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THEORY AND PHENOMENOLOGY OF THE GLUON PROPAGATOR FROM THE DYSON-SCHWINGER EQUATION IN QCD

by

J.R. Cudell
and
D.A. Ross†
CERN, CH-1211 Geneva 23, Switzerland

Abstract

A new solution is found for the Dyson-Schwinger equation for the gluon propagator in the axial gauge. Unlike previously found ones this propagator does not display a $1/k^4$ pole as $k \to 0$ but has a very soft singularity in this region. Thus it represents a gluon which is not confining but rather confined since in configuration space the propagator vanishes for large impact parameters. This solution is relevant for processes in which soft gluons are exchanged, such as physics at low x or equivalently the construction of the QCD pomeron. Some of the phenomelogical consequences are discussed.

† On leave of absence from Department of Physics, University of Southampton, Southampton SO9 5NH, U.K.

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1 Introduction

Although perturbative QCD has had many successes at high momentum transfers, its applicability to soft and semisoft processes is questionable. The main problem comes not so much from the size of the coupling but rather from the infrared and collinear singularities emerging from the gluon and quark propagators. Most perturbative calculations need to introduce some kind of regulator, be it a gluon mass or a minimum transverse momentum and the predictivity of these calculations is ruined by their cut-off dependence. Nevertheless, it seems that as a rule of thumb perturbative QCD with a 1 to 3 GeV cutoff gives reasonable answers.

On the other hand, soft processes are remarkably well reproduced by Regge calculus, and in particular the high-energy behaviour of hadron scattering is very well predicted by the pomeron. This means that hadronic amplitudes are proportional to $s^{\alpha(t)}$ where $\alpha(t)$ is the pomeron trajectory. It is essentially linear $\alpha(t) = \alpha(0) + \alpha' t$ with its intercept $\alpha(0) \approx 1$ and its slope $\alpha' \approx 0.25$ GeV^{-2}. This behaviour is well reproduced in models where the pomeron is described by the exchange of a single object coupled to quarks via a γ_μ vertex and with the quantum numbers of the vacuum. Low and Nussinov [1] suggested that the lowest order QCD construction possessing the correct quantum numbers was two gluon exchange. Explicit perturbative calculations using such a model [2, 3] are infrared finite, but their exact predictions are dependent on the form assumed for the hadronic wave functions and more importantly, they cannot reproduce the details of the elastic differential distributions [4].

The question as to whether perturbative QCD can actually give rise to the correct pomeron behaviour and trajectory is more complex, as the calculation of the slope α' involves the evaluation of higher-order corrections and eventually the resummation of an infinite number of diagrams in the colour singlet channel. Lipatov et al. [5] have developed a formalism making such an estimate possible. This machinery uses perturbative QCD to determine the sum of effective ladder diagrams in the leading $\log(s)$ limit and the resulting scattering amplitudes first had a cut rather than a Regge pole behaviour. It was then suggested by Lipatov [6] that this cut would be converted into a pole by incorporating renormalisation group effects, i.e. by accounting for the running of the strong coupling constant. However if one merely runs the coupling constant without fixing the phase of the scattering amplitudes in the infrared regime then the resulting behaviour does not correspond to a single Regge pole but rather to a dense or continuous distribution of poles, and thus cannot give rise to the simple $s^{\alpha(t)}$ dependence experimentally observed [7]. Since in order to determine the behaviour of scattering amplitudes in the infrared limit one needs information about the

nonperturbative behaviour of gluons one is drawn to the conclusion that both at lowest order and for infinite ladders nonperturbative aspects of QCD are crucial. As the high energy behaviour of hadronic cross sections is closely linked with the small x dependence of structure functions, the resolution of this problem might turn out to be of utmost importance to make reliable predictions at future colliders.

Such difficulties with perturbative QCD have prompted Landshoff and Nachtmann [8] to assume that the gluon propagator is intrinsically modified at low momentum transfers. Although entirely phenomenological, this model has been very successful [9], relating diffractive scattering, gluon structure functions, elastic scattering, gluon condensates and total cross sections. One of us [10] has shown that such modifications to the gluon propagator could in principle lead to a viable description of the pomeron after resummation in the manner suggested by Lipatov et. al. We thus address the question whether one can derive such a behaviour from QCD.

The tools available for such an investigation are very restricted. The problem is non perturbative, and we need to use exact methods to solve it. The Dyson-Schwinger equation is the best place to start, as it can be approximated and truncated into a closed form for the gluon propagator. Baker, Ball and Zachariasen (BBZ) [11] have produced such a truncated equation in the axial gauge, and investigated its solutions. They have found that at very low momentum transfer k^2 the gluon propagator behaves like $1/k^4$. They interpret this as a *confining* solution. Such a behaviour is rather difficult to manage in practical situations, as all the above problems are only made worse by a stronger singularity. Furthermore, it is surprising that gluons have any type of pole at zero k^2, as this should correspond to a force of infinite range. Finally, although such a behaviour is acceptable for the truncated gluon propagator resulting from the approximations of the BBZ equation, it was pointed out by West[12] that analyticity arguments (namely the positivity condition on the spectral functions) forbid it for the full propagator in the axial gauge. We thus expect *confined* gluons to have a behaviour at the origin softer than a pole.

