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Soft Pomeron and Lower-Trajectory Intercepts

J.R. Cudell[a1], V.V. Ezhela[b2], K. Kang[c3], S.B. Lugovsky[b4], and N.P. Tkachenko[b5]

a. Inst. de Physique, Université de Liège, Bât. B-5, Sart Tilman, B4000 Liège, Belgium

b. COMPAS Group[6], IHEP, Protvino, Russia

c. Department of Physics, Brown University, Providence RI 02912 USA

(Presented by **Kyungsik Kang**[7])

Abstract

We present a preliminary report on the determination of the intercepts and couplings of the soft pomeron and of the ρ/ω and f/a trajectories from the largest data set available for all total cross sections and real parts of the hadronic amplitudes.[8] Factorization is reasonably satisfied by the pomeron couplings, which allows us to make predictions on $\gamma\gamma$ and γp total cross sections. In addition we show that these data cannot discriminate between fits based on a simple Regge pomeron-pole and on an asymptotic $\log^2 s$-type behaviour, implying that the effect of unitarisation is negligible. Also we examine the range of validity in energy of the fit, and the bounds that these data place on the odderon and on the hard pomeron.

[1]JR.Cudell@ulg.ac.be
[2]ezhela@mx.ihep.su
[3]kang@het.brown.edu; Supported in part by DOE Grant DE-FG02-91ER40688 - Task A
[4]lugovsky@mx.ihep.su
[5]tkachenkon@mx.ihep.su
[6]COMPAS Group supported in part by RFBR Grant 96-07-89230
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[8]A short preliminary version of this work with some variation has been presented by three of us (VVE, SBL and NPT) in the 1998 Review of Particle Physics [3].

I Introduction

Irrespectively of the true nature of the pomeron, the Regge parametrisation [1, 2] plays an important role in the experimental analysis of diffractive processes at HERA [4] and in $\bar{p}p$ studies at the Tevatron and at CERN [5]. It also offers a successful phenomenological starting point at low x and low Q^2 or at the soft limit of the hadronic interactions, from which QCD evolution as well as the soft process theory based on QCD can in principle be developed or to which one would have to add a hard (leading-twist) QCD contribution [6].

Remarkably, the experience over the past several decades in phenomenological analysis of experiments has shown that the pomeron is, to a good approximation, a simple Regge pole with intercept $1+\epsilon$; despite the apparent complications of being non-perturbative in QCD, $t=0$ data can be described by particularly simple models– namely a sum of simple powers of the center-of-mass energy $\sqrt{s}$:

$$ImA_{h_1h_2}(s,t) = \sum_i (-1)^{S_i} C_{h_1h_2}(t) \left(\frac{s}{s_0}\right)^{\alpha_i(t)} \tag{1.1}$$

with S_i the signature of the exchange. The total cross section is then given by

$$\sigma_{tot}^{h_1h_2}(s) = ImA_{h_1h_2}(s,0)/s. \tag{1.2}$$

The trajectories $\alpha_i(t)$ are universal, and the process dependence is present only in the constants $C_{h_1h_2}$ (which absorb the scale s_0). The trajectories are manifest (and approximately linear) in the case of mesons, but can only be assumed in the case of the pomeron (despite the existence of strong glueball candidates [7]).

Adopting the viewpoint that simple Regge pole exchanges should account for all soft data up to the presently accessible energies, Donnachie and Landshoff (DL) [9] advanced a model for the total cross sections with just two Reggeons, i,e., an additional exchange-degenerate Reggeon representing both $C=\pm 1$ (ρ, ω, a, f) exchanges besides the pomeron:

$$\sigma_{tot}(s) = Xs^{\epsilon} + Ys^{-\eta} \tag{1.3}$$

with the intercepts given by $\alpha_p = 1+\epsilon$ and $\alpha_R = 1-\eta$. The simplicity of 1.3 has made the DL model fit of total cross sections a standard reference for models of total, elastic and diffractive cross sections [10].

Although the DL model fares reasonably well when fitting to pp and $p\bar{p}$ total cross sections, its χ^2/d.o.f. becomes considerably larger that that of other models [11] when fitting both the total cross sections and the real parts of the scattering amplitudes. This lead two of us (JRC and KK) with S. K. Kim to propose a slight generalisation of the DL model, lifting the degeneracy of the reggeon trajectories[12]:

$$\sigma_{tot}(s) = Xs^{\epsilon} + Y_+ s^{-\eta_+} \pm Y_- s^{-\eta_-} \tag{1.4}$$

The last two terms represent the exchanges of non-degenerate $C=+1(a,f)$, and $C=-1(\rho,\omega)$ meson trajectories, with intercepts $\alpha_p = 1-\epsilon$ and $\alpha_\pm = 1-\eta_\pm$ respectively. The sign of the Y_- term flips when fitting $p\bar{p}$ data compared to pp data. The real parts of the

forward elastic amplitudes are calculated from analyticity[13]. We shall refer to this model as CKK.

