

# GBOML: A modelling tool for structured MILPs

## TB-06: Software for Optimization 2: Modelling

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University of Liège Belgium

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The 23rd Conference of the International Federation of Operational Research Societies

July 10-14 • SANTIAGO, CHILE



# Examples of structured MILPs

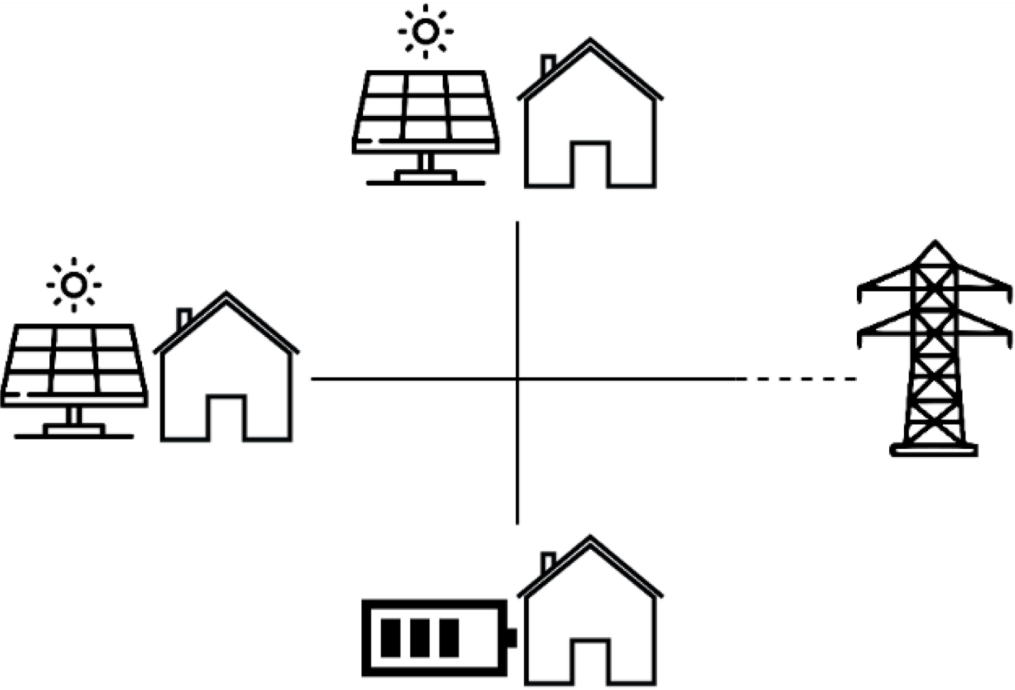


Figure 1: Renewable energy community

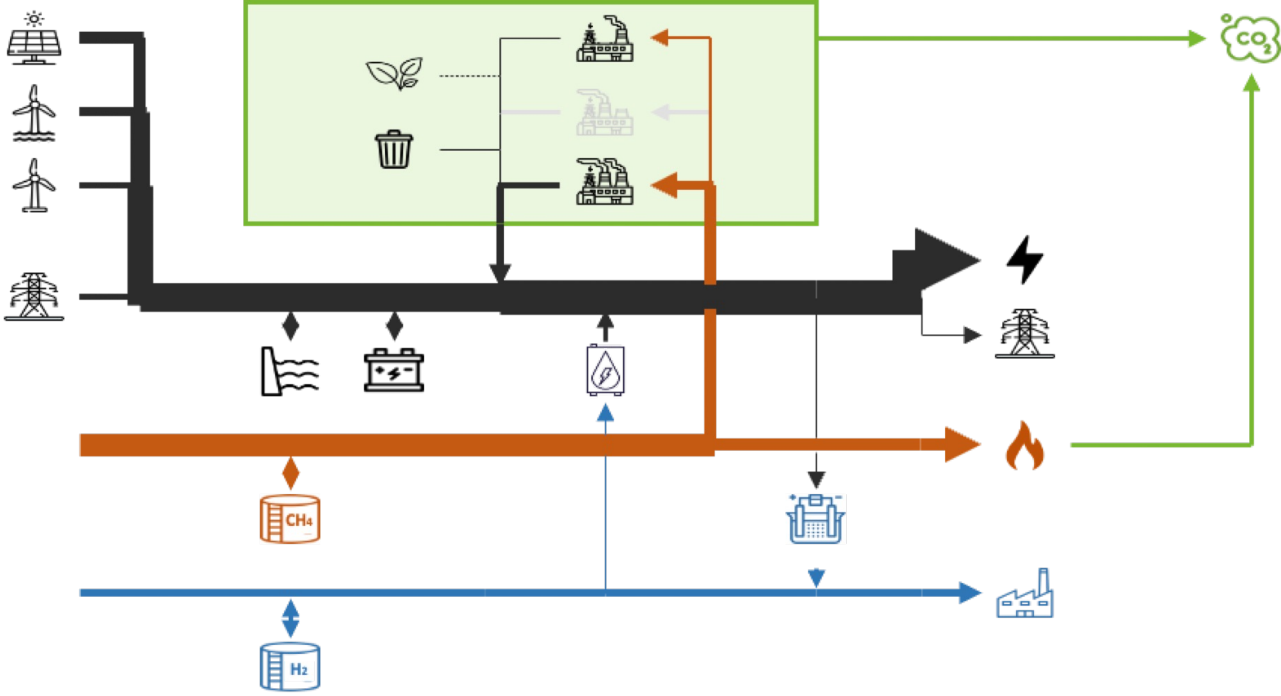


Figure 2: Belgian energy model

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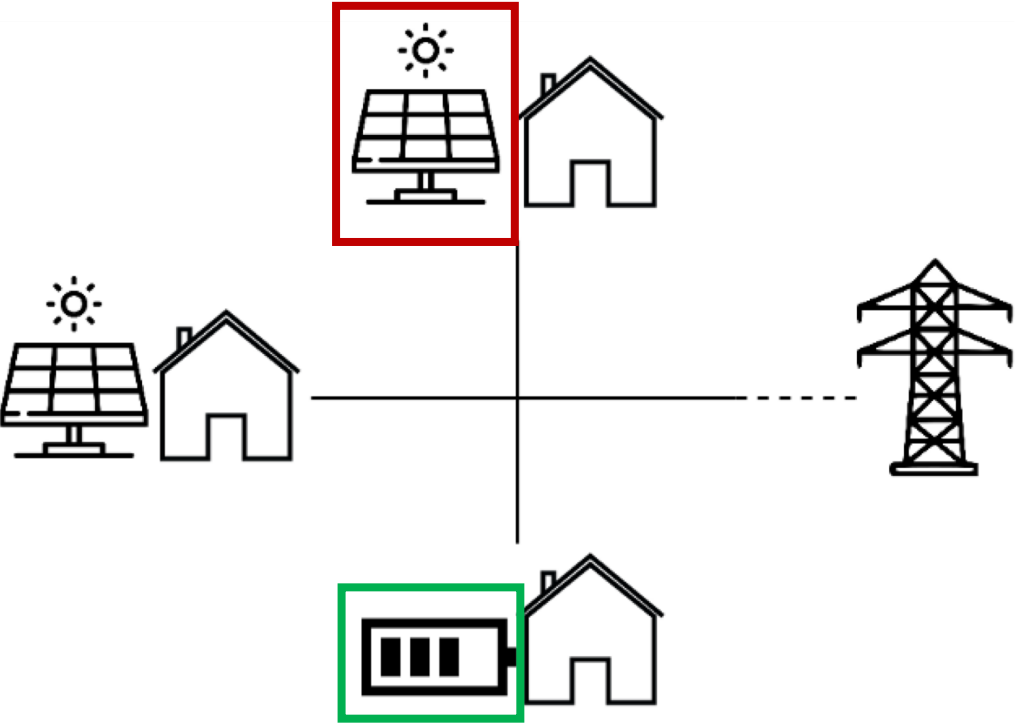


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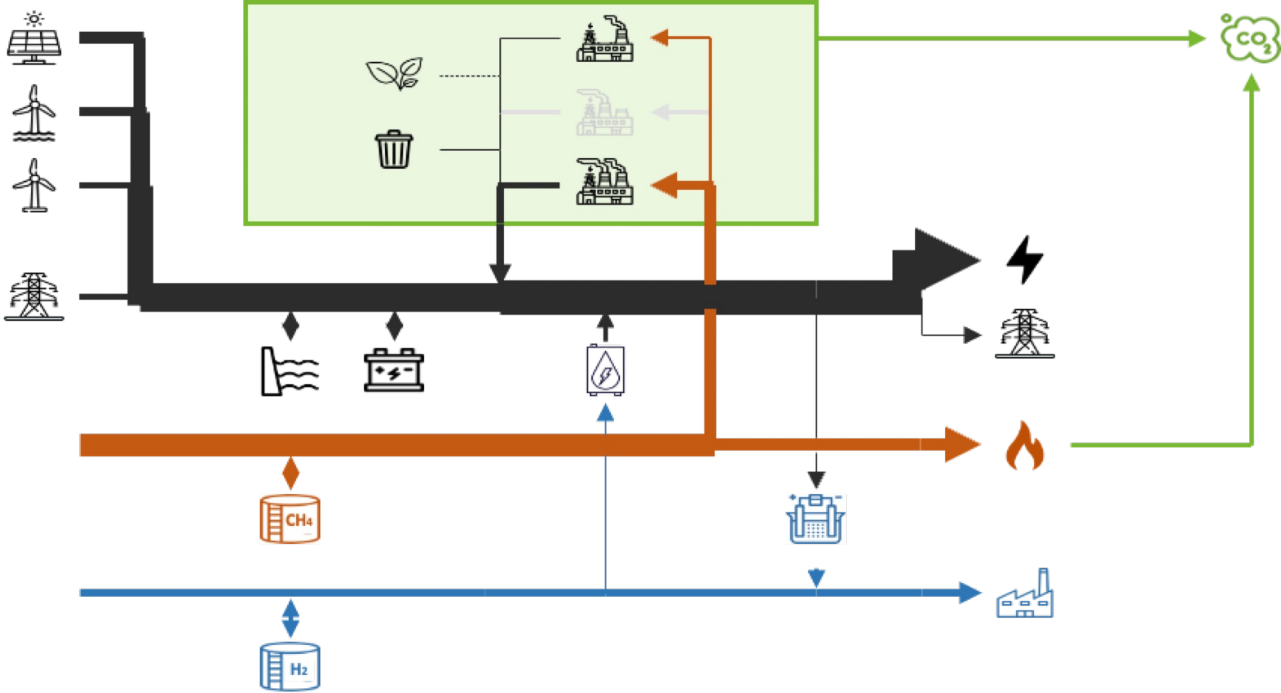


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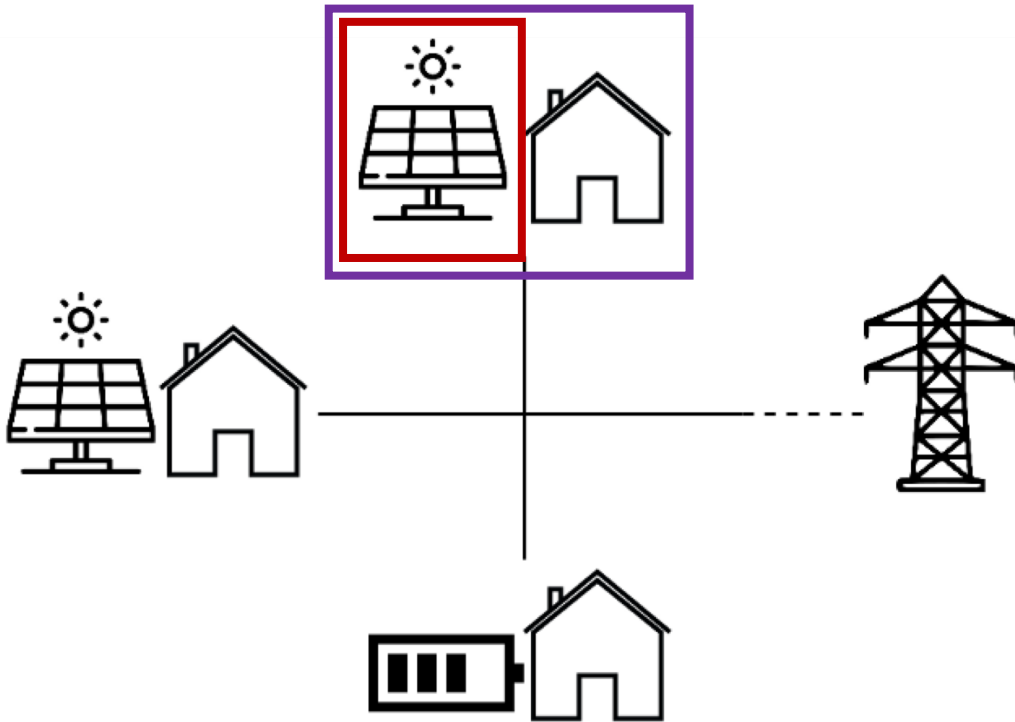