Concerning the relevance of the first solution, West showed[14] that the existence of a gauge in which the *full* propagator behaves like $1/k^4$ near the origin leads to a proof of confinement. Thus BBZ were searching specifically for a solution which had a behaviour more singular than $1/k^2$ as $k^2 \to 0$. As suggested in ref. [12] it might be that this very singular solution for the *truncated* propagator, although not representative of the full propagator, be a signal for confinement. Such solutions have also been found in the Landau gauge (where they are valid candidates for the full propagator), in the approximation of neglecting the effects of Fadeev-Popov ghosts[15].

Hints of a softer behaviour at the origin can be found in lattice calculations. Both in the Landau[16] and in the axial gauges[17], the gluon propagator seems to behave like that of a massive particle and therefore vanishes at large distances. Since the BBZ equation is non linear, it can certainly accomodate several solutions and is indeed known[13], in the light cone gauge to possess at least two. All this suggests that notwithstanding the existence of a highly singular solution to the Dyson-Schwinger equation, which may imply confinement, a solution with a singularity softer than a pole near $k^2 = 0$ is the one which is important for the description of processes involving the exchange of one or more soft gluons.

It is the derivation and properties of this second, *confined* solution that will be considered in this paper. We shall first give a brief summary of the equation derived by Baker Ball and Zachariasen, and explain how it can be further simplified using the results of Schoenmaker [18]. We shall then discuss the subtractions, the coupling of gluons, and its asymptotic behaviour which we shall need later. In the third section, we shall explain what ansatz we choose, and what our strategy is. In the fourth part, we shall optimistically examine the phenomenological consequences of our work and from this phenomenology we shall decide that the scale at which perturbative QCD becomes valid is of the order of 1 GeV.

2 The Baker Ball Zachariasen equation

We shall not engage in a rederivation of this equation, which is spelled out in references [11, 18], but rather remind the reader of the assumptions and approximations that are built into it. Most of these become exact at zero momentum transfer, and the BBZ equation should correctly describe the infrared behaviour of the propagator. Firstly, BBZ assume that quarks can be neglected, and that the gluon propagator can be well approximated by looking at gluon interactions only. Secondly, they choose to consider the gluon propagator in the axial gauge. The advantage of this gauge is that the Dyson-Schwinger equation will not involve ghosts. The disadvantage is that the gauge parameter, being a fixed four-vector n_μ, gives rise to a new kinematic variable $\gamma = (q.n)^2/(q^2\, n^2)$, besides the spacelike momentum squared, $-q^2$. We shall assume in the following that there exists a solution that is independent of γ, which we take to be negligibly small.

In the axial gauge, the propagator depends on two independent scalar functions, and the Dyson-Schwinger equation can be written as a set of two coupled equations for these two functions. At this point, BBZ assume that the IR behaviour is dominated by one of these functions, or equivalently that the full propagator $D_{\mu\nu}(q^2)$ has the same spin structure as

the bare one, namely that the propagator in Euclidean space is:

$$D_{\mu\nu}(q^2) = -\frac{Z(q^2)}{q^2}\left(\delta_{\mu\nu} - \frac{q_\mu n_\nu + n_\mu q_\nu}{q.n} + \frac{q_\mu q_\nu n^2}{(q.n)^2}\right) \equiv Z(q^2)D^{(0)}_{\mu\nu}(q^2) \tag{2.1}$$

or that the inverse† propagator is:

$$\Pi_{\mu\nu} = -\frac{q^2}{Z(q^2)}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \tag{2.2}$$

BBZ show that using the fact that from Eq. (2.1) $n_\mu D^{\mu\nu} = 0$ enables them to find a Dyson-Schwinger equation for Z that does not involve the four-gluon vertex. It however involves the (unknown) triple gluon vertex. This in turn can be decomposed into a longitudinal and a transverse part. The longitudinal part is related to the gluon propagator via the Ward-Slavnov-Taylor identities, and the transverse part is assumed to be negligible. This is the final step that leads to the BBZ equation, which in Euclidean space takes the form:

$$\begin{aligned}
\frac{n_\mu \Pi_{\mu\nu} n_\nu}{n^2} &= -\frac{q^2}{Z(q^2)} \\
&= -q^2 - G_0 \int dk \frac{n.(k-k')\, n.k'}{n^2} D^{(0)}_{\lambda\sigma}(k) D^{(0)}_{\lambda\sigma'}(k') \\
&\times \left\{ -\frac{Z(k)}{Z(q)} \frac{Z(k') - Z(q)}{k'^2 - q^2}(q+k')_\sigma q_{\sigma'} \right. \\
&+ \left. \frac{Z(k) - Z(k')}{k^2 - k'^2}(k.k'\delta_{\sigma\sigma'} - k'_\sigma k_{\sigma'}) + Z(k)\delta_{\sigma\sigma'} \right\} \\
&+ G_0 \int \frac{dk}{k} Z(k) \left(2 + \frac{k^2 n^2}{(k.n)^2}\right)
\end{aligned} \tag{2.3}$$

where $G_0 = 3\alpha_S^0/\pi$, $dk = d^4k/(4\pi^2)$ and $k' = k + q$. Let us also note that in order to have a solution consistent both with the Dyson-Schwinger equation and with the Ward-Slavnov-Taylor identities, one can derive that the solution has to obey [11]:

$$\Pi_{\mu\nu}(0) = 0 \tag{2.4}$$

We further simplify this equation using the results of Schoenmaker [18]. The angular integrals can be performed after symmetrization over k and k', in the limit that $k'^2 \approx k^2 + q^2$ which becomes exact at large and small q^2. The equation can then be symbolically written:

$$\begin{aligned}
q^2\left[\frac{1}{Z(q^2)} - 1\right] &= \alpha_S^0 \int_0^{q^2} dy F^{(l)}_<(q^2, y) + \frac{1}{Z(q^2)} F^{(q)}_<(q^2, y) \\
&+ \alpha_S^0 \int_{q^2}^{\infty} F^{(l)}_>(q^2, y) + \frac{1}{Z(q^2)} F^{(q)}_>(q^2, y)
\end{aligned} \tag{2.5}$$

†We use the term "inverse" in the sense defined in reference[11], namely $\Pi_{\mu\lambda} D_{\lambda\nu} = g_{\mu\nu} - q_\mu n_\nu / q \cdot n$

where< (>) refer to different functional forms and $F^{(q)}$ is quadratic in Z and $F^{(l)}$ linear. The explicit forms of these kernels can be found in [18].