Despite its successes, the above parametrisation [12] left some questions unanswered. First of all, one had to introduce a filtering of the data to produce a reasonable χ^2/d.o.f., irrespectively of the model. Such a filtering inevitably leads to a bias, which we could not evaluate precisely. The present analysis uses an expanded and revised dataset [14] which does not call for a selection of datapoints. Secondly, the analysis in [12] was limited to pp and $\bar{p}p$ data, and could not consider questions related to factorisability and universality of trajectories.

We present here the preliminary results of a joint fit to pp, $\bar{p}p$, $\pi^{\pm}p$, $K^{\pm}p$, γp and $\gamma\gamma$ cross sections and hadronic ρ-parameters. The latter two processes are particularly important in the study of DIS events at HERA. We will see that the soft-pomeron intercept obtained in [12] is reproduced by global fits to all available soft data from these reactions. Also, in order to see if the soft data are enough to establish the existence of a simple Regge pole for the pomeron, we present also fits to a typical analytic amplitude model [15], i.e., Model A2 out of many possible parameterizations in [11],

$$\sigma_{tot}(s) = \Lambda\left(A + B\ln^2(s/s_0)\right) + Y_+ s^{-\eta_+} \pm Y_- s^{-\eta_-} \tag{1.5}$$

which we shall call $RRL2$ in the following, and where the last two terms represent the lower Regge trajectory terms of $C = \pm 1$ as before. We will see that the $RRL2$ fit is indistinguishable from the CKK simple-pole one. Thus while one cannot conclude that the pomeron is a simple pole, the claims concerning eventual problems with unitarity [17] is not supported either.

II Results of the simple-pole fit: the CKK model

We first give the results for the simple-pole fit (model CKK). As before [12] we fit the data above an energy cut-off $\sqrt{s_{min}}$ and require that the results be stable w.r.t. variations in that cut-off, and that the χ^2/d.o.f. be of order 1. Furthermore, in order to get a (slightly) better fit at low energy, we use the variable $\tilde{s} \equiv (s-u)/2$ in eqs. (1.4, 1.5).

We assume the intercepts ϵ, η_+, and η_- to be universal, and the couplings are then related through charge conjugation by: $X_{h_1h_2} = X_{h_1\bar{h}_2}$, $Y_{\pm}^{h_1h_2} = \pm Y_{\pm}^{h_1\bar{h}_2}$. Hence $Y_-^{h\gamma} = 0$. We shall rewrite the pomeron couplings in the following forms, which make their properties more transparent:

$$X_{pp} = x_{pp} \times \frac{3}{2} X_{\pi p}, \qquad X_{Kp} = x_{Kp} \times X_{\pi p}, \tag{2.6}$$

$$X_{\gamma p} = x_{\gamma p} \times {g_{elm}}^2 \left[\frac{1}{f_\rho^2} + \frac{1}{f_\omega^2} + \frac{1}{f_\phi^2}\right](1+\delta) X_{\pi p} \approx x_{\gamma p} \times \frac{X_{\pi p}}{213.9}, \tag{2.7}$$

$$X_{\gamma\gamma} = x_{\gamma\gamma} \times \frac{X_{\gamma p}^2}{X_{pp}}. \tag{2.8}$$

The parameters $x_{h_1h_2}$ are expected to be of order 1, because (2.6) reflect the additive quark counting, (2.7) comes from generalised vector-meson dominance (GVMD) [18], where the

