

## IS A LOW-MASS TOP QUARK RULED OUT?

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## ABSTRACT

In view of the recent ARGUS measurement of  $B_d^0$ - $\bar{B}_d^0$  mixing, we carefully re-examine the mass limits for the top quark. The actual limit depends critically on the value of the confinement parameters for the  $B_d^0$ - $\bar{B}_d^0$  and the  $K^0$ - $\bar{K}^0$  systems, as well as on the structure of the Kobayashi-Maskawa matrix which we reconsider *ab initio*. The large uncertainties associated with these parameters as well as with input experimental errors allow us to accommodate any top mass above the present PETRA limit of 23 GeV. We also discuss other measurements relevant to the top mass and conclude that the experimental search for small top mass at TRISTAN and SLC is still relevant.

Recently, the ARGUS collaboration announced a measurement of mixing in the  $B_d^0$ - $\bar{B}_d^0$  system.<sup>1</sup> Their value  $r_{B_d} = 0.21 \pm 0.07$  seems surprisingly large for the standard model with three generations.<sup>2</sup> Several new analyses<sup>3</sup> have appeared, some of which suggest that this result, along with known values of  $\epsilon, \dots$  requires the top quark mass  $m_t$  to be larger than 45 GeV. The existence of such a bound for  $m_t$  is crucial not only in view of speculations on the value of  $m_t$  in the range 23 to 35 GeV but even more importantly for TRISTAN and SLC where this is precisely the range of masses that can be explored. With this in mind, we re-examine the calculation of  $\epsilon$  and  $r_{B_d}$ . We conclude that for a reasonable range of parameters, it is possible to accommodate all the data for  $m_t$  in the range 25 to 35 GeV.

The measurement of  $r_{B_d} \equiv \frac{\Gamma(B_d^0 \rightarrow l^+ l^- X)}{\Gamma(B_d^0 \rightarrow l^+ l^- \bar{X})}$  can be translated into  $|\delta m|/\Gamma = \sqrt{2r_{B_d}/(1-r_{B_d})} = 0.73 \pm 0.18$ , with  $\delta m$  the eigenstate mass difference and  $\Gamma$  the mean width of the  $B_d^0$ - $\bar{B}_d^0$  system. This quantity has been analysed theoretically<sup>4</sup> and depends on the following parameters: the  $B_0$ -lifetime and mass,  $\tau_B$  and  $m_B$ , the top quark mass  $m_t$ , a confinement factor  $B_B$ , the decay constant  $f_B$  and the Kobayashi-Maskawa matrix elements  $U_{tb}$  and  $U_{td}$ . Moreover, the box-diagram factor further depends on the  $W$  mass  $m_W$  and the QCD corrections on  $\Lambda_{QCD}$  and the bottom quark mass  $m_b$ . The Kobayashi-Maskawa elements of interest are not experimentally known and one has to extract them from unitarity constraints and from the known values of the other elements. We assume 3 generations and no supersymmetry and use the following parametrization, in terms of mixing angles  $\theta_1, \theta_2, \theta_3$ , and CP-violating phase  $\delta$ , with  $0 \leq \theta_i \leq \pi/2$  and  $0 \leq \delta \leq 2\pi$  by convention. We write  $s_i = \sin \theta_i$ ,  $c_i = \cos \theta_i$ , so the matrix  $U$  can be written

$$\begin{bmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{bmatrix}.$$

The experimental constraints<sup>5</sup> on the absolute values are summarized in Table I.

A recent analysis of the CLEO data<sup>6</sup> gives a new bound for the ratio of moduli:  $|U_{ub}|/|U_{cb}| \leq 0.20$  and we consider only this limit in the following.  $|U_{cb}|$  itself can be determined by the  $B$ -width  $\Gamma_B$  and the  $B$  semileptonic branching ratio  $B_{sl}$ , as given by the formula:

$$\Gamma_B(\text{GeV}) = \frac{1}{B_{sl}} \times \frac{|U_{cb}|^2 G_F^2 m_b^5}{192 \pi^3} \times f \left( \frac{m_c}{m_b} \right) \times X_{\text{QCD}}$$

with

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$$

$$X_{\text{QCD}} = 1 - \frac{2\alpha_S(m_b)}{3\pi} \left( \pi^2 - \frac{25}{4} \right)$$

and  $B_{sl}$  the semileptonic branching fraction of the  $B$  meson.