At this point the equation contains both linear and logarithmic divergences, which we need to remove through renormalization of the mass and of the coupling. The linear divergences come from the constant part of the integrand of Eq. (2.5) and can be removed by subtracting from the right hand side its value at $q^2 = 0$. This in turn guarantees that Eq. (2.4) is obeyed. We should point out here that in the case of the solution $Z(q^2) \sim 1/q^2$ one extra subtraction needs to be performed, as for such a singular solution one gets an extra finite contribution to $\Pi_{\mu\nu}(0)$. As we shall eventually be looking for softer solutions near the origin, these extra terms are not included in the equation we solve. Note also that in the case of the BBZ solution, the extra piece coming from the pole in Z becomes infinite if Z depends on γ. The solutions we are looking for do not exhibit this feature.

In order to deal with the logarithmic divergence, one needs to perform charge renormalization. One chooses a renormalization point m where, by definition:

$$Z(q^2) \equiv Z(m^2)\; Z_R(q^2) \quad \text{or} \quad Z_R(m^2) \equiv 1 \tag{2.6}$$

One can then show that in order to absorb all the $Z(m^2)$ into the coupling, the latter needs to be defined as:

$$\alpha_S(m^2) = \frac{\alpha_S^0 Z(m^2)}{1 + \alpha_S^0 Z(m^2) \left[\int_0^{q^2} F_<^{(l)} + \int_{q^2}^\infty F_>^{(l)}\right]_{Z=Z_R}} \tag{2.7}$$

With this definition, and after some rearranging of terms, the net effect of charge renormalization is to replace all Z by Z_R, the coupling α_S^0 by $\alpha_S(m^2)$ and to subtract from the right hand side of the equation its value at $q^2 = m^2$. This leads to:

$$\begin{aligned} \frac{1}{Z_R(q^2)} &= 1 + \frac{3\alpha_S(m^2)}{\pi}\, T_1 + \frac{3\alpha_S(m^2)}{\pi}\, \frac{T_2}{Z_R(q^2)} \\ T_1 &= \int_0^{q^2} F_1(q^2,y)dy - \int_0^{m^2} F_1(m^2,y)dy + \int_{q^2}^\infty F_2(q^2,y)dy - \int_{m^2}^\infty F_2(m^2,y)dy \\ T_2 &= \int_0^{q^2} F_4(q^2,y)dy - \int_0^{m^2} F_4(m^2,y)dy + \int_{q^2}^\infty F_5(q^2,y)dy - \int_{m^2}^\infty F_5(m^2,y)dy \end{aligned} \tag{2.8}$$

with

$$\begin{aligned} F_1(x,y) &= Z_R(x+y)\left(\frac{5\,y}{12\,x^2} + \frac{2}{3x} + \frac{2}{3(y+m^2)} - \frac{y^2}{12\,x^3}\right) + Z_2(x,y) \\ &\times \left(\frac{y^3}{24x^3} - \frac{y^2}{4x^2} - \frac{y}{4x}\right) + \frac{3}{4x}\left(Z_R(y) + y\,\frac{dZ_R}{dy}\right) \\ F_2(x,y) &= -\frac{Z_R(x+y)}{4y} - \left(\frac{3\,y}{4x} + \frac{1}{3} + \frac{x}{8y}\right) Z_2(x,y) + \frac{2\,Z_R(x+y)}{3(y+m^2)} + \frac{3y}{4x}\frac{dZ_R}{dy} \end{aligned}$$

$$\begin{aligned} F_4(x,y) &= \left(-\frac{y}{6x^2} - \frac{2}{3x} - \frac{2}{3(y+m^2)}\right) Z_R(x+y)\; Z_R(y) - Z_R(x+y) \\ &\quad Z_4(y,x)\, \frac{2(x+m^2)}{3(y+m^2)} \\ F_5(x,y) &= Z_R(x+y)\; Z_R(y)\left(\frac{7}{6y} - \frac{2}{3(y+m^2)}\right) - Z_R(x+y)\; Z_4(y,x)\, \frac{2(x+m^2)}{3(y+m^2)} \end{aligned}$$

We have used the shorthand notation $Z_2(x,y) = (Z_R(x+y) - Z_R(y))/x$, $Z_4(x,y) = (Z_R(x) - Z_R(y))/(x-y)$ and in the following we shall speak only of the renormalized $\alpha_S(m^2)$ and Z and drop the subscript R.

Note that the renormalized coupling, which is given by Eq. (2.7) is not the usual axial gauge running coupling constant, but rather corresponds to another subtraction scheme, which we shall refer to as the BBZ scheme. Its value, and the corresponding Λ_{BBZ}^{QCD} can then be different from the usual $\overline{MS}$ values. With this in mind, we can now turn to the description of the solution we have obtained.

