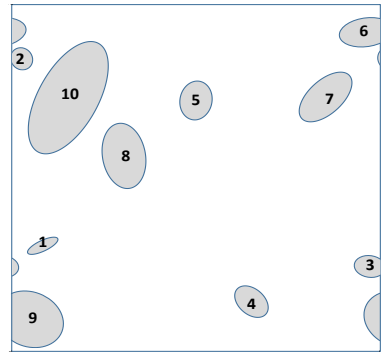
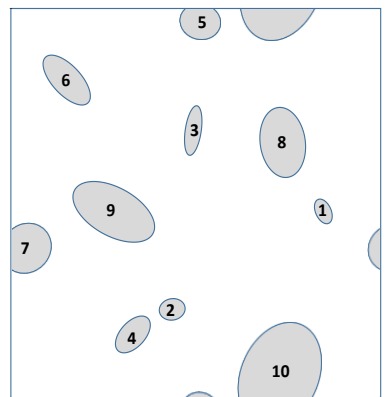
ductile failure with damage nucleation sites = mainly particle-matrix decohesion

— Experiment    — FEM only Matrix    - - - FEM with Inclusion no cohesive model

Comparison between the experiment and simulation of tensile test (big RVE no cohesive element)

# Smaller Representative Volume Element RVE

2D or 3D → 2.5D  
 Representative Size → 10 particles  
 Mesh density → medium



**Target:** representativity of a macro tensile test in Y dir.  
*Macro level* :  $\epsilon_x \approx \epsilon_z$  isotropic material  
*Local level*: strength in z dir  
 Identical for all particles  
*Interface*: Cohesive elements