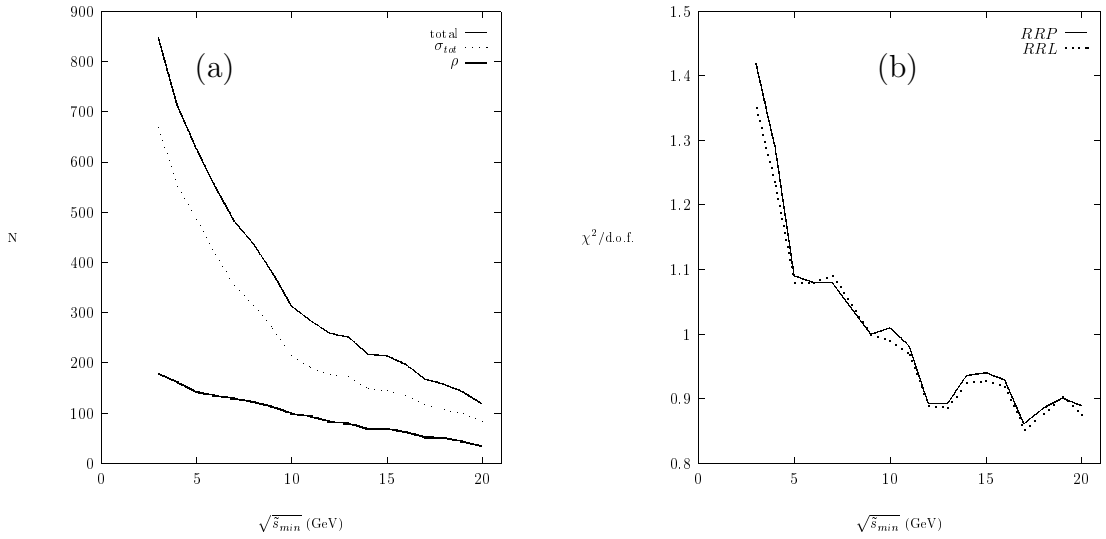

Figure 1: The number of points included in the fit (a) and the resulting χ^2/d.o.f. as a function of the minimum energy $\sqrt{s_{min}}$.

contribution of off-diagonal terms δ is expected to be about 15%, and (2.8) is a prediction from the factorisation property of the pomeron couplings.

The number of data points is shown in Fig. 1(a), and the resulting χ^2/d.o.f. in Fig. 1(b), upon changing $\sqrt{s_{min}}$ from 3 to 20 GeV. Clearly, the fit is bad for small energies. This is expected, as there is no reason to neglect the effects of lower trajectories and thresholds then. We also see that values of 1 or smaller for the χ^2/d.o.f. can be achieved for $\sqrt{s_{min}} \geq 9$ GeV.

The second criterion concerns the stability of the parameters. We show in Figure 2 the intercepts of the three trajectories entering (1.4). One sees that these parameters are stable once the energy is above 9 GeV. Also one obtains a larger pomeron intercept for a smaller energy cut-off. The intercepts from the global fit to all soft data are the same as the results of [12] above 9 GeV which is based on pp and $p\bar{p}$ data alone, and this justifies to some extent the statistical data treatment and numerical procedure employed in [12].

With the increased dataset from all available reactions, the errors of the parameters can be narrowed, compared to those of [12] where the errors correspond to a change of 5 units in χ^2. As in [12], we need both $C = \pm 1$ meson trajectories, which are non-degenerate, primarily because of the constraints coming from fitting the ρ parameters.

We show in Fig. 3 how the value of the pomeron coupling $X_{\pi p}$ together with those of $x_{h_1 h_2}$ depends on the minimum energy.

Note that the pomeron couplings become stable also for energies greater than 9 GeV. As the intercepts and pomeron couplings are the most important parameters we give our best fit results in Table 1 for $\sqrt{s} \geq 9$ GeV, where we have most statistics. If we set

$$X_{pp,p\bar{p}} \;=\; 9\beta_{qp}^2, \quad X_{\pi^\pm p} = 6\beta_{qp}^2, \quad X_{K^\pm p} = (\beta_{sp} + \beta_{qp})(3\beta_{qp})$$