One further constraint on the Kobayashi-Maskawa matrix comes from the measurement of  $\epsilon$  in the  $K^0-\bar{K}^0$  system. This quantity depends<sup>7</sup> on  $|U_{ab}|$ ,  $a = c, t, b = d, s$ , on the  $W$  mass, on the top and charm quark masses and through radiative corrections on  $\Lambda_{\text{QCD}}$  and  $m_b$ . Furthermore, a confinement parameter  $B_K$  appears for the  $K^0-\bar{K}^0$  system.

The range considered for all parameters is given in Table II. The main unknowns of the problem are the quark masses and the matrix elements  $B_K$  and

$$f_B^2 \times B_B:$$

$$\left\langle K^0 | \bar{s}_L \gamma^\mu d \bar{s}_L \gamma_\mu d | \bar{K}^0 \right\rangle = \frac{4}{3} m_K^2 f_K^2 B_K$$

$$\left\langle B^0 | \bar{b}_L \gamma^\mu d \bar{b}_L \gamma_\mu d | \bar{B}^0 \right\rangle = \frac{4}{3} m_B^2 f_B^2 B_B$$

Lattice calculations as well as QCD sum rules yield a value for  $B_K$  of order 1.

We allow it here to vary<sup>8</sup> between 0.33 and 2. Likewise, we allow the product  $f_B^2 \times B_B$  to vary between 0.01 and 0.1  $\text{GeV}^2$ . The quark masses entering most formulae are ambiguous as one does not know whether to use the current mass or the constituent mass. We allow them to range from the smallest current mass to the lightest meson mass.

One could in principle further constraint the problem by calculating  $\epsilon'/\epsilon$  and  $\delta m_K$ . However, the theoretical calculations predict<sup>9</sup> for a large range of  $m_t$   $\epsilon'/\epsilon \approx (0.2 \text{ to } 0.8) \times 10^{-2}$  subject to large uncertainties. The current experimental value<sup>10</sup>  $\epsilon'/\epsilon = (3.5 \pm 3.0 \pm 2.0) \times 10^{-3}$  is well within this range. The presence of uncalculated long-range contributions to  $\delta m_K$  prohibits any reliable theoretical estimate of that quantity.

One is thus confronted to a multi-parameter fit to the set of data given in Table I. One can define a probability density for a given set of parameters, assuming that the errors on the data are uncorrelated and gaussian. One then varies all the parameters of the problem to find the maximum of that density for a given top quark mass. This maximum probability gives a measure of the likelihood of a given top quark mass and is plotted in Fig. 1 where 1 is the maximum possible value with all the gaussians calculated at the mean. One curve is for the bag parameters allowed to vary in the full interval given in Table I and the other is for the somewhat more restrictive ranges  $f_B^2 \times B_B \leq 0.06 \text{ GeV}^2$  and  $B_K \leq 1.5$ . The maximum probability density is realized for a given set of values of the parameters and so defines a "best fit." Fixing all the parameters but one at their value at maximum, we calculate for each top quark mass an error at the 90% confidence level. These are shown in Figs. 2 and 3, for each considered range of bag parameters.

0.58 and 0.75 respectively. A more detailed discussion including information on the best values of our fits will be published elsewhere.

We close with some comments regarding the observation of a light top in  $p - \bar{p}$  collider experiments. For a top mass as low as 25 GeV the cross-sections become sizeable and one might naively assume that its detection becomes an easy task. Reality is quite different as the problem is not a question of rate but of signal to background with the background related to the production of the lighter charm and bottom quarks. For a light top the decay leptons are not sufficiently energetic to use prompt electrons as a signature. One inevitably relies on muon and multimMuon data. Working at a nominal muon minimum transverse momentum of 3 GeV the inclusive rate  $d\sigma/dp_T$  for prompt muon decay from a 25 GeV top is indeed a few nb/GeV inside 1.5 units of rapidity.<sup>12</sup> The corresponding rate from charm and bottom decay is however two orders of magnitude larger. Although cuts can be designed to increase the signal to noise one ultimately has to rely on isolation cuts to reduce the background by at least a factor of 10. One requires the absence of hadronic activity in the direction of the lepton thus eliminating semileptonic decays of charm and bottom quarks with large transverse momentum faking a top quark. While this might be possible for a heavy top it is a difficult procedure for a 25 GeV quark. Not only is one dealing with accompanying jet activity which is relatively weak, but the problem of fake isolation resulting from mismeasurement of the accompanying jet axis becomes more severe for the softer jets associated with the background to a top quark as light as 25 GeV. It should also be pointed out that any limits from such experiments eventually depend on the calculation of the production cross-section. These calculations are still ambiguous because potentially large contributions of

