



Workshop Summary: Exoplanet Orbits and Dynamics

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Abstract

Exoplanetary systems show a wide variety of architectures, which can be explained by different formation and dynamical evolution processes. Precise orbital monitoring is mandatory to accurately constrain their orbital and dynamical parameters. Although major observational and theoretical advances have been made in understanding the architecture and dynamical properties of exoplanetary systems, many outstanding questions remain. This paper aims to give a brief review of a few current challenges in orbital and dynamical studies of exoplanetary systems and a few future prospects for improving our knowledge. Joint data analyses from several techniques are providing precise measurements of orbits and masses for a growing sample of exoplanetary systems, both with close-in orbits and with wide orbits, as well as different evolutionary stages. The sample of young planets detected around stars with circumstellar disks is also growing, allowing for simultaneous studies of planets and their birthplace environments. These analyses will expand with ongoing and future facilities from both ground and space, allowing for detailed tests of formation, evolution, and atmospheric models of exoplanets. Moreover, these detailed analyses may offer the possibility of finding missing components of exoplanetary systems, such as exomoons, or even finding new exotic configurations such as co-orbital planets. In addition to unveiling the architecture of planetary systems, precise measurements of orbital parameters and stellar properties—in combination with more realistic models for tidal interactions and the integration of such models in N -body codes—will improve the inference of the past history of mature exoplanetary systems in close-in orbits. These improvements will allow a better understanding of planetary formation and evolution, placing the solar system in context.

Unified Astronomy Thesaurus concepts: [Exoplanets \(498\)](#); [Orbit determination \(1175\)](#); [Exoplanet dynamics \(490\)](#)

1. Introduction

Understanding the formation and evolutionary processes at play in exoplanetary systems is key to getting a comprehensive picture of planetary formation and evolution and in understanding the place of the solar system in this picture. Exoplanetary systems have shown very diverse architectures: gas giant planets in very close-in orbits (e.g., Mayor & Queloz 1995), tightly packed systems of rocky planets (e.g., Gillon et al. 2017), massive gas giant planets and brown dwarfs in wide orbits (e.g., Marois et al. 2008). Several formation mechanisms have been proposed to explain these various architectures such as core accretion (e.g., Pollack et al. 1996), pebble accretion (e.g., Lambrechts & Johansen 2012), disk gravitational instability (e.g., Boss 1997), and fragmentation of a protostellar disk (e.g., Bate 2009). To constrain these mechanisms, orbital monitoring plays a fundamental role in the analysis of exoplanetary architectures, dynamical states and evolution, and formation and migration processes.

No single observational method can probe all of the various types of exoplanetary architectures and the diversity of underlying dynamical processes. Radial velocities (RVs) and transits have been better suited to study planets close to the stars ($\lesssim 5$ au) and around mature stars (e.g., Winn & Fabrycky 2015). Nevertheless, recent breakthrough achievements have been made with the detection of young close-in planets with RV instruments (e.g., Donati et al. 2016) and with the transit technique (e.g., David et al. 2016). Direct imaging

has been better suited to study planets in wide orbits ($\gtrsim 10$ au) around young stars and to study planet-disk interactions (e.g., Boccaletti et al. 2020). Astrometry has mostly been limited to confirming and refining the orbits of known exoplanets due to stringent precision requirements (e.g., Malbet & Sozzetti 2018), but the ongoing Gaia mission (Gaia Collaboration et al. 2016) is expected to discover a large number of exoplanets around nearby stars of all ages (Perryman et al. 2014). Whenever possible, combining several techniques is beneficial for orbital and dynamical studies because of ambiguities when inferring the parameters using a single technique. This has been a major challenge due to the poor overlap between the techniques, but progress has been made recently, which will expand in the coming years.