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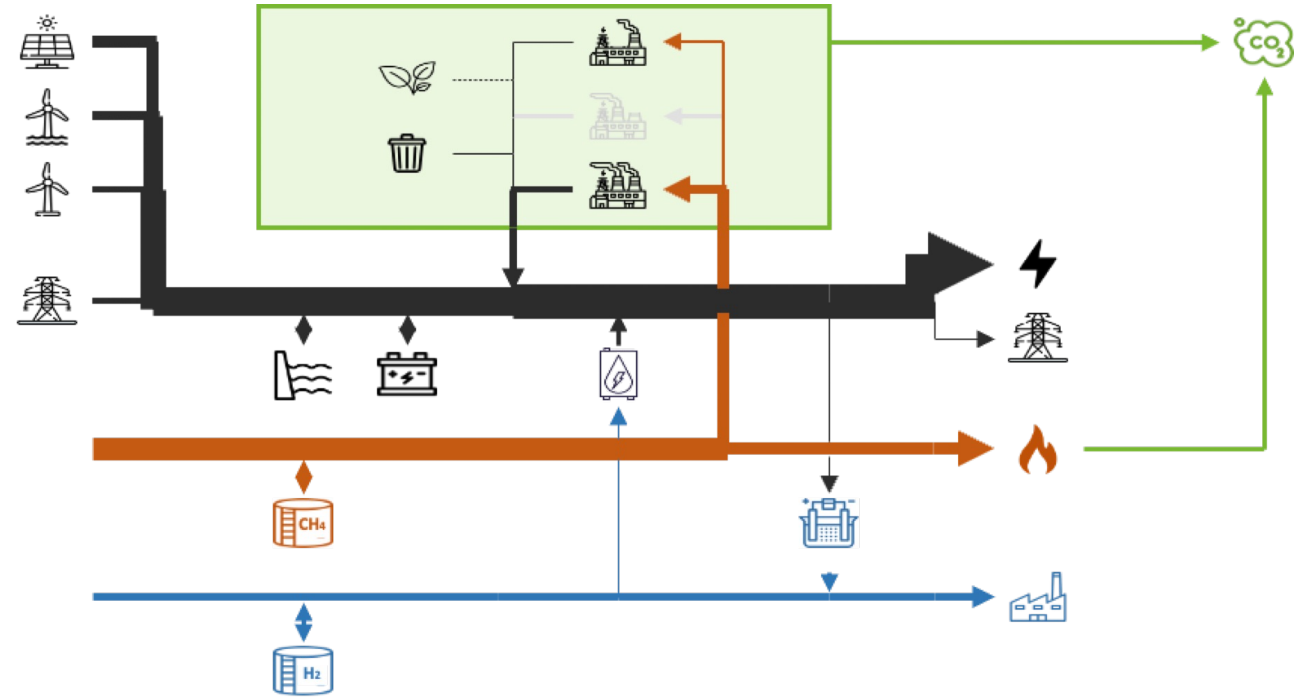


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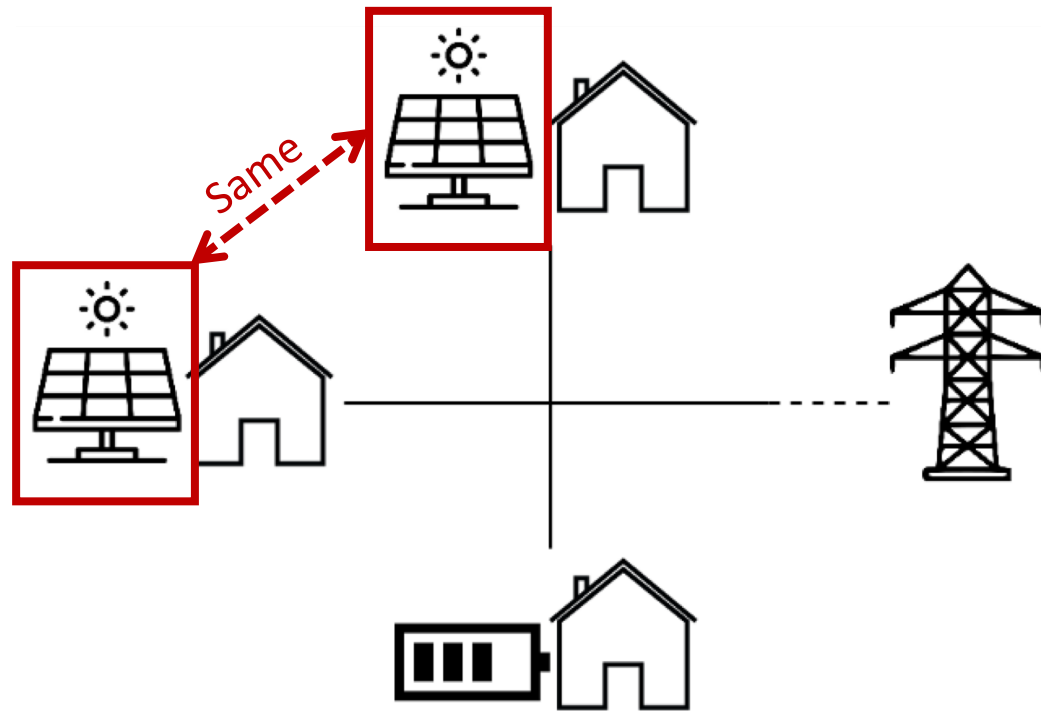


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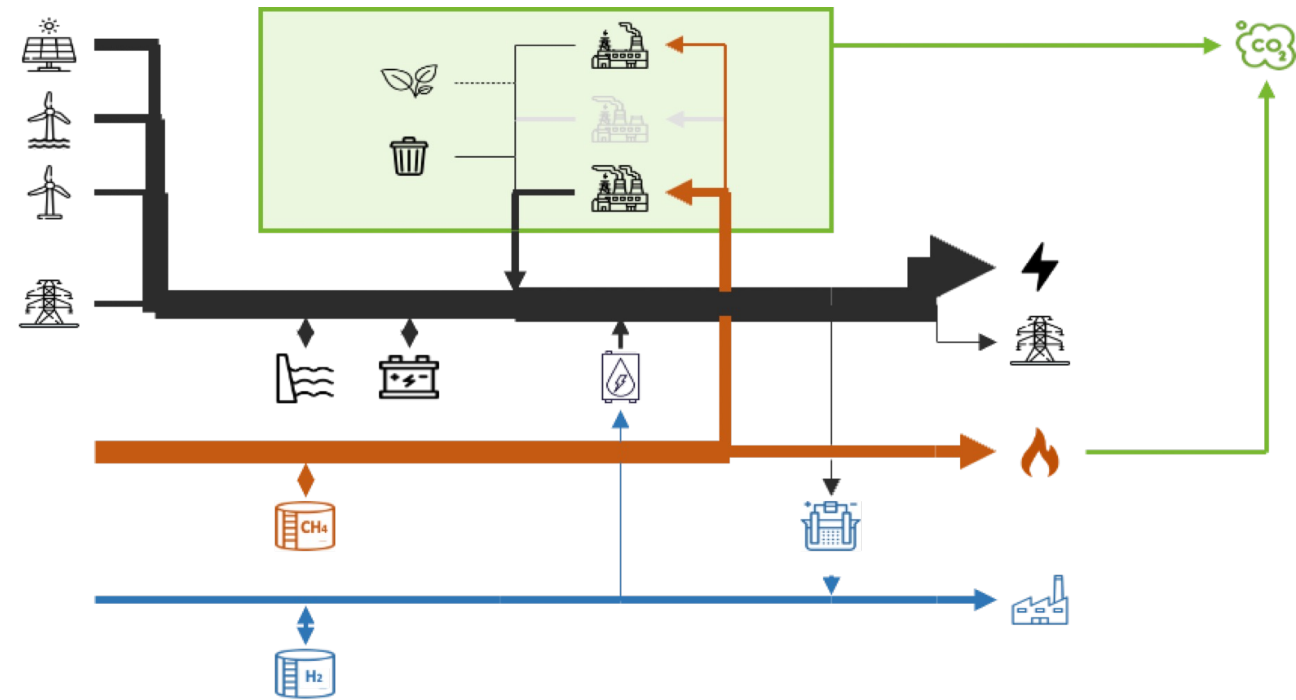


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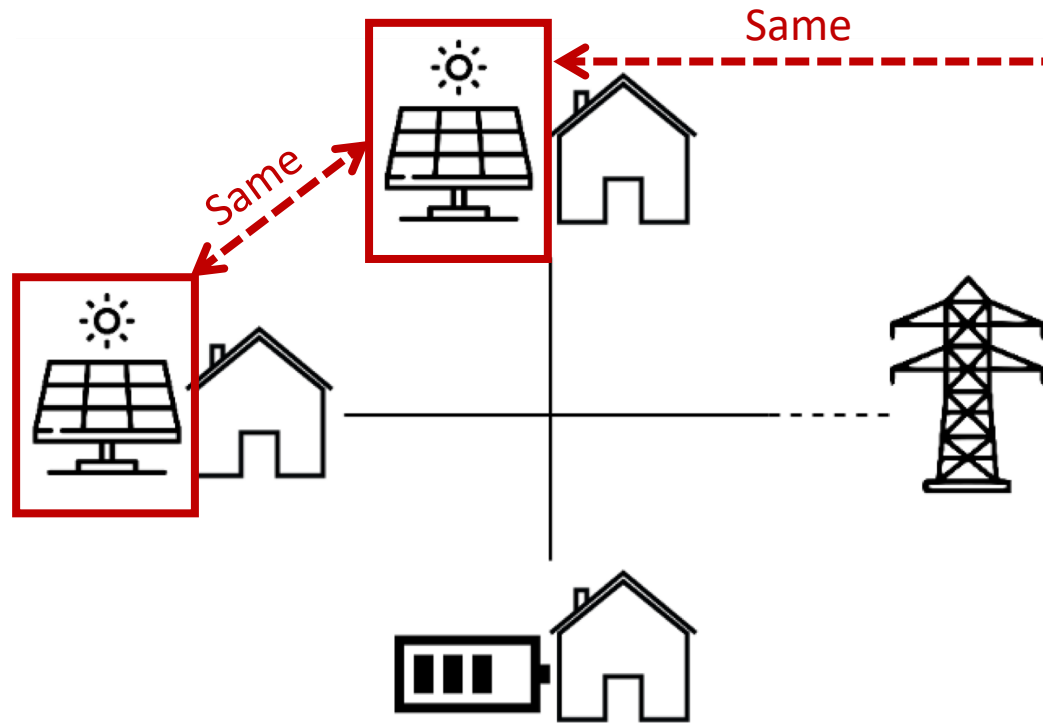


