Time-varying dynamics in financial macroeconomics and markets

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Hamid Babaei

Supervised by Georges Hübner

HEC Liège, Management School-University of Liège, Belgium

Jury members

Prof. Marie Lambert (HEC Liège, Management School-University of Liège, Belgium)

Prof. Aline Muller (Luxembourg Institute of Socio-Economic Research: LISER, Luxembourg)

Prof. Thorsten Lehnert (University of Luxembourg, Luxembourg)

Prof. Mikael Petitjean (Université Catholique de Louvain, Belgium)

Prof. Sébastien Laurent (Aix-Marseille Université, France)

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Chapter 1

Introduction

1.1 Introduction

Most of the literature in empirical economics and finance is concerned with statistical inference on static relationships. As a specific example, we can address the well-known relationship between consumption and income. The theory of purchasing power parity and the pricing models of capital assets are among other examples.

The assumption of Ceteris Paribus is a basic assumption in micro and even macroeconomics. As a clear example, it can be said that integrated variables that form a cointegration relation, such as consumption and income, or money demand and supply, are based on the assumption that no change will occur in the consumption attitudes or for example in the exchange rates. A shift in household consumption habits causes a change in the equilibrium relationship between consumption and income. Similarly, exchange rate fluctuations or economic uncertainty can lead to changes in liquidity preferences and this leads to the failure of the equilibrium relationship between money supply and demand. In the case of purchasing power parity, it postulates that prices in different countries should have a long-term convergence. As another example in financial economics, the cointegration relation between financial series assumes that the conditions governing the surrounding environment of the economy are almost constant.

Our premise in this thesis is that the world is not stable and is constantly changing and these shifts are affecting the relationships between economic variables. These changes can be caused by exogenous shocks including economic or political decisions, political conflicts, international cooperation and treaties, globalisation, and other financial or economic variables. As a result, investment decisions are affected and this will overshadow the theoretical relations postulated between variables of interest.

The extent of this influence can differ depending on the nature of the changes. For example, a development in information technology can lead to a change in the consumption attitude of households and transform the relationship between consumption and income. The same can happen in financial markets. The development of information and communication technology can lead to a rapid flow of capital between financial markets and cause a change in the long-term equilibrium relationship between markets. A floating exchange rate regime can also provoke cross-border speculative flow of money and disrupt price convergence across economies. An asset manager rebalances portfolios when he/she obtains new information related to future economic perspectives. In this case, a static factor model fails to capture the exposure of the portfolio to risk factors.

The main objective of this thesis is to analyse the instantaneous changes between financial and economic variables. This work addresses three classical topics in financial economics by considering time variation in the relations. Investigating the time variation in the cointegration relationship between stock market indices of the G7 countries is one of the main topics of this research. The prevalence of purchasing power parity after the introduction of the euro and investigating the impacts of speculative money, economic uncertainty, and interest rate differential is another question that is tackled in this work. Finally, we relate the adaptive exposures of the hedge funds to the disagreement between the options and stock markets.

1.2 Motivation for dynamic cointegration

Stock market interdependence has been evidenced in the literature to bear structural breaks or variations in the relation. The causes of variation in the linkage between stock market indices can be related to a range of macroeconomic events, technological shifts and innovations, and other exogenous shocks. These events involve stock market crashes, liberalisation of capital flow, shifts between fixed and floating exchange rate regimes, development in informatics technology, accumulation of capital due to higher productivity of labour and capital, digitalisation of capital markets, and cross-listing. The focus of this dissertation is on cointegration of the G7 stock market indices. We examine the evolution of bivariate as well as multivariate cointegration by estimating time-varying cointegrating vectors.

Empirical investigations find weak evidence for static cointegration of the G7 stock market indices. A few number of the works identify a certain event leading to a structural break in the cointegration relation. Taylor and Tonks (1989) find that abolition of exchange rate control in the UK in 1979 leads to stronger pairwise cointegration between the UK stock market and those of West Germany, Netherlands, Japan, and the US.

Conflicting conclusions on origins and causes of the stock market crash in October 1987 motivates Arshanapalli and Doukas (1993) to focus on the degree of co-movement among stock markets of France, UK, Germany, the US, and Japan during pre-crash and post-crash periods. Their analysis reveals that the degree of markets' interdependence during the post-crash subperiod becomes stronger except for the one involving Japan in which the bivariate link disappears.

In line with the development of estimation methods for cointegration, the literature has provided a broad range of empirical works investigating the long-run equilibrium relation between the world's developed stock markets. A particular focus of the empirical works has been on the stock market cointegration of the G7 member countries (i.e., the US, the UK, France, Germany, Italy, Canada, and Japan).

Covering a longer period of time, an investigation by Kanas (1998) fails to find bivariate cointegration between the US stock market and the European ones. Subperiod analysis on pre-crash and post-crash periods distinguished by the stock market crash of 1987 fails to identify cointegration either. Using 36-year-long data from 1973 to 2009, Menezes et al. (2012) apply the test developed by Gregory and Hansen (1996) to the G7 stock market indices and find distinct time-invariant cointegration in different subperiods based on unknown breakpoints.

Alternatively, to investigate the evolution of the long-run market co-movements over time, numerous works have also applied methods based on recursive as well as rolling time windows. Rangvid (2001) applies the recursive cointegration test developed by Hansen and Johansen (1999) to the European stock markets over the period 1960-1999. They find no evidence of an increasing number of common trends over time. Pascual (2003) uses the rolling window as well as recursive methods to investigate the cointegration of the major stock markets of the world from 1960-1999. Recursive estimation of trivariate cointegration including stock markets of the UK, France, and Germany results in an upward trending trace statistic. However, it does not reject the null of no cointegration. Only weak evidence of co-movement of the stock markets of Italy, Germany, France, and Spain is found by Mylonidis and Kollias (2010) over the short period of 1999-2009. Rolling trace statistics shows a slightly upward trending, however, it is not stabilised over the critical value. In a recent work, Agoraki et al. (2019) find poor evidence for bivariate cointegration of the US, UK, Germany, and Japan using recursive estimation over the period 1980-2019.

Empirical works in the literature provide evidence seldom for time-variation in cointegration of the G7 stock markets. However, the reasons for disruptions in the long-run equilibrium relation of stock markets have not been specifically investigated. On the other hand, the time-variation assumed in the literature involves either a structural break or estimation in a recursive window which does not reveal instantaneous variation in cointegration relation. We believe that long-run equilibrium relations are subject to instantaneous changes as a result of repeated changes in economic circumstances. By state-space modeling of the single equation cointegration model in chapter 3, we focus on the evolution of cointegration of the G7 stock markets over the period 1990-2022. State-space representation of cointegration relation enables us to assess the evolution of the cointegrating vectors by a diverse range of economic as well as political events. The potential reasons impacting cointegration relations may involve a broad range of reasons including developments in multinational operations of companies, further accumulation of capital, liberalisation of capital flows, improvements in informatics and communication technology, and digitalisation of financial markets that have all enhanced cross-border information flows and have lowered financial transactions. The extent to which this set of potential reasons provokes variation in the long-run equilibrium relation of the stock markets can better be explained through a state-space formulation of cointegration relation.

We address here the important factor of uncertainty, as recently evidenced in the economics and finance literature, to substantially influence economic aggregates and, in turn, to drive security prices. There is indeed a broad range of papers investigating the effects of economic uncertainty on a number of subjects including investment, economic growth, firm-level managerial decisions, and risk management. We form our hypothesis relying on the findings in the literature implying that uncertainty can be a significant driver of economic aggregates as well as security prices. This question could be of special interest particularly because expectations of future economic activity can be significantly shifted during periods of rising or declining uncertainty.

Among the variables that have been proposed as proxies for economic uncertainty, the index of economic policy uncertainty (EPU) proposed by Baker et al. (2016) has received extensive attention. This variable aims to overcome some identified weaknesses of earlier attempts to measure uncertainty. It introduces a news-based index that covers articles containing three categories of pre-specified words pertaining to uncertainty, the economy, and policy.

Since introduction of the geopolitical risk (GPR) index developed by Caldara and Iacoviello (2022), a number of studies have used it as a proxy for the uncertainty associated with political instabilities, as it is found to be a key determinant of stock market dynamics (Antonakakis et al., 2013; Balcilar et al., 2018; Das et al., 2019; Kannadhasan and Das, 2020). As indicated by Caldara and Iacoviello (2022), being exogenous to business and financial cycles, the GPR index can provoke financial volatility and policy uncertainty. For the same reasons as those justifying the use of the EPU variable as a direct explanatory factor of the time variations in market cointegration levels, we adopt the GPR index introduced by Caldara and Iacoviello (2022) as the most convincing proxy to date for the uncertainty associated with geopolitical instability.

1.3 Motivation for cointegration strength and instability

The extant literature does not support static cointegration of the G7 stock markets. Therefore, we approach the issue in Chapter 4 from a new perspective which allows us to perform an in-depth scrutiny of time-variation in cointegration relations. Stock markets' long-run equilibrium relation can be intensified or weakened as a result of stronger or weaker market integration. A high economic integration coupled with high capital mobility can lead to a strong cointegration relation and vice versa. On the other hand, repeated changes in economic circumstances including technological developments, political conflicts, international cooperations, and treaties may lead to repeated shocks to cointegration relations and result in an unsteady relation. We approach these questions in chapter 4 by introducing two new measures of cointegration strength and instability. We estimate the new measures for multivariate cointegration relations over 1970-2022. The evolution of the new measures illustrates how well multivariate cointegration has been intensified or loosened over time. The instability measure illustrates to what extent a time-varying cointegrating vector involving a 7-tuple or 5-tuple vector is disturbed over time.

Chapter 3 differs chapter 4 in several ways. First, chapter 3 focuses on the evolution of bivariate and multivariate cointegrating vectors and examines the impact of uncertainty and a set of risk factors on the evolution of the vectors. Second, it tackles the impact of globalisation by examining the evolution of two augmented versus unaugmented cointegrating vectors. There exist two aspects of cointegration relation that remain unexplored in Chapter 3. A constantly varying cointegrating vector may form a strong or a weak cointegration relation. It can also bear repeated shocks even though cointegration remains strong or not. This is a question that we aim to specifically answer in chapter 4. The two measures associated with each cointegration relation can also reveal the impact of globalisation. Chapter 3 aims to investigate dynamic cointegration by relating time-varying cointegrating vectors to EPU, GPR and financial risk factors. Chapter 4 investigates the strength and stability of cointegration by estimating time-varying canonical R-squares in the VECM and instability in the space spanned by cointegrating vectors.

1.4 Motivation for joint PPP and UIP modeling

Purchasing price parity (PPP) has been since long investigated by empirical researchers. The efforts to find evidence supporting PPP involve two shortcomings. The first shortcoming concerns sample bias. The data sample of the empirical works covers periods during which exchange rate fluctuations were capped by the central banks to a narrow range. Evident examples include several treaties, agreements, and pacts among the world's top economies. Bretton Woods system of exchange rate management, the Smithsonian agreement, the European monetary system, and the Maastricht treaty stipulate controlled regimes of exchange rate to fluctuate within a narrow range. The range was generally determined according to a parity depending on price indices in the two countries. The inclusion of these periods of controlled exchange rate regimes forces empirical investigations to confirm PPP.

Second, the availability of more capital in recent decades and easier capital flow across countries has led to a voluminous flow of speculative money across international financial markets. The enormous speculative money can definitely disrupt price convergence across countries. A few empirical works account for fluctuations in interest rate differential (IRD), a major factor that impacts liquidity preferences and shifts investment opportunities. Juselius (1995) and the recent work by Dahlquist and Pénasse (2022) are the ones who emphasise the importance of IRD in international price convergence.

Our data sample covers the post-euro period starting from January 1999 to February 2023. During this period, no control over exchange rates was enforced by central banks. The sample involves a regime of floating exchange rates and an empirical investigation on this period can reveal whether or not price convergence holds. Our objective is to present a model that accounts for the interaction of goods and capital markets. Such a model can disentangle dynamics driven by speculative money from dynamics related to goods price convergence. To this end, we first relate PPP to UIP following Juselius (1995). We arrive at a new parity between the real exchange rate (RER) and IRD. Then we develop the parity to a model in which the RER is decomposed into two components. The first component is an unobservable state variable that is augmented by IRD and is assumed to capture RER dynamics caused by speculative money. The second component is assumed to follow an autoregressive process and represents the dynamics of price convergence.

The decomposition model is also augmented by policy-related economic and monetary uncertainties. Since the EPU introduced by Baker et al. (2016) has recently received extensive attention by empirical researchers, we examine if uncertainty has any impact on fluctuations in RER. Uncertainty in the monetary policy of the central banks should have a negative impact on currency valuation. More generally, uncertainty in economic policy should also lead to currency depreciation. By augmenting the decomposition model with three uncertainty indicators of domestic and foreign EPU and monetary policy uncertainty (MPU) in the US, we hypothesise the impact of the uncertainty factors on exchange rates.

1.5 Motivation for dynamic factor model of hedge funds

It is well-documented that hedge funds follow active trading strategies. The nonlinear risk-return characteristics have been investigated in a number of ways including examination of varying exposures, introduction of trading strategies, and use of information by hedge fund managers. For example, Fung and Hsieh (2001) and Fung and Hsieh (2007) rely on ICAPM to capture trend-following strategies. Agarwal and Naik (2004) show that hedge fund returns are exposed to high left tail risk because funds managers use a combination of buy and hold and option-based strategy. Cai and Liang (2012) use dynamic linear regression to reduce residual autocorrelation which suggests that imperfect predictions by a static factor model have vanished. Billio et al. (2012) estimate a factor model in which factor loadings vary depending on different market regimes.

A number of the works relate market timing to exogenous sources of information. These information involve market liquidity (Cao et al., 2013), macroeconomic aggregates (Brandt et al., 2019; Lambert and Platania, 2020) and sentiment index (Zheng et al., 2018; Chen et al., 2021).

Fund managers are also known to seek mispriced assets to profit from prospective returns. Ma, Li and Tee (2022) develop a model to investigate hedge funds' abilities to benefit from mispriced assets. They find that hedge funds are able to make returns by either short-selling or bearing market risk. The options market can be a valuable source of information involving information on relative mispricing of options versus the stock market. Using put-call parity, we derive an index representing the relative mispricing of the two markets. The monthly index equals the percentage of the disagreement days in a month where a disagreement day is identified when the difference between the S&P 500 implied and spot prices exceed a certain amount. To the best of our knowledge, there is no paper in the hedge funds literature investigating the use of mispricing information implied by the options market.

To evaluate funds managers' ability in the favorable identification of mispricing opportunities, we estimate a dynamic factor model in which factor loadings follow an autoregressive process. In the first step, we use the Kalman filter to estimate time-varying factor loading and in the second step, we obtain the time-varying factor loadings that are predicted by the disagreement rate.

Chapter 2

A review on fundamental developments in cointegration

2.1 From spurious regression to cointegration

A common anomaly that arises in the regression of integrated series happens when unrelated series in the regression leads to high R^2, significant slopes, and low Durbin-Watson statistic. Such a regression can not be used for prediction because the error term follows a unit root process which has infinite variance when the prediction horizon extends infinitely.

The puzzle of spurious regression was known for almost half a century before it was theoretically tackled by Granger and Newbold (1974). The puzzle has been first addressed by Yule (1926):

> "It is fairly familiar knowledge that we sometimes obtain between quantities varying with the time (time-variables) quite high correlations to which we cannot attach any physical significance whatever, although under the ordinary test, the correlation would be held to be certainly "significant." As the occurrence of such " nonsense correlations" makes one mistrust the serious arguments that are sometimes put forward on the basis of correlations between time series-my readers can supply their own examples-it is important to clear up the problem how they arise

and in what special cases."

He continues that interpreting such a correlation as causation is a result of ignoring the common influence of time. He underlines that such a correlation has no meaning at all and that it is simply a coincidence and if one observes the two variables over a rather long time then the correlation vanishes.

The question of spurious regression in regression models with level variables (i.e., variables that are unit root processes) was theoretically investigated by Granger and Newbold (1974). They provide new insight into the problem by showing that the usual significance tests on coefficients of such a regression are invalid. They argue that R^2 distribution in a regression of independent autoregressive (AR) processes is not unimodal at zero, rather it is multimodal when the variance of the R^2 exceeds a certain amount. One implication of this multimodal distribution is that higher R^2 values can occur with a high probability. Overall, it implies that the high R^2 does not represent evidence for a significant relation between the variables. Spurious regression can arise in regressions between return variables if they are persistent, i.e., series that are highly autocorrelated. The case spurious regression on stationary and persistent series are discussed in Ferson et al. (2003), Giot and Petitjean (2011), and Deng (2014). Since the focus of this dissertation is on unit root processes, we skip the spurious regressions on stationary persistent series.

Although statistical reasons of a spurious regression were outlined by Granger and Newbold (1974), the special case of a valid relation between integrated series was not postulated until the work by Granger (1981) was published. He uses spectral analysis to introduce the case of cointegrated series. Generally, if x_t and z_t are two series of integration orders d_x and d_z, then integration order of the linear combination $y_t = cx_t + gz_t$ equals $max(d_x, d_z)$. Granger (1981) introduces conditions derived from the spectrum of y_t for the special case of $d_y < max(d_x, d_z)$ where the integration order of the resulting series is lower than either of x_t or z_t. He refers to this special case as cointegrated series. He concludes that if x_t and z_t are cointegrated, then for any finite lagged filters $a(L)$ and $b(L)$, the series $a(L)x_t$ or $b(L)z_t$ are cointegrated. Given this landmark development in the literature of time series, valid estimation methods were not developed until 1987 when theoretical relations between three representations of cointegrated series were postulated. Before then, the joint use of first differences and level variables was known to yield better fits. In these models which are analogous to error correction models (ECM) the first differences of a certain series was used as the dependent variable and the difference between lagged level variables coupled with first differences of other series were used as independent ones. Sargan (1964) and Davidson et al. (1978) use such a modeling approach.

2.2 Estimation methods and statistical inference

2.2.1 Early estimation methods

The first test method for a valid linear relation between cointegrated series was introduced by Engle and Granger (1987). Before then, an effort to develop an estimation and test method was performed by Granger and Weiss (1983). They propose an estimation of the ECM and test for the significance of the lagged-level variables. Given that the asymptotic theory was not yet developed, they acknowledge that the test will have unsatisfactory results in medium-sized samples. As an alternative method, they propose a single equation estimation method followed by examining the correlogram of the residuals for cointegration. In another work, an estimation method was proposed by Stock (1987) in a working paper that has been available since 1984. The method presented in Stock (1987) is the well-known two-step method that is also used by Engle and Granger (1987) who then propose seven test methods for cointegration. More details on this paper will be discussed later in this section.

Engle and Granger (1987) present a seminal theorem that illustrates the links between cointegration relation and two other representations, namely vector error correction model (VECM) and vector autoregressive (VAR) model. The representation theorem states that if $\boldsymbol{X}_t$ is a $(N \times 1)$ vector of cointegrated series with r independent cointegrating vectors, there exists a VAR and a VECM representation of the vector process $\boldsymbol{X}_t$. More specifically, for the vector process $\boldsymbol{X}_t$, they derive a vector autoregressive moving average (VARMA) representation of the form:

$$\boldsymbol{A}(L)\boldsymbol{X}_t = d(L)\boldsymbol{\epsilon}_t, \tag{2.1}$$

where L denotes the lag operator, $\boldsymbol{A}(L)$ is a $(N \times N)$ matrix of lag polynomials with $\boldsymbol{A}(0) = \boldsymbol{I}_N$ and $\boldsymbol{A}(1)$ a matrix of rank r, $d(L)$ is a scalar lag polynomial with $d(1)$ being finite. The VECM representation can be derived as:

$$\boldsymbol{A}^*(1-L)\boldsymbol{X}_t = -\boldsymbol{\gamma}\boldsymbol{z}_{t-1} + d(L)\boldsymbol{\epsilon}_t, \tag{2.2}$$

where $\boldsymbol{z}_t = \boldsymbol{\alpha}' X_t$ is an $(r \times 1)$ vector of stationary variables and is referred to as equilibrium error, $\boldsymbol{\gamma}$ is an $(N \times r)$ matrix of rank r which is referred to as adjustment matrix and $\boldsymbol{A}^*$ satisfies:

$$\boldsymbol{A}(L) = \boldsymbol{A}(1) + (1-L)\boldsymbol{A}^*(L). \tag{2.3}$$

On the other hand, if there exists a VECM specification for $\boldsymbol{X}_t$, then the $\boldsymbol{\gamma}\boldsymbol{z}_{t-1}$ must be stationary which implies the prevalence of cointegration. If there is a VAR on $\boldsymbol{X}_t$, one can not necessarily conclude the existence of a cointegration relation, because in case of insignificant cross-dependence of the variables in the VAR, they are not cointegrated.

Engle and Granger (1987) propose a two-step method[1] to estimate cointegration relation. The first step involves ordinary least squares (OLS) estimation of the single equation model on level variables to obtain cointegrating vector and in the second step, the cointegrating vector is used in ECM to estimate adjustment parameters using OLS. Distribution of the coefficients in a regression including integrated series is a fundamental issue that is not discussed by Engle and Granger (1987), rather they refer to the work by Stock (1987) which was then a working paper on the asymptotic properties of OLS estimators.

To test for cointegration, Engle and Granger (1987) apply seven test methods with the null hypothesis of no cointegration five of which use the errors obtained from the first step. The two other tests are two unrestricted VAR models (differing in the use of a number of lagged first differences) which involve the estimation of the ERM without using the errors obtained from the first step. The first five tests include Durbin-Watson (DW), Dickey-Fuller

[1] As discussed earlier in this section, the two-step estimation method is first introduced by Stock (1987) as a working paper in 1984.

(DF), augmented Dickey-Fuller (ADF) tests, and two models of restricted VAR and unrestricted VAR. The latter models involve an ECM in which the estimated error term of the first step is used with a different number of lagged first differenced terms. They compare the power of the seven tests for bivariate cointegration. Simulation results of bivariate cointegration show that the first three tests of DW, DF, and ADF have higher power than others. The power of these tests for higher order vector process $\boldsymbol{X}_t$ remains unexplored.

Since cointegrating vectors are unknown, the standard ADF critical values can not be used to test for cointegration in systems containing numerous variables. Critical values of Engle and Granger (1987) cointegration test which involves an ADF test on residuals are tabulated by Engle and Yoo (1987). The critical values are obtained for different sample sizes. Further developments on critical values of cointegration models will be reviewed in subsection 2.3.

2.2.2 Asymptotic theory

In the following sections, we review the fundamental developments in cointegration theory which are essential to investigate dynamics in stock market cointegration. Prior to Engle and Granger (1987), two important theoretical works by Phillips and Durlauf (1986) and Stock (1987) illustrated asymptotic distributions of regression equations including integrated series. Once a well-known working paper in 1984, the theoretical development in the paper by Stock (1987) is cited by Engle and Granger (1987) before being published. However, Stock (1987) also cites Engle and Granger (1987) in the published paper. The paper contributes to the cointegration literature by deriving the asymptotic distribution of two OLS and nonlinear least squares (NLS) estimators of cointegration parameters. Since the paper illustrates technical complexities associated with cointegrated series, we discuss the findings of this paper in more detail. The OLS estimator involves two-step estimation of parameters by estimating the single equation cointegration model in the first step and using the resulting equilibrium errors in the ECM to estimate, in the second step, to estimate the parameters of the short-run dynamics.

Let $\boldsymbol{\alpha}$ be a matrix having cointegrating vectors of the vector process $\boldsymbol{X}_t$ as columns. Each single equation cointegration model can be estimated by choosing $\boldsymbol{X}_{i,t}$ as the dependent variable and other components of $\boldsymbol{X}_t$ as the independent ones. Therefore, the corresponding cointegrating vector can be expressed as:

$$\boldsymbol{\alpha}_i = \boldsymbol{e}_{(i)} + \boldsymbol{R}_i\boldsymbol{\theta}_i \qquad (i = 1, \ldots, r), \tag{2.4}$$

where $\boldsymbol{\alpha}_i$ is a $(N \times 1)$ vector with 1 in the i-th component, $\boldsymbol{e}_{(i)}$ is the N dimensional unit vector with 1 at the same place appearing in $\boldsymbol{\alpha}_i$, $\boldsymbol{R}_i$ is a $(N \times k)$ matrix of known constants (i.e. 0 and -1) with $k \leq N-1$ and $\boldsymbol{\theta}_i' = (\theta_{i,1}, \ldots, \theta_{i,k})$. Stock (1987) adopts a normalisation by fixing the i-th element of the cointegrating vector $\boldsymbol{\alpha}_i$ to unity. If $i = 1$, then the normalisation in (2.4) implies $\boldsymbol{\alpha}_i = (1, -\theta_{i,1}, \ldots, -\theta_{i,N-1})'$ which is simply regressing $X_{1,t}$ on $X_{j,t}$s with $j \neq 1$. The OLS estimator of cointegrating vector is obtained by the following minimisation problem:

$$\min_{\boldsymbol{\theta}_i} \sum_{t=1}^{T} (\boldsymbol{\alpha}_i'\boldsymbol{X}_t)^2 \qquad \boldsymbol{\alpha}_i = \boldsymbol{e}_{(i)} + \boldsymbol{R}_i\boldsymbol{\theta}_i. \tag{2.5}$$

The NLS estimator is simply the OLS estimation of the following equations of the ECM representation:

$$\Delta X_{1,t} = \boldsymbol{\beta}'\boldsymbol{\xi}_{t-1} + \sum_{j=1}^{N} \delta_j X_{j,t-1} + \boldsymbol{u}_t, \tag{2.6}$$

where $\boldsymbol{\xi}_t = (\Delta\boldsymbol{X}_t', \ldots, \Delta\boldsymbol{X}_{t-p+1}')'$, $\boldsymbol{\beta}$ is a $(N \times 1)$ vector, δ_is are scalar parameters and $\boldsymbol{u}_t$s are i.i.d normally distributed random variables with zero mean. Note that, Equation (2.6) is an unbalanced equation in which the left-hand side variable is stationary and the right-hand side includes integrated level variables. Stock (1987) estimates the first equation in the VECM in Equation (2.6) then uses the normalisation in Equation (2.4). The subscript 1 used in Equation (2.6) stands for the first equation in the VECM representation. Likewise, each equation in the VECM can be used as an NLS estimator. Note that, Equation (2.6) is an unconstrained model in which the estimated equilibrium errors of the OLS estimator are not used. The coefficients of the $X_{j,t}$s estimated by the NLS estimator form the cointegrating vector.

A number of fundamental theorems are proved by Stock (1987) which settle statistical inference on regressions with $I(1)$ processes. The OLS estimator in (2.5) can be expressed by $\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i = -\boldsymbol{V}_T^{-1}\boldsymbol{U}_T$ where $\boldsymbol{V}_T = T^{-2}\sum_{t=1}^{t=T} \boldsymbol{R}_i'\boldsymbol{X}_t\boldsymbol{X}_t'\boldsymbol{R}_i$ and $\boldsymbol{U}_T = T^{-2}\sum_{t=1}^{t=T} \boldsymbol{R}_i'\boldsymbol{X}_t\boldsymbol{X}_t'\boldsymbol{\alpha}_i$.

Stock (1987) prove that $\boldsymbol{V}_T - \boldsymbol{D}_1'\boldsymbol{\Gamma}\boldsymbol{D}_1 \xrightarrow{p} 0$ and $T\boldsymbol{U}_T - \boldsymbol{M} - \boldsymbol{D}_1'\boldsymbol{\Psi}\boldsymbol{D}_2 \xrightarrow{p} 0$[2] where $\xrightarrow{p}$ denotes convergence in probability. This result implies that OLS estimators of a regression on $\boldsymbol{I}(1)$ processes asymptotically follow nonstandard distributions.

Superconsistency of the OLS estimator is the second important property of the OLS estimator introduced by Stock (1987) meaning that the OLS estimator $\hat{\boldsymbol{\theta}}_i$ introduced in (2.5) converges to the true value $\boldsymbol{\theta}_i$ at $O(T^{-1})$ order. In other words $T^{1-\delta}(\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i) \xrightarrow{p} 0$ for all $\delta > 0$.

The third important property proved by Stock (1987) concerns asymptotic properties of the NLS estimator. The NLS estimator in (2.6) can be expressed by $\tilde{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i = -\tilde{\boldsymbol{V}}_T^{-1}\tilde{\boldsymbol{U}}_T$, where $\tilde{\boldsymbol{V}}_T = T^{-2}\sum_{t=p+1}^{t=T} \boldsymbol{R}_i'\boldsymbol{X}_{t-1}\boldsymbol{X}_{t-1}'\boldsymbol{R}_i$ and $\tilde{\boldsymbol{U}}_T = T^{-2}\sum_{t=1}^{t=T} \boldsymbol{R}_i'\boldsymbol{X}_{t-1}(\Delta\boldsymbol{X}_{1,t} - \hat{\boldsymbol{\beta}}'\boldsymbol{\xi}_{t-1} + \tilde{\gamma}_1\boldsymbol{z}_{i,t-1})/\tilde{\gamma}_1$. The scalar parameter $\tilde{\gamma}_1$ is the first element in (2.2) after normalisation of the cointegrating vector according to Equation (2.4). Stock (1987) proves that $T\tilde{\boldsymbol{V}}_T - \boldsymbol{D}_1'\boldsymbol{\Gamma}_T\boldsymbol{D}_1 \xrightarrow{p} 0$, $T\tilde{\boldsymbol{U}}_T - \boldsymbol{D}_1'\boldsymbol{\Psi}_1\boldsymbol{D}_3 \xrightarrow{p} 0$ where $\boldsymbol{D}_3' = \boldsymbol{e}_1'\boldsymbol{G}^{1/2}\gamma_1^{-1}$ and $\boldsymbol{e}_1' = (1, 0, \ldots, 0)$. Superconsistency of the NLS estimator is proved likewise, i.e. $T^{1-\delta}(\tilde{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i) \xrightarrow{p} 0$ for all $\delta > 0$.

An important implication of the asymptotic distribution of the NLS estimator is that unlike $T\boldsymbol{U}_T$, the limiting distribution of $T\tilde{\boldsymbol{U}}_T$ does not include the matrix $\boldsymbol{M}$ which suggests that using ECM to estimate cointegrating vectors results in less bias than using the single equation cointegration model. However, if γ_1 is small, the NLS estimator will have a higher variance.

The last property of the two estimators concerns the asymptotic distribution of the short-run parameters, i.e. $\boldsymbol{\beta}$ in (2.6). The parameters representing the short-run dynamics can be estimated using two approaches. The two-step approach and the NLS method. Stock (1987) proves that parameters of the short-run dynamics estimated from either of the two approaches have a limiting normal distribution.

A good deal of theoretical developments in regressions including integrated series were also published before Engle and Granger (1987). Phillips and Durlauf (1986) develop a general asymptotic theory for distributions of parameters in regression models including integrated

[2] $\boldsymbol{\Gamma} = \int_0^1 \boldsymbol{W}(t)\boldsymbol{W}(t)'dt$, $\boldsymbol{\Psi} = \int_0^1 \boldsymbol{W}(t)d\boldsymbol{W}(t)'$ where $\boldsymbol{W}(t)$ denotes N dimensional Wiener process. $\boldsymbol{D}_1' = \boldsymbol{R}_i'\boldsymbol{C}(1)\boldsymbol{G}^{1/2}$, $\boldsymbol{D}_2' = \boldsymbol{\alpha}_i'\boldsymbol{C}^*(1)\boldsymbol{G}^{1/2}$, $\boldsymbol{M} = \boldsymbol{D}_1'\boldsymbol{D}_2' + \boldsymbol{R}_i'\sum_{j=0}^{t=\infty}\boldsymbol{C}_j^*\boldsymbol{G}\boldsymbol{C}_j^{*\prime}\boldsymbol{\alpha}_i$. The matrix $\boldsymbol{C}$ is given by the Wold representation of the stationary process $\Delta\boldsymbol{X}_t$, i.e. $\Delta\boldsymbol{X}_t = \boldsymbol{C}(L)\boldsymbol{\epsilon}_t$ where $\boldsymbol{C}(L)$ is a matrix of lag polynomials and $\boldsymbol{C}^*$ is given by $\boldsymbol{C}(L) = \boldsymbol{C}(1) + (1-L)\boldsymbol{C}^*$ and $\boldsymbol{G}$ is the variance-covariance matrix of the vector process $\boldsymbol{\epsilon}_t$

processes. More specifically, they derive asymptotic distributions of the following VAR(1) model:

$$\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{u}_t \qquad \boldsymbol{A} = \boldsymbol{I}_n \tag{2.7}$$

and the following multiple regression model:

$$\boldsymbol{y}_t = \boldsymbol{B}\boldsymbol{x}_t + \boldsymbol{u}_t \tag{2.8}$$

where $\boldsymbol{y}_t$ and $\boldsymbol{u}_t$ are $(n \times 1)$ vector process, $\boldsymbol{x}_t = \boldsymbol{x}_{t-1} + \boldsymbol{v}_t$ is a $(m \times 1)$ vector process, $\boldsymbol{A}$ is a $(n \times n)$ matrix and $\boldsymbol{B}$ is a $(n \times m)$ matrix.

Park and Phillips (1988) develop the asymptotic theory to models having drift and time trend. Park and Phillips (1989) generalise the asymptotic theory to regressions having variables of different integration order. Regressions having stationary and integrated series is a special case where the OLS estimates of the coefficients of stationary regressors are asymptotically biased, and consistent, and they follow asymptotically normal distribution at the convergence order $o(T^{-1/2})$. This estimator is consistent if it is uncorrelated with the error term $\boldsymbol{u}_t$, otherwise, it will be inconsistent. The same conclusion can be achieved after the inclusion of variables of higher-order integration.

2.2.3 Further estimation and test methods

The representation theorem introduced by Engle and Granger (1987) states that for a vector process $\boldsymbol{X}_t$ of $(N \times 1)$ dimension, there can be r cointegrating vectors with $0 \leq r \leq N$. The two-step estimation method proposed by Engle and Granger (1987) does not guarantee estimating distinct cointegrating vectors. The two OLS and NLS estimators proposed by Stock (1987) do not yield independent cointegrating vectors either. Johansen (1988) develops two test methods that allow testing for a definite number of cointegrating vectors. The test methods are known as trace and maximum eigenvalue tests. Both test statistics are calculated from canonical correlations of the VECM. The VECM of the vector process X_t can be written as:

$$\Delta\boldsymbol{X}_t = \boldsymbol{\mu} + \boldsymbol{\Pi}\boldsymbol{X}_{t-1} + \boldsymbol{\Gamma}_1\Delta\boldsymbol{X}_{t-1} + \cdots + \boldsymbol{\Gamma}_p\Delta\boldsymbol{X}_{t-p+1} + \boldsymbol{\epsilon}_t \tag{2.9}$$

where $E[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}'_\tau] = \boldsymbol{\Omega}$ when $t = \tau$ and $E[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}'_\tau] = 0$ otherwise. Let $\hat{\boldsymbol{u}}_t$ denote the residual vector obtained from regressing $\Delta \boldsymbol{X}_t$ on a constant and the lagged differenced terms in (2.9) and $\hat{\boldsymbol{v}}_t$ denote the residuals from regressing $\boldsymbol{X}_{t-1}$ on a constant and the same lagged differenced terms. Regressing $\hat{\boldsymbol{u}}_t$ on $\hat{\boldsymbol{v}}_t$ yields the impact matrix $\boldsymbol{\Pi}$. The rank of this matrix represents a number of cointegrating vectors. Let $\hat{\boldsymbol{\Sigma}}_{vv}$, $\hat{\boldsymbol{\Sigma}}_{vu}$, $\hat{\boldsymbol{\Sigma}}_{uu}$ and $\hat{\boldsymbol{\Sigma}}_{uv}$ represent sample variance-covariance matrices of the vector processes $\hat{\boldsymbol{u}}_t$ and $\hat{\boldsymbol{v}}_t$ and let the ordered scalars $\hat{\lambda}_1 > \hat{\lambda}_2 > \cdots > \hat{\lambda}_N$ represent eigenvalues of the matrix $\hat{\boldsymbol{\Sigma}}_{vv}^{-1} \hat{\boldsymbol{\Sigma}}_{vu} \hat{\boldsymbol{\Sigma}}_{uu}^{-1} \hat{\boldsymbol{\Sigma}}_{uv}$. It can be shown that $\hat{\lambda}_i$s are canonical correlations between two vectors $\hat{\boldsymbol{u}}_t$ and $\hat{\boldsymbol{v}}_t$. Johansen (1988) proves that under the null hypothesis of r cointegrating vector, the maximum log likelihood function of (2.9) equals:

$$\mathcal{L}^* = -\frac{TN}{2}(1 + log(2\pi)) - \frac{T}{2} log|\hat{\boldsymbol{\Sigma}}_{uu}| - \frac{T}{2} \sum_{i=1}^{r} log(1 + \hat{\lambda}_i). \tag{2.10}$$

From (2.10), it can be shown that the likelihood ratio test statistic of the null hypothesis of r cointegrating vectors against the alternative of $r+1$ cointegrating vectors equals $-Tlog(1 - \hat{\lambda}_{r+1})$. This test is referred to as the maximum eigenvalue test. The likelihood ratio test statistic corresponding to the null hypothesis of less than r cointegrating vectors against the alternative of m cointegrating vectors equals $-T \sum_{r+1}^{m} log(1 - \hat{\lambda}_{r+1})$. This test is referred to trace test. Johansen (1988) derives an asymptotic distribution for the trace statistic that does not depend on the covariance matrix of the residuals in (2.9), rather it depends only on N. Critical values of these distributions are obtained by MacKinnon et al. (1999).

Johansen (1988) suggests that the eigenvectors $\hat{\boldsymbol{\beta}}$ of the matrix $\boldsymbol{\Sigma}_{vv}^{-1} \boldsymbol{\Sigma}_{vu} \boldsymbol{\Sigma}_{uu}^{-1} \boldsymbol{\Sigma}_{uv}$ be normalised by $\hat{\boldsymbol{\beta}}' \boldsymbol{\Sigma}_{vv} \hat{\boldsymbol{\beta}} = I$. Then the impact matrix satisfies $\boldsymbol{\Pi} = \hat{\boldsymbol{\alpha}} \hat{\boldsymbol{\beta}}'$ where $\hat{\boldsymbol{\alpha}} = \boldsymbol{\Sigma}_{vv} \hat{\boldsymbol{\beta}}$.

One major shortcoming associated with Johansen's estimation is that the variance of the estimated cointegrating vectors can be high. On the other hand, Pesaran et al. (2000) shows that the power of the two rank tests is not high enough. They propose adopting bootstrap methods to obtain critical values.

In another effort to test for and estimate independent cointegrating vectors, Stock and Watson (1988) represent the vector $\boldsymbol{X}_t$ in terms of a reduced number of common trends. They introduce two test methods which are known as common trends tests. The tests involve a rank test aimed at determining the number of cointegrating vectors. First, they deduce a common trend representation for vector process $\boldsymbol{X}_t$ of dimension $(N \times 1)$. The common trend representation can be expressed as:

$$\boldsymbol{X}_t = \boldsymbol{X}_0 + \boldsymbol{A}\boldsymbol{\tau}_t + \boldsymbol{a}_t, \qquad \boldsymbol{\tau}_t = \boldsymbol{\pi} + \boldsymbol{\tau}_{t-1} + \boldsymbol{\nu}_t, \tag{2.11}$$

where $\boldsymbol{\tau}_t$ is a $(k \times 1)$ vector composed of random walks having independent innovations, $\boldsymbol{A}$ is a $(N \times k)$ matrix of constants, $\boldsymbol{a}_t = \boldsymbol{C}^*(L)\boldsymbol{G}^{1/2}\boldsymbol{v}_t$ with $\Delta\boldsymbol{X}_t = \boldsymbol{C}(L)\boldsymbol{\epsilon}_t$ where $\boldsymbol{C}(L)$ is a matrix of lag polynomials and $\boldsymbol{C}^*$ is given by $\boldsymbol{C}(L) = \boldsymbol{C}(1) + (1-L)\boldsymbol{C}^*$ and $\boldsymbol{G}$ is the variance-covariance matrix of the vector process $\boldsymbol{\epsilon}_t$ in Equation (2.1). Equation (2.11) implies that any vector process $\boldsymbol{X}_t$ of dimension $(N \times 1)$ can be decomposed into two components the first of which is a linear combination of k random walks and the second component is a vector of stationary processes. If (2.11) holds, then there are $r = N - k$ independent cointegrating vectors.

The common trend representation in (2.11) provides it possible to perform a test of k versus m common trends. Stock and Watson (1988) propose the use of principal component analysis to obtain independent candidates of cointegrating vectors. The first test is simply a unit root test on the first k integrated principal components. Let $\boldsymbol{\alpha}$ denote a $(N \times r)$ vector of cointegrating vectors (here, principal components) of $\boldsymbol{X}_t$ and let $\boldsymbol{\alpha}^\dagger$ denote a $(N \times k)$ vector $(k = N - r)$ with $\boldsymbol{\alpha}\boldsymbol{\alpha}^\dagger = 0$. Assuming $\boldsymbol{Y}_t = [\boldsymbol{\alpha}\boldsymbol{\alpha}^\dagger]'\boldsymbol{X}_t$, a rank test can be performed by regressing the final k integrated components of $\boldsymbol{Y}_t$ (i.e. $\boldsymbol{V}_t$) on $\boldsymbol{V}_{t-1}$. A major anomaly associated with this regression arises when errors are autocorrelated. To avoid this anomaly, Stock and Watson (1988) propose an approach analogous to Dickey and Fuller (1979). The anomaly is then resolved by a VAR representation of $\boldsymbol{V}_t$ (i.e. $\boldsymbol{\Pi}(L)\Delta\boldsymbol{V}_t = \boldsymbol{\gamma} + \boldsymbol{\eta}_t$). Now let $\boldsymbol{\zeta}_t = \boldsymbol{\Pi}(L)\boldsymbol{V}_t$, then the unit root test on the k integrated components of $\boldsymbol{Y}_t$ can be performed by regressing $\boldsymbol{\zeta}_t$ on $\boldsymbol{\zeta}_{t-1}$. The AR coefficient matrix is given by:

$$\hat{\boldsymbol{\Phi}} = [\Sigma\boldsymbol{\zeta}_t\boldsymbol{\zeta}'_{t-1}][\Sigma\boldsymbol{\zeta}_{t-1}\boldsymbol{\zeta}'_{t-1}]^{-1} \tag{2.12}$$

The test is performed based on the ordered eigenvalues of $\hat{\boldsymbol{\Phi}}$. Under the null hypothesis, $\boldsymbol{\Phi}$ has k unit roots or it has k unit eigenvalues and under the alternative, it has only

m unit eigenvalues. Stock and Watson (1988) point out that $T(\hat{\boldsymbol{\Phi}} - \boldsymbol{I}_k) \xrightarrow{p} \boldsymbol{\Psi}_k' \boldsymbol{\Gamma}_k^{-1}$ where $\boldsymbol{\Gamma}_k = \int_0^1 \boldsymbol{W}_k(t)\boldsymbol{W}_k(t)'dt$, $\boldsymbol{\Psi}_k = \int_0^1 \boldsymbol{W}_k(t)d\boldsymbol{W}_k(t)'$ and $\boldsymbol{W}_k$ denotes a $(k \times 1)$ vector of Brownian motions. The probability convergence $T(\boldsymbol{\lambda}_{\boldsymbol{\Phi}} - \boldsymbol{l}) \xrightarrow{p} \boldsymbol{\lambda}_*$ will follow immediately where $\boldsymbol{l} = (1, \ldots, 1)'$ and $\boldsymbol{\lambda}_*$ represents the vector of ordered eigenvalues of $\boldsymbol{\Psi}_k' \boldsymbol{\Gamma}_k^{-1}$.

The second test is performed on the corrected value of $\hat{\boldsymbol{\Phi}}$. The correction is performed to remove the bias inherent in the OLS estimation of $\boldsymbol{\Phi}$ due to endogenous lagged principal component. They use Monte Carlo simulation to obtain critical values of the tests.

So far, estimation methods of cointegrated series lead to biased, however consistent cointegrating vectors. The amount of bias can be considerable in small sample sizes. Phillips and Hansen (1990) introduce a single equation estimation approach known as the fully modified OLS (FMOLS) which estimates and corrects the bias of the cointegrating vector.

Pesaran et al. (1995) obtain asymptotic distribution of autoregressive distributed lag (ARDL) models of the from:

$$\phi(L)y_t = \alpha_0 + \alpha_1 t + \boldsymbol{\beta}'\boldsymbol{x}_t + u_t \tag{2.13}$$

where $\boldsymbol{x}_t$ is a vector of $I(1)$ processes that are not cointegrated. Pesaran et al. (1995) show that if $|\phi| < 1$, then the short run parameters i.e. α_0, $\alpha_1 t$, $\boldsymbol{\beta}$ and the long run parameters i.e. $\delta = \alpha_1/\phi(1)$ and $\boldsymbol{\theta} = \boldsymbol{\beta}/\phi(1)$ asymptotically follow normal distributions. Moreover, δ is $T^{3/2}-$consistent and $\boldsymbol{\theta}$ is $T-$consistent.

Pesaran et al. (2001) develop a test method for cointegration of a vector process when there is no presumption on trend stationarity or first difference stationarity of the underlying variables.

Shin et al. (2014) decompose $\boldsymbol{x}_t$ in (2.13) to $\boldsymbol{x}_t = \boldsymbol{x}_0 + \boldsymbol{x}_t^+ + \boldsymbol{x}_t^-$ where $\boldsymbol{x}_t^+$ and $\boldsymbol{x}_t^-$ denote partial sum processes obtained from $\boldsymbol{x}_t$ based on positive and negative changes respectively. The ARDL model in (2.13) is the rewritten with distinct matrices $\boldsymbol{\beta}^+$ and $\boldsymbol{\beta}^-$ corresponding to $\boldsymbol{x}_t^+$ and $\boldsymbol{x}_t^-$. The resulting asymmetric model is referred to as nonlinear ARDL (NARDL).

2.3 Critical values of static cointegration tests

Engle and Granger (1987) obtain critical values for seven bivariate cointegration tests as explained in subsection 2.2.1. A multivariate single equation cointegration and a bivariate one in the first step of Engle and Granger (1987) differ in the number of the unknown parameters and therefore the τ statistic in the ADF test on the resulting residuals should have a different distribution. Engle and Yoo (1987) obtain critical values for multivariate single equation cointegration test (i.e. up to 5-variate regressions) proposed by Engle and Granger (1987). MacKinnon (1991) extends these values to regressions including 12 independent variables and deterministic trends. MacKinnon (1996) obtains critical values for finite sample models in addition to the asymptotic values. They provide a computer program to obtain these critical values for the cointegration model of Engle and Granger (1987).

Johansen and Juselius (1990) obtains critical values for the maximum eigenvalue and trace statistic for processes of dimensions $N = 1, \ldots, 5$. Osterwald-Lenum (1990) extends these critical values to the processes of dimensions $N = 1, \ldots, 11$. Osterwald-Lenum (1992) obtain critical values of models involving linear trends and seasonal dummies. MacKinnon et al. (1999) calculates critical values for the maximum eigenvalue and trace tests in a model incorporating exogenous $I(1)$ variables. The simulation involves processes of dimensions $N = 1, \ldots, 20$ including at most 8 exogenous and 12 endogenous variables in the VECM. Ericsson and MacKinnon (2002) normalise cointegrating vector in the ECM in such a way that the first component is 1 and obtain the adjustment coefficient γ_1 (i.e. once the adjustment matrix in 2.2). They obtain the asymptotic distribution of the t-ratio of γ_1 which is referred to as κ.

2.4 Test and estimation methods for time-varying cointegration

Since in this dissertation, we aim at estimating time-varying cointegration models, an instability test is indispensable. The null hypothesis of the instability tests is a static cointegration relation. Thus, we provide a review of tests of parameter instability, then we review estimation methods for time-varying cointegration.

A number of theoretical works develop statistical tests of instability in cointegration relations (see Hansen, 1992*b*, Quintos and Phillips, 1993, Gregory and Hansen, 1996 and Koedijk et al., 2004). Estimation of smoothly varying cointegrating vectors has also been in the spotlight of a number of theoretical works (Park and Hahn, 1999; Bierens and Martins, 2010; Li et al., 2020). Quandt (1960) suggested a test based on an unknown timing of the change in the parameter. Hamilton (1989) assumes discrete shifts in the dynamics of the cointegration relation. He uses Goldfeld and Quandt (1973) Markov switching regression to identify structural changes in the cointegration parameters. He assumes that the economy switches between two regimes of fast growth and slow growth. Hansen (1992*b*) extended the test of parameter change to the context of cointegration relation of fully modified cointegration introduced by Phillips and Hansen (1990). They argue since an alternative hypothesis of a random walk in the intercept implies no cointegration relation, their test statistic represents a null of cointegration against the alternative of no cointegration. Hansen (1992*b*) develops distributional theory for three test statistics namely F_{nt}(or L_c test first proposed by Gardner, 1969), $SupF$ (proposed by Quandt, 1960) and $MeanF$. The null corresponding to all three test statistics is cointegration. The alternative hypotheses are respectively a known structural break, an unknown structural break, and a random walk in the parameters. Hansen emphasises that rejection of the null implies that the standard cointegration model with stable coefficients is rejected. Quintos and Phillips (1993) propose an approach to test parameter constancy assuming a random cointegrating parameter. Their approach corresponds more or less with that of Hansen (1992*b*) where cointegrating coefficient follows a random walk. Their test is applicable to a subset of cointegrating parameters while that of Hansen (1992*b*) is applicable to the entire set of cointegrating coefficients. They argue that a constant coefficient implies a zero variance of the innovations that drive the random walk. Shin (1994) modifies Hansen (1992*b*) test to account for the alternative of a spurious regression. Assuming the intercept follows no random walk process, he tests this hypothesis versus the alternative of a random walk implying a spurious regression.

Gregory and Hansen (1996) provide a residual-based test for the null of no cointegration versus a regime change in either the intercept alone or the cointegrating vector. The structural shift in the cointegrating vector is unknown.

Park and Hahn (1999) investigate smooth evolution of cointegrating coefficient. They use a nonparametric method to estimate time-varying coefficients. They provide two specification tests to test the time-varying cointegration relation. In the first specification test, they test the null of the constant cointegration relation against the alternative of time-varying coefficients. The second specification test is used to test the null of time-varying cointegration versus the alternative of a time-varying spurious regression.

Hansen and Johansen (1999) use the recursive estimation of cointegration parameters to derive graphical procedures for the evaluation of parameter constancy. They use the time path of the recursively estimated parameters to derive the limiting distribution of fluctuation test by Ploberger et al. (1989) and LM-type proposed by Nyblom (1989). The fluctuation test is used to test the constancy of recursively estimated eigenvalues.

Hansen (2003) models the structural change in the Johansen cointegration model. His framework allows for structural change in any subset of the parameters. He assumes the change point and the number of cointegration relations are known a priori.

Saikkonen and Choi (2004) study asymptotic theory of a time-varying cointegration model that incorporates a smooth function of coefficients. They call their model cointegrating smooth transition regression.

Bierens and Martins (2010) express cointegrating vectors in terms of smoothly varying Chebyshev time polynomials and rewrite VECM in terms of the approximated vectors by Fourier's expansion. This formulation provides a way to estimate smoothly-varying cointegrating vectors. They develop a test based on Johansen (1988) to test the null hypothesis of time-invariant cointegration against the alternative of time-varying cointegration.

Li et al. (2020) specify a single equation cointegration model where coefficients are smoothly-varying functions of time. They apply standard Nadaraya-Watson Kernel methods to estimate time-varying cointegrating vector. They develop an asymptotic theory to show that the limit distribution of the time-varying vector converges to a normal distribution.

Eroğlu et al. (2022) represent the single equation cointegration model in state-space formulation with the coefficients following AR processes. They use the Kalman filter to estimate time-varying cointegrating vectors and provide four possible conclusions on time-varying cointegration. Possible conclusions are no cointegration, cointegration with fixed coefficients, smoothly varying cointegration which relies on very small error variances of the state variables, and stochastically varying cointegration.

Chapter 3

The effects of uncertainty on the dynamics of stock market interdependence: Evidence from the time-varying cointegration of the G7 stock markets[1]

Hamid Babaei[1], Georges Hübner[1], Aline Muller[1,2]

[1] *HEC Liège, Management School-University of Liège, 14 rue Louvrex, B-4000 Liège, Belgium*

[2] *Luxembourg Institute of Socio-Economic Research, Luxembourg*

Abstract

This chapter investigates the dynamic pattern of interdependence among the stock markets of the G7 member countries over the period from 1990 to 2023. The state-space formulation of the time-varying cointegrating coefficient makes it possible to examine the potential drivers of disruption in the long-run co-movement of markets. Results reveal that variations in a number of financial risk factors, economic policy uncertainty (EPU), and world geopolitical risk (GPR) have a significant impact on cointegrating coefficients. Further analysis of the co-movement of the augmented and unaugmented cointegrating coefficients suggests that globalization has reduced the causes of market segmentation to our risk factors.

JEL classification: F360, F020, E520, D89

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3.1 Introduction

Following the introduction of the news-based measure of economic policy uncertainty (EPU) by Baker et al. (2016), there has been an extensive body of literature on the impacts of uncertainty on asset prices. A large volume of literature has considered EPU as an influential risk factor on stock returns. The majority of these investigations, however, have focused on the role of EPU on cross-country market volatility (Balcilar et al., 2019; Chen and Chiang, 2020; Chang, 2021), on time-varying volatility spillover (Balli et al., 2021; Li et al., 2023), and volatility prediction (Liow et al., 2018; Ma, Guo, Chevallier and Huang, 2022). Additionally, there has been a growing number of studies investigating the effects of EPU on stock prices with a resulting consensus that uncertainty does indeed have a negative impact on stock prices. A number of the papers have examined the interdependence of EPU and stock prices with a focus on the G7 member countries (Nusair and Al-Khasawneh, 2022), the cross-border effect of the US EPU on Asian stock markets (Liang et al., 2020), co-movement of the EPU and stock markets in BRICS countries (Wang and You, 2022), and causality between EPU and stock prices in BRIC countries (Aydin et al., 2022).

This study investigates the impact of uncertainty on the dynamics of market interdependence. The finding in the literature that stock prices across different countries heterogeneously respond to the shocks in the uncertainty (see Das et al., 2019 and Aydin et al., 2022) highlights the question that markets' linkages can also be affected by uncertainty. Market linkages can also be destabilised by uncertainty through shifts in international portfolio allocation (Nguyen et al., 2022; He et al., 2022; Chada, 2023) and asymmetric response of stock markets to uncertainty (Hoque and Zaidi, 2019). Given these empirical findings, we aim at examining how economic uncertainty indicators have influenced stock markets' co-movement. This question is of particular importance because cointegration of the world's major stock market indices has been evidenced to vary over time (see Mylonidis and Kollias, 2010; Menezes et al., 2012, and Pascual, 2003). Despite empirical evidence of time-varying cointegration, the specific reasons for repeated disruptions in the long-run equilibrium relation of stock markets have never been investigated. We address here the key factor of uncertainty and its impacts on the dynamics of markets' co-movement, as it has been evidenced in the literature, to substantially influence security prices.

As the uncertainty associated with political instabilities is also found to be a key factor in stock market dynamics (Antonakakis et al., 2017; Balcilar et al., 2018; Das et al., 2019; Kannadhasan and Das, 2020), the geopolitical risk (GPR) index introduced by Caldara and Iacoviello (2022) is chosen as the most convincing proxy for the uncertainty cast by geopolitical instability. GPR is found to predict stock market returns (Ma, Lu and Tao, 2022), impact international investment (Yu and Wang, 2023), and reduce covariance between oil and stock markets (Antonakakis et al., 2017). After considering the extant empirical evidence on the impacts of GPR on stock markets, we aim to investigate the role of GPR along with EPU as two main potential causes of the dynamic behaviour in the markets' interdependence.

Even though uncertainty is evidenced to propagate across markets, impact asset prices, and hamper cross-border investment, there is no paper that tries to make a clear and direct link between uncertainty and the dynamics of cointegration. We present a state-space formulation of a single equation cointegration model in which the slopes depend on uncertainty and a number of diverse risk factors. In parallel with this model, we estimate an unaugmented time-varying cointegration in which the slopes follow a random walk. The state-space formulation of the augmented and the unaugmented time-varying cointegration models makes it possible to gain insight into the relative long-run risk-adjusted return of the US market vis-à-vis other stock markets of the G7 member countries. We apply a Kalman filter to estimate the time-varying cointegration models. As the existing works have either used nominal price return (NPR) indices or real total return (RTR) indices, we use both sets of indices over the period 1990-2023.

This work adds to the existing literature in two directions. First, this study contributes to the literature on long-run stock markets' co-movement by shedding light on the variables that disrupt cointegration of the stock markets of the G7 member countries. We form our hypothesis based on the findings in the literature that uncertainty can be a significant driver of security prices. Our empirical findings build on the literature on uncertainty by relating the evolution of cointegration relation to variations in EPU and GPR indicators. The dynamics of the cointegrating coefficients are found to be significantly influenced by variations in uncertainty and financial indicators. Secondly, we examine the role of globalisation in the evolution of the co-movement of stock markets. Further analysis on the augmented and unaugmented cointegrating vectors reveals that growing globalisation over recent decades has led to stronger cointegration relations.

This paper is divided into the following sections: Section 2 briefly provides a review of the literature on uncertainty by presenting empirical works on various aspects of the impacts of uncertainty variables on stock markets. Section 3 presents the state-space model used for the purposes of this paper to study the drivers of time-varying cointegration. Section 4 explains the data and preliminary analysis of bivariate cointegration. The results of a stability analysis on bivariate and multivariate cointegration are presented in section 5 and section 6 discusses the drivers of cointegration dynamics. Section 7 examines, in detail, the incremental information added by the augmented model, with conclusions forming section 8 of the paper.