This paper is the product of an online workshop held in 2021 January which aimed to bring together observers and dynamical modelers of exoplanetary systems to discuss challenges and prospects for the study of their orbital and dynamical properties (<https://sites.google.com/view/exodynamics/home>). The video recordings of the talks can be accessed on www.exoplanet-talks.org. An increasing demand for improved accessibility for non-experts to understand exoplanet modeling and dynamics has motivated in the recent years the development of publicly available orbital fitting tools able to handle multiple-planet systems and different types of data sets. Some of them were presented at the workshop and are referenced here. This paper is not meant to be a comprehensive review of the field. For the discussion of challenges in dynamical characterization, we chose

the topics of the inference of histories of planetary systems from current orbital parameters, tidal analysis, formation and detection of co-orbital worlds, and resonances and planet-disk interactions in young directly imaged exoplanetary systems. We acknowledge that the distribution of the represented topics is biased and does not cover in detail several topics related to dynamical characterization, e.g., resonances in mature exoplanetary systems, migration, Kozai–Lidov evolution, obliquities. The paper is organized as follows. In Section 2, we discuss challenges faced in the determination of the orbital parameters of exoplanets in close orbits and in wide orbits. In Section 3, we outline some of the key open topics in the dynamical studies of exoplanetary systems. Section 4 emphasizes a few key facilities and data that are expected to improve our knowledge of system architectures. Finally, we provide a summary and an outlook in Section 5.

2. Challenges in the Estimation of the Orbital Parameters

2.1. RVs and Transit Timing Variations: Agreement and Complementarity

The measurement of planetary masses using RVs or transit timing variations (TTVs) can be affected by various sources of noise or uncertainty. In the RV case: the presence of additional unknown planets in the system contaminating the signal, the stellar noise such as stellar jitter, and the instrument systematics. In the TTV case: the presence of additional unknown planets in the system contaminating the signal, the mass-eccentricity or eccentricity-eccentricity degeneracy, the stellar noise such as photospheric inhomogeneities, and the instrument systematics. This motivates a comparison between the two techniques. Early discussions reported a possible discrepancy: the masses measured by RVs appeared to be systematically larger than those measured via TTVs for planets of similar radii (e.g., Weiss & Marcy 2014). However, Steffen (2016) and Mills & Mazeh (2017) suggested that this difference may simply be related to some sensitivity biases: the planets measured by the TTV method have on average a longer orbital period than the ones measured by the RV method, so the two techniques are actually probing slightly different exoplanet populations, which may just follow different mass–radius relationships (e.g., short-period planets may be denser due to atmospheric loss).

To better understand these effects, bright systems for which *both* the RV and TTV methods can be applied are key. Until recently, prior to the launch of TESS (Ricker et al. 2015), the number of systems for which this was possible was very small due to the small transit probability of RV-detected systems and the poor RV precision of most TTV systems found by Kepler (Borucki et al. 2010) around relatively fainter stars. Furthermore, even when this is possible, independent comparative analyses between the two methods are not always presented in the literature, which often only reports the results of a

simultaneous fit of both data sets combined, as this approach yields the best constraints on the system parameters (see below). Among the few systems for which independent RV and TTV mass measurements are available, most of them show a good agreement between the two methods, for example Kepler-18 (Cochran et al. 2011), K2-19 (Barros et al. 2015; Dai et al. 2016), or WASP-47 (Weiss et al. 2017). However, a significant inconsistency persists for Kepler-89 d, for which the RV mass (Weiss et al. 2013) is about twice as large as the mass obtained by TTVs (Masuda et al. 2013; Jontof-Hutter et al. 2022). As TESS is detecting new compact planetary systems around bright stars, it is thus important to perform more comparative studies between the two methods, to better understand their possible biases and limitations.

This also offers the prospect of being able to combine both data sets in a simultaneous fit (using, e.g., a photodynamical model) for more systems, revealing more about their architecture. Constraints from RVs can for example help to break the mass-eccentricity degeneracy in a TTV system. TTVs can also help to better disentangle RV signals from the stellar noise for active stars. Combining both data sets also yields an increased detection sensitivity to additional unknown planets in the system. Finally, the absolute masses and radii of the star and planets can be obtained (e.g., Agol et al. 2005; Montet & Johnson 2013), without the need for stellar models.

2.2. Imaging

A precise and robust determination of the orbital parameters of directly imaged exoplanets faces several challenges. Some challenges are related to the measurements of the orbital position of the planets and others to the interpretation of the measurements in terms of possible ranges of orbital parameters. Due to the high flux ratios and the angular proximity between stars and planets, all directly imaged exoplanets detected with current 8–10 m ground-based telescopes are located at separations from the stars beyond ~ 10 au so have slow orbital motions and long orbital periods. Precise position measurements of a companion are mandatory to detect orbital motion at high significance. Because of the small field of view of high-contrast imaging instruments (typically $< \sim 10''$), absolute astrometry is not feasible and the positions of a companion are measured relative to the host star.