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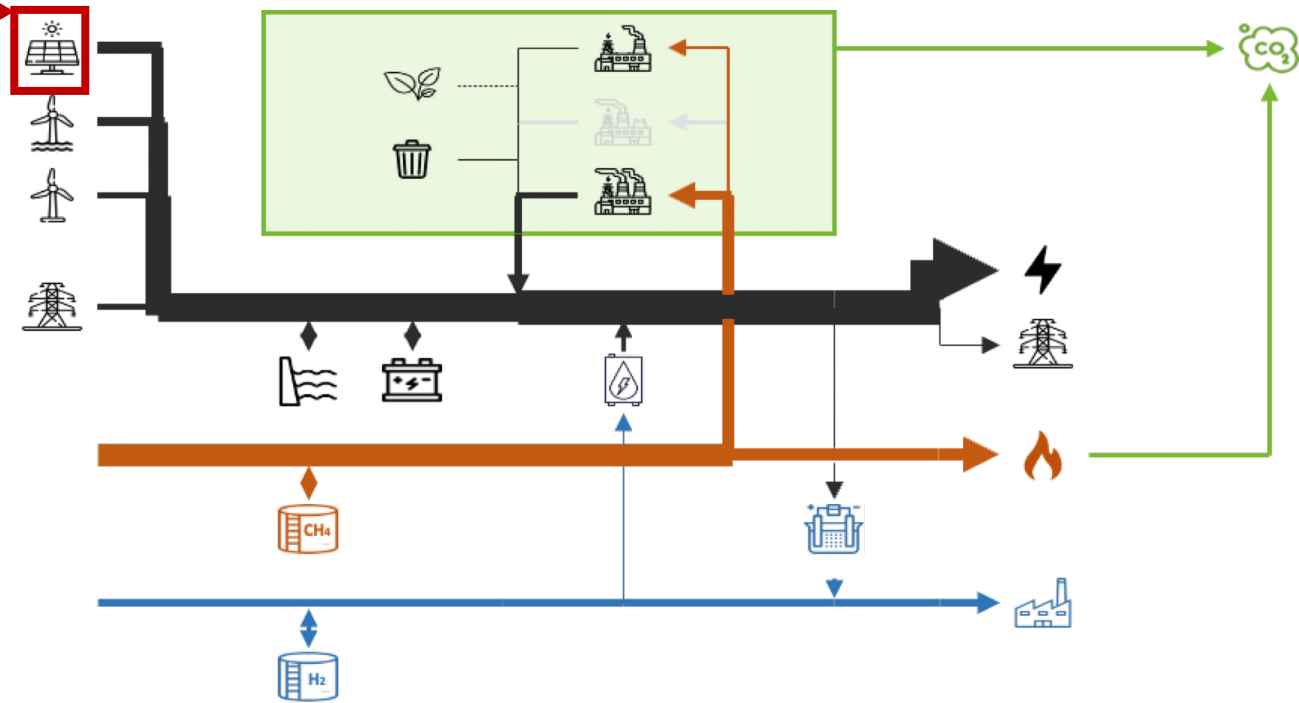
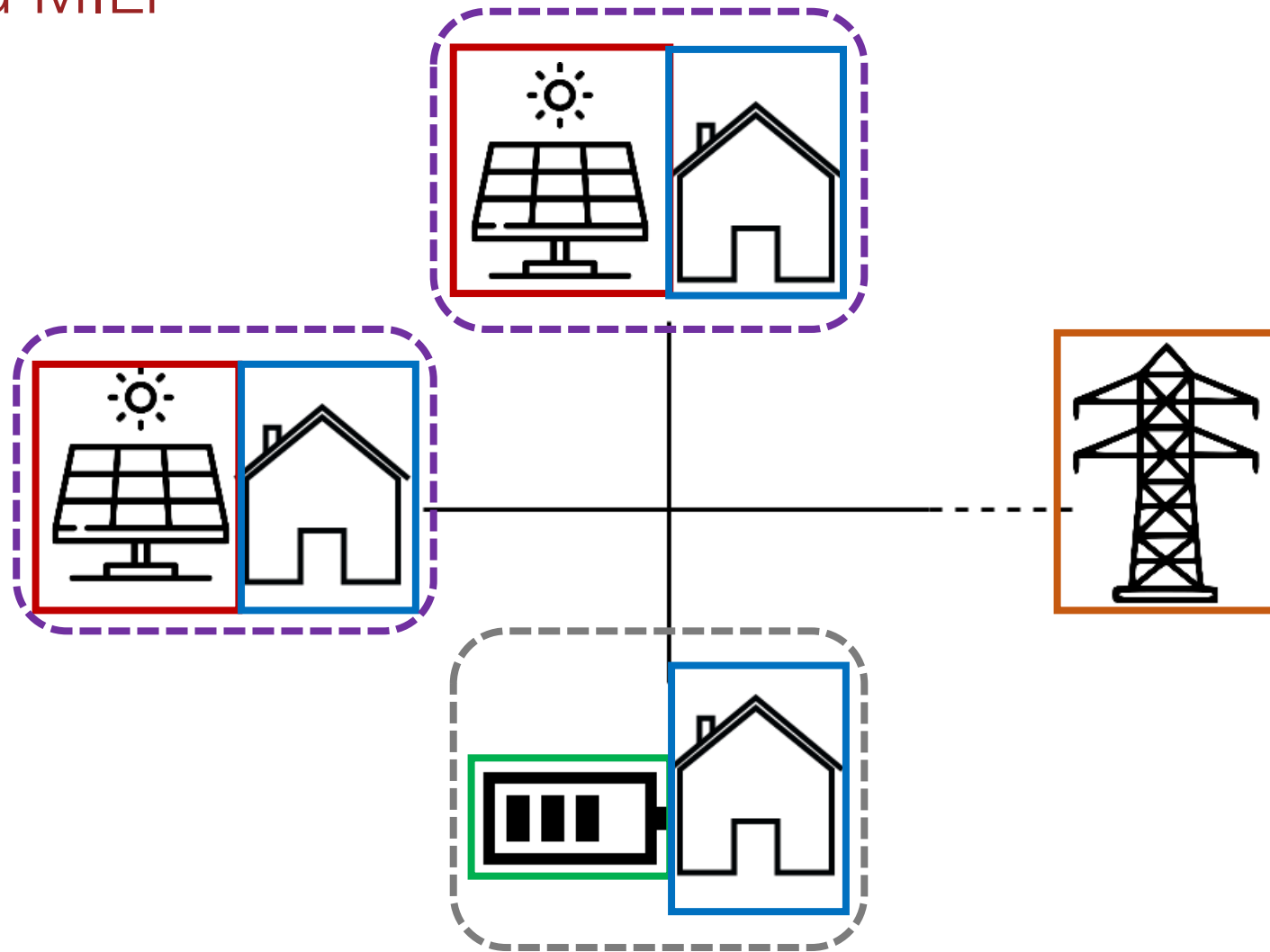


Figure 2: Belgian energy model

Let us formalize these problems

# A structured MILP

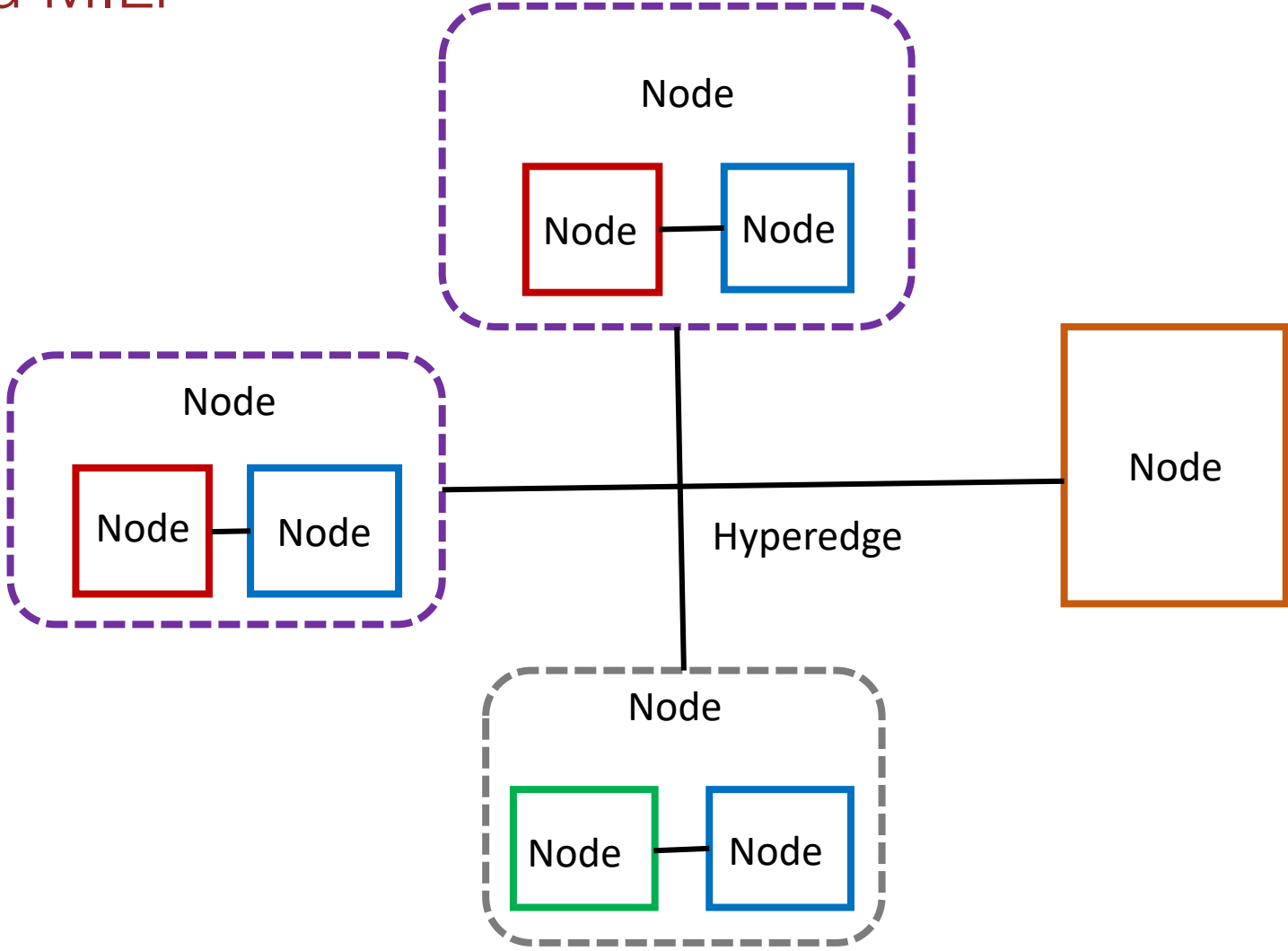


- PV Panels
- House
- Grid
- Battery
- - - Prosumer
- - - Bat-consumer

Figure 1: Renewable energy community



# A structured MILP



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Figure 3: Structured MILP

# Structured MILPs abstraction

**Hierarchical hypergraph** made of

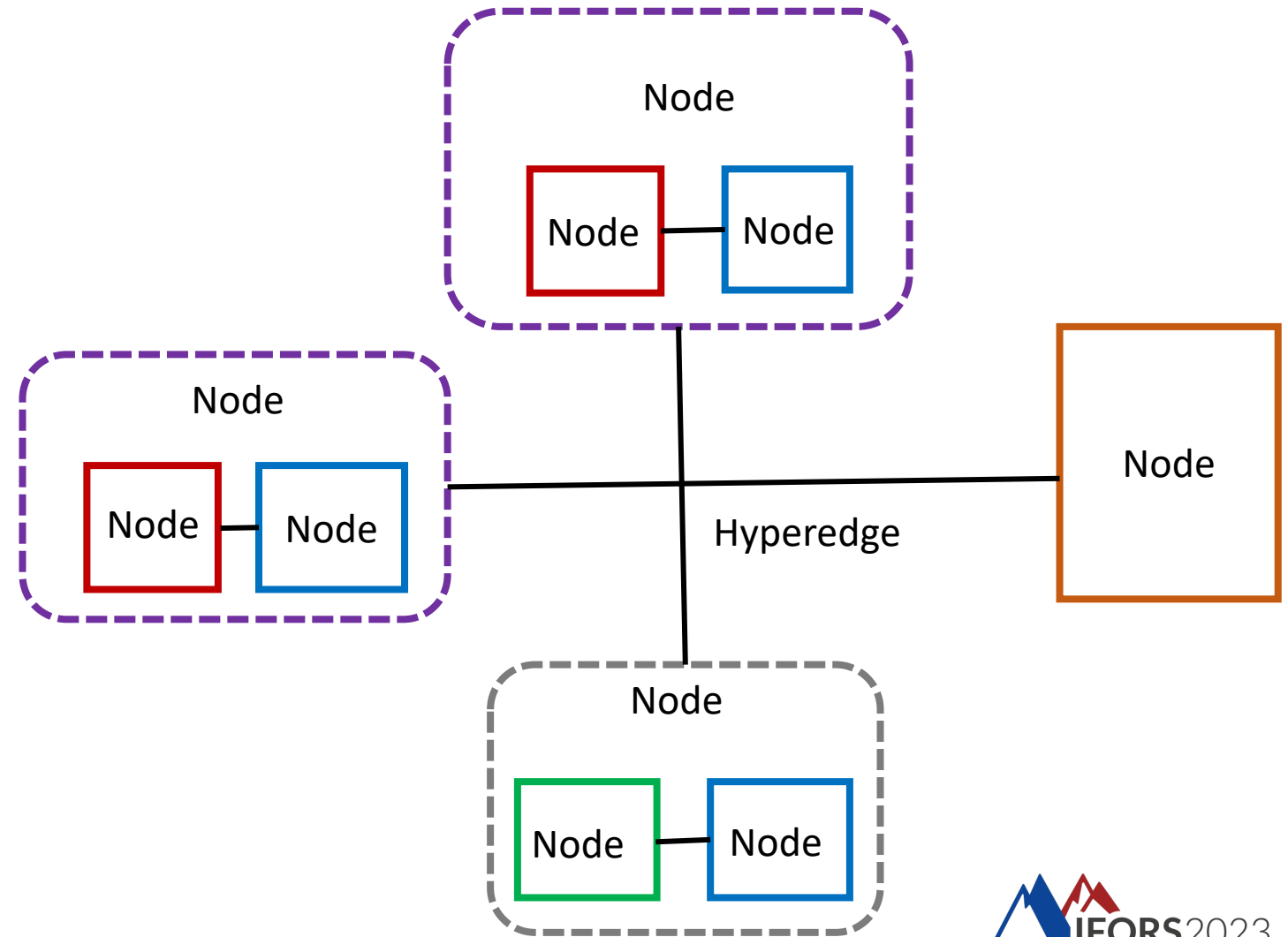
- **Nodes**
- **Hyperedges**

Each **node** is itself made of

- **Parameters**
- **Variables**
- **Constraints**
- **Objectives**
- **Hierarchical hypergraph**

