A one-leader multi-follower approach to distribution network development planning

Bailly Geoffrey
Montefiore institute
University of Liège
Liège, Belgium
gbailly@uliege.be

Cornet Manon
Montefiore institute
University of Liège
Liège, Belgium
mcornet@uliege.be

Glavic Mevludin
Montefiore Institute
University of Liège
Liège, Belgium
Mevludin.Glavic@uliege.be

Cornélusse Bertrand
Montefiore Institute
University of Liège
Liège, Belgium
Bertrand.Cornelusse@uliege.be

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***Abstract*—The growing electrification of transportation, heating, and cooling will largely impact electricity distribution networks. To determine how to develop distribution networks, it is paramount to consider jointly the multi-year distribution network development plan and the grid users' energy infrastructure evolution. To this end, we formulate a bilevel program in a one-leader multi-follower setting with the distribution network development plan as the upper level, while the lower level minimizes grid users' energy costs. Solving this optimization problem allows for assessing the impact of exogenous factors such as grid tariffs on network development plans and grid user investment in distributed energy sources and storage. Some initial results are reported using a small test system.**

***Index Terms*—distribution network, development plan, co-optimization, bilevel programming**

I. INTRODUCTION

The transition towards a decarbonized energy sector requires a significant increase in the deployment of renewable energy sources (RES) and the electrification of other sectors, such as mobility, heating, and industrial processes. Electricity networks in this process have to develop in an effective way [1]. In this respect, co-optimization appears to be a key to successful electricity network development.

This work is concerned with the distribution network development plan as an important aspect of the efficient transition toward a decarbonized energy sector. In Europe, the importance of this problem is identified, and the European Commission sets the rule requiring distribution system operators (DSOs) to conduct their network development plan at least every two years for a time horizon from five to ten years [2]. Several approaches have been considered for distribution network development planning [3]. In this work, we are interested in the use of bilevel programming [4].

Bilevel programming is an optimization approach that fits the context of co-optimization by formulating an upper-level (leader) and one or more lower-level (follower) optimization problems [5]. This is well recognized in the power system research community when concerned with the problem of distribution network development planning [6], [7]. The work presented in [6] formulated generation and distribution network development as an upper-level problem while the lower-lever focused on demand response. In [7], the upper-level problem is set to optimize distribution network development, while the lower-level ones include RES and demand aggregators.

This paper proposes a comprehensive distribution network development planning problem as a bilevel program in a one-leader and multi-follower setting. Initial results are provided based on a standard test system used in the literature. We finally provide a discussion on possible extensions of the problem setting.

The problem formulation is presented in Section II, while Section III presents the results using a small test system. Section IV discusses the results. In Section V some conclusions and future research are presented.

II. PROBLEM FORMULATION

For ease of presentation, we do as if we were developing a totally new distribution network. Some impositions can be added to represent an existing network. One input of our problem is a graph $\mathcal{G} = \{\mathcal{B}, \mathcal{E}\}$ where nodes in $\mathcal{B}$ are electrical buses and edges in $\mathcal{E}$ are routes between buses where conductors can be placed to develop the distribution network. Some electrical buses are candidate substations where the distribution network under consideration can connect to a higher-voltage network, which is assumed already developed, and withdraw or inject power. Buses are indexed as $1, 2, ..., n$ and the candidate substations buses $\mathcal{B}_s$ are the first $n_s < n$ indices. Users can connect to the distribution network at buses that are not candidate substations, that is, buses $\mathcal{B} \setminus \mathcal{B}_s$.