3.2 Literature review

A growing body of literature provides evidence that uncertainty associated with economic policy and political risks does affect stock markets. To date, the literature on the impacts of EPU and GPR indices on stock markets goes in four main directions. The first strand of the papers introduces uncertainty as a risk factor that impacts stock return volatility. These papers perform a wide range of analyses including risk dynamics as well as the spillover associated with uncertainty. The second strand of the papers focuses on the co-movement of uncertainty variables and stock prices. These papers use two autoregressive distributed lag (ARDL) and nonlinear autoregressive distributed lag (NARDL) cointegration models to investigate the short-run and long-run effects of uncertainty on stock prices. The third category of papers deals with the consequences of uncertainty for foreign investments in financial markets. The last category of the papers investigates how, and to what extent, uncertainty drives the dynamics of market integration. The focus of this category of papers involves short-run market integration in which returns are used. To the best of our knowledge, this latter category of papers does not consider the impact of uncertainty on the long-run dynamics of markets' co-movement.

3.2.1 Uncertainty as a risk factor

Since the introduction of EPU, a number of papers have explored its impact on equity market volatility. The literature has documented that EPU is positively associated with higher volatility and higher risk premium of stock prices (see Brogaard and Detzel, 2015; Raza et al., 2018 and Ma, Guo, Chevallier and Huang, 2022). However, the main focus of this class of papers has been on the role of EPU in cross-country volatility prediction. Liow et al. (2018) explore the dynamic interdependence between EPU spillover and financial risk spillover in seven countries and find that changes in EPU spillover lead to changes in financial stress spillover. Balcilar et al. (2019) emphasise the importance of EPU indicators for predicting cross-country stock market volatility. Balli et al. (2021) examine the role of EPU in the US on the time-varying spillover from the US stock market to global markets and find that US uncertainty variables drive spillovers from the US to global stock markets. Li et al. (2023) investigate the spillover effect of EPU on stock markets and find that Asian stock markets react strongly to the innovations of EPU in the US. Han and Li (2023) study the asymmetric impact of global EPU on long-run asset volatility and find a negative relation between shocks to global uncertainty and long-run volatility. Chang (2022) find evidence supporting the asymmetric impact of EPU on the joint volatility cycle between the US and Japan.

3.2.2 Co-movement of uncertainty and stock prices

Two cointegration models of ARDL, developed by Pesaran et al. (2001) and NARDL developed by Shin et al. (2014), are widely used to study the long-run and short-run impact of EPU on stock prices. Nusair and Al-Khasawneh (2022) examine the effect of EPU on G7 member country stock prices using an ARDL as well as a NARDL model. They find that EPU has a negative and asymmetric impact on stock prices in both the short-run and long-run. The causality between EPU and international stock markets has been the subject of several investigations in BRIC (Aydin et al., 2022), BRICS (Wang and You, 2022) and OECD countries (Chang et al., 2015). Similar studies show asymmetric as well as diverse patterns in the causality of EPU on stock markets (see Bahmani-Oskooee and Saha, 2019; Liang et al., 2020 and Bairagi, 2022).

3.2.3 Effects of uncertainty on foreign portfolio holdings

Globalisation has resulted in a significant expansion of free capital flows and international investments across the world's stock markets. International portfolio diversification and foreign direct investment are among the major drivers of stock market integration. The literature has evidenced that a major determinant of capital flows is the level of uncertainty in the host country. As indicated by Andrikopoulos et al. (2023), global EPU impacts capital inflow by decreasing portfolio investment. EPU has been evidenced by He et al. (2022) to adversely impact foreign institutional investment in Chinese listed firms. In a similar study, Chada (2023) find that there exists a positive relationship between exposure to both global and domestic EPU and institutional foreign investment in Indian firms. This finding contradicts the empirical findings by He et al. (2022) in China. Yu and Wang (2023) document the negative impact of geopolitical risks on foreign capital inflow to 41 countries over the period 2003-2020. Using a large sample over 1995-2018, Le and Tran (2021) find that the uncertainty induced by GPR leaves an adverse impact on firm investments in Asian economies. Julio and Yook (2016) show that political uncertainty negatively impacts cross-border capital flow.

3.2.4 The role of uncertainty in market integration

There is a clear lack of empirical work on the issue of whether uncertainty can have an influential role in the dynamics of markets' linkages. Aladesanmi et al. (2019) examine the hypothesis that EPU convergence drives the integration of stock markets in the UK and the US. They find that macroeconomic policy convergence led to a higher integration of the US and UK stock markets over the period 1935-2015. Youssef et al. (2021) focus on the impact of EPU on the dynamic connectedness of stock markets during the COVID-19 crisis. Their findings reveal the positive impact of EPU on the connectedness index, particularly during the pandemic crisis. Using an error correction model (ECM), Aladesanmi et al. (2019) investigate the evolution of the conditional correlation between the US and UK stock markets and find that the conditional correlation between the EPU of the two countries significantly influences the conditional correlations of their stock markets.

3.2.5 Uncertainty and time-variation in the long-run equilibrium

Several key issues arise from the extant literature. These issues highlight the question of the impact of uncertainty on the dynamics of cointegration of stock markets. First, the literature confirms, however, that uncertainty shocks are transmitted to domestic and foreign stock markets: Stock prices and returns are shown to be profoundly affected by uncertainty regarding economic policies. The repercussions of these effects on the long-run equilibrium between stock market indices remain an unexplored dimension of time-varying cointegration. Second, the relaxation of controls on capital movements over the last few decades has resulted in an expansion of international portfolio investment and fostered market integration. On the other hand, in the integrated stock markets of the G7 member countries, rising uncertainty can provoke capital outflows which can hamper long-run co-movement of stock market indices. Third, a few works have also provided evidence for time-variation in the nexus between EPU and stock prices (Ko and Lee, 2015; Xiong et al., 2018; Nguyen et al., 2020). In line with the poor empirical findings on static cointegration in the G7 member country stock markets (Kanas, 1998; Agoraki et al., 2019), a number of works find evidence for variation in the cointegration relation (Rangvid, 2001; Pascual, 2003; Menezes et al., 2012). On the other hand, empirical studies on cointegration of global stock markets emphasise that economic and political events lead to structural breaks in the equilibrium relations (Taylor and Tonks, 1989; Arshanapalli and Doukas, 1993). This body of empirical evidence highlights the importance of EPU and GPR indicators to examine the impact of economic and political uncertainties in an instantly varying cointegration relation.

Our study is the one to link the dynamics of stock markets' co-movement to EPU and GPR indices. State-space modelling of the single equation cointegration model allows us to shed more light on the causes of instabilities in bivariate and multivariate cointegration of stock markets. The time-variations in stock market cointegration are estimated in the literature by three main approaches; abruptly discontinuous change (Taylor and Tonks, 1989; Arshanapalli and Doukas, 1993), recursive estimation (Rangvid, 2001; Pascual, 2003), and rolling-window estimation (Mylonidis and Kollias, 2010). The state-space model of this paper captures instant changes of the cointegrating coefficients. Moreover, by relating cointegrating vectors of the G7 stock markets to a set of uncertainty and financial variables, we investigate whether cointegration relations are susceptible to changes caused by these variables.

3.3 Time-varying cointegration model

To the best of our knowledge, only a very few papers apply the Kalman filter in the context of long-run equilibrium relations between economic or financial variables. Haldane and Hall (1991) apply the Kalman filter to study the central role of the Deutschmark-Dollar exchange rate on the Dollar-Sterling and Deutschmark-Sterling exchange rates. They find a high degree of variation in bivariate cointegration parameters. Serletis and King (1997) measure stock market integration by regressing the difference between a specific market index with Germany on a constant and the difference between Germany and the United States. They estimate the time-varying parameters of this bivariate regression model using a Kalman filter.

Despite the growing volume of theoretical and empirical literature on time-varying cointegration, the reasons for underlying causes in variations in the cointegration relations over time between the largest global stock markets remain an open question. A state-space formulation of cointegration relation allows us to tackle this question by relating the varying cointegrating vector to the presumed driving factors of the dynamic interdependence between market indices.

To shed more light on the mixed findings in the literature, we study two sets of indices, namely, the NPR and the RTR, of each country, as both sets of variables are used in the literature.[2] The use of RTR indices is motivated by a number of works that investigate cointegration of return indices. Richards (1995) argues that return indices are not cointegrated because of permanent country-specific components that drive their evolution. In a theoretical work, Lence and Falk (2005) investigate the interrelationship between integrated markets and cointegrated prices. Having defined integrated markets based on the absence of arbitrage

[2]Taylor and Tonks (1989), Arshanapalli and Doukas (1993), Rangvid (2001), Mylonidis and Kollias (2010), and Menezes et al. (2012) are among those who use price indices. Richards (1995) and Gilmore et al. (2008) are among those who use total return indices.

opportunities, they conclude that, the necessary (but not sufficient) condition for two price indices to be cointegrated, regardless of market integration, is that their supply processes are themselves cointegrated. They theoretically derive the same conclusion for the return indices.

In this current study, we address three potential variables that can presumably change investors' expectations concerning firms' future cash flows, and hence lead to a new equilibrium relation. As noted in the literature, the EPU and GPR indices are candidate proxy variables for economic and political uncertainty conditions, which can potentially cast serious doubts on future economic activity.

In the time-varying cointegration model of this work, the innovations in the cointegrating vectors are driven by the innovations in the uncertainty variables. This is represented by the following two variations of the state-space model:

$$log(Y_t^{RTR}) = a + \boldsymbol{b}_t' log(\boldsymbol{X}_t^{RTR}) + \epsilon_t, \tag{3.1a}$$

$$log(Y_t^{NPR}) = a + \boldsymbol{b}_t' log(\boldsymbol{X}_t^{NPR}) + \epsilon_t, \tag{3.1b}$$

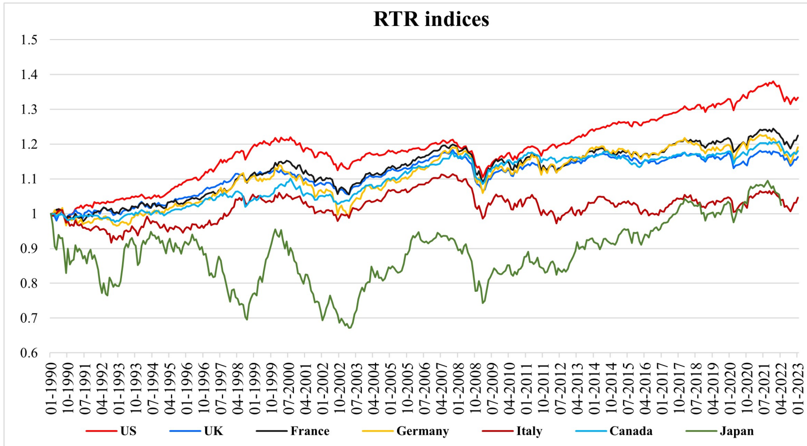
$$\boldsymbol{b}_t = \boldsymbol{b}_{t-1} + \sum_{l=0}^{L} \boldsymbol{\gamma}_l' \boldsymbol{U}_{t-l} + \boldsymbol{\omega}' \boldsymbol{F}_t + \boldsymbol{\nu}_t. \tag{3.2}$$

Equations (3.1a) and (3.1b) represent Engle and Granger (1987) cointegration relations between RTR and NPR indices, respectively. Y_t represents the dependent stock market index and $\boldsymbol{X}_t$ is a $(n \times 1)$ vector of the G7 stock market indices with $1 \leq n \leq 6$, a is a scalar, $\boldsymbol{b}_t$ represents a $(n \times 1)$ vector of the time-varying cointegrating coefficients, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and $\boldsymbol{\nu}_t$ is a $(n \times 1)$ vector of normally distributed innovations with $E[\boldsymbol{\nu}_t] = 0$ and $E[\boldsymbol{\nu}_t \boldsymbol{\nu}_\tau'] = \boldsymbol{\Omega}$ when $t = \tau$ and $E[\boldsymbol{\nu}_t \boldsymbol{\nu}_\tau'] = 0$ otherwise. $\boldsymbol{\Omega}$ is a diagonal matrix and the disturbances ϵ_t and $\boldsymbol{\nu}_t$ are assumed to be uncorrelated for all lags. In the state Equation (3.2) which is applied to either of the RTR or NPR models, the line vector $\boldsymbol{\gamma}_l' = [\gamma_{US,l} \quad \gamma_{EU,l} \quad \gamma_{GPR,l}]$ gathers the coefficients of the l-th lag uncertainty variable and $\boldsymbol{U}_{t-l}' = [\Delta log(EPU_{US,t-l}) \quad \Delta log(EPU_{EU,t-l}) \quad \Delta log(GPR_{t-l})]$ forms the vector of the uncertainty variables. The variables $EPU_{US,t}$ and $EPU_{EU,t}$ represent the EPU in the US and European Union, respectively, at time t, and GPR_t represents the world GPR index at time t. The column vector $\boldsymbol{F}_t$ contains a number of stock return determinants, including the US bond-to-equity ratio (BEER), US credit spread, US dividend yield, change in the US implied volatility index (VIX), US and domestic inflation rates, and US and domestic term spreads. The vector $\boldsymbol{\omega}$ contains the slopes of the financial risk determinant. Note that when $n \geq 2$, the slopes of a specific uncertainty or risk factor (i.e. the vectors $\boldsymbol{\gamma}_l$ and $\boldsymbol{\omega}$) are identical for all state variables. This assumption reduces the computational complexity of the model in the multivariate case and makes the estimation practical.

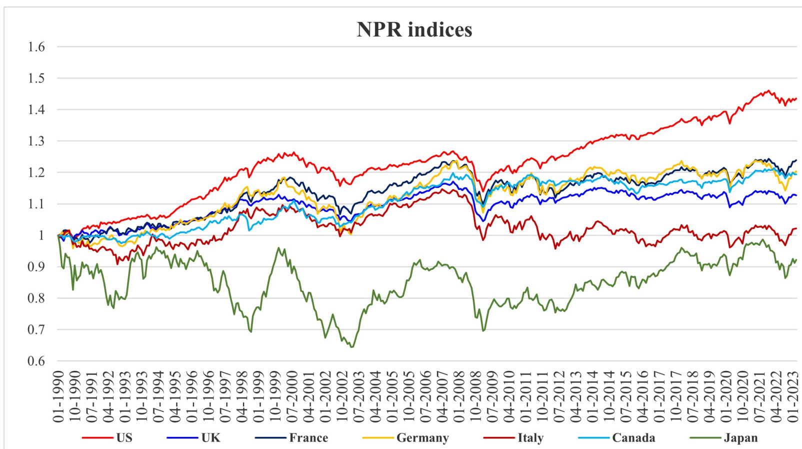
As Equation (3.2) indicates, we assume that a shock to the uncertainty variables can trigger a disruption in the cointegrating vectors. We also include lagged values of the innovations in the uncertainty variables because we let the earlier values of the uncertainty innovations impact the cointegrating vectors. The intercept in Equations (3.1a) and (3.1b) is constant because otherwise if it is added to the state-space, Equations (3.1a) and (3.1b) can no longer be referred to as a cointegration relation.

3.4 Data and preliminary evidence

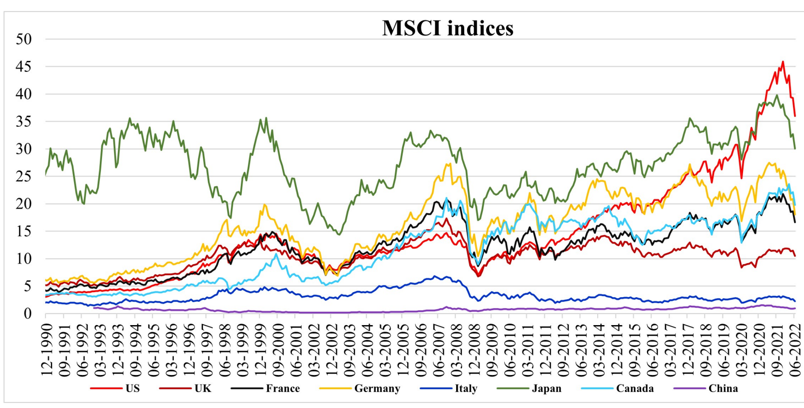
We use monthly stock market indices of the US (S&P 500), France (MSCI index), Germany (MSCI Germany), the UK (FTSE all-share index), Italy (MSCI Italy), Canada (S&P TSX composite index), and Japan (TOPIX all share index). Data cover the period from January 1990 to February 2023 and are retrieved from DataStream. These indices are selected to cover a large part of the market capitalization throughout the sample period. All indices are expressed in terms of US dollars as a common numeraire to avoid any contamination by a currency effect. The sample paths of the time series of these indices, with normalised starting points, are represented in Figure 3.1. We also include nominal values of the MSCI indices in our analysis because these indices are used in Chapter 4. The preliminary analysis on these indices are presented in Chapter 4. The US credit spread is calculated as the Moody's Aaa minus Baa corporate bond yield. The US term spread is calculated as the 10-year minus 3-month treasury constant maturity rates and those of the other G7 member countries are calculated as the long-term government bond yield minus 3-month interbank rates. The US BEER is calculated as the ratio of the US 10-year treasury constant maturity rate to the S&P 500 earnings yield. The VIX, the inflation rates, and all the interest rates used in the calculation of the financial factors are downloaded from the Federal Reserve Bank of St. Louis.



(a) RTR indices



(b) NPR indices



(c) MSCI indices

Figure 3.1: RTR, nominal NPR, and MSCI indices of the G7 countries. All indices are in terms of US dollars and are transformed by natural logarithms. The first sub-graph exhibits inflation-adjusted total return indices. The second sub-graph exhibits nominal price return indices.

We apply four commonly used unit root tests of Augmented Dickey-Fuller (ADF) (Dickey and Fuller, 1979), Phillips-Perron (PP) (Phillips and Perron, 1988), Elliot-Rothenberg-Stock (ERS) (Elliott et al., 1996) and NG-Perron (Ng and Perron, 2001). The null of a unit root in the indices is not rejected in the majority of cases, although the results of ADF and PP tests on Japan's NPR index reject the null of a unit root. These results are not confirmed by ERS and NG-Perron tests. In the case of Italy, both ERS and Ng-Perron tests reject the null hypothesis for RTR and NPR indices. The results of ADF and PP, however, give contrasting results to those of ERS and Ng-Perron tests.

Given this set of results, we subsequently perform standard time-invariant cointegration tests. The tests involve both bivariate and multivariate relations. The results of these tests are displayed in Tables 3.2 and 3.3. The bivariate cointegration tests are performed

Table 3.1: Unit root tests on the indices transformed by logarithms

	RTR indices				NPR indices			
	ADF		PP		ADF		PP	
	t-statistic	lag	t-statistic	bandwidth	t-statistic	lag	t-statistic	bandwidth
US	-0.80	0	-0.81	6	-0.73	0	-0.74	6
UK	-1.78	0	-1.82	5	-2.14	0	-2.21	5
FR	-1.16	0	-1.15	2	-1.51	0	-1.51	2
DE	-1.45	0	-1.45	3	-1.74	0	-1.78	4
IT	-2.12	0	-2.19	5	-2.16	0	-2.22	5
CA	-0.83	0	-0.83	0	-1.02	0	-1.05	1
JP	-1.98	3	-2.24	7	-3.05**	1	-3.23**	7
	ERS		Ng-Perron		ERS		Ng-Perron	
	t-statistic	lag	MZa	lag	t-statistic	lag	MZa	lag
US	145.63	0	-1.31	0	167.76	0	1.40	0
UK	57.48	0	0.31	0	37.08	0	-0.16	0
FR	52.94	0	0.74	0	38.75	0	0.36	0
DE	30.79	0	-0.09	0	26.71	0	-0.33	0
IT	3.56*	0	-7.56*	0	3.20**	0	-7.85*	0
CA	54.35	0	0.67	0	43.97	0	0.39	0
JP	6.92	3	-3.63	3	8.94	1	-2.99	1

All indices are in terms of US dollars and are transformed by natural logarithms. Unit root test is performed using the augmented Dickey-Fuller (ADF), Phillip-Perron (PP), Elliot-Rothenberg-Stock (ERS) and NG-Perron test methods. *, ** and *** represent significance at the 10%, 5% and 1% levels, respectively. The numbers in parentheses represent the lag length and the bandwidth of the PP test. The lag length is specified using the Schwarz criterion. In the PP test, Bandwidth is specified using the Newey-West method and Bartlett kernel is used as spectral estimation method. Critical values of the ERS test at 1%, 5% and 10% significance levels are 1.99, 3.26 and 4.48 respectively. Critical values of Ng-Perron test at 1%, 5% and 10% significance levels are -13.80, -8.10 and -5.70 respectively.

using Engle and Granger (1987) single equation model. Each bivariate relation in Table 3.2 involves two tests that differ on the choice of the dependent variable. The Engle and Granger (1987) cointegration test does not reject the null hypothesis of the absence of cointegration in most of the bivariate relations. Only in the single case of the bilateral relation of the US stock market with Japan, the null hypothesis is rejected at 10% significance level. The sample covers a relatively long period, and the weak results presented in Table 3.2 are therefore expected. During the period from 1990 to 2023, numerous political and economic crises impacted the evolution of the world's stock markets, some of which, according to the literature, are evidenced to have caused a structural break in the cointegration relations (Kanas, 1998; Mylonidis and Kollias, 2010). We do not consider any abruptly and exogenously discontinuous cointegration structure, as it is not always possible to pinpoint the onset or ending point of an economic or political crisis.

We estimate multivariate time-invariant cointegration relations using the fully modified ordinary least squares (FMOLS) approach developed by Phillips and Hansen (1990) in which a few anomalies associated with the single equation cointegration are resolved. Since, in the multivariate case, there are numerous relations depending on the choice of the depen-

Table 3.2: Bivariate time-invariant cointegration test

	US-UK		**US-France**		**US-Germany**		**US-Italy**		**US-Canada**		**US-Japan**	
	RTR indices											
Dependent	US	UK	US	France	US	Germany	US	Italy	US	Canada	US	Japan
tau-statistic	-0.83	-1.52	-2.05	-2.21	-1.69	-2.06	-1.55	-2.50	-1.18	-1.19	-2.91	-2.62
	(0.93)	(0.76)	(0.50)	(0.42)	(0.68)	(0.50)	(0.75)	(0.28)	(0.86)	(0.86)	(0.14)	(0.23)
z-statistic	-2.56	-6.32	-8.38	-9.71	-6.72	-9.14	-4.07	-12.04	-2.35	-2.69	-6.57	-13.38
	(0.90)	(0.71)	(0.47)	(0.38)	(0.59)	(0.41)	(0.80)	(0.25)	(0.91)	(0.90)	(0.60)	(0.20)
	NPR indices											
Dependent	US	UK	US	France	US	Germany	US	Italy	US	Canada	US	Japan
tau-statistic	-0.73	-1.84	-1.84	-2.25	-1.71	-2.28	-0.48	-2.09	-1.00	-1.22	-1.76	-3.28
	(0.94)	(0.61)	(0.61)	(0.40)	(0.67)	(0.38)	(0.96)	(0.48)	(0.90)	(0.90)	(0.65)	(0.06)
z-statistic	-2.29	-7.39	-7.24	-10.13	-7.35	-11.29	-0.55	-8.75	-2.06	-2.06	-2.18	-18.09
	(0.92)	(0.54)	(0.55)	(0.35)	(0.54)	(0.29)	(0.98)	(0.44)	(0.93)	(0.93)	(0.92)	(0.08)

All indices are in terms of US dollars and are transformed by natural logarithms. Engle and Granger (1987) cointegration is applied to examine bivariate cointegration relations. The numbers in parentheses are p-values. Each bivariate cointegration relation is examined twice depending on the choice of the dependent variable.

dent variable, we use FMOLS once for each cointegration relation. This approach takes the endogeneity of the independent variables into account and corrects the endogeneity bias in the estimated coefficients. We assume three multivariate relations as indicated in Table 3.3. The 4-variate cointegration relation takes the UK stock market index as the dependent variable and those of France, Germany, and Italy as the independent variables. The 5-variate model involves a relation between the US stock market and those of the European members of the G7. We also estimate a 7-variate cointegration relation including all the G7 stock market indices with the US stock market index as the dependent variable. A notable finding in Table 3.3 is that all the coefficients are significantly distinct from zero in only one relation between RTR indices of the European members of the G7. The ADF and PP tests on the residuals of this relation evidence a stationary process. Although there are insignificant coefficients in most of the multivariate relations, the resulting residuals of certain relations turn out to be stationary. The unit root tests on the residuals of the 7-variate relation unanimously support stationarity. The insignificant coefficients can possibly be a result of time-variation in the cointegration relation. This makes instability analysis of the cointegration relations worthy of examination.

Table 3.3: Multivariate time-invariant cointegration using FMOLS

	US		US		UK		US		US		UK	
	slope	p-value	slope	p-value	slope	p-value	slope	p-value	slope	p-value	slope	p-value
C	0.66	0.18	0.48	0.38	1.38	0.00	-0.01	0.99	-0.45	0.57	2.66	0.00
UK	0.43	0.02	0.15	0.45			0.36	0.09	0.25	0.31		
France	1.89	0.00	1.59	0.00	0.30	0.00	2.44	0.00	1.84	0.00	0.09	0.30
Germany	-0.12	0.47	-0.35	0.06	0.43	0.00	-0.27	0.16	-0.33	0.13	0.58	0.00
Italy	-0.76	0.00	-0.63	0.00	0.17	0.00	-1.07	0.00	-0.84	0.00	0.18	0.00
Canada	-0.62	0.00					-0.52	0.00				
Japan	0.11	0.12					0.03	0.67				
ADF	-4.87***	0.00	-2.04	0.27	-3.66***	0.01	-3.85***	0.00	-2.21	0.20	-4.41***	0.00
PP	-4.68***	0.00	-2.36	0.15	-3.45***	0.01	-5.22***	0.00	-2.65*	0.08	-4.41***	0.00
ERS	1.40***		3.72*		7.58		4.01*		3.30*		5.67	
Ng-Perron	-10.03**		-6.71*		-3.69		-6.56*		-7.53*		-5.06	

All indices are in terms of US dollars and are transformed by natural logarithms. C is the intercept in the single equation cointegration relation. The bottom panel represents four unit root tests of ADF, PP, ERS, and Ng-Perron. The unit root tests are applied to the residuals of the single equation cointegration model. Critical values of the ERS test at 1%, 5%, and 10% significance levels are 1.99, 3.26, and 4.48 respectively. Critical values of Ng-Perron test at 1%, 5%, and 10% significance levels are -13.80, -8.10, and -5.70 respectively.

3.5 Evidence of dynamic cointegration

Prior to the estimation of time-varying cointegration through the state-space formulation of Equations (3.1) and (3.2), we apply the Hansen stability analysis to the bivariate and multivariate cointegration relations.[3]

Hansen (1992*b*) develops a distributional theory for three test statistics namely F_{nt} (or the L_c test first proposed by Gardner, 1969), the $SupF$ test (proposed by Quandt, 1960) and the $MeanF$, in a regression with integrated processes. The underlying null hypothesis posits constant cointegration coefficients, whereas the alternative hypotheses are an unknown structural break and a random walk in the parameters. Hansen emphasises that rejecting the null hypothesis implies that a standard cointegration model with stable coefficients is rejected. We focus here on the $MeanF$ and L_c statistics as in the underlying alternative hypothesis, the cointegrating coefficients follow a random walk. Hansen (1992*b*) argues that since an intercept following a random walk implies the absence of cointegration, the corresponding alternative hypothesis of L_c is no cointegration. We apply the Hansen stability test to the bivariate cointegration of the US stock market with each of the six other selected developed markets. As Table 3.4 shows, the null hypothesis of time-invariant cointegration of the NPR indices is rejected in four relations out of the six bivariate relations. The $MeanF$ test statistic indicates that the null hypothesis of static cointegration is rejected in 4 bivariate relations. The L_c statistic of the NPR indices rejects the null of cointegration against the alternative of no cointegration, at the 10% level, in 3 bivariate relations. In the first panel of Table 3.4 corresponding to RTR indices, the null hypothesis is rejected in three relations by $MeanF$ or L_c test statistics.

Both sets of indices demonstrate a degree of instability in the cointegrating coefficient. As $SupF$ indicates, the bivariate cointegration relations between the US and certain nominal indices incur a sudden structural break in an unknown time in the cointegrating coefficient. We observe stronger evidence of instability in Table 3.5. In the 4-variate relation of NPR

[3]There are a number of tests aimed at examining parameter stability, most of which assume a regime change or smooth variation of the parameters. The Hansen stability analysis fulfills our objective, as it tests the null hypothesis of static cointegration against the alternative of a gradual change in the cointegrating vector.

Table 3.4: Hansen stability analysis of bivariate cointegration relations

	UK	France	Germany	Italy	Canada	Japan
	RTR indices					
L_c	0.12	0.26	0.13	0.32	0.06	0.53
	(0.2)	(0.19)	(0.2)	(0.13)	(0.2)	(0.03)
$MeanF$	2.24	4.39	3.54	3.67	2.92	5.77
	(0.2)	(0.06)	(0.12)	(0.10)	(0.18)	(0.02)
$SupF$	6.83	12.18	6.70	9.81	5.16	20.31
	(0.2)	(0.07)	(0.2)	(0.16)	(0.2)	(0.01)
	NPR indices					
L_c	0.44	0.40	0.19	0.08	0.07	0.64
	(0.09)	(0.08)	(0.2)	(0.2)	(0.2)	(0.02)
$MeanF$	5.54	5.68	3.85	0.96	3.28	7.08
	(0.03)	(0.02)	(0.09)	(0.2)	(0.14)	(0.01)
$SupF$	22.91	14.08	9.42	2.70	10.09	17.83
	(0.01)	(0.03)	(0.19)	(0.2)	(0.15)	(0.01)

All indices are in terms of US dollars and are transformed by natural logarithms. The Hansen test is applied to the bivariate cointegration relations between the US and other G7 member countries. The numbers in parentheses are p-values. p-values above 0.2 are indicated by 0.2. The k_1 and k_2 parameters are both taken to be zero. The null hypothesis of the three tests is constant cointegration. The alternative hypothesis corresponding to L_c and $MeanF$ is a random walk in the cointegrating coefficient. The alternative hypothesis corresponding to $SupF$ is an unknown structural break in the cointegration relation.

indices, the null hypothesis of static cointegration is not rejected, although the results of static cointegration in Table 3.3 indicate an insignificant slope. Given that the alternative hypothesis implies a joint instability of the parameters, the insignificant slope of the French stock market index in Table 3.3 can be a result of instability in the cointegration relation.

The state-space variables in Equation (3.2) are augmented by the lagged vector $\boldsymbol{U}_{t-l}$

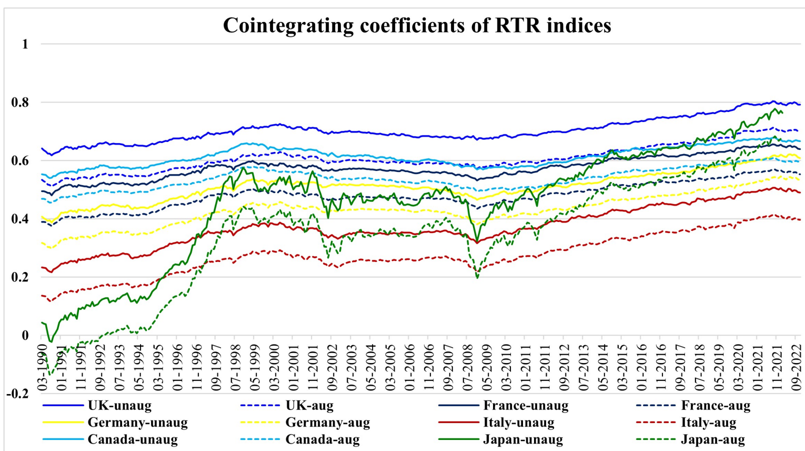
Table 3.5: Hansen stability analysis of multivariate cointegration relations

	RTR indices						NPR indices					
	US		US		UK		US		US		UK	
	stat	p-value	stat	p-value	stat	p-value	stat	p-value	stat	p-value	stat	p-value
L_c	1.08	0.05	3.96	0.01	1.09	0.01	1.40	0.01	1.30	0.01	0.30	0.2
$MeanF$	26.85	0.01	93.52	0.01	10.31	0.01	68.41	0.01	9.40	0.04	2.44	0.2
$SupF$	41.65	0.01	154.25	0.01	23.66	0.01	87.90	0.01	22.82	0.01	8.15	0.2

All indices are in terms of US dollars and are transformed by natural logarithms. The Hansen test is applied to the multivariate cointegration relations. p-values above 0.2 are indicated by 0.2. The k_1 and k_2 parameters are both taken to be zero. The null hypothesis of the three tests is constant cointegration. The alternative hypothesis corresponding to L_c and $MeanF$ is a random walk in all the cointegrating coefficients. The alternative hypothesis corresponding to $SupF$ is an unknown structural break in the cointegration relation.

and the concurrent vector $\boldsymbol{F}_t$, although it can also be formulated as a simple random walk. Estimating the time-varying cointegration relation excluding the vectors $\boldsymbol{U}_{t-l}$ and $\boldsymbol{F}_t$ results in a model that we refer to as the unaugmented model. The estimated state-spaces of the augmented and unaugmented models are denoted by $\hat{\hat{\boldsymbol{b}}}_t$ and $\hat{\boldsymbol{b}}_t$ respectively.

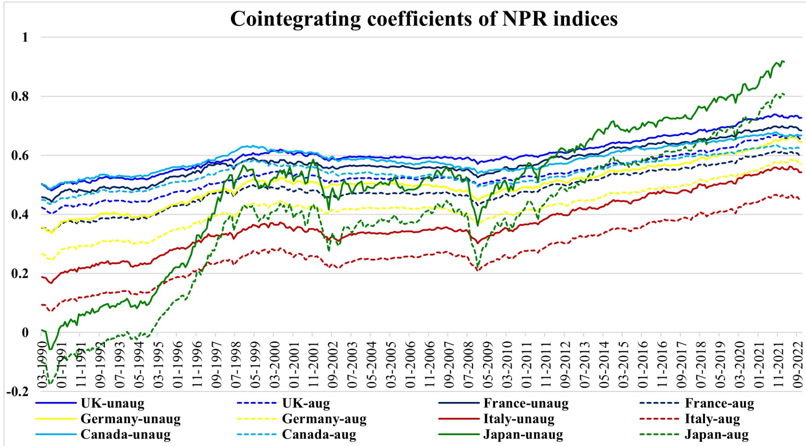
As Figure 3.2 shows, the cointegrating coefficients for all the bivariate relations appear to follow an upward trend that continues until the late 1990s when they stabilise until the economic recession of the early 2000s. The second increasing trend is associated with the economic growth preceding the global financial crisis of 2008. Following that crisis, the estimated cointegrating coefficients follow a relatively steep upward trend that continues up until the end of the sample period. Unlike earlier crises, the Covid-19 crisis does not appear to be associated with a drop in the cointegrating coefficients. Since the indices are in terms of the natural logarithms, one may treat the first differences as returns and try to relate the returns to the equilibrium error through the ECM representation. The contemporaneous rates of



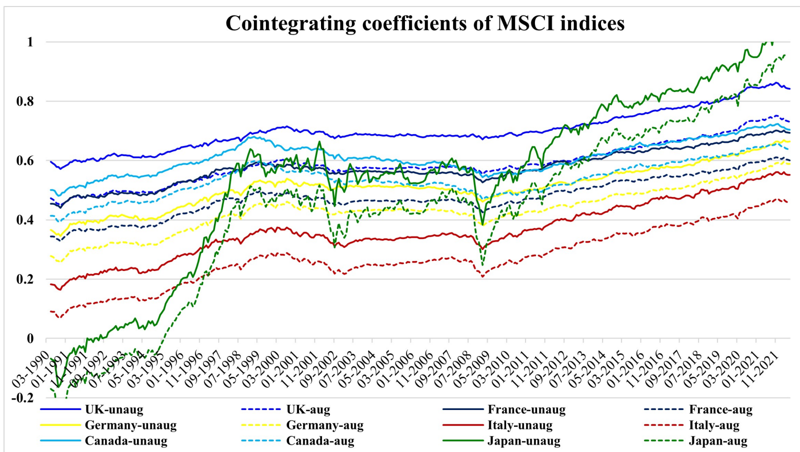
(a) RTR indices

returns of the underlying market indices cannot be related in this type of representation. For this reason, if we are to correlate the contemporaneous rates of returns of two markets, we have to use the estimated equilibrium relation in Equations (3.1a) or (3.1b) and refer to them as the relation between long-run rates of returns. Accordingly, the difference between the unaugmented and augmented coefficients (i.e. $\hat{\boldsymbol{b}}_t - \hat{\hat{\boldsymbol{b}}}_t$) has valuable information about the relative long-run rate of return of the indices.

The upward trending cointegrating coefficients over the three aforementioned periods



(b) NPR indices



(c) MSCI indices

Figure 3.2: Cointegrating coefficient of the bivariate relations between the US and each of the G7 members. The solid lines represent the results of the unaugmented model and the dashed lines represent the results of the augmented model. All indices are in terms of US dollars and are transformed by natural logarithms. The first sub-graph exhibits the estimated results using inflation-adjusted total return indices. The second sub-graph exhibits the estimated results using nominal price return indices. The third sub-graph exhibits the estimated results using MSCI indices.

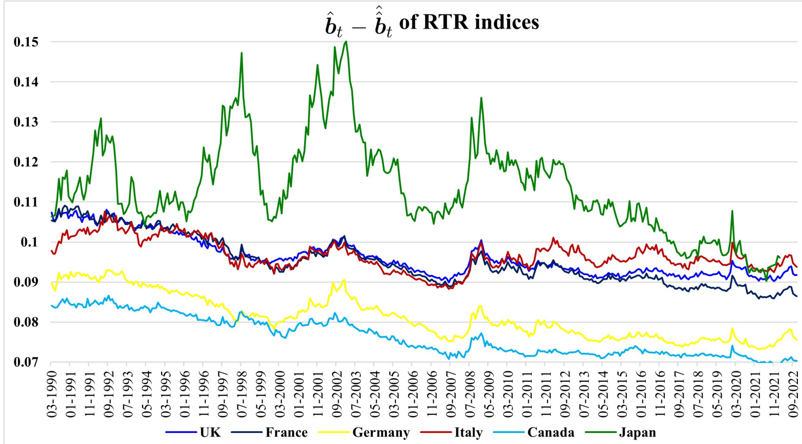
are the result of a higher long-run risk premium that the US market receives compared to the other G7 markets. As illustrated in Figure 3.2, during economic crises, the US market is also penalised more heavily, in the long run, than other markets. Hereafter, we refer to the augmented model as the long-run risk-adjusted equilibrium relation in which $\hat{\hat{\boldsymbol{b}}}_t$ subsumes the long-run relative risk-adjusted rates of return of both markets.

In Figure 3.3, we report the dynamics of the difference between the unaugmented and the augmented time-varying cointegration coefficients of the bivariate relations. During bearish periods, the augmented (i.e., the long-run risk-adjusted) cointegrating coefficient falls faster than the unaugmented coefficient. As the indices are in natural logarithms, the discrepancy between both sets of cointegrating coefficients yields insight into the relative long-run risk-adjusted return of the US market vis-à-vis other markets. An upward (downward) sloping discrepancy variable is associated with a faster downward (upward) sloping risk-adjusted cointegrating coefficient. This, in turn, leads to a lower (higher) relative long-run risk-adjusted rate of return for the US market. A notable implication of this finding is that the US market has higher exposure in the long run to uncertainty and financial risk factors than the other G7 markets as the discrepancy variable takes a downward trend during periods of economic growth and follows an upward trend during periods of economic recession.

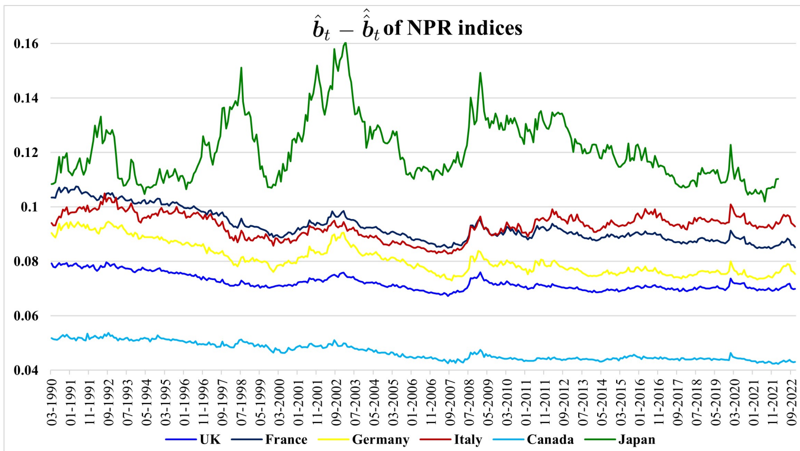
The discrepancy variable of the US-Japan relation behaves in a distinctive way. During the years before the global financial crisis, it exhibits substantial fluctuations. During the Asian crisis, the recession in the early 2000s, and the global financial crisis (GFC), it spiked and then followed a downward trend which continues until 2022. The relative long-run risk premium of the US to that of Japan is much higher than that of the other bivariate relations. Figure 3.3 suggests that the US market declined more sharply than all the other G7 markets, particularly Japan, during the global financial crisis. Moreover, the downward trending discrepancy variables imply a faster increasing trend in the relative long-run risk-adjusted rate of return of the US market. The discrepancy variable corresponding to the bivariate relation between the US NPR index and Canada appears to some extent distinct, as it remains relatively steady in comparison to the other discrepancy variables.

3.6 Drivers of the cointegration dynamics

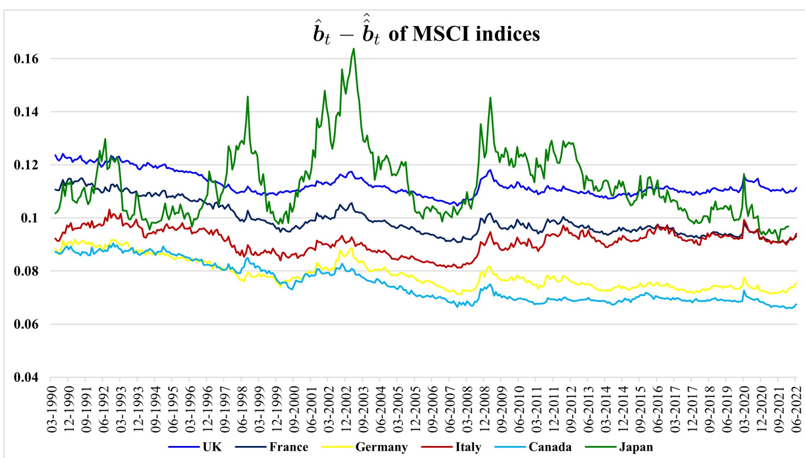
Although the literature, for the most part, documents a significant impact of EPU and GPR on stock market evolution, the impact of these variables on cointegration dynamics remains largely unexplored to date. Through our modelling approach, we associate, in Equation (3.2), the change in cointegrating coefficients not only with financial stock return determinants but also, more importantly, with innovations in uncertainty associated with



(a) RTR indices



(b) NPR indices



(c) MSCI indices

Figure 3.3: The discrepancy between the unaugmented and augmented cointegrating coefficients. Discrepancy is defined as $\hat{\boldsymbol{b}}_t - \hat{\hat{\boldsymbol{b}}}_t$, where $\hat{\hat{\boldsymbol{b}}}_t$ and $\hat{\boldsymbol{b}}_t$ represent the augmented and the unaugmented state-space variables, respectively. The first sub-graph exhibits the estimated results using inflation-adjusted total return indices. The second sub-graph exhibits the estimated results using nominal price return indices. The third sub-graph exhibits the estimated results using MSCI indices.

economic policy and political crises. This section examines the estimated parameters of the augmented model: International portfolio allocation and foreign investment as a whole, as well as economic growth, can be adversely affected by shocks to uncertainty which financial markets subsequently react to. This may then cause stock market indices to depart from existing, long-run co-movement relations. In other words, our hypothesis is motivated by the argument that a varying level of uncertainty may bring about a change in optimal investment decisions and hence lead to time-varying equilibrium relations, which translates into varying cointegrating vectors.

Table 3.6 presents the estimated results of Equations (3.1)-(3.2) up to the first lagged values of innovations in EPU and GPR. The results of the ADF test reveal that the residuals are strongly stationary. As Table 3.6 indicates, the innovations in the uncertainty variables are predominantly negatively correlated with the state-space variable, when the coefficients are significant. This finding suggests that in the bivariate relations, the US market is usually penalised more heavily in the long run than the markets of the other G7 members by positive innovations in the uncertainty variables. That is, the US stock market investors incur larger risks than investors in the other G7 markets. The estimation results of the RTR indices reveal that GPR is a significant driver in four bivariate relations with the strongest bivariate relation being that between the UK and Japan. The bivariate relation with Italy is, however, found to be significantly affected by innovations in GPR and by innovations in the EPU of the US. This relation is also significantly affected by the EPU of the US when applying our model to the NPR indices. The bivariate relations of the US stock market with those of the three European countries, i.e., the UK, France, and Germany, remain unaffected by innovations in either of the EPU indicators. The results of the estimation using NPR indices reveal that the US EPU induces a significant disruptive impact on the equilibrium relations with Italy. This would suggest that the European EPU does not significantly impact the bivariate cointegration relations. The credit spread, the US dividend yield and DVIX, however, are found to significantly impact bivariate cointegration relations. The slopes of the credit spread and DVIX are found to be negative for all bivariate relations. This finding indicates that the US stock market has higher long-run exposure to credit risk and changes to the implied volatility.

The positive slope of the US dividend yield indicates that the US stock market enjoys higher long-run return with higher dividend yield.

In the multivariate cointegration relations presented in Table 3.7, the 7-variate relation of the NPR indices can be seen to have a significant negative slope on the GPR index. The negative correlation of the GPR indicator in the 7-variate relation confirms the results of the bivariate analysis in Table 3.6 where the US stock market is found to bear higher geopolitical risk, in the long run than, the other G7 stock markets. The European EPU index is found to have a significant positive slope in the 5-variate model on the NPR indices and the 7-variate model on the RTR indices. The 5-variate model involves the US stock market as the dependent and the stock markets of the European G7 members as the independent variables. The positive slope of the European EPU in the 5-variate relation implies that the European stock markets bear a higher risk, in the long run than the US stock market. This finding is not, however, confirmed by the bivariate analysis. The 4-variate relation in Table 3.7 involves the UK stock market as the dependent and those of the three European member countries of the G7 as being the independent variables. The results of the estimation using both NPR and RTR indices evidence a significant positive slope on the GPR index. This finding shows that during periods of geopolitical tensions, the UK stock market is less risky in the long run than the other three European stock markets.

Regarding the financial variables, the DVIX is found to be strongly significant for all bivariate relations. Used as an endogenous measure of uncertainty in the literature (Bloom, 2009), implied volatility can significantly influence the co-movement of stock markets. The negative correlation of the volatility indicator with the cointegrating coefficients suggests that when markets become increasingly nervous, the US stock market is penalised, in the long run, to a greater extent than the other markets. The US credit spread is also found to be a major driver of the dynamics in all bivariate cointegration relations, with the exception of the UK. This finding suggests that the US market is riskier than the other G7 stock markets with respect to this risk factor.[4] The US dividend yield also has a positive significant correlation

[4] Naturally since the BEER, DIV, and DVIX variables are measured relative to the US market, we expect their impact to be greater on the US stock exchange than on the other stock exchanges. Nevertheless, the sign of their coefficients is informative regarding how the discrepancy reacts to their evolution.

Table 3.6: Estimated parameters of the time-varying bivariate cointegration of the US stock market with G7 stock markets

	UK	**France**	**Germany**	**Italy**	**Canada**	**Japan**
	RTR indices					
a	2.5715***	3.7111***	4.5475***	5.7083***	2.4995***	6.9967***
EPU_0^{US}	0.0004	0.0000	0.0000	-0.0005	-0.0004	-0.0008
EPU_0^{EU}	0.0008	0.0007	0.0010	0.0009	0.0005	-0.0019
GPR_0	-0.0015***	-0.0009	-0.0008	-0.0013**	-0.0008*	-0.0053***
EPU_1^{US}	-0.0003	-0.0006	-0.0009	-0.0013*	-0.0007	-0.0011
EPU_1^{EU}	0.0000	0.0001	0.0004	0.0007	0.0006	-0.0005
GPR_1	-0.0002	-0.0003	0.0003	-0.0008	-0.0003	0.0001
BEER	0.0002	0.0000	0.0000	0.0002	0.0000	0.0011
Credit spread	-0.0438	-0.1224***	-0.0949***	-0.1212***	-0.1026***	-0.3732***
DIVY	0.0514**	0.0880***	0.0723***	0.0779***	0.0910***	0.2777***
DVIX	-0.0002***	-0.0003***	-0.0003***	-0.0005***	-0.0002***	-0.0016***
US inflation	-0.0014***	-0.0018***	-0.0011***	-0.0017	-0.0022***	-0.0049***
Domestic inflation	0.0003	0.0013***	0.0002***	0.0014**	0.0007**	0.0031*
US Term spread	-0.0137	-0.0102	-0.0054	-0.0120	-0.0197	-0.0630
Domestic Term	0.0105	0.0258***	0.0238*	0.0181	0.0107	0.0831
$\hat{\sigma}_\epsilon$	0.0000	0.0000	0.0000	0.0001	0.0000	0.0002
$\hat{\sigma}_\nu$	0.0025	0.0025	0.0029	0.0031	0.0021	0.0101
SLL	883.51	885.52	879.84	817.24	886.48	768.15
ADF	-19.29***	-19.23***	-19.22***	-19.63***	-19.37***	-18.35***
	NPR indices					
a	2.6848***	3.5791***	4.3325***	5.2551***	2.1840***	6.1667***
EPU_0^{US}	0.0002	0.0001	-0.0004	-0.0006	-0.0002	-0.0007
EPU_0^{EU}	0.0007	0.0008	0.0013	0.0010	0.0007	-0.0014
GPR_0	-0.0015***	-0.0011*	-0.0012 **	-0.0017	-0.0008	-0.0057***
EPU_1^{US}	-0.0005	-0.0008	-0.0011	-0.0016 *	-0.0006	-0.0012
EPU_1^{EU}	0.0000	0.0001	0.0006	0.0009	0.0005	0.0000
GPR_1	-0.0002	-0.0004	-0.0001	-0.0010	-0.0004	0.0000
BEER	0.0003	-0.0001	0.0000	0.0002	0.0001	0.0010
Credit spread	-0.0560	-0.1378***	-0.1083**	-0.1373***	-0.1000**	-0.3570**
DIVY	0.0589**	0.0949***	0.0763 ***	0.0812**	0.0871***	0.2535***
DVIX	-0.0003***	-0.0003***	-0.0003***	-0.0005***	-0.0002***	-0.0017***
US inflation	-0.0003**	-0.0007	0.0004	-0.0005	-0.0012**	-0.0008
Domestic inflation	-0.0003	0.0008	-0.0003	0.0011	0.0000	0.0026
US Term spread	-0.0189	-0.0099	-0.0062	-0.0128	-0.0172	-0.0641
Domestic Term	0.0151	0.0321**	0.0322 *	0.0234	0.0077	0.0819
$\hat{\sigma}_\epsilon$	0.0001	0.0001	0.0001	0.0001	0.0000	0.0002
$\hat{\sigma}_\nu$	0.0027	0.0032	0.0036	0.0038	0.0032	0.0112
SLL	881.62	885.32	860.43	816.97	878.77	768.82
ADF	-19.45***	-19.77***	-19.27***	-19.63***	-21.66***	-18.50***

All indices are in terms of US dollars and transformed by natural logarithms. Each column corresponds to a bivariate cointegration relation of the US with individual G7 member countries. *, ** and *** represent significance at 10%, 5% and 1% levels, respectively. The numbers in parentheses are p-values. SLL stands for the maximised sum of the log-likelihood function. ADF represents the augmented Dickey-Fuller test statistic of the residuals of the cointegration relation.

with the cointegrating coefficient across all bivariate relations and is found to be a significant driver of time-varying cointegration when using either the RTR or NPR indices.

Table 3.7: Multivariate time-varying cointegration

	RTR indices			NPR indices		
	7-variate	5-variate	4-variate	7-variate	5-variate	4-variate
a	1.8125***	2.6018***	3.2891***	2.3966***	2.3401***	3.6522***
EPU_0^{US}	0.0001	0.0002	-0.0003	0.0000	0.0001	-0.0003
EPU_0^{EU}	0.0002*	0.0002	-0.0001	0.0002	0.0003*	-0.0001
GPR_0	-0.0002	-0.0002	0.0003*	-0.0002*	-0.0002	0.0004*
EPU_1^{US}	-0.0001	0.0000	-0.0002	-0.0001	-0.0001	-0.0003
EPU_1^{EU}	0.0000	0.0000	0.0001	0.0000	0.0000	0.0002
GPR_1	0.0000	0.0001	0.0000	0.0000	0.0000	-0.0001
BEER	0.0000	0.0001	0.0000	0.0000	0.0001	-0.0001
Credit spread	0.0110***	0.0009***	-0.0002	0.0160***	-0.0041***	-0.0220
DIVY	0.0027	0.0000	0.0007	-0.0021	0.0048	0.0018
DIVX	0.0000	0.0000	-0.0001	0.0000	-0.0001	-0.0001
US inflation	-0.0004***	-0.0004***		-0.0001***	-0.0001***	
UK inflation	0.0000	-0.0001	-0.0001***	-0.0001	-0.0002*	0.0003***
France inflation	0.0001	0.0003***	-0.0002*	0.0000	0.0003**	-0.0001
Germany inflation	0.0000	-0.0001	-0.0002	-0.0001	-0.0001	-0.0003
Italy inflation	0.0003***	0.0002*	0.0002*	0.0004***	0.0002*	0.0003**
Canada inflation	0.0000			-0.0001		
Japan inflation	0.0001			0.0001*		
US Term	-0.0027	-0.0030***		-0.0032***	-0.0021***	
UK Term	0.0021	-0.0034	0.0032***	0.0016	0.0030	0.0024***
France Term	0.0014	-0.0016***	0.0004	-0.0004	0.0053	-0.0002
Germany Term	0.0031	0.0034***	0.0009	-0.0021	-0.0010	0.0129
Italy Term	-0.0029	-0.0001	-0.0008	-0.0021	0.0007	0.0008
Canada Term	-0.0010			0.0015		
Japan Term	-0.0010			0.0064		
σ_ϵ	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001
$\sigma_{\nu-UK}$	0.0009	0.0011		0.0009	0.0001	
$\sigma_{\nu-France}$	0.0008	0.0002	0.0000	0.0000	0.0005	0.0000
$\sigma_{\nu-Germany}$	0.0010	0.0000	0.0000	0.0016	0.0000	0.0004
$\sigma_{\nu-Italy}$	0.0013	0.0024	0.0024	0.0021	0.0033	0.0028
$\sigma_{\nu-Canada}$	0.0006			0.0012		
$\sigma_{\nu-Japan}$	0.0041			0.0001		
SLL	928.60	913.62	933.60	903.30	910.84	935.73
ADF	-19.17***	-19.10***	-19.22***	-17.98***	-19.46***	-19.33***

All indices are in terms of US dollars and transformed by natural logarithms. Each column corresponds to a multivariate cointegration relation. The 7-variate cointegration relation involves the US stock market index as the dependent variable. The 5-variate cointegration relation involves the US stock market index as the dependent variable and those of the UK, France, Germany, and Italy as the independent ones. The 4-variate cointegration relation involves the UK stock market index as the dependent variable and those of France, Germany, and Italy as the independent ones. *, ** and *** represent significance at 10%, 5% and 1% levels, respectively. SLL stands for the maximised sum of the log-likelihood function. ADF represents the augmented Dickey-Fuller test statistic of the residuals of the cointegration relation.

3.7 Incremental information of the augmented model

Augmenting the time-varying cointegration model with the EPU, GPR indicators, and stock return determinants appears to be informative regarding the significance of the associated coefficients. It is, nevertheless, necessary to compare the evolution of the augmented state-space with that of a Kalman filter incorporating a simple random walk as the state-space. The fit of the augmented model vis-à-vis the unaugmented model can be examined by conducting an F test on the variance in the error terms of the measurement as well as the state-space variables. As indicated in Table 3.8, augmenting the state-space leads to a significant increase in the variance of the measurement error in all the bivariate cointegration relations. On the other hand, the augmented model significantly reduces the variation in errors of cointegrating coefficients. In other words, augmenting the state-space reduces error variance in the cointegrating coefficient although it increases the variance of the measurement error term in the cointegration equation.

So far, the unaugmented model has been shown to explain much of the variation in the

Table 3.8: Variance equality test of the residuals

Variance term	**UK**	**France**	**Germany**	**Italy**	**Canada**	**Japan**
	RTR indices					
σ_ϵ	70.9254	30.5793	3.6878	5.0812	80.6142	3.2467
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
σ_ν	0.6300	0.6026	0.7599	0.6117	0.4607	0.6015
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$Y_t - \hat{Y}_t$	95.1540	40.5883	4.3134	6.7349	185.2968	4.2813
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Variance term	**NPR indices**					
σ_ϵ	5.9506	2.6220	6.3870	12.2434	1.0012	5.1041
	(0.00)	(0.00)	(0.00)	(0.00)	(0.49)	(0.00)
σ_ν	0.7188	0.7852	0.7383	0.6178	1.000	0.6160
	(0.00)	(0.01)	(0.00)	(0.00)	(0.50)	(0.00)
$Y_t - \hat{Y}_t$	7.1423	2.9768	7.3588	16.1240	0.9199	6.5662
	(0.00)	(0.00)	(0.00)	(0.00)	(0.79)	(0.00)

The error terms of the augmented model and that of the unaugmented model are tested for equality of variance. The F statistic is obtained by dividing the estimated variance in the error terms of the augmented bivariate cointegration model by those of the unaugmented model. Each column corresponds to a bivariate cointegration relation of the US with individual G7 member country. The numbers in parentheses indicate p-values. The p-values of the first and third rows are right-tail values, and those of the second row are left-tail values

dependent market index by significantly increasing the variability of the cointegrating coefficient. This leads to the question as to whether the cointegrating vectors of the unaugmented and augmented models subsume any information about gradual market cointegration or segmentation over time, as globalisation has significantly developed over the last few decades. A more detailed analysis of the evolution of the unaugmented and augmented cointegrating vectors can provide insight into the effects of globalisation on the gradual convergence of the two vectors. We refer to the augmented and the unaugmented cointegrating vectors as the risk-adjusted and the unadjusted cointegrating vectors, respectively. As Table 3.8 also suggests, the unaugmented state-space can be used as the best-fitting time-varying cointegrating coefficient. Equation (3.2) presumes that both augmented and unaugmented cointegrating coefficients are unit root processes. Deeper analysis of the long-run co-movement of the $\hat{\hat{\boldsymbol{b}}}_t$ and the $\hat{\boldsymbol{b}}_t$ can shed more light on the effects of globalisation.

