Weak measurements under dissipation

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Abstract: We study weak measurements under the influence of dissipation. Even though dissipation harms the anomalous properties of the weak value, we found specific setups in which extracting information from the weak value is feasible. © 2023 The Author(s)

Weak measurements find many applications due to their amplification capacity and because the experimental observations are described by complex numbers. A quantum weak measurement is a four-step protocol involving a system of interest and a meter (an ancilla) [1]. Firstly, pre-selection is imposed, which implies choosing the initial state of the system. The ancilla wave function is centered at 0, to have an adequate reference. In a second step, a weak measurement is executed through a unitary operator, involving both the system and the ancilla, $\hat{U} = e^{-ig\hat{A}\otimes\hat{P}}$, where $\hat{A}$ is the system observable of interest, $\hat{P}$, is the meter momentum operator and g is the interaction strength. The third step consists of post-selection on the system, which involves a projective measurement, and filtering to keep a specific final state. Eventually, the meter wave function is read out. Two shifts appear in the meter wave function. The shift of the wave function in position representation is proportional to the real part of the weak value, $A_w = \frac{\langle\psi_f|\hat{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$, while the shift in momentum representation is proportional to the imaginary part of the weak value. The weak value is a complex and unbounded quantity. It is called anomalous when its value cannot correspond to an average value of the system observable.

A quantum system in the physical world cannot be completely isolated. It always interacts with the surrounding environment. Open quantum system theory studies how the dynamics of a system is modified due to the interaction with the environment. In the Born-Markov and secular approximations, the dynamics of the system can be described by a Lindblad master equation. This equation has a first term determined by the von Neumann equation for the isolated evolution of the density matrix, and a second term proportional to the application of a dissipator to the density matrix. The dissipator defines the non-unitary dynamics of the system, including dissipation, decoherence, and dephasing. It involves dissipation rates γ_i that are as numerous as the number of present channels.

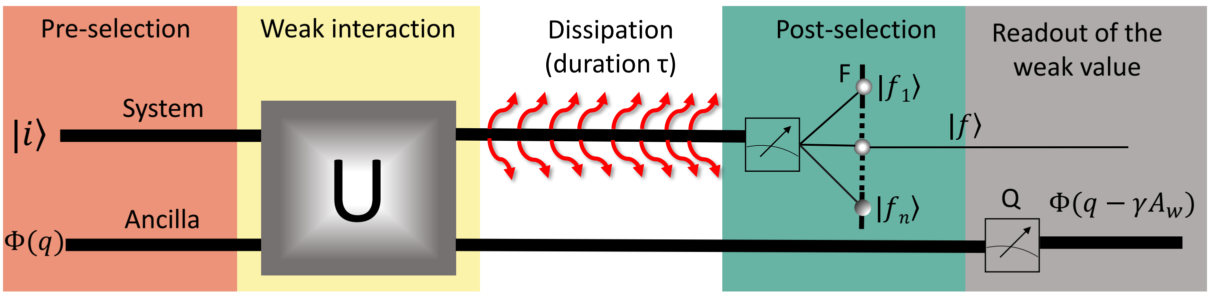

Fig. 1. The dissipative weak measurement involves: a pre-selection on the systems; the execution of the weak measurement through the unitary operator $\hat{U}$; dissipation on the system for a finite duration τ; post-selection on the system; and the correlated readout of the meter to extract the weak value.

We study weak measurements with dissipation occurring after the weak application of the unitary operator and before post-selection (Fig. 1). The procedure is the following: first, pre-selection is performed in the system. The general unitary operator $\hat{U} = e^{-ig\hat{A}\otimes\hat{N}}$ is consequently applied. Then, the system exhibits a dissipative dynamics for a duration τ, after which post-selection is executed. This scheme happens in any experimental setup with a time delay between the unitary evolution and post-selection. The duration of the weak interaction is assumed to be short in order to neglect dissipation during the application of $\hat{U}$.

Wiseman defined the dissipative weak value in the context of homodyne measurements [2]. Since then, the dissipation included within weak measurements in open quantum systems was found to damp the anomalous properties of weak values. We investigated alternative approaches that mitigate this damping. In contrast to previous studies, we consider the general limit of weak values at large dissipation time τ. In addition, we exploit the amplifying features at short dissipation time to extract any relevant information. Furthermore, we investigate degenerate fundamental states.