Each **hyperedge** is made of

- **Parameters**
- **Constraints**



**Figure 3: Structured MILP**

# Structured MILPs formalization

**Hierarchical hypergraph**  $G_g = (\mathcal{N}_g, \mathcal{E}_g)$

made of

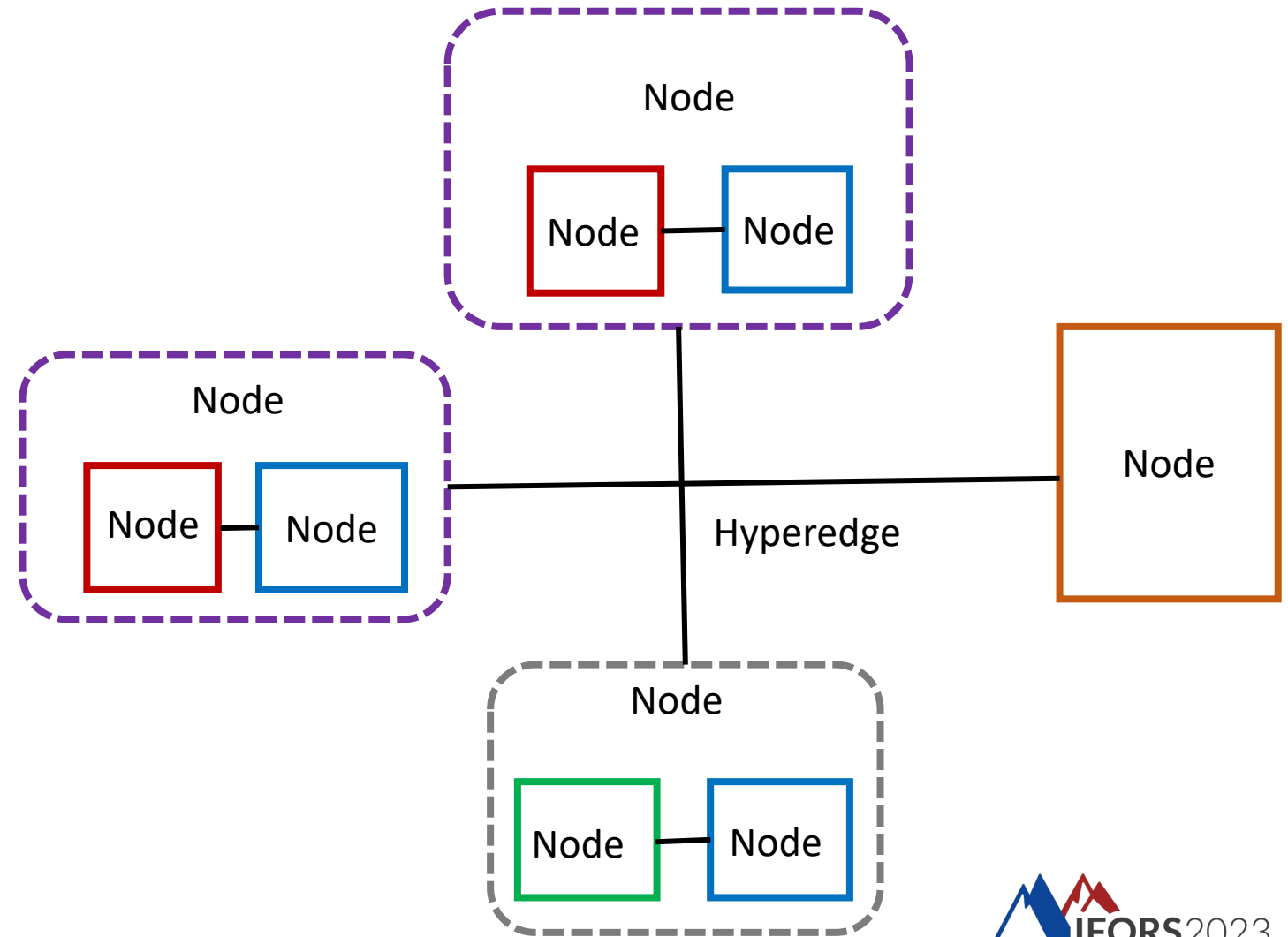
- A set of **nodes**  $\mathcal{N}_n$
- A set of **hyperedges**  $\mathcal{E}_n$

Each **node**  $\langle v_n^{ext}, v_n^{int}, G_n, H_n, G_n, O_n \rangle$  is itself made of

- **External variables**  $v_n^{ext}$
- **Internal variables**  $v_n^{int}$
- **Constraints matrices**  $G_n$  and  $H_n$
- **Objectives matrix**  $O_n$
- **Hierarchical hypergraph**  $G_n$

Each **hyperedge**  $\langle \mathcal{N}_e, G_e, H_e \rangle$  is made of

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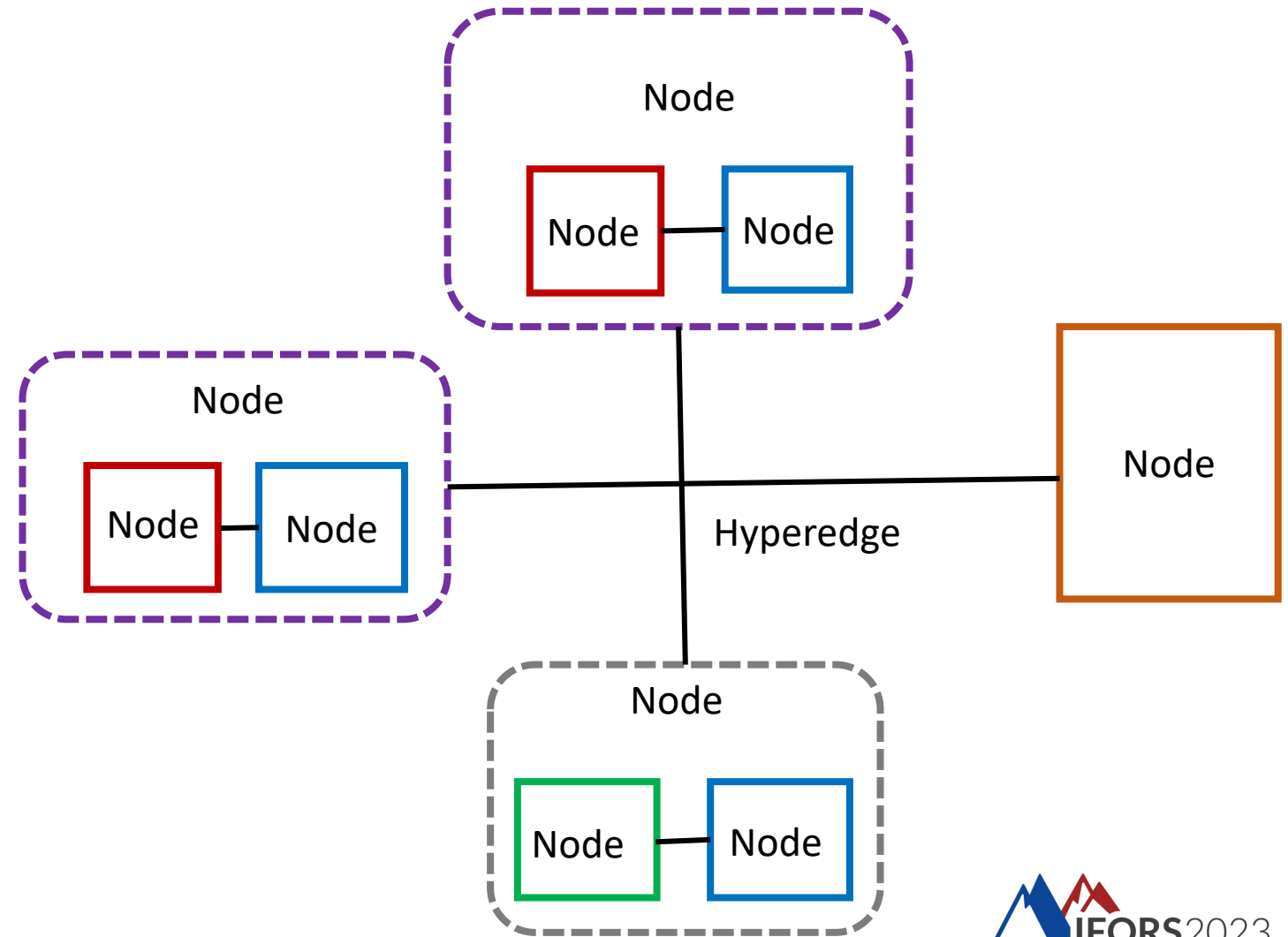
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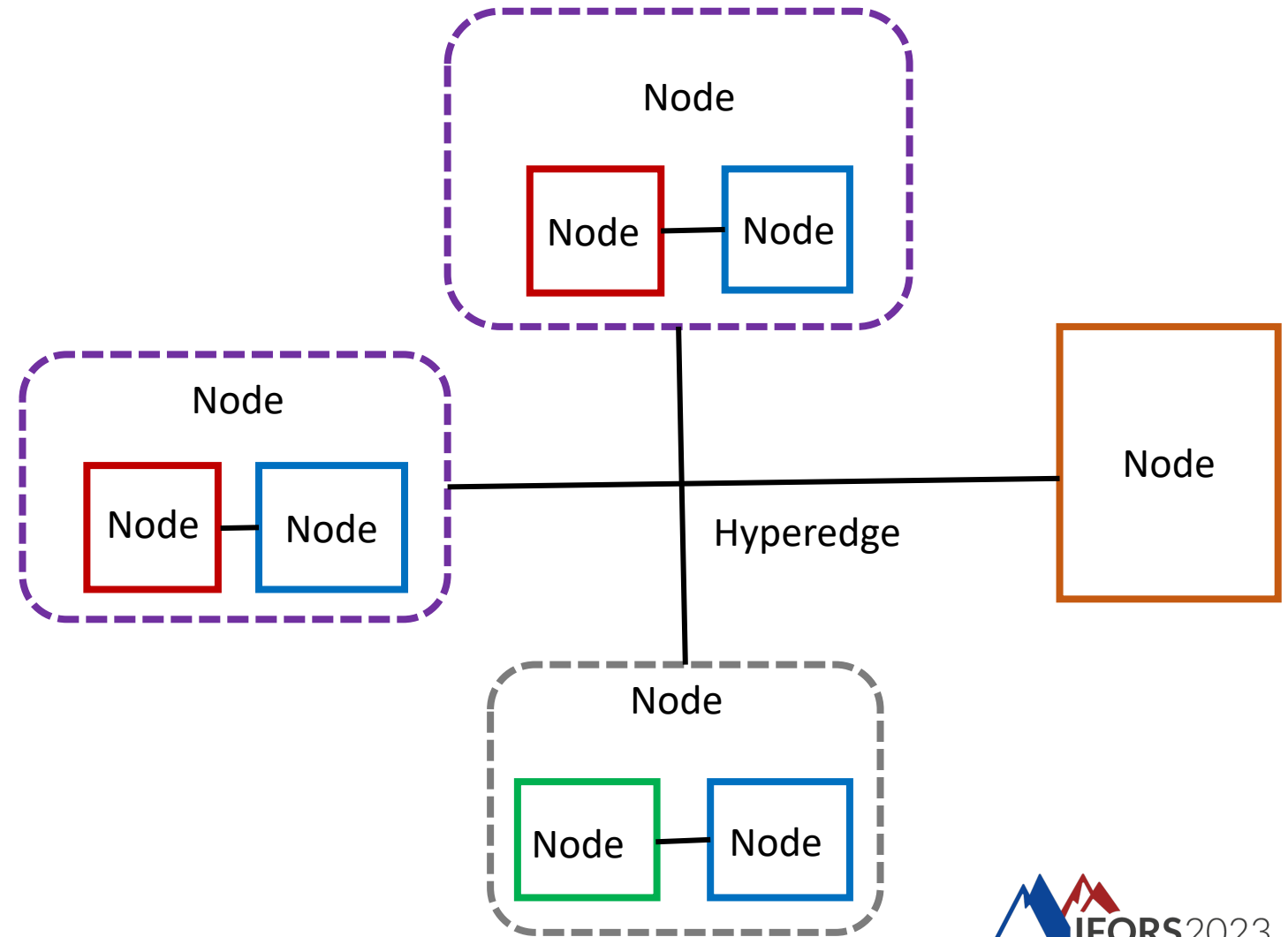
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# Structured MILPs formalization

We have a **hierarchical hypergraph**  $G_g = (\mathcal{N}_g, \mathcal{E}_g)$

For each node, we have

$$\begin{aligned} \min \quad & \mathbf{1}^{1 \times \sigma_n} \mathbf{O}_n [1 \ \mathbf{v}_n]^T \\ \text{s. t.} \quad & \mathbf{G}_n [1 \ \mathbf{v}_n]^T \leq \mathbf{0} \\ & \mathbf{H}_n [1 \ \mathbf{v}_n]^T = \mathbf{0} \end{aligned}$$

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And so on recursively for all sub-hypergraphs.