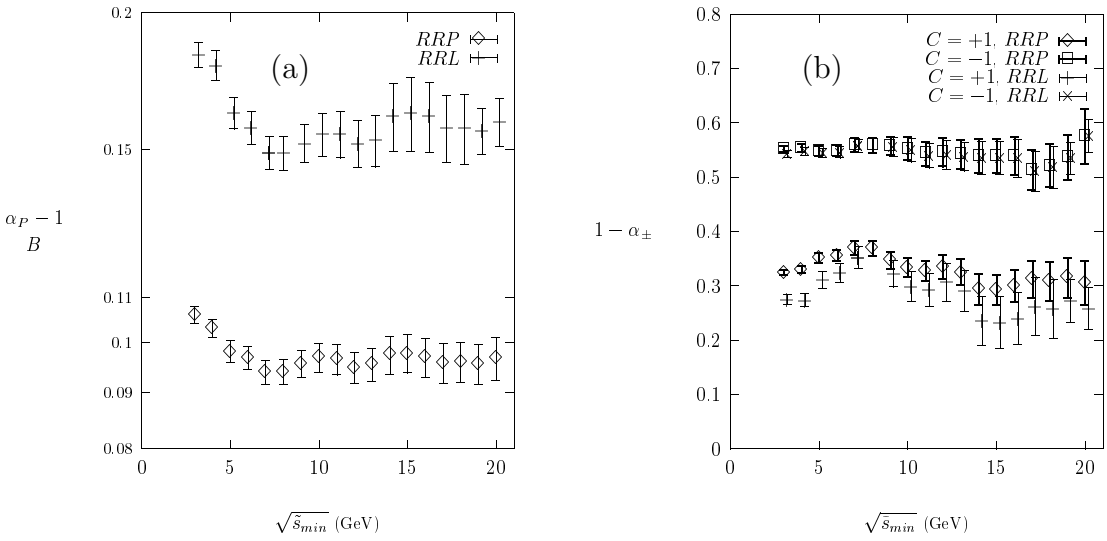

Figure 2: The value of the pomeron intercept (RRP) and of the coefficient of the $\log^2 s$ (RRL) (a) and of the intercepts of the a/f ($C = +1$) and ρ/ω ($C = -1$) trajectories.

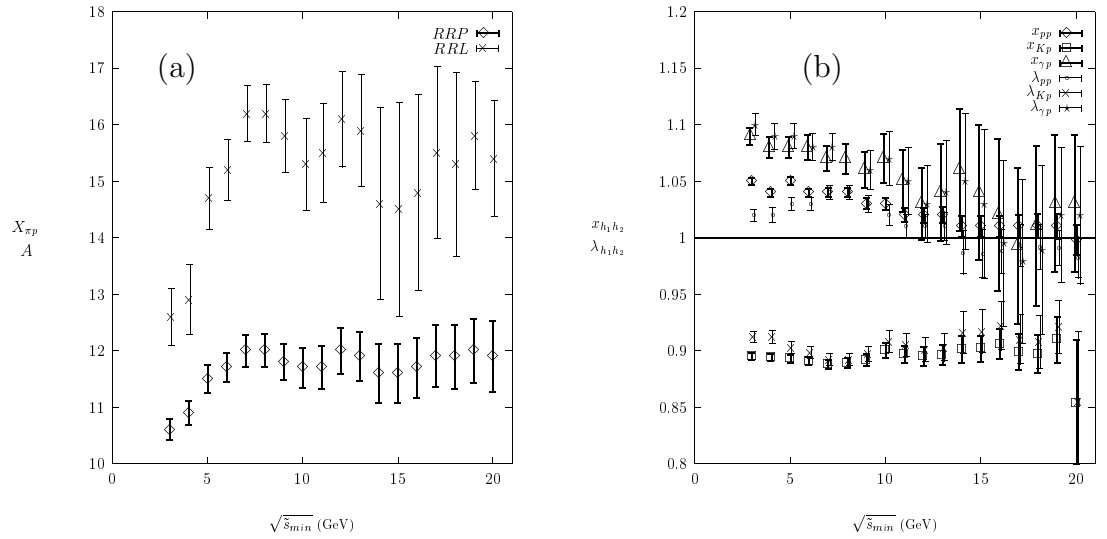

Figure 3: The value of the pomeron πp coupling (RRP) and of the constant term A (RRL) (a) and of the x and λ couplings defined in Eqs. (2.6 - 2.8) and (3.10).

ϵ	η_+	η_-	χ^2/d.o.f.	statistics	
0.096 ± 0.003	0.35 ± 0.02	0.56 ± 0.02	1.00	268	
	pp	πp	Kp	γp	$\gamma\gamma$
X (mb)	18.3 ± 0.6	11.8 ± 0.3	10.5 ± 0.3	0.059 ± 0.002	$(1.5 \pm 0.2) \times 10^{-4}$
Y_+ (mb)	61 ± 2	26 ± 1	14 ± 1	0.12 ± 0.01	$(4 \pm 3) \times 10^{-4}$
Y_- (mb)	36 ± 3	8 ± 1	14 ± 1		
process	χ^2/N, σ_{tot} (N)	χ^2/N ,ρ (N)	process	χ^2/N, σ_{tot} (N)	χ^2/N, ρ (N)
pp	1.02 (74)	1.25 (59)	K^+p	0.534 (22)	0.635 (7)
$\bar{p}p$	1.16 (33)	0.504 (11)	K^-p	0.824 (28)	1.97 (5)
π^+p	0.551 (24)	2.17 (7)	γp	0.627 (25)	
π^-p	1.13 (47)	0.939 (23)	$\gamma\gamma$	0.265 (15)	

Table 1: the values of the parameters of the hadronic amplitude in model CKK (1.4), corresponding to a cut off $\sqrt{s} \geq 9$ GeV, and the values of the individual χ^2 of the various processes, together with the number of points N.