*Bouffieux et al. ESAFORM 2022*

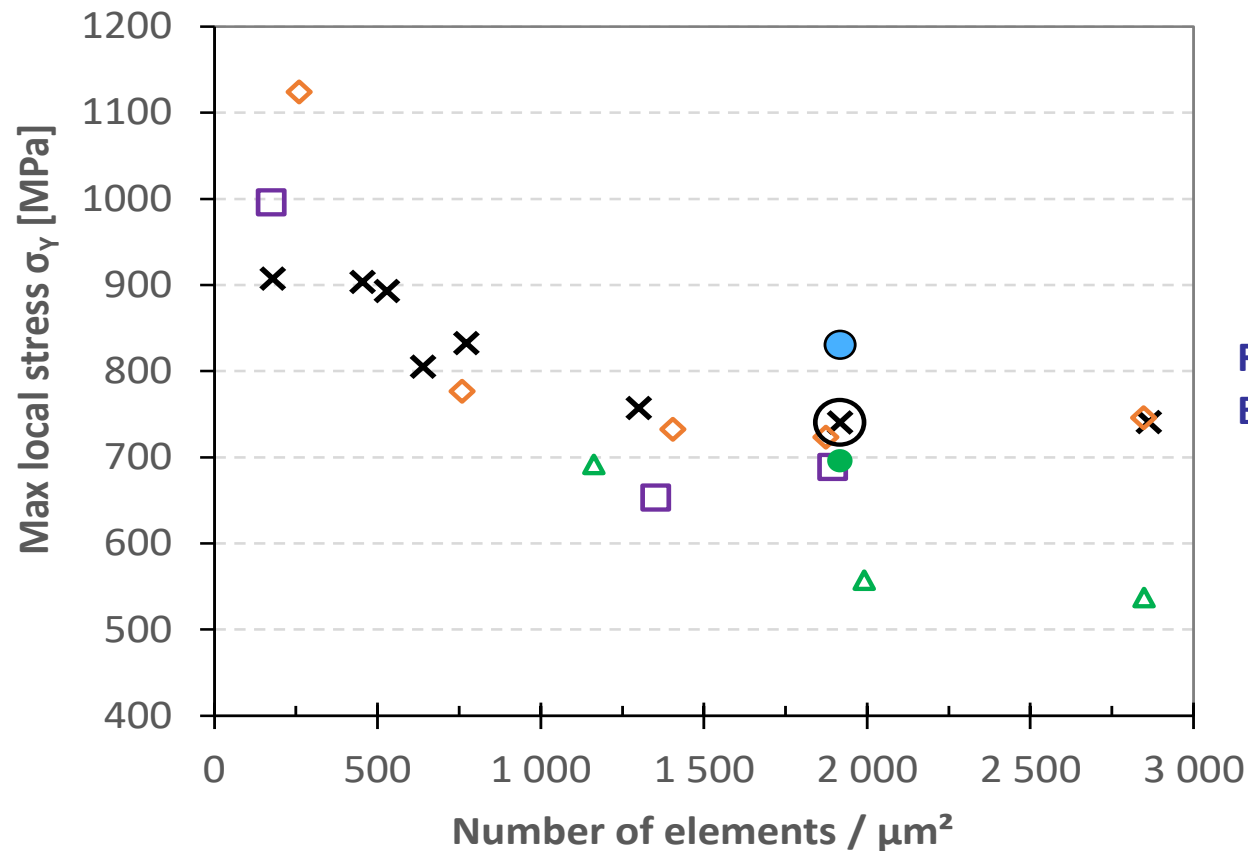


# Representative Volume Element RVE



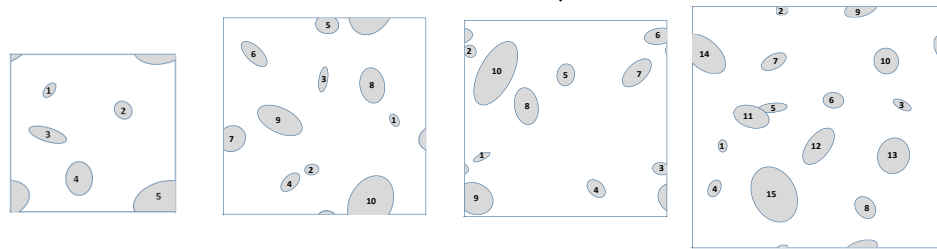
With periodic boundary conditions

RVE macro strain : 10 %



For Medium-A RVE  
Effect of boundary condition

△ Small 
 ✕ Medium-A 
 ◇ Medium-B 
 □ Large 
 ○ Ref case 
 ● Fns 
 ● Free 7

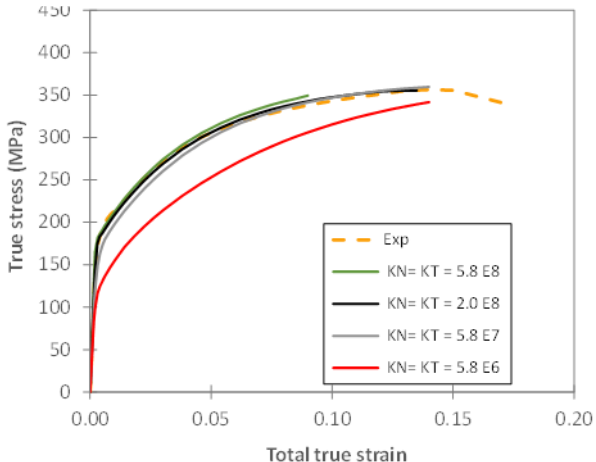
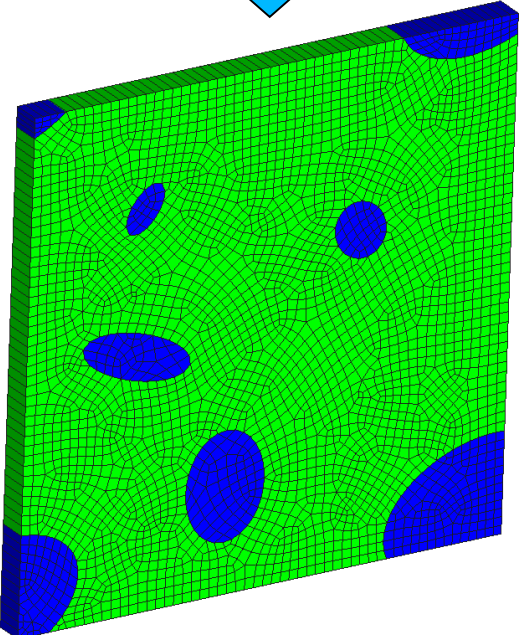
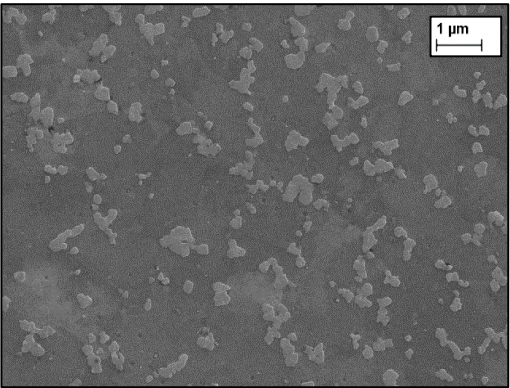