It is clear that if  $f_B^2 \times B_B$  and  $B_K$  are allowed to reach values as high as 0.1 GeV<sup>2</sup> and 2 respectively, one can adjust the other parameters of the problem to fit all data equally well for any top mass above the PETRA limit of 23 GeV. Even when one restricts the confinement factors to the smaller limits 0.06 GeV<sup>2</sup> and 1.5 respectively, the maximum probability density for masses of order 25 (20) GeV gets reduced by a factor  $\sim 0.53$  (0.22). This means that the experimental quantities are on the average 0.4 (0.6) standard deviation away from their mean value. In fact, the probability drop mainly comes from  $|U_{ud}| = 0.9752$  (0.9752) and  $\delta m/\Gamma = 0.53$  (0.23), which agree with the ARGUS data at the  $1\sigma$  ( $2\sigma$ ) level. Clearly, masses of order 23 to 40 GeV cannot be ruled out.

Our analysis differs from the previous ones in that we used the experimental constraints on  $|U_{us}|$  directly and allowed the bag factors to vary, along with all the other parameters of the problem. We want to point out here that the cross-hatched regions of Figs. 2 and 3 do not represent absolute bounds. They have been derived by integrating the probability density around the maximum that we found. As the probability density depends on 12 variables, the maximum value has several branches, as shown in Fig. 1 and we have only given one of the many possibilities. The allowed range of the phase of the Kobayashi-Maskawa matrix is however similar for different branches, and the new measurement of  $\tau_{B_d}$  shows that for values of  $m_t$  smaller than 80 GeV,  $\cos \delta$  has been established to be negative<sup>11</sup>.

We have also investigated the possible range for  $\tau_{B_s} \equiv \frac{\Gamma(B_s^0 \rightarrow l^+ l^- X)}{\Gamma(B_s^0 \rightarrow l^+ l^- X)}$ . We assume the same matrix element as for  $B_d^0$ , except for the Kobayashi-Maskawa mixing. Our best fit gives to  $\tau_{B_s} \geq 0.75$  for 25 GeV  $\leq m_t < 40$  GeV and  $\tau_{B_s} \geq 0.9$  for  $m_t \geq 40$  GeV. The corresponding numbers from our 90% C.L. regions are  $\tau_{B_s} \geq$

higher order 2-to-3 processes are not theoretically under control. A 90% C.L. on the existence of a top can be greatly modified by a factor of 2 change in the calculated expected cross-section.

Given these problems it has been suggested<sup>12</sup> that dileptons are a superior signature for searching for a relatively light top. The dimuon cross-section  $d\sigma/dM$  is indeed of the order of a pb/GeV for dimuon masses of order 10 GeV and  $m_t = 25$  GeV. Although this might correspond to an observable rate, the corresponding rate for dimuons from  $c\bar{c}$  and  $b\bar{b}$  pairs is again two orders of magnitude larger. The problem remains. It can now be solved by increasing the minimum momentum cut on the decay leptons. The associated reduction in cross section puts the top out of reach of present experiments. Nevertheless this procedure is very promising with the increased luminosity expected at CERN and Fermilab. It also allows a search for  $e\mu$  events which provide in this context the ultimate signature. For a cut on the muon and electron transverse momentum of 5 and 15 GeV respectively we estimate that  $d\sigma/dM$  exceeds 0.1 pb/GeV for  $M(e\mu) = 20-30$  GeV. These estimates correspond to  $m_t = 25$  GeV and  $\sqrt{s} = 630$  GeV. This cross section quickly rises with energy.

In conclusion one can safely say that it will be easier either to rule out or to establish the existence of a top quark from  $p\bar{p}$  collider data in the 30-50 GeV region than to close the window just above the PETRA limit of 23 GeV. We also recall<sup>13</sup> here the fact that the present information on the weak boson widths extracted from the ratio of  $W$ ,  $Z$  cross sections statistically favors a top quark mass below 60 GeV.