3 A new solution to the BBZ equation

As was already mentioned, the BBZ equation relies on a series of approximations which are supposed to become exact in the infrared regime. The equation however involves integrals to infinite momenta, so one needs to use an ansatz that will solve Eq. (2.8) for all q^2, but we shall trust it only for low q^2 values. Fortunately, the exact behaviour at $q^2 \to \infty$ does not influence the infrared part very much. Let us first discuss the general ansatz at low q^2, then explain how we can join it to the correct asymptotic behaviour.

For the reasons explained in the introduction, we investigate the solutions that do not exhibit a pole at the origin. Because of Eq. (2.4) we have to assume however that the propagator is infinite at $q^2 = 0$. We thus look for propagators that behave like $(q^2)^{-c}$ near the origin. In the following, c is constrained to be positive, but otherwise left free. So Z near the origin will behave like $(q^2)^{1-c}$ and we multiply this by a Taylor expansion in q^2. Such a solution is consistent as $q^2 \to 0$ provided that in Eq. (2.8) $T_2 \to \pi/3\alpha_S(m^2)$ and T_1 remains finite. As T_1 and T_2 involve integrals over all momenta, such a cancellation can come from the region of intermediate and large k^2 in $Z(k^2)$.

The asymptotic behaviour of Z has been studied by BBZ, and by solving the renormalization group equation resulting from (7) they found that

$$Z \sim \frac{1}{(\log(q^2/m^2))^{11/16}} \text{ for } q^2 \to \infty \tag{3.1}$$

We then assume that at some scale m this asymptotic behaviour becomes exact, and we define this value as the renormalization point and so above m^2 we write:

$$Z_R(q^2)|_{q^2>m^2} = \left(\frac{\log(m^2/\Lambda_{BBZ}^2)}{\log(q^2/\Lambda_{BBZ}^2)}\right)^{11/16} \tag{3.2}$$

where Λ_{BBZ} is the QCD scale in the BBZ subtraction scheme.

We then match smoothly the low and high q^2 behaviours, by writing:

$$\begin{aligned} Z_R(q^2)|_{q^2<m^2} &= e^{-A_1(m^2/q^2-1)}\left[\sum_{i=0}^{n_P} P_i\left(\frac{m^2}{q^2}\right)^i\right] \\ &+ \left(\frac{q^2}{\Lambda_{BBZ}^2}\right)^{1-c} e^{-A_2\frac{q^2}{\Lambda_{BBZ}^2}}\left[\sum_{i=0}^{n_B} B_i\left(\frac{q^2}{\Lambda_{BBZ}^2}\right)^i\right] \end{aligned} \tag{3.3}$$

The P_i are used to get a smooth match with the asymptotic behaviour. As it turns out, in order to get a continuous $Z(m^2)$ from Eq. (2.8) , one needs to match Z and its first two derivatives. We use this to eliminate three of the P_i. Apart from c which we impose positive, all other parameters are left free, including the actual value of the coupling $\alpha_S(m^2)$.

Z being dimensionless, it is a function of q^2/m^2 and m/Λ_{BBZ}, so that a rescaling of all the momenta produces an equally good solution. To decide what the actual value of m is, we shall need to calculate some dimensional quantity, such as the total pp cross section, and we shall examine this in the next section. Furthermore, the dependence of Z on m/Λ_{BBZ} comes only from the large q^2 behaviour, Eq. (3.2) and the value of the coupling at $q^2 = m^2$. The first dependence is very weak as it enters only through logarithms.

Our strategy to solve the equation is to input Z^{in} given by Eqs.(3.2) and (3.3) into the right hand side of Eq. (2.8). The left hand side is then Z^{out} and we choose the input parameters so that they minimize the difference between Z^{in} and Z^{out}.

We give in Table 1 the parameters values for such a solution, and we plot in Figure 1 Z^{in} and Z^{out}. We find that we get an acceptable solution for $n_P = n_B = 4$ in (13), for which the maximum difference between Z^{in} and Z^{out} is 4%. Note that this difference occurs only in the middle region (where the large q^2 and small q^2 behaviours are matched) and that it becomes better both at high and small q^2. As it turns out, if we let the coupling $\alpha_S(m^2)$ free, other solutions can be obtained for other values of m/Λ_{BBZ} and give similar curves. However, the value of the coupling at m^2 is always optimized to be about 0.9, and so, for consistency, we require that the coupling in the BBZ scheme be fixed at the value:

$$\alpha_S(m^2) = \frac{\pi}{8\log\frac{m}{\Lambda_{BBZ}}} = 0.9 \tag{3.4}$$

This in turn leads to $m/\Lambda_{BBZ} \approx 1.6$.

As the fit is established for $q^2 < m^2$, one might worry about the validity of the assumed asymptotic behaviour. As a consistency check, we show in Figure 2 that our solution automatically satisfies Eq. (2.8) up to values of q^2 considerably beyond m^2. It is also worth pointing out that the value of c, although left free, always converges to a very small number, of the order of 0.01. This means that Eq. (2.4) will be satisfied, but that for any practical purpose, the propagator will essentially be constant near the origin, as had been suggested in [8].