We are looking for one or several distribution networks out of $\mathcal{G}$ such that all users are connected to a substation. Each distribution network must contain only one substation and have a radial structure to be coherent with the usual distribution network operation rules. Mathematically, we want $\mathcal{G}^\star = \bigcup \mathcal{G}_i(\mathcal{B}_i, \mathcal{E}_i)$, $\mathcal{B} = \bigcup \mathcal{B}_i$, $\bigcap \mathcal{B}_i = \emptyset$, $\bigcup \mathcal{E}_i \subseteq \mathcal{E}$, where $\mathcal{E}_i$ contains selected routes that form a spanning tree of $\mathcal{B}_i$, $\forall i$, $\bigcap \mathcal{E}_i = \emptyset$, and each $\mathcal{B}_i$ contains one substation node. Additionally, we can choose among a set of conductors $\mathcal{K}$ in each selected route. The DSO should design $\mathcal{G}^\star$ to minimize costs while satisfying users' needs with sufficiently high reliability, imposed as operational constraints on grid voltages and line currents in the sequel.

Users can choose between several options to satisfy their energy needs that are assumed fixed, although their power

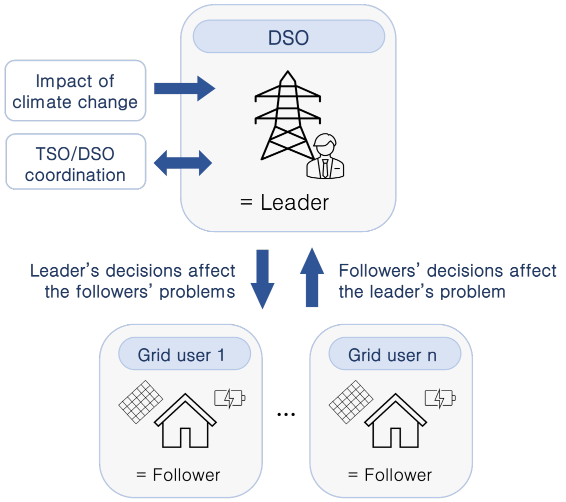

Fig. 1. Illustration of the considered bilevel optimization problem.

consumption can be considered flexible to some extent. They can buy from the DSO some capacity to withdraw or inject into the distribution network. They can also decide to invest in generation and storage. The total cost a user is minimizing is the addition of these investments and operational costs, such as the electricity purchased from the public grid.

We want to determine the impact of these options on the equilibrium point of the global system, that is, to which extent the DSO should invest in the distribution networks, and users should invest in their own generation and storage assets.

To this end, we formulate a bilevel program with the distribution network development plan as the upper level (a leader), while the lower level (followers) minimizes grid users' energy costs. It is illustrated in Fig. 1 and further detailed in the sections below. However, we limit the mathematical details of the presented models for conciseness and discuss possible extensions and their potential implications in Section V.

A one-leader multi-follower bilevel programming problem can be written in compact form as [5]:

$$
\begin{aligned}
\min_{x,y_u,y_{l1},\dots,y_{ln}} \quad & F(x, y_u, y_{l1}, \dots, y_{yln}) && \text{(1a)}\\
\text{s.t.} \quad & G(x, y_u, y_{l1}, \dots, y_{ln}) \leq 0 && \text{(1b)}\\
& \forall\, y_{lk},\ k = 1, \dots, n : \\
& \min_{y_{lk}} f(x, y_u, y_{l1}, \dots, y_{ln}) && \text{(1c)}\\
& \text{s.t.}\ \ g(x, y_u, y_{l1}, \dots, y_{yln}) \leq 0 && \text{(1d)}
\end{aligned}
$$

where x represents state variables (both upper and lower level), y_u and $y_{l1}, y_{l2}, \dots, y_{ln}$ upper and lower level decision variables (n followers are assumed). Equations (1a) and (1b) are upper level objective and constraints while (1c) and (1d) represent the lower-level objective and constraints.