2.5D

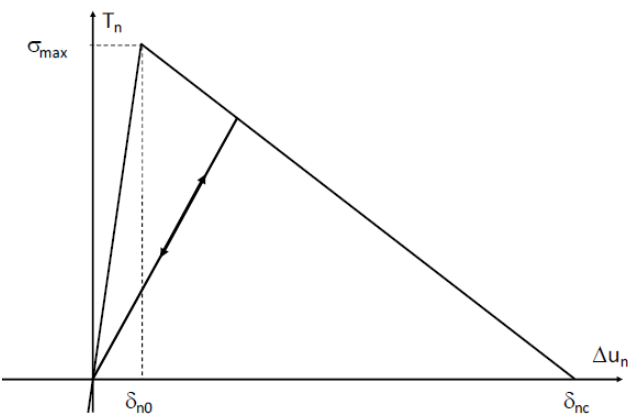
Plane strain

No macro stress however effect of strength heterogeneity (3D law 1 element layer mesh)

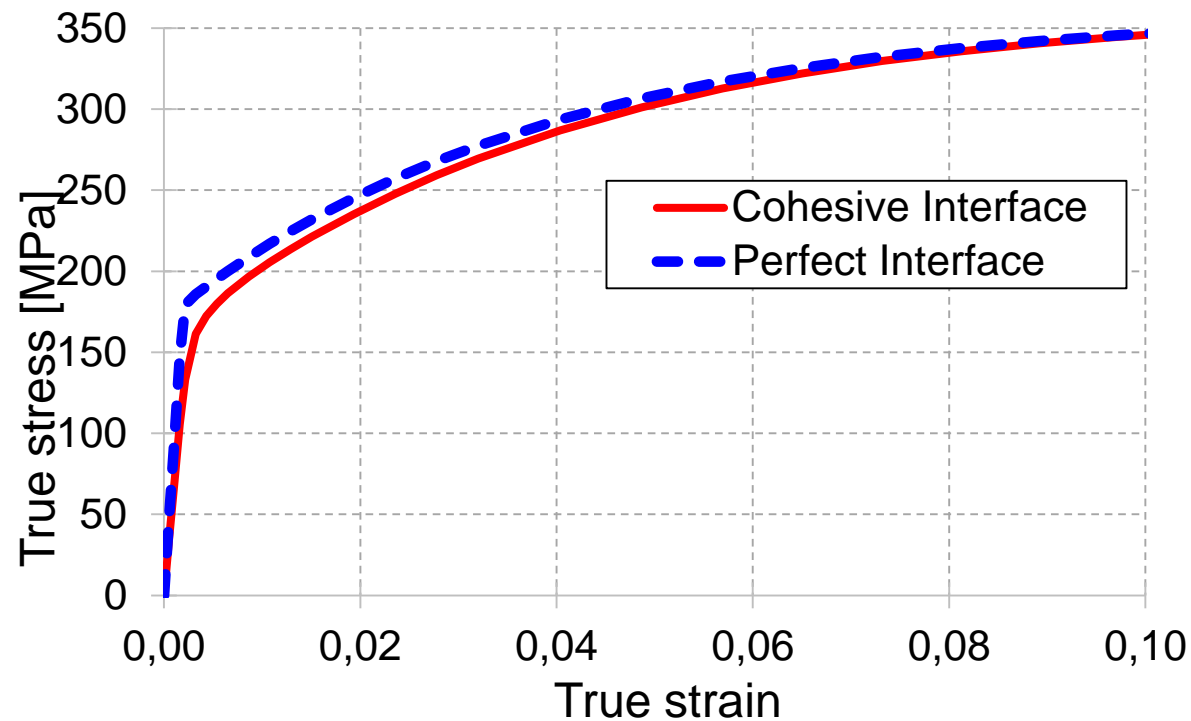
# Small RVE Application **with** cohesive element