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Node variables

Inequality constraints  
Equality constraints

Objective matrix

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 \end{aligned}$$

External variables of certain nodes

Inequality constraints

Equality constraints

And so on recursively for all sub-hypergraphs.

# Structured MILPs formalization

Let us define three recursive functions :

- the function  $f$  that takes a set of nodes  $\mathcal{N}$  as input and returns the sum of the objectives of the nodes and their subnodes recursively,

$$f(\mathcal{N}) = \sum_{n \in \mathcal{N}} \left( \mathbf{1}^{1 \times \sigma_n} \mathbf{O}_n [1 \ \mathbf{v}_n]^T + f(\mathcal{N}_n) \right).$$

- the Boolean-valued function  $g$  that takes a hypergraph  $G = (\mathcal{N}, \mathcal{E})$  as input and returns,

$$g(G) = [\mathbf{G}_e [1 \ \mathbf{v}_e]^T \leq 0 \ \forall e \in \mathcal{E}] \wedge \left[ \left( \mathbf{G}_n [1 \ \mathbf{v}_n]^T \leq 0 \wedge g(G_n) \right) \forall n \in \mathcal{N} \right].$$

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Applied recursively on its sub-nodes

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Hyperedge Node

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# Structured MILPs formalization

Given

- $f(\mathcal{N}) = \sum_{n \in \mathcal{N}} \left( \mathbf{1}^{1 \times \sigma_n} \mathbf{O}_n [1 \ \mathbf{v}_n]^T + f(\mathcal{N}_n) \right),$
- $g(\mathbf{G}) = [\mathbf{G}_e [1 \ \mathbf{v}_e]^T \leq 0 \ \forall e \in \mathcal{E}] \wedge \left[ \left( \mathbf{G}_n [1 \ \mathbf{v}_n]^T \leq 0 \wedge g(\mathbf{G}_n) \right) \forall n \in \mathcal{N} \right],$
- And,  $h(\mathbf{G}) = [\mathbf{H}_e [1 \ \mathbf{v}_e]^T = 0 \ \forall e \in \mathcal{E}] \wedge \left[ \left( \mathbf{H}_n [1 \ \mathbf{v}_n]^T = 0 \wedge h(\mathbf{G}_n) \right) \forall n \in \mathcal{N} \right].$

We have that our problems can be written as,

$$\begin{aligned} & \min f(\mathcal{N}_g) \\ & \text{s.t. } h(\mathbf{G}_g) \text{ is true} \\ & \quad g(\mathbf{G}_g) \text{ is true} \end{aligned}$$

How can we exploit this structure in a modelling tool?



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# Modelling tool workflow

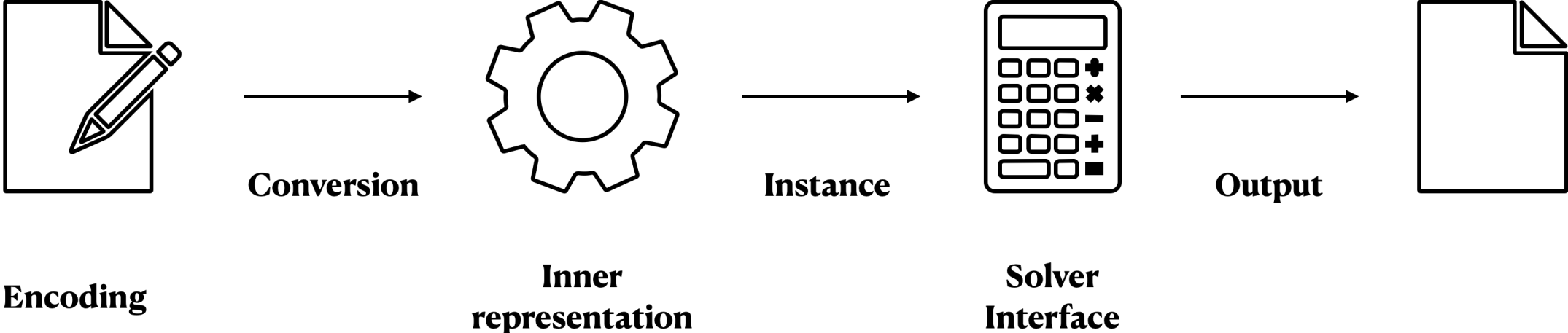
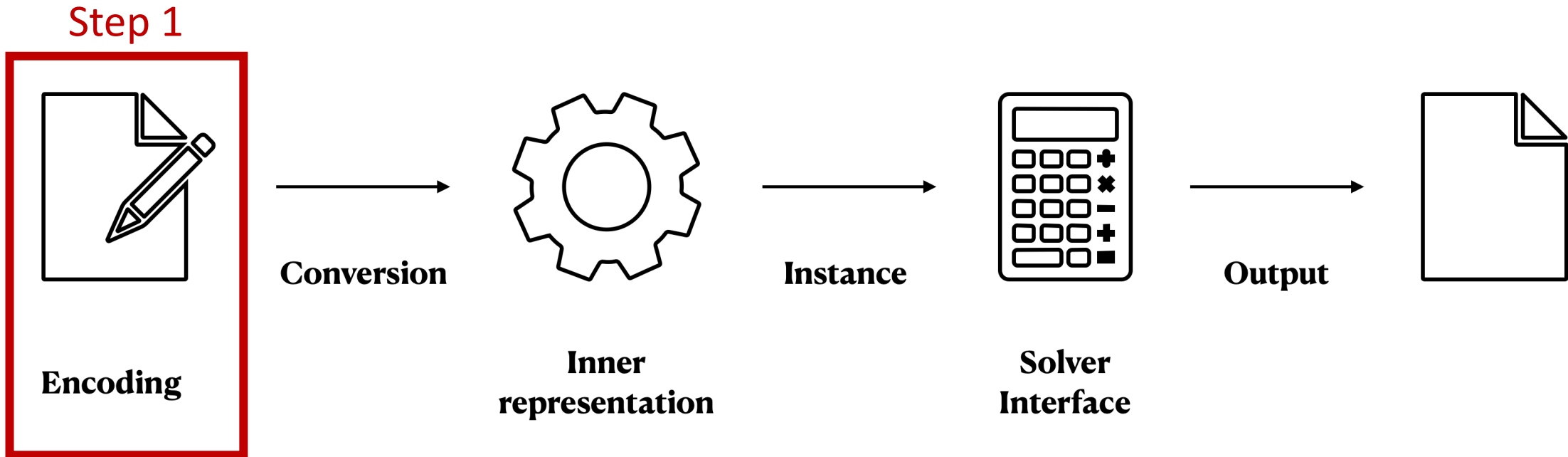


Figure 4: Modelling tool workflow

# The Graph-Based Optimization Modelling Language (GBOML)



**Figure 4:** Modelling tool workflow



## Encoding structure [1]

```
#NODE <node_name>  
#PARAMETERS  
<param_def>  
#VARIABLES  
<var_def>  
#CONSTRAINTS  
<constr_def>  
#OBJECTIVES  
<obj_def>
```

## #TIMEHORIZON

```
T = <value>;
```

```
#HYPEREDGE <edge_name>
```

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#CONSTRAINTS  
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## Encoding hierarchical structure [1]

**#NODE** <node\_name>

**#PARAMETERS**

<param\_def>

[**#NODE** <node\_name>]

[**#HYPEREDGE** <edge\_name>]

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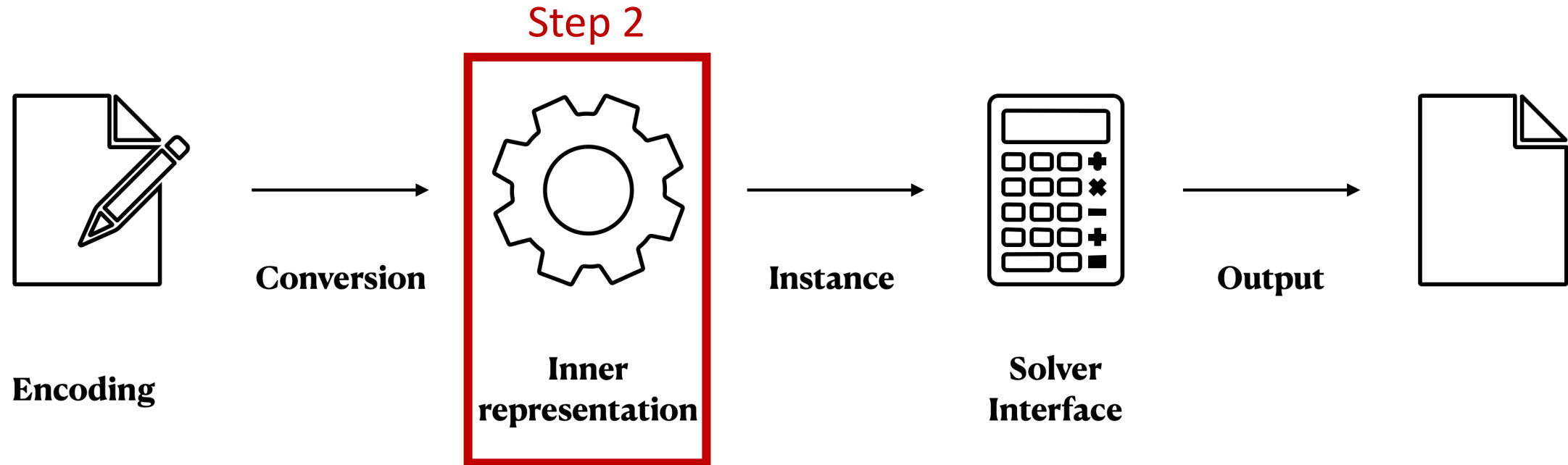
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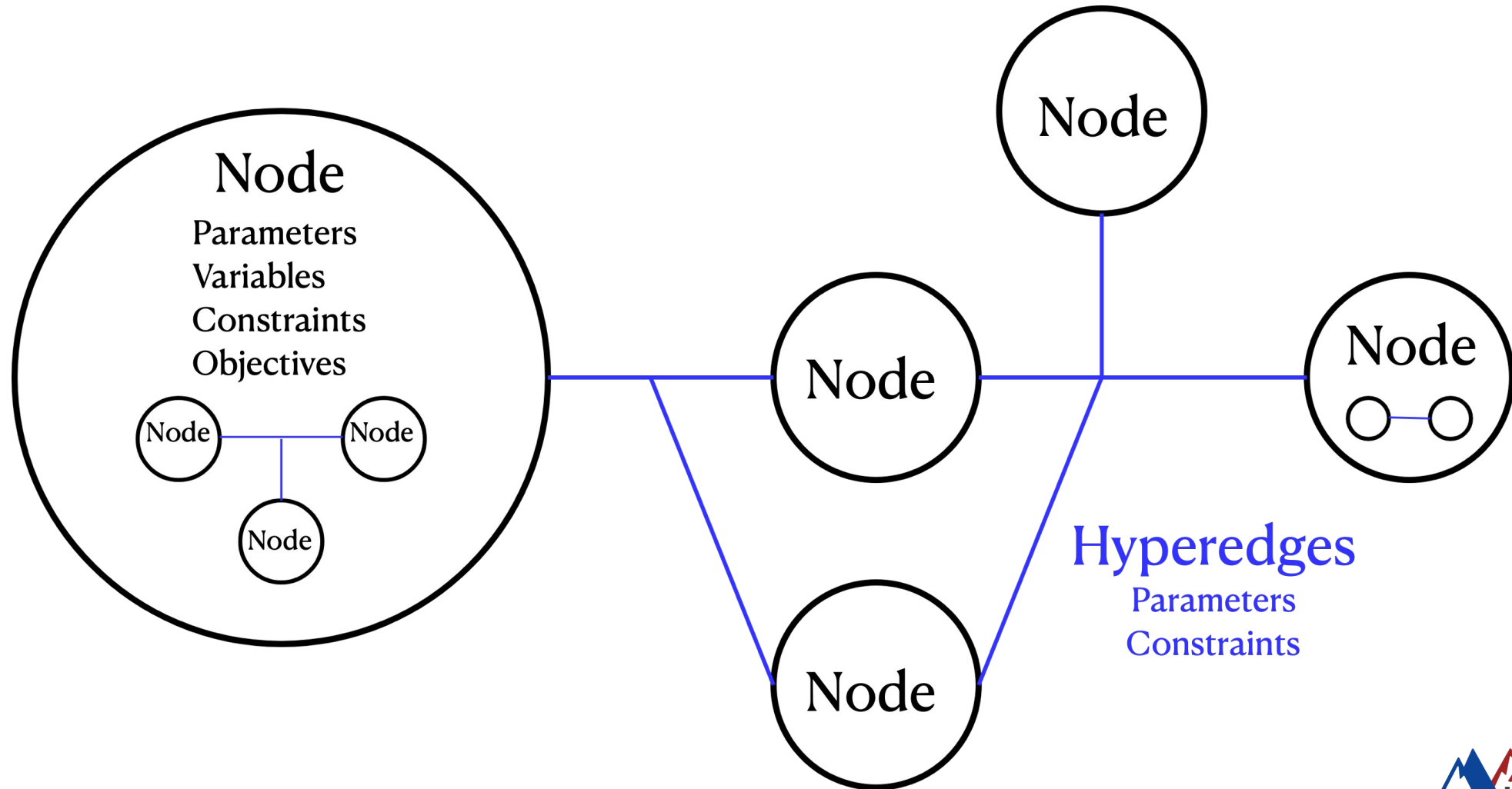
# The Graph-Based Optimization Modelling Language (GBOML)



