The internal rotation of low-mass stars from solar and stellar seismology

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The possibility of measuring the internal rotation of the Sun and stars thanks to helio- and asteroseismology offers tremendous constraints on hydro- and magnetohydrodynamical processes acting in stellar interiors. Understanding the processes responsible for the transport of angular momentum in stellar interiors is crucial as they will also influence the transport of chemicals and thus the evolution of stars. Here we present some of the key results obtained in both fields and how detailed seismic analyses can provide stringent constraints on the physics of angular momentum transport in the interior of low mass stars and potentially rule out some candidates.

Keywords: Stellar physics; Stellar evolution; Helioseismology; Asteroseismology

1. Introduction

Low mass stars ($\approx 1M_\odot$) are mostly characterized by the presence in their structure of an outer convective envelope during their main-sequence. As a result, they will exhibit efficient braking through the effect of magnetised winds and they will most often exhibit long rotation periods of a few days. As a result of their low mass, they exhibit long evolutionary timescales of typically a few Gy on the main sequence. This implies that, if a given dynamical process is at work in stellar interiors, it will have time to leave a durable imprint on the evolution of the star, even if its characteristic timescale is relatively long. A good example of such a process is microscopic diffusion, that plays a key role in both solar and stellar modelling.

Another consequence of the existence of an outer convective envelope in low mass stars is that they will exhibit solar-like oscillations. These oscillations are global acoustic modes of the star that have been measured in both the solar case,[1] giving birth to the field of helioseismology, and in the stellar case,[2–4] paving the way for the field of asteroseismology of solar-like oscillators. With the advent of space-based photometry missions, the field of asteroseismology experienced an exponential growth, with the detection of global acoustic oscillations in thousands of stars.

Moreover, the quality of the observations enabled to perform similar analysis to those of the Sun, but on distant stars, increasing the sample of objects for which the internal rotation could be measured. The detection of mixed oscillation modes also gave access to the deep core rotation of evolved stars, providing laboratories for angular momentum transport process at various evolutionary stages.

In addition, the fact that most solar-like oscillators exhibited slow rotation of their surface allowed modellers to use perturbative approaches applied in helioseismology.

These measurements proved to be very challenging to explain using classical self-consistent rotating models including the effects of meridional circulation and shear-induced turbulence. While there exists multiple candidates to explain the internal rotation of low-mass stars on the main-sequence, the internal rotation of post-main sequence stars is still impossible to fully reproduce. Behind these difficulties hide the limitations of stellar models, emphasizing the need to improve our physical representation of stellar interiors.

In this brief review, we will discuss how the internal rotation of low mass stars is measured using a perturbative formalism in the fields of helio- and asteroseismology in Section 2. We will discuss the implications for angular momentum transport of the seismic constraints obtained on the main-sequence and will discuss the link with the transport of chemicals in Section 3. We will focus here on the constraints from acoustic oscillations only. More complete reviews, discussing also the results of massive gravity mode pulsators can also be found in the litterature[5] and classical textbooks also provide a more detailed view and additional references.[6] In Section 4, we will briefly present the main results using the mixed oscillation modes observed in post-main sequence stars and briefly discuss in Section 5 the implications for the main candidates foreseen as the "missing angular momentum transport process".

2. Measuring the internal rotation of low mass stars

The surface rotation of stars is accessible in photometry through the observations of spots on the stellar surface, or in spectroscopy through the determination of the rotational broadening of spectral lines. This allows us to test for example the formalisms used for the magnetic braking of low mass stars or the correlation between the depletion of light elements such as lithium or beryllium and the rotation period of stars in clusters. However, to study the internal rotation of stars, the only method available is by using the constraints derived from global oscillation modes. Indeed, their properties are intimately linked to the internal structure and dynamics of stellar interiors.

In the case of a non-rotating, non-magnetic, isolated star, the global oscillation modes will exhibit spherical symmetry and their angular dependencies can be decomposed on the basis of spherical harmonics. The eigenfunction of an oscillation mode, denoted $\vec{\xi}$, will thus be written in spherical coordinates as

$$\vec{\xi} = a(r)Y_{\ell,m}(\theta,\phi)\vec{e}_r + b(r)\left[\frac{\partial Y_{\ell,m}(\theta,\phi)}{\partial\theta}\vec{e}_\theta + \frac{imY_{\ell,m}(\theta,\phi)}{\sin\theta}\vec{e}_\phi\right], \tag{1}$$

with r,θ and ϕ the radial and angular coordinates of the system, with their respective unit vectors $\vec{e}_r$, $\vec{e}_\theta$ and $\vec{e}_\phi$, $Y_{\ell,m}$ is the spherical harmonic of degrees ℓ and m, and $a(r)$ and $b(r)$ are the functions expressing the radial dependencies of the radial and angular components of the eigenfunctions.