$$KN = \sigma_{\max} / \delta_{N0} \text{ et } KT = \tau_{\max} / \delta_{T0}$$



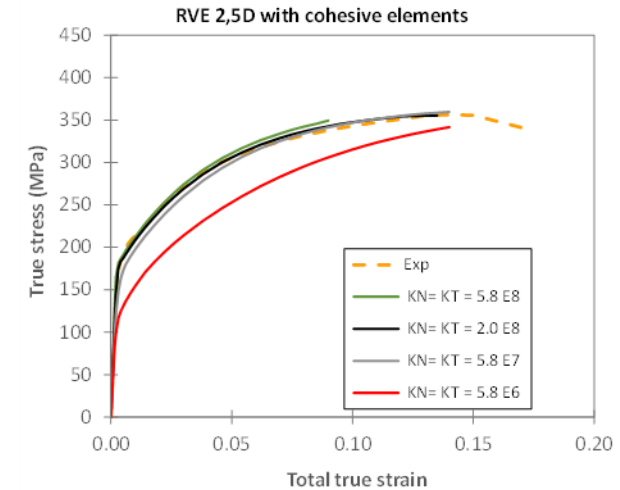
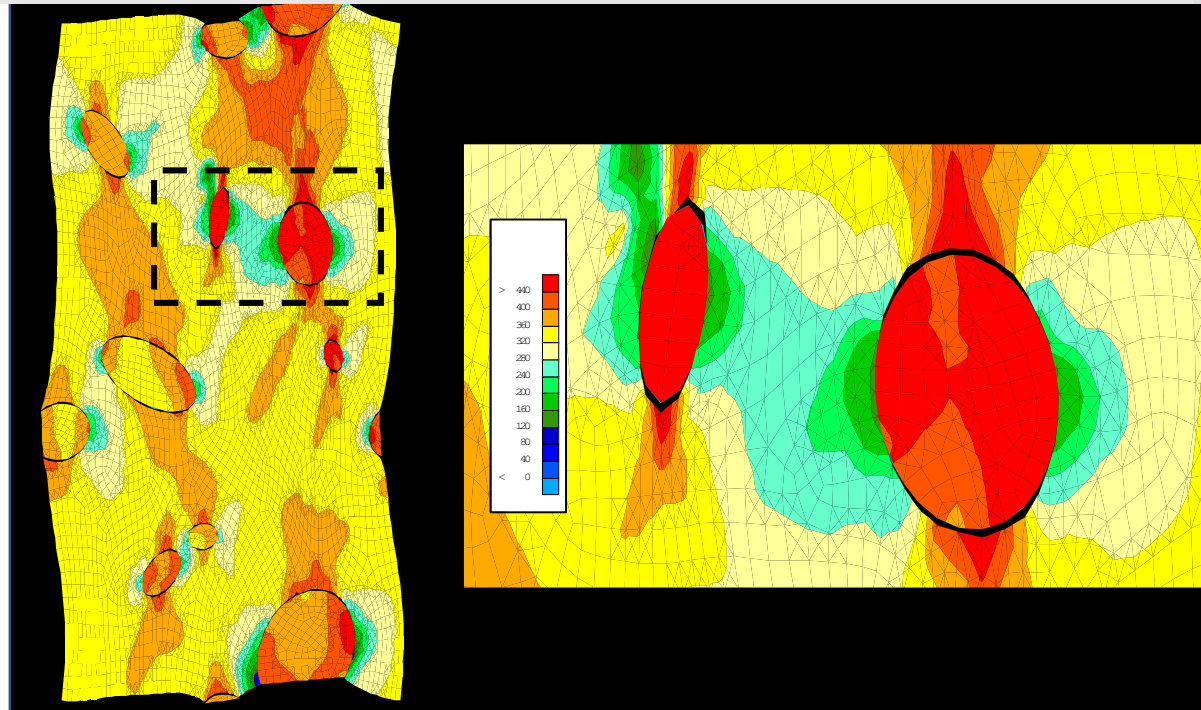
$\sigma_{\max}$	$\delta_{n0}$	$\delta_{nc}$	$\tau_{\max}$	$\delta_{t0}$	$\delta_{tc}$
550	5.5E-6	6E-6	550.	5.5E-6	6E-6