**Figure 4:** Modelling tool workflow



# Inner representation



**Figure 5:** Hierarchical hypergraph inner representation





# Inner representation

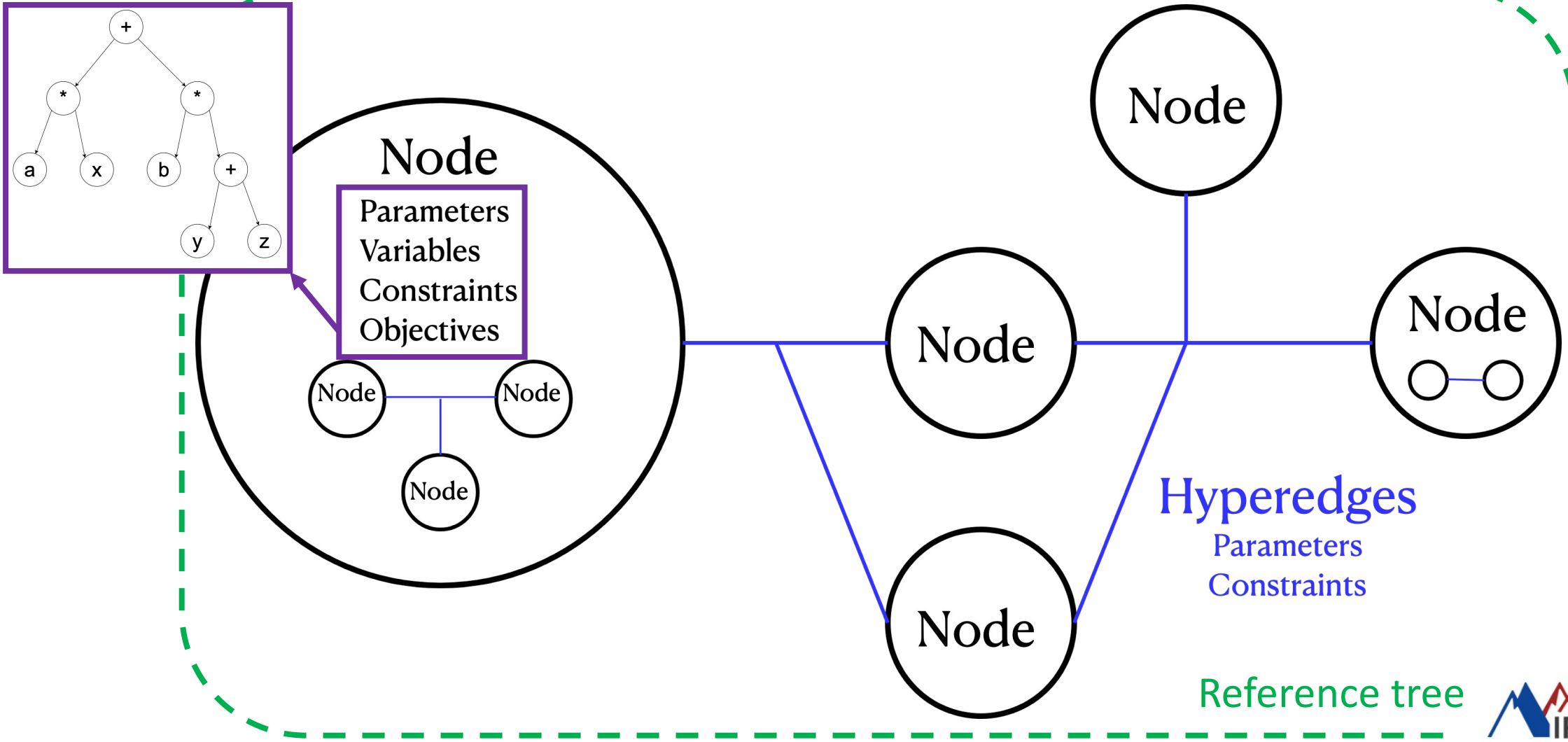


Figure 5: Hierarchical hypergraph inner representation



# Inner representation

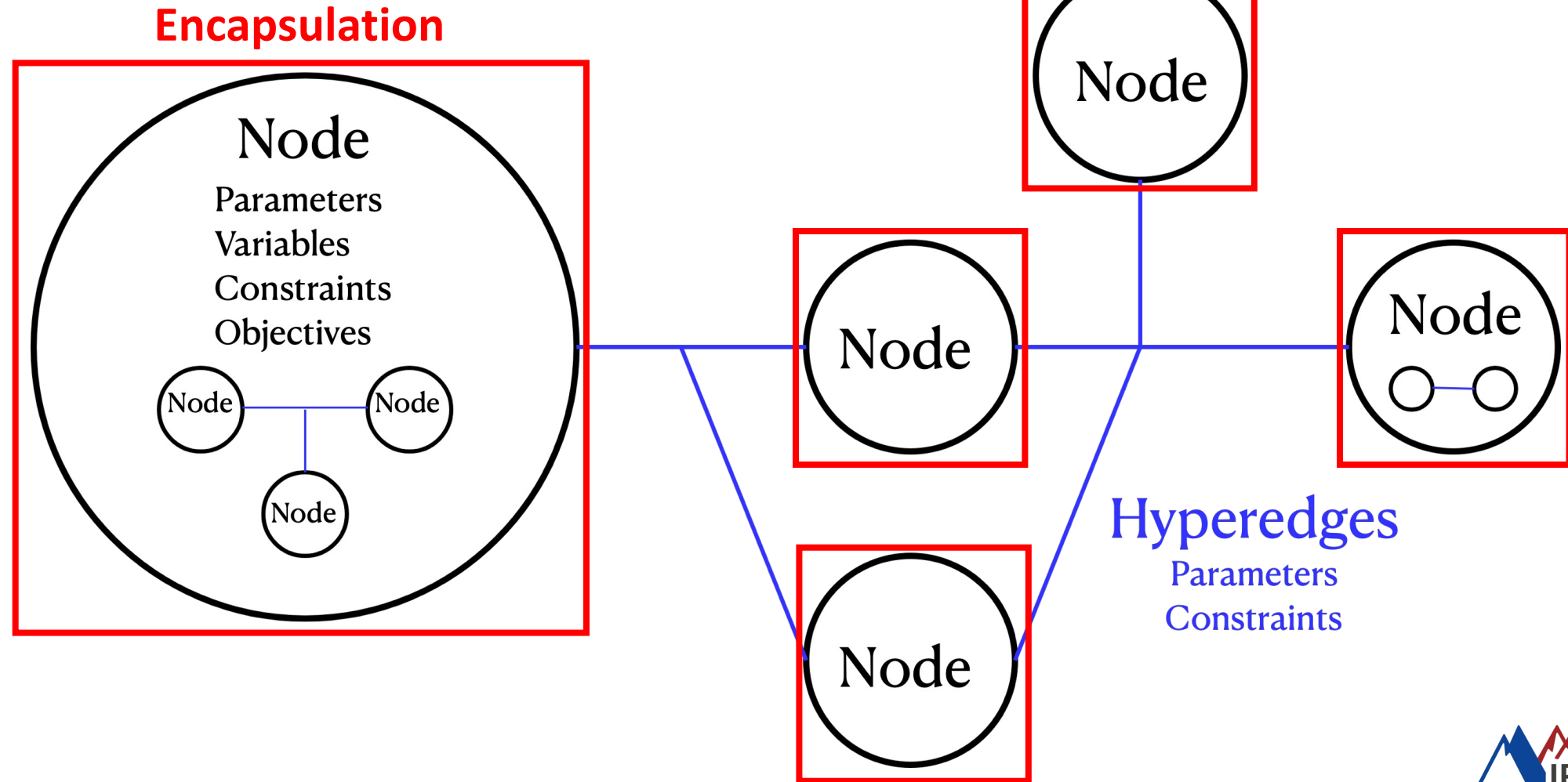


Figure 5: Hierarchical hypergraph inner representation



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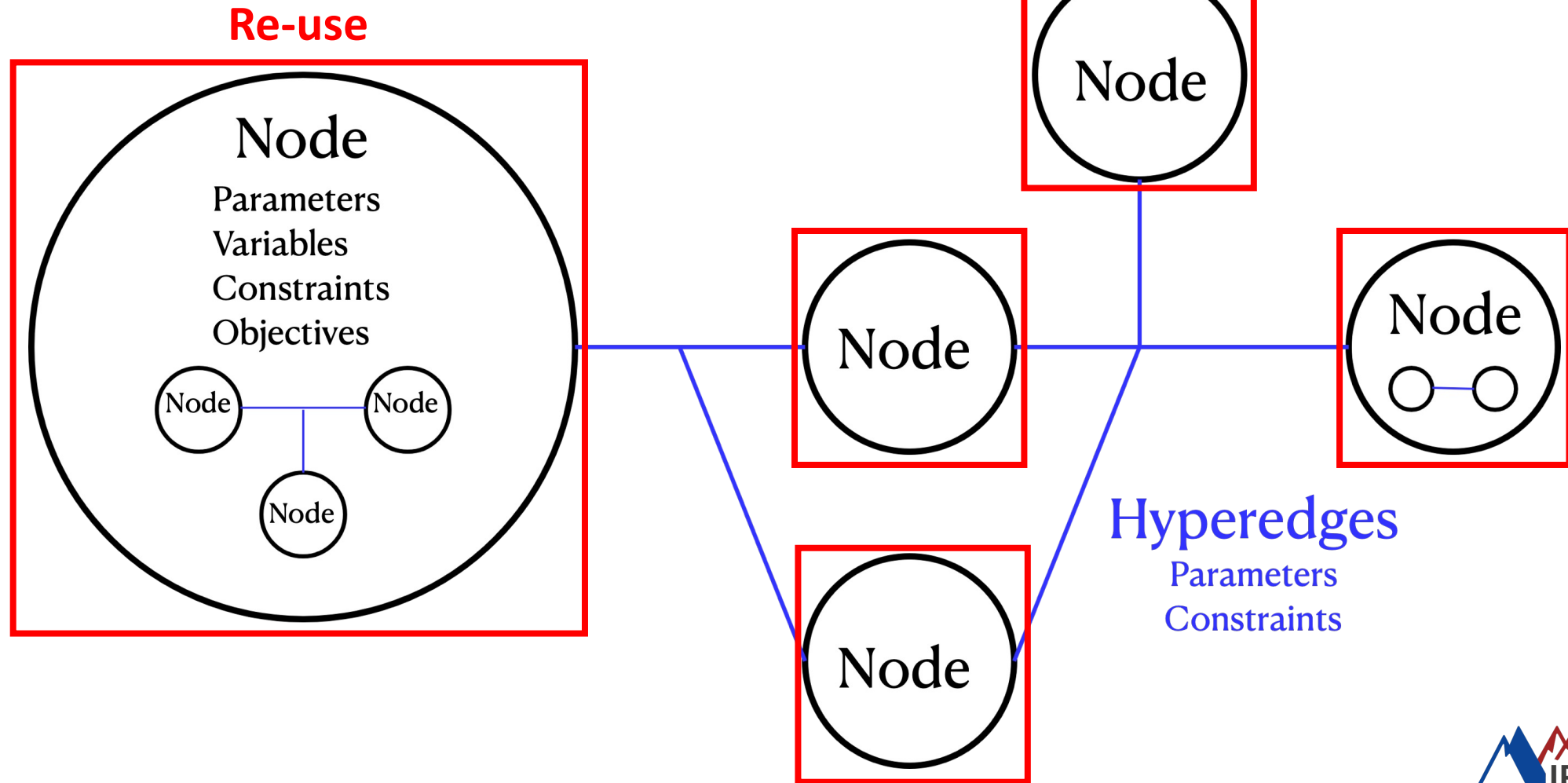