We know from Tables 3.6 and 3.7 and Figure 3.3 that the cointegrating vectors are driven by uncertainty and stock return determinants. Additionally, the risk-adjusted cointegrating vectors fluctuate substantially in periods of market turmoil compared to the unadjusted coefficients. This finding motivates the question as to whether globalisation has driven both cointegrating coefficients to follow a common stochastic trend. Markets can possibly be segmented as a result of a number of known phenomena including different technological developments, restrictions on capital flow, trade wars, etc. The causes of market segmentation, however, form a number of unknown factors for which there are no inclusive indicators. Globalisation can dominate these obstacles by reducing segmentation causes to a number of risk factors. It follows that if stronger market cointegration is the result of globalisation, then the unaugmented cointegrating coefficients have to display a common trend with the corresponding risk-adjusted cointegrating coefficients. In Table 3.9, it can be seen whether or not the two varying coefficients associated with a bivariate relation follow a common trend.

The bivariate cointegration test of the augmented and the unaugmented cointegrating coefficients reveals that in most cases, the two time-varying cointegrating coefficients follow distinct stochastic trends. When using NPR indices, the coefficients associated with the relations between the US and Germany, and Japan follow common trends. As the majority of the tests does not reject the null hypothesis of no cointegration, we conclude that, in the majority of cases, the state-space variables estimated by two different specifications follow two distinct stochastic trends. Therefore, the best fitting time-varying equilibrium relation (i.e.,

Table 3.9: Cointegration test of augmented and unaugmented cointegrating coefficients

	UK		**France**		**Germany**		**Italy**		**Canada**		**Japan**	
	RTR indices											
Dependent	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$
tau-statistic	-2.11	-2.16	-2.11	-2.08	-2.50	-2.49	-2.72	-2.70	-1.38	-1.36	-2.81	-2.75
	(0.47)	(0.45)	(0.47)	(0.49)	(0.28)	(0.28)	(0.20)	(0.20)	(0.81)	(0.81)	(0.17)	(0.18)
z-statistic	-8.90	-9.00	-9.08	-8.84	-12.72	-12.60	-14.21	-14.12	-3.76	-3.60	-14.94	-14.60
	(0.43)	(0.42)	(0.42)	(0.43)	(0.22)	(0.23)	(0.17)	(0.17)	(0.83)	(0.84)	(0.15)	(0.16)
	NPR indices											
Dependent	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$	$\hat{\boldsymbol{b}}_t$	$\hat{\hat{\boldsymbol{b}}}_t$
tau-statistic	-2.47	-2.49	-2.53	-2.53	-3.07	-3.07	-2.29	-2.29	-1.57	-1.57	-3.06	-3.02
	(0.29)	(0.28)	(0.27)	(0.27)	(0.10)	(0.10)	(0.38)	(0.38)	(0.73)	(0.74)	(0.10)	(0.11)
z-statistic	-12.30	-12.41	-12.62	-12.56	-19.12	-19.07	-10.55	-10.56	-5.00	-4.92	-17.85	-17.61
	(0.24)	(0.24)	(0.23)	(0.23)	(0.06)	(0.06)	(0.33)	(0.33)	(0.73)	(0.74)	(0.08)	(0.09)

Engle and Granger (1987) cointegration is applied to examine the bivariate cointegration relations between augmented and unaugmented cointegrating coefficients. $\hat{\boldsymbol{b}}_t$ represents the unaugmented cointegrating coefficient and $\hat{\hat{\boldsymbol{b}}}_t$ represents the augmented cointegrating coefficient. The numbers in parentheses are p-values. Each bivariate cointegration relation is examined twice, depending on the choice of the dependent variable.

unaugmented) follows a distinct unit root process probably driven by a number of unknown factors. This result implies that markets can be segmented beyond what can be explained by the known set of risk factors identified in this study.

3.8 Conclusions

Rapidly growing globalisation suggests that a stronger interconnectedness of global stock markets should exist whereas the stock markets of the G7 member countries do not exhibit a time-invariant equilibrium relation. Although the theoretical developments in time-varying cointegration are quite mature, the causes of time-varying cointegration among stock markets remain unexplored. The literature theoretically and empirically supports the effective role of uncertainty in the economy as well as in financial markets. Time-varying cointegration could be a consequence of repeated shocks to the uncertainty prompted by economic policy-makers or geopolitical risks. As the literature has provided evidence that uncertainty affects the evolution of stock markets, we investigated the hypothesis that a shock to uncertainty is likely to distort the co-movement of markets. Our empirical findings suggest that the time-varying nature of the long-run equilibrium relation is driven by changing geopolitical risks, a number of stock return determinants, and to some extent by economic policy uncertainty.

The US stock market is found to bear higher long-run geopolitical risk than the other stock markets of the G7. This finding is supported by the bivariate as well as the 7-variate cointegration of the NPR indices. The results of the estimation indicate that the US market has a higher (lower) long-run risk-adjusted return than the other member countries of the G7 in periods of economic growth (recession). This finding implies that higher exposure to certain risk factors makes bivariate cointegration relations more susceptible to segmentation. Moreover, our results confirm that the unknown factors disrupting cointegrating vectors have been reduced as a result of globalisation.

Co-movement of the augmented and unaugmented cointegrating vectors of the 5-variate cointegration relation is, to some extent, different to those of the bivariate and 4-variate relations. The stochastic volatility associated with some of the coefficients has been upward trending in the last years starting from 2017. The sources of the diverging cointegrating coefficients of the augmented and unaugmented models can be explored in future investigations. However, emerging economies can be a new source of the segmentation of the 5-variate relation. The possible impact of emerging China on the dynamics of the multivariate cointegration of the G7 stock markets can be worth exploring. On the other hand, the effects of trade conflict provoked by Trump's administration can also have a negative impact on market cointegration.

The fact that augmented and unaugmented cointegrating vectors follow distinct stochastic trends, entails important implications for policy makers. Economic policies including trade policies seem to provide potential for development. Trade barriers remain a major obstacle to the integration of the stock markets of the G7 members. Trade agreements that aim to improve regulatory coherence and reduce tariffs promote international trade and multilateral economic growth and integration. On the other hand, a number of factors including the limited flow of labour, protectionism, subsidies, and non-protectionist policies seem to be an obstacle to further market integration. A future in-depth study of the impacts of these factors could also be of particular interest.

Chapter 4

Co-movement dynamics and disruptions of the major stock markets

Hamid Babaei[1], Georges Hübner[1]

[1] *HEC Liège, Management School-University of Liège, 14 rue Louvrex, B-4000 Liège, Belgium*

Abstract

In the current chapter, we apply three approaches to investigate the evolution of the time-varying (TV) multivariate cointegration of the G7 stock markets from 1970 to 2022. Our work contributes to the empirical literature on cointegration by illustrating the extent to which market equilibrium relations fall apart over periods of time. The evolution of the market's interdependence is illustrated by investigating the instability and strength of the market's co-movement over time. We find that the growing globalization of the world's economies restrains market segmentation during recent economic crises, in particular the global financial and COVID-19 crises. We also find that during the 1980s, the cointegration of G7 stock markets was destabilized far more than during the following decades. The growing China was found to negatively impact the 7-variate cointegration of the G7 in the recent decade, whereas the 8-variate cointegration was strengthened. We also find that the quadrivariate cointegration of the European stock markets has been steady from the late 1990s to date[1].

[1]Integration of stock markets has been investigated from two distinct standpoints. Accordingly, theoretical methodologies have been developed in the literature. These two standpoints concern the short-run and the long-run integration. The short-run integration is a matter of the correlation between stock market returns, whereas the long-run integration is a matter of the correlation between prices (i.e., non-stationary level variables)

JEL classification: G15, F02, F36

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provided that residuals are stationary. The time-series literature has already developed models (e.g., DCC-GARCH model) to tackle the question of dynamics of stock market integration in the short run. The DCC-GARCH model can not be modified to estimate time-varying correlation coefficients between level variables. Otherwise, we could simply estimate time-varying correlation coefficients that fulfill our objective in this chapter. Our objective to investigate the long-run relation between stock market indices is motivated by the fact that cointegration can be interesting for an investor who follows a buy-and-hold strategy and wants to diversify his/her portfolio because he/she can allocate his/her budget to a number of cointegrated stocks to reduce his long-run risk.

4.1 Introduction

International stock market cointegration has been the subject of a wide range of empirical investigations[2]. As we observed in Chapter 3, empirical evidence suggests that market linkages do not follow a static linear relation over time, rather there is a complicated evolution of the linkage among stock markets. This complexity can be a result of continuous as well as concurrent changes in technological, economic, or political circumstances. In Chapter 3, we focused on factors that make instantaneous shocks to cointegration relations between the G7 stock markets. In the present chapter, we address two main questions that remain unanswered in Chapter 3. One major shortcoming of the empirical findings in the literature is that it ignores the question of intensity and weakness of the long-run equilibrium relation. Market linkages can also bear periods of stability and instability. For instance, the globalization and integration of the world economies on one hand, and the political as well as economic uncertainties associated with the persistence of the integration process on the other hand, can intermittently disrupt and boost the integration process. For example, markets' liberalization and abolition of the exchange rate control are found to leave a positive impact on markets' cointegration (see Taylor and Tonks, 1989).

The present chapter differs from Chapter 3 in the specification and methodological approach that aims at investigating two new measures of cointegration strength and instability. In order to investigate the strength and stability of the cointegration relation, we aim at a new time-varying estimation of the classical Vector Error Correction Model (VECM). A methodological innovation in the area of cointegration enables us to estimate a time-varying measure representing the evolution of the intensity in the multivariate cointegration. In order to tackle the question of varying intensity and weakness of the long-run equilibrium among

[2]The linkage among market indices is evidenced in the literature to bear either abrupt structural breaks or to be continuously varying. In a number of studies, it is assumed that the linear relation can be broken up following a shift in the economic circumstances. The papers that address a sudden pre-identified moment for the shift in the cointegrating vectors, include Taylor and Tonks (1989), Arshanapalli and Doukas (1993), Bachman et al. (1996), Kanas (1998), Aggarwal and Kyaw (2005), Aladesanmi et al. (2019) and Patel (2019). A regime shift or an unknown sudden shift in the cointegration relation has been also widely used in the literature to explore the dynamics in the market co-movement. See for example Voronkova (2004), Davies (2006), Lucey and Voronkova (2008) and Menezes et al. (2012). A number of studies assume a continuously varying cointegration and adopt a rolling time window approach or a recursive estimation to explore the dynamics of the cointegration. For example see Crowder and Wohar (1998), Pascual (2003), Gilmore et al. (2008), Awokuse et al. (2009), Mylonidis and Kollias (2010) and Chien et al. (2015).

stock market indices, we aim at estimating time-varying "lambdas" which, in the static form introduced by Johansen (1988), represent the strength of cointegration. The asymptotic distribution of the two well-known rank tests of the trace test and maximum eigenvalue test is shown be Johansen (1988) to depend on the canonical R-squared values. R-squared values, which are themselves measures of fitness, serve as a measure representing cointegration strength. As a measure of instability, the shifts in the normal vector of the spanning space of the time-varying cointegrating vectors fulfill our objectives. We measure the shift in the normal vector as the angle between two normal vectors associated with two consecutive cointegrating spaces.

To date, the econometric tools that have been developed and employed in order to study these time-varying relations have not enabled empirical researchers to fully uncover the nature of these linkage dynamics. The empirical works on the time-varying cointegration of stock markets suffer from three major drawbacks. First, the results depend on the underlying time window in all the approaches, ranging from the recursive estimation to the sub-period analysis. We overcome this shortcoming by state-space modelling and estimating by the Kalman filter. The state-space approach is preferred over the recursive or smoothly-varying estimation (e.g., Bierens and Martins, 2010), because on the one hand, the resulting parameters are not dependent on the underlying time interval, and on the other hand, they do not depend on the smooth function chosen by the model. The state-space modelling of the VECM allows us to estimate the time-varying eigenvalues which also paves the way to investigate the evolution of the strength of multivariate cointegration. Moreover, the time-varying cointegrating vectors estimated by state-space formulation make it possible to study the instability of the spanning space of the cointegrating vectors. The use of the Kalman filter is motivated by the fact that it delivers unbiased minimum variance estimation when the errors are assumed to be normally distributed. Second, the time-varying cointegration models present no time-varying measure standing for the strength of the linkage among markets. The few works aiming at investigating the integration of the stock markets use the trace statistic of the recursive cointegration or of the rolling time window, or identify an unknown structural shift in the relation to illustrate the integration progress (see for example Mylonidis and

Kollias, 2010; Menezes et al., 2012 and Chien et al., 2015). Third, the commonly used models of nonlinear cointegration, threshold cointegration, and regime switching cointegration add nothing to the question of instantaneously evolving cointegration strength and instability. In particular, filtered probabilities of a regime-switching cointegration should exhibit compliance with the course of the event which impacts stock markets. In practice, it is hard to relate the results of filtered regime probabilities to economic and political events.

In this chapter, our data spans a larger period than that of Chapter 3 starting at 1970 to date. We also study the impact of the growing China on the cointegration dynamics of the world's major economies. We compare the instability and strength of the long-run market interdependence over the past half-century after and before the inclusion of China. We find that the inclusion and exclusion of China in the cointegration relations of the G7 stock market indices makes a significant change in the trend of the strength and instability measures.

We believe that the approach used in this chapter is suitable in order to faithfully estimate the cointegration dynamics. To the best of our knowledge, the theoretical developments in the literature do not provide a satisfactory way to estimate time-varying lambdas of the VECM. For example in the work by Bierens and Martins (2010), the cointegrating vectors are smoothly varying, whereas the lambdas are assumed constant. The recent work by Eroğlu et al. (2022) uses a single equation model to estimate time-varying cointegration which lacks the possibility to estimate a time-varying measure of cointegration strength. Our work contributes to the literature by providing a way to estimate not only the time-varying cointegrating vectors but also the time-varying lambdas. This allows us to explore how the degree of markets' cointegration has evolved over time. We further address the impact of the surge of China on the evolution of the cointegration degree among the world's most developed stock markets. This approach can shed more light on the evolution of the extent to which G7 and China's stock markets have been cointegrated over the past decades starting from 1969 to 2022, as it has been evidenced in the literature that the long-run equilibrium relation can be broken up by a number of political, technological end economic events. In particular, we address the question of whether globalization has contributed to stronger and more stable cointegration over time. We also document to what extent the potential segmenting events of the markets lead to any adjustment in the long-run equilibrium relations.

We find that globalization has contributed to stronger interdependence among the G7 stock markets until 2010. The economic uncertainties in the 1980s and during the beginning of the 1990s are found to weigh on markets' co-movement. Since then, the cointegration of the G7 stock markets has intensified until shortly after the global financial crisis. We also find that the growing integration of China into the world economy is a source that has weakened the linkage of the G7 markets in the last decade. The uncertainties concerning the future of China's economic growth seems to have negatively impacted the international market cointegration. The impact of the economic crises on markets' segmentation seems to have diminished over time. A number of events including the technological innovation in the production of crude oil, the shift in the monetary policy, and the uncertainties concerning the trade war between China and the US, are found to disrupt markets' linkages.

This paper is organised as follows. Section 2 presents a review of the literature with a focus on empirical findings on the cointegration of the world's major stock markets. In section 3 our approach to estimate time-varying cointegration is explained. Section 4 discusses empirical findings on the evolution of the multivariate cointegration of G7 stock markets and section 5 concludes.

4.2 Literature review

4.2.1 *Empirical studies*

Since our objective is to study the evolution of the cointegration among the G7 stock markets, we focus on the empirical studies of the most developed stock markets. Given that the developments in the technological environment and changes in economic policies can lead to adjustments in the long-run relation among stock market indices, a number of works evidenced variation in the cointegration relation among stock market indices.

A series of works consider abrupt structural breaks in the cointegration relation and test for identifying the possible break point. Taylor and Tonks (1989) focus on the abolition of the UK exchange rate control in 1979 on the bivariate cointegration between the UK and of the US, Germany, Japan, and the Netherlands from 1973 to 1986. The null hypothesis of no cointegration in the pre-abolition period is not rejected, whereas during the post-abolition period, supporting evidence for cointegration with other markets else than the US is found. Arshanapalli and Doukas (1993) study the bivariate cointegration between the US stock market and either of the four markets of the UK, Germany, France, and Japan from 1980 to 1990 and focus on the stock market crash in 1987. They find no evidence of cointegration in the pre-crash subperiod, whereas the analysis supports cointegration during the post-crash subperiod in three bivariate relations. Bachman et al. (1996) examine multivariate cointegration of the G7 countries from 1970 to 1989 and find two cointegration in each of the two decade-long subperiods. The European countries are found to have one cointegration in each of the two decade-long subperiods. The entire period provides weaker evidence of cointegration. Kanas (1998) investigates bivariate cointegration between the US stock market and that of either of the European major markets including the UK, France, Germany, Italy, Netherlands, and Switzerland over the period ranging from 1983 to 1996. They find no significant cointegration in either the pre-crash or the post-crash sub-periods. The analysis of the entire period reveals no evidence of significant cointegration either. Aladesanmi et al. (2019) study the bivariate cointegration between the US and UK stock markets in three subperiods representing three monetary regimes from 1935 to 2015 and find stronger evidence of cointegration during the post-Bretton Woods regime.

A number of the works do not consider a priori known breaks in the long run co-movements, rather investigate variations in the relation or switching between equilibrium regimes. Crowder and Wohar (1998) perform the recursive cointegration analysis on the 5-variate cointegration, including stock market indices of the US, UK, Germany, Canada, and Japan. The results of the Johansen rank test on the empirical data from 1974 to 1990 support weak evidence of time-invariant (TI) cointegration; nevertheless, the constancy test statistic of the cointegrating vectors decline until the end of the period. Assuming two equilibrium regimes, Davies (2006) finds strong evidence of 7-variate time-varying cointegration among stock markets of the US, UK, Germany, Japan, Canada, Australia, and Switzerland. Using a 2-year rolling time window, Awokuse et al. (2009) apply Johansen's multivariate cointegration test to three developed stock markets (i.e. US, UK, and Japan) and nine Asian markets. Their findings support the positive impact of financial markets' liberalization during the early 1990s on markets' integration.

4.2.2 *Theoretical developments on time-varying cointegration*

The concept of cointegration and its relevancy to the actual economic variables was first introduced by Granger (1981). The representation theorem introduced by Engle and Granger (1987) illustrated the theoretical links between the vector autoregressive (VAR), the VECM, and the cointegration relation of the underlying series. The work also provided a two-step method to estimate and test for cointegration. The landmark work by Johansen (1988) provided a formulation to estimate the independent vectors spanning the space of the cointegrating vectors. The work also provided a rank test with the asymptotic theory to test for the number of the cointegrating vectors[3].

The empirical findings supporting time-varying cointegration relation motivated theoretical researchers to develop a methodology to estimate time-varying cointegration. Gregory and Hansen (1996) consider a sudden shift in the cointegration relation and develop a test for identifying the possible break point. Park and Hahn (1999) developed an approach to estimate time-varying cointegration with smoothly evolving coefficients. Saikkonen and Choi (2004) use the smooth transition model to estimate single equation time-varying cointegration. Hansen and Johansen (1999) fix the parameters associated with the short-term dynamics to the full sample and recursively estimate the long-run parameters of the VECM to identify possible structural breaks.

Bierens and Martins (2010) developed an approach to estimate multivariate time-varying cointegration in which the smoothly varying cointegrating vectors are expressed in

[3]A number of the works developed asymptotic theory for the distribution of the estimated parameters in cointegration relations (see the general asymptotic theory developed by Phillips and Durlauf (1986), the asymptotic properties of the OLS and the non-linear least squares developed by Stock (1987) and the single equation bias-corrected estimator known as the fully modified developed by Phillips and Hansen (1990)).

terms of the Chebyshev time polynomials. Their estimation theory is founded on Johansen (1988) with the time-varying cointegrating vector being estimated by an augmented form of VECM in which $\boldsymbol{Y}_{t-1}$ is replaced by a vector whose components are products of the Chebychev time polynomials and $\boldsymbol{Y}_{t-1}$.

Eroğlu et al. (2022) develop a single equation cointegration model in which the cointegrating coefficients and the stationary residual follow distinct autoregressive processes. Assuming a non-Gaussian distribution for the residuals, they develop a semi-parametric procedure to test the null hypothesis of static cointegration against the alternative of time-varying cointegration.

4.3 Multivariate time-varying cointegration

4.3.1 *Single equation time-varying cointegration*

We also estimate the single equation time-varying cointegration model developed by Eroğlu et al. (2022), which allows us to test the null hypothesis of no time-varying cointegration against the alternative of time-varying cointegration.

$$y_t = \alpha + \boldsymbol{X}_t'\boldsymbol{\beta}_t + w_t \qquad t = 1, \ldots, T \tag{4.1a}$$

$$\boldsymbol{\beta}_t = \phi\boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t \tag{4.1b}$$

$$w_t = \theta w_{t-1} + \sum_{i=1}^{k} \delta_i \Delta w_{t-i} + \epsilon_t \tag{4.1c}$$

where y_t is the stock market index chosen as the dependent variable, and $\boldsymbol{X}_t$ is a vector of regressors. The time-varying cointegrating vector which can be written as $[1 \ \ -\beta_{1,t} \ \ \ldots \ \ -\beta_{n-1,t}]'$, depends on the choice of the dependent variable. α, ϕ, θ and δ_is are scalars. The error vector $\boldsymbol{\eta}_t$ follows jointly *i.i.d.* normal distributions having zero mean and standard deviations σ_i for $i = 1, \ldots, n-1$. The error terms ϵ_t follow *i.i.d.* normal distributions having zero mean and standard deviations σ_w. We also assume that all the error terms have zero covariances.

4.3.2 *Smoothly varying cointegration*

Bierens and Martins (2010) develop a testing approach for the multivariate time-varying cointegration in which the cointegrating vectors are expressed in terms of Chebyshev time polynomials. The VECM can then be represented by:

$$\Delta \boldsymbol{Y}_t = \boldsymbol{C} + \boldsymbol{\alpha}(\sum_{i=0}^{m} \boldsymbol{\xi}_i P_{i,T(t)})' \boldsymbol{Y}_{t-1} + \sum_{i=1}^{p} \boldsymbol{\Gamma}_i \Delta \boldsymbol{Y}_{t-i} + \boldsymbol{\epsilon}_t \tag{4.2}$$

where $\boldsymbol{Y}_t$ is a $(n \times 1)$ vector and the parameters $\boldsymbol{C}$, $\boldsymbol{\alpha}$, $\boldsymbol{\Gamma}_i$ and $\boldsymbol{\epsilon}_t$ are as explained in equation (4.3). $P_{i,T(t)}$ is the i-th Chebyshev time polynomial and defined by $P_{0,T(t)} = 0$ and $P_{i,T(t)} = \sqrt{2}\cos(i\pi(t-0.5)/T)$ for $i = 1, 2, \dots$ where T is the sample size, m is the Chebyshev polynomial order and $\boldsymbol{\xi}_i$ is a $(n \times r)$ matrix. They generalise the limiting distribution of the ordered eigenvalues to a more general asymptotic theory in which the Wiener process is replaced by an augmented vector process composed of the multiplication of the Chebyshev polynomials and Wiener processes. Though their approach estimates time-varying cointegrating vectors, the eigenvalues do not vary with time and therefore the evolution of the strength of the multivariate cointegration remains unexplored.

4.3.3 *Time-varying* $\lambda_{i,t}$*s*

Johansen (1988) uses canonical correlation analysis to estimate the linearly independent cointegrating vectors from the vector error correction model (VECM). Canonical correlations are considered as measures standing for the strength of the linear relations, and the time-varying canonical correlations can then be used to investigate the evolution of the cointegration relations. Suppose $\boldsymbol{Y}_t$ is a vector containing n stock indices. The VECM can be represented by equation (4.3):

$$\Delta \boldsymbol{Y}_t = \boldsymbol{C} + \boldsymbol{\alpha}\boldsymbol{\beta}' \boldsymbol{Y}_{t-1} + \sum_{i=1}^{p} \boldsymbol{\Gamma}_i \Delta \boldsymbol{Y}_{t-i} + \boldsymbol{\varepsilon}_t \tag{4.3}$$

with $E[\boldsymbol{\varepsilon}_t] = 0$ and $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_\tau'] = \boldsymbol{\Omega}$ when $t = \tau$ and $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_\tau'] = 0$ otherwise. $\boldsymbol{\beta}$ is a $(n \times r)$ matrix composed of the TI cointegrating vectors and $\boldsymbol{\alpha}$ is a $(n \times r)$ matrix of adjustment coefficients. Johansen's approach to estimating independent cointegrating vectors involves two sets of regressions, the first of which is to regress the first differenced vector $\Delta\boldsymbol{Y}_t$ on a constant and its lagged vectors i.e. $\Delta\boldsymbol{Y}_{t-1},\ldots,\Delta\boldsymbol{Y}_{t-p}$, and the second is to regress the lagged level variable $\boldsymbol{Y}_t$ on a constant and the lagged first differenced variables $\Delta\boldsymbol{Y}_{t-1},\ldots,\Delta\boldsymbol{Y}_{t-p}$. The residuals of these two regressions are stored in $\boldsymbol{u}_t$ and $\boldsymbol{v}_t$ respectively. Let $\boldsymbol{\Sigma}_{uu}$, $\boldsymbol{\Sigma}_{uv}$, $\boldsymbol{\Sigma}_{vv}$ and $\boldsymbol{\Sigma}_{vu}$ represent the mutual variance-covariance matrices associated with $\boldsymbol{u}$ and $\boldsymbol{v}$. Let $\hat{\boldsymbol{\beta}}$ denote the eigenvectors of the matrix $\boldsymbol{\Sigma}_{vv}^{-1}\boldsymbol{\Sigma}_{vu}\boldsymbol{\Sigma}_{uu}^{-1}\boldsymbol{\Sigma}_{uv}$ normalised by $\hat{\boldsymbol{\beta}}'\boldsymbol{\Sigma}_{vv}\hat{\boldsymbol{\beta}} = I$ with the corresponding eigenvalues $\lambda = (\lambda_1,\ldots,\lambda_n)$ ordered as $\lambda_1 \geq \cdots \geq \lambda_n$. Johansen (1988) proved that the normalised eigenvectors $\hat{\boldsymbol{\beta}}$ and the matrix $\hat{\boldsymbol{\alpha}} = \boldsymbol{\Sigma}_{vv}\hat{\boldsymbol{\beta}}$ maximise the likelihood function of the VECM in equation (4.3).

As the λ_is equal the squared canonical correlations between the first differenced variables and the lagged level ones, each λ_i represents the fit of the corresponding equilibrium error associated with a certain cointegrating vector and a stationary series. Thus the time-varying λ_is provide us with the information on the evolution of the fit associated with each equilibrium and a stationary series. We aim at the estimation of the $\lambda_{i,t}$s by state-space representation of the VECM. As it will be explained in the next subsections, the state-space approach provides us with the $\lambda_{i,t}$s associated with the time-varying cointegrating vectors.
We base our estimation of time-varying cointegration on state-space formulation of the VECM in equation (4.3). Particularly we aim to estimate time-varying impact matrix by the following specification of the VECM, assuming that the impact matrix follows a random walk:

$$\Delta\boldsymbol{Y}_t = \boldsymbol{C} + \boldsymbol{\Pi}_t\boldsymbol{Y}_{t-1} + \sum_{i=1}^{p}\boldsymbol{\Gamma}_i\Delta\boldsymbol{Y}_{t-i} + \boldsymbol{\varepsilon}_t \tag{4.4a}$$

$$\boldsymbol{\Pi}_t = \boldsymbol{\Pi}_{t-1} + \boldsymbol{\delta}_t \tag{4.4b}$$

where $\boldsymbol{\Pi}_t = \boldsymbol{\alpha}\boldsymbol{\beta}_t'$ is a $(n \times n)$ time-varying impact matrix, $\boldsymbol{\beta}_t$ is a $(n \times r)$ matrix composed of the time-varying cointegrating vectors normalised by $\hat{\boldsymbol{\beta}}_t'\boldsymbol{\Sigma}_{vv}\hat{\boldsymbol{\beta}}_t = I$, and $\boldsymbol{\alpha}$ is a $(n \times r)$ matrix of adjustment coefficients. $\boldsymbol{\varepsilon}_t \overset{iid}{\sim} N(0,\sigma)$, $E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_\tau'] = \boldsymbol{\Omega}$ when $t = \tau$ and $E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_\tau'] = 0$ otherwise, and $\boldsymbol{\delta}_t \overset{iid}{\sim} N(0,\sigma_\delta)$. The intercept in equation (4.4a) is fixed to full sample whereas $\boldsymbol{\Pi}_t$ is varying. Having fixed a part of the parameters to the full sample, and letting the other parameters vary, is an approach used in the literature to investigate time-varying cointegration (see Hendry and Ericsson, 1991, and Hansen and Johansen, 1999).

The well-known estimation procedure of Johansen ensures the estimation of the independent vectors spanning the space of the cointegrating vectors. Since in this paper, our objective is to investigate the sources of markets' segmentation and the evolution of the interdependence of the markets in light of globalisation, the state-space modelling of the VECM meets our objectives. In other words, the evolution of the $\mathbf{\Pi}_t$ can shed light on the periods of instability in the cointegrating vectors, which allows us to identify sources of the shocks to the long-run equilibrium relation.

We apply the Kalman filter independently to each row of the time-varying VECM to estimate the state variables in Equation (4.4b). Kalman filter delivers minimum variance unbiased estimator if the errors of the Equation (4.4a) are normally distributed. Given that this approach involves no binding condition to estimate independent cointegrating vectors, the rank of the impact matrix $\mathbf{\Pi}_t$, at each time, can vary from 1 to n. This fact can be more illustratively explained in terms of the common trend representation by Stock and Watson (1988). Stock and Watson (1988) prove that the vector $\boldsymbol{Y}_t$ can be represented as follows:

$$\boldsymbol{Y}_t = \boldsymbol{Y}_0 + \boldsymbol{A}\boldsymbol{\tau}_t + \boldsymbol{a}_t \tag{4.5a}$$

$$\boldsymbol{\tau}_t = \boldsymbol{\pi} + \boldsymbol{\tau}_{t-1} + \boldsymbol{\nu}_t \tag{4.5b}$$

where $\boldsymbol{\tau}_t$ is a $(k \times 1)$ vector of random walks with $k = n - r$, $\boldsymbol{A}$ is a $(n \times k)$ matrix of rank k, $\boldsymbol{a}_t$ is a $(n \times 1)$ vector of stationary processes, $\boldsymbol{Y}_0$ and $\boldsymbol{\pi}$ are the intercepts and $\boldsymbol{v}_t$ is the vector of the innovations. The rank of the matrix $\boldsymbol{A}$ represents the number of the common trends. If the matrix $\boldsymbol{A}$ is assumed to vary, then there is, by construction, no constraint on the rank of the matrix and thus it can also change over time taking any value from 1 to n.

Johansen normalised the impact matrix by $\hat{\boldsymbol{\beta}}'_i = \hat{\mathbf{\Pi}}_i/(\hat{\mathbf{\Pi}}_i\hat{\mathbf{\Sigma}}_{vv}\hat{\mathbf{\Pi}}'_i)$ in which $\hat{\mathbf{\Pi}}_i$ represents the i-th row of the impact matrix $\hat{\mathbf{\Pi}}$, to obtain the cointegrating vectors satisfying $\hat{\boldsymbol{\beta}}'\mathbf{\Sigma}_{vv}\hat{\boldsymbol{\beta}} = \boldsymbol{I}$. Our primary objective to study the evolution of the cointegrating vectors can be fulfilled by considering the evolution of the space spanned by the impact matrix $\hat{\mathbf{\Pi}}_t$, because the space remains invariant by the normalization.

The state-space modelling of the VECM allows us to examine the evolution of the extent to which markets are cointegrated in periods of economic and political instability. The time-varying cointegrating vectors span a space having a dimension equal to their rank. Let $rank(\hat{\boldsymbol{\beta}}_t) = p$, then the space spanned by $\hat{\boldsymbol{\beta}}_t$ forms a hyperplane in $\mathbb{R}^{p+1}$. This hyperplane is characterised by its normal vector which can serve as an indicator signaling any shift in the space of the cointegrating vectors. We take the shifts in the normal vector of the hyperplane as a proxy signaling the instability of the spanned space of the time-varying cointegrating vectors. The shift in two normal vectors of equal length is measured as the angle between the two vectors (see Figure 4.1). In the n-variate case, without loss of generality, we take $(n-1)$ number of the time-varying cointegrating vectors to investigate the variability of the spanned space. The angle between two consecutive normal vectors associated with the spanned space of the cointegrating vectors illustrates how severely the space is destabilised.

In addition to the instability, we estimate time-varying $\lambda_{i,t}$ as a set of measures

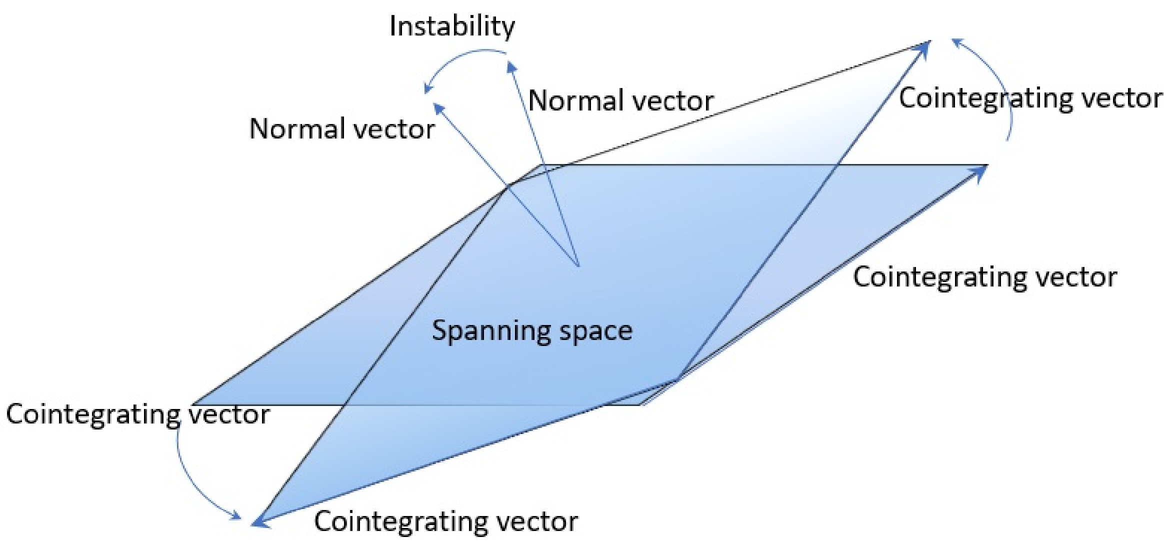


Figure 4.1: Instability degree between two consecutive cointegrating vectors.

representing the strength of markets' co-movement. Johansen (1988) proved that the matrix $\boldsymbol{\beta}$ composed of the r cointegrating vectors and the corresponding eigenvalues λ_i satisfy equation (4.6):

$$\frac{|\boldsymbol{\beta}'(\boldsymbol{\Sigma}_{vv} - \boldsymbol{\Sigma}_{vu}\boldsymbol{\Sigma}_{uu}^{-1}\boldsymbol{\Sigma}_{uv})\boldsymbol{\beta}|}{|\boldsymbol{\beta}'\boldsymbol{\Sigma}_{vv}\boldsymbol{\beta}|} = \prod_{i=1}^{n}(1-\lambda_i) \tag{4.6}$$

where λ_is are the eigenvalues of $\boldsymbol{\Sigma}_{vv}\boldsymbol{\Sigma}_{uu}^{-1}\boldsymbol{\Sigma}_{uv}$ with respect to $\boldsymbol{\Sigma}_{vv}$ ordered as $\lambda_1 \geq \cdots \geq \lambda_n$. The matrix $\boldsymbol{\beta}$ minimizing the left hand side of equation (4.6) together with the matrix $\boldsymbol{\alpha} = \boldsymbol{\Sigma}_{vv}\boldsymbol{\beta}$ are the best-fit matrices of the VECM in equation (4.3). The minimizing matrix of the quotient is composed of the eigenvectors associated with the eigenvalues λ_i. Moreover, given an eigenvector $\boldsymbol{\beta}_i$, the value of the quotient equals $1 - \lambda_i$ in which the λ_i is the i-th squared canonical correlation of equation (4.3).

The quotient in equation (4.6) is the generalised Rayleigh quotient and can simply be reduced to a Rayleigh quotient $|\boldsymbol{x}'\boldsymbol{A}\boldsymbol{x}|/|\boldsymbol{x}'\boldsymbol{x}|$ where the matrix $\boldsymbol{A}$ can be written as $\boldsymbol{A} = \boldsymbol{H}^{-1}(\boldsymbol{\Sigma}_{vv} - \boldsymbol{\Sigma}_{vu}\boldsymbol{\Sigma}_{uu}^{-1}\boldsymbol{\Sigma}_{uv})(\boldsymbol{H}')^{-1}$, $\boldsymbol{\Sigma}_{vv} = \boldsymbol{H}\boldsymbol{H}'$ and $\boldsymbol{x} = \boldsymbol{H}'\boldsymbol{\beta}$. For a given vector $\boldsymbol{x}$, the Rayleigh quotient of a covariance matrix can be expressed as the weighted sum of the eigenvalues in which the weights are the squared coordinates of $\boldsymbol{x}$ in the eigenbasis. It can also be expressed as the weighted sum of the squared cosines of the angles between $\boldsymbol{x}$ and the eigenvectors of the covariance matrix in which the weights are the eigenvalues. Moreover, the Rayleigh quotient lies between the minimum and the maximum eigenvalues of the underlying matrix.

Let $\boldsymbol{\beta}_{i,t}$ denote the i-th time-varying cointegrating vector estimated by the state-space modelling of the VECM in equation (4.4b). We calculate the time-varying parameter $\lambda_{i,t}$ by virtue of the time-varying $\boldsymbol{\beta}_{i,t}$ in equation (4.7):

$$\lambda_{i,t} = 1 - \frac{|\boldsymbol{\beta}_{i,t}'(\boldsymbol{\Sigma}_{vv} - \boldsymbol{\Sigma}_{vu}\boldsymbol{\Sigma}_{uu}^{-1}\boldsymbol{\Sigma}_{uv})\boldsymbol{\beta}_{i,t}|}{|\boldsymbol{\beta}_{i,t}'\boldsymbol{\Sigma}_{vv}\boldsymbol{\beta}_{i,t}|} \tag{4.7}$$

Note that the whole sample variance-covariance matrices are used in equation (4.7). The reason is that the quotient $\frac{|\boldsymbol{\Sigma}_{vv} - \boldsymbol{\Sigma}_{vu}\boldsymbol{\Sigma}_{uu}^{-1}\boldsymbol{\Sigma}_{uv}|}{|\boldsymbol{\Sigma}_{vv}|}$ converges regardless of the fact that there exists significant cointegration relation or not. Consider multiplying the numerator and the denominator of the quotient by $1/T$. The matrix $\boldsymbol{\Sigma}_{vv} - \boldsymbol{\Sigma}_{vu}\boldsymbol{\Sigma}_{uu}^{-1}\boldsymbol{\Sigma}_{uv}$ in the numerator is the variance-covariance matrix of the residuals of an orthogonal projection. Divided by T, it converges according to the central limit theorem even in the case of a spurious regression. Similarly, divide the denominator by T, the matrix $\boldsymbol{\Sigma}_{vv} = 1/T\sum_{i=1}^{n}\boldsymbol{v}_t\boldsymbol{v}_t'$ can also be written as the random variable $1/T^2\sum_{i=1}^{n}\boldsymbol{v}_t\boldsymbol{v}_t'$ which converges in distribution to $\boldsymbol{\Sigma}^{1/2}\int_0^1 \boldsymbol{W}_t\boldsymbol{W}_t'\,dt\boldsymbol{\Sigma}^{1/2}$ where $\boldsymbol{W}_t$ is a Wiener process and $\boldsymbol{\Sigma}$ is the long run variance-covariance matrix of $\boldsymbol{v}_t$ (for a detailed discussion see Phillips and Durlauf, 1986).

The measure $\lambda_{i,t}$ introduced in equation (4.7) lies between λ_1 and λ_n and serves us to examine the strength of the markets long-run link over time. Note that the maximum value attained by $\lambda_{i,t}$ equals λ_1 because the variance-covariance matrices in 4.7 are the whole sample ones. The use of the whole sample variance-covariance matrices involves the implicit assumption that the components of the vector $\boldsymbol{v}_t$ follow unit root processes in which there is no structural break. Such a structural break in the evolution of the $\boldsymbol{v}_t$, if exists, is reflected in the evolution of $\boldsymbol{\beta}_t$.

4.4 Empirical analysis

4.4.1 *Data*

We use the monthly values of the MSCI indices of G7 stock markets, namely the US, UK, France, Germany, Italy Japan, and Canada, downloaded from Datastream. The MSCI indices are adjusted for the effect of cross-listing. The analysis period starts from 1969 until June 2022 covering 631 months. All the indices are in terms of the US dollars and are transformed by the natural logarithms. In order to further analyze the impact of the Growing China on the multivariate cointegration of the G7 stock markets, we add the MSCI index of China's stock market to the analysis. Figure 4.2 exhibits the normalised G7 and China stock market indices.

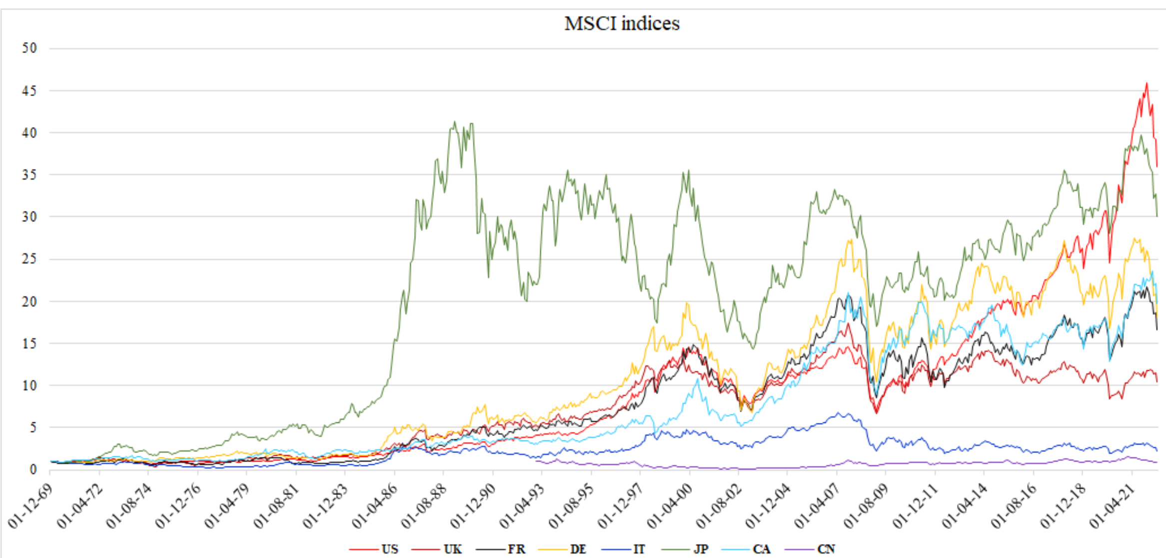


Figure 4.2: MSCI indices of G7 and China stock markets. The indices are in terms of the US dollars and are transformed by the natural logarithms

The descriptive statistics of the indices transformed by the natural logarithms are exhibited in Table 4.1. As the historical data of the China stock market index started in December of 1992, the observation number of the index is 355 months. Since the indices are reported in terms of the US dollars, that of Japan takes negative values after having transformed by the natural logarithms.

Table 4.1: Descriptive statistics of the G7 and China stock market indices

Descriptive statistic	MSCI indices							
	US	UK	FR	DE	IT	JP	CA	CN
mean	6.21	7.04	6.43	5.77	6.31	1.35	6.13	1.94
std.	1.19	1.00	1.17	1.11	0.82	1.08	0.99	0.61
median	6.41	7.44	6.73	6.05	6.58	1.86	5.97	2.18
Kurtosis	-1.36	-1.08	-1.40	-1.32	-0.70	-0.24	-1.39	-0.44
Skewness	-0.08	-0.65	-0.43	-0.43	-0.66	-1.08	-0.03	-0.85
Min	4.17	4.60	4.34	3.68	4.39	-1.45	4.37	0.50
Max	8.43	8.34	7.97	7.21	7.66	2.45	7.70	2.89
Sample No.	631	631	631	631	631	631	631	355

4.4.2 *Unit root test*

The results of the unit root test are shown in Table 4.2. The augmented Dickey-Fuller test is applied twice for each index in two specifications, assuming an intercept with no trend and an intercept coupled with a trend. Only in the single case of Japan, the null is rejected at 8% significance level.

Table 4.2: Augmented Dickey-Fuller test

	Intercept			Intercept and trend		
	t-statistic	p-value	Lag lengths	t-statistic	p-value	Lag lengths
US	0.02	0.96	0	-2.35	0.40	0
UK	-1.35	0.61	0	-1.30	0.89	0
FR	-0.89	0.79	0	-1.80	0.71	0
DE	-1.41	0.58	0	-1.76	0.72	0
IT	-1.20	0.68	0	-1.52	0.82	0
CA	-0.95	0.77	0	-3.13	0.10	0
JP	-2.68	0.08	0	-1.53	0.82	0
CN	-1.45	0.56	0	-2.20	0.49	0

The augmented Dickey-Fuller test is applied to the G7 and China stock market indices. The first panel assumes an intercept with no time trend in the model. The second panel includes both an intercept and a time trend in the model. The lag length is selected by BIC.

Table 4.3: Stationarity test using ERS and Ng-Perron methods

	ERS		Ng-Perron				
	P_T	lag	MZ_α	MZ_t	MSB	MP_T	lag
US	216.31	0	1.49	2.50	1.68	203.78	0
UK	85.55	0	0.43	0.50	1.15	80.14	0
FR	86.81	0	0.68	0.77	1.13	81.89	0
DE	108.16	0	0.59	0.76	1.28	100.95	0
IT	15.32	0	-1.48	-0.78	0.53	14.90	0
CA	108.34	0	0.97	1.20	1.23	101.45	0
JP	209.98	0	0.54	0.87	1.61	153.58	1
CN	9.89	0	-2.59	-1.14	0.44	9.45	0
1%	1.99		-13.80	-2.58	0.17	1.78	
5%	3.26		-8.10	-1.98	0.23	3.17	
10%	4.48		-5.70	-1.62	0.28	4.45	

Unit root test is performed using the Elliot-Rothenberg-Stock (ERS) and NG-Perron test methods. d_t is assumed to be $d_t = 1$. P_T denotes point optimal test statistic. MZ refers to the modified version of PP's Z test statistic. $MZ_\alpha = Z_\alpha + (T/2)(\hat{\alpha} - 1)^2$ where Z_α is PP's Z test statistic and $\hat{\alpha}$ is the estimated AR coefficient. $MSB = (T^{-2} \sum_{t=1}^{t=T} q_{t-1}^2)^{1/2}/s^2$ where s^2 is the residual variance and $Z_t = MSB * Z_\alpha$. MP_T denotes the modified point optimal test statistic and has the same asymptotic distribution as P_T. *, ** and *** represent significance at the 10%, 5% and 1% levels, respectively. The bottom panel shows the critical values.

4.4.3 *Evidences on static cointegration and time-varying cointegration tests*

We also perform a TI cointegration test using the Johansen trace as well as the maximum eigenvalue test statistics. The test is inclusive, accounting for a number of specifications in the cointegration equation and in the corresponding VAR model. The evidence supporting static 7-variate cointegration in Table 4.4 is quite weak, showing only one significant relation at 5% significance level and no significant relation at 1% significance level. The results of the trace and maximum eigenvalue statistics at 10% significance level show one significant relation for only three model specifications. Given that the underlying time period is long enough, covering almost half a century, these results are not surprising. A number of events, potentially impacting markets' co-movement have been realised in the past half-century, some of which are the floating exchange rate regime following Bretton Woods, the crude oil crisis in the aftermath of the political instability in the Middle East, the foundation of OPEC, the introduction of Euro, liberalization of free capital flow and growing foreign investment.

To gain insight into the variability of the cointegrating vectors, we estimate the multivariate time-varying cointegration test developed by Bierens and Martins (2010) to investigate parameter instability of the 7-variate cointegration relation. The corresponding null hypothe-

Table 4.4: The number of cointegrating vectors by applying Johansen TI cointegration test

1% significance level					
Cointegration relation	No trend No intercept	No trend Intercept	No trend Intercept	Trend Intercept	Trend Intercept
VAR	No intercept No trend	No intercept No trend	Intercept Trend	Intercept Trend	Intercept Quadratic trend
Trace	0	0	0	0	0
Max-Eigenvalue	0	0	0	0	0
5% significance level					
Cointegration relation	No trend No intercept	No trend Intercept	No trend Intercept	Trend Intercept	Trend Intercept
VAR	No intercept No trend	No intercept No trend	Intercept Trend	Intercept Trend	Intercept Quadratic trend
Trace	1	0	0	0	0
Max-Eigenvalue	1	0	0	0	0
10% significance level					
Cointegration relation	No trend No intercept	No trend Intercept	No trend Intercept	Trend Intercept	Trend Intercept
VAR	No intercept No trend	No intercept No trend	Intercept Trend	Intercept Trend	Intercept Quadratic trend
Trace	1	0	0	0	0
Max-Eigenvalue	1	1	1	0	0

Johansen (1988) rank test is applied to MSCI market indices of G7 countries from Dec 1969 to Jun 2022. Both trace and maximum eigenvalue test statistics are estimated in five specifications. Each specification differs from the others in assuming a time trend in the cointegration relation and an intercept in the cointegration relation or in the VAR.

sis is TI cointegration and the alternative hypothesis is time-varying cointegration. As Table 4.5 indicates, the null of TI cointegration is strongly rejected, given any number of cointegrating vectors and Chebyshev polynomial order m.

We also estimate the time-varying single equation cointegration model developed by Eroğlu et al. (2022) to test for time-varying cointegration in the 7-variate and 4-variate models. The results are presented in Table 4.6. The results of this model should depend on the choice of the dependent variable. Therefore, we perform the model in equation (4.1) to the 7-variate cointegration depending on the choice of the dependent variable. The results of the test are highly consistent with the results of the time-varying model developed by Bierens and Martins (2010) presented in Table 4.5. The autoregressive coefficient of the stationary residual series, i.e. the parameter θ, determines the existence of the cointegration. The $|\theta| < 1, T = 1$ and small standard deviations σ_i imply a smoothly varying cointegration relation. The coefficient of the autoregressive series w_t is estimated to be less than 1 in 6 relations

Table 4.5: Likelihood ratio test of multivariate time-varying cointegration

	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$
			$lags=1$		
$r=1$	20.94	36.02	58.23	94.71	125.85
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=2$	47.18	81.00	131.71	175.26	228.77
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=3$	67.62	117.20	188.80	246.30	317.80
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=4$	94.75	161.35	239.31	309.67	399.11
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=5$	110.43	198.84	284.09	370.24	469.82
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=6$	121.86	226.88	329.26	420.03	536.69
			$lags=2$		
$r=1$	30.68	47.53	69.98	96.60	126.80
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=2$	53.19	97.55	140.12	185.27	241.27
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=3$	76.19	135.83	208.35	267.80	342.07
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=4$	101.82	176.58	258.66	329.73	426.04
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=5$	116.03	212.54	301.96	390.67	496.81
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=6$	129.08	238.54	343.47	438.87	561.98
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)

The likelihood ratio test of time-varying cointegration, developed by Bierens and Martins (2010), is applied to MSCI stock market indices of the G7 countries from Dec 1969 to Jun 2022. The parameters m and r represent Chebyshev polynomial order and the number of time-varying cointegrating vectors respectively. The test statistic follows Chi-squared distribution with mrn degrees of freedom where m denotes Chebyshev polynomial order, r denotes the number of independent cointegrating vectors, and n stands for the number of the variables. The numbers in the parentheses represent the p-values.

out of the seven cointegration relations, whereas the null hypothesis of $\theta = 1$ is rejected in 5 cointegration relations. The autoregressive coefficient of the cointegrating vector i.e. $\hat{\phi}$ is significantly different than 1 in the case of the relation having Italy as the dependent variable. The estimated standard deviation of the $\hat{\phi}$ equals 0.0008 which is the lowest among all the cointegration relations.

We estimate the quadrivariate single equation time-varying cointegration among European countries of the G7 in Table 4.7. The null hypothesis of $\theta = 1$ against the alternative

Table 4.6: The estimation of the 7-variate cointegration using time-varying single equation model

Model parameters	US	UK	FR	DE	IT	JP	CA
$\hat{\beta}_{1,0}$	-0.1856	-0.2461	-0.5225	0.0362	-0.3862	3.3921	0.0502
$\hat{\sigma}_1$	0.100	0.0057	0.0067	0.0065	0.0042	0.0053	0.0048
$\hat{\beta}_{2,0}$	0.0698	0.0831	-0.3470	-0.7488	-3.3794	-1.2168	-0.0428
$\hat{\sigma}_2$	0.0052	0.0052	0.0047	0.0044	0.0037	0.0049	0.0042
$\hat{\beta}_{3,0}$	0.1961	0.0948	0.5775	0.5296	0.2860	-1.7654	-0.0029
$\hat{\sigma}_3$	0.0054	0.0049	0.0077	0.0062	0.0041	0.0063	0.0046
$\hat{\beta}_{4,0}$	-0.0310	-0.0132	0.2584	0.1102	1.2393	0.9587	0.0089
$\hat{\sigma}_4$	0.0041	0.0052	0.0062	0.0055	0.0052	0.0063	0.0051
$\hat{\beta}_{5,0}$	0.2835	0.1042	-0.2304	1.6516	-5.7239	-0.4058	-0.0105
$\hat{\sigma}_5$	0.0104	0.0127	0.0092	0.0182	0.0155	0.0059	0.0047
$\hat{\beta}_{6,0}$	0.2344	0.1582	0.1404	0.6712	1.4826	-0.4450	-0.1457
$\hat{\sigma}_6$	0.0045	0.0049	0.0069	0.0050	0.0045	0.0050	0.0165
$\hat{w}_0$	0.0205	0.0096	0.0028	0.0014	-0.0024	0.0088	0.0147
$\hat{\sigma}_w$	0.0182	0.0213	0.0116	0.0112	0.0427	0.0324	0.0268
Intercept	3.8887***	5.2908***	4.5160***	3.7416***	5.6022***	-0.8768***	4.3957***
	(0.1293)	*(0.1030)*	*(0.0771)*	*(0.0854)*	*(0.0009)*	*(0.1196)*	*(0.0927)*
$\hat{\phi}$	1.0010	1.001	1.0021	1.0027	0.9940***	1.0042	1.002
	(0.0494)	*(0.0510)*	*(0.0343)*	*(0.0369)*	*(0.0008)*	*(0.0484)*	*(0.0463)*
$\hat{\theta}$	0.7372	0.6285***	0.1415***	0.2503***	1.1167	0.6462***	0.4962***
	(0.2464)	*(0.1286)*	*(0.3135)*	*(0.2608)*	*(0.1004)*	*(0.0562)*	*(0.1373)*
max Log likelihood	839.23	842.22	813.68	861.04	701.59	787.66	896.26
BIC	-1568.85	-1574.84	-1517.76	-1612.48	-1293.57	-1465.71	-1682.91

The 7-variate cointegration is estimated for the G7 stock market indices using time-varying single equation in equation (4.1). The estimated relations are exhibited in the columns, each of which corresponds to different choices of the dependent variable. The $\hat{\beta}_{i,0}$s for $i = 1, \ldots, 6$ stand for the initial values of the cointegrating vectors in which the first component is normalised to one. The $\hat{w}_0$ stands for the initial value of the stationary component w. The $\hat{\sigma}_i$s for $i = 1, \ldots, 6$ and $\hat{\sigma}_w$ stand respectively for the standard deviation of the cointegrating vector and of the stationary residual series. The numbers in parentheses represent the standard deviations. The number of the lagged terms Δw_{t-i} are chosen according to BIC. *, ** and *** respectively stand for significance at 1%, 5% and 10% levels. The test on ϕ is performed on the null hypothesis of $\phi = 1$ against the alternative of $\phi \neq 1$. The test on θ is performed on the one-sided null hypothesis of $\theta = 1$ against the alternative of $\theta < 1$.

hypothesis $\theta < 1$ is strongly rejected in four models differing on the choice of the dependent variable. The autoregressive coefficient $\hat{\phi}$ of the cointegrating vectors is not significantly different than unity. The stationarity of the residuals, as evidenced in the parameter $\hat{\theta}$, and the presence of the unit root in the cointegrating vector imply smoothly varying cointegration in the quadrivariate relation.