$$X_{\gamma p} \;=\; 3\beta_{qp}\beta_{\gamma p}, \quad X(\gamma\gamma) = \beta_{\gamma p}^2 \tag{2.9}$$

where β_{qp} denotes the pomeron coupling to any of the non-strange quarks u and d, β_{sp} the pomeron-strange quark coupling, and $\beta_{\gamma p}$ the pomeron-photon coupling, we get numerically from the global fit result of Table 1 that $\beta_{qp} = 1.39(2), \beta_{sp} = 1.12(2)$ and $\beta_{\gamma p} = 0.0139(3)$. We see from this and also from Fig. 3(b) that the pomeron couplings respect factorizability based on the additive quark counting within a few percents, and that the pomeron coupling to the s quark is 15% lower than that to u and d. Also GVMD works well.

However, it is worth pointing out that the couplings of the lower trajectories are not as stable as those of the pomeron. The quark counting and factorisation are violated by about 50% for $C = +1$ Reggeon couplings, although GVMD still works well. The quark counting is totally off for the $C = -1$ Reggeon couplings. This problem of instability is easy to understand: the Regge couplings are basically representing the low energy nature of the data where they have to compete with the secondary Regge contributions and multi-Regge correction terms. At high-energies they compete with the pomeron, which determines most of the cross section. Therefore there seems to be no best cut-off for their determination. Only a model including more trajectories (and many more parameters!) and elastic and inelastic threshold effects might achieve the stability. The couplings in Table 1 are those determined with this 9 GeV energy cutoff where the χ^2, the pomeron parameters and Regge intercepts are showing stability. In this respect, the error determination based on a χ^2 variation of one unit may be underestimating the true errors, and certainly is in the case of the couplings of the lower trajectories. We also show the χ^2 per data points, and the number of data points, for each process fitted to. One can see that, as in our previous work [12], the χ^2 is a little high for some of the sub-processes. We have shown in [12] that this has nothing to do with the model, but rather with the dispersion of the data. Filtering the data for these two processes did not change the determination of the parameters. We shall demonstrate in another way in the next section that this is probably due to inconsistencies within the data, and that this does not affect our conclusions. The fits for the total cross sections and