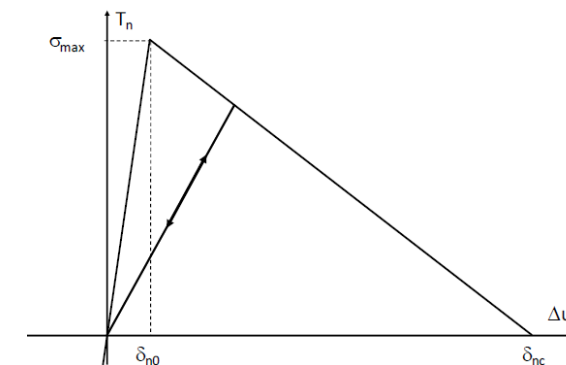
# RVE Application – with cohesive element

2.5D RVE (10 Si particles )  
with in plane : periodic  
boundary conditions

2.5D behavior  
// 3D RVE field



$$KN = \sigma_{\max} / \delta_{N0} \text{ et } KT = \tau_{\max} / \delta_{T0}$$



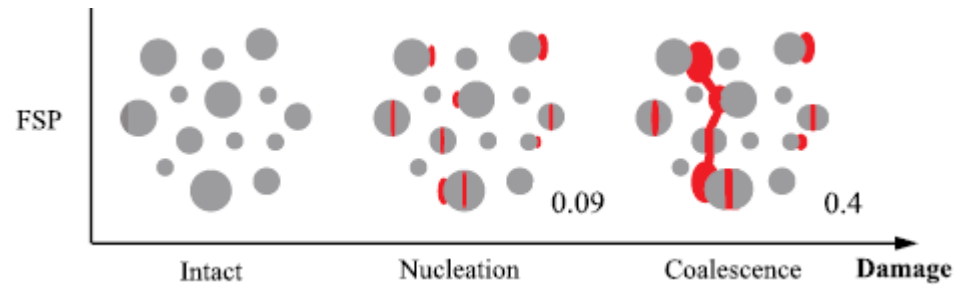
$\sigma_{yy}$  stress in tensile direction [MPa] - a macro strain of 10 %.  
Displacement multiplied by 10 to enhance decohesion.

*Cohesive element offers tensile and shear rupture*

*Identified set of parameters*

*(same strength in tensile and shear) predicts tensile cracks happen first*

Experiment analysis and RVE simulations gave the same rupture mechanism



Zhao et al. MSEA 2019

AlSi10MG processed by LPBF + FSP  
Laser Powder Bed Fusion +  
Friction Stir Processing

- Initial intention: fatigue Lemaitre & Chaboche parameters to  $\alpha$  phase damage parameters in our cohesive law identified through cyclic tests on uniaxial and notched samples
  - to predict Wöhler curves with close but different microstructures, different notches...
  - use 2.5D RVE to be able to do multi-scale simulations FE<sup>2</sup> to compute different part behaviour
- Today : energy dedicated to generate sound large FSP samples more focused on understanding process upscaling.....





## Introduction

- Motivation
- Lemaitre and Chaboche model

## Welded structures

- Microstructural evolution under operational conditions
- Macromechanical behaviour under creep

## Bi-Phasic material

- Numerical model
- Methodology
- Results & discussion

## Concluding remarks

- Conclusions
- Future prospects



- Damage models can provide quantitative information but still requires **experimental campaigns** for reliable application:
- Remind in civil engineering **scaling factors** about damage parameters
- At smaller scale accurate identification of an RVE can take different approaches → **cross your experiences** → **multiply the validations**
- Is RVE prediction correct for Baushinger effect?
- Is a coupled damage model essential or decoupled is OK ?
- Is my length scale = a tuning parameter or guided by microscopic approach?