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Table I. Data and errors.

quantity	mean value	error
$ U_{ud} $	0.9744	0.001
$ U_{us} $	0.220	0.002
$ U_{cd} $	0.207	0.024
$ U_{cs} $	0.95	0.14
$\frac{ U_{us} }{ U_{cs} }$	$\leq 0.20$	-
$B_{sl}$	0.11	0.01
$\tau_B$ ( $10^{-12}$ s)	1.16	0.14
$m_B$ (GeV)	5.2752	0.0028
$m_W$ (GeV)	81.8	1.5
$\epsilon$ ( $10^{-3}$ )	2.275	0.021
$\frac{\delta m}{T}$	0.73	0.18

Table II. Parameters of the problem.

parameter	minimum value	maximum value
$s_1$	0.21	0.23
$s_2$	0.	0.15
$s_3$	0.	0.1
$\delta$	0.	$2\pi$
$\Lambda_{\text{QCD}}$	0.1	0.4
$m_c$	1.1	1.8
$m_b$	4.4	5.2
$m_t$	20	100
$B_K$	0.4	2.0 - 1.5
$f_B^2 \times B_B$	0.01	0.1 - 0.06

## FIGURE CAPTIONS

Fig. 1 Maximum probability density as a function of the top quark mass, leaving all other parameters free. The plain curve is for  $f_B^2 \times B_B \leq 0.1 \text{ GeV}^2$  and  $B_K \leq 2.0$ , the dashed curve for  $f_B^2 \times B_B \leq 0.06 \text{ GeV}^2$  and  $B_K \leq 1.5$ .

Fig. 2 90% C.L. regions on all the parameters as a function of the top quark mass, obtained by fixing all parameters but one at their value at maximum probability density. The cross-hatched regions correspond to the plain curve of Fig. 1.

Fig. 3 Same as Fig. 2, the cross-hatched regions corresponding to the dashed curve of Fig. 1.