4.4.4 *Empirical findings on $\lambda_{i,t}$s*

The theoretical developments applied so far, provide us with evidence of the variation of the cointegrating vectors. Though the shifts take place in the markets' linkages, the fact that

Table 4.7: The estimation of the 4-variate cointegration using time-varying single equation model

Model parameters	UK	FR	DE	IT
$\hat{\beta}_{1,0}$	0.0459	-0.7370	-0.6129	-1.8998
$\hat{\sigma}_1$	0.0083	0.0084	0.0083	0.0083
$\hat{\beta}_{2,0}$	0.0340	0.3844	0.3976	-0.1007
$\hat{\sigma}_2$	0.0093	0.0102	0.0092	0.0090
$\hat{\beta}_{3,0}$	-0.0295	0.4978	0.3033	2.8282
$\hat{\sigma}_3$	0.0088	0.0094	0.0093	0.0101
$\hat{w}_0$	0.0180	0.0001	0.0005	0.0018
$\hat{\sigma}_w$	0.0466	0.0391	0.0446	0.0421
Intercept	5.2466***	4.4434***	3.5858***	5.5367***
	(0.0875)	*(0.0808)*	*(0.0877)*	*(0.0546)*
$\hat{\phi}$	0.9999	1.0021	1.0024	1.0040
	(0.0426)	*(0.0350)*	*(0.0358)*	*(0.0486)*
$\hat{\theta}$	0.3598***	-0.2371***	-0.0197***	0.0390***
	(0.1343)	*(0.1484)*	*(0.1799)*	*(0.0853)*
max Log likelihood	722.72	690.20	706.77	663.49
BIC	-1335.83	-1270.79	-1303.94	-1217.37

The 4-variate cointegration is estimated for the G7 stock market indices using time-varying single equation in equation (4.1). The estimated relations are exhibited in the columns, each of which corresponds to different choices of the dependent variable. The $\hat{\beta}_{i,0}$s for $i = 1, \ldots, 3$ stand for the initial values of the cointegrating vectors in which the first component is normalised to one. The $\hat{w}_0$ stands for the initial value of the stationary component w. The $\hat{\sigma}_i$s for$i = 1, \ldots, 3$ and $\hat{\sigma}_w$ stand respectively for the standard deviation of the cointegrating vector and of the stationary residual series. The numbers in parentheses represent the standard deviations. The number of the lagged terms Δw_{t-i} are chosen according to BIC. *, ** and *** respectively stand for significance at 1%, 5% and 10% levels. The test on ϕ is performed on the null hypothesis of $\phi = 1$ against the alternative of $\phi \neq 1$. The test on θ is performed on the one-sided null hypothesis of $\theta = 1$ against the alternative of $\theta < 1$.

these linkages are intensified or not is unexplored. The time-varying measures introduced in equation (4.7) illustrate the process of convergence of the market's interdependence. We focus on the $\lambda_{i,t}$s and the instability measure to gain a more in-depth understanding of the dynamics in the market interdependence.

Figures 4.3, 4.7 and 4.10 exhibit respectively the time-varying impact matrix in equation (4.4b) for the 7-variate, 4-variate and 3-variate cointegration relations. Though the time-varying cointegrating vectors exhibit periods of smoothly varying cointegrating vectors interrupted by periods of high variability, the evolution of the orthogonal vector on the spanning space of all the cointegrating vectors can reveal the instability of the long-run relation of the underlying markets.

4.4.5 *The 1970s*

During the decade of 1970s, the time-varying cointegrating vectors in Figure 4.3, Figure 4.7, and Figure 4.10 look relatively stable across all the cointegration vectors. The stability is more apparent in the 7-variate case in Figure 4.3. Nevertheless, a closer look at the angular shift of the normal vector of the space of the cointegrating vectors in Figure 4.4, shows severe instability during the oil crisis and the stock market crash of 1973-1974. The short period of the bull market following the crash exhibits a rather stable cointegration relation until the inception of the stagflation in the late 1970s.

The components of the cointegrating vectors can be treated as elasticity of the indices because indices are transformed by natural logarithms. In 1970, as Figure 4.3 shows, the coefficients of the UK and Germany get distance from that of the US and Canada. In certain subgraphs, that of France and Japan are realising larger coefficients. However, the range of the coefficients during the 2010s is smaller than 1970s and 1980s. In the first subgraph, the elasticity of the UK and the German stock market indices to the US and Canada stock market indices is smaller during the 1970s than 2010s. If we assume that the stock market return of France, Italy, and Japan remains zero, then the magnitude of future stock market returns of the UK and Germany are smaller than that of the US and Canada during these decades. Thus, there could be a risk factor to which the US and Canadian stock markets are more exposed during the 1970s. In Figure 4.7, the same can be inferred for the UK and the German stock markets. The UK stock market to that of Germany is more elastic during the 1970s than 2010s. Then the UK stock market bore more risk than Germany during this period. In Figure 4.10, much information can not be inferred.

4.4.6 *The 1980s*

During the early years of the 1980s, the cointegrating vectors suffer from severe instability as evidenced in Figure 4.4. Although the economies recover by the mid-1980s, the stable 7-variate cointegration does not last long until the end of the decade. This finding is consistent with the work by Menezes et al. (2012) where they find a level and a regime shift in the long-run equilibrium relation of the G7 markets during the decade. The shift identified by two distinct methodological approaches in the work by Menezes et al. (2012) happen respectively on black Monday in 1987 and in 1984-1985. The instability during this period of time is also evidenced in the literature in the bivariate cointegration relations. Kanas (1998) finds no evidence of bivariate cointegration between the US and either of the European members of the G7 in the period from 1986 to 1996. This finding is consistent with the work by Arshanapalli and Doukas (1993), who find no bivariate cointegration between the US stock market and either of France, Germany, Japan, and the UK in the period starting from 1980 until 1987.

As Figure 4.3 shows, The magnitude of the stock market return of the UK and Germany is smaller than those of the US and Canada during the 1980s if other markets realise zero return. The US and Canada stock markets are exposed more to risk factors during these periods than the UK and German stock markets. In Figure 4.7, the same conclusion can be drawn in the 1980s on the relative riskiness of German and the UK stock markets.

The high instability during the late 1980s and early 1990s, evidenced in Figure 4.4, is notable because firstly, the global financial crisis leads to a lesser instability, and secondly, the period is a long-lasting period of unstable market linkage across the whole time interval. The long-lasting period of the frail linkage among G7 stock markets can be explained by a set of pivotal economic policies across G7 countries. During the early years of the 1980s, the economic recession was coincident with raging inflation, causing an unprecedented stagflation and encouraging monetary contraction in all the G7 countries. The decade is also concurrent with a number of economic legislation to liberalise the economy and also to overcome the double-dip recession. A number of new economic policies were also put in force by the US in an effort to bolster economic growth[4].

The decade of the 1980s is also identified as a turning point in the macroeconomic policy of the states in which the policy of budget deficit is abandoned by the countries and state intervention is mostly concentrated on decreasing public spending, rising taxes, and broad

[4]The money market of the US was further liberalised during the decade by lifting the ceiling on the deposit interest rate and legalizing mortgage lending. Deregulation acts in the banking sector of the US, which liberated deposit interest rates and legalised adjustable-rate mortgage loans, are among the liberalization measures which were sought to encourage economic recovery. The two well-known acts on the deregulation of the banking sector, though perceived as deregulating measures, led the financial sector to collapse including numerous bank failures. The Saving and Loan Crisis (S&L Crisis) in the second half of the 1980s could have also affected the linkage.

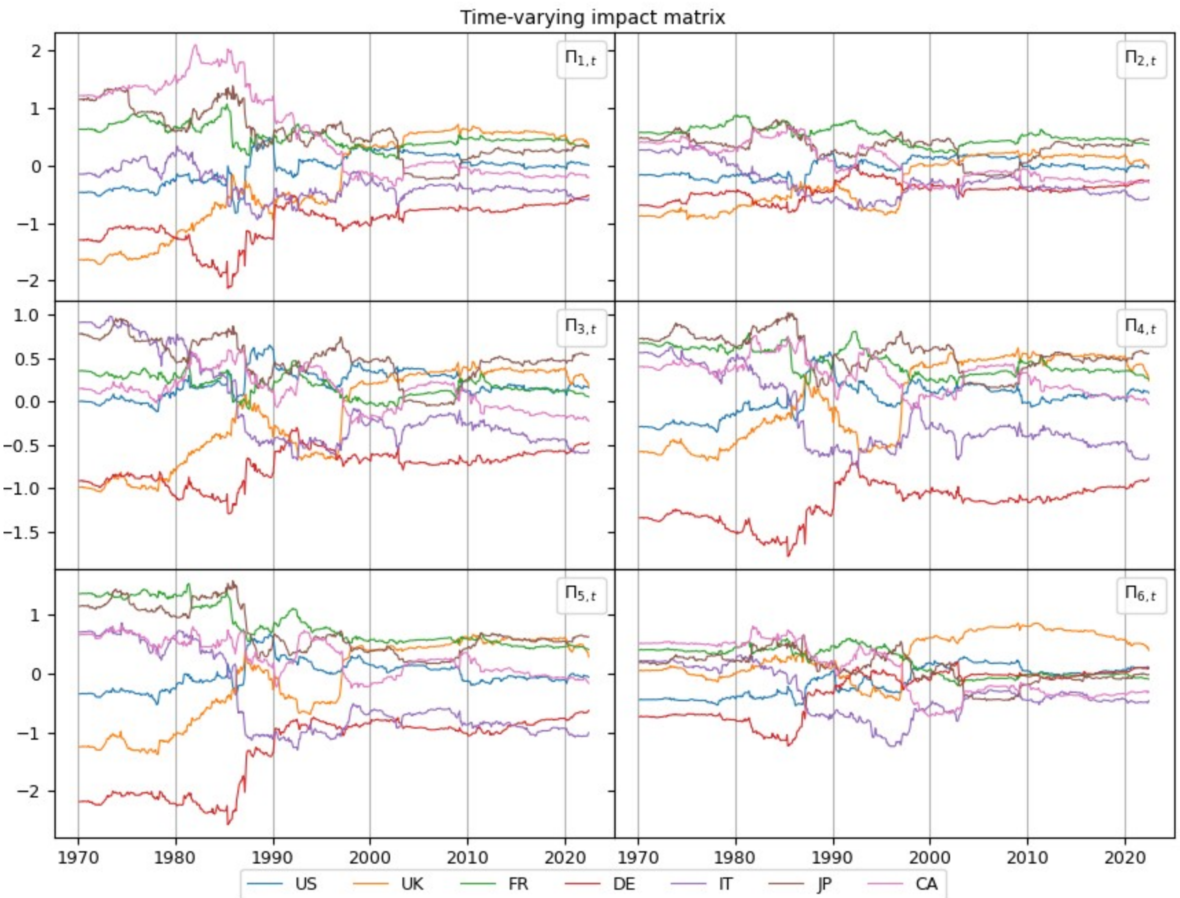


Figure 4.3: The time-varying impact matrix of the 7-variate cointegration model. The 7-variate time-varying cointegrating vectors of the G7 stock market indices are estimated by the state-space modelling of the VECM. Each subplot corresponds to an equation of the VECM or in other words to $\mathbf{\Pi}_{i,t}$ which represents the i-th row of the impact matrix.

privatization of the state-owned firms. The decade is generally marked by the evolution of the G7 economies from state-owned firms and regulated economies into the regime of private ownership and liberalised integrated markets.

The 1980s decade is also distinguished by the contractionary monetary policy as opposed to the expansionary policy in the other economic recessions. All the G7 countries responded to the inflation by raising interest rates. The heterogeneous interest rates of G7 countries in particular that of the UK during the late 1980s and early 1990s until the UK currency crisis could have adjusted expectations against investment in the UK market. The US interest rate follows a downward trend in this period, whereas those of the other G7 countries remain upward trending. The heterogeneous trend of the interest rate persists across the decade of the 1970s. These results are comparable to the findings by Hansen and Johansen

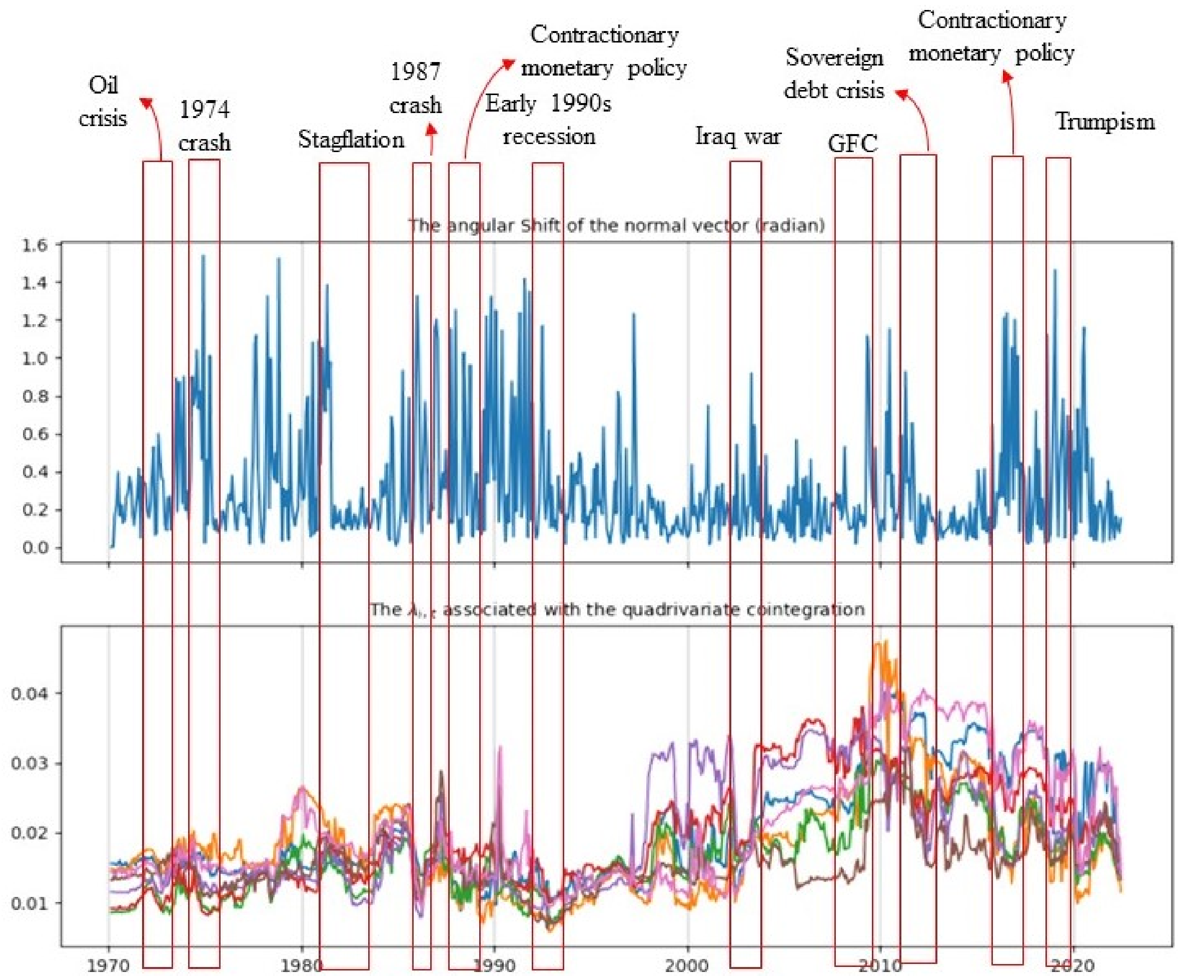


Figure 4.4: The instability and the strength of 7-variate cointegration. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radians.

(1999) who find that the 1 to 4-month US treasury yields bear a structural break in the early 1980s. The work confirms the influence of the changing procedures of the Federal Reserve on the interest rates.

4.4.7 *The 1990s and 2000s*

The early years of the 1990s are also characterised to a high degree by instability in the cointegrating vectors. The recession at the beginning of the decade destabilises stock market co-movements of the G7 countries, not less than the faltering period of the preceding decades. Overall, it seems that the turmoil period of the unsteady cointegration is not present after the UK and Italy currency crisis in 1992.

Table 4.8: The sub-period regressions of the $\lambda_{i,t}$s

Sub-period	coefficients	Time-varying $\lambda_{i,t}$s						
		$\lambda_{1,t}$	$\lambda_{2,t}$	$\lambda_{3,t}$	$\lambda_{4,t}$	$\lambda_{5,t}$	$\lambda_{6,t}$	$\lambda_{7,t}$
1969–2022	c	0.0102	0.0141	0.0100	0.0098	0.0001	0.0134	0.0115
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
	Trend	0.00003	0.00004	0.00002	0.00001	0.00003	0.00000	0.00003
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
1992–2010	c	-0.0168	-0.0264	-0.0072	-0.0219	-0.0213	0.0010	-0.0219
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
	Trend	0.00010	0.00012	0.00007	0.00012	0.00012	0.00004	0.0001
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
2010–2022	c	0.0787	0.1192	0.0697	0.0562	0.0573	0.0385	0.1000
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
	Trend	-0.00009	-0.00017	-0.00009	-0.00006	-0.00005	-0.00004	-0.00010
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)

The $\lambda_{i,t}$s of the 7-variate cointegration model are regressed on a constant and a trend variable. The numbers in the parentheses represent the p-values.

In the aftermath of the currency crisis during the early 1990s, the 7-variate co-movement is intensified. As Table 4.8 shows, the deterministic trend in the period from 1992 until 2010 is strongly significant. This is a major result which also finds support in the work by Awokuse et al. (2009) who investigate 12-variate cointegration and find evidence for stronger cointegration in the aftermath of the financial liberalization policies. The findings by Taylor and Tonks (1989) also imply the positive impact of market liberalization, in particular the floating exchange rate regime, on market co-movement.

4.4.8 *The 2010s*

The market linkage during the two decades of the 1990s and 2000s enjoys a steady intensification. As it is apparent in Figure 4.4, the strength of the 7-variate cointegration takes a downward trend in the last decade after the peak in the post-crisis period. There is also a remarkable rise in instability, a couple of years before the covid-19 crisis. This finding is surprising in particular because the liberalization of the markets and international collaboration have reached maturity in the recent decade. The sovereign debt crisis and the technological innovation in the production of crude oil that led to a reduction in a range of commodity prices lead to a severe decline in the $\lambda_{i,t}$s. However, they revert to the decade-long downward trend. The contractionary monetary policy in 2016 and Trump's administration led to weaker market cointegration.

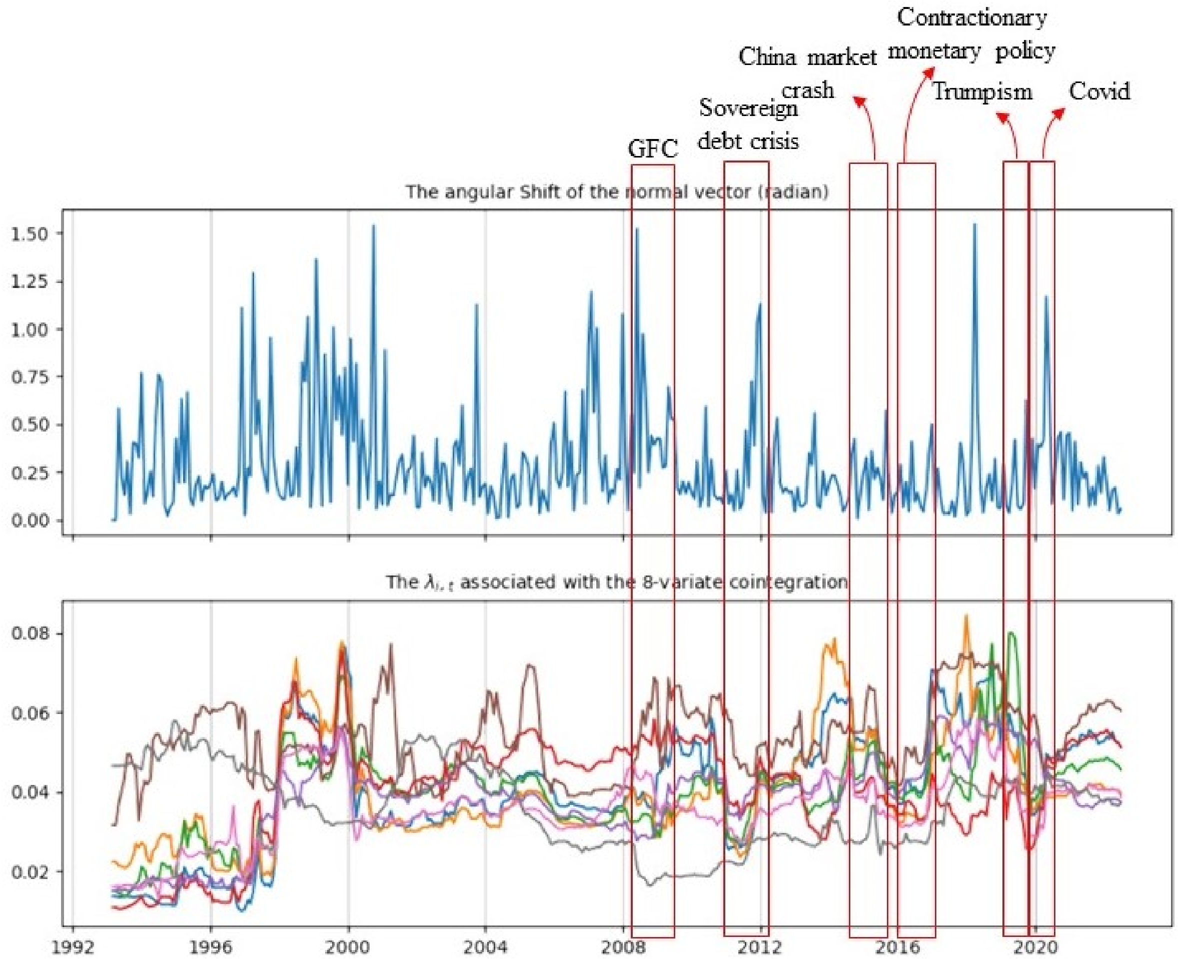


Figure 4.5: The instability and the strength of 8-variate cointegration of the G7 and China. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radians.

4.4.9 *The integration of China and the particular sub-period of 2010-2022*

We conduct an 8-variate cointegration including China to bring this finding into more scrutiny. As exhibited in Figure 4.5, following the global financial crisis, the strength measure is upward sloping until the innovation in crude oil production which significantly lowered energy prices. The downward trend of the 7-variate model after 2010 turns into a rather rising trend of the 8-variate model in Figure 4.6. The results in Table 4.9 evidence significant deterministic trend in most of the $\lambda_{i,t}$s during the sub-period of 2010-2022 as well as in the whole period. From 2017 on, when the crude price stabilises, the strength measure is restored until the trade wars between the US and China which lasts almost until the covid-19 crisis. As a proxy for the overall evaluation of the 7-variate and 8-variate cointegration relations, we compare the summation of the time-varying $\lambda_{i,t}$s in Figure 4.6. It is evident that China has enjoyed a steady integration into the world economy since the early 1990s. During the late 1990s, the summation of the 8-variate $\lambda_{i,t}$s spikes, and during the following decade it returns to the long-run trend.

Table 4.9: The sub-period regressions of the $\lambda_{i,t}$s

Sub-period	coefficients	Time-varying $\lambda_{i,t}$s							
		$\lambda_{1,t}$	$\lambda_{2,t}$	$\lambda_{3,t}$	$\lambda_{4,t}$	$\lambda_{5,t}$	$\lambda_{6,t}$	$\lambda_{7,t}$	$\lambda_{8,t}$
1993–2022	c	-0.0027	0.0159	0.0136	0.0213	0.0084	0.0394	0.0159	0.0549
		(0.35)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
	Trend	0.00010	0.00005	0.00006	0.00004	0.00007	0.00003	0.000041	-0.00004
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
2010–2022	c	-0.0010	0.0389	-0.0115	0.0204	0.0221	0.0217	0.0050	-0.0483
		(0.93)	(<0.01)	(0.22)	(0.02)	(<0.01)	(0.02)	(0.48)	(0.48)
	Trend	0.00009	0.00001	0.0001	0.00004	0.0004	0.00006	0.00006	0.0001
		(<0.01)	(0.59)	(<0.01)	(0.02)	(<0.01)	(<0.01)	(<0.01)	(0.48)

The $\lambda_{i,t}$s of the 8-variate cointegration model are regressed on a constant and a trend variable. The numbers in the parentheses represent the p-values.

4.4.10 *The dynamic interdependence of the European countries*

There is also remarkable evidence that the long-run linkage among the European countries faces instability in the period of the late 1980s and early 1990s, whereas it remains relatively stable during the decade of 1970 (Figures 4.7 to 4.8). The quadrivariate time-varying impact matrix of the European members of the G7, as exhibited in Figure 4.7, shows higher instability in the short period of time preceding the currency crisis. The onset of the currency crisis of 1992 which led to the UK and Italy's withdrawal from the European monetary system, provokes sharp variability in the quadrivariate cointegrating vectors. In the aftermath, during the late 1990s and after the introduction of the Euro, the quadrivariate time-varying cointegration remains considerably stable. The small shifts in the cointegrating vectors made by the global financial crisis of 2008 and the Covid-19 crisis seem far less extensive than the earlier ones in particular the one triggered by the currency crisis of 1992. The European level cointegration enjoys a tightening linkage from the mid-1990s to 2022 so that the economic recessions or the political rifts among the countries in particular Brexit do not impact the quadrivariate co-movement of the four European countries (Figure 4.8).

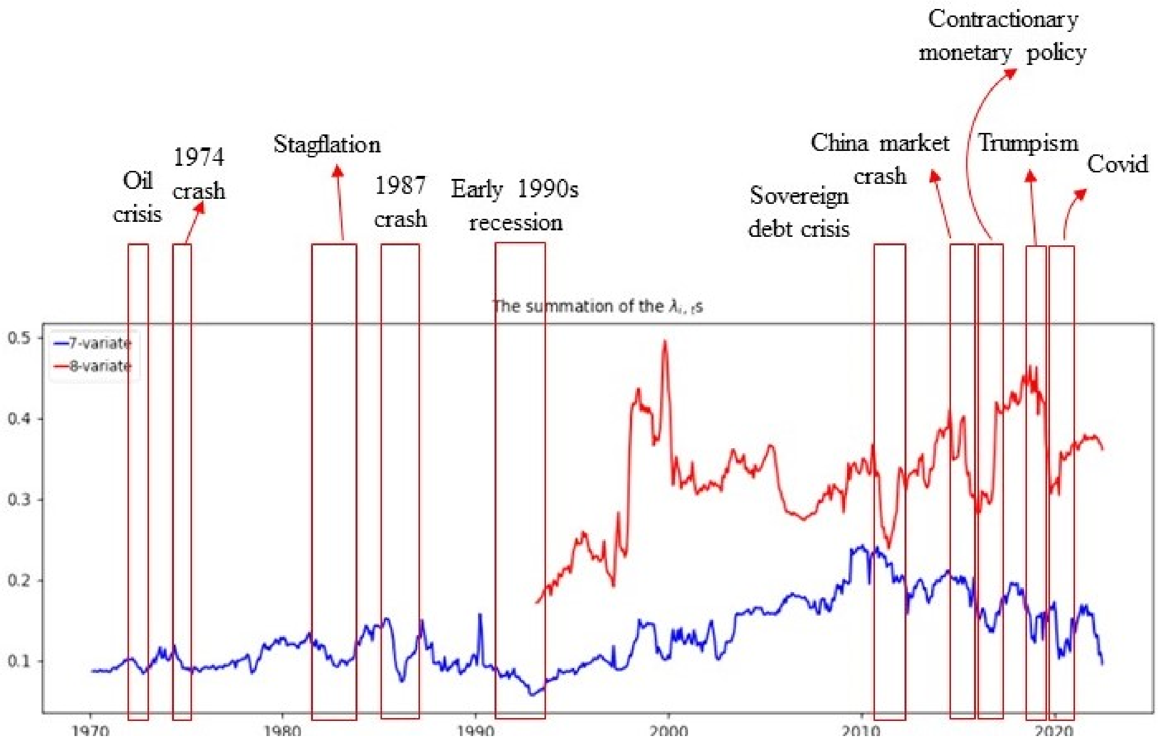


Figure 4.6: The Summation of the time-varying $\lambda_{i,t}$s.

As Figure 4.8 exhibits, the strength measure of the quadrivariate co-movement is downward sloping after 2005. Two events of the sovereign debt crisis and the technological innovation in the production of crude oil negatively impact the strength measure, though it is not restored to the levels earlier than 2005. We explore again the impact of China's economy on the quadrivariate cointegration of the European markets. It is evident that China has been integrated into the European economies from 2006 onwards when the country enjoyed repeated records of unprecedented economic growth. The crude oil innovation only impacts the strength of two cointegrating vectors, whereas the other one becomes stronger after 2016.

The 5-variate cointegration of the European markets and China as shown in Figure 4.9 seems to have been adversely impacted in 2018, possibly by Trumpism policy in which higher tariffs were imposed on some European and Chinese industries. The US-Europe and US-China trade frictions have shocked the linkage of the European and Chinese markets in particular after the bear market of 2018 in China when the fears of negative impacts of the trade friction were escalated.

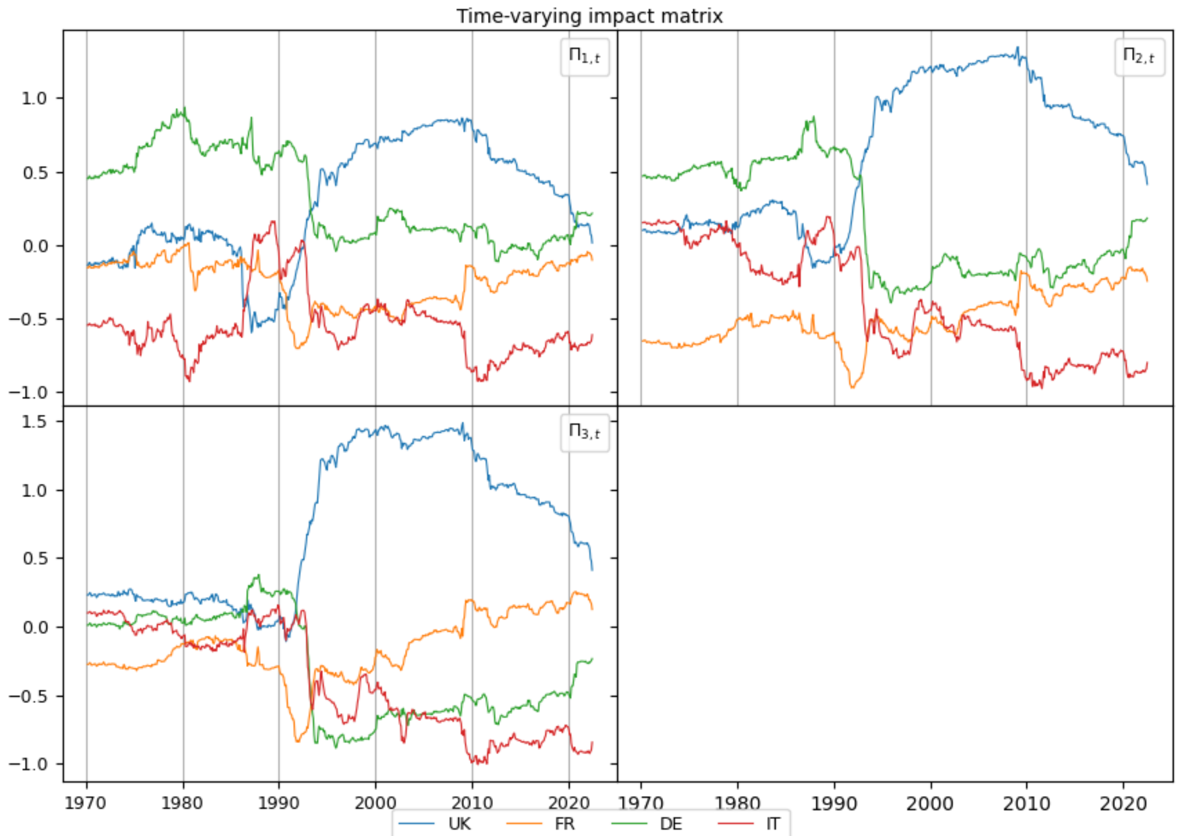


Figure 4.7: The time-varying impact matrix of the quadrivariate cointegration model. The quadrivariate time-varying cointegrating vectors of the European countries of the G7 are estimated by the state-space modelling of the VECM. Each subplot corresponds to an equation of the VECM or in other words to $\mathbf{\Pi}_{i,t}$ which represents the i-th row of the impact matrix.

We also conduct a trivariate time-varying cointegration among three European countries other than the UK. The trivariate cointegration among France, Germany, and Italy confirms again the fact that the decade of the 1980s is the paramount period across the underlying time horizon. Although the quadrivariate cointegration among the European members of G7 becomes unsteady from the second half of the decade, it seems that the trivariate cointegration is destabilised from the beginning of the period. This fact evidences the disrupting role of the UK market on the trivariate cointegration relation of the other European countries during the crucial period of the economic reversal of the Western countries. Both trivariate and quadrivariate cointegration at the European level become steady in the second half of the 1990s, with the trivariate relation more stable than the quadrivariate one, and seemingly the UK market has no longer a disruptive effect on the trivariate relation.

We see the impact of Brexit on neither of the multivariate cointegration relations.

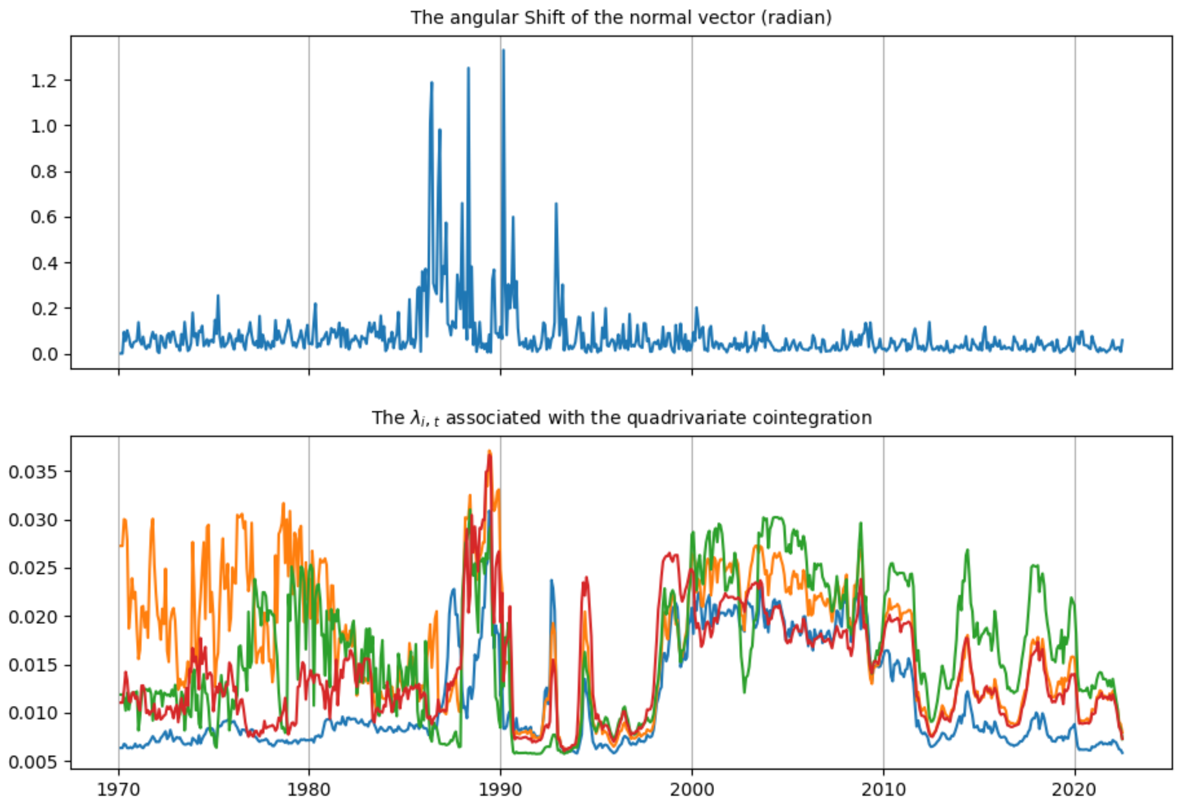


Figure 4.8: The instability and the strength of quadrivariate cointegration of the European countries of the G7. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radians.

The quadrivariate, as well as the trivariate cointegration relations, are stabilised in the early 1990s after the constitution of the European Union. One cannot certainly reject the inference that the liberalization of capital flow in the European countries, enforced in 1988, promoted markets' co-movement. The fact that notable crises in the era following these dates do not affect adversely the stability of the cointegrating vectors, could be the consequence of both free capital flow and the institution of the European Union.

From the standpoint of the theory of interest rate parity, the persistent instability in the equilibrium relation can be attributed to the varying disparities. The differential interest rates, ceteris paribus, do not ensue any shift in the cointegration relation, whereas the varying disparities between onshore and offshore interest rates can shift the cointegration relation. The disparities between interest rates reflect exchange rates or other economic uncertainties caused by political risks. The results of Figure 4.11, which evidence high instability in the period prior to 1990 and a few years afterward, indicate the high uncertainty associated with

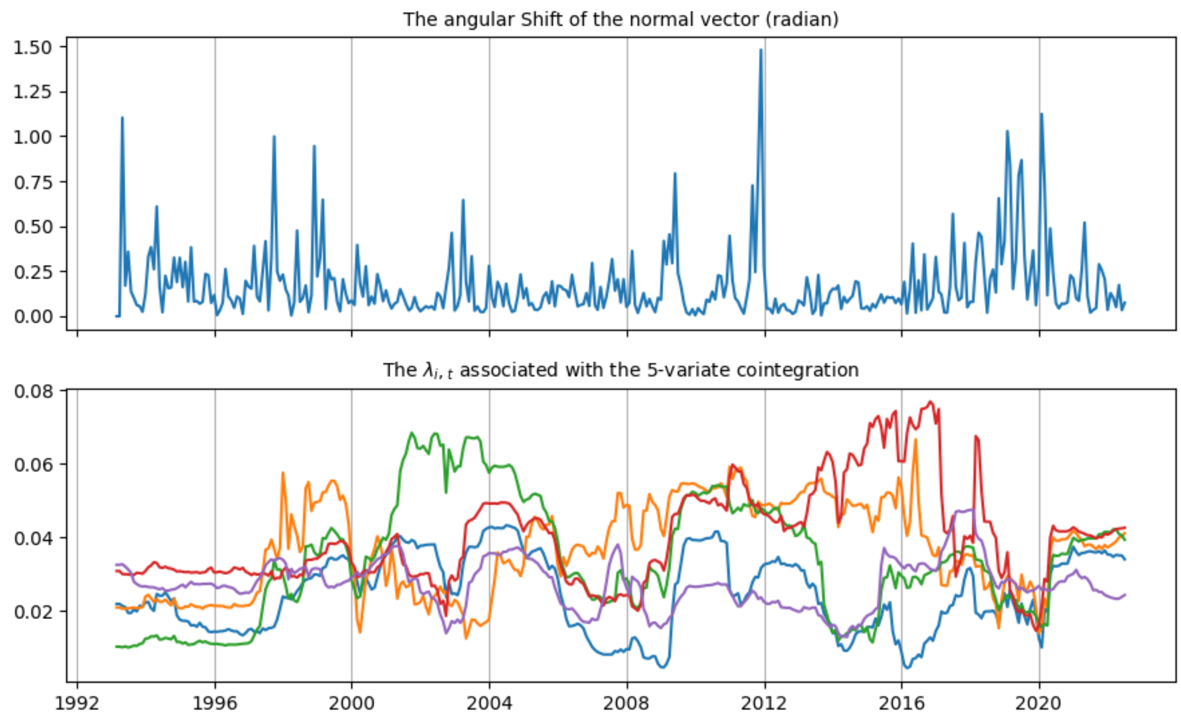


Figure 4.9: The instability and the strength of 5-variate cointegration of the European countries and China. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radian.

the future exchange rates of the European countries. The uncertainty associated with the economic reversals at the state level and the uncertain future of the talks to establish the European Union are among the potential factors causing repeated adjustments in the equilibrium relation among European members of the G7.

In order to further bring the evolution of the time-varying cointegration estimated by the single equation model under scrutiny, we investigate the instability in the cointegrating vectors as in the model estimated by the time-varying VECM in equation (4.4). We measure the shift in two consecutive cointegrating vectors by the angle between two vectors.

Figure 4.12 represents the shift in the consecutive 7-variate cointegrating vectors measured as the angle between two consecutive vectors in terms of radian with different choices of the dependent variable. One can conclude that in 6 cointegration models, the cointegrating vector is more volatile during the 1980s decade and to some extent during the 1990s. Only one model has Japan as the dependent variable, the results of the model are not in compliance with the other models. The conclusion on the 7-variate time-varying cointegration is the fact that the cointegrating vectors incur lower instability during the global financial crisis in 2008.

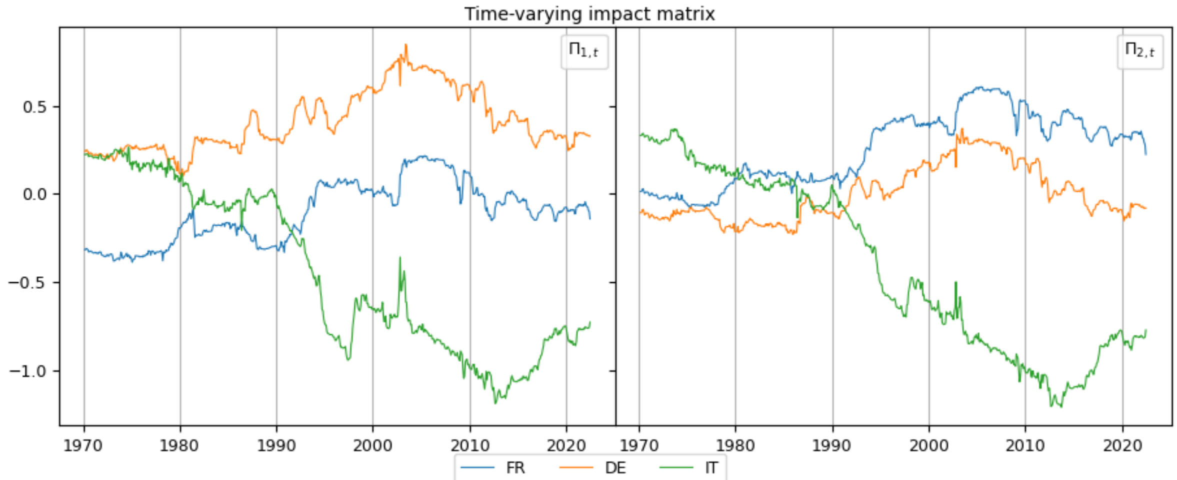


Figure 4.10: The time-varying impact matrix of the trivariate cointegration model. The trivariate time-varying cointegrating vectors of Germany, France, and Italy are estimated by the state-space modelling of the VECM. Each subplot corresponds to an equation of the VECM or in other words to $\mathbf{\Pi}_{i,t}$ which represents the i-th row of the impact matrix.

The Covid-19 crisis leaves significantly less instability in the 7-variate cointegration among G7 countries. Overall, the positive impact of globalisation on the markets' co-movement is in line with the conclusions made on the time-varying cointegrating vectors estimated by the VECM.

The results of the cointegration instability exhibited in Figure 4.13, in the cases having the UK and France stock markets as the dependent variables, evidence high variability of the cointegrating vector during the period from 1970 to 1990. The positive impact of globalisation on the steadiness of the markets' co-movement is also evident in the relations with the UK and France as the dependent variables. The cointegration equation having the German stock market as the dependent variable also evidences, to a lesser degree, the positive impact of globalisation on markets' convergence. The Global financial crisis and the Covid-19 crisis segment European markets far less than the earlier crises, in particular those of the decades 1970s and 1980s.

4.4.11 *The role of states and central banks*

The results of applying two methodologies to multivariate cointegration of the G7 stock markets reveal a number of landmark facts in the evolution of the stock markets. Both methodological approaches evidence that segmenting the impact of the economic crises on the market

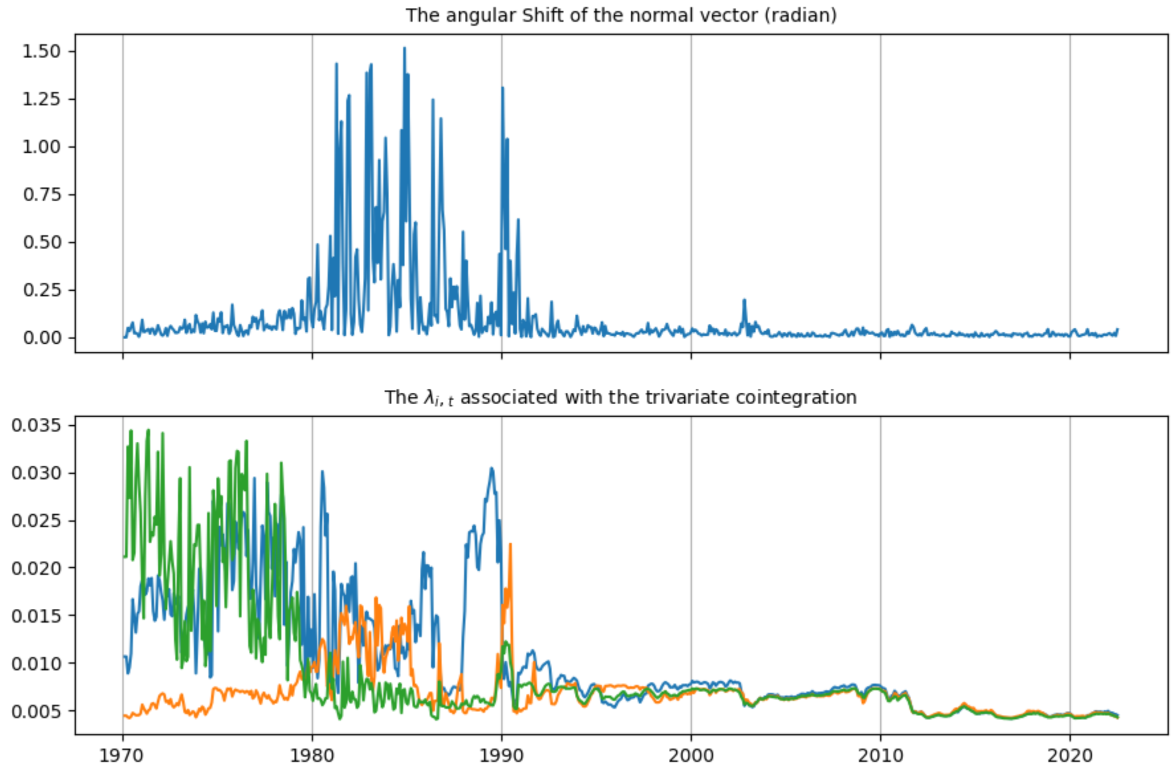


Figure 4.11: The instability and the strength of trivariate cointegration of FR, DE, and IT. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radians.

co-movement has considerably declined over time. Markets deregulation and extensive privatization, following the recession of 1982-83 that marked the decade as a transition to a new regime of market economy, are accompanied in the 1990 decade by more capital availability and free capital movements across markets. Capital availability is promoted by a number of events including macroeconomic and technological achievements. The US budget policy shifts into tax raising during President Clinton administration which stepped further back from the deficit policy. The lower deficit reduced state borrowing and enhanced capital availability in the financial markets. The innovation in information technology in the second half of the decade created new jobs and produced accumulated capital during the dot-com bubble. The revolution in information technology provoked further capital mobility across markets. Despite the positive advancements, it seems the vectors are not considerably stable until the second half of the 2000s.

The prevalent high uncertainty during the period 1980 to mid-1990, as the empirical

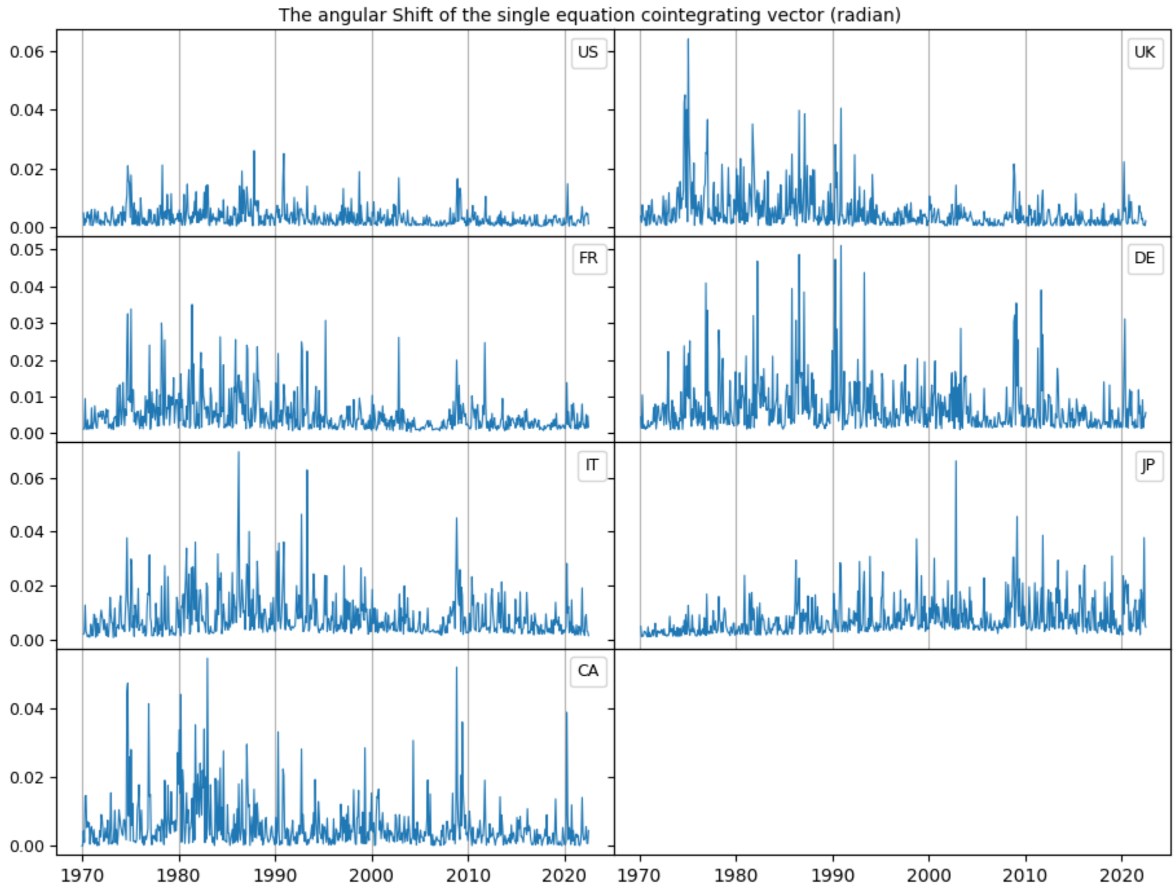


Figure 4.12: The angular shift of the cointegrating vector of the 7-variate cointegration model. The shift in two consecutive cointegrating vectors is expressed as the angle between two vectors in terms of radians.

investigation strongly confirms, seems to have calmed down in particular after the 2000s. The possible political reasons for retracting the rifts in market co-movement comprise a number of momentous political strategies and technological innovations including extensive liberalization of the markets, the constitution of the European Union, free movement of labor in the European Union, free capital flow across G7 countries, the introduction of the euro currency and the technological revolution in the information technology. The political reversals and the tightening political and economic ties among G7 countries paved the way for more integrated world economies and substantially mitigated the uncertainty associated with the economies.

The smaller segmentation during serious economic depressions in recent decades can be also attributed to the more active role of the central banks in taking crucial courses of action to bring the economic turmoil into control. The central banks' interventions during the stagflation of the 1980s were limited to the conventional monetary policy of increasing interest

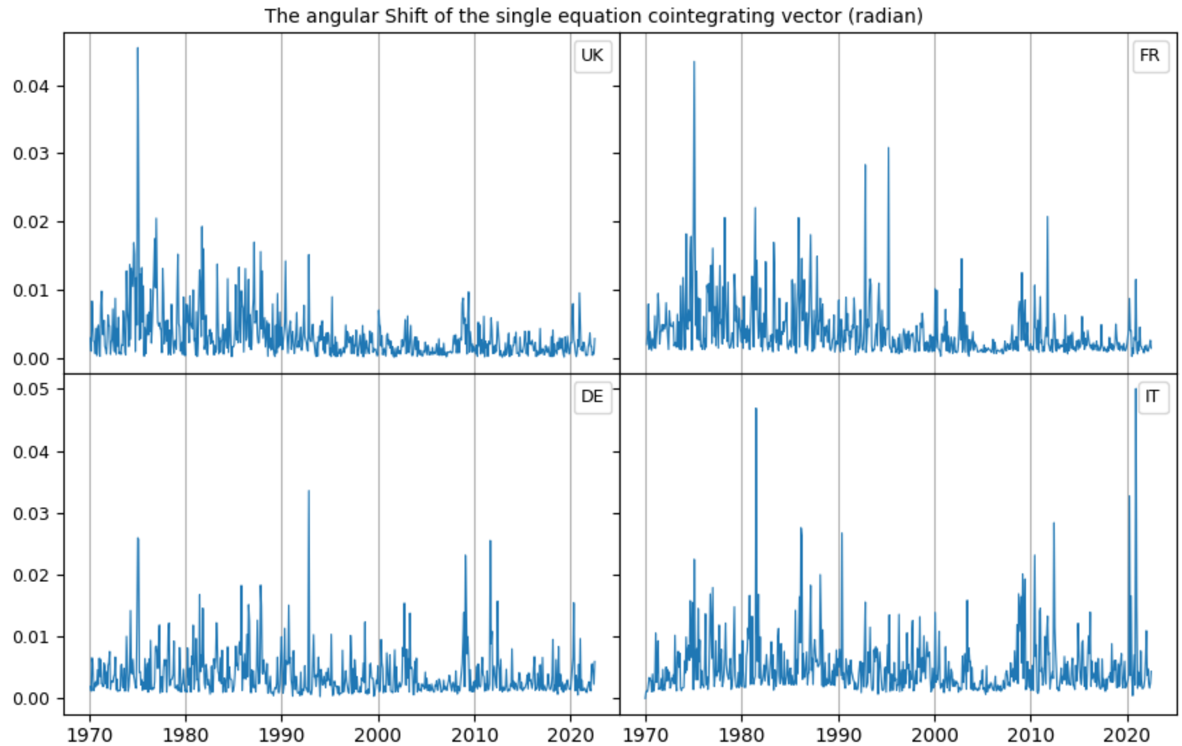


Figure 4.13: The angular shift of the cointegrating vector of the 4-variate cointegration model. The shift in two consecutive cointegrating vectors is expressed as the angle between two vectors in terms of radian.

rates to fight inflation. Higher interest rates spawn uncertainty in the market by creating a gloomy future for the aggregate demand. Overall, markets' co-movement suffered from the uncertain economic future and low globalization during the decade of 1980s. It seems that from 2000 onwards, the uncertainty associated with the interest rates vanished as a result of the more active role of the central banks in particular in taking expansionary policy of quantitative easing. This happened during the global financial crisis of 2008 when central banks of the G7 resorted to the quantitative easing policy to stimulate the economy. During covid-19 crisis, world central banks promptly reacted to the crisis by asset buying programs and pumping unprecedented volumes of liquidity into the economy.

4.5 Conclusions

The development of Johansen's estimation procedure of multivariate cointegration paved the way to investigate multi-variate long-run equilibrium relations among stock markets. State-space modelling of the VECM, though not formulated to estimate independent cointegrating vectors, can reveal the variation in the space of the time-varying cointegrating vectors. We analyzed the instability of the spanning space by examining the shifts in the normal vector of the space. As Johansen's λs proxy the strength of the equilibrium relations, we introduced time-varying λs to investigate the time-varying strength of the equilibrium relations. In an effort to further support our empirical conclusions, we also estimated the two existing approaches in the literature including the single equation cointegration model developed by state-space modelling and the smoothly varying approach developed by the Chebyshev time polynomials.

The influential events in favor of the markets' long-run co-movement can be cited as the momentous economic policies to deregulate markets and liberalising capital flow leading to more integrated capital markets. The pivotal period of the economic policy reversal from the deficit policy and state-owned economies to the liberalised economies during the 1980s ensues high variability of the markets' co-movements. The availability of capital in the aftermath of the right-wing economic policies of the states and the developments in information technology and free cross-border information flow, higher foreign trade liberalization, and easier international financial transactions have strengthened markets' co-movements.

The more active role of the central banks has overshadowed the uncertainty arising from the economic crisis. The extensive expansionary measures in terms of quantitative easing during the global financial crisis have dominated the gloomy outlook and motivated investment and therefore it has limited the extent to which markets' co-movements are destabilised during the recession.

Chapter 5

Purchasing power parity and uncovered interest parity revisited: Evidence from G7 exchange rates using a dynamic joint model

Hamid Babaei[1]

[1] *HEC Liège, Management School-University of Liège, 14 rue Louvrex, B-4000 Liège, Belgium*

Abstract

This chapter investigates purchasing power parity (PPP) between the G7 countries after the advent of the euro currency. A dynamic joint model is introduced to decompose the real exchange rate (RER) into two components, based on which the prevalence of PPP is discussed. Analysis of the evolution of price convergence reveals that the UK parities with Eurozone countries are steadily strengthening, whereas the parities with Japan undergo a fragile process of price convergence. The impacts of the monetary policy uncertainty (MPU) and economic uncertainty are found to be significant on the RER. A structural VAR on the IRD and uncertainty indicators confirms that deviations from PPP are caused by the IRD and uncertainty.

JEL classification: C32, F15, F31, F65

Keywords: Purchasing power parity, Uncovered interest parity, State-space, SVAR

5.1 Introduction

The theory of purchasing power parity (PPP) states that if there are price discrepancies between countries, then according to the Law of One Price (LOP) nominal exchange rates and prices will move to eliminate the arbitrage opportunities. Given that most empirical studies have evidenced a lack of support for PPP particularly in the post-euro period, a wide range of the works in the literature add ad hoc hypotheses to test for the stationarity of the real exchange rate (RER). We contribute to the literature by supporting the fact that the permanent shocks to the RER during the post-euro period are related to the interest rate differential (IRD) and a number of uncertainty indicators including economic policy uncertainty (EPU) and monetary policy uncertainty (MPU). We also find that domestic EPU leads to relative currency depreciation in most of the parities.