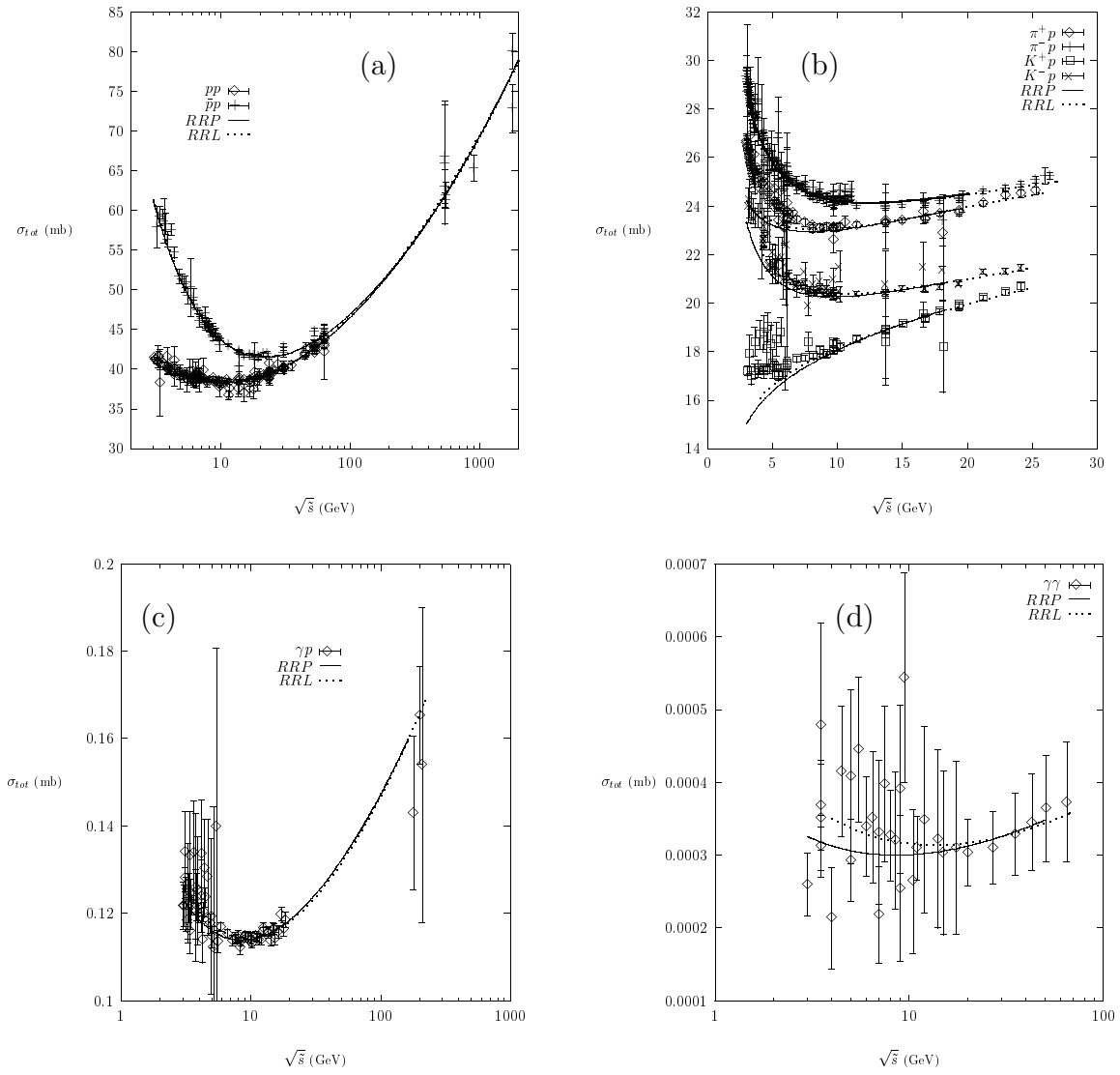

Figure 4: The total cross sections corresponding to the parameters of Table 1. The fits have been performed for $\sqrt{s} \geq 9$ GeV.

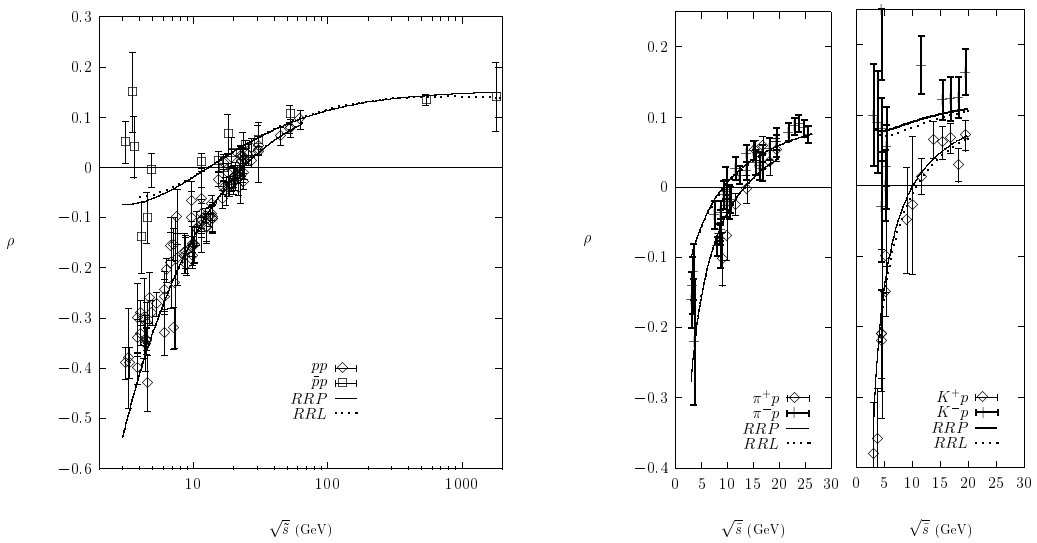

Figure 5: The ρ parameters corresponding to the parameters of Table 1. The fits have been performed for $\sqrt{s} \geq 9$ GeV.

ρ-parameters respectively for $\sqrt{s_{min}} \geq 9$ GeV, and extrapolated to $\sqrt{s_{min}} \geq 3$ GeV. are shown in Figs. 4 and 5. Although the value of χ^2/d.o.f. is bad in the low-energy region (it goes above 2), and thus is statistically unacceptable, the fits look deceptively satisfactory.

III Comparison with the $RRL2$ model

While the Regge pole hypothesis works surprisingly well with the soft data up to the Tevatron energy, this is not the only parameterization that is successful in term of χ^2/d.o.f. : there have been a number of successful analytic amplitude representations at the phenomenological level [11] based on analyticity and with the asymptotic behavior $\ln^2 s$ or $\ln s$ for the total cross sections, when appropriately modified by the meson trajectory contributions as in (1.4), which could give equally good or better fits to the pp and $p\bar{p}$ data as shown in [11]. In these models, one may also regard the asymptotic $\ln^2 s$ or $\ln s$ terms as an effectively unitarised form [19] of the bare pomeron term of s^ϵ. We know that a simple pole will eventually violate the Froissart-Martin bound. We show here first that in the region of available data, the two descriptions of Eq. (1.4) and (1.5) are indistinguishable. Furthermore, the instability present in the lower trajectory couplings of the simple-pole fit has nothing to do with the assumptions regarding the pomeron.