In non-degenerate systems, at long dissipation time, we show that the anomalous properties of weak values cannot be preserved. Weak values at infinite dissipation time tend to the expectation values of the observable,

$$A_w(\tau) = \frac{\mathrm{Tr}\left[\hat{\sigma}_f e^{L\tau}\left(\hat{A}\hat{\sigma}_i\right)\right]}{\mathrm{Tr}\left[\hat{\sigma}_f e^{L\tau}\hat{\sigma}_i\right]} \qquad A_w(\tau \to \infty) = \mathrm{Tr}\left[\hat{A}\hat{\sigma}_i\right], \tag{1}$$

where $\hat{\sigma}_i$ and $\hat{\sigma}_f$ represent the pre- and post-selected density matrices, $\hat{A}$ is the observable to measure weakly, and L is the Lindbladian governing the dissipative dynamics. However, in degenerate systems, dissipation can preserve anomalous weak values, even at infinite dissipation time. For example, in a fluorescent transition between degenerate energy levels, the weak value of the angular momentum operator in the z-direction, $\hat{J}_z$ as a function of the dissipation time can exhibit a non-zero imaginary part (Fig. 2). As a result, the weak value is anomalous at infinite dissipation time, unlike the non-degenerate case. A second possibility to extract information from weak

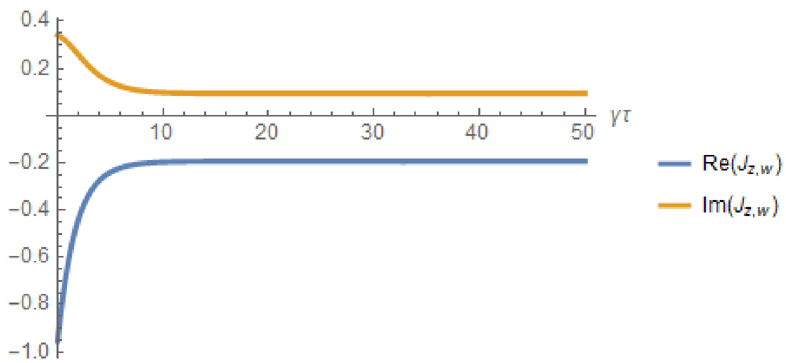

Fig. 2. The weak value of the angular momentum operator in the z-direction, $\hat{J}_z$, exhibits anomalous properties at infinite dissipation time in a transition between two degenerate energy levels with a single dissipation channel. The real and imaginary parts of this weak value were depicted as functions of the product of the dissipation time and the dissipation rate.

values in dissipative systems is to investigate their behavior at short dissipation time. To this end, we consider a two-level atom as the system, and a single excitation cavity field, as the meter. The weak measurement is obtained via the short transit of the atom through the cavity. Then, the atom suffers dissipation via spontaneous emission of photons. After post-selection, the real and imaginary parts of the weak value are measured by reading out the $\hat{Q}$ and $\hat{P}$ quadratures of the cavity field. Considering a Markovian dissipative dynamics, at short dissipation time, the weak value linearly depends on the dissipation time. The slope is proportional to the dissipation rate. Consequently, by measuring the weak value at different dissipation times, the dissipation rate can be extracted.

When considering a non-Markovian dynamics, under certain circumstances, at short dissipation time, the weak value does not depend linearly on the dissipation time, but quadratically. In consequence, if by measuring the weak value at different dissipation times, we observe a quadratic dependance without any linear dependance regardless of the dissipation time, the dynamics is non-Markovian.

In conclusion, we have shown two examples where dissipative weak measurements extract relevant information. On the one hand, when the system is degenerate, the anomalous weak value can be preserved at infinite dissipation time. On the other hand, at short dissipation time, the measurement of the weak value allows the extraction of information on the dynamics such as the dissipation rate or the system Markovianity.

References

1. Yakir Aharonov, David Z Albert, and Lev Vaidman. How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. *Physical Review Letters*, 60(14):1351, 1988.
2. HM Wiseman. Weak values, quantum trajectories, and the cavity-qed experiment on wave-particle correlation. *Physical Review A*, 65(3):032111, 2002.