Though the free mobility of goods and the LOP form the theoretical underpinning of PPP, free capital flow for speculative purposes can also considerably impact the exchange rate and undermine the prevalence of the PPP. Uncovered interest parity (UIP) postulates a forward-looking theory in the capital market that links the expected and spot exchange rates to IRD and can be used to formulate a relation incorporating both goods and capital markets. The nexus between PPP and IRD is also theoretically discussed in light of the monetary theory of exchange rates by MacDonald (2007) where the exchange rate is related to the differentials in economic output, money supply, and interest rate. A number of empirical works have also focused on the interaction between PPP and IRD to solve the puzzle of the RER evolution (see for example Juselius, 1995; Taylor and Sarno, 2004 and Dahlquist and Pénasse, 2022). We introduce a new dynamic model that decomposes the RER into two components. The first component depends on IRD and the aforementioned uncertainty indicators and the second component is an autoregressive (AR) process. The stationarity of the second component ensures that the stochastic trend of the RER depends on the IRD, uncertainty variables, and possibly a number of unknown variables. A few works use time-varying modeling to study the speed of adjustment of the RER (see for example Cho and Doblas-Madrid, 2014 and Rabe and Waddle, 2020). We investigate the gradual tendency of price convergence by estimating the time-varying AR coefficient of the second component of the RER. We find that globalisation has led to stronger price convergence over time.

The empirical efforts to find evidence for price convergence after the introduction of the euro are mixed. For example Rogers (2007) and Wu and Lin (2011) find that during post-Euro, LOP fails to hold. A few works focus on the nonlinearities and structural breaks in the mechanism of price convergence. Kanas (2006) uses a regime-switching model to examine a regime-dependent stationarity test of the RER. Yang and Zhao (2020) apply quantile nonlinear unit root test to the PPP in the period 2000-2018 and find stronger support for price convergence. The weak empirical evidence on PPP in the pre-euro decades during which exchange rates were forced to fluctuate within a band (see for example Christidou and Panagiotidis, 2010 and Koedijk et al., 2004), encouraged researchers to relax the PPP by having resort to cointegration (see for example Patel, 1990; Falk and Wang, 2003 and Ma et al., 2017). We believe that a dynamic approach accounting for money flow for the two motives of goods and investment transactions, which is also augmented with economic uncertainty variables, disentangles the unit root component from the stationary component of the RER and is illustrative of the permanent shocks to the RERs.

The bias in the conclusions of papers arising from the inclusion of the managed exchange rate mechanisms enforced by states during the decades before the introduction of the euro (e.g. snake in the tunnel) puts forward an anomaly associated with many studies and questions their conclusions. The sample used by most works including the recent ones, e.g. Christidou and Panagiotidis (2010) and Rabe and Waddle (2020) covers pre-euro periods and is also subject to sample bias. The work by Yang and Zhao (2020) uses the sample from 2000 through 2018 whereas the process assumed for the RER combines a smooth transition process having a middle and an outer regimes with a process for the errors which is a distributed lag model of a stationary process and is ultimately estimated by a quantile regression, making the approach ad hoc. We aim to investigate PPP using simpler hypotheses on the evolution of the RERs during the post-euro for which the extant literature has not provided supporting evidence.

Our first motivation for conducting this work is that most of the empirical works involve the managed currency periods which leave a bias in the statistical inference. Following the collapse of the Bretton Woods, the European countries agreed on maintaining limiting bands for the exchange rates obtained from the price indices to establish monetary stability. For instance, the Exchange Rate Mechanism (ERM) was a system in which the currencies were kept within a band of 2.5% around a central parity. The literature evidencing PPP within European countries covers these managed currency periods (see for example Koedijk et al., 2004 and Rabe and Waddle, 2020). On the other hand, the empirical works focusing on price convergence of the industrialised economies after the advent of the euro use ad hoc modeling approach which involves overcomplication of the processes and thus are less realistic (for example see Yang and Zhao, 2020).

Secondly, being the most developed economies of the world, having liberalised capital accounts, and enjoying a high degree of integration, we raise the question if the exchange rates of the G7 countries can be substantially impacted by speculations arising from interest rate differentials.

The contribution of this paper is twofold. First, departures from the PPP are related, through joint modeling with the UIP, to the IRD and the uncertainty indicators, and price convergence is shown to hold in a number of the parities. We find that IRD and economic policy as well as the monetary policy uncertainties are the sources of permanent shocks to the RER. It is also shown that the price convergence is getting stronger over time in a number of the parities. The parities between European countries are found to get stronger over time in terms of the mean reversion speed. The parities with Japan are found to be more susceptible to disruptions in the evolution of the mean reversion speed. Our second contribution concerns the sample bias in most of the empirical works. As noted by Rabe and Waddle (2020), the results of the investigations on PPP are susceptible to data samples. A major shortcoming of Rabe and Waddle (2020) is that their sample covers four decades prior to 2000 during which the enforced control systems on the exchange rates forced countries to maintain a narrow band around a certain parity. Thus their approach produces results with a low half-life of RER convergence. This study uses the data after the introduction of the euro during which the exchange rates are floating without controls put in force by central banks. A structural vector autoregressive (SVAR) sheds clearer light on the short-run and the long-run interaction between the UIP and PPP by showing that the structural shocks to IRD leave long-run effects on the RER.

This paper is organised as follows. In section 5.2 the literature review is discussed. Section 5.3 involves the methodological approaches including the dynamic joint modeling and the SVAR model. Section 5.4 presents our empirical findings and section 5.5 concludes.

5.2 Literature review

The empirical investigations on PPP appear in abundance with a broad range of methodological approaches. The works using the most classical unit root tests end up with mixed conclusions. Koedijk et al. (2004) apply univariate unit root tests to the RER of the Eurozone countries and world major currencies. Overall, they find weak evidence in favor of PPP between European and non-European countries. Using the century-long data of the twenty most industrialised countries, Taylor (2002) finds support for the stationarity of the RERs of the parities with the US.

A range of the papers use cointegration to examine price convergence. Patel (1990) emphasise the use of cointegration-based approaches to study PPP. Their argument relies on the monetarism standpoint in which the permanent shocks to prices are of monetary sources. Their empirical findings on the major industrialised economies do not support PPP in the majority of the pairs ranging from 1974 to 1986. Mark (1990) address PPP in the industrialised economies over the period of 1973-1988 and find unfavorable evidence for PPP using cointegration tests. Falk and Wang (2003) investigate PPP between the United States and either of the most developed economies of the world using residual-based and likelihood-ratio-based cointegration tests. They find sufficient evidence of PPP over the period of 1973-1999. Ma et al. (2017) use quantile cointegration to investigate the PPP between the United States and either of the three major Asian economies of China, Japan, and South Korea. The quantile cointegration reveals a significant relation, whereas the cointegration approach and the unit root test on the RERs do not support the hypothesis of PPP.

A few works introduce time-varying cointegration models and find evidence of time-varying PPP for the developed economies of the world as the empirical illustration. Bierens and Martins (2010) and Martins (2018) reject the null hypothesis of time-invariant cointegration against the alternative of time-varying cointegration in the vector composed of the price indices and the nominal exchange rate. da Fonseca et al. (2019) find strong evidence of time-varying cointegration for most of the pairs composed of the US and either of the developed economies including the G7 countries. Despite the evidence in favor of the significant time-varying PPP, the variables explaining the time-varying nature of the PPP remain unexplored. Assuming the expected exchange rate to equal the price difference, Juselius (1995) formulates a cointegration-based joint model of PPP and UIP, and investigates the prevalence of PPP between Germany and Denmark.

However, the major body of the literature investigates the evolution of the RER mostly by applying variants of unit root tests. The approaches range from sub-period analysis to the models accounting for structural breaks. Kanas (2006) implements the Markov-switching regression of the augmented Dickey-Fuller (ADF) equation. He applies the regime-switching model to the RER obtained from the century-long data of 16 major economies. Although he does not provide sufficient facts to support filtered probabilities of the stationary and non-stationary regimes, he concludes that during the recent period of floating exchange rates until 1998, the RERs of most of the countries appear to be non-stationary for a large part of the period. Golit et al. (2019) formulate a non-linear autoregressive distributed lag (ARDL) model with structural breaks and find a significant relation between exchange rates, price difference, and IRD of the G7 countries. Yang and Zhao (2020) develop a new unit root test in which the RER follows a smooth transition process having two regimes. A quantile regression evidences favorable evidence in support of PPP for the Eurozone, Japan, the UK, and Korea.

Gradual convergence of prices is examined by several works for the data covering both the pre-euro and post-euro periods. Cho and Doblas-Madrid (2014) focus on trade intensity as a measure affecting the mean reversion of the RER. Using an exponential smooth transition function, they find that the average half-life of RER for high trade intensity countries is lower than the low trade intensity ones. Huang and Yang (2015) examine PPP by conducting a panel unit root test on the RER of the European economies after the introduction of the euro currency. The analysis of the mean reversion properties of the RER reveals that the half-lives of the RER reversion during the post-euro are substantially higher than the earlier sub-periods. Using long-run data from 1960, Rabe and Waddle (2020) explore the adjustment speed of the RER to the long-run mean. They introduce a time-trending panel regression model to investigate the half-life associated with the reversion of the RER to the mean. They find that half-lives of the deviations from PPP fall by two years over the last five decades.

In a number of the works, the introduction of euro currency is brought under scrutiny and PPP is examined in the sub-periods preceding and succeeding the advent of the single European currency. Wu and Lin (2011) apply panel unit root test to the RER in two sub-periods before and after the introduction of the euro currency and find that PPP holds before the introduction of the Euro, whereas it does not hold afterward. Christidou and Panagiotidis (2010) perform six tests including univariate as well as panel unit root tests to examine PPP between the US and fifteen European countries over different sub-periods ranging from 1973 until 2009 the last of which covers the post-euro period. Their empirical investigation fails to find support for PPP for most of the countries.

Several works focus on the interaction of the RER and IRD as an indicator that drives the level of the RER. Dahlquist and Pénasse (2022) introduce a state-space model in which the RER depends on the IRD and an unobservable component stands for the missing risk premium. This approach implies a decomposition of the RER into two functionals the IRD and the unobservable risk premium. They find that the missing risk premium explains much of the variation in the RER of the G10 countries from 1976 to 2020. Our approach differs from this model in a number of ways. First, in line with Juselius (1995), we link the PPP and the UIP by assuming that the expected exchange rate converges in the long run to the price difference in the two countries. Dahlquist and Pénasse (2022) relate the exchange rate to the IRD through an investing strategy that borrows in the domestic currency and pays off through the foreign currency appreciation and the IRD. In our model, the varying intercept that depends on the IRD (with a slope of unity) and uncertainty variables captures the effect of the speculative flow of money. Second, unlike Dahlquist and Pénasse (2022), the IRD in our model is not forced to follow an AR(1) process. This is a less restrictive assumption on the data-generating process of the underlying variables. Finally, the IRD appears with a negative coefficient in their model which contradicts UIP.

5.3 Methodological approaches

5.3.1 Dynamic joint modeling

Let $p_{1,t}$ and $p_{2,t}$ denote the price indices in domestic and foreign countries transformed by the natural logarithms and let $s_{12,t}$ denote the natural logarithm of the spot exchange rate in terms of the domestic currency per unit of the foreign currency. The RER is defined as follows:

$$q_t = p_{1,t} - p_{2,t} - s_{12,t}. \tag{5.1}$$

If PPP holds, then q_t should be stationary. As there is a lack of empirical evidence for the stationarity of q_t in particular after 2000, in line with Juselius (1995), our premise is that the RER evolution is partly driven by the interest rate differentials. This can be of paramount importance as more capital availability over the recent decades has led to a voluminous share of speculative money compared to the total money in circulation. A factor that shifts investment opportunities is the interest rate. Both UIP and the quantity theory of money relate the exchange rate to the interest rate. The functional form of the quantitative theory of money that relates money volume, velocity, price, GDP, and interest rate all together needs to be presumed which may impact results. Unlike the quantitative theory of money, UIP states that the compounded interest rate difference should equal the relative change in the exchange rate. It can be expressed by:

$$i_{1,t} - i_{2,t} = E_t[s_{12,t+1}] - s_{12,t}, \tag{5.2}$$

where i_{1t} and i_{2t} denote the compounded interest rates denominated respectively in the domestic and foreign currencies over the period $(t, t+1)$, and $E_t[.]$ represents the expected value at time t. In order to link the two parities, in line with Juselius (1995), we assume that the expected value of the exchange rate in the long run is formed according to the price differences. This is a fundamental assumption that is also used by Dahlquist and Pénasse (2022) who assume that PPP holds in the long run which implies that the expected long-run exchange rate is constant and equals the actual price difference in the two countries. Combining Equation (5.1) and (5.2) in light of the assumption $E_t[s_{12,t+1}] = p_{1,t} - p_{2,t}$ yields the joint static formula:

$$i_{1,t} - i_{2,t} = p_{1,t} - p_{2,t} - s_{12,t}. \tag{5.3}$$

Equation (5.3) represents the static form of the joint PPP and UIP parities and implies that the RER equals the IRD. It should presumably hold in the long run as the expected exchange rate is assumed to converge in the long run to price difference. Thus the underlying variables evolve so that in the short run they deviate from Equation (5.3).

In the dynamic formulation, we decompose the RER into two components both of which are assumed to evolve as unobservable state-space variables. Equation (5.4a) represents a decomposition of the RER into two components both of which have an AR component. This decomposition disentangles the effects of the speculative money from the money circulating in the goods market. The dynamic specification of the RER in equations (5.4a) to (5.4c) involves the IRD as a proxy for speculative demand of the domestic currency relative to foreign currency. The higher the value of the IRD, the more demand for the domestic currency vis-à-vis the foreign currency. According to UIP, the expected exchange rate is positively associated with the IRD. The first component s_t depends on the IRD and the lagged s_t which translates into a process evolving by the lagged values of the IRD and a moving average (MA) process. Assuming that IRD is integrated, the stationary MA process implies that s_t converges in the long run to the IRD. This component represents a portion of the RER that is evolving in the long run by the speculative flow of money. The second component w_t is assumed to follow an AR process. This component is augmented by no variable. However, it evolves by a set of latent factors that impact the prevalence of PPP. The influence of these factors over time is also worthy of investigation because it sheds light on the impacts of globalisation on PPP. The state-space representation of the dynamic formulation can be expressed as follows:

$$q_t = \mu + s_t + w_t, \tag{5.4a}$$

$$s_t = \phi s_{t-1} + (i_{1,t} - i_{2,t}) + \eta_t, \tag{5.4b}$$

$$w_t = \theta w_{t-1} + \sum_{k=1}^{p} \delta_k \Delta w_{t-k} + \epsilon_t, \tag{5.4c}$$

where $\eta_t \sim N(0, \sigma_\eta)$, $\delta_t \sim N(0, \sigma_\delta)$, q_t is the RER, μ is a constant, s_t is the first component of the deviation emanating from speculations on interest rates, w_t is the error term defined as an AR process given by Equation (5.4c), p is the number of the lagged Δw_t in Equation (5.4c), and δ_t and ϵ_t are jointly i.i.d. random variables. ϕ, ψ, θ and δ_ks are scalars, $i_{1,t}$ and $i_{2,t}$ are the domestic and foreign interest rates.

The recursive substitution of Equation (5.4b) yields:

$$s_t = \phi^t s_0 + \sum_{j=0}^{t} \phi^j (i_{1,t-j} - i_{2,t-j}) + \sum_{j=0}^{t} \phi^j \eta_{t-j}. \tag{5.5}$$

If $|\phi| < 1$, then IRD and s_t will be of identical integration order. A $|\phi| > 1$ implies that s_t will be of higher integration order than the IRD. If the IRD is integrated and if $|\phi| < 1$, then the unit root component of s_t is driven by the concurrent as well as the lagged values of the IRD. In this case, if $|\theta| < 1$, then the stochastic trend of the RER is driven by the concurrent and lagged values of the IRD. In the case of $|\phi| = 1$, s_t will be integrated regardless of the integration order of IRD. In this case, the prevalence of PPP can be inferred provided that $|\theta| < 1$. This will be discussed later in this section.

We also assume that θ varies as w_t is getting far away from the long-run equilibrium level. The smooth transition model provides an effective way to identify regime changes in the mean reversion process of the w_t. We allow for two regimes using a smooth transition autoregressive process to identify the deterministic changes in the reversion to the mean.

$$w_t = \theta w_{t-1} + \sum_{k=1}^{p} \delta_k \Delta w_{t-k} + \left[\mu^* + \theta^* w_{t-1} + \sum_{k=1}^{p} \delta_k^* \Delta w_{t-k} \right] G(w_{t-1}; \gamma, c) + \epsilon_t. \tag{5.6}$$

We choose the lagged w_t as the transition variable. We also let the transition be governed by the popular exponential function:

$$G(w_{t-1};\gamma,c) = 1 - e^{-\gamma(w_{t-1}-c)^2}, \tag{5.7}$$

where c denotes the location parameter and $\gamma > 0$ is the transition parameter. Equations (5.6) and (5.7) represents an exponential STAR (ESTAR) specification on w_t. The exponential transition function is used by Omay et al. (2020) and Cho and Doblas-Madrid (2014) to investigate PPP.

We extend the Kalman filter applied by Eroğlu et al. (2022) to estimate the dynamic model in equations (5.4) in which w_t bears regime changes as given by (5.6). Defining the state vector $\boldsymbol{\xi}_t$ by

$$\boldsymbol{\xi}_t = \begin{cases} [s_t' \quad w_t \quad w_t]' \quad if \quad p=0 \\ [s_t' \quad w_t \quad \Delta w_t \quad \dots \quad \Delta w_{t-p+1} \quad w_t \quad \Delta w_t \quad \dots \quad \Delta w_{t-p+1}]' \quad if \quad p>0 \end{cases} \tag{5.8}$$

the RER can then be expressed as:

$$q_t = \mu + \begin{cases} \mathbf{1}'\boldsymbol{\xi}_t \quad if \quad p=0 \\ [\mathbf{1}' \quad 0 \quad 0 \quad \dots \quad 0]\boldsymbol{\xi}_t \quad if \quad p>0 \end{cases} \tag{5.9}$$

where $\mathbf{1}' = [1 \quad 1 \quad 0]$ and the new state Equation becomes $\boldsymbol{\xi}_t = \boldsymbol{A}_t + \boldsymbol{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{Q}\boldsymbol{E}_t$ with $\boldsymbol{A} = [(i_{1,t} - i_{2,t}) \quad \mu^* G(.) \quad \mu^* G(.)]'$, $\boldsymbol{E}_t = [\eta_t \quad \epsilon_t \quad \epsilon_t]'$. The matrices $\boldsymbol{F}$ and $\boldsymbol{Q}$ are given by the following equations:

$$\boldsymbol{F} = \begin{cases} \begin{bmatrix} \phi & 0 & 0 \\ 0 & \theta & \theta^* G(.) \\ 0 & \theta & \theta^* G(.) \end{bmatrix} & if \quad p = 0 \\ \begin{bmatrix} \phi & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \theta & \delta_1 & \dots & \delta_p & \theta^* G(.) & \delta_1^* G(.) & \dots & \delta_p^* G(.) \\ 0 & \theta - 1 & \delta_1 & \dots & \delta_p & (\theta^* - 1) G(.) & \delta_1^* G(.) & \dots & \delta_p^* G(.) \\ 0 & 0 & 1 & & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & 1 & 0 & 0 & \dots & 0 \end{bmatrix} & if \quad p > 0 \end{cases} \tag{5.10}$$

$$\boldsymbol{Q} = \begin{cases} I_2 \quad if \quad p = 0 \\ \begin{bmatrix} I_2 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix} & if \quad p > 0 \end{cases} \tag{5.11}$$

where I_2 denotes (2×2) identity matrix.

The state Equation (5.4b) involves an IRD term with a restricted slope to unity. The model can also be relaxed by assuming an unknown slope ψ. We augment the first component of the RER in Equation (5.4b) by the EPU of the countries in the parity. The uncertainty associated with the US monetary policy is an indicator found to impact the exchange rates. We also include this uncertainty indicator in the state Equation (5.4b). In the augmented dynamic joint model, the RER evolves as follows:

$$s_t = \phi s_{t-1} + \psi(i_{1,t} - i_{2,t}) + \boldsymbol{\beta}' \boldsymbol{U}_t + \eta_t, \tag{5.12}$$

where $\boldsymbol{\beta}$ is a (3×1) vector of constants and $\boldsymbol{U}_t' = [log(EPU_{1,t}) \quad log(EPU_{2,t}) \quad log(MPU_t)]$ forms the vector composed of the uncertainty indicators.

The two parameters ϕ and θ in equations (5.4b) and (5.4c) are of special importance because these parameters determine price convergence in two countries. The time-varying cointegration model introduced by Eroğlu et al. (2022) and the role of θ in concluding the existence of cointegration can be extended to the dynamic model in equations (5.4). In an unaugmented decomposition model where s_t follows an AR(1) process, a stationary w_t implies the existence of PPP in the long run. If $\theta = 1$, then w_t follows a unit root process and the null hypothesis of PPP is not rejected. If $|\theta| < 1$ and $\phi = 1$ with a small σ_η, accordingly we conclude a smoothly varying PPP. If $|\theta| < 1$ and $\phi < 1$ with $\sigma_\eta \neq 0$, we conclude in favor of a stochastically varying PPP. This can be further extended to the case of an augmented s_t process in Equations (5.4b) and (5.12).

5.3.2 Structural factorisation

Cheung and Lai (2000) assume an ARIMA model for the RER of the four European countries of the G7 and analyze impulse response function in an AR model. Their results are also subject to the policy of the ERM as their data covers the 1973-1996 period. Using 14 currencies over the period 1980-2008, Cho and Doblas-Madrid (2014) fit a smooth transition AR process to the RERs using an exponential function as the transition process. They find that generalised impulse response functions for parities with high trade intensity decay faster. A more general AR process for the variables of interest in this study can corroborate our findings by the dynamic model in (5.4). The decomposition model (5.4) and the augmented s_t given by (5.12) reveal no information about the impact of innovations in the IRD or the uncertainty variables on the RER. The impact of structural shocks on the evolution of the RER is informative in particular they can shed more light on the short-run and long-run evolution of the RERs under study. An SVAR analysis can reveal the impact of exogenous shocks to the EPU and IRD on the RER. To this end, we estimate the following structural model:

$$\mathbf{\Gamma} X_t = \mathbf{\Gamma}_0 + \sum_{i=1}^{l} \mathbf{\Gamma}_i \boldsymbol{X}_{t-i} + \boldsymbol{B}\boldsymbol{u}_t, \tag{5.13}$$

where $\boldsymbol{X}_t = [EPU_{1,t} \quad EPU_{2,t} \quad MPU_t \quad (i_{1,t} - i_{2,t}) \quad q_t]'$, $\mathbf{\Gamma}_0$ and $\boldsymbol{u}_t$ are (5×1) matrices and $\mathbf{\Gamma}$, $\mathbf{\Gamma}_i$s and $\boldsymbol{B}$ are (5×5) matrices. $E[\boldsymbol{u}_t \boldsymbol{u}_\tau'] = I$ when $t = \tau$ and $E[\boldsymbol{u}_t \boldsymbol{u}_\tau'] = 0$ otherwise.

As indicated in Baker et al. (2016), the uncertainty indicators are exogenous compared to other volatility indices (e.g., VIX). This assumption implies the following restricted VAR model in Equation (5.14) in which the uncertainty variables follow an AR(l) process:

$$X_t = \boldsymbol{A_0} + \sum_{i=1}^{l} \boldsymbol{A}_i \boldsymbol{X}_{t-i} + \boldsymbol{e}_t, \tag{5.14}$$

where $\boldsymbol{A_0}$ and $\boldsymbol{e}_t$ are (5×1) matrices and $\boldsymbol{A_i}$s are (5×5) matrices. We further add the restrictions $[\boldsymbol{A}]_{ij} = 0$ where $(i, j) \in \{(i, j) \mid 2 \leq i \leq 4 \wedge i = j\}$ to ensure that the uncertainty variables are not impacted by the RER or the IRD.

5.4 Empirical analysis

5.4.1 Data

We use monthly data of the G7 countries from Jan-1999 to Nov-2022. The consumer price indices (CPI) are downloaded from Federal Reserve Bank of St. Louis. All the CPI indices include all item indices. The base year of all the CPI indices is 2015. The exchange rates and the interest rates are downloaded from the Datastream. The interest rates are the market yield on the 1-year treasury bill of the US, Canada, UK, France, Germany, Italy, and Japan. We also downloaded the EPU of the G7 countries and the MPU of the US from the database of economic policy uncertainty [1].

5.4.2 Kalman smoothing

As the keyword-based policy uncertainty indices contain noise, we use smoothed indices in our empirical investigation. First, we fit the local level model in Equation (5.15) to each component of $\boldsymbol{U}_t$. Then, a Kalman smoother is applied to obtain smoothed values of the indices. The local level model on uncertainty variables is given by:

$$U_{i,t} = \pi_{i,t} + \eta_{i,t}, \tag{5.15a}$$

$$\pi_{i,t} = \pi_{i,t-1} + \delta_{i,t}, \tag{5.15b}$$

[1] https://www.policyuncertainty.com/

where $i = 1, \ldots, m$, $U_{i,t}$ denotes the i-th component of $\boldsymbol{U}_t$, $\pi_{i,t}$ is a scalar, $\eta_{i,t}$ and $\delta_{i,t}$ are jointly i.i.d. random variables with $\eta_{i,t} \sim N(0, \sigma_{\eta_i})$, $\delta_{i,t} \sim N(0, \sigma_{\delta_i})$. The smoothed estimate $\hat{\pi}_{i,t|T} = \hat{E}(\pi_{i,t} \mid \boldsymbol{\mathcal{U}}_{i,T})$ of the local level model in Equation (5.15) is estimated conditional on the entire sample $\boldsymbol{\mathcal{U}}_{i,T} = (U_{i,1}, \ldots, U_{i,T})$ of the uncertainty variables. Following Hamilton (2020), the final smoothed value $\hat{\pi}_{i,T|T}$ is set to the last filtered value of $\hat{\pi}_{i,t|t-1}$. Then the generated sequence of matrices $\boldsymbol{J}_{i,t} = \boldsymbol{P}_{i,t|t}\boldsymbol{I}_{1\times 1}\boldsymbol{P}^{-1}_{i,t+1|t}$ [2] are used in the formula $\hat{\pi}_{i,T-1|T} = \hat{\pi}_{i,T-1|T-1} + \boldsymbol{J}_{i,T-1}(\hat{\pi}_{i,T|T} - \hat{\pi}_{i,T|T-1})$ where $\boldsymbol{P}_{i,t+1|t}$ denotes the variance-covariance matrix of the state variable. Proceeding backward using the same formula, the set of $\hat{\pi}_{i,t|T}$ is obtained for $t = 1, \ldots, T$.

Figure 5.1 displays the smoothed uncertainty indicators. The graphs exhibit an increasing uncertainty until the end of the sluggish economy in 2003. Following a declining period of economic uncertainty, the EPU indicators show an upward trend until 2022. The US MPU seems to be stable which realises growing periods interrupted by periods of decline.

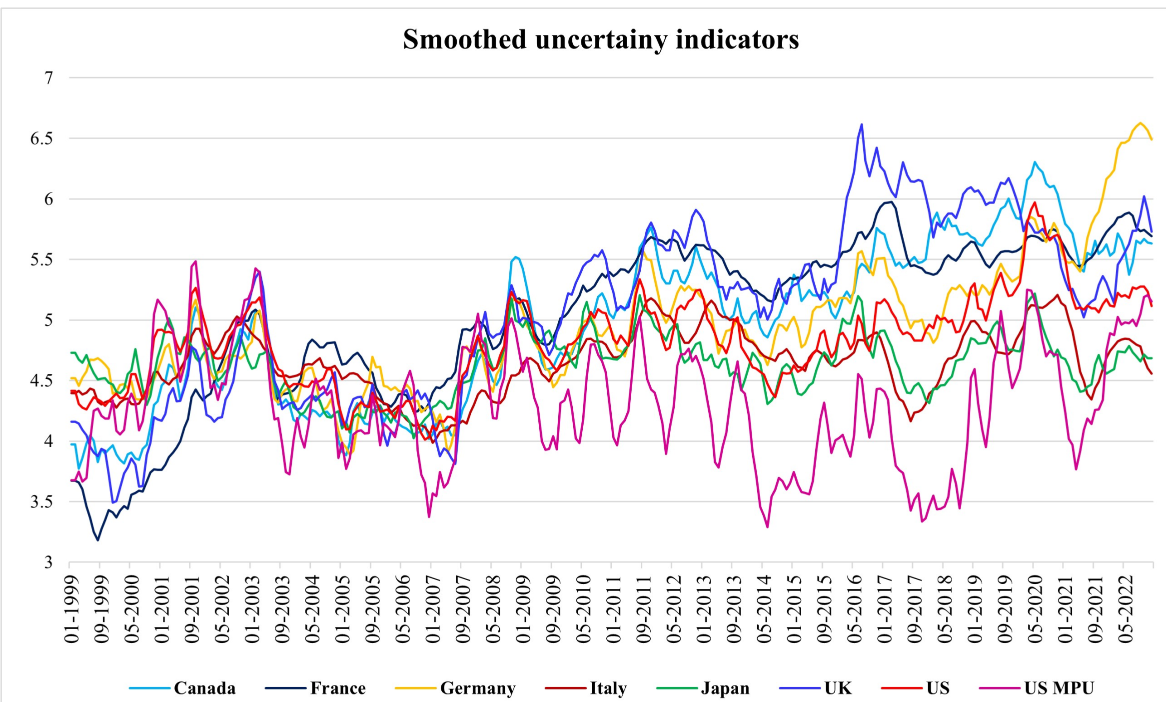


Figure 5.1: The EPU of G7 countries and the US MPU. All the uncertainty indicators are transformed by natural logarithms. The indicators are smoothed by Kalman smoother of the local level model.

[2]matrix $\boldsymbol{F}$ as explained in Hamilton (2020) equals identity matrix in the local level model.

5.4.3 Static PPP

The three countries of the Eurozone use the same currency and are not mutually considered in parities. We identify 18 parities between G7 member countries. Various unit root tests are extensively used in the literature to test the stationarity of the RERs. Table 5.1 shows results of the two well-known tests of Augmented Dickey-Fuller (ADF) (Dickey and Fuller, 1979), Phillips-Perron (PP) (Phillips and Perron, 1988) on 18 bilateral parities. The Canadian parity with three Eurozone countries is mean reverting in 5% significance level. The PP test rejects the null hypothesis strongly at 1% significance level. However, neither the ERS's feasible point optimal test nor NG-Perrons modified point optimal tests do not reject the null of unit root for these parities.

We also apply two other commonly used unit root tests developed by Elliot-Rothenberg-

Table 5.1: Stationarity test of the RERs using ADF and PP tests

	Augmented Dickey-Fuller test				Phillips–Perron test		
	t-statistic	p-value	Lag lengths	BIC	t-statistic	p-value	Bndwidth
US JP	-0.24	0.93	1	-1313.23	-0.01	0.96	2
US CA	-1.48	0.54	1	-1400.02	-1.49	0.54	7
US UK	-0.92	0.78	1	-1338.37	-1.05	0.73	6
US FR	-1.10	0.71	1	-1310.18	-1.10	0.72	7
US DE	-1.54	0.51	1	-1308.52	-1.46	0.55	6
US IT	-1.35	0.61	1	-1310.78	-1.22	0.67	6
JP CA	-1.47	0.55	1	-1161.51	-1.57	0.50	5
JP UK	-1.82	0.37	1	-1171.50	-1.71	0.42	2
JP FR	-1.57	0.50	1	-1225.10	-1.39	0.59	2
JP DE	-1.34	0.61	1	-1207.39	-1.22	0.67	3
JP IT	-1.37	0.60	1	-1222.37	-1.20	0.67	2
CA UK	-1.95	0.30	0	-1334.27	-2.03	0.28	2
CA FR	-3.06	0.03	0	-1319.14	-3.33	0.01	4
CA DE	-3.63	0.01	0	-1319.84	-3.80	0.00	2
CA IT	-3.32	0.01	0	-1316.46	-3.58	0.01	2
UK FR	-1.57	0.50	0	-1415.28	-1.57	0.49	0
UK DE	-1.52	0.53	0	-1398.58	-1.66	0.45	4
UK IT	-1.43	0.57	0	-1406.14	-1.55	0.51	3

The augmented Dickey-Fuller and Phillips–Perron tests are applied to the real exchange rates of the G7 countries. The lag length is automatically specified using the Bayesian information criterion. Bandwidth is specified using the Newey-West method and Bartlett kernel is used as spectral estimation method.

Stock (hereafter ERS) (Elliott et al., 1996) and NG-Perron (Ng and Perron, 2001). The ERS test outperforms ADF and PP tests in terms of power when the AR coefficient in the underlying process is close to unity. The unit root test in Table 5.1 motivates us to estimate ERS and NG-Perron test. The ERS test assumes a specification of the form $q_t = d_t + u_t$ where d_t is a deterministic term and $u_t = \rho u_{t-1} + \nu_t$ with $\nu_t \overset{iid}{\sim} N(0, \sigma)$. ERS show that the test statistic is optimal if the point alternative $\bar{\rho} = 1 + \bar{c}/T$ is chosen as an alternative to unity[3]. The alternative hypothesis in Table 5.2 is then $\rho = 1 - 7/287 = 0.9756$. Following ERS, we obtain quasi-differenced values of q_t and d_t using $1 - \rho L$ operator. Define $S(\rho)$ as sum of squares from the regression of $(1 - \rho L)q_t$ on $(1 - \rho L)d_t$. The point optimal test statistic is defined as $P_T = (S(\bar{\rho}) - \bar{\rho}S(1))/\hat{\lambda}^2$ where $\hat{\lambda}^2 = T^{-1}\sum_{t=1}^{T} \hat{\nu}_t^2$. The NG-Perron test involves ERS specification with a modified version of PP's Z test which is robust to size distortions. Despite the fact that the more classical tests in Table 5.1 evidence stationarity of three RERs, the two tests in Table 5.2 do not show rejection of the null hypotheses for any parity. As Table 5.2 shows, these two tests do not reject the null hypothesis of a unit root in the RERs. In one single parity of Japan and the UK, the MSB test rejects the null hypothesis at 10% significance level.

5.4.4 Restricted vs. unrestricted decomposition of the RERs

Table 5.3 presents results of the decomposition model (5.4). If we apply DF test using the standard formula $std(\hat{\theta}) = \hat{\sigma}_\epsilon\sqrt{\hat{w}_t'\hat{w}_t}$, then the error term $\hat{w}_t$ of nine parities in Table 5.3 will be stationary. However, $\hat{\theta}$ may vary due to the variation in the unobservable state variable s_t. This motivates us to obtain standard deviations from the inverse of the Hessian matrix of the maximised likelihood function. An important issue concerning the decomposition in Equation (5.4a) is that the first varying component in the unaugmented model depends only on the IRD corresponding to the underlying countries in the parity. Applying the DF τ test to the second component in Table 5.3 supports the stationarity of the last three parities. Given that the IRD slope is restricted to unity, these findings imply that, unlike FMOLS results, IRD impacts the evolution of the RER. It turns out that PPP holds between Canada and the European countries after controlling for the IRD. The surprising finding is that the US prices do not converge to the prices in Canada. The parity between the US and European countries seems not to hold either.

[3]ERS show that when the specification involves a constant intercept with no trend, the optimal value of $\bar{c} = 7$ has 50% rejection rate of a wrong null hypothesis

Table 5.2: Stationarity test of the RERs using ERS and Ng-Perron tests

	ERS		Ng-Perron				
	P_T	lag	MZ_α	MZ_t	MSB	MP_T	lag
US JP	26.85	1	1.94	0.93	0.48	24.50	1
US CA	10.81	1	-2.43	-1.10	0.45	10.09	1
US UK	13.41	1	-0.51	-0.18	0.36	12.42	1
US FR	8.46	1	-2.63	-0.78	0.30	8.05	1
US DE	6.63	1	-4.16	-1.16	0.28	6.27	1
US IT	5.95	1	-4.51	-1.28	0.28	5.86	1
JP CA	30.00	1	0.57	0.33	0.58	26.15	1
JP UK	4.65	2	-4.70	-1.27	0.27*	5.76	1
JP FR	9.82	1	-2.32	-0.81	0.35	8.99	1
JP DE	10.64	1	-1.77	-0.61	0.34	9.80	1
JP IT	14.12	1	-1.00	-0.41	0.41	12.74	1
CA UK	39.51	0	0.14	0.10	0.72	33.74	0
CA FR	24.56	0	-0.25	-0.14	0.54	20.34	0
CA DE	23.12	0	-0.74	-0.41	0.55	18.82	0
CA IT	14.40	0	-1.58	-0.68	0.43	11.93	0
UK FR	6.09	0	-4.12	-1.42	0.35	5.97	0
UK DE	6.26	0	-4.08	-1.35	0.33	6.11	0
UK IT	11.77	0	-2.05	-0.91	0.44	10.96	0
1%	1.94		-13.80	-2.58	0.17	1.78	
5%	3.21		-8.10	-1.98	0.23	3.17	
10%	4.39		-5.70	-1.62	0.28	4.45	

Unit root test is performed using the Elliot-Rothenberg-Stock (ERS) and NG-Perron test methods. d_t is assumed to be $d_t = 1$. P_T denotes point optimal test statistic. MZ refers to the modified version of PP's Z test statistic. $MZ_\alpha = Z_\alpha + (T/2)(\hat{\alpha}-1)^2$ where Z_α is PP's Z test statistic and $\hat{\alpha}$ is the estimated AR coefficient. $MSB = (T^{-2}\sum_{t=1}^{t=T} q_{t-1}^2)^{1/2}/s^2$ where s^2 is the residual variance and $Z_t = MSB * Z_\alpha$. MP_T denotes the modified point optimal test statistic and has the same asymptotic distribution as P_T. *, ** and *** represent significance at the 10%, 5% and 1% levels, respectively. The bottom panel shows the critical values.

A prominent aspect of the findings of the unaugmented model in Table 5.3 is the restricted slope of the IRD to unity which settles the forward premium puzzle if the expected exchange rate converges to price difference as assumed in Equation (5.3). These findings endorse the presumption that growth in the expected exchange rate should equal the relative value of prices. The AR coefficient i.e. $\hat{\phi}$ is also found to be positive in the majority of the parities. This translates into the positive impact of the lagged IRD on the RER. The particular finding in Table 5.3 is that the unit root process of the s_t is due to the integrated process of IRD because $|\hat{\phi}| < 1$. This holds across all the parities. The parameter $\hat{\theta}$ in Table 5.3 is found to be very close to unity in all the parities. However, in seven parities it turns out to be significantly less than unity. $\hat{\sigma}_\eta$ is found to be very smaller than $\hat{\sigma}_\epsilon$ across all the parities. As indicated by Eroğlu et al. (2022), a small $\hat{\sigma}_\eta$ can be interpreted as a smoothly-varying s_t

provided that IRD is constant. This fact emphasises that the variation in IRD is a major source of permanent shocks in the RER.

We estimate the unrestricted model to further examine the impacts of the IRD on the evolution of the RERs. Table 5.4 presents results of unrestricted dynamic model (5.4). The results in this table are very similar to those in Table 5.3. This fact implies that our presumption of the expected exchange rate is not far from reality. We perform an ERS test on w_t in addition to the ADF test. The high values obtained for $\hat{\theta}$ provoke an ERS test as it has higher power than the ADF test. The ERS test on $\hat{w}_t$ of the US-Italy parity rejects the null of $\theta = 1$ against $\theta = 0.9756$. The Japan-UK and the Japan-Canada parities turn out to substantially improve the likelihood value of the filtered data whereas only the Japan-Canada parity is found to hold. The notable finding in Table 5.4 is that the slopes of the IRD are unanimously positive. This finding complies with the restricted model in that both models are positively associated with the IRD. This finding contrasts with the forward premium puzzle which indicates that the assumption of the equality of the expected exchange rate to price differences and the joint dynamic model brings theory and reality into compliance.

Table 5.3: Restricted dynamic joint model of PPP and UIP without smooth transition component

	$\hat{\mu}$	$\hat{s}_0$	$\hat{W}_0$	$\hat{\sigma}_\eta$	$\hat{\sigma}_\epsilon$	$\hat{\phi}$	$\hat{\theta}$	$std(\hat{\theta})$	max LL
US-JP	3.5740	-0.0140	0.8011	0.0001	0.0226	0.4093***	1.0024	0.0072	678.09
US-CA	0.2074**	0.0064	0.1341	0.0002	0.0180	0.5251**	0.9894	0.0085	742.25
US-UK	-0.3028**	-0.0086	-0.2167	0.0000	0.0202	0.6168***	0.9889	0.0110	709.39
US-FR	-0.1135	-0.0725	-0.0958	0.0000	0.0215	0.5865***	0.9909	0.0103	691.65
US-DE	-0.1593	-0.0819	-0.0477	0.0000	0.0215	0.5611***	0.9861	0.0109	692.22
US-IT	-0.0643	0.3209	-0.0842	0.0000	0.0219	-0.0891***	0.9925	0.0097	684.42
JP-CA	-4.6138***	-0.0098	0.5968***	0.0000	0.0278	0.4321***	0.9892	0.0089	618.22
JP-UK	-5.0788***	-0.0913	0.2431	0.0000	0.0275	0.5958***	0.9855	0.0110	621.23
JP-FR	-4.9459***	-0.0893	0.3598*	0.0000	0.0258	0.3698***	0.9900	0.0088	640.31
JP-DE	-5.0125***	-0.1071	0.4381	0.0001	0.0264	0.4262***	0.9920	0.0087	633.68
JP-IT	-4.9791***	-0.0174	0.4305	0.0001	0.0262	-0.2769***	0.9912	0.0086	633.14
CA-UK	-0.5746***	0.0129	-0.3092***	0.0002	0.0198	0.7572***	0.9724**†††	0.0127	716.06
CA-FR	-0.4023***	-0.0270	-0.1835***	0.0000	0.0204	0.6843*	0.9478***†††	0.0185	707.23
CA-DE	-0.3980***	-0.0421	-0.1851***	0.0002	0.0203	0.5593**	0.9406***†††	0.0188	708.05
CA-IT	-0.3636***	-0.1300	-0.1408***	0.0001	0.0207	0.1760***	0.9457***†††	0.0190	700.68
UK-FR	0.1709***	-0.0577	0.1481*	0.0000	0.0171	0.7702*	0.9757*††	0.0126	758.04
UK-DE	0.1635***	-0.0635	0.1570*	0.0000	0.0177	0.7430*	0.9780*†	0.0123	747.67
UK-IT	0.2091***	-0.5097	0.1754***	0.0000	0.0178	0.0700***	0.9828*††	0.0101	742.85

The coefficient of the IRD is restricted to $\psi = 1$. The null hypotheses of the tests on the autoregressive parameters of $\hat{\phi}$ and $\hat{\theta}$ are respectively $\hat{\phi} = 1$ and $\hat{\theta} = 1$. The corresponding alternative hypotheses are $\hat{\phi} \neq 1$ and $\hat{\theta} \neq 1$ respectively. The lag length in W_t is selected by BIC. The superscripts *,** and *** represent respectively rejection of the null hypothesis by the ADF τ test at 10%, 5% and 1% levels. The superscripts †,†† and ††† represent respectively rejection of the null hypothesis by the ERS test at 10%, 5% and 1% levels. max LL denotes maximised sum of log-likelihood function.

A comparison of these findings and those of Dahlquist and Pénasse (2022) is worthy of consideration. Dahlquist and Pénasse (2022) formulate a decomposition of the RER in which q_t depends concurrently on the IRD and a missing risk premium that is presumable to be an unobservable process. The IRD and the unobservable process are assumed to follow distinct AR processes. Dahlquist and Pénasse (2022) find an AR coefficient -associated with the error term- of 0.971 with a standard deviation of 0.01 for the US and UK parity which is significantly less than 1. The slope is found to be 0.979, 0.988, and 0.977 for the parities between the US and either of Canada, Japan, and Germany respectively. The standard deviations of the slopes are all found to be 0.01 [4]. These findings do not support price convergence with Japan. The evidence found by Dahlquist and Pénasse (2022) to support PPP between the US and UK, Germany, and Canada contrast with our results in Tables 5.3 and 5.4. Given that the data sample in the work covers the period 1976-2020, the stationarity of the second component

[4] see the appendix of Dahlquist and Pénasse (2022) at https://www.jfinec.com/internet-appendices

in the parities with Germany and UK can possibly be induced by the regimes of snake-in-the-tunnel and ERM. Another major anomaly associated with the results of Dahlquist and Pénasse (2022) is that the RER is negatively correlated with the IRD in all parities. This is definitely a puzzle and is deepened by their empirical results.

Table 5.5 shows the results of the augmented model. The dominant pattern concerning the EPU variables is that the slope of the foreign EPU, whenever significant, is negative. This is an important finding that implies the impact of uncertainty cast by economic policy on the depreciation of the currencies. The significant slopes on the domestic EPU in the parities of CA-DE and UK-IT are negative. The pattern of the US MPU is interesting because it leads to the relative depreciation of the European currencies compared to the Japanese Yen. In one single parity of US-JP, it turns out to be negative.

Table 5.4: Unrestricted dynamic joint model of PPP and UIP without smooth transition component

	$\hat{\mu}$	$\hat{s}_0$	$\hat{w}_0$	$\hat{\sigma}_\eta$	$\hat{\sigma}_\epsilon$	$\hat{\phi}$	$\hat{\psi}$	$\hat{\theta}$	$std(\hat{\theta})$	max LL
US-JP	5.6276*	-0.0188	-1.3556	0.0000	0.0221	0.1666***	3.1595***	0.9977	0.0069	683.63
US-CA	0.2072**	-0.0023	0.1435	0.0002	0.0177	0.2628***	3.1565***	0.9891	0.0084	746.99
US-UK	-0.3120***	-0.0084	-0.2076	0.0000	0.0202	0.5874***	1.2581***	0.9879	0.0114	709.53
US-FR	-0.1234	-0.0646	-0.1339	0.0004	0.0214	0.3132***	2.4002***	0.9896	0.0102	693.78
US-DE	-0.1691*	-0.0921	-0.0974	0.0001	0.0212	0.2532***	2.9797***	0.9846	0.0109	696.50
US-IT	-0.1909***	-0.4483	0.4594	0.0004	0.0217	0.9428*	0.2055	0.9816†††	0.0122	686.57
JP-CA	-4.4992***	-0.0201	0.7192***	0.0000	0.0260	0.1112***	5.8947***	0.9883*	0.0065	638.12
JP-UK	-5.0326***	-0.2222*	0.3586**	0.0000	0.0267	0.4162***	3.6474***	0.9883	0.0088	629.81
JP-FR	-4.9396***	-0.1246	0.4155*	0.0001	0.0253	0.1515***	3.7161***	0.9917	0.0074	645.48
JP-DE	-5.0256***	-0.1791	0.5303	0.0000	0.0256	0.1572***	4.4660***	0.9936	0.0069	641.85
JP-IT	-4.9855***	-0.0162	0.4351	0.0000	0.0262	-0.2782***	0.9393**	0.9914	0.0087	633.15
CA-UK	-0.5812***	0.0106	-0.3030***	0.0005	0.0196	0.6386***	1.9762***	0.9687***†††	0.0130	718.27
CA-FR	-0.4101***	-0.0061	-0.2460***	0.0000	0.0198	0.2150***	3.4697***	0.9470***†††	0.0168	714.92
CA-DE	-0.4141***	-0.0652	-0.2413***	0.0000	0.0197	0.1511***	3.7192***	0.9444***†††	0.0170	717.54
CA-IT	-0.3639***	-0.1702	-0.1497***	0.0000	0.0206	0.1215***	1.3120***	0.9462***†††	0.0189	701.13
UK-FR	0.1721***	-0.0097	0.0857	0.0000	0.0169	0.5680***	2.1529***	0.9773*†	0.0122	760.12
UK-DE	0.1636***	-0.0303	0.1039	0.0001	0.0176	0.5311***	2.1159***	0.9794*	0.0120	749.55
UK-IT	0.2045***	-0.1188***	0.2186***	0.0001	0.0177	0.5219	0.3298	0.9823***	0.0101	742.85

The null hypotheses of the tests on the AR parameters of ϕ and θ are respectively $\hat{\phi} = 1$ and $\theta = 1$. The corresponding alternative hypotheses are $\phi \neq 1$ and $\theta \neq 1$ respectively. The null and alternative hypotheses of the test on ψ are respectively $\psi = 0$ and $\psi \neq 0$. The lag length in W_t is selected by BIC. The superscripts *,** and *** represent respectively rejection of the null hypothesis by the ADF τ test at 10%, 5% and 1% levels. The superscripts †,†† and ††† represent respectively rejection of the null hypothesis by the ERS test at 10%, 5% and 1% levels. max LL denotes maximised sum of log-likelihood function.

Table 5.5: Unrestricted augmented decomposition of the RERs without smooth transition component.

	US JP	US CA	US UK	US FR	US DE	US IT	JP CA	JP UK	JP FR
$\hat{\mu}$	3.1387	-0.2500**	-1.0464***	-0.6213***	-0.5450***	-0.3725*	-4.9846***	-5.3411***	-5.3391***
$\hat{s}_0$	0.0481	0.3264***	0.5625***	0.2813*	0.2052	1.5184***	0.2227***	0.0426**	0.0982*
$\hat{w}_0$	1.0989	0.2590***	-0.0550	0.0140	-0.0377	-0.0300	0.8635	0.3990	0.5896
$\hat{\sigma}_\eta$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{\sigma}_\epsilon$	0.0217	0.0165	0.0194	0.0209	0.0205	0.0216	0.0247	0.0261	0.0246
$\hat{\phi}$	0.2301***	0.3824***	0.6888***	0.3693***	0.2591***	-0.0269***	0.1155***	0.4372***	0.1442***
$\hat{\psi}$	2.8834***	3.5051***	1.1936***	2.4966***	2.9253***	0.7304**	5.0813***	3.0908***	3.2515***
$\hat{\beta}_1$	0.0057	0.0087	0.0408***	0.0221	0.0182	0.0341	-0.0012	0.0031	-0.0114
$\hat{\beta}_2$	0.0187	0.0413***	0.0073	0.0401**	0.0454***	0.0316	0.0479***	0.0077	0.0397
$\hat{\beta}_3$	-0.0217**	0.0045	-0.0056	-0.0051	-0.0120	-0.0048	0.0243***	0.0229***	0.0242***
$\hat{\theta}$	1.0019	0.9871*	0.9758††	0.9901	0.9831	0.9918	0.9898*	0.9885	0.9928
$std(\theta)$	0.0051	0.0077	0.0177	0.0095	0.0110	0.0097	0.0059	0.0091	0.0069
max LL	687.20	764.22	719.38	697.47	703.02	688.35	650.35	634.70	650.67

	JP DE	JP IT	CA UK	CA FR	CA DE	CA IT	UK FR	UK DE	UK IT
$\hat{\mu}$	-5.4686***	-5.3348***	-0.6553***	-0.3203***	-0.3260***	-0.3241***	0.2339**	0.4379	0.3222
$\hat{s}_0$	-0.0187	0.2688	0.0597	-0.0663	-0.0673	-0.1786	9.9850	-0.2356	-0.9528
$\hat{w}_0$	0.8096	0.4912*	-0.2804***	-0.2638***	-0.2802***	-0.1695***	0.0784	0.0585	0.9745
$\hat{\sigma}_\eta$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0171
$\hat{\sigma}_\epsilon$	0.0250	0.0254	0.0195	0.0197	0.0194	0.0206	0.0172	0.0175	0.0000
$\hat{\phi}$	0.1714***	-0.2012***	0.6206***	0.2174***	0.1744***	0.1124***	-0.0028***	0.7385	0.9496***
$\hat{\psi}$	3.8688***	1.0309***	1.9622***	3.4624***	3.6233***	1.2424***	2.0345***	1.5036	0.1092
$\hat{\beta}_1$	-0.0177	-0.0109	-0.0018	-0.0191	-0.0267***	-0.0114	-0.0114	-0.0086	-0.0047***
$\hat{\beta}_2$	0.0302*	0.0680***	0.0118	0.0106	0.0221	0.0117	0.0054	-0.0040	0.0125***
$\hat{\beta}_3$	0.0188**	0.0293***	-0.0057	-0.0060	-0.0113	-0.0071	-0.0007	-0.0018	-0.0096***
$\hat{\theta}$	0.9958	0.9916	0.9651***†††	0.9509***†††	0.9444***†††	0.9493***†††	0.9793*†	0.9693††	0.9688**
$std(\theta)$	0.0069	0.0084	0.0145	0.0165	0.0162	0.0185	0.0121	0.0192	0.0158
max LL	646.51	642.43	717.02	714.41	718.73	702.44	753.51	748.97	755.43

The null hypotheses of the tests on the AR parameters of $\hat{\phi}$, $\hat{\theta}$ and $\hat{\psi}$ are respectively $\phi = 1$, $\theta = 1$ and $\psi = 0$. The corresponding alternative hypotheses are $\phi \neq 1$, $\theta \neq 1$ and $\psi \neq 0$ respectively. The lag length in W_t is selected by BIC. The superscripts *,** and *** represent respectively rejection of the null hypothesis by the ADF τ test at 10%, 5% and 1% levels. The superscripts †,†† and ††† represent respectively rejection of the null hypothesis by the ERS test at 10%, 5% and 1% levels. max LL denotes maximised sum of log-likelihood function.

5.4.5 ESTAR

Given that empirical findings do not evidence PPP in a considerable number of parities, we estimate the joint model with an ESTAR specification on w_t as outlined in Equation (5.6). Table 5.6 shows the results of the unaugmented decomposition of the RERs with an ESTAR specification on the missing risk premium. ESTAR specification disentangles two regimes of evolution for the w_t. The first regime is identified by periods when w_t fluctuates around the long-run equilibrium, and the second regime is identified when w_t is getting distance from the long-run level. As Table 5.6 shows, w_t behaves explosive in a few parities. As w_t is moving away from the equilibrium, the process becomes strongly mean reverting. This finding is unanimously observed by all the parities. Although we find strong smooth transition behavior of the PPP, the maximised sum of log-likelihood of the parities involving the US and Japan do not appear to be improved by the ESTAR modeling in Table 5.6 compared to the augmented model in Table 5.5. The inclusion of uncertainty indicators leads to a better fit than an ESTAR specification. This fact encourages the examination of uncertainty indicators in a model with ESTAR specification. As Table 5.7 shows inclusion of uncertainty variables leads to a noticeable improvement in the likelihood value. The significant impact of the US MPU on all the parities with Japan is reiterated by ESTAR specification. Furthermore, the ESTAR model exhibits a stronger impact of the US MPU on the parities with Japan.

It is generally reasonable to expect a negative correlation between EPU and currency price. However, the US-UK and the US-Italy parities are the ones with a positive loading on domestic EPU. This fact is confirmed by $\hat{\beta}_2$ in Table 5.7. It seems that MPU leaves a stronger negative impact on the US dollar. The Western currencies are also found to be depreciated by a rise in the US MPU. This fact can be deduced from the parities of Japan with other G7 member countries.

Table 5.6: Unrestricted decomposition of the RERs with smooth transition component.

	US JP	US CA	US UK	US FR	US DE	US IT	JP CA	JP UK	JP FR
$\hat{\mu}$	11.1480***	0.2735***	-0.6423	8.8723***	-1.7362***	-1.8015	-5.3238***	-6.9876	-4.5848***
$\hat{s}_0$	-0.0324	0.0111	-0.0145	-0.2601	-0.1031	-0.1450	-0.0599	-0.2189	-1.9744
$\hat{w}_0$	-6.8771	0.0713	0.1270	-9.1062	1.4759	1.6762	1.5477	2.3116	0.0293
$\hat{\sigma}_\eta$	0.0005	0.0007	0.0007	0.0002	0.0002	0.0006	0.0001	0.0002	0.0003
$\hat{\sigma}_\epsilon$	0.0072	0.0068	0.0068	0.0119	0.0065	0.0071	0.0092	0.0068	0.0061
$\hat{\phi}$	0.1605***	0.2883***	0.5641***	0.1545***	0.2453***	0.2834	0.1238***	0.4187***	-0.0032***
$\hat{\psi}$	3.1991***	3.2162***	1.3471***	2.3131***	2.7901***	0.4246	5.9077***	3.6485***	3.6438***
$\hat{\theta}$	1.0002***	1.0270	1.0070	1.0000	0.9999	0.9993	0.9945	0.9985	1.0406**
$\hat{\theta}^*$	-2.1078***	-1.7626***	-2.0448***	-1.1453***	-2.2280***	-2.0928***	-1.9061***	-2.7117***	-2.8502***
$\hat{\mu}^*$	0.8453	0.0062	0.4140	-10.2704***	3.6508***	3.8619	3.7405	6.3400	-0.1190
$\hat{\gamma}$	0.0002	0.7093	0.3381	0.4687	0.3439	0.4263	0.0200	0.0362	0.1689
$\hat{c}$	-5.3045***	-0.0469	0.2180	-9.0606***	1.5562***	1.5840	1.2195	2.1963	-0.0186
max LL	684.50	749.24	710.72	695.92	699.23	688.89	640.15	631.17	650.12
	JP DE	JP IT	CA UK	CA FR	CA DE	CA IT	UK FR	UK DE	UK IT
$\hat{\mu}$	-4.5827***	-7.9162	-0.5646***	-0.3462***	-0.3654***	-0.2971*	-5.5764***	-5.0812	-5.0055
$\hat{s}_0$	-0.1415	-0.0164	0.1608	0.2183	0.2698	0.0073	-0.0003	-0.0330	-0.1808
$\hat{w}_0$	0.0814	3.3653	-0.4484	-0.4060	-0.3979	-0.2701	5.8265	5.3493	5.5240
$\hat{\sigma}_\eta$	0.0001	0.0008	0.0012	0.0008	0.0008	0.0013	0.0008	0.0009	0.0007
$\hat{\sigma}_\epsilon$	0.0106	0.0059	0.0046	0.0056	0.0059	0.0044	0.0057	0.0056	0.0061
$\hat{\phi}$	0.1660***	-0.2619***	0.6217***	0.1981***	0.1420***	0.0948***	0.6057***	0.5025***	0.8104
$\hat{\psi}$	4.6229***	0.9436**	1.9574***	3.5111***	3.7589***	1.2570***	2.1597***	2.0542***	0.3770*
$\hat{\theta}$	1.0312**	0.9988	0.9804	0.9767	0.9748	0.9905	1.0001***	0.9996	0.9997
$\hat{\theta}^*$	-1.6088***	-3.0532***	-2.8917***	-2.4233***	-2.2703***	-3.1682***	-2.0000***	-2.1279***	-1.9463***
$\hat{\mu}^*$	-0.0470	10.7290	0.0863	-0.3769	-0.2923	-0.4061	11.3722***	11.5269	10.1184
$\hat{\gamma}$	0.1778	0.0849	0.2576	1.0215	1.0371	2.2065	2.1002	1.4524	0.7877
$\hat{c}$	-0.0109	3.1714	-0.1162	-0.0960	-0.0718	-0.0542	5.8533***	5.2651	5.3502
max LL	645.18	635.93	719.42	717.22	719.54	705.65	765.36	754.21	748.82

The null hypotheses of the tests on the AR parameters of $\hat{\phi}$, $\hat{\theta}$ and $\hat{\psi}$ are respectively $\phi = 1$, $\theta = 1$ and $\psi = 0$. The corresponding alternative hypotheses are $\phi \neq 1$, $\theta \neq 1$ and $\psi \neq 0$ respectively. The null hypothesis of the test on θ^* is $\theta^* = 0$ against the alternative of $\theta^* \leq 0$. The lag length in W_t is selected by BIC. The superscripts *,** and *** represent respectively rejection of the null hypothesis by the ADF τ test at 10%, 5% and 1% levels. max LL denotes maximised sum of the log-likelihood function.