4 Phenomenology

We shall now attempt to estimate the value of m by studying the total cross section. It must be emphasized at this point that this part of our work is very tentative. Firstly, we shall assume that at very high energy the main effect of confinement is to modify the gluon propagator, and can otherwise be absorbed in quark wave functions. Secondly, we shall only consider the lowest order Born approximation to the processes at hand. This is admittedly very crude, and one can certainly expect large corrections from higher order terms (i.e. the exchanged pomeron should be built out of ladders involving nonperturbative gluons). Finally, as the gluon propagator gets modified, the quark propagator as well as the quark-gluon and gluon-gluon vertices will also receive nonperturbative contributions. We assume here that as a first approximation these can be neglected, and take a coupling γ_μ for the quark-gluon vertex. We intend to come back to these questions in a future publication, but want to show here that the phenomenology, even at lowest order, is roughly right for a range of values of m.

Nevertheless, the use of our solution, which takes into account some of the nonperturbative effects of QCD should be better than the usual purely perturbative ansätze. A somewhat convenient feature is that the value of the coupling is determined by Eq. (3.4) and that the running is included in the propagator. There is thus no further question as to which value the coupling should be run down to.

The BBZ equation is valid at small q^2 only, and for the derivation of Eq. (3.2) we have assumed the asymtotic form dictated by the equation, both for the propagator and for the coupling. The value of the coupling, and of the corresponding QCD scale, depend on the subtraction scheme, and we shall assume that one can extend the BBZ scheme to higher values of q^2, and we thus shall keep the previous value of Λ_{BBZ} and $\alpha_S(m^2)$. However, for phenomenological applications, we know that the asymptotic behaviour is that of perturbative QCD, and that in the axial gauge the running of the coupling constant is contained entirely in the q^2 behaviour of $Z(q^2)$. We therefore assume that at high q^2, $Z(q^2 > m^2) = \log(m^2/\Lambda_{BBZ}^2)/\log(q^2/\Lambda_{BBZ}^2)$.

We shall now examine the elastic cross section $d\sigma/dt$ for pp collisions, and its corollary, the total cross section. One can write the lowest order approximation to the elastic amplitude from 2 gluon exchange as [2, 3]:

$$\mathcal{A}(q^2) = is\frac{8}{9}n_p^2\alpha_S^2 \times (\mathcal{T}_1 - \mathcal{T}_2) \tag{4.1}$$

with

$$\mathcal{T}_1 = \int_0^s d^2k D_g(q/2+k)D_g(q/2-k)[G_p(q,0)]^2 \tag{4.2}$$

$$\mathcal{T}_2 = \int_0^s d^2k D_g(q/2+k)D_g(q/2-k)G_p(q,k-\frac{q}{2})[2G_p(q,0) - G_p(q,k-\frac{q}{2})] \tag{4.3}$$

where n_p=3 is the number of quarks in the proton, $D_g(l) = Z(l^2)/l^2$ is the gluon propagator and $G_p(q,l)$ is a convolution of proton wave functions:

$$G_p(q,l) = \int d^2p d\alpha \psi^*(\alpha,p)\psi(\alpha,p-l-\alpha q) \tag{4.4}$$

with the wavefunction $\psi(\alpha,p)$ being the amplitude for the quark to have transverse momentum p and fraction α of the longitudinal momentum. $\mathcal{T}_1$ comes from diagrams where both gluons are attached to the same quark within one proton, whereas $\mathcal{T}_2$ comes from diagrams in which the gluons are attached to different quarks. $G_p(q,0)$ is equal to the elastic form factor, $F_1(q^2)$. If we assume that the wavefunction is heavily peaked at $\alpha = \alpha_0$ then $G_p(q,l)$ is equal to $F_1((q+l/\alpha_0)^2)$. In the case of purely perturbative propagators it is necessary to choose $\alpha_0 = 1/2$ in order to ensure a finite amplitude (this is implicit in ref. [3]). In the case of pp scattering a more reasonable value is $\alpha_0 = 1/3$.

From this amplitude, one can get the total cross section $\sigma_T = \mathcal{A}(0)/is$ and the elastic cross section $d\sigma/dt = |\mathcal{A}(t)|/16\pi s^2$. Furthermore, the elastic cross section behaves like $e^{-B|t|}$ and the logarithmic slope B can also be estimated. The situation in perturbative QCD is that the total cross section is of the right order of magnitude, but the shape of the elastic one is wrong, in particular $B(0) = \infty$. In other words, if $D_g(k) \sim 1/k^2$, then $\mathcal{T}_1$ and $\mathcal{T}_2$ diverge individually and only their difference becomes finite, but the cancellation takes place only in $\mathcal{A}$ and not in its logarithmic derivative B. On the other hand, in the model proposed by Landshoff and Nachtmann [8] $\mathcal{T}_2$ is supposed to be negligible, and the other terms are regulated by assuming a finite propagator near the origin. For the propagator discussed

here, using the optimal value of $m = 0.8$ GeV (see below), taking $\alpha_0 = 1/3$ and using the elastic form factor [19], we find $\mathcal{T}_2/\mathcal{T}_1 = 0.37$.

In Figures 3 and 4 we show respectively the total cross section for pp scattering and the logarithmic slope, B, against the scale m at which the propagator becomes asymptotic, with the ratio $m/\Lambda_{BBZ} = 1.56$. We have assumed that the wavefunction is heavily peaked at $\alpha = 1/3$ and taken the form factor from ref. [19]. We emphasize here that this is by no means a unique prescription and uncertainties in the wavefunction lead to uncertainties of at least a factor of two in the cross section. Nevertheless a comparison with data for the logarithmic slope, $B_{exp} \approx 12$ GeV^{-2}, suggests a value of 0.8 GeV for m giving a cross section of 38 mb, consistent with experiment. Using this value of m, we plot in Figure 5 our calculation for the differential cross section $d\sigma/dt$ together with data at $\sqrt{s} = 53$ GeV [20]. As can be seen, the fit is reasonable out to $t \approx -0.5$ GeV^2, beyond which we do not expect single pomeron exchange to dominate.