There exists actually a degeneracy in the quantum numbers describing the eigenvalues, as they are identified by three quantum numbers, ℓ, the spherical degree, n, the radial order and m, the azimuthal degree in the general case, but only two are needed to describe the eigenvalue in the non-rotating case.

In the case of low-mass stars, we can assume that we are studying a slow rotator. Consequently, the effects of rotation will thus be treated as a perturbation of the operator describing the global acoustic modes, in a similar fashion to the treatment of a weak magnetic field in the case of the hydrogen atom in quantum mechanics. Just as the degeneracy in some eigenstates is lifted by the presence of the magnetic field in the hydrogen atom, the addition of rotation to the eigenvalue problem of stellar oscillations lifts the degeneracy of the eigenvalues, namely the oscillation frequencies of the star. They appear now as multiplets in the oscillation spectrum defined by their three quantum numbers, this time with m taking values between $-\ell$ and ℓ. This breaking of symmetry stems from the fact that adding rotation to the problem leads to the definition of an equator to the star and that one can now differentiates between the lines of the eigenfunction crossing the equator and those parallel to the equator, that define the quantum numbers ℓ and m in the base of spherical harmonics.

In other words, the eigenvalues will now be described as

$$\nu_{n,\ell,m} = \nu_{n,\ell,0} + \delta\nu_{n,\ell,m} \tag{2}$$

Mathematically, the perturbative approach applied to the stellar oscillation problem leads to a simple integral relation linking the so-called rotational splittings to the internal rotation as function of the radial position r and the latitude θ[7]

$$\delta\nu_{n,\ell,m} = m \int_0^R \int_0^\pi K_{n,\ell,m}(r,\theta)\Omega(r,\theta) dr d\theta \tag{3}$$

with $\delta\nu_{n,\ell,m}$ the rotational splitting, $\Omega(r,\theta)$ the internal rotation profile and $K_{n,\ell,m}(r,\theta)$ the so-called kernel function, depending on the internal structure and the eigenfunction of the oscillation mode.

This integral relation can be solved for the internal rotation profile, using specific numerical techniques (see e.g.[8]). Given that the linear perturbative approach is valid, the internal rotation profile can be determined independently from the stellar model used to perform the inversion. In other words, a non-rotating stellar model is sufficient to perform an analysis of the internal rotation of the star, if rotation is slow enough to be treated as a perturbation.

Equation 3 gives the general $2D$ form of the perturbative relations, but the ability to carry out $2D$ inversions is limited to a few exceptionnal targets, and was only possible for the Sun until very recently. In most seismic analyses, the hypothesis of spherical symmetry will be made and the rotation profile will be inferred as a function of r only. The rotation kernels are then a function of r only and show no explicit dependency in m, as shown for example in classical textbooks.[9] Moreover, the availability of only low ℓ modes for asteroseismic targets will also limit the resolution of the inferences. The positions at which the rotational profile can be inferred are intrinsically bound to the physical nature of the observed oscillation modes and the behaviour of their kernel functions.

3. Results on the main sequence

Main-sequence low mass stars are known to exhibit purely acoustic modes called solar-like oscillations. As indicated by their name, these oscillations have been observed for the first time in the Sun. The main feature of these modes is that they have a much higher amplitude in the outer layers, as indicated in Fig 1 by the $1D$ rotational kernels computed here for a standard solar model.

Consequently, most of the inferences we will have for all main-sequence solar-like oscillators will focus on the upper radiative layers and the outer convective envelope. The inner 20% of the star will not be accessible to the pressure modes and thus will not be constrained by the inversion procedure. This statement is also true for the Sun, despite the wealth of seismic data accumulated in the last 30 years and has motivated the search for the elusive solar gravity modes.

3.1. *The solar case*

In the solar case, thousands of oscillation modes with degrees ranging from $\ell = 0$ to $\ell = 1000$ have been observed.[10–12] This tremendous amount of data has allowed a full scan of the internal solar rotation profile, down to $\approx 0.2R_{\odot}$ (see e.g[13]).