Table 5.7: Unrestricted augmented dynamic joint model of PPP and UIP with smooth transition

	US JP	US CA	US UK	US FR	US DE	US IT	JP CA	JP UK	JP FR
$\hat{\mu}$	5.0340***	-0.5355	-0.9316	-0.6640***	-0.6360***	-0.5030***	-6.5212***	-3.3827***	-3.0230***
$\hat{s}_0$	0.0123	0.3193	0.6708	0.3474	0.3174	-0.1845	0.2267	0.1300	0.1165
$\hat{w}_0$	-0.7543	0.5501	-0.2780	-0.0001	-0.0469	0.0261	2.3716	-1.6357	-1.7371
$\hat{\sigma}_\eta$	0.0003	0.0007	0.0014	0.0007	0.0007	0.0007	0.0001	0.0003	0.0003
$\hat{\sigma}_\epsilon$	0.0098	0.0062	0.0045	0.0062	0.0062	0.0063	0.0152	0.0097	0.0097
$\hat{\phi}$	0.2153	0.3737***	0.7333***	0.4199***	0.3166***	0.0688	0.1343	0.4523***	0.1667
$\hat{\psi}$	2.5265***	3.5843***	0.8885**	2.5873***	2.8171***	0.7896**	4.8515***	2.7575***	2.9652***
$\hat{\beta}_1$	0.0070	0.0093	0.0420***	0.0319	0.0286	0.0379*	0.0020	0.0054	-0.0105
$\hat{\beta}_2$	0.0145	0.0400***	0.0080	0.0335***	0.0515***	0.0393	0.0473***	0.0084	0.0372
$\hat{\beta}_3$	-0.0236***	0.0051	-0.0060	-0.0068	-0.0161***	-0.0068	0.0245***	0.0250***	0.0250***
$\hat{\theta}$	1.1000	0.9976***	1.1169*	1.0848***	1.0791***	1.0658***	1.1188	1.1793**	1.2469***
$\hat{\theta}^*$	-1.4241***	-1.8049***	-2.9145***	-2.2444***	-2.2068***	-2.2897***	-0.9966***	-1.8070***	-1.6652***
$\hat{\mu}^*$	9.9964	0.6460	17.8760***	0.0240	-0.0080	0.0138	-11.6133	15.6054	23.7103
$\hat{\gamma}$	0.0044	0.1624	0.0087	0.9859	1.2709	0.8489	0.0010	0.0011	0.0011
$\hat{c}$	0.5333	0.3693	0.1648	0.0399	0.0510	0.0132	-2.2166	2.1423	2.1605
max LL	694.59	764.86	722.69	706.45	710.91	696.64	652.62	639.04	659.04

	JP DE	JP IT	CA UK	CA FR	CA DE	CA IT	UK FR	UK DE	UK IT
$\hat{\mu}$	-6.0740***	-8.8692	-0.7007***	0.5717***	-2.3699	1.9344	0.2316*	3.0505***	0.3834***
$\hat{s}_0$	0.0139	0.2620	0.2363	-0.0151	-0.0032	-0.1392	0.0294	-0.3854	7.6611
$\hat{w}_0$	1.3892	4.0283	-0.3886	-1.1719	1.7455	-2.4545	-0.0043	-2.3948	0.0427
$\hat{\sigma}_\eta$	0.0002	0.0006	0.0012	0.0015	0.0009	0.0008	0.0012	0.0016	0.0008
$\hat{\sigma}_\epsilon$	0.0134	0.0072	0.0049	0.0044	0.0056	0.0058	0.0048	0.0043	0.0060
$\hat{\phi}$	0.1827	-0.2228	0.6209***	0.2172*	0.1723	0.1326	0.6504***	0.9050***	-0.0038
$\hat{\psi}$	3.7170***	1.0561***	1.9365***	3.4225***	3.5948***	1.2009***	2.0787***	0.8388 ***	0.5385**
$\hat{\beta}_1$	-0.0157	-0.0091	-0.0011	-0.0190	-0.0264**	-0.0094	-0.0115*	-0.0049*	0.0039
$\hat{\beta}_2$	0.0307*	0.0676*	0.0136	0.0113	0.0221	0.0161	0.0168	0.0012	-0.0100
$\hat{\beta}_3$	0.0198**	0.0280***	-0.0069	-0.0065	-0.0113	-0.0070	-0.0024	-0.0035	-0.0042
$\hat{\theta}$	1.1000	0.9990	0.9657	1.0287***	0.9949	0.9970	1.0267	1.0000***	1.0334
$\hat{\theta}^*$	-1.1846***	-2.4375***	-2.7281***	-3.0986***	-2.3869***	-2.4233***	-2.3754***	-2.6271 ***	-1.9813***
$\hat{\mu}^*$	-11.9977	9.9852	0.8041	1.9662***	9.9987	-11.9904	-0.2713	-6.8482***	-0.1558
$\hat{\gamma}$	0.0014	0.0652	0.1611	0.0070	0.0097	0.0197	1.3498	2.6014	0.9991
$\hat{c}$	-1.3621	3.8548	-0.0353	-0.0208	2.4950	-2.5192	0.0035	-2.4776	-0.0213
max LL	651.49	644.32	718.44	715.47	720.13	704.22	764.36	753.85	749.79

The null hypotheses of the tests on the AR parameters of $\hat{\phi}$, $\hat{\theta}$ and $\hat{\psi}$ are respectively $\phi = 1$, $\theta = 1$ and $\psi = 0$. The corresponding alternative hypotheses are $\phi \neq 1$, $\theta \neq 1$ and $\psi \neq 0$ respectively. The null hypothesis of the test on θ^* is $\theta^* = 0$ against the alternative of $\theta^* \leq 0$. The lag length in W_t is selected by BIC. The hypothesis test is performed using the τ statistic. The superscripts *,**, and *** represent respectively rejection of the null hypothesis by the ADF test at 10%, 5%, and 1% levels. max LL denotes maximised sum of the log-likelihood function.

The estimated $\hat{\theta}$s in Tables 5.3, 5.4 and 5.5 are analogous to the AR slopes estimated by Koedijk et al. (2004) where the values of the AR term in the RER are large and generally not significantly less than unity. They find an AR coefficient of 0.9835 with a standard deviation of 0.0068 for the RER of the parity between the US and Eurozone area over the period 1981-2003. Our findings do not support strong evidence of PPP between the US and the Eurozone countries of the G7. The other parities between the UK, Canada, and Japan are found to have an AR coefficient of 0.9684, 0.9853, and 0.9676 respectively with a standard deviation not as small as to reject the null hypothesis of an AR slope of less than unity. Our results on these countries in Tables 5.4 and 5.5 seems to be stronger evidence of PPP.

The findings in Tables 5.1 and 5.2 are definitely in compliance with those of Koedijk et al. (2004) who find weak evidence of PPP. Koedijk et al. (2004) use the synthetic euro rate before 1999 which is definitely affected by the ERM system. The results of the decomposition in Tables 5.5 and 5.7 suggest that incorporating the IRD and uncertainty variables is crucial to the examination of PPP. These findings imply that the stochastic trend of the RER is explained by the dependent variables in Equation (5.12). The fact that the error terms in (5.12) and (5.4c) are independent implies that the moving average component of s_{t_1} and w_{t_2} are uncorrelated for all choices of t_1 and t_2 and thus the summation of the moving average component and w_t is stationary. This implication confirms the conclusion that the stochastic trend of the RER is a linear function of the IRD and the uncertainty indicators.

Despite stronger evidence of stationarity of the AR components in Table 5.5, the $\hat{\theta}$ remains high and as a result, the half-lives of reversion to mean remain high. This finding motivates further analysis of the evolution of θ.

5.4.6 Time-variation in θ and impact of globalisation

Though the second component in Equation (5.4b) is not dependent on any variable, it can also be impacted by an unknown set of factors. A broad range of factors can affect w_t, some of which are the ones influencing goods markets. These factors can be trade barriers including tariff and non-tariff barriers, trade wars, the cost of transportation between countries, protectionism measures, etc. If globalisation has left a favorable impact on the integration of goods markets, then evidence for PPP has to get stronger over time. The gradual impact of globalisation on the prevalence of PPP can be investigated by the time-varying AR parameter of w_t. We use the following specification to estimate the time-varying θ_t by a Kalman filter:

$$w_t = \theta_t w_{t-1} + \sum_{k=1}^{p} \delta_k \Delta w_{t-k} + \epsilon_t, \tag{5.16a}$$

$$\theta_t = \theta_{t-1} + \nu_t, \tag{5.16b}$$

Figures 5.2 and 5.3 exhibit time-varying $\hat{\theta}_t$ of the estimated $\hat{w}_t$. The US parities with all the countries other than Japan are generally getting stronger. $\hat{w}_t$ values of these parities except do not exceed unity, although there are very short periods of spiking. After a downward trending $\hat{\theta}_t$ until the global financial crisis (GFC), the parities with Canada, France, and Italy are destabilised during the crisis. The US debt-ceiling crisis in 2011 that led to a downgrade of credit rating by Standard & Poor's seems to destabilise the currency parity of the US with Canada more than those with other countries. The credit rating downgrade that provoked a sell-off in financial markets led to a shift in liquidity preferences by shifting to less risky currencies. During this period of hiking $\hat{\theta}_t$, the Canadian dollar was appreciated against the US dollar and euro throughout 2011-2012. The fears of sovereign default throughout 2011 seem to have disrupted US-Canada parity until the fiscal cliff. The US parities with the European countries seem to have boosted after a period of destabilisation during the sovereign debt crisis. The US price convergence with Germany as the pillar of the Eurozone is persistently strengthened. Prices in the US and Germany pursue a persistent trend to convergence in comparison to the US-France and the US-Italy parities. The US price convergence with Italy is repeatedly destabilised during crises periods over 2008-2014. Following the debt crisis, the US-Italy parity enjoys a steady period of stronger price convergence.

Among the time-varying $\hat{\theta}_t$s in Figures 5.2 and 5.3, those of Japan exhibit decreasing evolution until the global financial crisis when they abruptly spike. In some cases $\hat{\theta}_t$ exceeds unity. The θ_t of the parity between the US and Japan is downward trending until after the GFC. However, it exceeds unity again until 2022. Japan's parity with the UK and Canada is persistently strengthening. We observe no strengthening evidence of PPP between Japan and other G7 countries. Parities including Japan are the only ones for which time-varying $\hat{\theta}_t$ exceeds unity. The notable particularity in the parities with Japan is that the evolution of $\hat{\theta}_t$

is driven by more volatile innovations.

The trade volume between countries seems to play an important role in time-varying mean reversion of the RER. For instance, prices in Canada and Germany do not seem to follow a strengthening process to convergence. The trade intensity between Germany and Canada is lower than between Germany and the US. The US has also a more voluminous trade with Germany than with France and Italy. This finding confirms the importance of trade intensity found by Cho and Doblas-Madrid (2014).

The European countries evidence strengthening PPP with the US over time in particular in the years following the GFC. Although the θs of these parities in Table 5.5 are not significantly less than unity, the θ_ts of these parities do not exceed unity. The US-Germany parity is remarkable as it goes through a persistently strengthening price convergence. The price convergence of the US with Germany the third biggest economy of the G7 is more noticeable than with Japan as the second largest economy of the G7. This finding can be attributed to the geographical distance and non-tariff barriers including higher transportation costs. However, the Japan-UK and the Japan-Canada parities are strengthening over time.

During the legislation process of Brexit (i.e. the European Union referendum act) in 2015, $\hat{\theta}_t$ of the UK parities with three European countries in Figure 5.3 spikes. Shortly after, it pursues a downward trend until the end of the period. The PPP of the UK-Germany and the UK-France is strengthened until the period after Covid-19 crisis. Among the European parities, the UK-Germany parity has been adversely impacted by Brexit, however θ_t of the two countries returns to the long run downward trending evolution. $\hat{\theta}_t$. Brexit seems to have adversely impacted the UK-Canada parity. However, this event is not persistent as it evidences new lows. Overall, it seems that θ_t of the UK with Eurozone countries bears the least volatile innovations.

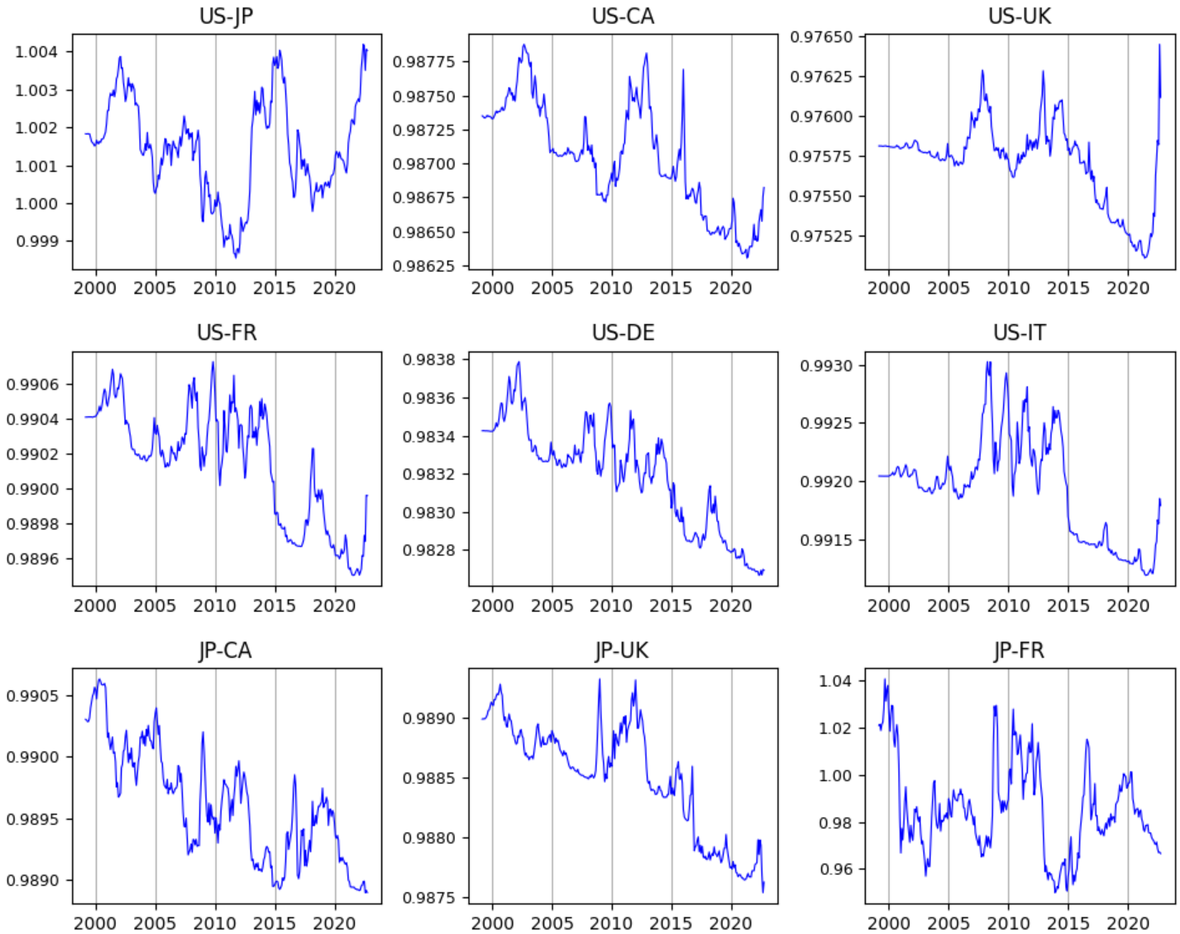


Figure 5.2: The time-varying AR slopes associated with the augmented model (Part 1).

Among the six parities with Canada, Italy is the only one following a strengthening PPP over time. The parities of Canada with Eurozone countries exhibit a spiking $\hat{\theta}_t$ in 2011 and 2012. The escalating Sovereign debt crisis seems to have destabilised PPP between the euro currency and the Canadian dollar as the euro was depreciated by fears cast out by a downgrade of Greece's credit rating and postponed European decision on the bailout. Canada-UK parity in Figure 5.3 enjoys a strengthening trend, in particular following the spike during Brexit legislation in 2015 until 2022. Although the results in Table 5.5 provide weak evidence of PPP between European countries, the time-varying $\hat{\theta}_t$ indicates strengthening evidence of PPP between them.

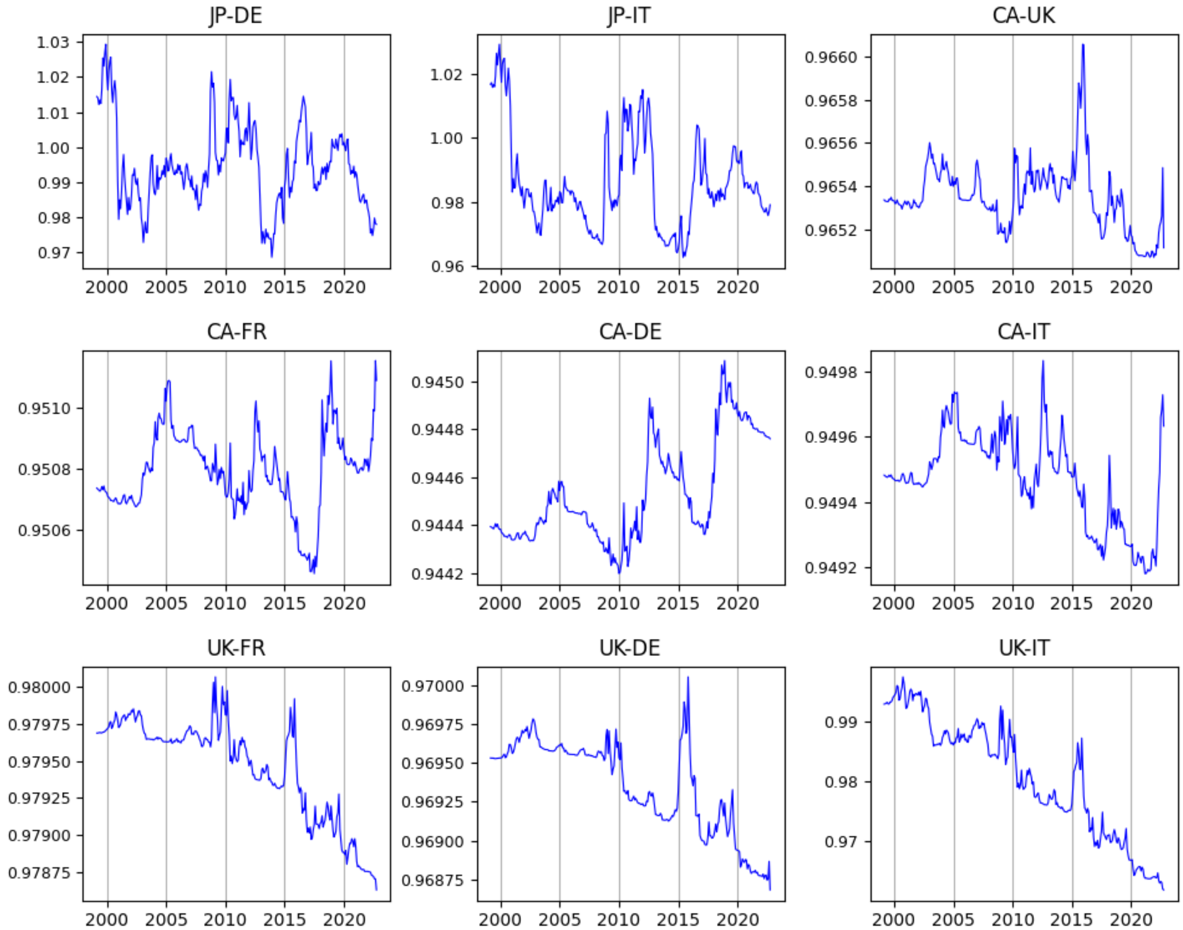


Figure 5.3: The time-varying autoregressive slopes associated with the augmented model (Part 2).

5.4.7 Structural decomposition

The finding that the stochastic trend of the RER in a number of the parities can be expressed by an AR process depending on the IRD and the uncertainty indicators motivates an in-depth analysis of the impacts of the structural shocks in these variables on the RERs. A structural decomposition will shed more light on the fact that the exogenous shocks to IRD leave long-term impact on the RER. We estimate the SVAR model in Equation (5.13) assuming a Cholesky decomposition in which $\mathbf{\Gamma}$ is a lower triangular matrix with the diagonal elements of unity. The structural model for each parity is just-identified. Figure 5.4 presents the impulse response functions of the RERs corresponding to the structural model (5.13). Since the estimated θs in Tables 5.3, 5.4 and 5.5 are high and the half-lives of reversion of the w_ts to zero is high, we set the response horizon of q_t to 100 months.

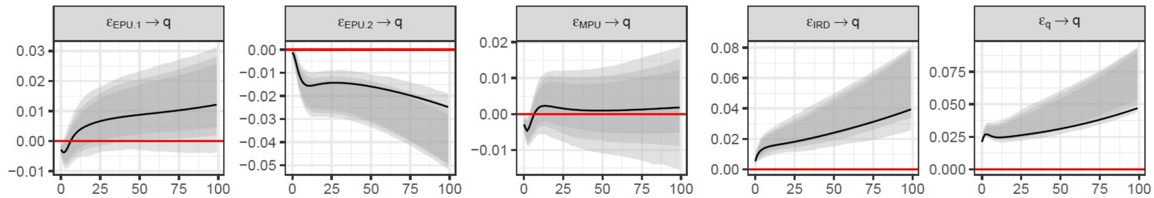
The Cholesky restriction on the structural shocks illustrates, to a substantial extent, the impacts of the innovations in the uncertainty and IRD on the RER. In Figure 5.4, there is no instantaneous interaction between the EPU of the first country in the parity and the other uncertainty and IRD variables. To further investigate the impact of the exogenous shocks in the IRD, we obtain the impulse response functions of the RER using Cholesky decomposition in Figure 5.5.

The exogenous innovation in the EPU of Japan is found to leave a long-run significant impact on the RER in the parities. The exogenous shock in the EPU of Japan on RER of the JP-US parity seems to be persistent.

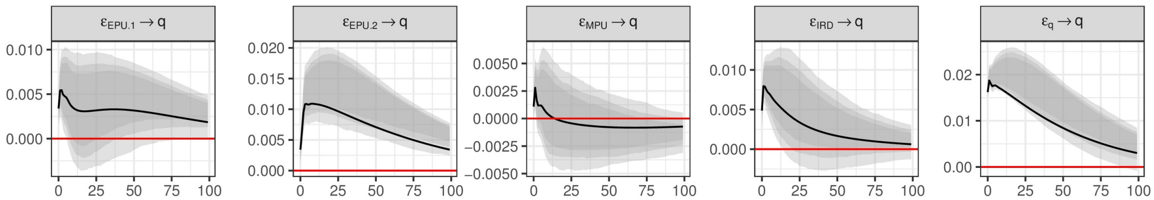
No parity with the US is found to be affected in the long run by innovations in the US EPU. Among the parities with the US, the long-run impact of the shocks to the EPU in Germany, Canada, and the UK significantly impacts the RER. The two parities of the US with Canada and the UK which are found to have a stationary second component in Table 5.5, seem to be persistently responding only to the variations in the EPU in Canada and the UK. The IRD of these two countries in Figure 5.5 does not significantly impact the RER at 1% level over the long run. As Figure 5.5 shows, the US parities with France, Italy, and Japan receive long-lasting impact by the exogenous shocks to IRD. Overall, structural factorisation shows that the unit root component of the corresponding RER of the parities with the US are driven by uncertainty variables and the IRD.

Uncertainty in the UK is also found to have left long-run impact on the RER with Germany and Italy. In a weaker significance level, the same conclusion is reiterated on the UK-France parity. The impact of the shocks to the UK uncertainty in the Canada-UK parity vanished after a relatively long period of four years. Since we use Cholesky decomposition and on the other hand the EPU in Canada does not significantly impact the RER of Canada-UK parity, the significant impact of the structural shocks to the EPU in the UK is ensured. The German EPU in the UK-Germany parity can probably be correlated with the UK EPU and thus its exogenous significant impact at 10% level can not be concluded. The exogenous shock in the German EPU significantly drives the RER in the US-Germany parity, because the innovations in the US EPU do not have a significant impact on the RER. The US-UK parity does not confirm the significance of the exogenous EPU in the UK, because innovations in the

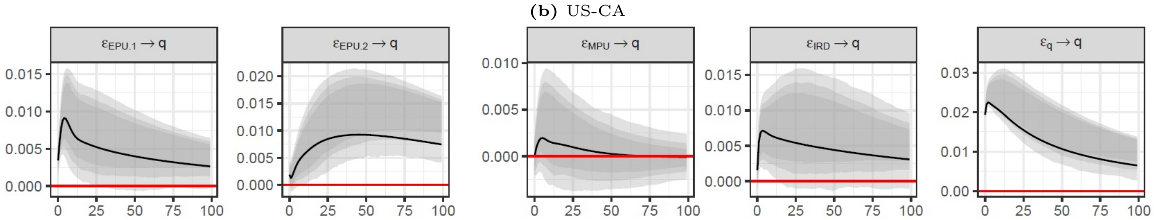
US EPU are significant at 1% and 5% significance levels. However, at the 10% significance level, the significance of the exogenous innovations in the UK EPU can be verified. The UK EPU leaves a significant impact in the long run on the parities with the G7 member countries excluding Japan.



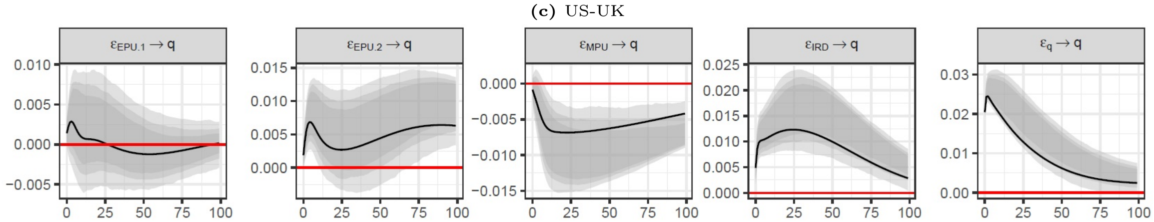
(a) US-JP



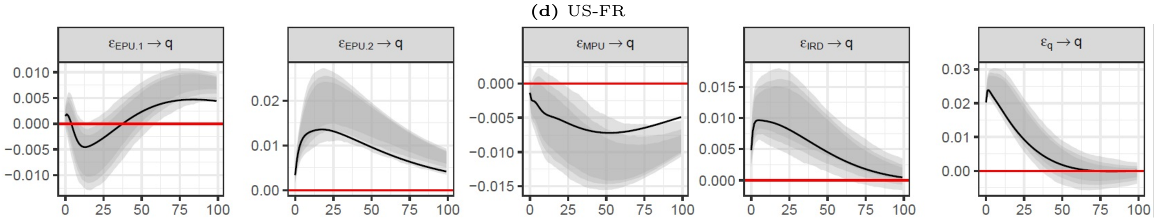
(b) US-CA



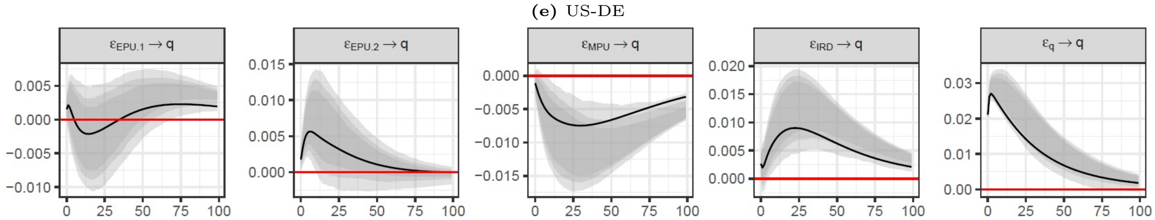
(c) US-UK



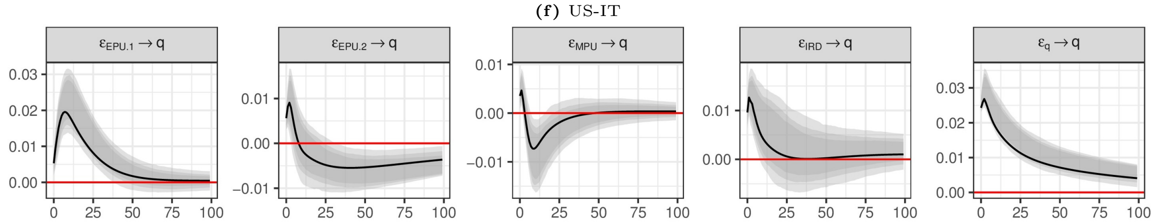
(d) US-FR



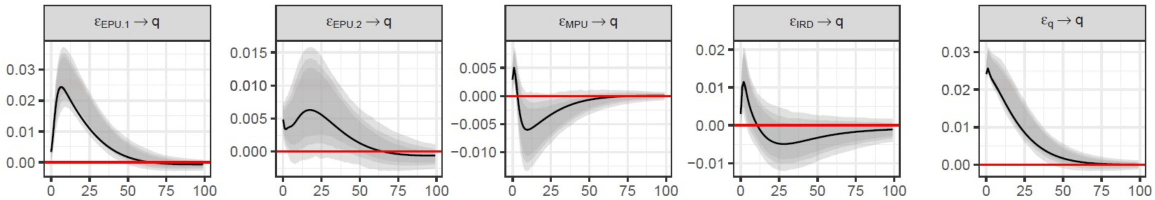
(e) US-DE



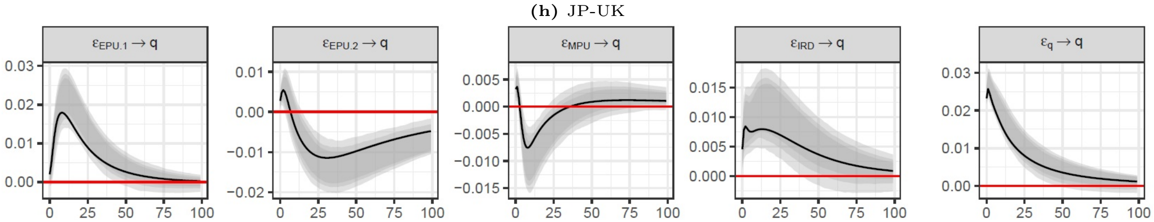
(f) US-IT



(g) JP-CA



(h) JP-UK



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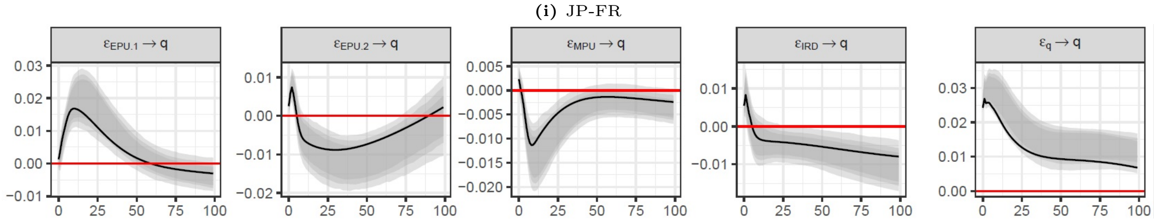
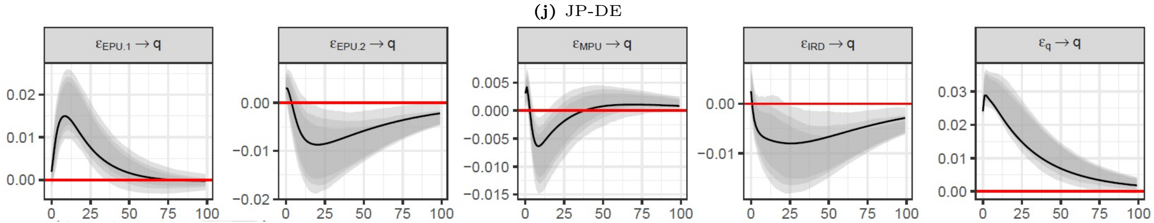
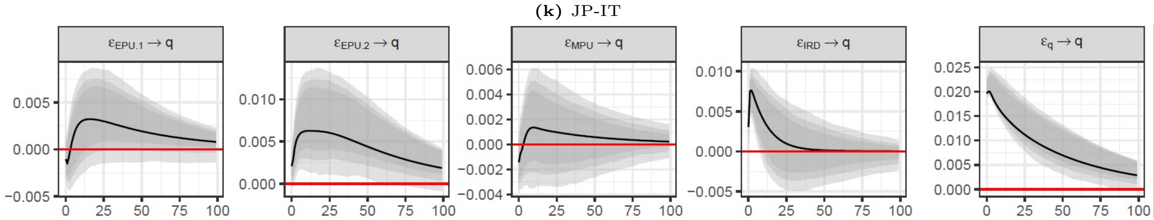


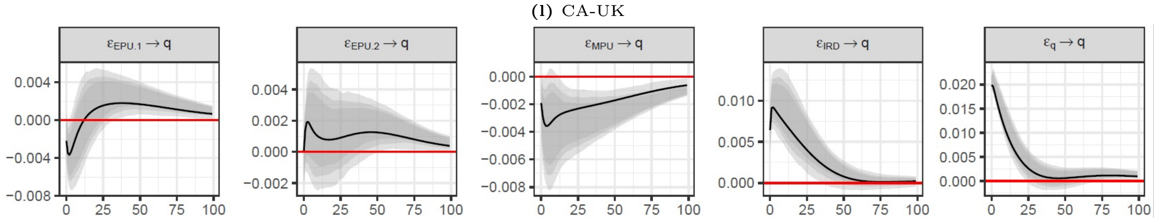
(j) JP-DE



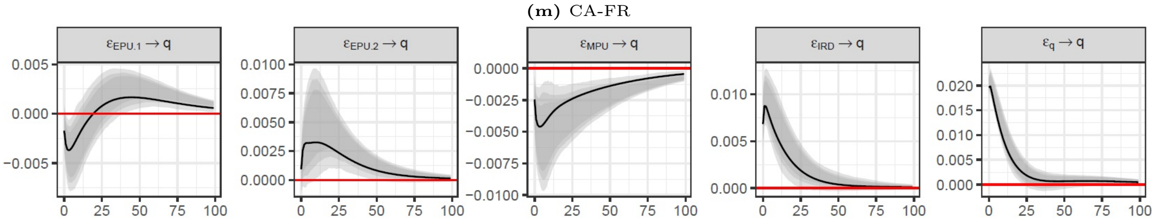
(k) JP-IT



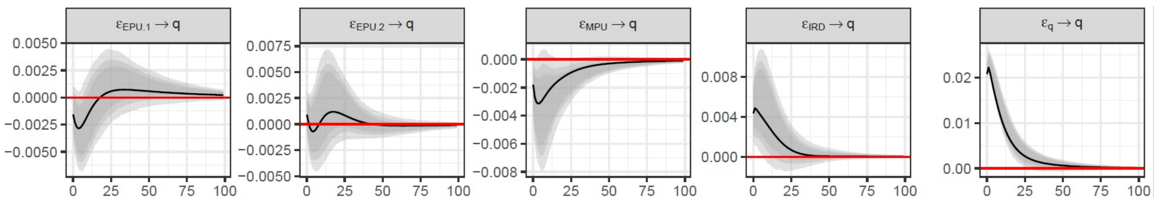
(l) CA-UK



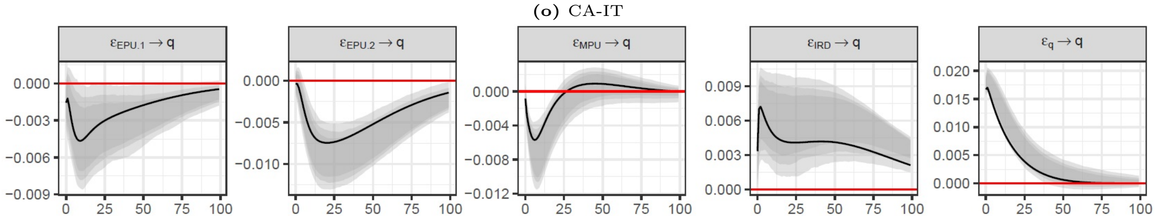
(m) CA-FR



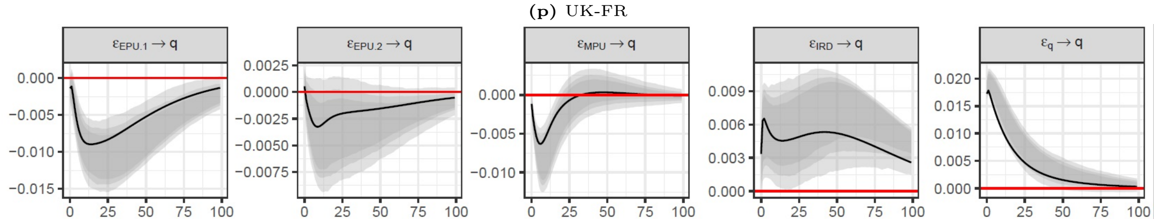
(n) CA-DE



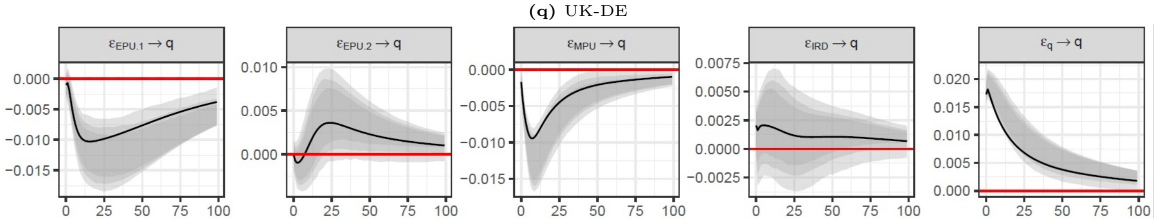
(o) CA-IT



(p) UK-FR



(q) UK-DE



(r) UK-IT

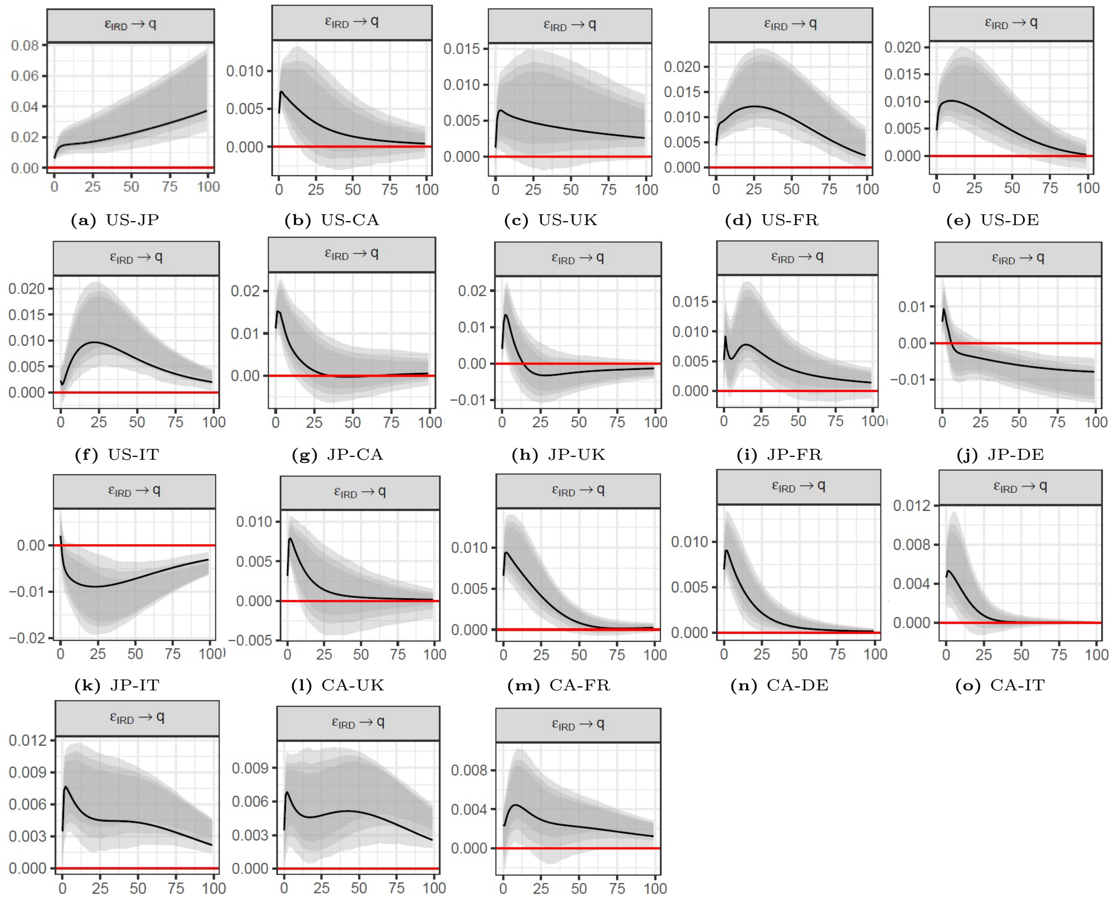
Figure 5.4: Impulse response functions. Confidence bands are obtained by moving block bootstrap (block length $= T^{1/3}$ with 16-84 (darker band), 10-90 (lighter band) and 5-95 (lighter band) Hall's percentile. The lag lengths are selected by the Akaike information criterion.

We see no clear evidence supporting the significant and persistent impact of the EPU in France on the RERs. In the parity with Japan, the innovations in France EPU may be correlated with that of Japan. In the parity with the US, the impulse response vanishes shortly following the innovation. In the parity with the UK, when the impulse responses to the UK EPU are not significant in the 5% significance level, the significant impact of France's EPU is confirmed.

Even though the RER of France and Germany exhibit similar evolution, the German EPU is found to be more influential on the RER than France's EPU. It appears to leave a significant impact on the RER of the parities with Canada and the US.

Since in the impulse response functions in Figure 5.4, the innovations in the IRD are correlated with the uncertainty variables, we perform an alternative structural factorisation to examine the long-run impact of exogenous innovations in IRD. As Figure 5.5 shows, the impulses in the IRDs do not change in comparison to Figure 5.4. As Figure 5.5 indicates, an exogenous innovation in the IRD leaves a long-run impact on the majority of the RERs. In few parities the innovation vanishes after a short time. The Japan-UK, Japan-Germany, US-Canada, Canada-Italy, and Canada-UK are among the parities on which IRD does not appear to be the source of price divergence. In 5% significance level, the IRD impulse has a long-run impact on the RER.

The majority of the impulse response functions in Figure 5.5 imply a positive correlation between IRD and RERs. This finding complies with the findings by the decomposition model and contrasts with the forward premium puzzle that is extensively evidenced by empirical data (see Dahlquist and Pénasse, 2022).



(a) US-JP (b) US-CA (c) US-UK (d) US-FR (e) US-DE (f) US-IT (g) JP-CA (h) JP-UK (i) JP-FR (j) JP-DE (k) JP-IT (l) CA-UK (m) CA-FR (n) CA-DE (o) CA-IT (p) UK-FR (q) UK-DE (r) UK-IT

Figure 5.5: Response of q_t to the IRD impulse. Confidence bands are obtained by moving block bootstrap (block length = $T^{1/3}$ with 16-84 (darker band), 10-90 (lighter band), and 5-95 (lighter band) Hall's percentile. The lag lengths are selected by the Akaike information criterion. The structural factorisation is estimated using Cholesky decomposition.

5.5 Conclusions

We presented a decomposition of the RER into two components, the first of which represents the variation in the RER that has evolved by the speculative flow of money between countries and the second component is an AR process. The decomposition uses a parity derived from PPP and UIP by Juselius (1995). A fundamental assumption of parity is that the expected exchange rate equals the price difference between two countries. Our findings contrast with those of Dahlquist and Pénasse (2022) who find that variation in the RER is negatively correlated with the IRD. Our SVAR model, which controls for economic uncertainty in both countries, reveals a set of empirical findings that are free of the forward premium puzzle. These findings also comply with those of the dynamic joint model in which the slope on IRD is restricted to unity.

The long-run impact of the unknown disruptive factors of PPP associated with a number of the parities is found to decrease over time. The parities between the UK on the one hand and the three Eurozone countries on the other hand are found to be steadily strengthened. The disruptions made to price convergence of these countries are the least volatile ones among all the parities. Price convergence of the G7 countries with Japan is found to be fragile by undergoing the highest innovations in $\hat{\theta}_t$.

The first component in the decomposition is found to have a strongly stationary AR slope. The stochastic trend of the first component is also found to be driven by the IRD and the uncertainty variables. The second component, which is assumed to follow an AR process, represents deviations caused by factors affecting the goods market. Further research on the second component of the RER can be performed to analyse the latent risk factors that drive the deviations from PPP. These factors involve a diverse set of factors affecting goods markets. Trade barriers including both tariffs and non-tariff barriers, trade volume between countries, and the distance between countries can represent potential candidates driving the evolution of the second component. The gravity model of trade which relies on the distance and economic size of the countries can be incorporated into the joint dynamic model. As fears of default can widen the spread in long-term and short-term bond yields, the impact of term structure can also be investigated on the dynamics of PPP in future investigations.

The results of the smooth transition model imply that the decomposition model can be used to formulate a trading strategy. As the results show, the tendency of reversion to the location parameter in the ESTAR model increases when the second component is getting far enough away from the parameter. The trading strategy should trigger a currency exchange when the second component gets a certain distance from the location parameter. It should account for the effect of uncertainty parameters and shifts in the monetary policies of the countries that change the IRD. The performance of such a strategy should be examined in the out-of-sample data in future works.

Chapter 6

Hedge funds responsiveness to disagreement between options and stock markets

Hamid Babaei[1], Georges Hübner[1], Marie Lambert[1], Federico Platania[2]

[1] *HEC Liège, Management School-University of Liège, 14 rue Louvrex, B-4000 Liège, Belgium*

[2] *ISG International Business School 147, Avenue Victor Hugo - 75116 Paris, France*

Abstract

Implied moments extracted from options have been shown to provide forward-looking information for the stock markets. Stock and options markets can thus, at specific times, reflect differing information, called 'disagreement periods'. We first define a disagreement measure between markets over the period January 1997–December 2015. We then investigate the role that hedge funds, a proxy for sophisticated investors, play in the price discovery process between stock and option markets during disagreement or agreement periods. We investigate the timing ability of hedge funds by relating the discrepancy between options and stock markets to fund performance. We observe that hedge fund managers time their portfolios efficiently around periods of high disagreement. A threshold regression analysis reiterates the same conclusion for higher levels of disagreement.

JEL classification: C32, G32, G12

Keywords: hedge funds, gamma trading, market frictions, options market, informed trad-

ing,

6.1 Introduction

Hedge funds are commonly perceived to employ dynamic trading strategies as opposed to the buy-and-hold strategy, predominant in the mutual fund industry. A great deal of the literature emphasises the use of adaptive trading strategies by hedge funds in response to changing market conditions (see Fung and Hsieh, 1997; Agarwal and Naik, 2004; Kuenzi and Shi, 2007; Bollen and Whaley, 2009; Billio et al., 2012; Patton and Ramadorai, 2013 and Christoffersen and Langlois, 2013). In short, hedge fund managers attempt, to time the financial markets.

As evidenced in the recent literature, hedge fund managers identify periods of mispricing to exploit accordingly the prospective returns. Cao et al. (2016) find that hedge funds have a pioneering role in identifying mispriced assets and dissipation of alpha. Ma, Li and Tee (2022) investigate hedge fund managers' abilities to seek mispriced assets and to improve subsequently market efficiency. Fund managers also analyse the information flow having implications to assets' future prices. Brandt et al. (2019) use an index summarizing the real-time information flow about macroeconomic aggregates estimated by principal component analysis to evaluate hedge funds market timing. Chen et al. (2020) investigate the use of continuously evolving information by skilled investors including hedge funds to adapt their trading strategies.

A key question about market timing is: what type of information can hedge fund managers exploit? A number of the works address the ability of fund managers to process superior information on indicators such as market volatility, liquidity risk, or macroeconomic data. Cao et al. (2013) explore liquidity timing by hedge funds and find that market exposure is positively associated with market liquidity. Lambert and Platania (2020) implement a factor model in which unobservable factor loadings depend on the macroeconomic variables including GDP, implied volatility index, the relative T-bill rate, and aggregate dividend yield. They find that macroeconomic variables significantly explain exposure variation. Recent literature goes further in sophistication in signal processing and focuses on market sentiment. Zheng et al. (2018) examine hedge funds' ability to time market sentiment by disentangling high sentiment and low sentiment periods and relating market beta to an investor sentiment index. Chen et al. (2021) analyze sentiment timing skill and find high sentiment beta and alpha for sentiment timer funds.

We believe that, before trying to extract relevant information from very complex analytics based on interpretations, hedge fund managers would be naturally inclined to reap the "low-hanging fruits" signaling informational inefficiencies. The link between options and stock markets is a natural candidate for this purpose. This route has already been adopted with some success in the hedge fund literature. Based on the method of Bakshi et al. (2003) to retrieve the intrinsic values of the risk-neutral variance, skewness, and kurtosis payoffs from option prices, Hübner et al. (2015) develop a conditional higher-moment asset pricing model that is shown to complement the leading specification applied to hedge funds. Shin et al. (2019) further explore the timing of the option-implied tail risk by hedge funds. They use four proxies for the option-implied tail risk factors including skewness, kurtosis, and two other factors measured as the slope of the regression of out-of-money put-implied volatility on option moneyness.

In this work, we focus on observable discrepancies between options and stock markets, and investigate whether hedge fund managers account for such information to time their strategies. Hereafter, we refer to the dynamics of the option-stock distortions as the "disagreement rate". Even though one may anticipate that relative mispricing of options compared to their underlying would represent valuable information about the foreseeable market evolution, this question surprisingly represents an unexplored dimension of market timing.

Periods of high volatility are likely to discourage arbitrageurs from initiating trading which can itself provoke mispricing. Relative mispricing of S&P 500 futures during a highly volatile market is also evidenced by Tu et al. (2016). Jacoby et al. (2022) hypothesise that heterogeneous beliefs among investors induce mispricing. They relate investors' heterogeneous beliefs to periods of high uncertainty market and find that investors' disagreement beta explains a cross-section of hedge funds' return only in high uncertainty months.

Empirical investigations across different hedge funds' investment styles show that managers formulate less accurate predictions in reaction to sudden shifts in the information set during market turmoil and highly accurate predictions in market normal times. Dragomirescu-Gaina et al. (2021) analyse hedge fund managers' decision-making during periods of high and low uncertainty. By disentangling low-accuracy and high-accuracy betas, generalised impulse response functions show that innovations in the implied volatility index (VIX) entail stronger and faster reactions of the low-accuracy betas than high-accuracy ones.

We approach the issue of hedge funds market timing in two steps. First, we capture the dynamics of our factor model for each hedge fund trading strategy by an unrestricted state-space modelling of the pricing equation in which factor loading follows a random walk.

In the second step, we relate the hedge funds' market timing behavior to the information sent from the options markets in distinctive considerations. To this end, we obtain the factor loadings adjusted by the market disagreement rate and then evaluate hedge funds performance through the returns explained by the adjusted factor loadings. The important merit of our proposed state-space modelling compared to the methodologies usually employed in the literature is that we do not restrict the varying factor loadings by relating them to a priori known indices. It is common practice in the literature to approximate varying exposure by a first-order Taylor expansion series expressed in terms of the supposedly timing variable. The exposure is in fact reduced to a linear model in terms of the underlying timing variable. This modelling approach can cause serious biases because not only the complex nonlinearities hidden in the variation of the factor loading are ignored but also other timing skills that might be used by fund managers are ignored. The autoregressive specification of the exposures overcomes these shortcomings by capturing the dynamics of the timing strategies without the inclusion of any hypothetical timing variable.

We find that option implied disagreement rates contain information about contemporaneous and future market volatility and thus can be used by informed traders to anticipate future market returns. Our analysis on the estimated varying factor loadings and the contemporaneous relation to the disagreement rate reveals that market frictions are an important source of information that are efficiently used by hedge funds in their market timing strategies.

The remainder of the paper is organised as follows: Section 2 reviews the literature regarding the connection and information flow between stock and option markets. Section 3 describes the data on options and hedge funds used in the empirical study. Section 4 explains the methodology employed to detect times of disagreement between stock and option markets and provides some economic insights regarding those specific periods. Section 5 describes the methodology used to infer hedge funds' adaptive trading strategies. Section 6 analyses the global impact of the disagreement rate on funds performance. Finally, Section 7 concludes.

6.2 Information flows between stock and options market

When markets are complete, option trades should not transmit any new information in particular. They can be replicated through a portfolio of stocks and bonds. However, when market frictions appear, a price discovery process might occur between option and stock markets, with the result that both markets transmit different information.

There is a common perception that informed investors might first trade in the options market in order to benefit from the limited downside and the leverage effect. There is pervasive evidence that supports the close connection between the option and stock markets (Black, 1975; Mayhew et al., 1995; Easley et al., 1998; Arnold et al., 2000; Cao et al., 2005; Pan and Poteshman, 2006; Lee et al., 2021). For example, the levels of the expected market volatility, skewness, and kurtosis for the next 30 trading days could be extracted from a cross-sectional series of out-of-the-money option prices Bakshi et al. (2003). These parameters are interpreted as follows. Option-implied volatility levels are powerful predictors of future realised volatility, while a decrease (respectively an increase) in the skewness (respectively kurtosis) of the market portfolio suggests an increase in the probability of experiencing strongly negative (respectively extreme) returns.

Information about general market conditions could have hence an effect on the expected market risk related to both an investment or a company. These risk estimates implied by the options market have been shown to be able to anticipate asset allocation and thus risk exposures of fund managers (Hübner et al., 2015). Investors with private information will preferably trade in options markets (lower short-selling costs, highly leveraged bets). Changes in option prices may therefore reveal information about the underlying asset that is not incorporated in earnings expectation disclosures. Diavatopoulos et al. (2012) have shown the predictive power of changes in expected skewness and kurtosis as implied in option prices on stock returns prior to earnings announcements. These changes in implied moments reflect anticipated information of informed investors or analysts which, therefore, have predictive power for future returns. Besides, Chang et al. (2012) express their forward-looking measure of beta as a function of the variance and the skewness of the underlying distributions. They demonstrate the ability of option-implied moments to anticipate changes in future betas. Furthermore, Chang et al. (2012) shows the significance of option-implied moments for explaining the cross-section of stock returns. Gharghori et al. (2017) evaluate the future stock return and volatility predictability by options traders around stock split periods and find that options traders anticipate volatility and stock returns because of information leakage.[1]

Chakravarty et al. (2004) investigate price discovery by introducing a measure of information share as the contribution of either of the markets to the total variance of the common trend component of the cointegrated spot and implied price series. They also relate the information share to market characteristics including volume, volatility, price spread, and excess return, and find evidence supporting the fact that the options market is more informative during periods of high volume. Lee et al. (2021) examine the information content of options order imbalances and find evidence of predictive power of order imbalances in particular that of the foreign investors has longer time and more significant predictive power.

In times of high volatility, noise trading can be important and make stock prices deviate from their fundamental values as shown by De Long et al. (1990). This could push informed investors to perform sophisticated investment trades in the option markets in order to benefit from leveraged bets. The information content of their trades will therefore be transmitted first into the option prices and create a disagreement event between the two markets. A disagreement day is defined as an event in which the stock and option market disagree about the price of the stock.

Hedge fund dynamic trading around dates of disagreement provides indeed a natural experiment as these managers trade volatility and convexity (Fung and Hsieh, 1997, 2001; Agarwal and Naik, 2004; Chen and Liang, 2007; Agarwal et al., 2017). Besides, there is strong

[1]For further readings about the information content of risk estimates implied by options, see Goncalves-Pinto et al. (2020), Fodor et al. (2017)

evidence supporting the ability of hedge funds to exploit mispricing caused by noise traders. This is supported by Giannetti and Kahraman (2016) who qualify hedge funds as "rational arbitrageurs" or in Jank and Smajlbegovic (2017) who show that short sellers, especially hedge funds, trade against mispricing. Large hedge funds appear to trade on private fundamental information (see Irwin and Holt, 2004). If indeed they first trade in the options market, as would informed traders, those markets should be the first to reflect the information, thereby creating a disagreement.

Some recent works argue against the dominance of informed trading on the information content of the options market. Goncalves-Pinto et al. (2020) document that disagreement between options and the stock market arises as a result of the price pressure in the stock market. According to Hiraki and Skiadopoulos (2021), disagreement between two markets arises as a result of market friction rather than informed trading in the options markets. Patel et al. (2020) relate the disagreement in both markets to the differential speed of information propagation and prosecuted insider trading. Thus, having laid aside the causes of the disagreement between option and stock markets, we raise the question of whether or not the disagreement between options and stock markets can be a source of information that hedge fund managers can use to anticipate future market performance.