We present here the results of the fit to all soft data of total cross sections and ρ-parameter for the modified Amaldi-Schubert ($RRL2$) model (1.5) suitably factored to exhibit the factorization property by the $\ln^2 s$ term. In (1.5), Λ will be different for different reactions and will follow the factorization property based on the additive quark counting, if the $\ln^2 s$ term is to represent the pomeron exchange contributions. Here, the parameters A and B and the meson trajectory intercept parameters η_+ and η_- are taken to be universal

A (mb)	B (mb)	η_+	η_-	χ^2/d.o.f.	
15.8 ± 0.3	0.15 ± 0.06	0.32 ± 0.02	0.56 ± 0.02	1.00	
	pp	πp	Kp	γp	$\gamma\gamma$
Λ	1.55 ± 0.01	1	0.90 ± 0.07	$4.96 \pm 0.08) \times 10^{-3}$	$(1.3 \pm 0.2) \times 10^{-5}$
Y_+ (mb)	51 ± 2	20 ± 1	10 ± 1	0.088 ± 0.008	$(3 \pm 3) \times 10^{-4}$
Y_- (mb)	36 ± 3	7 ± 1	14 ± 1		

Table 2: the values of the parameters of the hadronic amplitude in model $RRL2$ (1.5), corresponding to a cut off $\sqrt{s} \geq 9$ GeV. and the values of the individual χ^2 of the various processes. The number of points, the statistics and the χ^2 for each process are the same as for Table 1.

for all reactions, while the factorization parameter Λ which will be set to 1 for $\pi^{\pm}p$. In analogy with (2.6 - 2.8) we define:

$$\Lambda_{pp} = \lambda_{pp} \times \frac{3}{2} \ , \ \Lambda_{Kp} = \lambda_{Kp},$$
$$\Lambda_{\gamma p} \approx \frac{\lambda_{\gamma p}}{213.9} \ , \ \Lambda_{\gamma\gamma} = \lambda_{\gamma\gamma} \times \frac{\Lambda_{\gamma p}^2}{\Lambda_{pp}} \tag{3.10}$$

In order to simplify our discussion, and in order to have the same number of parameters for both fits, we set[9] $s_0 = 1$ GeV^2. We again use $\tilde{s}$ instead of s in Eq. (1.5) to improve the fit at small energy.

We proceed as for the simple-pole fit. We see from Figs. (1-5) that identical problems and successes are present in this case. It is interesting to observe that the couplings of the a/f trajectory go down a little, but remain unstable, whereas the $C = -1$ contribution remains identical. One amazing outcome is that despite this variation, the two fits are identical. As shown in Fig. 1, the χ^2/d.o.f. are the same. If we again settle on the parameters corresponding to $\sqrt{s}_{min} = 9$ GeV, shown in Table 2, we obtain the dashed curves of Figs. 4 and 5, which are almost identical to those of the simple-pole fit. This has two important consequences. First, the simple-pole assumption is one of the possibilities, but not the only one. One has to realise however that the property of quark counting and factorisation, exhibited by both fits, is hardly understandable outside of a simple-pole ansatz. Hence it is the physics that must make us prefer this fit and the possibility to extend it to elastic and diffractive events, and not simply the quality of reproduction of the $t = 0$ data. Furthermore, as already mentioned, the indistinguishability of the simple pole from the $\log^2 s$ proves that effectively there cannot be any problem with unitarisation. As the difference in total cross sections at the LHC energies is at most 6 mb it seems unlikely that such an effect will be detectable, even if we assume that the total cross section will be reliably measurable within the approved CERN program.

[9]It is possible to get slightly better fits below $s = 9$ GeV^2 if one lets this parameter free, but it reaches unphysical values of the order of 100 MeV^2 or smaller, and the stability of the fit is not improved.

IV Concluding remarks

There is no room in any of the considered models for another $C = +1$ Regge trajectory with intercept $0.8 \leq \alpha_{new} \leq 1.2$. All couplings are then set to a very small value. Note however that this conclusion is possible only after inclusion of the real parts in the dataset. Total cross sections only allow (and slightly favour) an extra flat contribution.