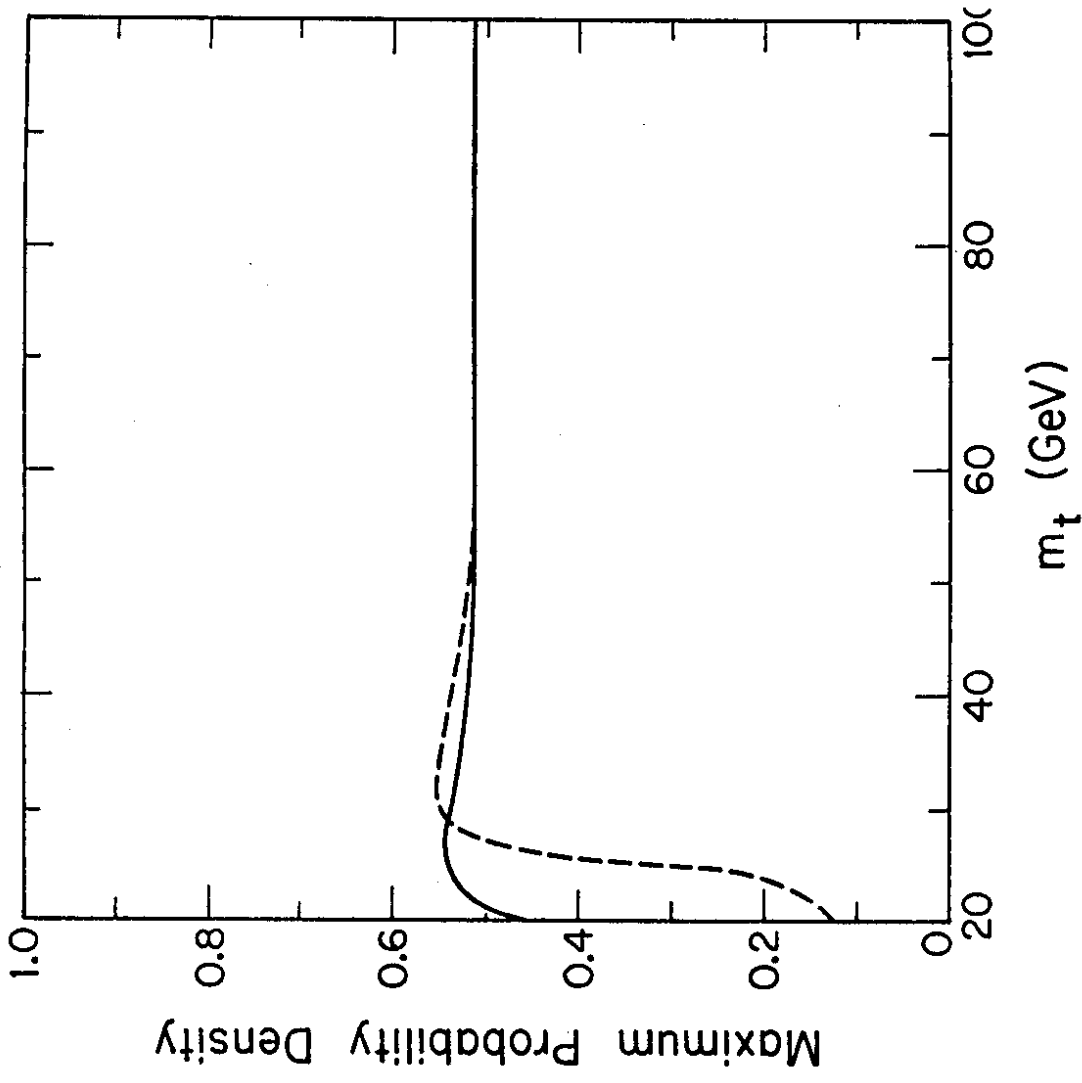
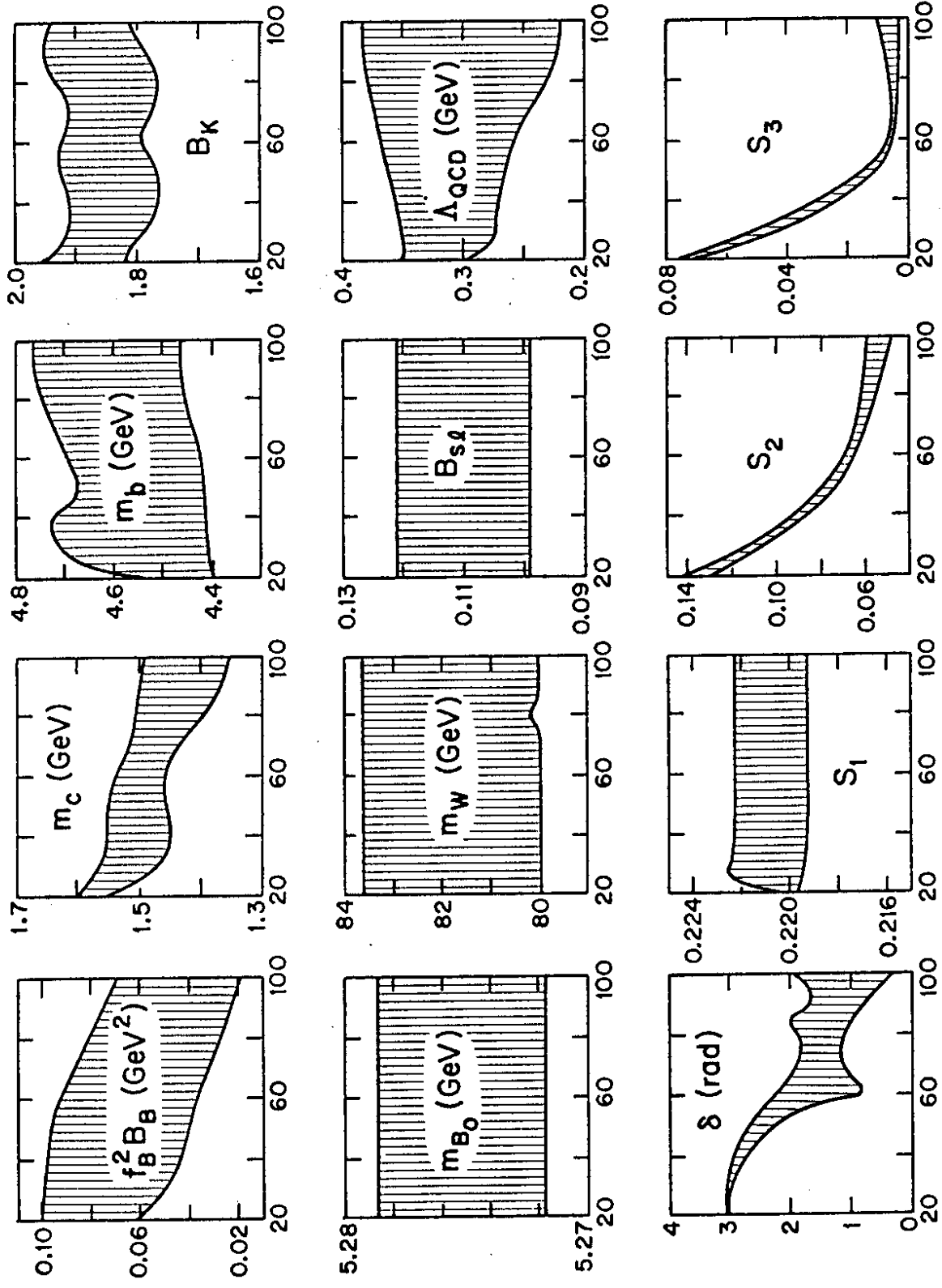