The obtained rotation profile exhibits three main features. First, approximately solid-body rotation in the inner radiative layers of the Sun. Second, latitudinal differential rotation in the outer convective zone. Third, these two zones are connected by a narrow region, the so-called tachocline,[14, 15] where the transition between the two rotation profiles occur.

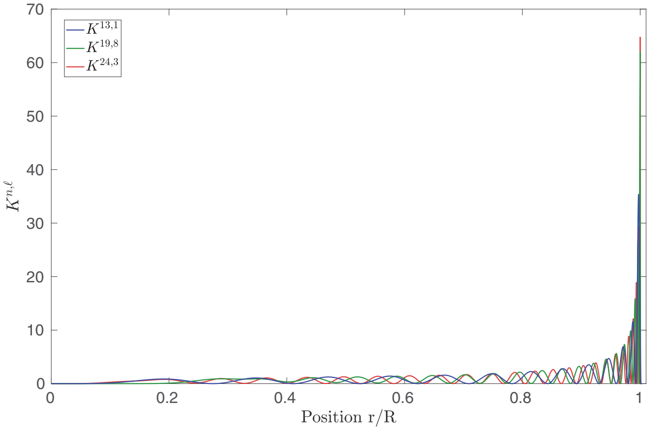

Fig. 1. $1D$ rotational kernels for various acoustic oscillaton modes of a standard solar model.

Here, we will focus mostly on the properties of the rotation profile in the inner solar radiative zone. The observed nearly uniform rotation of the inner solar layers is actually in complete contradiction with rotating models taking into account the effects of meridional circulation and shear-induced turbulence. Despite attempts at revising the prescriptions for these hydrodynamical processes,[16] it appears that their efficiency is too low to prevent the appearence of the characteristic strong differential rotation observed in these models.

Such a strong differential rotation is in disagreement with observations of the internal rotation of low-mass stars and also leads to a transport of chemicals by the shear-instability in disagreement with helioseismic inversions of the structure of the Sun and the depletion of light elements in low-mass stars. Therefore, additional processes have to been invoked to explain the discrepancies between rotating models and observations. The transport process involved should be very efficient at transporting angular momentum, but not chemical elements.

The first candidate is a large scale fossil magnetic field,[17] the second one are magnetic instabilities[18–20] and the third one are internal gravity waves.[21] We will further discuss the properties of these candidates in Section 5. They remain, to this day, the main suspects to explain the efficient angular momentum transport in stellar interiors.

3.2. *Results for Kepler and CoRoT targets*

With the advent of the CoRoT and *Kepler* missions, inferences on the internal rotation of main sequence stars could be carried out for a small sample of the best asteroseismic targets. Various authors[22–24] have shown that in the case of solar-like oscillators, the measurement of the average rotation seen by acoustic modes was consistent with the surface rotation derived from spots or spectroscopic measurements. In other words, no significant degree of differential rotation was found inside solar-like stars. This again accredited the existence of a very efficient transport of angular momentum acting in these stars, allowing to flatten their internal rotation profile.

However, a word of caution is required when stating that solar-like stars rotate as “solid-bodies”. The constraints derived from asteroseismic data are not able to fully resolve the internal rotation of distant stars, but rather provide an average measurement of their internal rotation. Just as for the solar case, the purely acoustic oscillations observed for main-sequence solar-like oscillators do not allow to probe the deep core of these objects. As we will see in Section 5, this has important consequences to select one of the three candidates for additional transport mentioned above[a]. Recently, two studies[25,26] have also shown that solar-like oscillators also exhibited latitudinal differential rotation in their convective envelopes.

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[a] We note that a similar situation is present also in more massive main-sequence stars such as γ Doradus stars, where the observed gravity modes only constrain the near core region, but not the convective core itself, as they do not propagate in these regions. Getting constraints on this region requires the analysis of inertial modes, recently observed.

3.3. *Link with the transport of chemicals*

An important consequence of the presence of an efficient angular momentum transport mechanism is its potential impact on the transport of chemical elements inside the star. For example, the effects of shear-induced instability in the presence of strong radial differential rotation in the radiative layers will be to erase the effects of microscopic diffusion and inhibit the settling of heavier elements from the convective layers to the deep interior.[27,28] While this effect should be limited as a result of the very low amplitude of rotation gradient in stellar radiative zones, it might not be fully negligible. In addition, a certain degree of additional mixing is required to reproduce the lithium abundance of the Sun[29] and solar-like stars.[30–33]

This is illustrated in Fig. 2 for the solar case and young solar-like stars.[34] The results show that mixing by the shear instability and the effects of magnetic instabilities will lead to a certain degree of mixing at the base of the convective zone thus changing both the helium and lithium abundances.