Deriving precise relative astrometry of directly imaged exoplanets with the first generation of high-contrast imaging instruments has been challenging due to the poor contrast and angular resolution performance of the instruments and large uncertainties in the measurement of the location of the star, which is either saturated on the detector or masked behind a coronagraph to allow for searching for faint companions close to the stars (e.g., Chauvin et al. 2012; Konopacky et al. 2016a). The first dedicated exoplanet imaging instruments, such as GPI (Macintosh et al. 2014), SPHERE (Beuzit et al. 2019), and

SCEXAO+CHARIS (Jovanovic et al. 2015; Groff et al. 2016), have allowed a significant gain in astrometric precision from ~ 10 mas to ~ 1 – 2 mas thanks to the high contrasts and diffraction-limited performance that they deliver and the implementation of optimized strategies to measure the location of the star in the science images. In addition to these coronagraphic instruments, the interferometric instrument GRAVITY (GRAVITY Collaboration et al. 2017) has recently achieved precise relative astrometry at ~ 40 – $80 \mu\text{as}$ on directly imaged exoplanets thanks to its dual-fiber observing mode, adaptive optics capabilities, and internal metrology system (GRAVITY Collaboration et al. 2019). Diffraction-limited coronagraphic instruments on extremely large telescopes (~ 30 – 40 m) are expected to achieve sub-mas astrometric precisions thanks to the gain in angular resolution and sensitivity provided by larger telescope diameters (assuming $\sigma \sim \frac{\lambda/D}{S/N}$ with σ the astrometric precision, λ/D the width of the point-spread function, λ the observing wavelength, D the telescope diameter, and S/N the signal-to-noise ratio; Lindegren 1978).

More precise measurements are more sensitive to poorly estimated biases. Biases in measurements can be caused by the methods used to extract and/or calibrate relative astrometric measurements by the observing teams, even for a given instrument. Various differential imaging algorithms have been proposed over the years to reach high contrasts close to the stars while minimizing and/or calibrating photometric and astrometric biases on the measured companions due to self-subtraction effects (the stellar speckle pattern is modeled using the science images, which contain the signal of the companions), see the algorithms proposed by, e.g., Lafrenière et al. (2007), Soummer et al. (2012), Cantalloube et al. (2015), Flasseur et al. (2018), Gomez Gonzalez et al. (2018). The extracted relative astrometry needs to be calibrated by correcting for the optical distortion of the images due to the telescope and instrument optics (before the high-contrast imaging processing), the pixel scale, and the orientation to the North. Biases in the calibration can arise from our limited knowledge of the instrument metrology, overheads, and/or stability (De Rosa et al. 2020b; Maire et al. 2021). Most high-contrast imaging instruments rely on angular differential imaging techniques, which require an accurate derivation of the parallactic angles of the images for their proper derotation before combination (Marois et al. 2006). Measurements of pixel scale and orientation to the North are performed by observing close to the science observations an astrometric field for which positions have been measured with a well-calibrated instrument (e.g., Hubble Space Telescope) and comparing the measured positions. Biases can arise from the use of different astrometric fields and/or reference catalogs between the observing teams (Chauvin et al. 2012; Ginski et al. 2014). Using measurements obtained in a homogeneous way in orbital

fits is critical to avoid potential biases in the inferred orbital parameters (Konopacky et al. 2016a). Given the long time-scales needed to cover a significant fraction of the orbit, a regular and homogeneous calibration strategy is mandatory to assess the astrometric stability of an instrument over time in particular after technical interventions (Chauvin et al. 2012; De Rosa et al. 2020b; Maire et al. 2021). Such a calibration strategy also needs to be optimized to minimize the use of nighttime observations so that most of the latter can be used for science observations (Maire et al. 2021). Showing the astrometric stability on sky of the upcoming imaging instruments for exoplanets on extremely large telescopes will be key for precise orbital studies at the expected sub-mas precisions.