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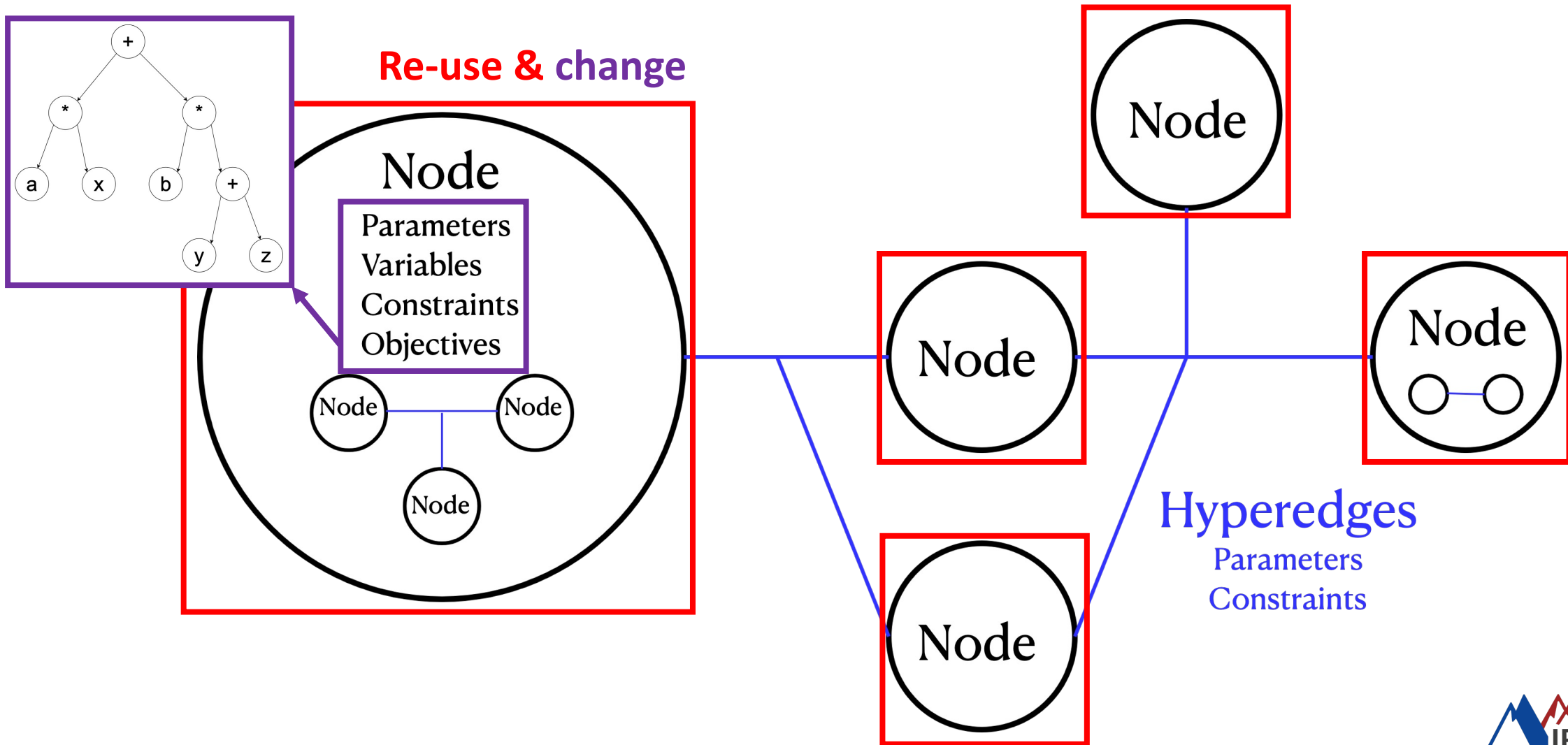


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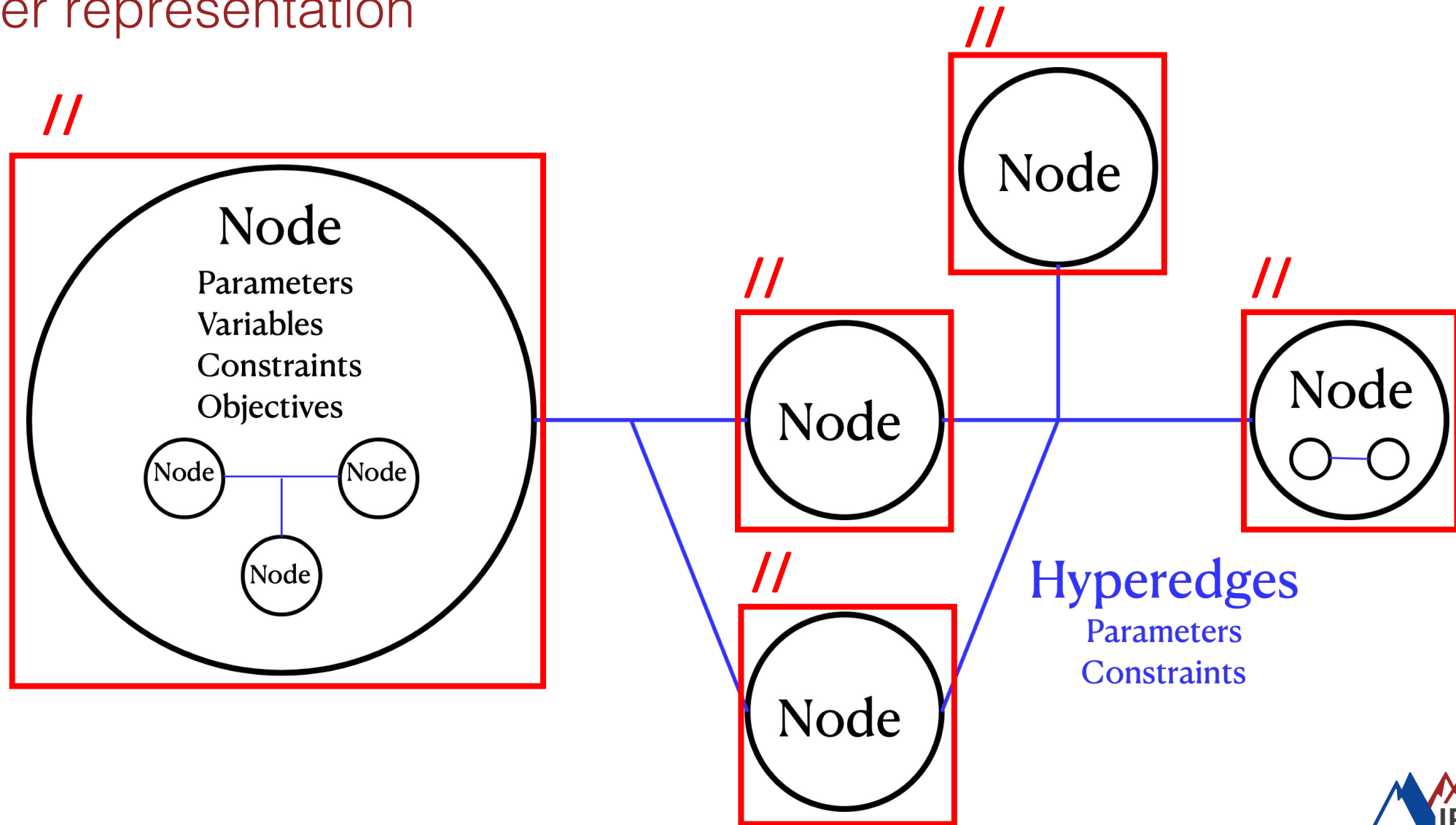
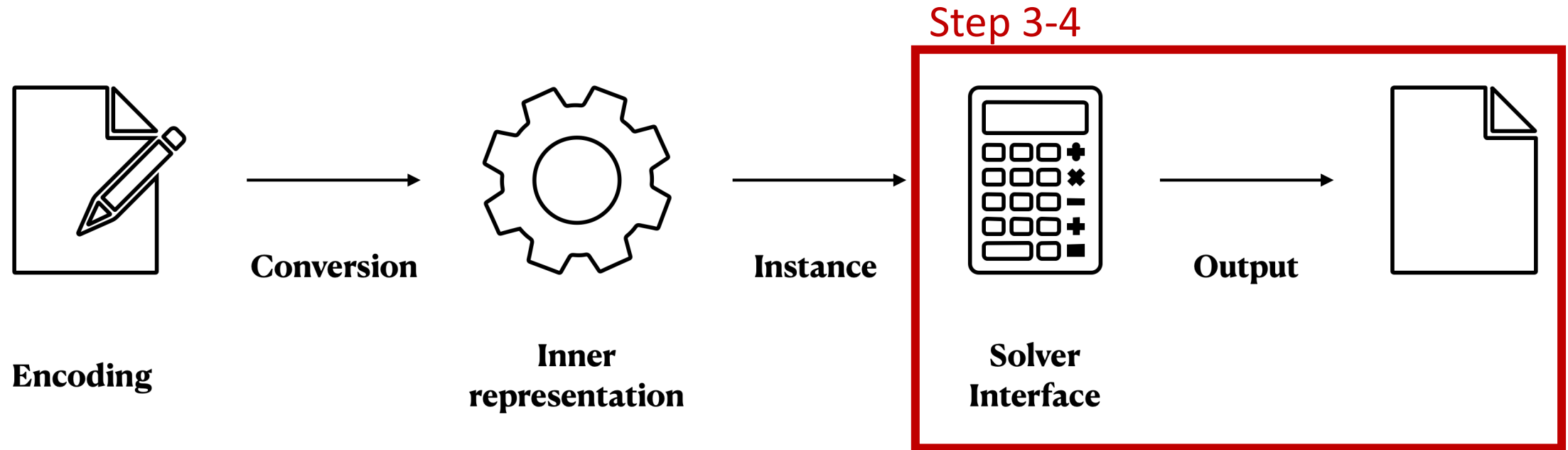


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# The Graph-Based Optimization Modelling Language (GBOML)