A. Upper level objective and constraints

The upper level represents the optimization problem solved by a DSO aiming to minimize (2a), i.e. the costs of new conductors c^c, substations c^s, and losses c^l. The optimization problem spans a number of time steps over a planning horizon T (discretized in a number of equal steps) that should be sufficiently large to cover representative system conditions. We consider a single-phase equivalent network, assuming a balanced three-phase regime. The structure of the network is determined by constraints (2b), with (2c) ensuring the network is radial at every time step. The power flows are dictated by (2d) using the *distflow* model [8], [9]. Grid constraints are imposed by (2e) and (2f). The money invested by the DSO, plus a margin to remunerate its activities, has to be recovered through the network tariffs applied to the grid users (2g).

$$
\begin{aligned}
\min \quad & c^c + c^s + \sum_{i=1}^{T} c_i^l && && \text{(2a)}\\
\text{s.t.} \quad & \text{Conductor selection} && && \text{(2b)}\\
& \text{Radial operation} && && \text{(2c)}\\
& \text{Power flow equations}_i && \forall i \in T && \text{(2d)}\\
& \text{Voltage limits}_i && \forall i \in T && \text{(2e)}\\
& \text{Current limits}_i && \forall i \in T && \text{(2f)}\\
& \text{Budget balance} && && \text{(2g)}
\end{aligned}
$$

The main decision variables of the upper level are the substations' capacity, the routes, and the conductors that are selected. The main parameters are the topology $\mathcal{G}$, the routes' length, the substation costs, and the conductor's costs and electrical characteristics. As will be discussed in Section II-B, the lower level sets the power profiles at the buses where users are connected.

B. Lower-level objective(s) and constraints

In this work, we assume that users behave as perfectly rational agents, thus, are able to optimize their investment and energy usage as a function of equipment costs, grid connection capacity, etc., and have a perfect knowledge of the future. For simplicity and conciseness, the demand of the users is assumed fixed (demand-side flexibility will be a future work).

The lower-level optimization problem is formulated as

$$
\begin{aligned}
\min \quad & c^{PV} + c^{grid} + \sum_{i=1}^{T} (c_i^{imp} - c_i^{exp}) && && \text{(3a)}\\
\text{s.t.} \quad & \text{Active power balance}_i && \forall i \in T && \text{(3b)}\\
& \text{Reactive power balance}_i && \forall i \in T && \text{(3c)}\\
& \text{Grid injection limit}_i && \forall i \in T && \text{(3d)}\\
& \text{Grid withdrawal limit}_i && \forall i \in T && \text{(3e)}\\
& \text{PV active power limit}_i && \forall i \in T && \text{(3f)}\\
& \text{PV PQ diagram}_i && \forall i \in T && \text{(3g)}
\end{aligned}
$$

The objective function is the sum of the investment costs in local generation c^{PV} here assumed to be only photovoltaic-based for simplicity, the grid connection costs c^{grid}, and the electricity import costs c^{imp} minus the revenues from electricity injected in the grid c^{exp}.

The main decision variables are the grid connection capacity, the PV installation size and the active and reactive powers exchanged with the grid at every time step.

The constraints comprise bus active and reactive power balances (3b) and (3c) for all time steps, respectively, constraints (3d) and (3e) to bound the apparent power injected in the grid at every time step, which also define the grid connection capacity, and constraints limiting the active and reactive power generated by the PV plant at all time steps (3f) and (3g), also defining the PV installation size as a byproduct.

Since we are not considering local energy communities in this work, users are independent of each other as they have no possibility to exchange energy. The lower-level problem can be equivalently expressed user by user or for all the users in a single problem by adding their objective functions and collecting all their constraints.

C. Comments

Our bilevel program formulation carries some specifics, making it different from existing works [6], [7]. It fits the European context where a DSO is not allowed to own DERs [2], [3]. Every network user is represented in the lower level by an optimization problem setting according to his own interest (a one-leader multi-follower setting was not studied in the literature). This makes the formulation easily extensible to new potential users. Furthermore, connection costs of network users, wherever appropriate, are explicitly considered.

III. Initial results

We conduct a detailed analysis of the results on a base case, then a sensitivity analysis on several important parameters.