Deriving precise and robust orbital parameters for directly imaged exoplanets is challenging because only a small part of the orbit is typically measured ($< \sim 20\%$) for these companions in wide orbits from the stars ($> \sim 10$ au). Markov-chain Monte Carlo methods may be slow to converge, depending on the choice of the fitted parameters (Ford 2006). Alternative fitting methods have been exploited, such as least-squares Monte Carlo (e.g., Schaefer et al. 2006; Ginski et al. 2013; Maire et al. 2018) and Bayesian rejection sampling (Blunt et al. 2017). Some approaches are publicly available in the open-source packages *orbitize!* (Blunt et al. 2020), *orvara* (Brandt et al. 2021b), and more recently *Exo-Striker* (Trifonov 2019). Fitting undersampled orbits is prone to biases in the inferred orbital parameters because of degeneracies between these parameters (Konopacky et al. 2016a; Ferrer-Chávez et al. 2021). It is difficult to distinguish between eccentric orbits close to a face-on configuration and circular orbits close to an edge-on configuration (Ferrer-Chávez et al. 2021). A bias encountered frequently is characterized by highly eccentric orbits with a time at periapsis passage near the epoch of the data (e.g., Konopacky et al. 2016a). Konopacky et al. (2016a) mitigate this bias by considering that there is a low probability of observing the companion near periapsis, where the orbital motion is fastest. Because the data typically provide limited constraints, the choice of the prior distributions can also affect the inferred parameters and introduce biases. O’Neil et al. (2019) propose observable-based priors as an alternative to model-based priors. These priors are based on the uniformity of the observables and more objective. When applying such priors for the orbital fitting of the four giant exoplanets around HR 8799, they find less eccentric orbits and orbits closer to a coplanar configuration. Another example is the controversy about the orbital eccentricity of the giant exoplanet β Pic b, which has been due to the nearly edge-on configuration and the lack of measurements sampling one side of the orbit until recently (e.g., Dupuy et al. 2019; Lagrange et al. 2019; Nielsen et al. 2019). Also, because direct imaging is biased toward the detection of long-period planets and measures their relative position to the host star, biases on the measured eccentricity can occur if there is an unknown companion orbiting close to

the star and inducing a stellar wobble around the center of mass of the system (Pearce et al. 2014). Such biases can be tested by comparing the predicted mass for the putative inner companion with detection limits, e.g., from imaging (e.g., Ginski et al. 2014; Maire et al. 2019).

2.3. Simultaneous Fits

2.3.1. RVs-TTVs-transits

Extracting the dynamical information contained in both RVs and TTVs (when they are both available for a given system) can be done via two main approaches. The first one is to separate the photometric and dynamical analyses: the TTVs are first extracted from the transit photometry, using, e.g., the open-source packages *Exo-Striker* (Trifonov 2019) and *allesfitter* (Günther & Daylan 2019, 2021), and then fitted simultaneously with the RVs using an N -body integrator, such as *TTVFast* (Deck et al. 2014) or *REBOUND* (Rein & Liu 2012). The second approach is to use a photodynamical model to fit the whole transit photometry and RVs consistently, by coupling N -body integrations to predict the motions of the planets, with a model of the corresponding flux variations. Unlike the N -body fit to TTVs and RVs (first approach), the photodynamical fit must reproduce not only the transit time, but also the transit depth, duration, and shape at each epoch. It is much more computationally intensive, since in this case the N -body model has to be computed for each single photometric measurement, and the number of fitted parameters is also larger. However, it has the advantages of being more self-consistent, precise, and accurate than a separate analysis. It is also potentially more informative, as it enables an exploration of the inclinations and longitudes of the ascending nodes of the planets, which can be constrained by the transit depths and durations. These reasons motivate a more widespread use of photodynamical modeling. While this approach can be challenging for non-experts, the development of some publicly available tools, such as *PhoDyMM* (Ragozzine et al. 2023, <https://github.com/dragozzine/PhoDyMM>) or soon *Exo-Striker*, should make it more accessible to the community.