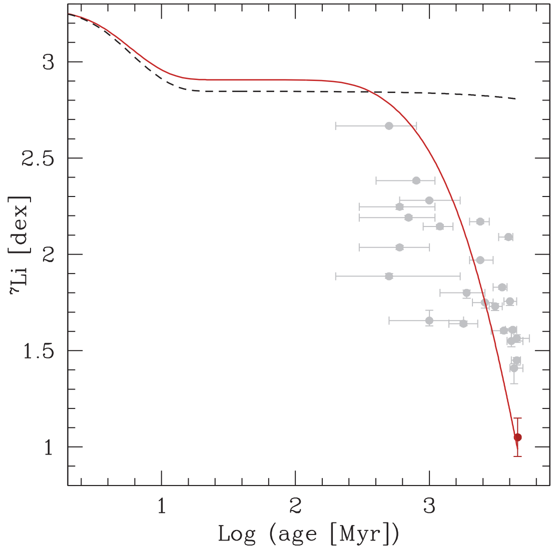

Fig. 2. Lithium abundance evolution as a function of the logarithm of age. The red dot correspond to the solar Lithium abundance[35] and the grey dots correspond to the abundance of solar-like stars.[34]The dashed line indicates the evolution of the surface Li abundance for a standard solar model, while the continuous red line corresponds to a model with hydrodynamic and magnetic instabilities.

Some of the stars observed by the *Kepler* satellite offer excellent testbeds for such effects, as for example the 16Cyg binary system,[36–39] where the binarity also allows to test the effects of planetary formation on the abundances of light elements. Moreover, seismic analyses of the helium abundance in the envelope of other *Kepler* targets[40] have shown the need for a mecanism reducing the efficiency of microscopic diffusion, that could be due to the effects of small rotation gradients.

4. Results for evolved stars

The determination of the internal rotation of subgiant and red giant stars[41–44] has been one of the main successes of the space-based photometry missions. These results have been achievable thanks to the observation of so-called mixed oscillation modes. The peculiarity of these modes is that they are of double nature and have high amplitudes both in the deep core and in the outer layers of post-main sequence solar-like oscillators. They exhibit a gravity-like character in the core and an acoustic nature in the envelope. Therefore they can be easily detected and carry information on the deepest layers of evolved stars, regions that are even unaccessible in the Sun due to the purely acoustic nature of the observed oscillations, despite the quality of helioseismic data.

From the analysis of these oscillation modes, it has been shown that subgiant and red giant stars exhibited radial differential rotation.[41–44] Namely, that their cores rotated faster than their outer layers. While this was expected, as a result of the contraction of the core and the expansion of the convective envelope undergone as the star leaves the main sequence, the degree of differential rotation observed is far lower than what is predicted by theoretical models.[45–47]

Recent results[48] have also indicated that young subgiant stars seem to exhibit almost solid-body rotation. Later on in the subgiant phase, they show some degree of differential rotation that develops but it appears that on the red giant phase, the stellar core does not spin up with the evolution.[49, 50] In later stages of evolution such as the red clump or the secondary clump, similar conclusions are drawn regarding the need for a very efficient transport process.

A crucial point to note here is that to study the efficiency of the missing angular momentum transport process, core rotation is not entirely sufficient, and a measurement of the surface rotation is also required. This is available for a few targets, either from seismology, or from spot modulations. However in the latter case, it seems that the sample is biased towards active, fast rotators, who could perhaps not be quite representative of the bulk of red giants.

The main issue posed by the results obtained for post-main sequence stars is that there is, to this day, no single mechanism capable of explaining all the observations. Analyses of the average efficiency of the missing transport process[51, 52] have shown that it should exhibit a drop in efficiency during the subgiant phase but reincrease as the star ascends the red giant branch. This could possibly point at either a change of regime in the transport process or at multiple transport processes acting at different evolutionary stages. For example, it has been shown that mixed modes themselves would lead to an efficient transport of angular momentum on the upper red giant branch and could potentially take over at stages where another process sees its efficiency reducing.[53]

5. Transport mechanism candidates

Apart from the transport by mixed modes, the main suspects under investigation are the processes already invoked to explain the solar rotation profile. Namely, these are large scale fossil magnetic fields, internal gravity waves and magnetic instabilities.