A. Test case description

We consider the 23-node test system of [10], illustrated in Fig. 2. There are two possible substations and 21 nodes with loads that we consider independent users of the network. The available conductors are listed in Table I. Demand and generation profiles come from [12]. In the base case, the demand (cf. Figure 3) is scaled so that the peak load is 7 MVA on a five-minute time scale, pro-rata of the load data in Table VI of [10]. The total energy consumed in the network is 16440 MWh/year. The amortization periods are 50 years for the DSO investment, 30 years for the PV panels, and ten years for the PV inverters. The other relevant parameters are summarized in Table II, where case 1 is the base case.

TABLE I
All Aluminum Conductors (data from [11]).

Code Word	q	i_{max}	r	x_l	cost
	mm^2	kA	Ω/km	Ω/km	k€/km
Poppy	53.5	0.23	0.5502	0.429	10
Oxlip	107.3	0.34	0.2747	0.402	12
Daisy	135.3	0.46	0.2180	0.394	15
Tulip	107.3	0.53	0.1732	0.381	20

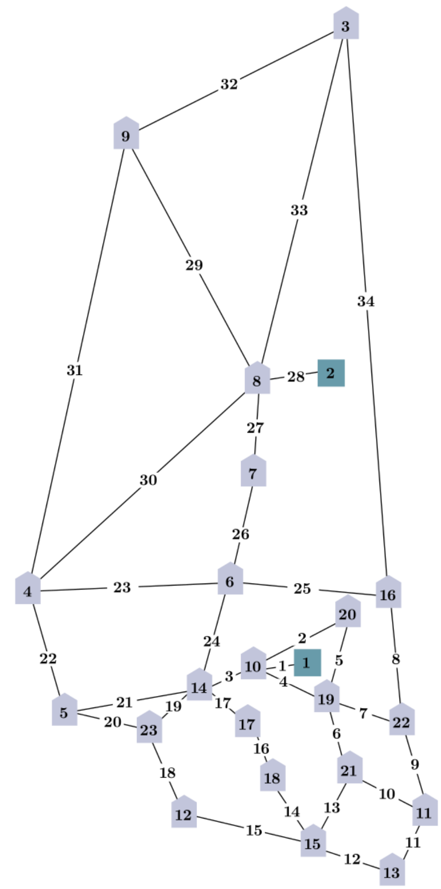

Fig. 2. Test network with bus and edge numbers.

B. Results for the base case

The problem is coded in Julia using the bilevel programming library BilevelJump [13]. The lower level is a linear program and is replaced by its Karush–Kuhn–Tucker conditions, which are linearized using the strong-duality property of linear programs. The choice of this solution approach is based on our initial investigation of different approaches [4] to solve the bilevel program, and the chosen approach revealed to be appropriate for the specific problem we are considering. BilevelJump automatically performs the reformulation task.

The data contains one typical summer day and one typical winter day. Data is averaged with a granularity of one hour. There is no heat pump and no electric vehicle. The resulting bilevel program is reformulated as a mixed-integer second-order cone program and contains 37170 continuous variables, 6699 integer variables, and 1728 cone constraints.

Figure 4 shows the selected routes and the power flows for the peak consumption period. Table III summarizes the results. The two substations are built for a total of 5.658 MVA. 21 routes are selected to form two radial distribution networks with the smallest conductors for all the routes. A maximum amount of PV is installed on all buses (8.4 MVA in total). PV generation allows users to cover almost 43% of their needs with an LCOE of 0.037 €/kWh.