2.3.2. Imaging-RVs-astrometry

One way to improve orbital constraints of directly imaged exoplanets is to include the orbital fitting measurements from other exoplanet detection techniques. In particular, imaging data cannot uniquely constrain the longitude of the ascending node, leading to an ambiguity of 180 deg in this parameter and the argument of the periapsis. Until recently, combining imaging with other techniques has been challenging due to the small overlap in terms of parameter space (separation to the star, stellar age). Directly imaged exoplanets are detected in wide orbits around young stars. Such systems are difficult

targets for a follow-up with RV due to the strong stellar activity and the small stellar wobble induced by the companions (for β Pic b, see Lagrange et al. 2013, 2020). Astrometry is less sensitive to stellar activity but the stringent requirements on the measurement precision (at least a few tens of μ as) have limited the developments of this technique for exoplanet detection until the advent of the Gaia mission (e.g., Malbet & Sozzetti 2018). As of early 2021, two exoplanet discoveries have been reported using ground-based instruments (Muterspaugh et al. 2010; Curiel et al. 2020). Brown dwarf companions have also been detected (e.g., Pravdo et al. 2005; Sahlmann et al. 2013). Other studies focused on constraining the inclination and mass of RV-discovered companions exploiting the Hubble Space Telescope (e.g., Benedict et al. 2002), Hipparcos (e.g., Reffert & Quirrenbach 2011; Sahlmann et al. 2011), and/or ground-based instruments (e.g., Sahlmann et al. 2016).

The releases of the Gaia data have stimulated studies to further constrain the orbital parameters and/or mass of known companions (e.g., Snellen & Brown 2018; Kiefer et al. 2019; De Rosa et al. 2020a; Lagrange et al. 2020; Nielsen et al. 2020; Nowak et al. 2020) or to infer the presence of companions for target selection of direct imaging surveys (Fontanive et al. 2019; Currie et al. 2020). Brandt (2018) and Kervella et al. (2019) derived catalogs of proper motion anomaly measurements using the ~ 24 yr baseline separating the Hipparcos and Gaia data. These catalogs have been used to constrain the dynamical mass of the giant exoplanets β Pic b and/or c (Dupuy et al. 2019; Brandt et al. 2021a), the RV exoplanet Proxima c (Kervella et al. 2020), the cold Jupiter orbiting the system of π Men hosting an inner transiting super-Earth (Xuan & Wyatt 2020), brown dwarfs (Brandt et al. 2019, 2020; Maire et al. 2020), and RV exoplanets (Li et al. 2021). An open-source tool for joint imaging-RV-astrometry fits is available in the packages *orvara* and *orbitize!*, and is planned in the package *Exo-Striker*.

Measuring the mass of directly imaged companions with dynamical methods is crucial to test evolutionary models of substellar companions (e.g., Baraffe et al. 2003; Marley et al. 2007; Saumon & Marley 2008). Because of their wide separations to the stars, the mass of most directly imaged companions are estimated from their measured luminosity and the system age with evolutionary models. However, the models suffer from large uncertainties in the assumptions for the atmosphere of the companions (clouds, molecular opacities; e.g., Baraffe et al. 2003; Saumon & Marley 2008) and for the accretion process (post-formation luminosity, which sets up the initial condition for the subsequent thermal evolution; e.g., Marley et al. 2007; Mordasini et al. 2017; Marleau et al. 2019).

Imaging, RV, and/or astrometric data can also be combined to improve the constraints on putative additional planets in a system (e.g., Lannier et al. 2017; Kervella et al. 2019).

2.4. *Non-Keplerian Motions in Directly Imaged Multi-planet Systems*

Orbital fits of directly imaged exoplanetary systems usually assume Keplerian orbits. Planet-planet interactions in multi-planet systems produce small deviations in the position of the planets from the Keplerian motion and small changes in the orbital parameters over time. This reflex motion can be used to constrain the true masses and mutual inclinations solely with RV data (e.g., Correia et al. 2010). It can also be used to detect the presence and to constrain the dynamical mass of an inner planet perturbing the orbit of a known planet. Such measurements for directly imaged exoplanetary systems require a high astrometric precision which has been enabled by the GRAVITY instrument. Lacour et al. (2021) use this approach on the β Pic system to detect and constrain the mass of the inner planet (semimajor axis ~ 2.7 au) based on the orbital motion of the outer planet (semimajor axis ~ 9.9 au). The development of such studies may require the inclusion of N -body integrations in orbital fits, methods which are currently available in separated packages.