However, all of them needed to be revised to explain the results observed in post-main sequence stars. We will here briefly summarize the main results and remaining issues with each of these processes.

5.1. *Fossil magnetic fields*

The first candidate introduced to explain solar rotation is a large-scale fossil field present in the radiative zone of the Sun.[17] It was demonstrated that the field, if properly confined, could lead to a rigid rotation profile in the interior of the Sun.

The main difficulty for the fossil field solution in the solar case is indeed linked to the confinement of the field, as it should not extent in the solar convective zone where differential rotation in latitude is observed. This requires the field to reach a very specific configuration for which simulations give conflictual results regarding its confinement to the radiative zone.[54–57] In addition, recent work[58] has shown that, in addition to the confinement problem, simulations required a significant increase in the viscosity of the plasma to avoid a dead zone of the field where the profile would not reproduce at all the observed data.

Nevertheless, a second issue with the fossil field configuration used to explain the solar rotation profile is that it enforces a strictly solid-body rotation in radiative zones. As shown before, this is not what is found in later evolutionary stages and thus another configuration, or another physical process, is required to explain asteroseismic data. One possible configuration for red giants is to have a solid-body rotation profile in radiative zones, combined to a profile in the power law of the radius in convective zones,[59, 60] of the form $\Omega(r) \propto r^{-\alpha}$, with $\alpha \in [1.0, 1.5]$. These studies[59, 60] showed that such a rotation profile could explain the core rotation rate of red giant branch stars inferred from seismic data.[49, 50]

However, these results are in contradiction with recent studies[61–63] who seem to indicate that the transition from the slow rotating envelope to the fast rotating core in red giant stars seems to be located in the radiative zone, close to the hydrogen burning shell. A detailed analysis of the case of Kepler 56 was recently performed,[64] including a full dedicated modelling of the internal structure of the star and an MCMC analysis of the rotational splittings using simple parametric profiles, including the power law solution provided by large scale fossil fields. Their results show that such a rotation profile cannot be used to explain the individual rotational splittings unless α is allowed to reach higher values of the order of 3.5, in contradiction with theoretical values.[59] This suggests that fossil magnetic fields are not the explanation for the efficient angular momentum transport process acting in evolved stars, and other processes must also be invoked.

5.2. *Internal gravity waves*

The effect of internal gravity waves has been studied already early on as being of potential interest to the modelling of stellar interiors and applied to the solar case.[21, 65]

Later studies[66, 67] indicated that the process was not efficient enough to actually flatten the solar rotation profile and pointed that the effect of waves excited by Reynold's stresses would leave a strong imprint of the wavefront on the internal solar rotation profile that is incompatible with observations. When applied to later evolutionary stages, the gravity wave model used on the main-sequence[21] is also found to be not efficient enough to reproduce the internal rotation of subgiants and red giant stars.[68]

However, hydrodynamical simulations[69] underlined the importance of the effects of convective plumes in the generation of gravity waves. These effects were later studied, showing that gravity waves induced by the impact of convective plumes could be much more efficient at transporting angular momentum both in the Sun and subgiant stars.[70, 71] They also showed that it would likely not be able to operate efficiently on the red giant branch, where another process would then be needed. It would then be particularly interesting to compute full evolutionary models with plume-induced gravity waves to investigate their effects on the rotation profile of solar-type and subgiant stars.

5.3. *Magnetic instabilities*

The last main candidate to explain the efficient transport of angular momentum in stellar radiatives zone are magnetic instabilities. Work by H. Spruit[72] showed that the first instability likely set in stellar conditions would be the Tayler instability. In the presence of a differentially rotating fluid, the winding up of a seed magnetic field together with the Tayler instability has been shown to lead to an efficient angular momentum transport mechanism, the so-called Tayler-Spruit dynamo.[73] This transport mechanism has been shown to be able to reproduce the internal solar rotation profile.[19, 20]

While a promising candidate, numerical simulations have shown contrasting results about the apparition of the Tayler-Spruit dynamo.[74, 75] The main difficulty with such conclusions is that such simulations are never in fully realistic stellar conditions.