We can also test the ratio of the total cross section for pp and πp scattering. For the pion we assume that the wave function is peaked at $\alpha = 1/2$ and take the estimate of the pion form factor from ref. [3]. We find $\sigma_{\pi p}/\sigma_{pp} \approx 0.5$, which is somewhat below the experimental value of 0.62. We point out that this value is very sensitive to our choice of α which differs from that of refs. [2, 3]. Indeed, if we also take a proton wavefunction peaked at $\alpha = 1/2$, the ratio turns out to be 0.6.

5 Conclusion

We have studied the Dyson-Schwinger equation in the truncated approximation proposed by Baker Ball and Zachariasen [11] and simplified by Schoenmaker [18]. We have shown that this equation admits a solution that is softer than a pole at the origin, and which as such is a candidate for the full QCD gluon propagator [12]. It assumes its asymptotic form at a scale m and from the value of the coupling $\alpha_S(m^2)$, we estimated that m is about 1.6 times the value of the QCD scale in the BBZ subtraction scheme. Lowest order phenomenology leads to a value for m of the order of 1 GeV.

These results explain why perturbative calculations have been very successful down to momentum transfers of the order of 1 GeV, and at the same time incorporate some of the phenomenological ideas of Landshoff and Nachtmann [8] and put them on a theoretical footing. Moreover, the method presented here provides a theoretical framework which can easily be extended to explore nonperturbative effects on the quark propagators and the

QCD vertices, and can in principle lead to a consistent framework for the evaluation of nonperturbative effects in high energy amplitudes. One can also, using analyticity agument, address the question of the gluon and quark propagators in the s channel, and thus estimate their density of states, as well as their fragmentation properties. Finally, we expect to reexamine the question of higher order corrections and of resummation in this context.

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parameter	value	parameter	value
A_1	4.08	A_2	4.39
P_0	5.65	c	0.014
P_1	-12.6	B_0	1.33
P_2	6.08	B_1	10.4
P_3	2.52	B_2	-30.3
P_4	-0.700	B_3	72.7

Table 1: parameter values of our fit for $m/\Lambda_{BBZ} = 1.56$

Figure Captions

Figure 1: Plot of the propagator as a function of the squared momentum q^2 scaled by m^2 for the region $0 < q^2 < m^2$. The solid line comes from our ansatz (3.3) with parameters given in Table 1. The circles are the output obtained when this propagator is inserted into the right hand side of (2.8).

Figure 2: Plot of the propagator as a funtion of q^2 within and beyond the fitted region. Again the solid line is our ansatz (3.3) and the circles are the output from (2.8). The region where the fit is performed is shaded. The optimal value of $m = 0.8$ GeV is used.

Figure 3: Plot of our calculation of the total cross section for pp scattering against m. The experimental data suggest a value of m of about 0.8 GeV.

Figure 4: Plot of our calculation of the logarithmic slope B at $t = 0$ for the differential cross section in pp elastic scattering against m. Again data suggest a value of m of about 0.8 GeV.

Figure 5: Plot of the differential cross section for pp elastic scattering against $-t$. The optimal value of m (0.8 GeV) is used. The solid line is our calculation and the crosses are data points from ref. [20].