*Thank You*

for your attention

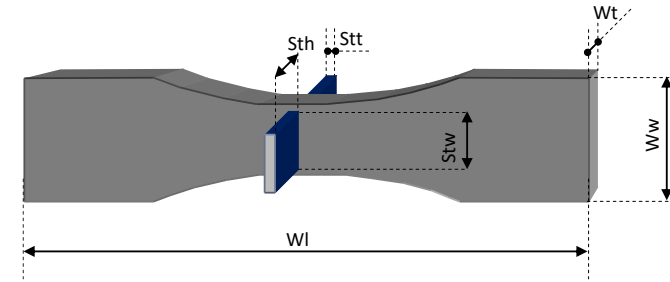
[anne.habraken@uliege.be](mailto:anne.habraken@uliege.be)





# WP 2: Fatigue study – Tests on small case specimen

Case	Test	Post treat.	Web		Stiffeners			Effect
			Wt	Wm	Stt	Stw	Sth	
			Thick	Red. width	Thick	Length	Height	
<b>A</b>	Fatigue	PIT	25	135	15	60	40	Ref. case, thick 25 mm, stiff 15 mm, welded, HFMI, edge distance. Effect of stress ratio
<b>B</b>	Fatigue	PIT	15	105	15	60	40	Smaller plate thickness // A
<b>C</b>	Fatigue	TIG	15	105	15	60	40	TIG remelting effect // B (PIT) on plate 15 mm
<b>D</b>	Fatigue	TIG	25	135	15	60	40	TIG rem. effect // A (PIT) on plate 25 mm
<b>E</b>	Fatigue	PIT	25	60	15	60	40	No distance to edges // A
<b>F</b>	Fatigue	TIG	25	115	15	40	40	TIG rem., decrease of the stiffeners length // D (TIG)
<b>G</b>	Fatigue	TIG	15	105	6	60	40	TIG rem., decrease of the stiffeners thickness // C (TIG)
<b>H</b>	Fatigue	PIT	40	60	15	60	40	Increase of the plate thickness // E
<b>I</b>	Fatigue	No	15	105	15	60	40	Fatigue behaviour of welding without post treatment
<b>K (=A)</b>	Static	PIT	25	135	15	60	40	To validate the simulation and the characterisation
<b>L (=B)</b>	Static	PIT	15	105	15	60	40	To validate the sensitivity to the sample sizes
<b>BM</b>	Fatigue		25	135				To study the fatigue behaviour on the base material alone



## Effect of:

- plate thickness
  - distance stiffener - edge
  - stiffener length & thickness
  - stress ratio
  - post-treatment (PIT / TIG rem. / nothing)
  - no welding, no post-treatment
- + static tests for characterisation validation

## Yield stress calculation

Alloys

- AlSi10Mg
- AlMg

➤ NaMo model  $\sigma_y = \sigma_0 + \sigma_{disl} + \sigma_{sol} + \sigma_{bp}$

➤ Quadratic model

$$\sigma_y = \sigma_0 + \sigma_{gb} + \sigma_{sol} + \sqrt{\sigma_{sh}^2 + \sigma_{bp}^2 + \sigma_{disl}^2}$$