Nevertheless, in its original form, the Tayler-Spruit dynamo still proves not efficient enough to counteract the spin up of the core at the end of the main-sequence. Models computed taking it into account still predict faster core rotation rates than the seismically inferred values.[76, 77]

Recently, a modification of the Tayler-Spruit instability was proposed that led to a more efficient braking of the stellar cores.[78] Further analyses[79] showed however that this variant could not reproduce all the observational constraints. Namely, it was found to be too efficient for subgiants if one wished to reproduce the red giant cores, or not efficient enough for red giants if the calibration parameter was set to reproduce subgiants, as shown in Fig 3. This calibration parameter, denoted α has to be varied from 0.5 to 1.5 to reproduce respectively the subgiants and the red giants. However, as it is set to the third power in the revised formalism, such a variation is far from anecdotic. A similar issue was found for secondary clump stars and white dwarfs.[80]

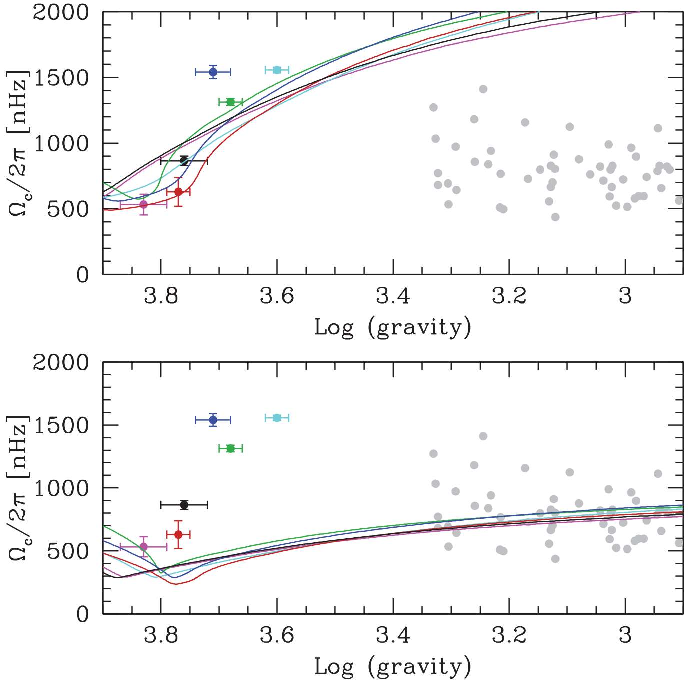

Fig. 3. Core rotation of subgiant and red giant stars as a function of the logarithm of the surface gravity. The colored points correspond to subgiant stars[43] and the grey dots to red giant stars.[49] In the upper and lower panels, the calibration parameter α in the revised version of the Tayler-Spruit dynamo is set to 0.5 and 1.5, respectively.

6. Conclusion

In this brief review, we have described the current state of the vision solar and stellar seismology brought on the internal rotation of low-mass stars in various evolutionary stages. We have focussed on the inferences brought by acoustic and mixed oscillations modes only, trying to provide a complete vision of the current issues at hand regarding the missing angular momentum transport mechanism found to act in stellar radiative zones.

The formalism for seismic inferences within the first-order perturbative formalism has been introduced in Section 2, while results for the main-sequence and post-main sequence stars have been discussed in Sections 3 and 4. We have also briefly introduced the three main candidates currently under extensive investigations in Section 5.

The main conclusion to be drawn from the current situation is a sort of stalemate between multiple processes, none of them providing a unifying satisfactory solution. In that respect, further investigations on the effiency of the missing transport process are required, analysing its dependencies with stellar properties such as mass and metallicity. To this end, better characterizing the location of rotation gradients inside stars is crucial, as well as providing both surface and core rotation measurements from a consistent seismic inference technique to fully constrain the efficiency of the transport process.

It should also be noted that the potential strong influence of chemical composition gradients in the development and efficiency of instabilities (magnetic or hydrodynamic) also calls for a better depiction of the chemical structure of stars. Therefore, the potential solution to the angular momentum transport problem is also tightly linked to the reliability of our vision of the internal structure of stars, that also relies on improving seismic inference techniques. At stakes is likely the recipe for a new generation of solar and stellar models, which would take into account both the effects of rotation, the depletion of light elements and better reproduce seismic and spectroscopic constraints. Moreover, while a unifying solution is tempting, the variations seen at different evolutionary stages might be pointing at various processes taking over during the evolution. To provide a clearer picture, further efforts must be made in the improvement of seismic inference techniques and in the exploitation of the current and future datasets from the CoRoT,[81] *Kepler*,[82] TESS[83] and Plato[84] missions.

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