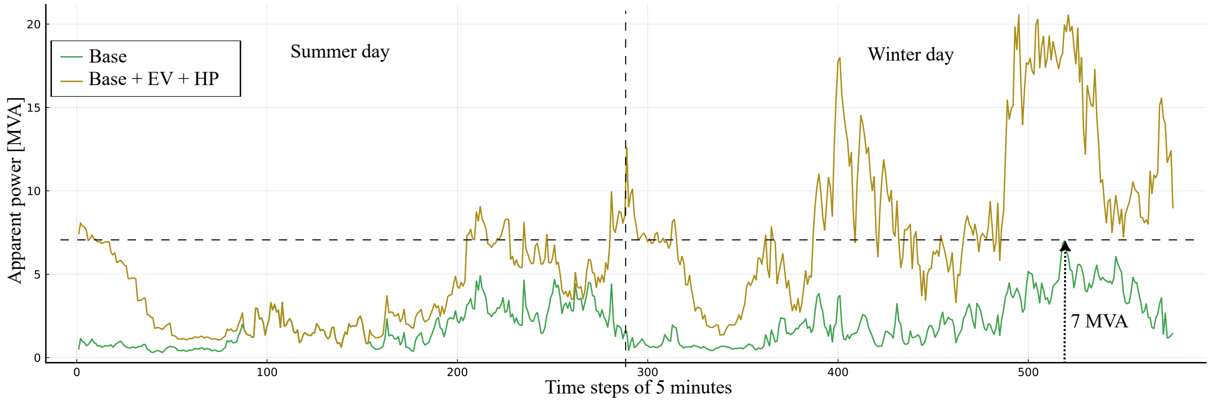

Fig. 3. Illustration of the power consumption.

TABLE II
DESCRIPTION OF TEST CASES.

EV: add electric vehicles' consumption, HP: add heat pumps' consumption, MPV: Maximum PV capacity per bus (MVA), EIP: energy import price (k€/MWh), GT: grid tariff (k€/MWh), EEP: energy export price (k€/MWh), GCC: grid connection cost (k€/MVA/y). False (F), true (T).

Case	EV	HP	MPV	EIP	GT	EEP	GCC
Base	F	F	0.4	0.3	0.1	0.1	80
Worst	**T**	**T**	**0**	0.3	0.1	0.1	80
Best	F	F	**0.8**	0.3	0.1	0.1	80
EIP inc.	F	F	0.4	**0.6**	0.1	0.1	80
GT inc.	F	F	0.4	0.3	**0.2**	0.1	80
EEP inc.	F	F	0.4	0.3	0.1	**0.2**	80
GCC inc.	F	F	0.4	0.3	0.1	0.1	**120**

C. *Sensitivity analysis*

We have performed a sensitivity analysis on some of the main parameters, and we compare the results to the base case (Table III). In the *Worst* case, the demand is very high because of heat pumps and electric vehicles, and we assumed no PV can be installed. The main consequence is a great increase in the DSO costs, with almost half of the lines that need an upgrade and both substations that must be significantly reinforced. Users' costs are multiplied by almost 5, although the energy demand has only tripled (the peak load is now 18.85 MVA, and the total energy consumed is 53110 MWh/y).

The *Best* case allows users to double the size of their PV installation, which they do. It does not significantly increase the DSO costs, but it decreases user costs and makes them more self-sufficient (up to almost 50 %).

Case *EIP inc.*, where the energy import price is doubled, essentially leads to more user costs, as in the case *GT inc.* where the grid tariff is increased. This is unsurprising since the base case's budget balance is already satisfied. Thus, an increase in the network tariff is essentially the same as an increase in the commodity price from a user's perspective, and the DSO does not need to invest more.

Case *EEP inc.* causes users to install more PV capacity and export excess PV generation to the grid. It slightly impacts the DSO costs and generates revenues for the users (a decrease in

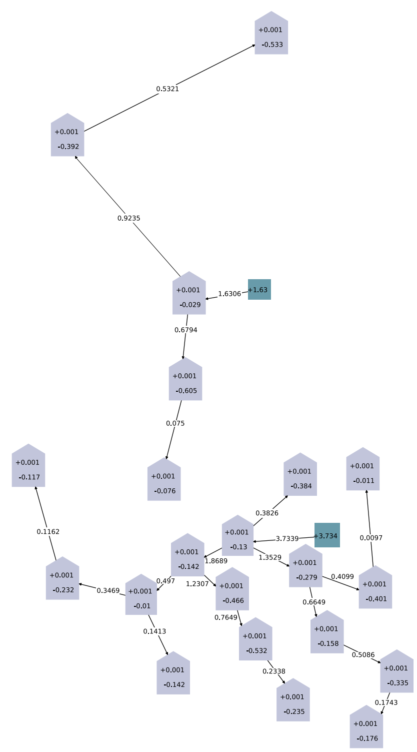

Fig. 4. Resulting network and power flow at 19:00 of the winter day for the base case.

the UNEEC).