- Hall-Petch's law

$$\sigma_{gb} = \sigma_{Al} = \sigma_0 + \frac{k}{\sqrt{d}}$$

$$\sigma_{disl} = M\alpha Gb\sqrt{\rho}$$

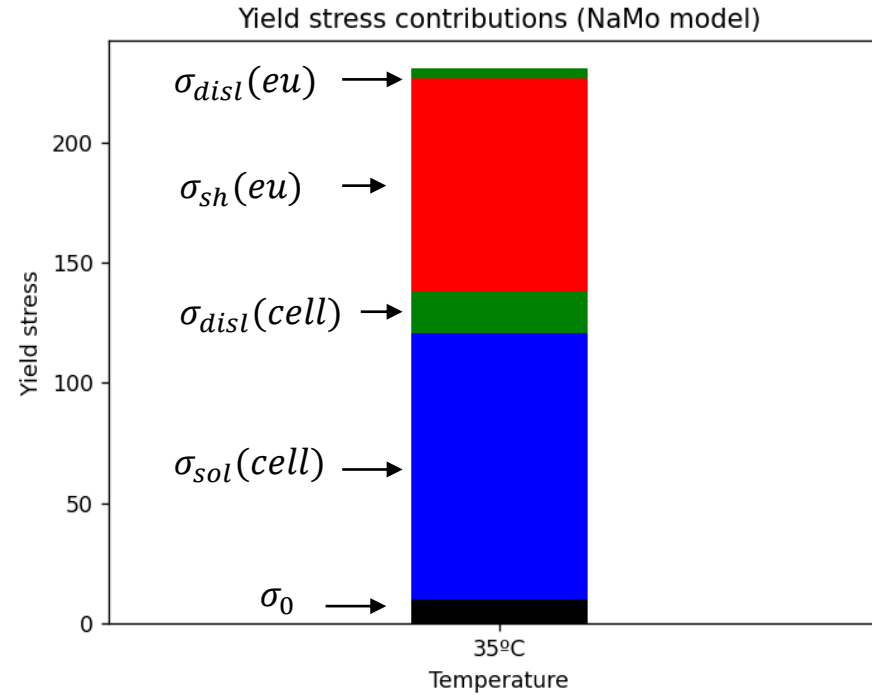
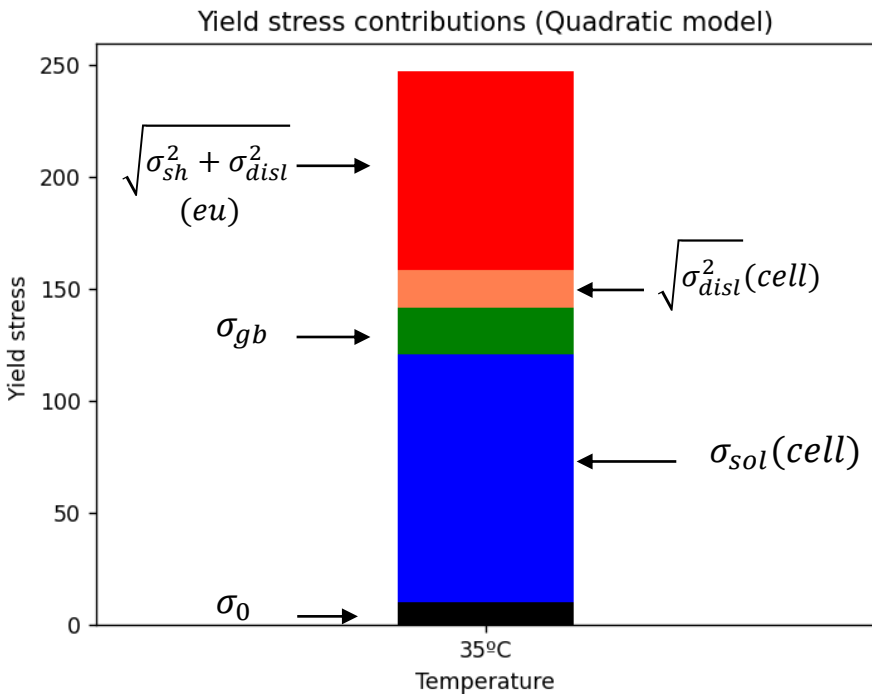
$$\sigma_{sol}(T) = M \cdot \tau_{y0} \cdot \exp\left(-\frac{1}{0.55} \frac{k_B T}{\Delta E_b} \ln \frac{\dot{\epsilon}_0}{\dot{\epsilon}}\right)$$

$b$  → Burger's vector  
 $G$  → shear modulus  
 $\alpha$  → geometric constant close to 0.5  
 $M$  → Taylor factor

$M$  → Taylor factor  
 $k_B$  → Boltzmann constant  
 $\dot{\epsilon}_0$  → Reference strain rate  
 $\dot{\epsilon}$  → Nominal strain rate  
 $\tau_{y0}$  → Zero-temperature stress  
 $\Delta E_b$  → The energy barrier.

# Representative Volume Element RVE

## Results (sample 35°C)



$\sigma_0$  : Pure aluminium yield strength  
 $\sigma_{disl}$  : Dislocations contribution  
 $\sigma_{sol}$  : Solid solution contribution  
 $\sigma_{gb}$  : Grain boundaries contribution  
 $\sigma_{sh}$  : Sheared precipitates contribution  
 $\sigma_{bp}$  : By-passed precipitates contribution

Comparison of results:

Yield stress calculation		
Experimental	NaMo model	Quadratic model
$244 \pm 12$ [MPa]	231.0 [MPa]	252.3 [MPa]