6.3 Data

6.3.1 Options data

We collect options data from OptSum. This dataset provides an end-of-day index option summary (bid, ask, volume, and price of the options as well as the price of the underlying asset) for CBOE traded options in SPX from May 1990 to November 2015. To calculate the option closing prices, we follow the methodology recommended by the data supplier[2]. Following Muravyev et al. (2013), we adopt three conditions that need to be satisfied for the

[2]To calculate the option closing prices we look at three components, the last bid, last ask, and the last sale of an option: (i) If the last sale is between the last bid and last ask the close is on the last sale; (ii) If the last sale is less than or equal to the last bid the option series is closed on the last bid and similarly if the last sale is greater than or equal to the last ask, the close is on the last ask; (iii) In the case where there is no last sale for an option series the previous day's close is looked at as if it were the last sale and the same rules are applied; (iv) In the case of a newly listed series having no last sale the close is on the last ask.

options to be included in the study.

1) Liquidity condition: There needs to have been at least one transaction for both the call and the put option of corresponding strike and maturity. If trading volume is nil for either, both options are eliminated.

2) Moneyness condition: $|log(S/K)| < 0.1$, where S is the underlying price and K denotes the strike price. Since volatility smirk is a strong source of mispricing, we include at-the-money options in parallel with the moneyness screening condition.

3) Maturity condition: All options must have a remaining maturity of between 7 days and 90 days.

6.3.2 Hedge funds data

We are interested in market-wide disagreement rates whose dynamics are estimated using state-space estimation techniques. Even though using individual hedge funds data would enable us to gain granularity in the results, aggregation of individual results at the index level might lead us to lose a substantial part of the information because aggregation of the parts -whatever technique is used- does not necessarily represent the true connection between their associated strategy and the disagreement rate on option and stock markets. Thus, using indices appears here to provide a better warranty of a meaningful set of results. The empirical part is conducted on EDHEC alternative indices: Long Short Equity, Market Neutral, Event Driven, Merger Arbitrage, Distressed Securities, Relative Value, Fixed Income, Convertible Arbitrage, CTAs, Global Macro, Emerging Markets, and Short Selling. We excluded Funds of Funds as these funds do not directly trade in the markets. Table 6.1 shows the descriptive statistics of the 226 monthly observations for each of the 12 trading strategies.

The data are retrieved from EDHEC Risk Institute. Contrary to other hedge fund indices, they are neither equal nor value-weighted. The indices come from a Principal Component Analysis, which extracts co-movements within hedge fund styles from several databases. This avoids not only style drift from individual hedge funds when they report to databases but also selection biases from using one single dataset. The databases used differ for each hedge fund style according to coverage and representativity of the datasets. This methodology is called the "indices of indices" as first used in Amenc et al. (2003).

Table 6.1: Descriptive statistics of the returns associated with the hedge fund trading strategies

	mean	median	max	min	std.	skewness	kurtosis	obs.
Convertible arbitrage	0.006	0.008	0.061	-0.124	0.018	-2.63	21.68	226
CTA	0.005	0.004	0.069	-0.054	0.024	0.17	2.86	226
Distressed securities	0.007	0.009	0.050	-0.084	0.018	-1.36	7.92	226
Emerging markets	0.007	0.010	0.123	-0.192	0.034	-1.18	8.72	226
Equity market neutral	0.005	0.006	0.025	-0.059	0.008	-2.38	19.32	226
Event driven	0.007	0.009	0.044	-0.089	0.017	-1.45	7.94	226
Fixed income arbitrage	0.005	0.006	0.037	-0.087	0.012	-3.86	27.47	226
Global macro	0.006	0.005	0.074	-0.031	0.015	0.90	5.33	226
Long/Short equity	0.007	0.009	0.075	-0.068	0.021	-0.39	4.27	226
Merger arbitrage	0.006	0.006	0.027	-0.054	0.010	-1.39	8.59	226
Relative value	0.006	0.008	0.039	-0.069	0.012	-1.89	11.78	226
Short selling	0.000	-0.005	0.246	-0.134	0.049	0.73	5.98	226

6.4 Disagreement events

6.4.1 Identifying disagreement rates

Since SPX options present no early exercise, that is European-style exercise. Similar to Muravyev et al. (2013), we use the Put-Call parity relation to compute daily estimates of the S&P 500 implied stock price. The put-call parity on day t can be presented as:

$$C_t(K,\tau) + e^{-r(\tau-t)}K = P_t(K,\tau) + e^{-q\tau}S_t, \tag{6.1}$$

where $C_t(K,\tau)$ and $P_t(K,\tau)$ respectively represent the price of a call option and a put option with the strike price of K and maturity τ on day t. r represents market yield on the US 3-Month treasury bill at constant maturity[3], and q and S_t respectively represent the monthly dividend yield[4] and the daily spot price of the S&P 500. The next step is to minimise the errors in (6.1) for a given day t to obtain the associated implied price $S_{implied,t}$. The daily

[3]Downloaded from Federal Reserve Bank of St. Louis.
[4]Downloaded from datastream.

error $\varepsilon_t(K, \tau)$ is given by:

$$\varepsilon_t(K, \tau) = C_t(K, \tau) - P_t(K, \tau) + e^{-r(\tau - t)}K - e^{-q\tau}S_{implied,t}. \tag{6.2}$$

Given a certain t, we use an OLS to estimate the $S_{implied,t}$ that corresponds to day t. With daily implied price estimates, we define a disagreement day when the S&P 500 index price and the estimated implied price differ by more than 0.2%. A similar relative threshold has been used in Muravyev et al. (2013). They stipulate two limits for disagreements. If the implied price and spot price differ by 0.02 dollars or they differ by 0.05% a disagreement day is identified. We stipulate three threshold levels for the disagreement day and continue our evaluation of fund managers' responsiveness using three distinct thresholds.

Furthermore, we define monthly disagreement rate as the ratio of the disagreement days to the total number of trading dates in the month, that is:

$$Disagreement\ rate = \frac{Disagreement\ days}{Total\ days\ in\ a\ month}. \tag{6.3}$$

We obtain two sets of disagreement rates with and without at-the-money options (see Figure 6.1). This is motivated by volatility smile which implies demand for in-the-money and out-of-the-money options is greater than at-the-money options. Larkin et al. (2012) evidence volatility smile in the Australian S&P ASX 200 index options. The presence of mispricing is supported by an exploitable delta-neutral trading strategy in the period preceding the global financial crisis. Constantinides et al. (2009) relate mispricing caused by volatility smile in the S&P 500 index options to the distribution of the index return used in the Black-Scholes model. A steeper smile happens when higher price movement is expected in the market.

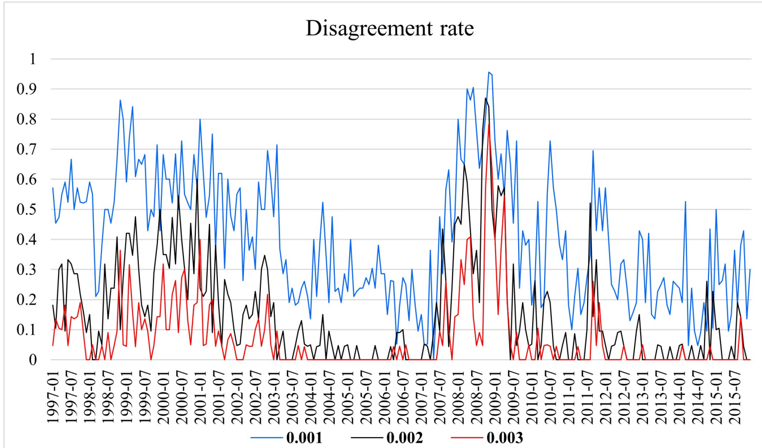
The threshold level of 0.01% delivers 75 months with a disagreement rate of more than 50% for the options with at-the-money options ranging from January 1997 to July 2015. We might relate this mixed evidence to times of high-risk aversion (bearish markets), which command less price discovery as informed investors actively trade within the spot markets and therefore command less disagreement between the information content of the two markets. The threshold levels of 0.02% and 0.03% lead respectively to 11 and 4 months with a disagreement rate of more than 50%. We count 109 months with a disagreement rate of more than 40% corresponding to the 0.01% threshold level. The 0.02% and 0.03% threshold levels lead to 25 and 6 months corresponding to a disagreement rate exceeding 40%. Only one month in which the 0.01% threshold delivers a zero disagreement rate. The number of months with no disagreement corresponding to the 0.02% and 0.03% threshold levels is 62 and 121 respectively. The disagreement between the two markets is significantly reduced in 2001 despite the continuation of the recession. It seems that the disagreement rate hikes during the global financial crisis in 2008 and 2009 and the debt ceiling crisis in 2011.

An interesting finding concerns the different evolution of disagreement rates in two subgraphs in Figure 6.1 during the recession in the early 2000s. During the dot-com bubble burst, mispricing becomes more apparent when at-the-money options are not screened out. All three curves in the second subgraph in Figure 6.1 spike from April until August 2000. Demand for at-the-money options around this period in particular demand for put options must be higher due to fears of future market downward movement. In general disagreement rates are higher after the inclusion of at-the-money options.

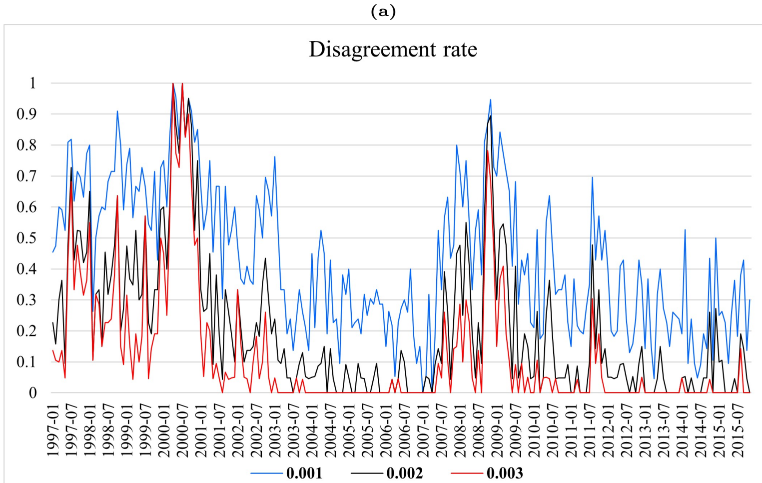
6.4.2 Robustness of the disagreement methodology

Different alternatives could have been considered for defining the disagreement rate: use of intraday data and use of bid-ask prices. We rejected the use of intraday data as we infer hedge fund trades on a monthly basis. Capturing a disagreement day whose disagreement resolves at the end of the day would not be relevant for understanding the hedge fund rebalancing on a monthly basis. We have therefore decided to stick to the latest prices.

We could also have defined disagreement by using implied (last) bid and ask prices. A spot price outside of the implied bid-ask range would define a disagreement day. The problem occurs in times of high volatility in the market, where gamma traders could widen the bid-ask spread as they do not want to carry gamma risks overnight in such risky conditions. This method would therefore be biased by the conservative strategies implied by these trades.



(a)



(b)

Figure 6.1: The disagreement rate. The first subgraph represents the implied price estimated by involving the moneyness condition. The second subgraph represents the implied price estimated by the inclusion of at-the-money options.

The behavior of the series during the early months of the dot-com bubble burst and the global financial crisis is of special interest. It seems that the information content of our disagreement rate has predictive power on the subsequent market crash. This finding is not surprising as Diavatopoulos et al. (2012) find that implied moments can predict future market volatility. Our disagreement rate represents the anticipated information of the informed investors in options markets. This finding around the two important dates is also in line with Gharghori et al. (2017), who argue option traders anticipate future return and volatility ahead of earnings announcements.

6.5 State-space modelling of hedge funds trading strategies

In this section, we obtain time-varying exposures toward a certain set of risk factors. For this endeavor, we use the set of factors presented in Billio et al. (2012), which has been widely used in the hedge fund literature (see for instance Fung and Hsieh, 2002 and Agarwal and Naik, 2004, among others). Each factor is defined as follows:

1. SP: The S&P 500 index, characterizing the US equity market risk factor.[5]

2. SMB: Small minus Big index is computed as the monthly return difference between the MSCI world small minus MSCI world large.[5]

3. HML: High minus Low index is computed as the monthly return difference between the MSCI world value minus MSCI world growth.[5]

4. UMD: The MSCI world momentum factor or the relative performance of winner over loser stocks.[5]

5. EM: MSCI Emerging markets.[5]

6. DVIX: First difference in the implied volatility of the US equity market.[6]

7. GSCI: S&P Goldman Sachs Commodity Index.[5]

8. Term: Term spread measured as the difference between yields on 10-year and 3-month Treasury bills.[6]

9. DEF: Default spread measured by the difference between yields on Moody's Seasoned Aaa rated and Baa rated corporate bond yield.[7]

Trend-following strategies have payoffs that are nonlinear functions of the risk factors. Fung and Hsieh (2001) show that these nonlinearities can be replicated by lookback straddles. Since these factors sought to capture nonlinearities in exposures to risk factors, we assume a constant exposure to these factors, which are referred to as option-like factors. We add five option-like factors that are of growing popularity in the literature during the last few years. These are five trend-following factors consisting of lookback straddles on bond futures (PTFSBD), on currencies (PTFSFX), on commodity futures (PTFSCOM), on the short-term interest rate (PTFSIR), and on the stock market (PTFSSTK). (For further theoretical and

[5]Obtained from Thomson Financial Datastream Inc.
[6]Obtained from CBOE website
[7]Obtained from FRED (Federal Reserve Bank of St. Louis) database.

empirical discussions about option-like factors see Fung and Hsieh, 2001; Agarwal and Naik, 2004; Fung and Hsieh, 2007; Fays et al., 2018 and Chen et al., 2021). All factors range from February 1997 to August 2015. Table 6.2 shows the descriptive statistics of the 226 monthly observations for each of the 14 risk factors.

We rely on Kalman filter to dynamically estimate the unobservable time-varying risk

Table 6.2: Descriptive statistics of the risk factors return

	mean	median	max	min	std.	skewness	kurtosis	obs.
SP	0.006	0.010	0.157	-0.168	0.048	-0.50	4.22	226
EM	0.005	0.007	0.222	-0.273	0.072	-0.41	4.46	226
GSCI	0.001	0.004	0.195	-0.295	0.068	-0.41	4.18	226
Mom	0.005	0.007	0.184	-0.344	0.055	-1.45	11.76	226
SMB	0.003	0.003	0.112	-0.091	0.025	-0.08	5.23	226
HML	-0.001	-0.003	0.081	-0.071	0.022	0.44	5.49	226
CS	0.018	0.020	0.037	-0.007	0.012	2.90	13.48	226
TS	0.010	0.009	0.034	0.005	0.004	-0.27	1.94	226
DVIX	0.000	-0.004	0.205	-0.153	0.047	0.84	7.02	226
PTFSBD	-0.019	-0.040	0.689	-0.266	0.151	1.38	5.68	226
PTFSFX	-0.005	-0.041	0.692	-0.300	0.185	1.25	4.82	226
PTFSCOM	0.000	-0.029	0.648	-0.247	0.146	1.08	4.60	226
PTFSIR	-0.015	-0.067	2.219	-0.351	0.270	4.33	30.04	226
PTFSSTK	-0.049	-0.075	0.666	-0.302	0.147	1.62	7.44	226

SP, EM, GSCI, Mom, CS, TS and DVIX stand for S&P 500, emerging markets, Goldman Sachs commodity index, momentum, credit spread, term spread, and the change in VIX respectively.

exposures. In more detail, we assume a state-space representation, where the observation equation describes the dynamic evolution of each hedge fund's returns, and the state equation defining the unobservable risk exposure evolution is given as an autoregressive process. In contrast to Lambert and Platania (2020), this specification allows mean reversion of the exposures because fund managers may revert to their earlier holdings after a change in the exposures due to market circumstances. Our dynamic factor model is given by:

$$R_t = \alpha + \sum_{i=1}^{N} \beta_{i,t} F_{i,t} + \sum_{i=1}^{M} \gamma_i H_{i,t} + \epsilon_t, \tag{6.4a}$$

$$\beta_{1,t+1} = \phi_1 \beta_{1,t} + \varepsilon_{1,t+1}, \tag{6.4b}$$

$$\vdots$$

$$\beta_{N,t+1} = \phi_N \beta_{N,t} + \varepsilon_{N,t+1}, \tag{6.4c}$$

where $\beta_{i,t}$ represents the time series of risk exposure to factor i, $F_{i,t}$ represents the time series of returns to factor i, ϕ_i represents autoregressive coefficient of exposure to $F_{i,t}$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$, $\varepsilon_{1,t+1} \sim N(0, \sigma_{\varepsilon_1}^2)$, $\ldots$, $\varepsilon_{N,t+1} \sim N(0, \sigma_{\varepsilon_N}^2)$. We pass a set of intervals as initial values of the state variables and other model parameters to an optimization routine[8], where the log-likelihood function is maximised. The intervals corresponding to the standard deviations are $(0, 1)$, and those corresponding to the intercept and factor loadings are $(-5, 5)$. α is the intercept and R_t stands for the time series of returns for a given hedge fund strategy. $H_{i,t}$ is the i-th option-like factors and γ_i denotes the time invariant exposures to the i-th option like factor. Also, as in Agarwal and Naik (2000), since hedge funds exhibit a great deal of flexibility in terms of asset allocation (i.e., short-selling, cash holding, etc) we allow for negative exposure to risk factors and relax the constraint that the style weights have to add up to one.

We perform an analysis on a selection of the time-varying factor loadings. We examine the variable selection information criterion (VIC) developed by Zhang and Wu (2012) to select the risk factors to which the exposures vary. The estimated factor loadings in Equation (6.4a) are, to some extent, robust in the sense that applying the variable selection procedure according to VIC leads to more or less the same evolution of the exposures. Table 6.3 exhibits the factors to which the exposures are selected by the VIC to vary. In the model chosen for each hedge fund style in Table 6.3, the exposures to the other risk factors and the intercept are constant.

In Equation (6.4a), all the loadings on the factors other than option-like factors vary over time, however, we choose, according to Table 6.3, the exposures delivering higher VIC to vary with time. Other exposures are taken constant.

[8] We use the "differential evolution" method to optimise the likelihood function. The algorithm is implemented in Python in Scipy.optimise package. This method searches a larger space of candidate solutions and delivers better optimum points compared to the well-known "fminsearch" procedure.

Table 6.3: Selection of time-varying risk loadings according to VIC

	The selected factors with time-varying exposure	VIC
Convertible arbitrage	EM-CS	-6.90
CTA	EM-GSCI	-2.41
Distressed securities	SP-EM-SMB- CS	-3.97
Emerging markets	EM-CS	-3.15
Equity market neutral	SP -EM-HML-CS	-5.77
Event driven	SP-EM-SMB-CS	-3.96
Fixed income arbitrage	EM-GSCI-CS	-9.01
Global macro	SP-EM- SMB -CS	-3.67
Long/Short equity	SP- EM-SMB- HML -CS	-4.20
Merger arbitrage	SP-EM- SMB -CS	-4.64
Relative value	SP-EM-CS	-5.82
Short selling	SP-SMB-HML	-1.92

SP, EM, GSCI, Mom, CS, TS and DVIX stand for S&P 500, emerging markets, Goldman Sachs commodity index, momentum, credit spread, term spread and the change in VIX respectively.

6.6 Examining hedge funds individual dynamic asset allocation

These new perspectives on stock and options market joint equilibria have the potential to give insightful explanations about hedge fund dynamic trades. We infer hedge fund positions from their dynamic beta. In this section, we examine whether hedge fund managers significantly alter their trades, when there is an imbalance between the information content of option and stock markets.

6.6.1 Test hypotheses

In section 6.5, we assumed a state-space representation, where the unobserved risk exposure to each factor follows a random walk as in Equations (6.4b) and (6.4c), and the filtered coefficients at time $t+1$ are optimally computed by Kalman filter. Such representation provides crucial information about the β's distribution and statistical properties in particular. We formulate two hypothesis tests. The first one concerns the dynamic portfolio management by fund managers. The second one aims to test the implications of the information content of the options market for fund managers. In the second hypothesis, we test for the impact of the disagreement on portfolio rebalancing. Hence, we define the following set of hypotheses to be tested.

Hypothesis 1:

- Null hypothesis $H1_0$: Hedge fund managers do not dynamically rebalance their portfolio.

- Alternative hypothesis $H1_a$: Hedge fund managers dynamically rebalance their portfolio resulting in a factor model in which each exposure (except γ_is) follows a random walk.

In order to perform the instability test on the factor loadings, we first regress each strategy on a constant and the option-like factors. Then the residuals of these regressions are examined in a second regression including the factor set $F_{i,t}$ as explanatory variables for possible dynamic instability. We use the L_c test statistic developed by Hansen (1992a) to test for varying exposures.

Hypothesis 2:

- Null hypothesis $H2_0$: A disagreement has no impact on hedge fund trades.

- Alternative hypothesis $H2_a$: A disagreement triggers an unexpected reallocation hedge fund

Given the state-space model, the abnormal or unexpected allocation to factor i is defined as:

$$AA_{i,t} = |\beta_{i,t} - E[\beta_{i,t}]| = |\beta_{i,t} - \beta_{i,t-1}|. \tag{6.5}$$

These values are then projected on the disagreement rates to investigate the contribution of the information in the options market in anticipating future stock market performance. To this end, we estimate the following regression to evaluate hedge funds' responsiveness to the disagreement rates:

$$AA_{i,t} = c + b\hat{D}_t + \nu_t. \tag{6.6}$$

where c and b are scalars, $\hat{D}_t$ is the estimated disagreement rate, and $\nu_t \overset{iid}{\sim} N(0, \sigma_\nu)$

6.6.2 Market timing

We further investigate whether hedge fund managers can time their trading using the information content of the disagreement rate. We perform a two-step analysis on the filtered factor loadings. First, we simply relate the absolute change in the factor loadings to the disagreement rate. This analysis aims to reveal whether there is any significant relation between the information stemming from the options market and the trading strategies performed by fund managers. Second, we investigate whether hedge fund managers can strategically time their trades to realise returns outperforming the market. To this end, we obtain the explained time-varying exposures after having projected them on the disagreement rates. The explained exposures are then used in the pricing Equation (6.4a) to yield the returns attributable to the disagreement rates.

We examine funds' performance by projecting the estimated factor loadings on the estimated disagreement rate or the uncertainty index. We use the projected factor loadings to predict hedge fund returns. The two-step estimation is given by:

$$\hat{\beta}_{i,t} = c + b\hat{D}_t + \eta_t, \tag{6.7}$$

$$\hat{R}_{i,t} = \hat{\alpha} + \sum_{i=1}^{N} \hat{\hat{\beta}}_{i,t} F_{i,t} + \sum_{i=1}^{M} \hat{\gamma}_{i,t} H_{i,t}, \tag{6.8}$$

where $\eta_t \sim N(0, \sigma^2_\eta)$, $\hat{\beta}_{i,t}$ denotes the estimated time-varying loading on i-th factor, $\hat{D}_t$ denotes the estimated disagreement rate. $\hat{\hat{\beta}}_{i,t}$ stands for the projected $\hat{\beta}_{i,t}$ on $\hat{D}_t$ as predicted by Equations (6.7). $\hat{\gamma}_{i,t}$ denotes the estimated time-invariant loading on the option-like factors by Equation (6.4a). $F_{i,t}$ and $H_{i,t}$ are as in Equation (6.4a) and c, b are scalars and $\hat{\alpha}$ is the estimated intercept in Equation (6.4a). $\hat{R}_t$ in Equation (6.8) represents the hedge fund return predicted by the disagreement rate. $\hat{R}_t$ subsumes information on the way fund managers use the disagreement rate to time their trading strategies. A threshold regression in Equations (6.7). We obtain Sharpe ratios of the predicted returns associated with different thresholds of the two variables of interest.

6.6.3 Empirical findings

We apply the L_c instability test developed by Hansen (1992a). The L_c statistic is calculated from the cumulative first-order condition of the ordinary least squares (OLS). In the simple univariate regression, the L_c statistic is calculated as the ratio of the average sum of squares of the cumulative first-order condition to the sum of squares of the first-order condition. The test statistic can be generalised to test for joint instability of the coefficients. The asymptotic distribution of the test statistic depends on the number of the parameters under the instability test.

We first regress the series of each trading strategy on a constant and the five option-

Table 6.4: Hansen instability test

strategy	convertible arbitrage	CTA	distressed securities	emerging markets
L_c	2.53^{*}	1.88	3.15^{***}	3.96^{***}
strategy	equity market neutral	event driven	fixed income arbitrage	global macro
L_c	3.96^{***}	2.57^{**}	1.84	3.40^{***}
strategy	long/short equity	relative value	short selling	merger arbitrage
L_c	2.58^{**}	3.98^{***}	2.67^{**}	5.06^{***}

The critical values at 1%, 5% and 10% significance levels are 3.05, 2.54 and 2.29 respectively. First, we regress the series of each trading style on a constant and the option-like factor. Then we use the residual series and the risk factors to perform the Hansen instability test. The superscripts *,** and *** represent respectively the significance at 10%, 5% and 1% levels.

like factors to obtain residuals. Then we use the residuals in a second regression on the nine risk factors to test for joint instability of the coefficients. As noted by Hansen (1992*a*), if the model includes numerous regressors, the L_c test statistic for the individual instability tests is a small number, whereas the test statistic of the joint instability test would be large. In that case, as indicated by Hansen (1992*a*), the joint test is more reliable than the individual tests. Table 6.4 shows the results of applying the Hansen instability test on the null hypothesis of the joint stability of the 9-factor loadings. In the individual stability test, the null hypothesis on the stability of each factor is not rejected for either of the factor loadings. However, we observe evidence of joint instability for most of the trading strategies.

Table 6.5 presents the estimated parameters of the univariate regression of AA_is on a constant and the disagreement rate. A significant intercept in the regression implies a varying exposure regardless of the fact that there is a significant slope on the disagreement rate or not. Because a significant intercept implies a significant abnormal allocation which translates, through Equation (6.5), to variation in the exposure. We observe strong evidence of the significance of the intercept in both Table 6.5 and Table 6.6. The strongly significant constant across the strategies and factor loadings implies the re-allocation of the portfolios by fund managers. The slope of the disagreement rate turns out to be significant across many factor loadings and hedge fund investment styles. As Table 6.5 shows, the exposures to S&P

500 factor are significant with only one exception related to relative value investment styles. The abnormal allocation of the shortselling strategy has a negative slope on the disagreement rate. This can be explained by the fact that during periods of higher disagreement which is coincident with market crash, managers of this investment style do not rebalance their market exposure to benefit from downward price movements. SML is a risk factor to which exposures are not significantly adjusted during periods of relative mispricing. The same can be concluded on Table 6.6. GSCI is a risk factor to which exposure is not adapted during periods of high disagreement between options and stock markets. This can be more or less expected because CTA funds mostly trade in commodity markets and it is natural that there is no link between their exposure to the commodity risk factor and mispricing in equity markets. The exposure to credit spread covariates significantly with the disagreement rate. This is also expected because, during periods of a bearish market, investors fear default.

Table 6.5: Regression of abnormal allocations on the disagreement rate without at-the-money options

	Convertible arbitrage		CTA		Distressed securities		Emerging markets	
	C	coefficient	C	coefficient	C	coefficient	C	coefficient
SP					0.0513***	0.1200***		
EM	0.0140***	0.0391***	0.0746***	0.0121	0.0433***	0.0411**	0.1218***	0.0906*
GSCI			0.0267***	0.0111				
Mom								
SML					0.0034***	0.0001		
HML								
CS	0.7478***	0.4290			0.0057***	0.0068*	0.0094***	0.0160***
TS								
DVIX								

	Equity market neutral		Event driven		Fixed income arbitrage		Global macro	
	C	coefficient	C	coefficient	C	coefficient	C	coefficient
SP	0.0270***	0.1843***	0.0427***	0.0444**			0.0045***	0.0072**
EM	0.0056***	0.0134***	0.0263***	0.0227	0.0476***	0.0999***	0.0622***	0.0576**
GSCI					0.0026***	0.0049**		
Mom								
SML			0.0041***	0.0012			0.0011***	-0.0001
HML	0.0002***	0.0007***						
CS	0.0093***	0.0131*	0.7803***	-0.0010	0.0046***	0.0024***	0.0064***	0.0030**
TS								
DVIX								

	Long/Short equity		Relative value		Short selling		Merger arbitrage	
	C	coefficient	C	coefficient	C	coefficient	C	coefficient
SP	0.0921***	0.0359	0.0130***	0.0063	0.0003***	-0.0001*	0.0643***	0.1828***
EM	0.0729***	0.0595*	0.0984***	-0.0170	0.0242***	0.0758***	0.0938***	0.2278***
GSCI								
Mom								
SML	0.0045***	-0.0001	0.0006	0.0015				
HML	0.0000	0.0001						
CS	0.0060***	0.0028**	0.0120***	0.0167	0.0076***	0.0031**	0.0037***	0.0032*
TS								
DVIX								

The disagreement series corresponds to the 0.2% threshold level. SP, EM, GSCI, Mom, CS, TS, and DVIX stand for S&P 500, emerging markets, Goldman Sachs commodity index, momentum, credit spread, term spread, and the change in VIX respectively. The time-varying factor loadings are selected according to VIC. The superscripts *,** and *** represent respectively the significance at 10%, 5% and 1% levels.

Table 6.6: Regression of abnormal allocations on the disagreement rate with at-the-money options

	Convertible arbitrage		CTA		Distressed securities		Emerging markets	
	C	coefficient	C	coefficient	C	coefficient	C	coefficient
SP					0.0525***	0.0866***		
EM	0.0144***	0.0305***	0.0722***	0.0122	0.0420***	0.0387***	0.1050***	0.1394***
GSCI			0.0293***	-0.0020				
Mom								
SML					0.0037***	-0.0013		
HML								
CS	0.8070***	0.1330			0.0073***	-0.0016	0.0105***	0.0083**
TS								
DVIX								

	Equity market neutral		Event driven		Fixed income arbitrage		Global macro	
	C	coefficient	C	coefficient	C	coefficient	C	coefficient
SP	0.0321***	0.1249***	0.0285***	0.0846***			0.0042***	0.0071***
EM	0.0060***	0.0085***	0.0215***	0.0314***	0.0562***	0.0449*	0.0602***	0.0584***
GSCI					0.0027***	0.0031*		
Mom								
SML			0.0037***	0.0020			0.0010***	0.0002
HML	0.0001**	0.0008***						
CS	0.0030*	0.0349***	0.6270***	0.4367**	0.0041***	0.0040***	0.0054***	0.0064***
TS								
DVIX								

	Long/Short equity		Relative value		Short selling		Merger arbitrage	
	C	coefficient	C	coefficient	C	coefficient	C	coefficient
SP	0.0789***	0.0693**	0.0121***	0.0076	0.0003***	-0.0001***	0.0705***	0.1288***
EM	0.0635***	0.0757***	0.0936***	0.0011	0.0303***	0.0379***	0.0672***	0.2814***
GSCI								
Mom								
SML	0.0043***	-0.0001	-0.0001	0.0035***				
HML	0.0000	0.0002***						
CS	0.0051***	0.0060***	0.0031	0.0481***	0.0066***	0.0063***	0.0024***	0.0077***
TS								
DVIX								

The disagreement series corresponds to the 0.2% threshold level. SP, EM, GSCI, Mom, CS, TS, and DVIX stand for S&P 500, emerging markets, Goldman Sachs commodity index, momentum, credit spread, term spread, and the change in VIX respectively. The time-varying factor loadings are selected according to VIC. The superscripts *,** and *** represent respectively the significance at 10%, 5% and 1% levels.

Overall, what can be concluded from Tables 6.5 and 6.6 is that most of the hedge funds use the disagreement rate to change their exposure to the market risk. This analysis indicates the importance of the information associated with the options market for fund managers, nevertheless, it entails no implication on favorable use of the information by the fund managers.

6.6.4 Performance measurement of market timing

Quantile regression by Tu et al. (2016) confirms positive influence of VIX on futures mispricing. Depending on the extent of mispricing, fund managers should react heterogeneously. Dragomirescu-Gaina et al. (2021) argue that in highly uncertain markets managers focus on a smaller set of data whereas in normal periods higher attention is required to identify profit opportunities. Accordingly, If hedge fund managers care about the disagreement between options and stock markets, their reaction should differ depending on the extent to which the two markets disagree. This motivates us to obtain $\hat{R}_t$ using threshold regression on the data set for which the disagreement rate exceeds equally distanced levels. In this section, we present the main findings on the favorable use of the variable of interest by fund managers. We examine if the information implied by the options market is used by fund managers to anticipate future market returns and to improve their performance. As the results in Table 6.5 and Table 6.6 indicate, hedge funds' performance in different levels of disagreement is also worthy of investigation. This analysis will let us explore the efficient responsiveness of managers of the investment styles for which selected factor loadings do not covary with the disagreement rate (i.e., CTA, or relative value in Table 6.5).

Figures 6.2 and 6.3 show the performance of each strategy across different thresholds on the disagreement rate. Since the disagreement rate barely exceeds the 50% level (with only 11 months exceeding 50% out of the 226 months), we include the thresholds until this level. The graphs exhibit the Sharpe ratios predicted by three disagreement series obtained from three daily threshold levels of 0.1%, 0.2%, and 0.3%. The predicted returns of almost half of the strategies outperform the realised returns in the first two thresholds. The Sharpe ratios associated with the daily percentage differences of 0.2% and 0.3% exhibit, in more than half of the cases, a decreasing performance in higher levels of monthly disagreement. The results associated with the 0.1% daily difference are mixed. This is also expected, because in highly turmoil markets when the stock market realises huge losses, the positions of hedge funds, which involves mostly long positions, will result in loss. There are four strategies for which the predicted returns do not outperform the realised ones. These strategies are CTA, emerging markets, long/short, and short selling. This is to some extent expected because CTA funds trade commodities and seem not to follow stocks' relative mispricing. Hedge funds that trade in emerging markets also seem not to rebalance their exposure according to the relative mispricing in the US stock markets. Long/short strategies which take on relatively high long bias compared to short positions, are not able to benefit from mispricing. Their predicted performance in high levels of disagreement is even worth it when markets make huge losses. Short selling is a strategy for which the realised Sharpe ratio is negative and does not seem to have performed well at all. The performance of convertible arbitrage and equity market neutral is improved across a number of higher thresholds. There is also a good deal of evidence that the performance is improved during the months when the disagreement exceeds 10%. We find strong evidence that distressed securities funds perform poorly during periods of high disagreement. The ability of these funds to exploit mispricing in higher disagreement days seems to be limited. There is very weak evidence that a short selling strategy is able to exploit mispricing. It does not perform well even during months of high disagreement when markets become extremely bearish.

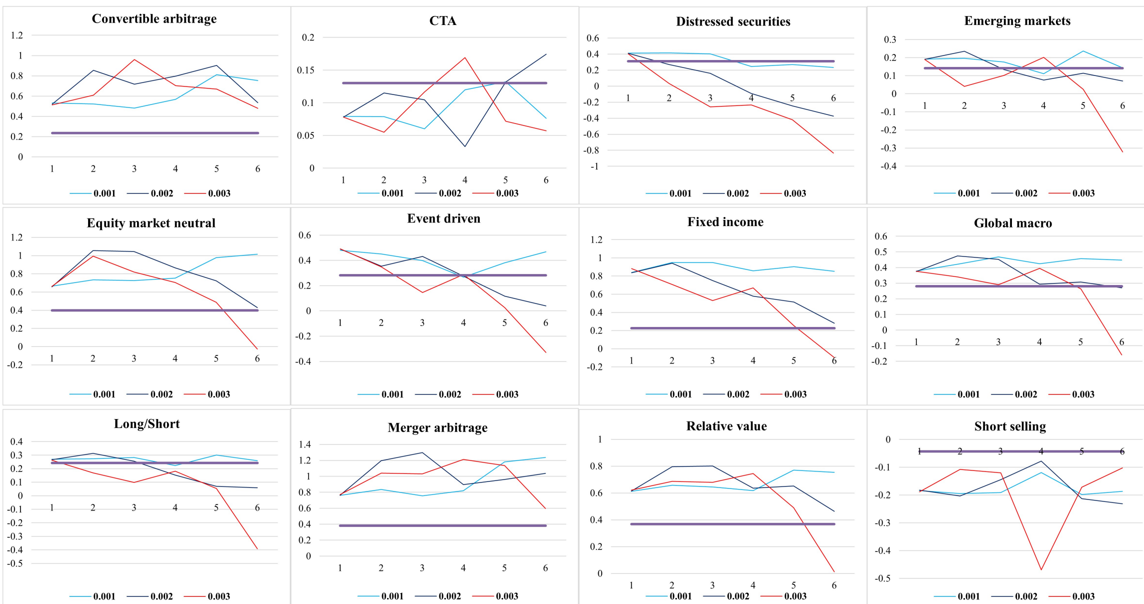


Figure 6.2: Sharpe ratios of the hedge fund returns predicted by the disagreement rate. The screening condition of the options includes at-the-money options. The vertical axis represents the Sharpe ratio, and the horizontal axis represents the threshold of the disagreement rate. The first threshold involves all data. In the next threshold, the months for which the disagreement rate is less than 0.1 are excluded. The fifth threshold excludes the months in which the disagreement rate is less than 0.5. The time-varying factor loadings are selected according to VIC.

6.7 Concluding remarks

Our paper's thesis is that if implied moments conduct forward-looking information, there should be active trading, which makes implied prices coming from options deviate from stock prices. We assumed an inclusive factor model comprising both option-like factors and nine widely used risk factors in the literature. Having assumed time-invariant exposures to the option-like factors, we examined the instability of the risk exposures through the Hansen in-

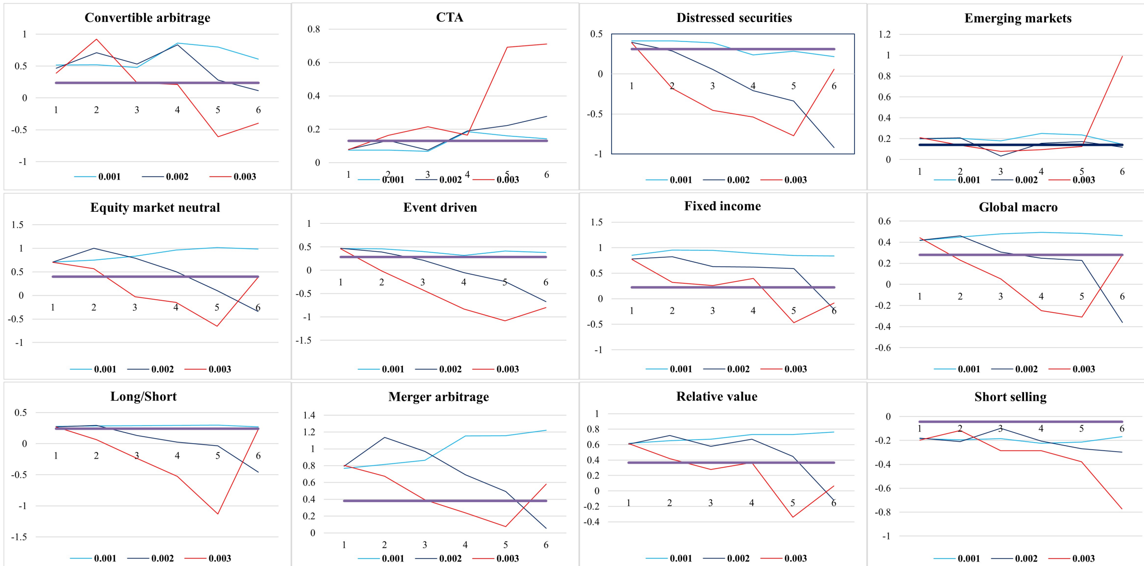


Figure 6.3: Sharpe ratios of the hedge fund returns predicted by the disagreement rate. The screening condition of the options includes moneyness condition. The vertical axis represents the Sharpe ratio, and the horizontal axis represents the threshold on the disagreement rate. The first threshold involves all data. In the next threshold, the months for which the disagreement rate is less than 0.1 are excluded. The fifth threshold excludes the months in which the disagreement rate is less than 0.5. The time-varying factor loadings are selected according to VIC.

stability test.

We related the asset positions (inferred from a time-varying multifactor model on hedge fund returns) to the disagreement between option-implied prices and stock prices. Therefore, this assumption implies that price discovery takes place and a flow of information comes from one market to the other. The disagreement between the two markets gets wider in periods of the bear market.

This paper also considers specific events, where we observe market frictions in the information transmission between option and stocks markets. We identified the periods that are associated with rising information imbalance between the two markets. Such disagreement or inefficient events are more likely to occur in times of high volatility and noise trading as well as in times of active gamma trading. Such events have significant implications for portfolio management by creating opportunities for traders who implement reversal strategies and tactical style allocation between small, large, value, and growth stocks. We implemented an analysis of the impact of the disagreement rate on hedge fund abnormal allocations. Our analysis shows that an imbalance of information between the stock and option markets, which we qualify as disagreement, is contemporaneously and positively correlated with abnormal portfolio rebalancements. Our work relates to Bernales et al. (2020) who recently demonstrated not only strong herding behavior in times of high market volatility or macroeconomic events in the underlying stock markets, but also in the most sophisticated option markets. In our framework, we show that such events, mostly related to macroeconomic events, cause disagreement between the two markets. Trading on market factor is by far the most dynamic strategy across all hedge fund styles. The reallocation in this strategy is positively associated with market frictions, which escalate during periods of economic slowdown. Active trades depending on the disagreement are also found in credit spread strategies.

We raised the question of hedge fund managers time the disagreement between options and stock markets and then adjust their exposures accordingly. Using the projected time-varying exposures on the disagreement rate, we obtained a series of hedge fund returns that can be used to evaluate the market timing of fund managers. The Sharpe ratios of the returns obtained from a threshold regression reveal that market frictions are of paramount importance for managers as they contain information that predicts future market returns. The threshold of 10% for the disagreement rate is considered as an important signal by fund managers to initiate portfolio rebalancing.

Chapter 7

Conclusions

This dissertation revisits two classical questions of financial economics. The first classical question concerns the stock markets' interdependence of the G7 countries which is investigated by a time-varying specification. The second question concerns PPP across the G7 member countries. In addition, the work targets the well-known issue of hedge fund managers' responsiveness from a new perspective that is new in the literature. The focus of the third and fourth chapters is on the dynamics of stock markets' bivariate as well as multivariate cointegration. The work examines the role of stock market return determinants, EPU, and GPR indies on the instantaneously varying cointegrating vectors. The fifth chapter links the real exchange rates of the G7 countries to the interest rate differential via uncovered interest parity and relates the evolution of the RER to two money-flow components associated with the speculative motive and goods transaction motive. The 6th chapter investigates fund managers' use of mispricing which is proxied by a disagreement index between options and stock markets.

The long-lasting question of cointegration between stock market indices is found in numerous works to bear variation as a result of changes in economic circumstances. As the literature study shows, the significance of a static relationship has been studied by numerous researchers. In these investigations, economic crises have been identified as a turning point in the equilibrium relationship. For example, the crisis of 1987 is found to have led to a structural break in the relationship. In addition, other structural changes in the economy and policy of control or liberalisation of the exchange rate have also affected the equilibrium relationship.

These findings present a new problem in front of us. That the circle of factors affecting the equilibrium relationship between stock market indices can be wide and not limited to one or more factors. This conjecture prompted us to model and study the evolution of the equilibrium relations with the state space method. In general, the results estimated with the Kalman filter show that numerous risk factors and macroeconomic variables significantly impact cointegrating vectors. Specifically, in Chapter 3, we focused on the effects of uncertainty indicators and geopolitical conflicts. The results of the study showed that, in general, in all periods, the US stock market index is more exposed to our set of risk factors than other stock markets. The study reveals that global geopolitical crises have a significant effect on the equilibrium relations between the G7 stock markets. In addition, the negative sign of the estimated coefficient showed that the value of the US stock market index bears a greater drop than other stock indices of the G7 countries. Another notable finding concerns the reaction of the equilibrium relation to this index in periods of a bearish market. During periods of bearish stock market, the US stock market undergoes a greater decline than other markets, and this indicates that this market is more exposed to risk factors than other markets of the G7 countries.

Although the results obtained in Chapter 3 are novel and interesting, the question can be examined from another standpoint, which Chapter 3 leaves unanswered. This aspect is related to the degree of cointegration, which is an idea analogous to the correlation coefficient. The degree of cointegration in the literature is sometimes stated as the number of cointegrating vectors. However, our meaning of cointegration degree in this thesis is to find a measure for the intensity of the cointegration relations. In Johansen (1988)'s model, the canonical correlation coefficient of the VECM is a suitable candidate for the degree of cointegration. In Chapter 4, we estimated the time-varying canonical correlation with a time-varying specification of the VECM. In addition, we defined and derived a measure representing the instability of the cointegration vector. The results showed that globalisation strengthened the equilibrium relations until 2010. In addition, China as an emerging country has had a disruptive effect on the equilibrium relations. Our findings show that economic crises or changes in the monetary policy of the central banks and trade wars adversely impact the degree of cointegration.

A particular finding in Chapter 4 is the evolution of the cointegrating vectors in Figures 4.3, 4.7, and 4.10. If we are to analyse the varying magnitude of the coefficients, we have to normalise them with respect to a certain country, otherwise, that would be hard to draw inferences about the evolution of relative elasticities of the indices. However, an analysis of the varying elasticities of the countries over time can be a result of varying exposure to unknown risk factors. Given the current results and the data we used in the analysis, the computational complexity does not let us get insight into the reasons for the growth and decay of the elasticities. This is an important question that can be investigated in the future. One possible approach would be to augment cointegrating vectors with exogenous variables. That will lead to a high computational complexity of the model. As an alternative method, the Bayesian model can be used to investigate this problem.

Current literature on price convergence is not limited to data on the floating exchange rate regime. In this dissertation, we focused on the post-euro era. The high accumulation of capital in the last few decades and the possibility of a large speculative flow of money between the developed economies of the world can be a potential reason for the non-convergence of prices. After deriving a specification between PPP and UIP, we find supporting evidence for price convergence between some of the countries. After augmenting our specification with uncertainty indicators, we find that domestic economic uncertainty can lead to currency depreciation. Regarding the uncertainty related to the US monetary policy, we find that this variable depreciates the US dollar against other currencies. However, the effect of this variable on the exchange rate of the euro and Japanese Yen is different. The results show that the uncertainty in the US monetary policy leads to the depreciation of European currencies in front of the Japanese yen. Further investigations show that the price convergence process between the British pound and G7 European countries has strengthened in the post-euro era. And Brexit has only had a temporary and transitory effect.

Certain macro indicators related to actual money demand, e.g., Business Confidence

Index (BCI), Consumer Confidence Index (CCI), or the ones that are related to the future money demand, e.g., Composite Leading Indicator (CLI), and Producer Price Index (PPI) can also possibly explain the variation of the exchange rates. CLI, BCI, CCI, and PPI provide early signals about business cycles and are related to the consumption of goods and services. Although they have nothing to do with foreign trade, they have information about the demand for money that impacts currency valuation. Currency demand is of course impacted by consumption. If we augment the model with an index related to consumption, then the resulting model will be a combination of PPP and the quantity theory of money. For future investigation, the current model can be augmented with these macroeconomic variables that are related to the demand for money. Special attention should be paid to the modelling because, in the current model, the second component evolves by the part of money demand related to the transactions motives, and these variables are all related to the transactions motive.

In chapter 6, hedge funds' market timing with respect to relative mispricing between options and stock markets is investigated. The chapter addresses the question of whether or not fund managers' time risk exposures according to the extent of the disagreement between the two markets. We quantified six series to represent the disagreement rate between the two markets. Our findings support the existence of a significant relationship between the rate of disagreement and the hedge funds' reaction. In addition, we found evidence of pertinent use of the disagreement rate by fund managers.

For future investigation, it is also interesting to obtain disagreement between the futures market and spot price. As Figure 6.1 shows, the disagreement rate rises in periods of high economic uncertainty. EPU is shown to be a risk factor that is priced in the stock markets. The interrelation between the EPU and the disagreement rate and whether or not the EPU spawns mispricing deserves more in-depth analysis in future investigations. Likewise, the returns predicted by time-varying projected exposures on the EPU can be obtained. The results of threshold regression can show if fund managers are inclined to time their strategies according to the EPU level.

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APPENDICES

Appendix A

Estimation of time-varying slopes by Kalman filter

The state-space models in this dissertation are implemented in Python. This section presents a number of simulated processes that are estimated by the Python programs. The first simulated model is a bivariate model in which the coefficient is a cosinusoidal function.

$$X_t = 2 + b_t Y_t + \delta_t \quad \delta_t \sim N(0, 0.1), \tag{A.1}$$

$$Y_t = Y_{t-1} + \epsilon_t \quad \epsilon_t \sim N(0, 0.1), \tag{A.2}$$

$$b_t = 0.5cos(t/30). \tag{A.3}$$

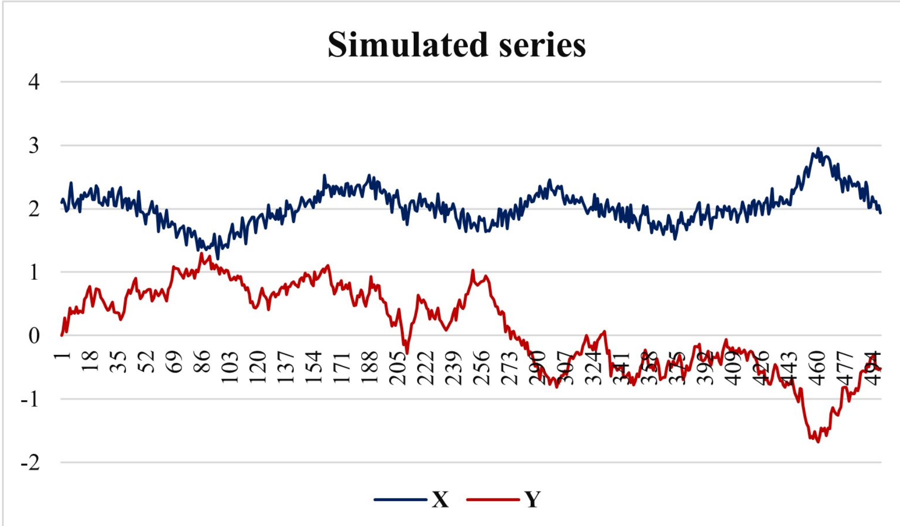


Figure A.1: he bivariate simulated series.

Figure A.2 presents the true slope in Equation (A.3). The graph involves the filtered and the smoothed state variables estimated by the Kalman filter.

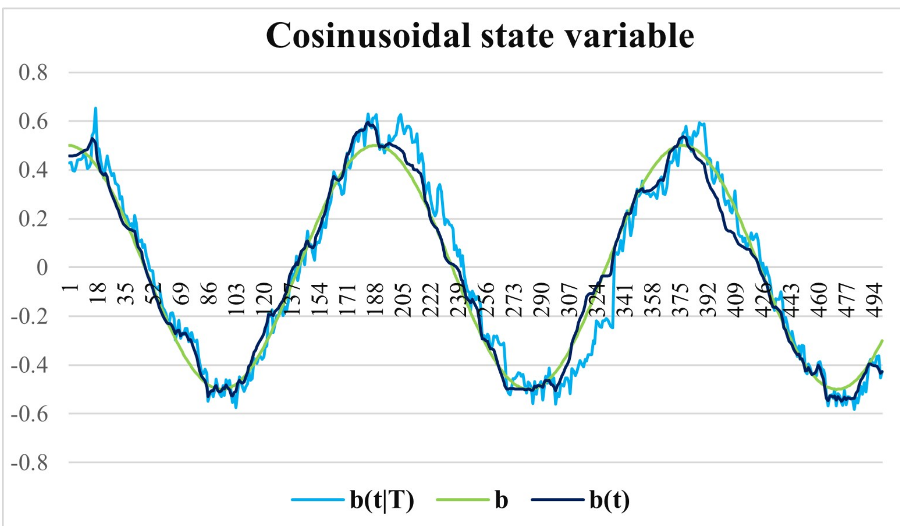


Figure A.2: The estimated varying slope.

Table A.1: The estimated parameters of the bivariate simulated series by Kalman filter

$\hat{c}$	$\hat{b}_{,0}$	$\hat{\sigma}_\delta$	$\hat{\sigma}_\epsilon$	max LL
2.0097***	0.4997	0.0912	0.0433	412.9813

*, ** and *** respectively stand for the significance at 1%, 5% and 10% levels.

In the next bivariate simulation, we assume the state variable follows a random walk.

$$X_t = 2 + b_t Y_t + \delta_t \quad \delta_t \sim N(0, 0.1), \tag{A.4}$$

$$Y_t = Y_{t-1} + \epsilon_{1t} \quad \epsilon_t \sim N(0, 0.1), \tag{A.5}$$

$$b_t = b_{t-1} + \epsilon_{2t} \quad \epsilon_t \sim N(0, 0.2). \tag{A.6}$$

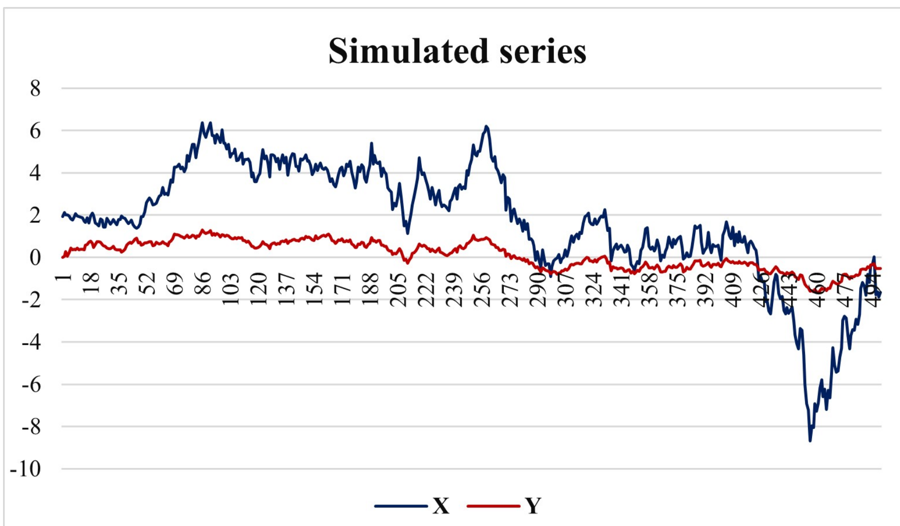


Figure A.3: he bivariate simulated series.

Figure A.4 exhibits the simulated slope in Equation (A.6) coupled with the filtered and the smoothed ones by Kalman filter.

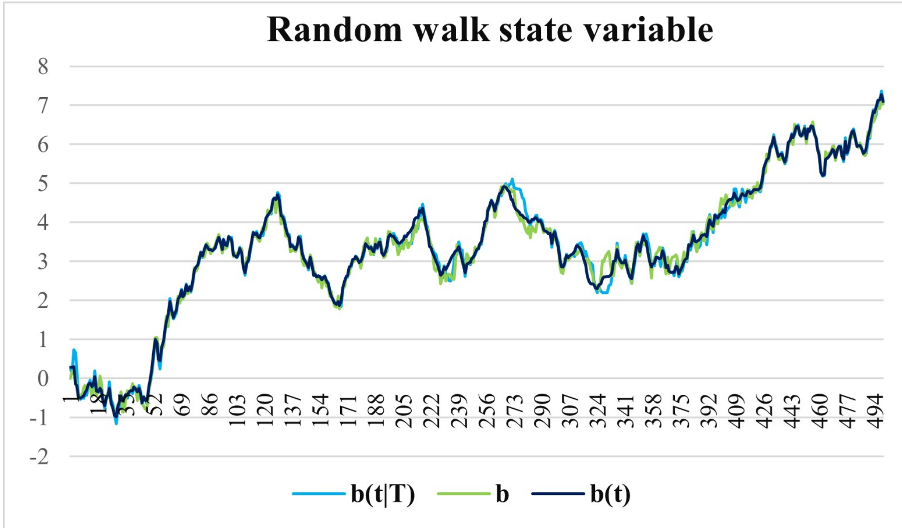


Figure A.4: The estimated varying slope.

Table A.2 shows the estimated parameter of the bivariate model (A.4), (A.5) and (A.6).

Table A.2: The estimated parameters of the bivariate simulated series by Kalman filter

$\hat{c}$	$\hat{b}_{,0}$	$\hat{\sigma}_\delta$	$\hat{\sigma}_\epsilon$	max LL
2.0002***	0.2880	0.0980	0.2091	155.0710

*, ** and *** respectively stand for the significance at 1%, 5% and 10% levels.

The second simulated model is a trivariate model in which the coefficients are a step function and a cosinusoidal function.

$$X_t = 2 + b_t Y_t + c_t Z_t + \delta_t \qquad \delta_t \sim N(0, 0.8). \tag{A.7}$$

$$Y_t = Y_{t-1} + \epsilon_{1,t} \qquad \epsilon_{1,t} \sim N(0, 0.5), \tag{A.8}$$

$$Z_t = Z_{t-1} + \epsilon_{2,t} \qquad \epsilon_{2,t} \sim N(0, 0.5), \tag{A.9}$$

and the state equations are:

$$b_t = \begin{cases} 5 & 0 \leq t \leq 500, \\ -5 & 500 \leq t \leq 1000, \end{cases} \tag{A.10}$$

$$c_t = 0.5\ cos(t/25). \tag{A.11}$$

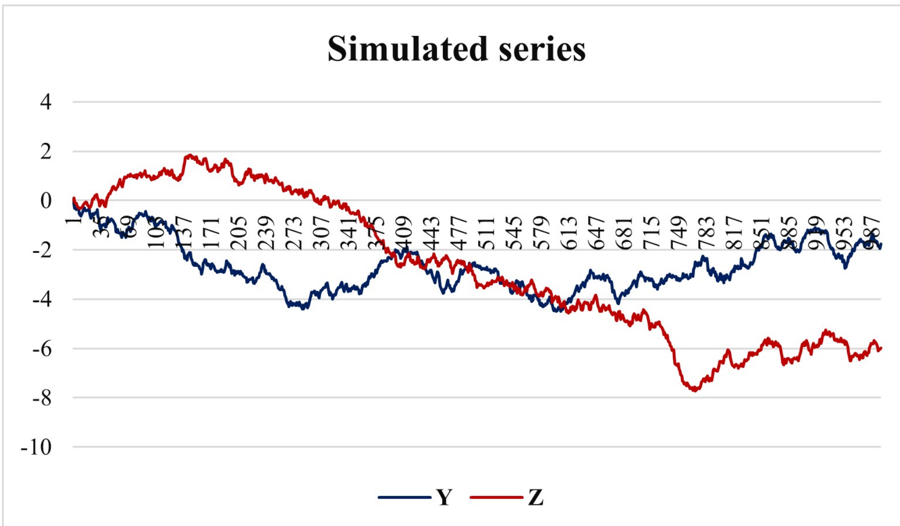


Figure A.5: The trivariate simulated series.