**Figure 4:** Modelling tool workflow



# Solver interface and output

- Commercial solvers



- Open-source solvers



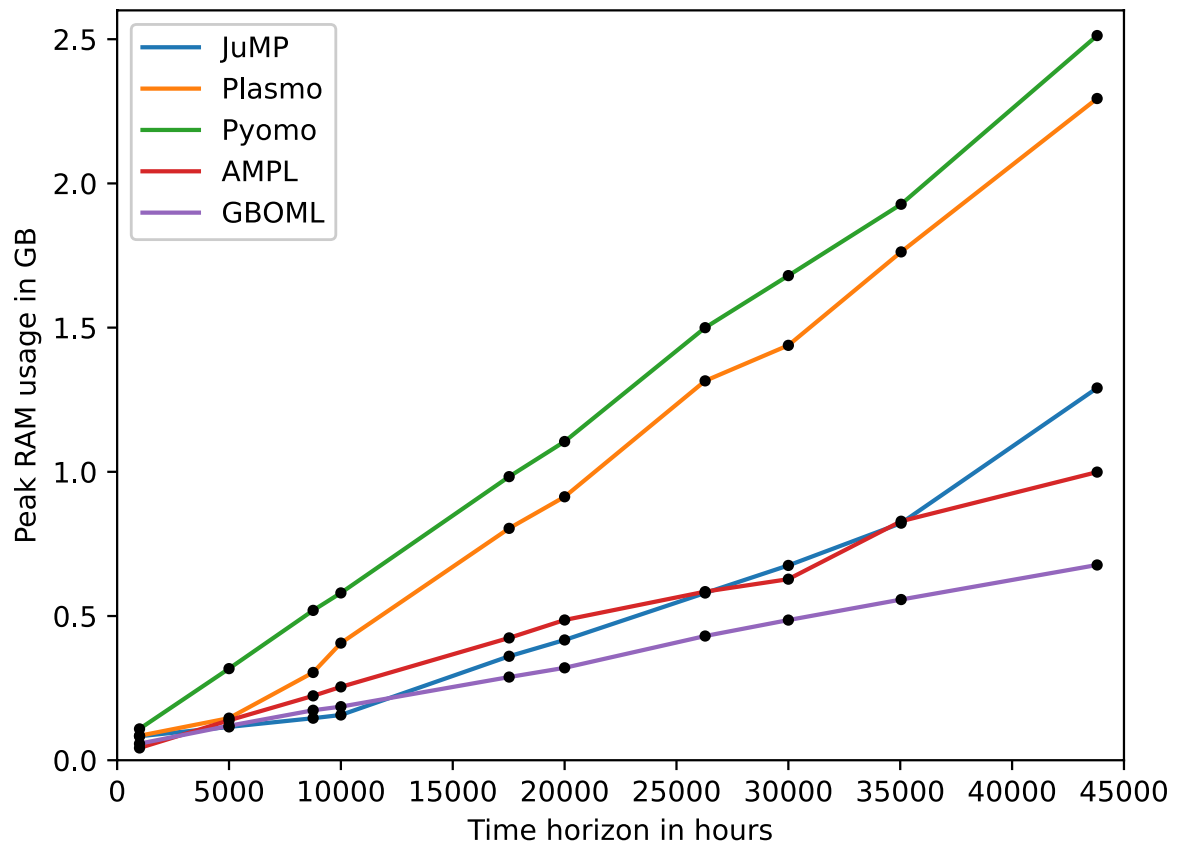
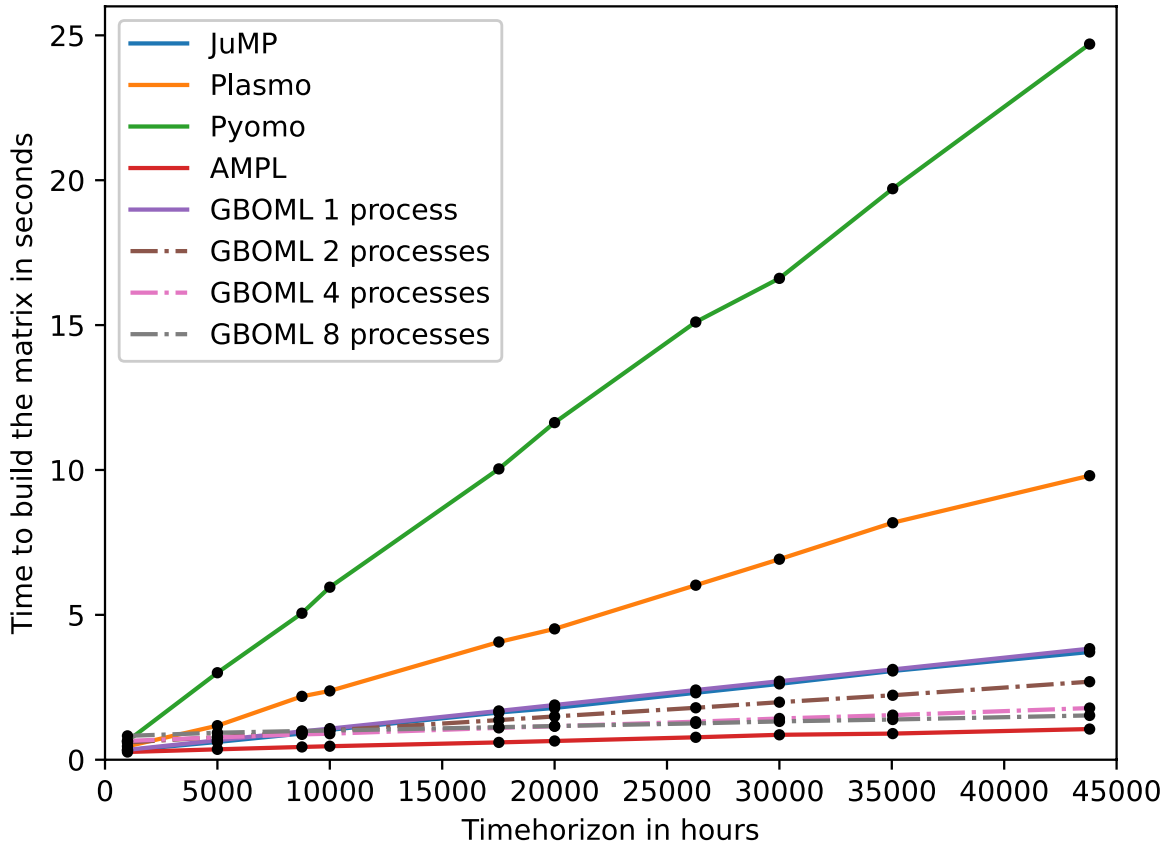
- Structure exploiting methods
  - DSP[7]: Dantzig-Wolfe decomposition
  - CPLEX: Benders decomposition
- Generates structure JSON and plain CSV

# Benchmarking





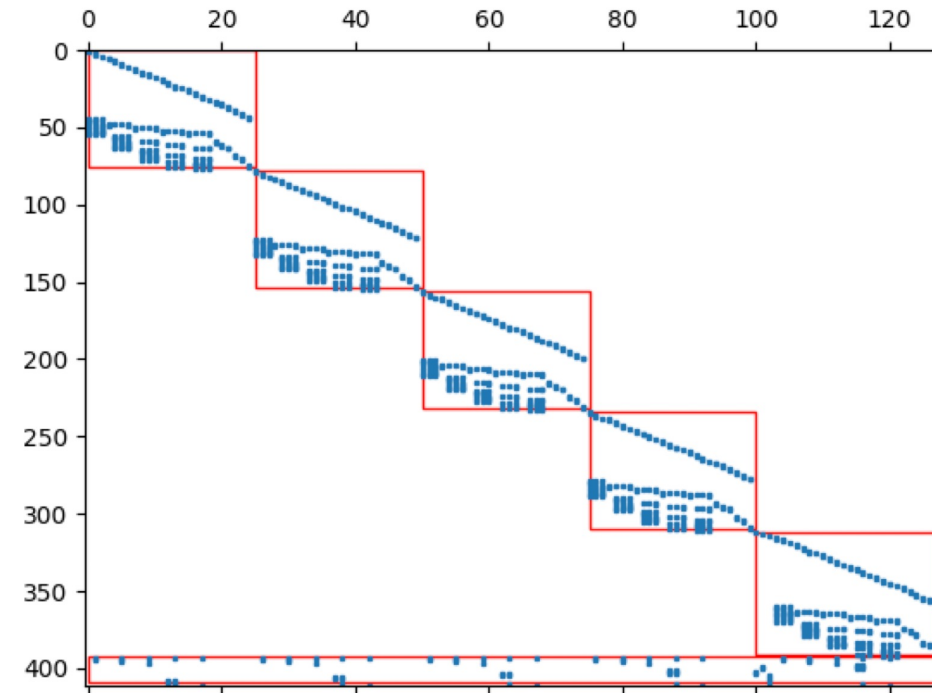
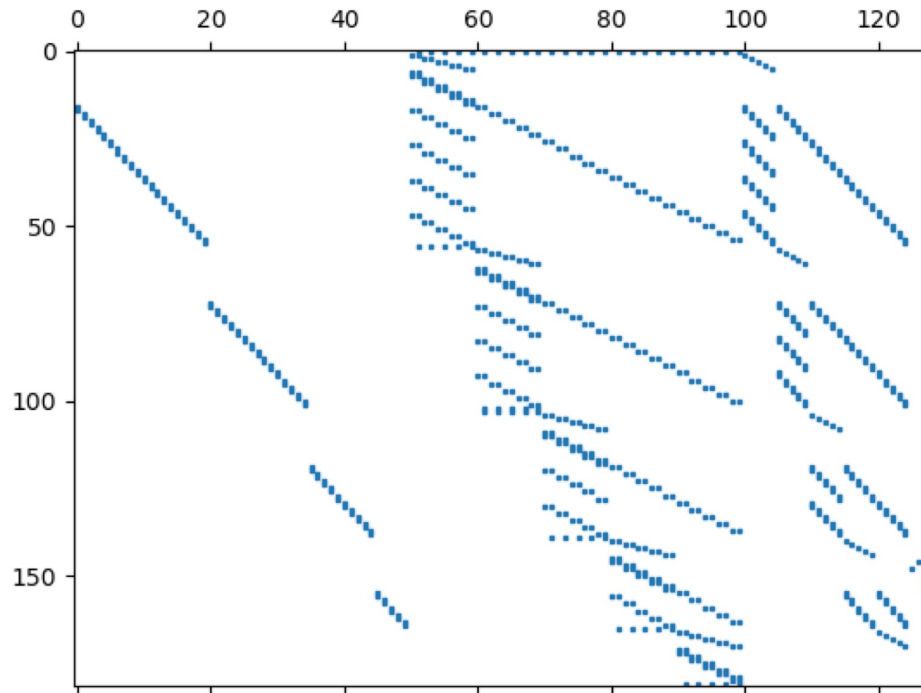
# Performances for generating an instance



**Figure 6:** (Left) time taken to generate an instance (right) peak RAM usage to generate an instance – both were done on the remote renewable energy hub[8] for a growing time horizon [9]



# Structure Exploiting methods



**Figure 7:** (Left) Plain no-swot problem from the MIPLIB[10] solved in 25s by Gurobi (Right) structured no-swot problem encoded in GBOML with a good decomposition solved by Dantiz-Wolfe in 2.5s.

# Conclusion

- Structure can help
  - Encode problems more naturally
  - Enable re-use and component assembling
  - Generate models faster
  - Interface with structure exploiting methods
- In terms of tool, GBOML exploits structure from model encoding to output

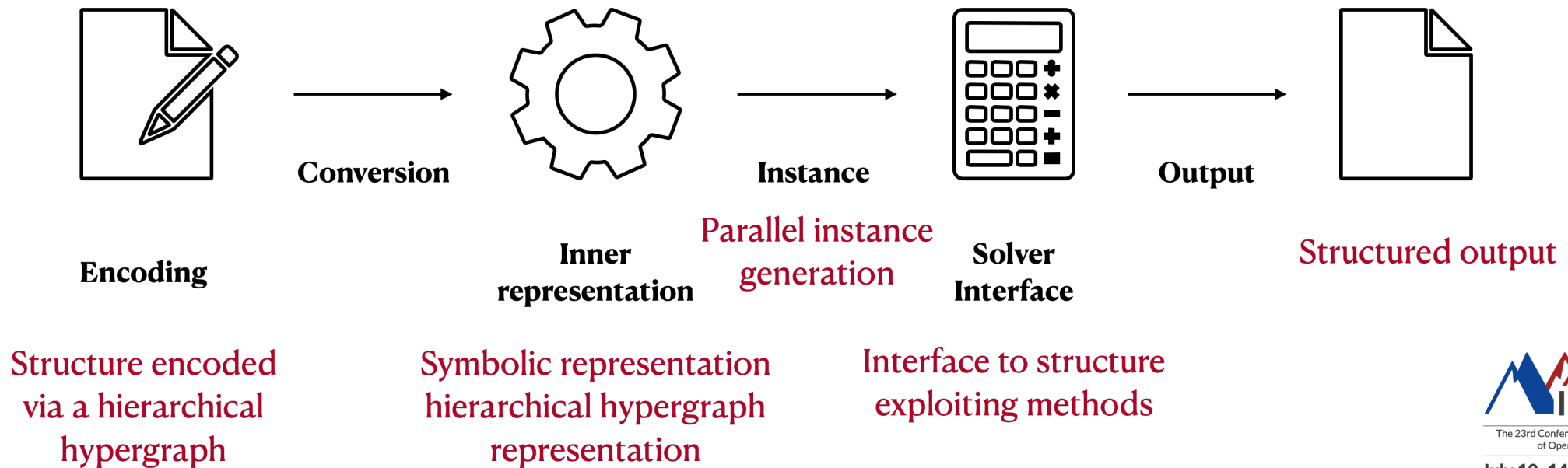


Figure 7: GBOML workflow

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# Appendix : GBOML inner working