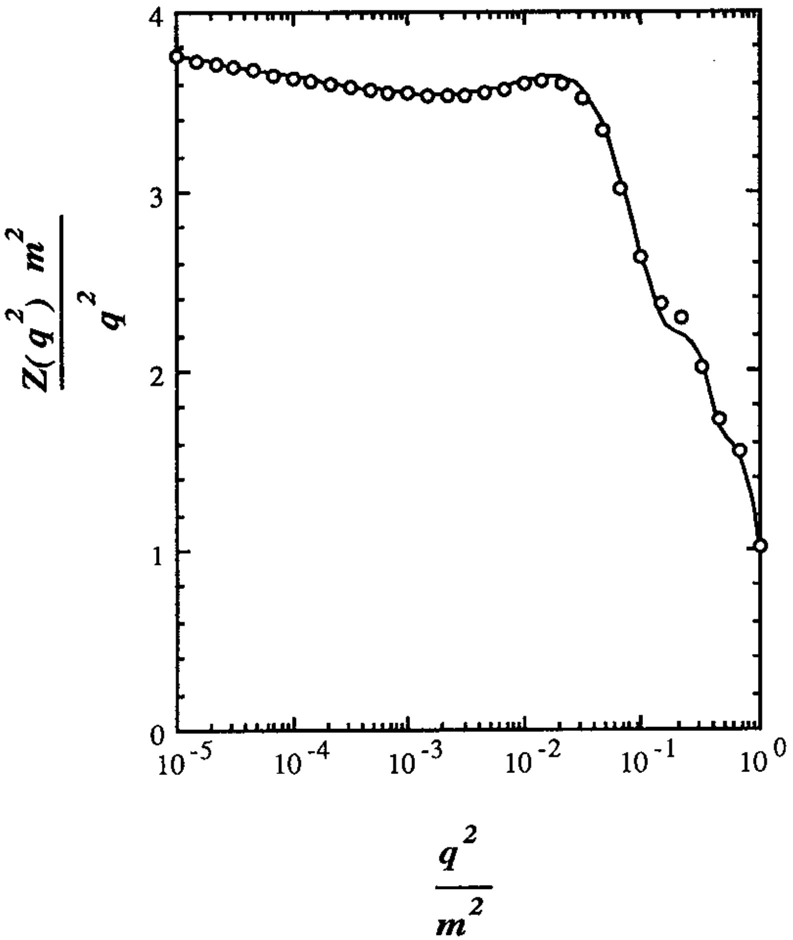

Figure 1

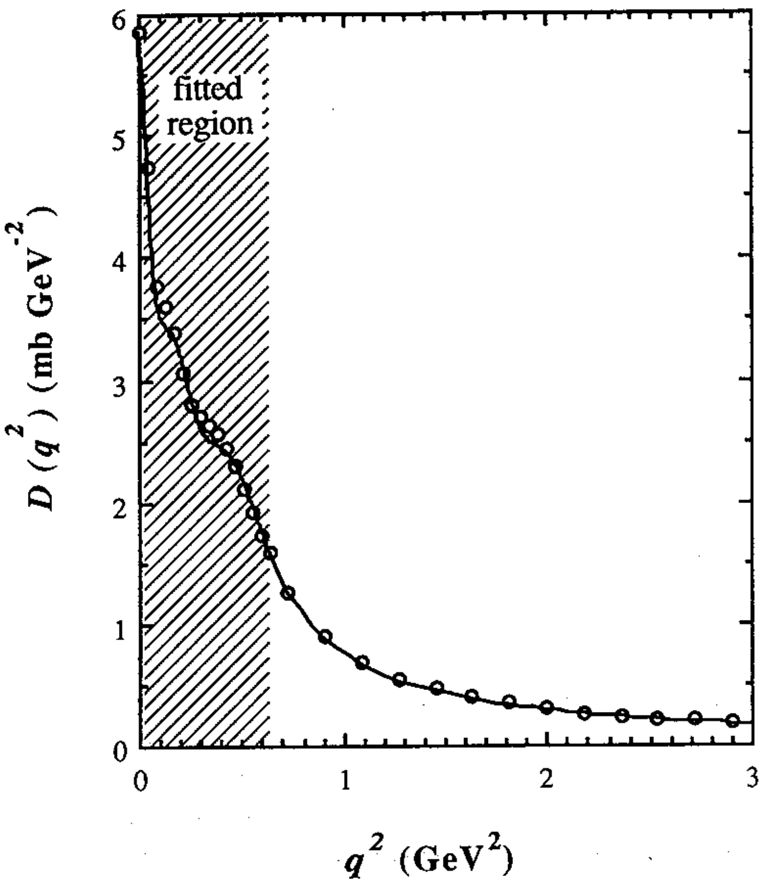

Figure 2

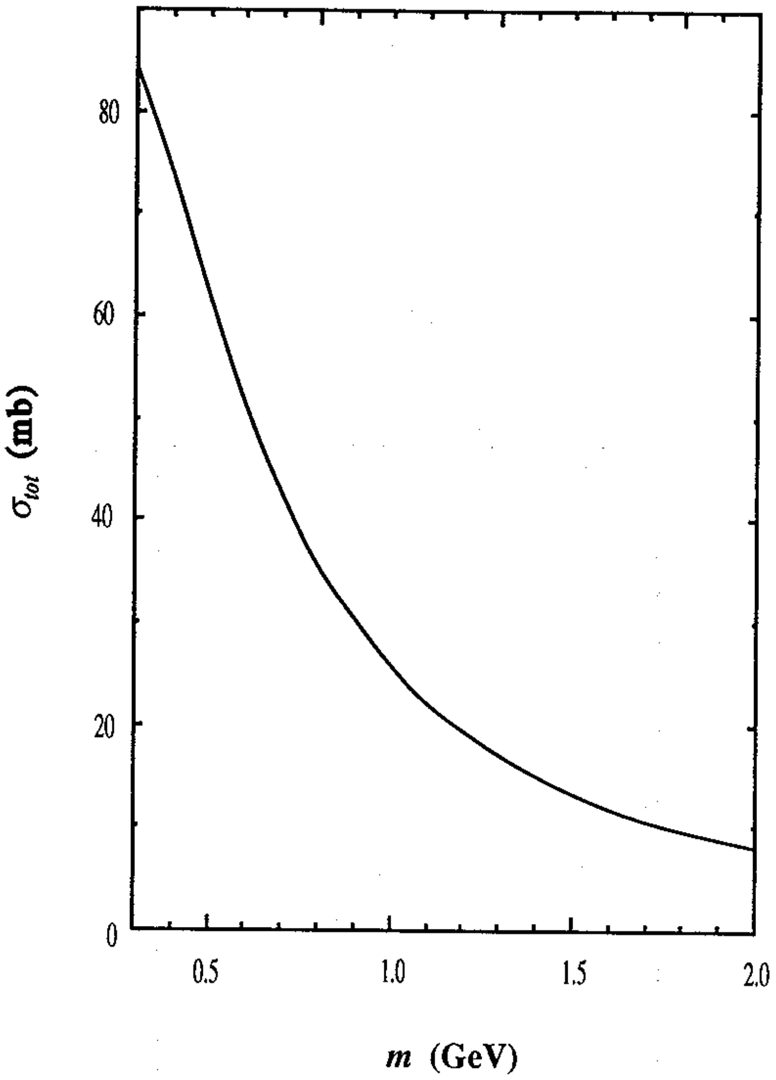

Figure 3