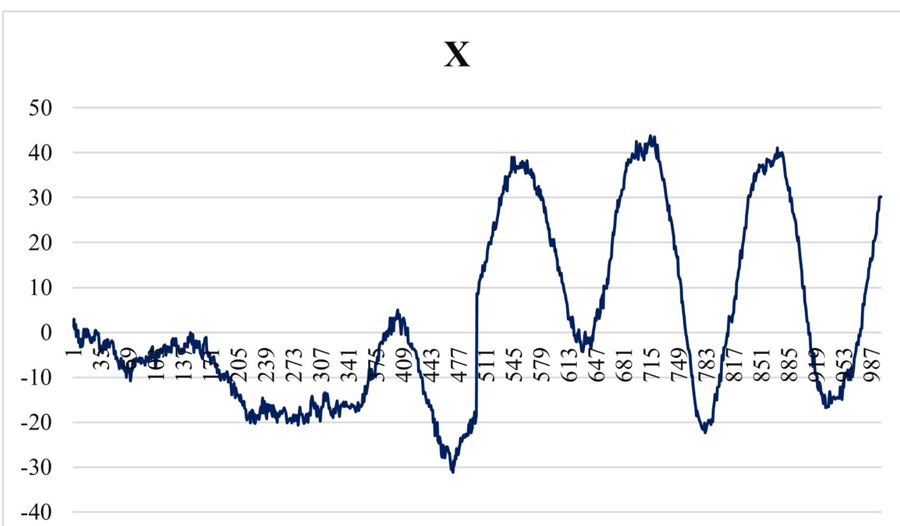


Figure A.6: The simulated X_t of the trivariate model.

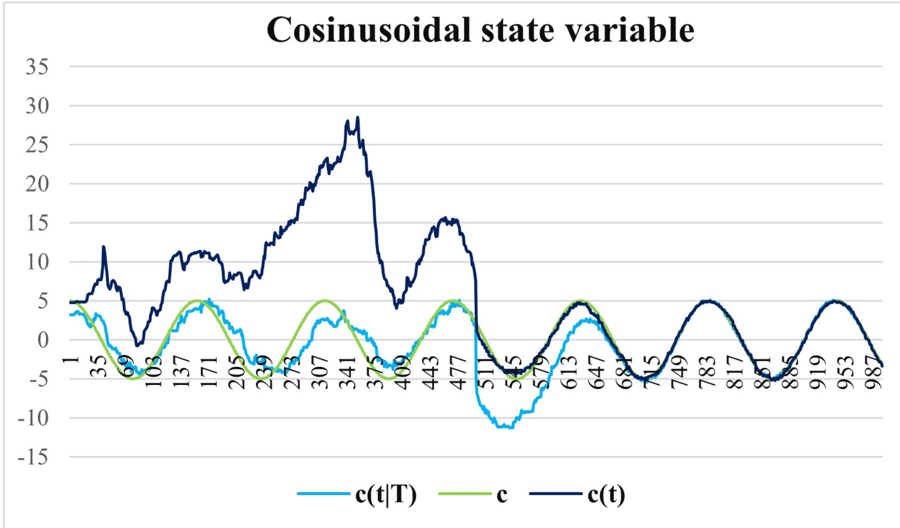


Figure A.7: The time-varying cosinusoidal coefficient in the trivariate model.

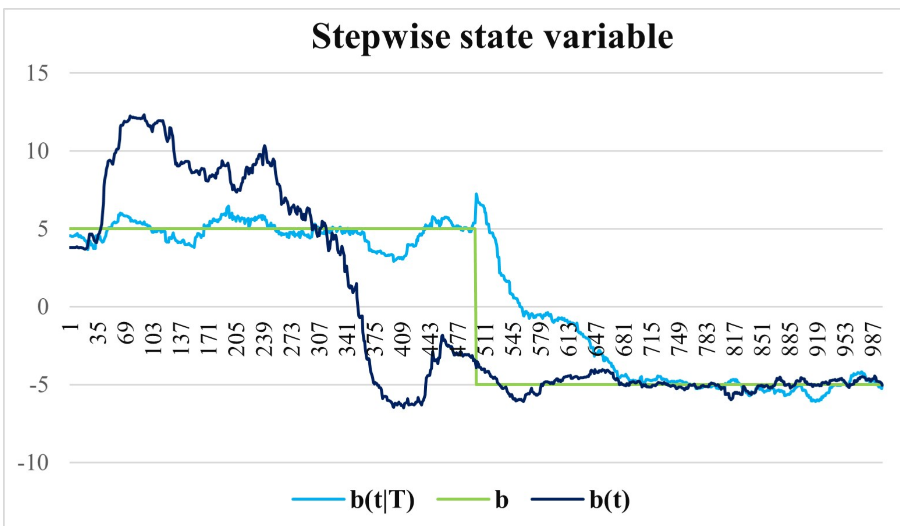


Figure A.8: The time-varying stepwise coefficient in the trivariate model.

We run the simulated trivariate model in Equation (7) and re-estimate it by the implemented Kalman filter. The smoothed state variables turn out to have less error than the conditional ones.

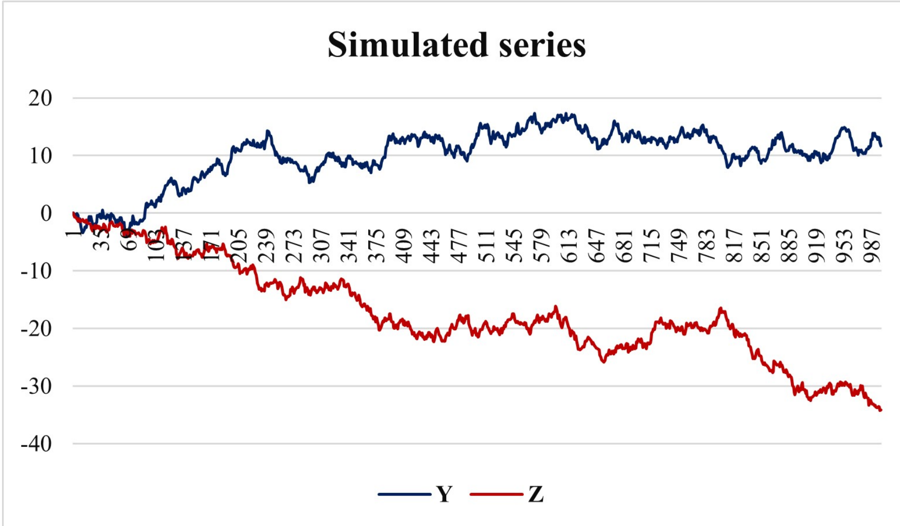


Figure A.9: The trivariate simulated series.

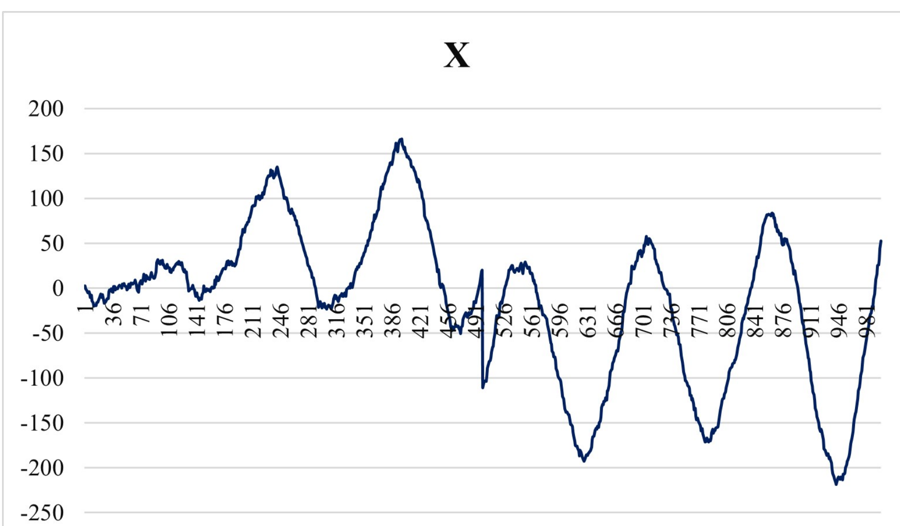


Figure A.10: The simulated X_t of the trivariate model.

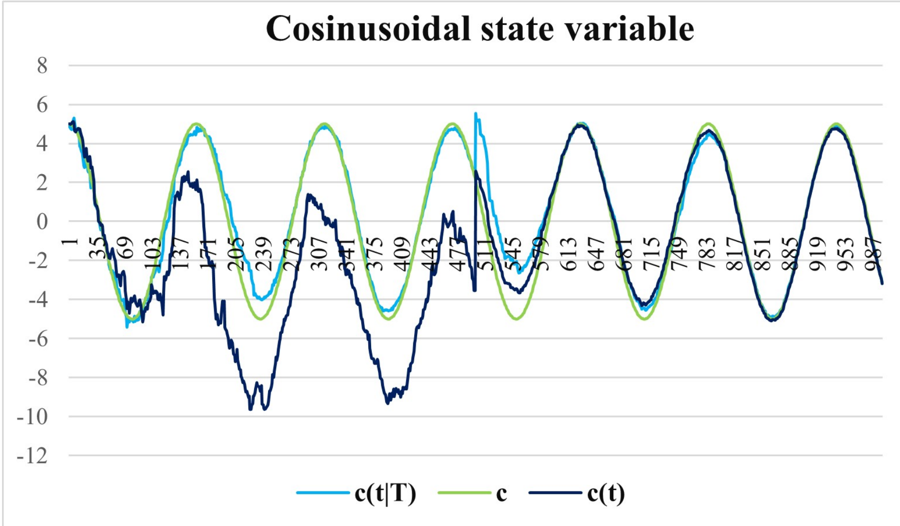


Figure A.11: The time-varying cosinusoidal coefficient in the trivariate model.

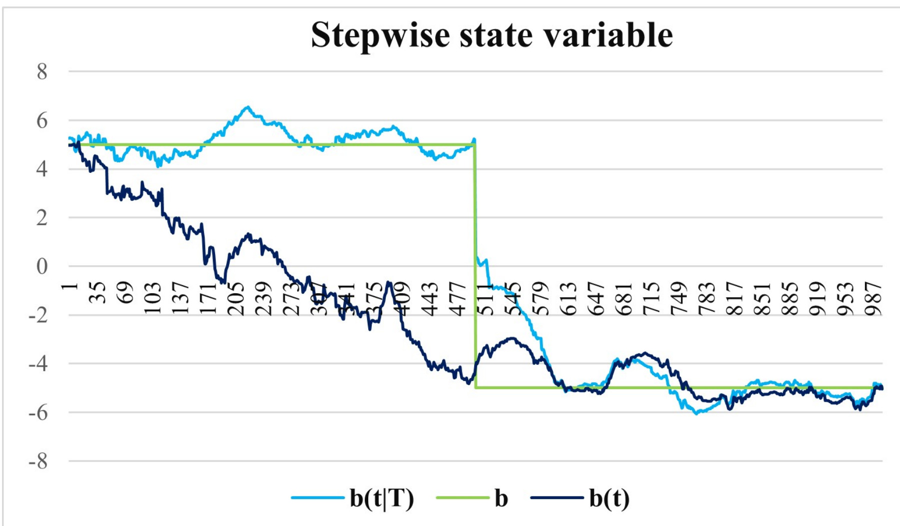


Figure A.12: The time-varying stepwise coefficient in the trivariate model.

Table A.3: The estimated parameters of the trivariate simulated series by Kalman filter

$\hat{c}$	$\hat{b}_{,0}$	$\hat{\sigma}_{\delta}$	$\hat{\sigma}_{\epsilon_1}$	$\hat{\sigma}_{\epsilon_2}$	max LL
1.7071***	4.9697	0.4981	0.1353	0.2541	-2827.5856

*, ** and *** respectively stand for the significance at 1%, 5% and 10% levels.

Appendix B

Simulation of time-varying cointegration

B.1 Deterministic series

As a trivial example, we assume two deterministic series including a time trend and a series with trigonometric oscillation around the time trend. The two series are generated by the following equation:

$$X_t = Y_t + 50cos(t/25). \tag{B.1}$$

$$Y_t = t, \quad t = 1, \ldots, 475. \tag{B.2}$$

The series are trivially cointegrated with the time-invariant cointegrating vector of $(1, -1)$. The resulting residual would then be a cosinusoidal function with zero mean. The assumed time series in Equations (B.1) and (B.2) are presented in Figure B.1. Figure B.2 presents the corresponding scatterplot.

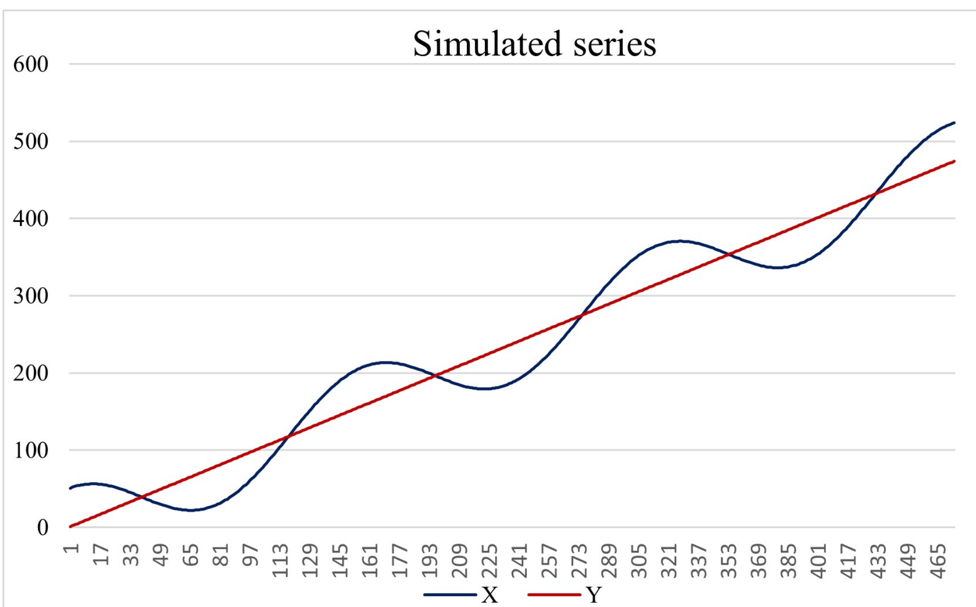


Figure B.1: The bivariate simulated series. Trigonometric oscillation around a linear trend.

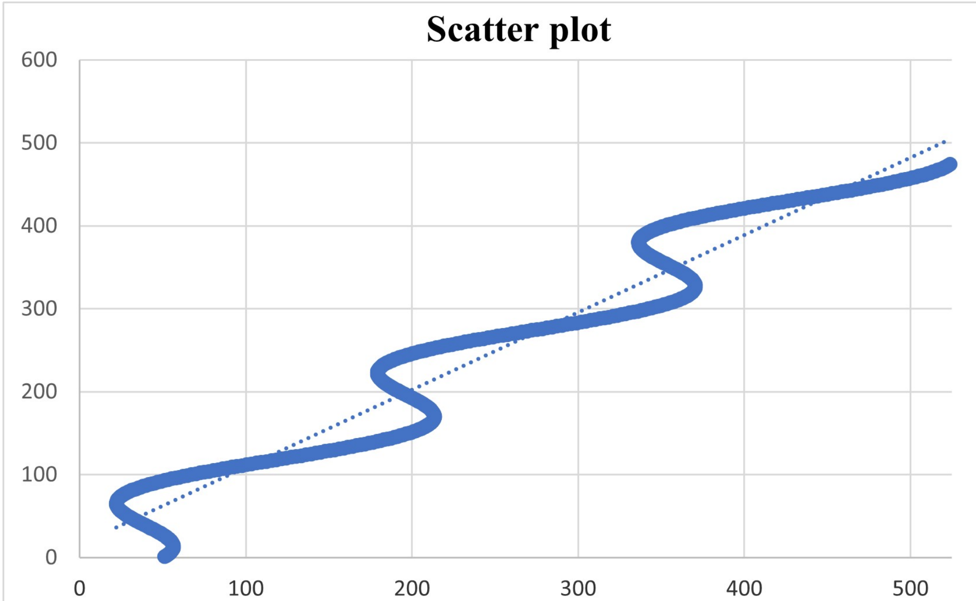


Figure B.2: The scatter plot of the simulated series. Trigonometric oscillation around a linear trend.

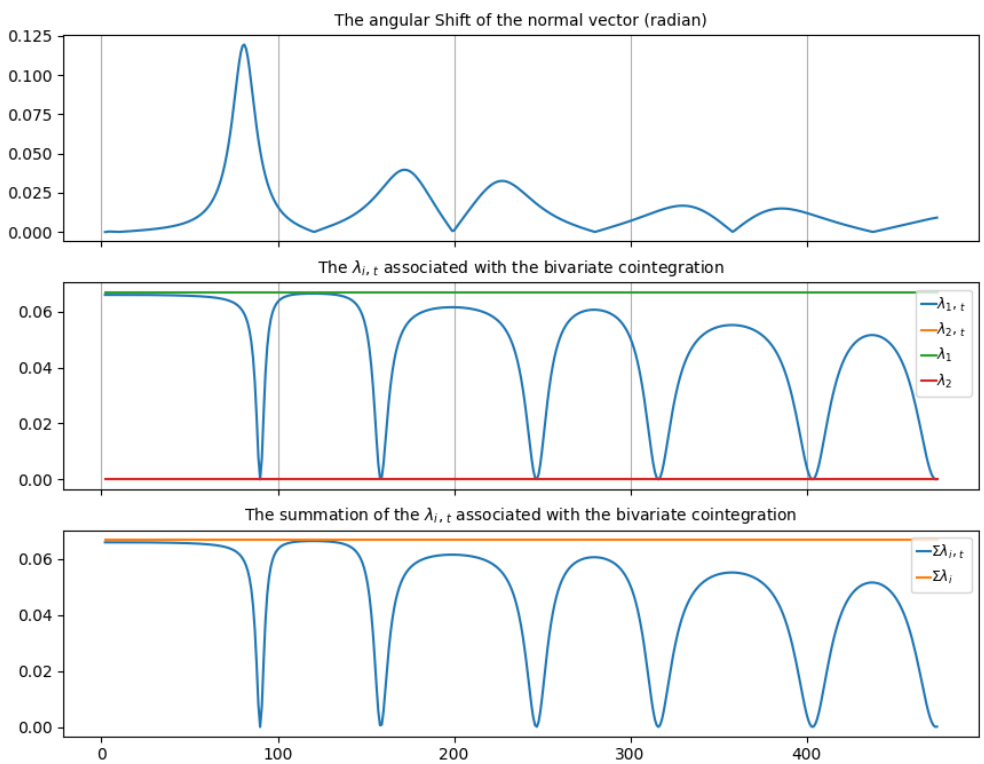


Figure B.3: The instability and cointegration strength of the simulated series. Trigonometric oscillation around a linear trend.

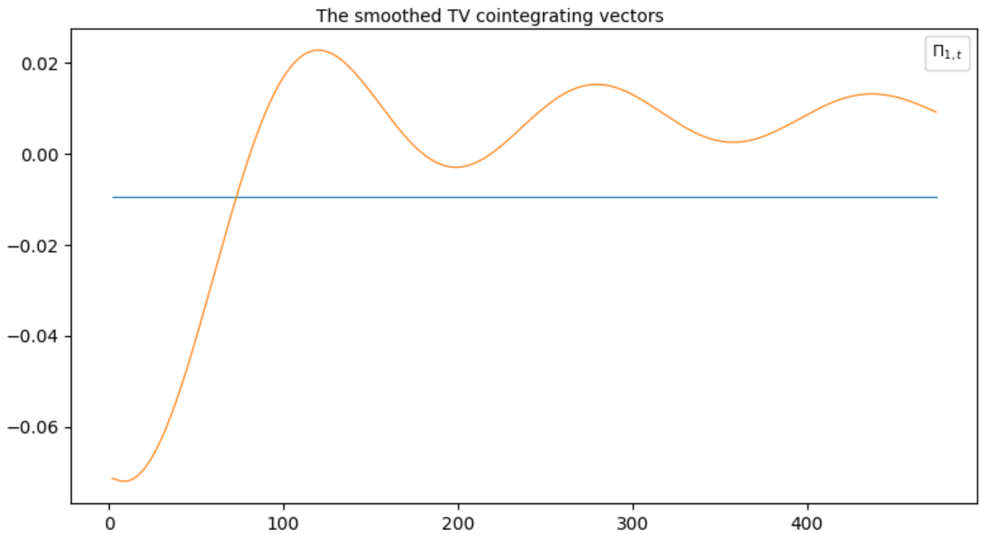


Figure B.4: The time-varying impact matrix of the bivariate VECM. Trigonometric oscillation around a linear trend.

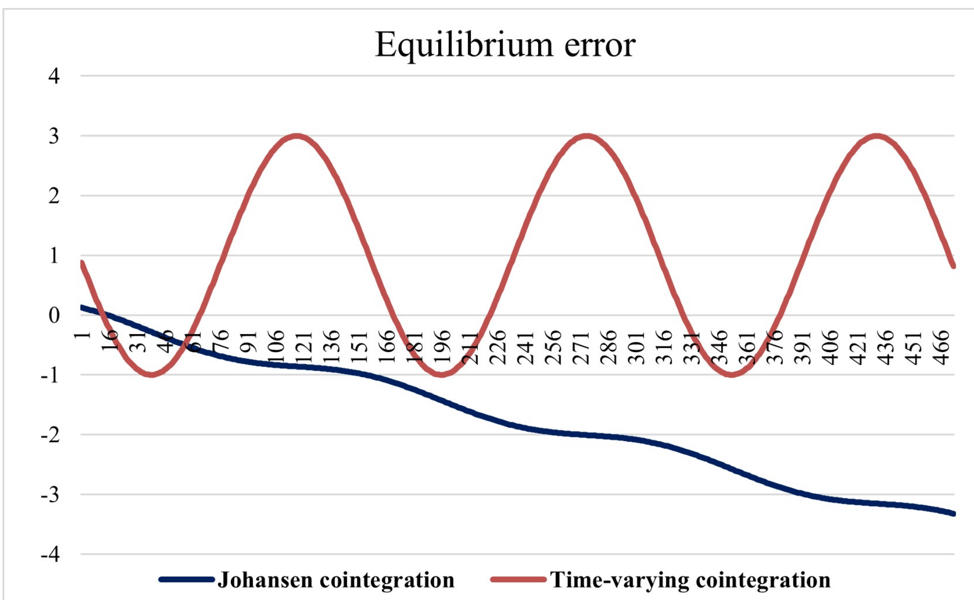


Figure B.5: Equilibrium error of time-varying versus Johansen cointegration.

Table B.1: The estimated parameters of the simulated series by Kalman filter

	$\hat{c}$	$\hat{\Pi}_{1,1,0}$	$\hat{\Pi}_{1,2,0}$	$\hat{\sigma}_{\eta 1}$	$\hat{\sigma}_{\eta 2}$	$\hat{\sigma}_{\epsilon}$	max LL
ΔX_t	1.4352	-0.0095	-0.0713	8.01×10^{-7}	3.54×10^{-9}	0.0005	466.2201
ΔY_t	1.0000	0.0000	0.0000	1.11×10^{-16}	1.11×10^{-16}	1.11×10^{-16}	14274.0404

Table B.2: Johansen time-invariant test

Rank test (Trace)					Cointegrating vector	
Null hypothesis	Eigenvalue	Trace statistic	5% critical value	P-value	X	Y
At most 0	0.0669	32.7646	15.4947	0.0001	0.0026	-0.0099
At most 1	0.0000	0.0000	3.8415	0.9998	0.0281	-0.0275

B.2 Stochastic series and coefficient

$$W_t = 0.9W_{t-1} + \eta_t \quad \eta_t \sim N(0, 0.1). \tag{B.3}$$

$$b_t = cos(t/25) + W_t. \tag{B.4}$$

$$Y_t = Y_{t-1} + \epsilon_t \quad \epsilon_t \sim N(0, 0.1). \tag{B.5}$$

$$X_t = 2 + b_tY_t + \delta_t \quad \delta_t \sim N(0, 0.1). \tag{B.6}$$

Figure B.6 shows the time-varying b_t in Equation (B.4). It involves a stationary fluctuation around cosinusoidal function.

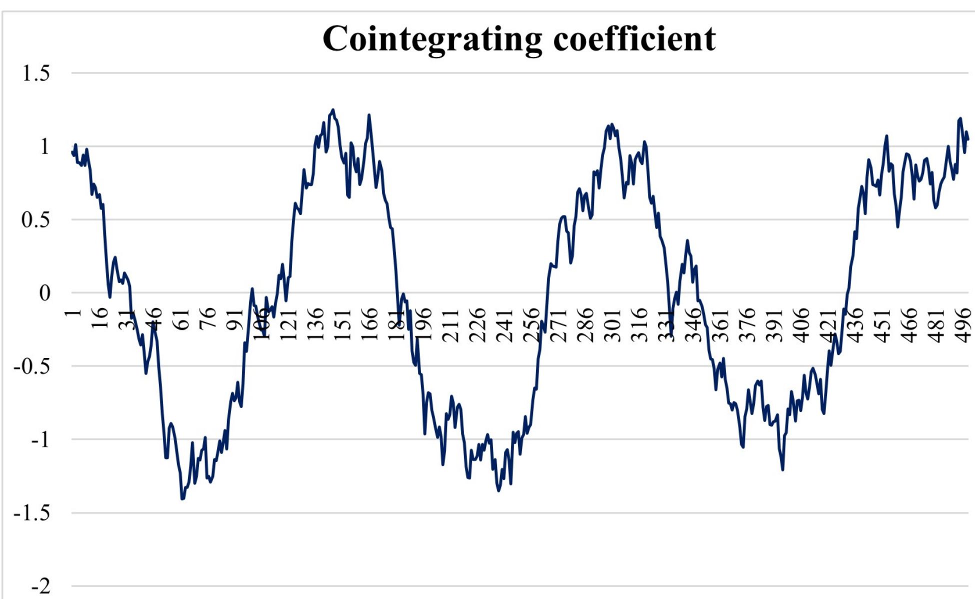


Figure B.6: The time-varying cointegrating coefficient.

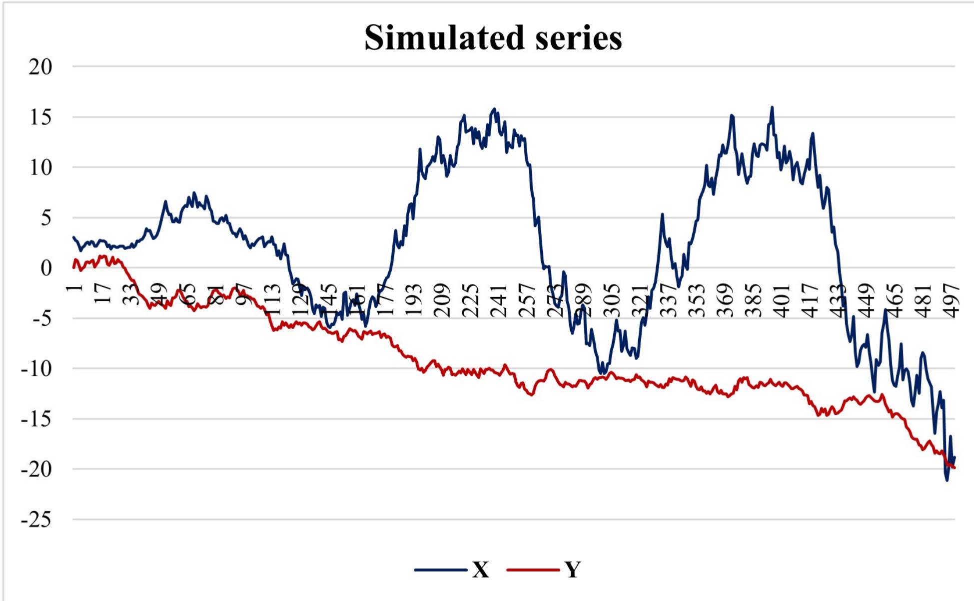


Figure B.7: The bivariate simulated series.

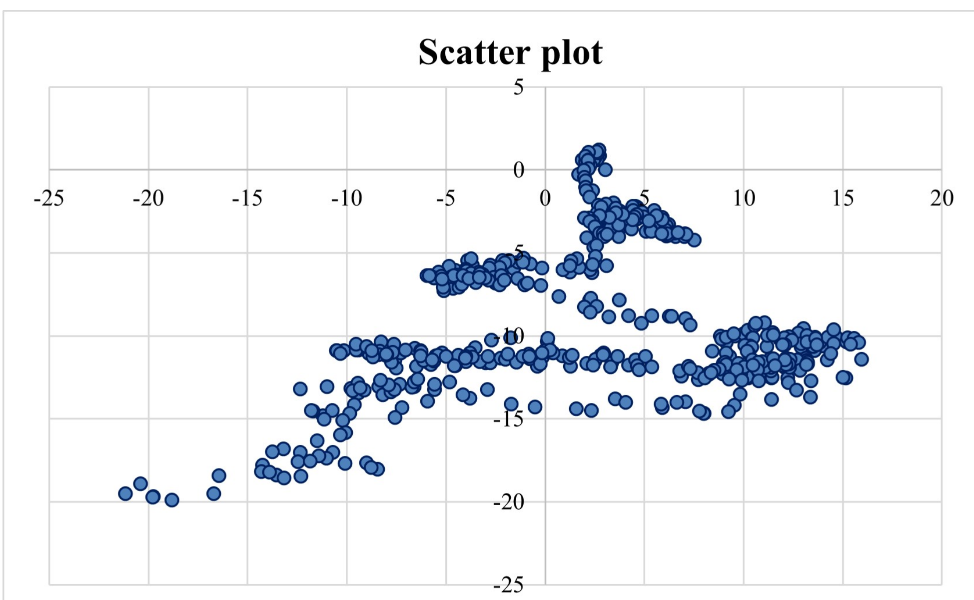


Figure B.8: The scatter plot of the simulated series.

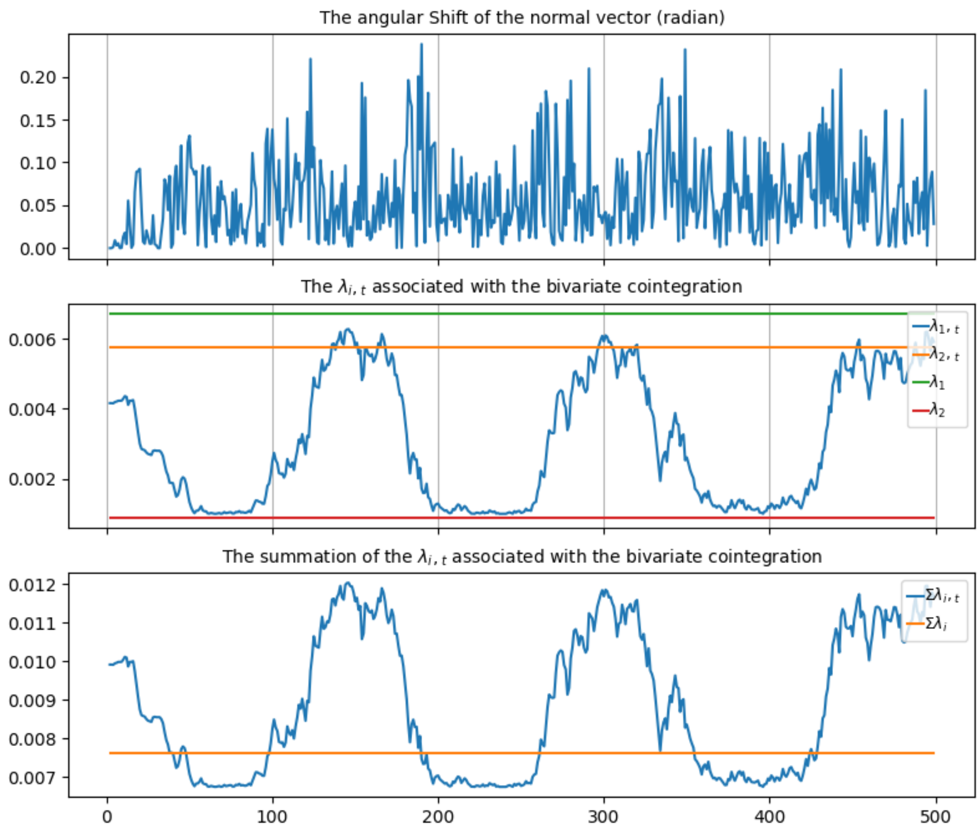


Figure B.9: The instability and cointegration strength of the simulated series.

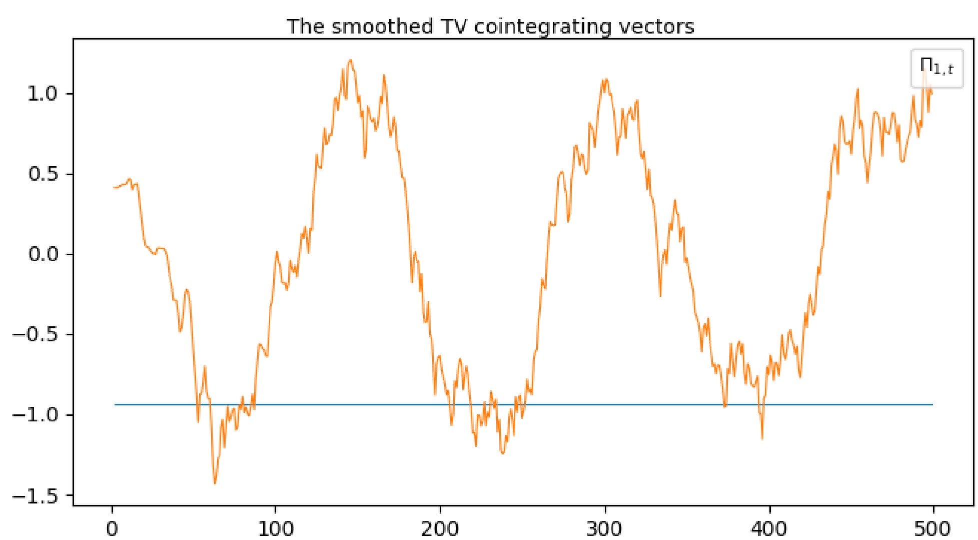


Figure B.10: The time-varying impact matrix of the bivariate VECM.

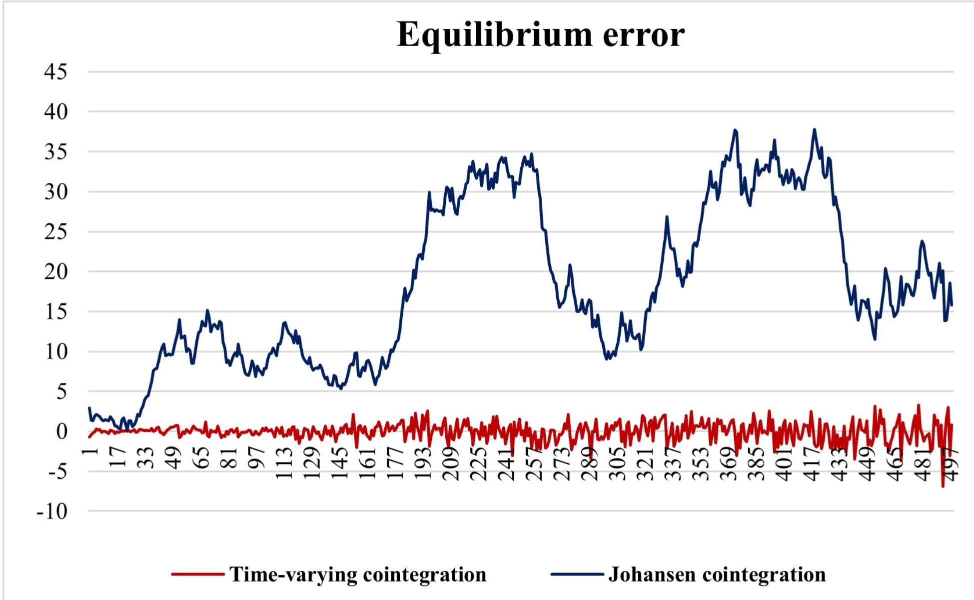


Figure B.11: Equilibrium error of time-varying versus Johansen cointegration.

Appendix C

Real exchange rates and EPU smoothing

Figure C.1 exhibits the RERs between currencies of the G7 members except the Japanese Yen. The RERs between the Japanese Yen and the G7 currencies are shown in Figures C.2 and C.3. The RER of a RER (e.g., that of the US-Japan parity) is calculated by:

$$q_t = Log(JPY/USD) + Log(CPI_{US}) - Log(CPI_{JP}) \tag{C.1}$$

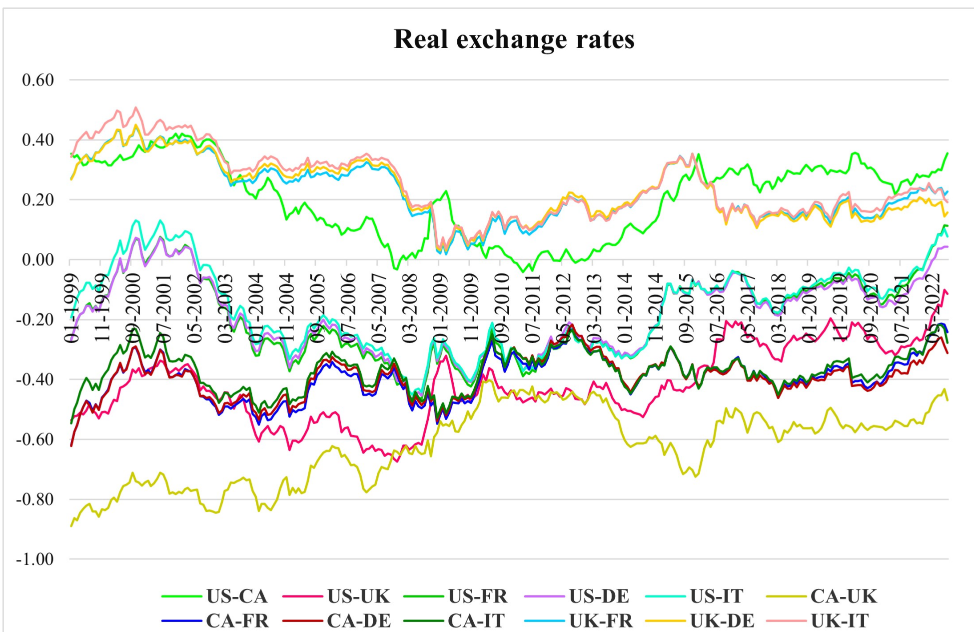


Figure C.1: RERs of the G7 countries (Part 1).

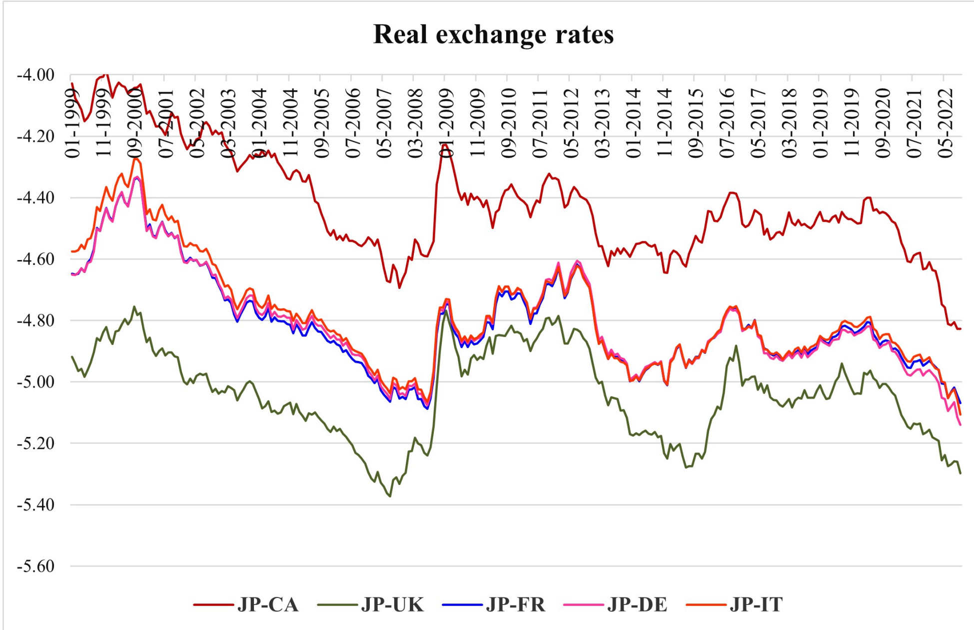


Figure C.2: RERs of the G7 countries (Part 2).

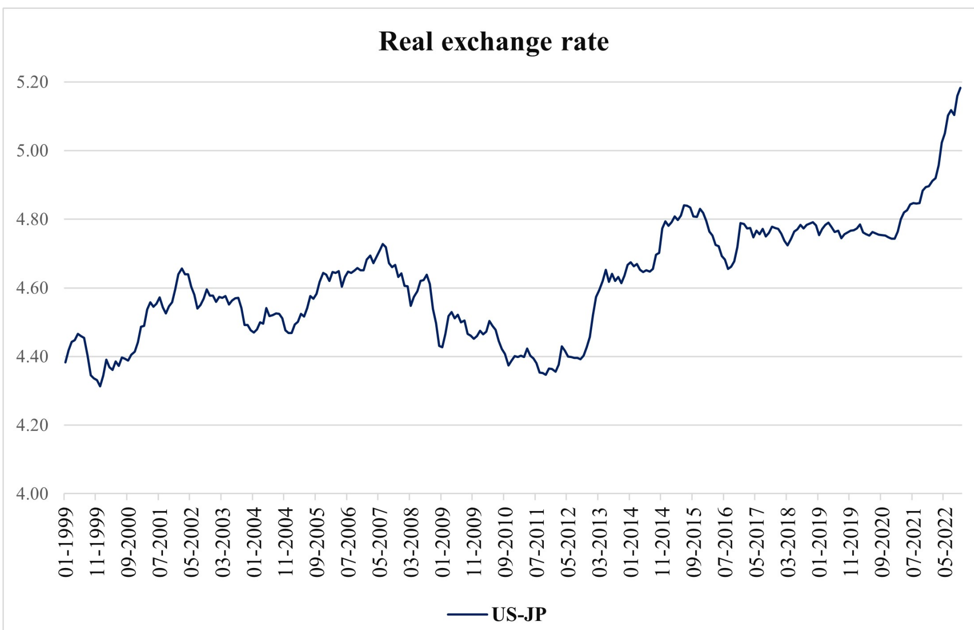
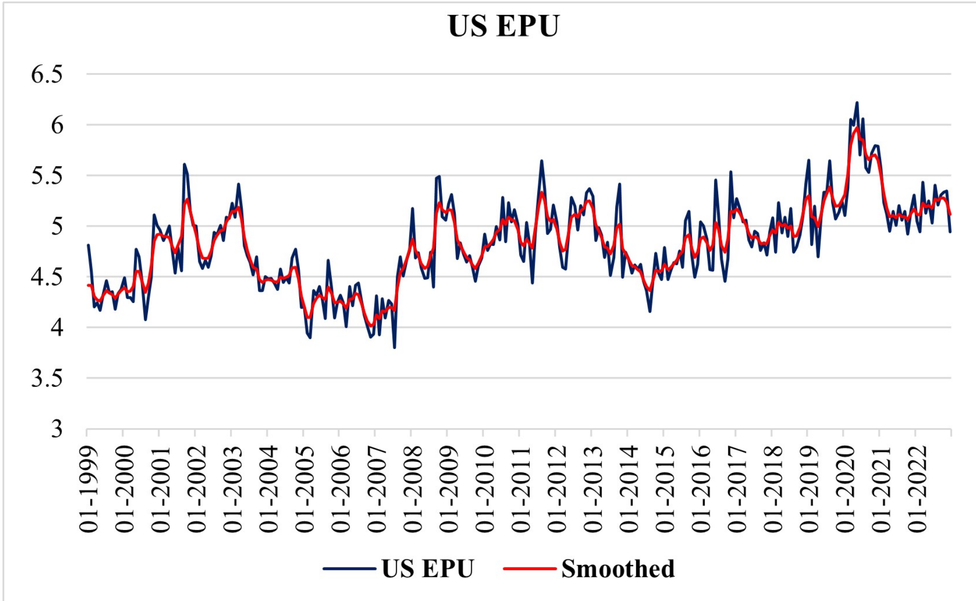
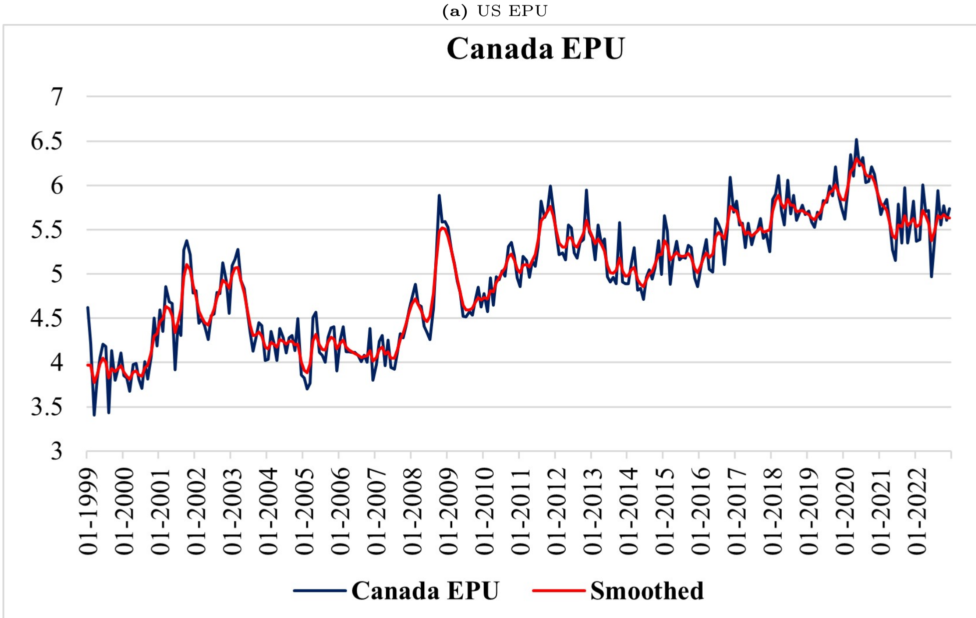


Figure C.3: RERs of the G7 countries (Part 3).

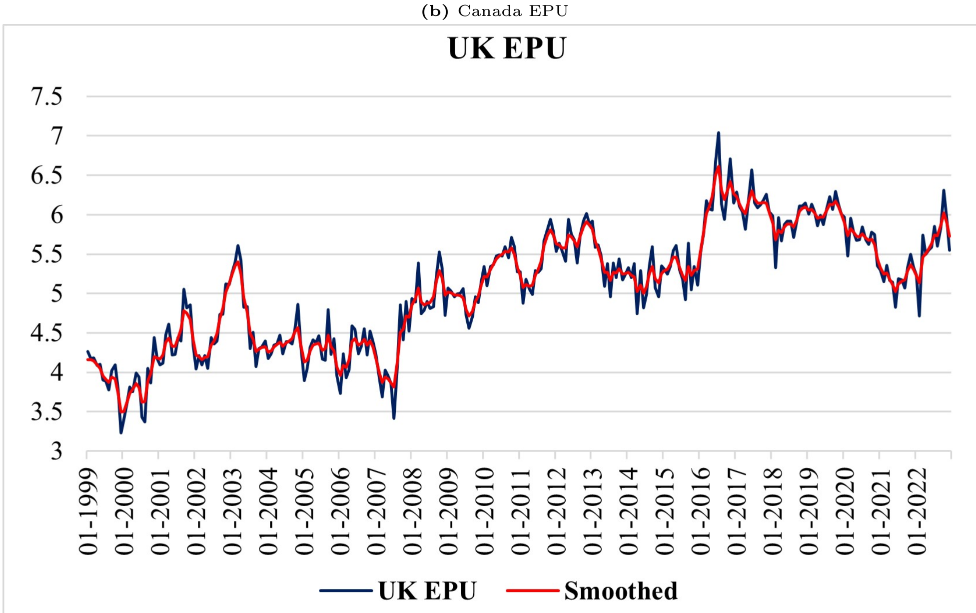
Figure C.4 shows the EPU indices of the G7 member countries. The indices are smoothed by the local level model in (5.15).



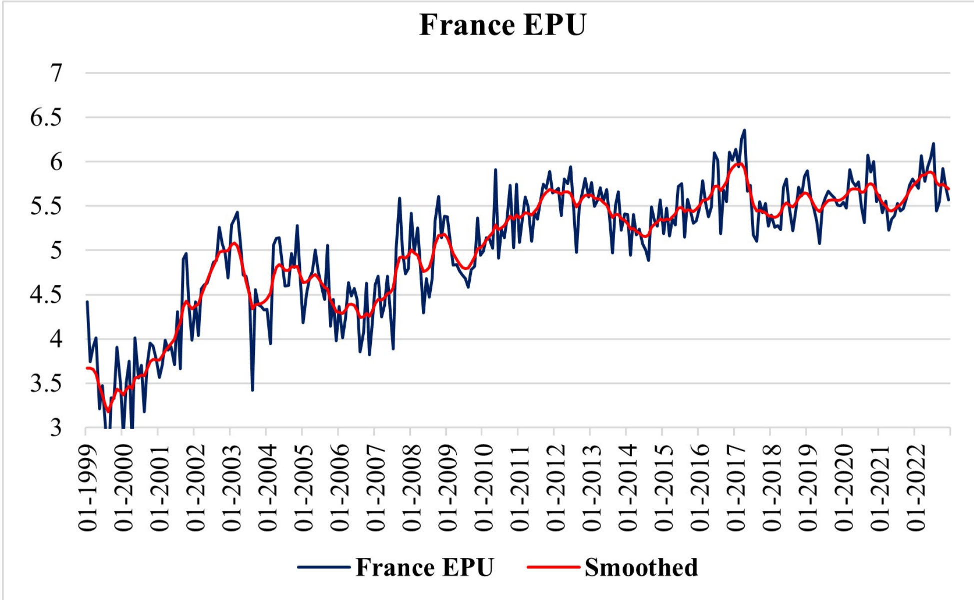
(a) US EPU



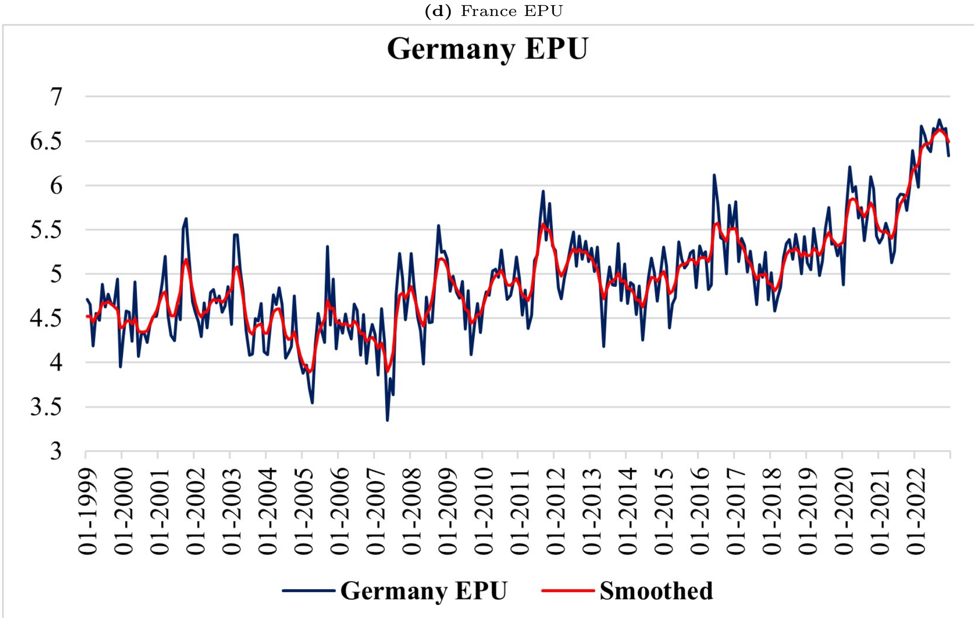
(b) Canada EPU



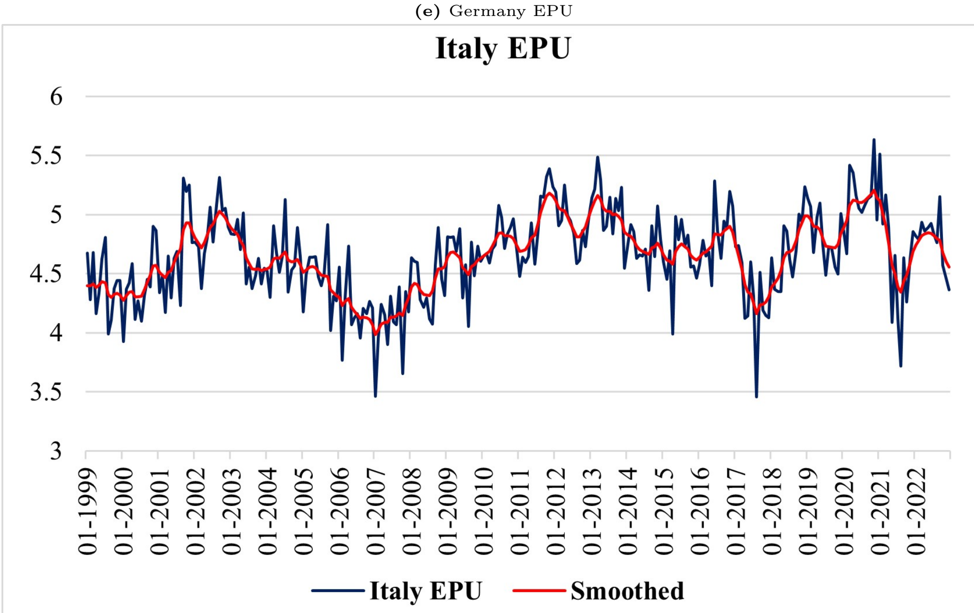
(c) UK EPU



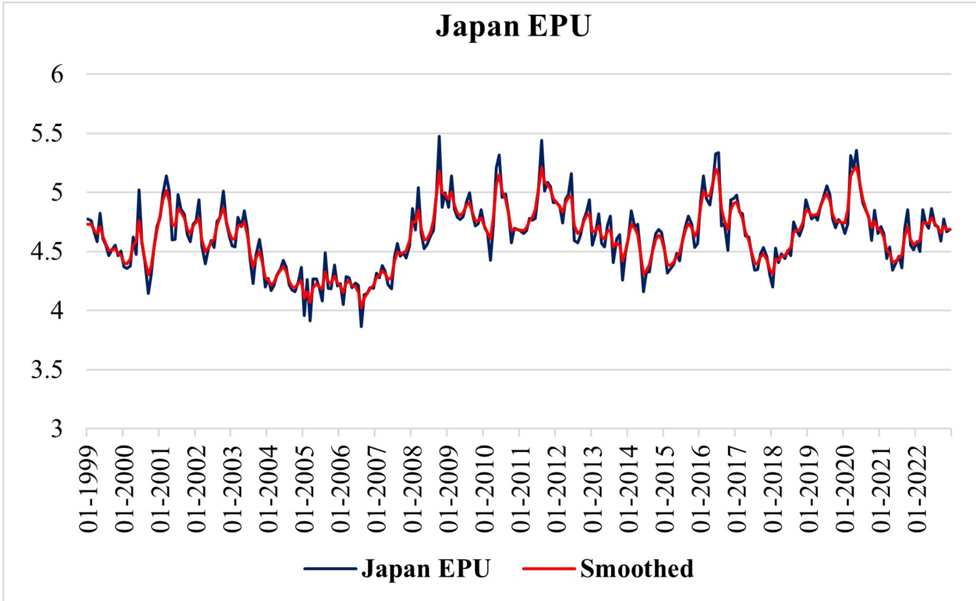
(d) France EPU



(e) Germany EPU



(f) Italy EPU



(g) Japan EPU

Figure C.4: Kalman smoothed EPU by the local level model.

Table C.1 shows the results of applying stationarity tests to the IRDs. As the results show, the IRDs follow unit root processes.

Table C.1: Stationarity test of the IRDs using ADF and PP tests

	Augmented Dickey-Fuller test			Phillips–Perron test		
	t-statistic	p-value	Lag lengths	t-statistic	p-value	Bandwidth
US JP	-1.66	0.45	2	-0.01	0.96	2
US CA	-2.47	0.12	2	-2.32	0.17	7
US UK	-2.11	0.24	4	-1.84	0.36	11
US FR	-2.55	0.10	4	-1.96	0.30	11
US DE	-2.15	0.22	3	-1.90	0.33	11
US IT	-2.32	0.17	0	-2.32	0.17	0
JP CA	-1.88	0.34	2	-1.76	0.40	9
JP UK	-1.54	0.51	1	-1.45	0.56	7
JP FR	-1.36	0.60	1	-1.42	0.57	9
JP DE	-1.36	0.60	1	-1.37	0.60	8
JP IT	-1.71	0.42	0	-1.71	0.42	0
CA UK	-1.90	0.33	1	-1.99	0.29	6
CA FR	-1.79	0.38	0	-2.30	0.17	9
CA DE	-2.02	0.28	1	-2.16	0.22	8
CA IT	-3.25	0.02	0	-3.21	0.02	5
UK FR	-2.25	0.19	0	-2.34	0.16	9
UK DE	-2.39	0.14	0	-2.43	0.13	7
UK IT	-2.85	0.05	0	-2.48	0.12	7

The augmented Dickey-Fuller and Phillips–Perron tests are applied to the interest rate differentials of the G7 countries. The lag length is automatically specified using the Bayesian information criterion. Bandwidth is specified using the Newey-West method and Bartlett kernel is used as spectral estimation method.