2.5. *Data and Computing*

An important set of challenges to address and which are part of all the previous discussions on the estimation of orbital parameters are those related to both the increasing amount of data on exoplanetary systems and the available computational resources to analyze them. On the one hand, the interest on keeping up-to-date ephemerides for known exoplanetary systems by the community has given rise to various projects aiming at gathering data for systems deemed as “high-priority” targets for, e.g., atmospheric characterization (see, e.g., Mallonn et al. 2019; Zellem et al. 2019; Kokori et al. 2022). Furthermore, a consequence of the various present wide-field ground-based (e.g., WASP, HATNet, HATSouth, NGTS; Bakos et al. 2004; Pollacco et al. 2006; Bakos et al. 2013; Wheatley et al. 2018) and space-based (e.g., TESS) photometric surveys observing at good enough cadences to be able to precisely measure exoplanetary transit events is to further enhance the amount of data produced for the community for known exoplanetary systems, which in some cases, such as TESS targets, are updated monthly. With the advent of missions such as PLATO (Rauer et al. 2014), and precise upcoming spectrophotometers such as JWST (Gardner et al. 2006) and ARIEL (Tinetti et al. 2021), which will also precisely constrain transit events, the data volume is destined to keep increasing further. Thus, the field is slowly approaching (or perhaps already is at) a state in which the available data for all exoplanetary systems is extremely large. So large that keeping all the planetary parameters at their most up-to-date status possible (i.e., parameters computed with all the available data at a given moment) is a lost cause by direct human interaction: by the time a full analysis is made on any single

exoplanetary system, there will most likely be more data gathered by ground-based and/or space-based instruments.

The above state of affairs suggests that there is currently a need for a system in which both new data can be easily ingested by any observer around the world (and credit should be given for this data), but also new analyses can be automatically performed given this new data set. This would efficiently solve the problem of constant human interaction to keep the most up-to-date exoplanetary parameters at any given time, and would save significant time for researchers performing these analyses on individual systems for their own use-cases (e.g., ephemerides updating), as well as benefit both population-level analyses that aim to search for correlations among planetary physical and/or orbital parameters and individual-system analyses that aim to search for varying parameters as a function of time (e.g., TTVs, transit duration variations or TDVs). Current cloud computing environments might be good avenues for these automated systems, and indeed, work is under way to explore how well these environments might be for such endeavors (Espinoza, Cooper & Bourque, private comm.).

3. **Challenges in the Dynamical Analyses**

3.1. *Challenges with Inferring Histories of Systems from Current Parameters*

Understanding planet formation requires connecting the formation processes and subsequent migration mechanisms to the resultant exoplanet system geometries. Since the range of outcomes technically allowed by physics generally greatly exceeds the range observed to occur in nature, it is imperative to use observations to both limit and guide the development of planet formation theories. Since the evolution of an exoplanet system cannot be observed in its entirety, there are two ways to infer the dynamical pathways that exoplanet systems take throughout their lifetimes: one option is to observe a large number of systems at different ages and construct a piecemeal understanding of how systems evolve in bulk over time, and the other option is to make inferences about the past of individual systems using the currently observed system parameters combined with theoretical models.

The first of these two strategies requires exoplanet systems to be discovered and characterized around stars of every age. Although exoplanets have been found around stars less than 10 Myr old (David et al. 2016; Donati et al. 2016), these planets are generally very difficult to discover because young stars rotate more rapidly and have higher levels of stellar activity than older stars (Lagrange et al. 2013; Gallet & Bouvier 2015), which results in significant RV variability (Queloz et al. 2001; Huerta et al. 2008; Luhn et al. 2020) and photometric variability (McQuillan et al. 2012; Biddle et al. 2021), often preventing secure planet detection and even resulting in false positives (Figueira et al. 2010; Damasso et al. 2020). Once stars mature to the 50–1000 Myr age range, they

are generally calmer and the disk has dissipated, which allows for a larger number of secure planetary detections (e.g., David et al. 2018; Hedges et al. 2021; Tofflemire et al. 2021; Zhou et al. 2021). Due to observational biases and the difficulty of planet discovery around younger stars, the vast majority of planets discovered have been found around stars with ages older than 1 Gyr (Silva Aguirre et al. 2015). Since the difficulty of detecting planets is much higher at younger stellar ages due to disk presence, stellar activity, and stellar rotation, there is a significant bias in the types of systems that can be discovered about young stars. As planets of different ages are subject to significantly different systematic effects, it is not straightforward to convert the planet occurrence rates by age to secure conclusions about the processes of planet formation.