Finally, the grid connection cost increase (case *GCC inc.*) does almost not impact the results.

IV. DISCUSSION

There are several other aspects that we need to take into account when tackling our problem. Almost all parameters are uncertain, and we cannot assume that grid users will always act rationally. Additionally, we need to consider the greenhouse gas emissions associated with equipment construction and operation. Finally, climate change may impact consumption and other aspects of the problem long-term.

TABLE III
RESULTS OBTAINED WITH THE BILEVEL MODEL

#LCi: Number of lines built with conductor i, ISC: Installed substation capacity (MVA), CAPEX: DSO annual amortized cost of investments (M€/y), OPEX: DSO cost of losses (M€/y), UPVC: Users' PV annual amortized cost of investments(M€/y), UGCC: Users' annual grid connection cost (M€/y), UNEEC: Users' net annual electricity exchange cost (M€/y), USS: Users' average self-sufficiency (%), USC: Users' average self-consumption (%).

Case	**Network topology**					**KPIs**						
	#LC1	#LC2	#LC3	#LC4	ISC	CAPEX	OPEX	UPVC	UGCC	UNEEC	USS	USC
Base	21	0	0	0	5.66	0.116	0.138	0.240	2.14	3.44	42.7	72.3
Worst	13	1	5	2	19.18	0.388	1.575	0	7.38	15.91	0	0
Best	18	1	0	2	5.64	0.117	0.125	0.405	2.16	2.96	49.4	69.2
EIP inc.	17	3	1	0	5.68	0.117	0.343	0.240	2.17	7.09	42.7	72.3
GT inc.	21	0	0	0	5.65	0.116	0.081	0.240	3.13	3.68	42.7	72.3
EEP inc.	21	0	0	0	5.88	0.121	0.134	0.302	2.73	2.06	42.7	72.3
GCC inc.	21	0	0	0	5.66	0.116	0.128	0.240	2.29	3.64	42.7	72.3

We can refer to [14] to handle uncertainties in a bilevel programming setting, which comprehensively surveys the approaches. Uncertainties can stem from data or decisions, and we can adopt either robust or stochastic approaches to handle data uncertainties. Decision uncertainties arise when a leader or a follower faces uncertainties about the decisions of other followers. Bounded rationality is another factor that we need to consider in many bilevel programming problems, and it is strongly connected to uncertainties in data and decisions.

When planning network development, it is crucial to take into account climate change and environmental factors on a global scale. This includes considering their impact on the distribution network and on users. For example, it is possible to incorporate greenhouse gas emissions related to assets and their usage into our model. However, this adds a multiobjective aspect to the problem since we aim to minimize both GHG emissions and costs. Instead of converting emissions into costs, we can analyze how the equilibrium shifts based on a total GHG emissions limit we set. This approach is outlined in [15].

V. CONCLUSIONS AND FUTURE RESEARCH

We have presented a model for studying how distribution networks should be planned as a function of users' reactions to changes in exogenous factors. This is formulated as a bilevel program representing a game between a leader, the DSO, and followers, the grid users. The model presented in this work allows to analyze the impact of some regulatory decisions (e.g., the limitation of PV installation sizes or grid tariff modifications) on the network development plan and the behavior of the users. Future work will focus on carrying out more analyses and enriching the model to account for the topics discussed in Section IV and other options available to the DSO and the users, such as storage. Expected large-scale development of energy communities [16] will impact the network development plan [16]. Therefore, it is important to account for this type of network user in future extensions.

The appropriateness of our approach will be further verified through the comparison with existing ones.

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