As for the presence of a hard pomeron with intercept ≈ 1.4 [6], there was no room in pp and $\bar{p}p$ data for such an object [12], and we obtain a (1σ) upper bound on its coupling: $X_{new}/X \leq 5 \times 10^{-9}$. However, such a large intercept will certainly call for a strong unitarisation. It is puzzling that in fact the presence of such a term is favoured both by the πp and Kp data, particularly for the measured ρ parameters. Notice from Table 1 that the individual χ^2 were rather high for the fits to the ρ parameters of π and K. These χ^2 per data point get lowered to from 2.17 (resp. 1.97) to 1.51 (resp. 1.42) in the presence of a hard pomeron. The global fit then gets a χ^2/d.o.f. of 0.89 instead of 1.00. The hard pomeron intercept is fitted to 0.38 ± 0.13, with a coupling of the order of 1.5% that of the soft pomeron. The γp data do not favour such a term, and the errors on the $\gamma\gamma$ data are large enough to allow for it. But as the total γp and $\gamma\gamma$ data are under reconsideration, and as unitarity effect may set in early, it is hard to draw any conclusions about these.

The data does not allow further $C = -1$ trajectories. The quality of the fit is not improved by the introduction of one such trajectory, and all couplings are less that 1 per thousandth of the soft-pomeron coupling. We have also tried to stabilize the couplings of lower trajectories through the introduction of new $C = \pm 1$ trajectories with lower intercepts. However, the fit depends then on 26 parameters, and the data is not constraining enough to draw any firm conclusion.

Concerning the existing conflicting data, first of all, the long-standing problem of the measurement of the total cross section at the Tevatron [20] cannot be resolved by this method. As can be seen from Figure 4, the fit chooses the middle points of the two measurements. Because of our choice of χ^2, which does not privilege higher-energy data, this conclusion is stable independently of removal of either the CDF or the E710 point from our dataset. Secondly, there is a well-known uncertainty about the HERA total cross section[21]. Our fit favours the H1 measurement. Imposing GVMD exactly leads to exactly the same conclusion. This is expected in view of the ZEUS analysis of low-Q^2 F_2 data [22]. Finally, as there may be some problem regarding the value of the $\gamma\gamma$ cross sections [23], we can remove the high-energy $\gamma\gamma$ data from our dataset, and we can predict the $\gamma\gamma$ cross section imposing factorisation $x_{\gamma\gamma} = y_{\gamma\gamma} = 1$. This leads to the dashed curve of Fig. 6. We see that the factorisation hypothesis favours higher numbers than the published L3 data but compatible with the preliminary OPAL measurement[10]

We have shown that the soft pomeron produces very good fits to $t = 0$ data, once the energy is bigger than 9 GeV. From our new compilation of data points, and from the 264 points above 9 GeV, we determined the pomeron intercept to be 1.096 ± 0.03, in agreement with the conclusions of [12]. Lower $C = \pm 1$ trajectories are non-degenerate, and have intercepts given in Table 1. The determination of these parameters is stable and reliable, as

[10]New L3 $183GeV$ and OPAL measurements are now consistent as reported to ICHEP'98 in Vancouver [24]. We would like to thank A. De Roeck for bringing this to our attention.

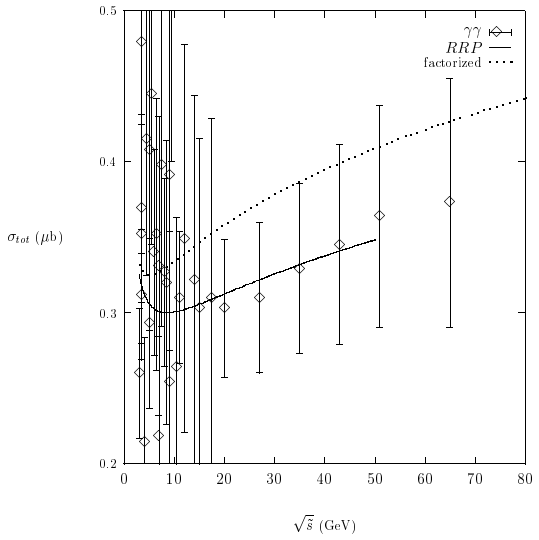

Figure 6: The $\gamma\gamma$ total cross section at LEP, assuming factorisation.

is that of the pomeron couplings, but the interplay between $C = +1$ contributions makes the determination of the couplings of the a/f trajectories problematic. Finally, $t = 0$ data are not sufficient to rule out other models of forward scattering amplitudes, but the factorisation and quark counting properties which seem to be well respected are difficult to be understood outside of the context of simple poles. Further details of the work along with the fits to other analytic amplitude models and the results of the efforts to ameliorate the instability of the lower Reggeon couplings will be reported elsewhere[25].

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