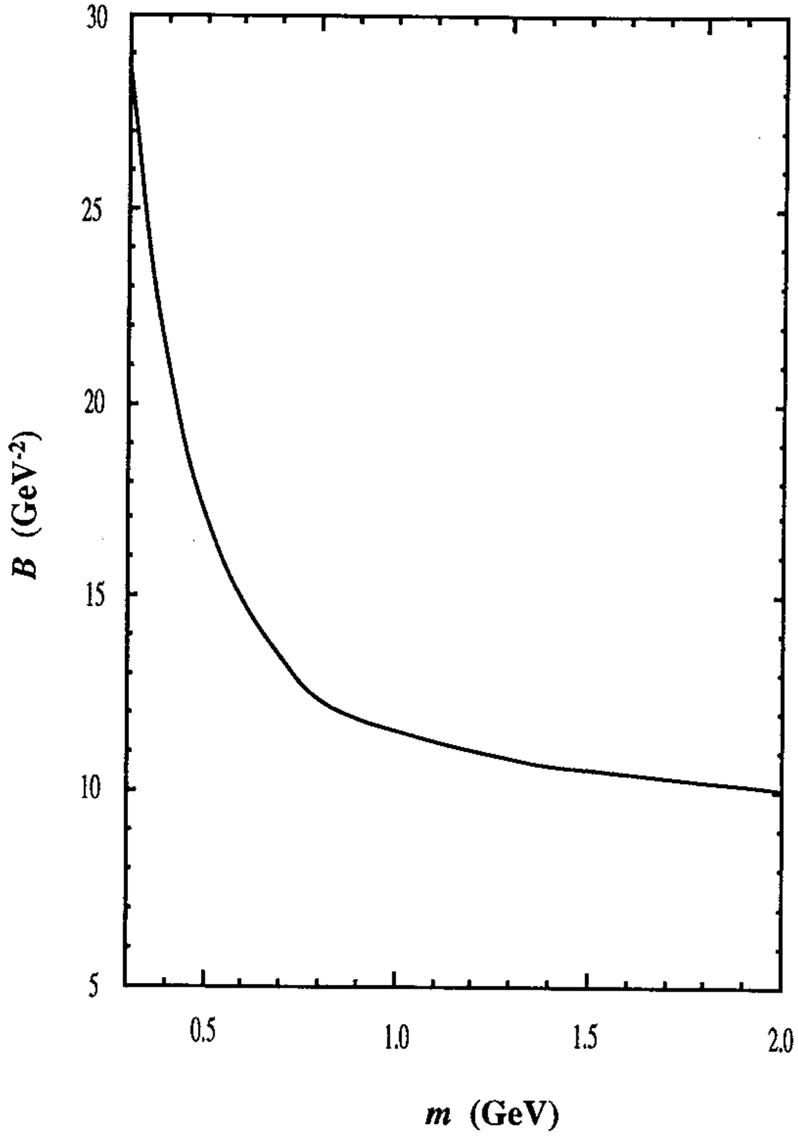

Figure 4

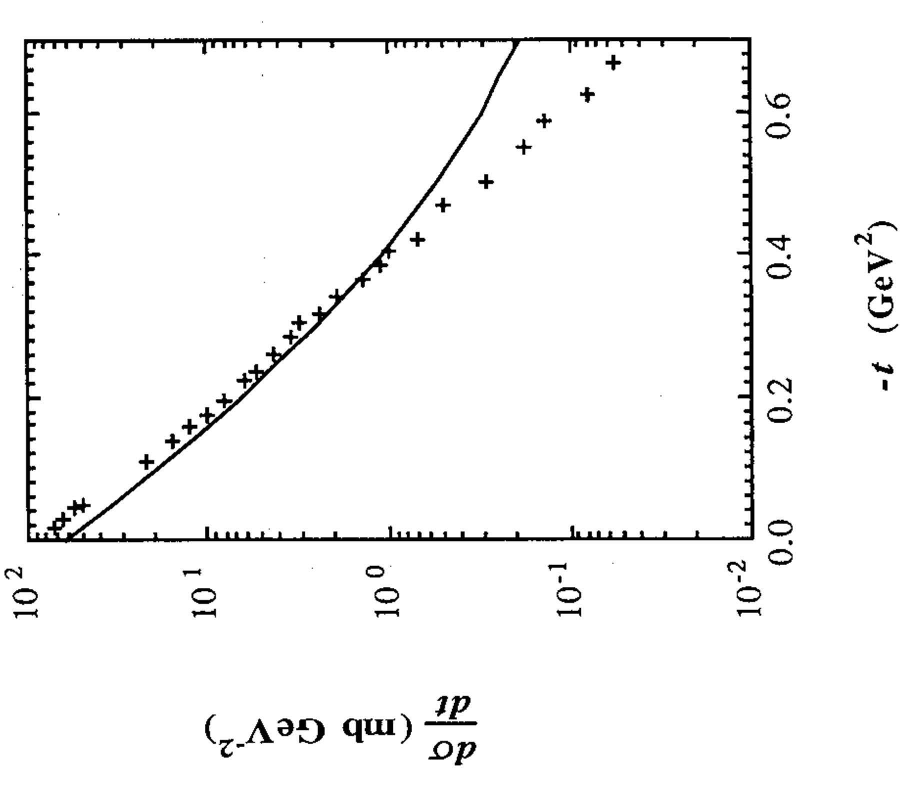

Figure 5

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