Fig. 1



$m_t$  (GeV)

Fig. 2



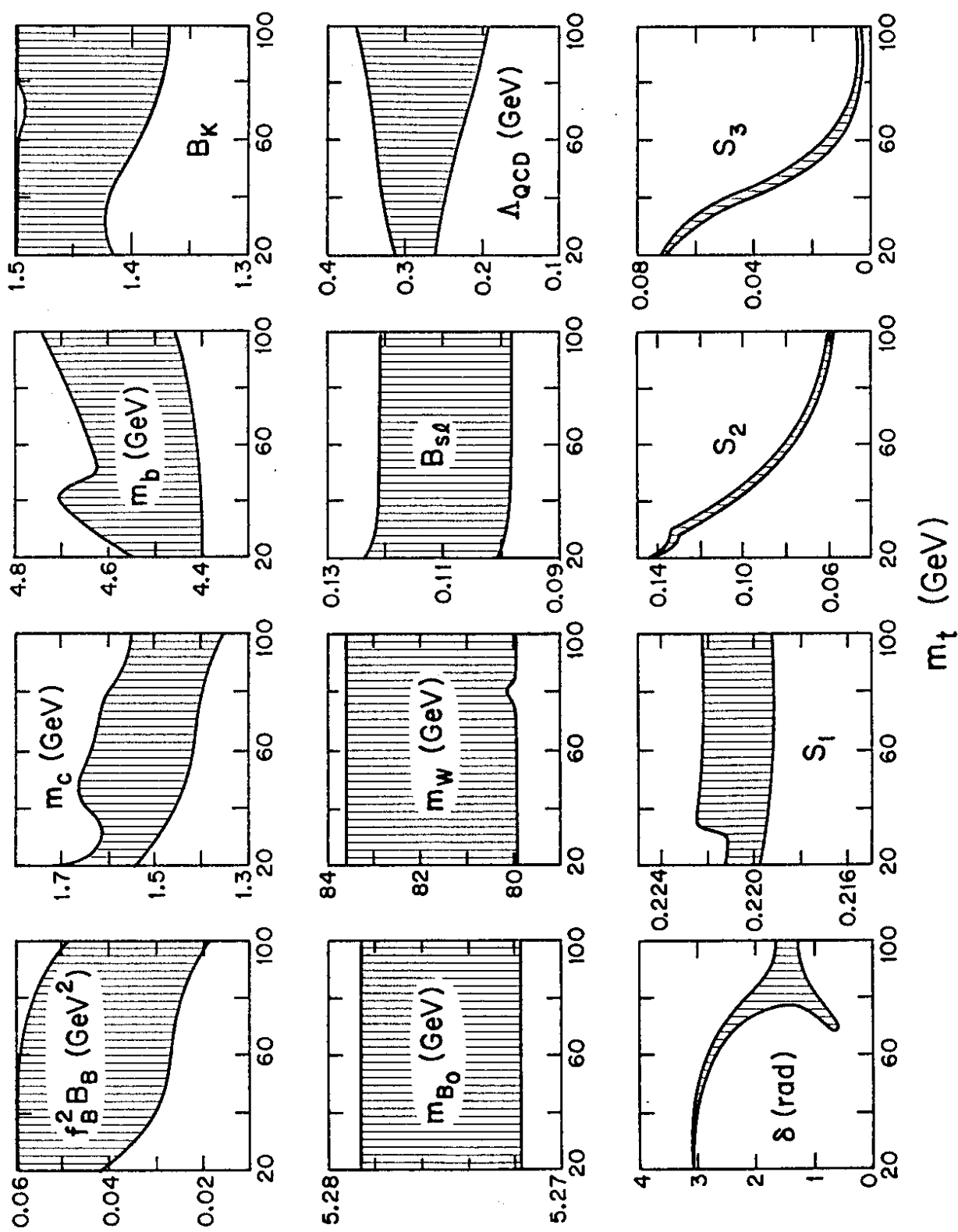


Fig. 3