Instead, for the time being, we can turn to the next potential strategy, which is to take systems discovered at a time when observational biases are minimal and then use theoretical methods to infer their past evolution. Mature exoplanet systems, those above 1 Gyr or so in age, have generally settled into dynamically stable configurations. At these ages, confounding stellar effects are minimal, allowing for the discovery and more robust characterization of planetary systems. In many cases, the modeling of planets' past dynamical histories can be done using straightforward theoretical methods, but again, there are limitations inherent in both our understanding of the system at current day and also in our understanding of how parameters scale backwards in time. These limitations arise due to both observational uncertainties (the physical parameters of the star and planets as well as the possibility of missing system components) and the irreversibility of many pieces of the evolution of planetary systems.

Observational uncertainties in the physical parameters of stars and planets may prevent secure determinations of their likely dynamical histories. When uncertainties are large enough, multiple current and past dynamical states may be consistent with the observed values. This is commonly seen in multi-planet systems near resonance, where the observational priors are wide enough that the true dynamical state of the system cannot be uniquely determined (e.g., Vanderburg et al. 2019). An additional complication arises due to the potential for unseen planets in the system. Even for well-characterized systems, planets may be missed because their orbital parameters do not allow them to be seen in the original system discovery method (Buchhave et al. 2016) or because their signal amplitudes reside below the detection capabilities of the original detection methods (Dawson & Fabrycky 2010; Shallue & Vanderburg 2018) or require more data to be seen (Rivera et al. 2005).

Additionally, the evolution of the host star will both affect the observability of its planets and alter their orbit. As a star ages, the stellar potential will change in ways that can affect nearby planets quite strongly. In particular, a combination of a non-zero stellar obliquity and quadrupole moment (J_2) provides

an additional direction of angular momentum to the system. The exact rate at which the stellar spin rate decreases can have significant effects on the dynamics of the exoplanet system (Spalding & Batygin 2016; Becker et al. 2020; Li et al. 2020; Brefka & Becker 2021; Schultz et al. 2021). However, the process of stellar spin-down cannot be directly integrated backwards in time due to the collapse of the area populated by all possible spin rates for a star of a given mass over time (Gallet & Bouvier 2013; Matt et al. 2015) combined with the (often significant) observational uncertainties on the current-day values. Although the expected spin rates for a group of stars can be reliably estimated, retrieving the past spin rate of individual stars is often not possible. Similarly, irreversible processes such as dynamical instabilities can be generally inferred (i.e., by the presence of substantial orbital eccentricity in remaining planets; Adams & Laughlin 2003; Raymond et al. 2010; Kane & Raymond 2014; Carrera et al. 2019) but not directly constrained for individual systems. In cases like this, where specific information is lost, inferences must instead be made for populations of planets with similar properties (Pu & Wu 2015) in order to derive their relevant formation mechanisms. Improvements may be made for individual systems if the data is high-quality enough that highly precise stellar modeling (Huber et al. 2022) and orbital determinations are possible.

3.2. Tidal Analysis

Tidal interactions can play an important role in the dynamics of planetary systems. This is especially important for close-in planetary systems, whose discovery motivates the improvement of tidal modeling.

Tides drive the spin evolution and influence the thermal state by internal friction (Henning & Hurford 2014; Bolmont et al. 2020; Bierson & Steinbrügge 2021). They also drive the orbital evolution by exchanging angular momentum, which leads to the migration of the planets, damping of their eccentricity and inclination, and precession of their orbit (e.g., Hut 1981; Bolmont et al. 2011, 2012). These latter effects have a strong impact on the dynamics of close-in planetary systems.

The precession of the argument of periapsis leads to TTVs in the transit events of exoplanets. TTVs are primarily due to planet–planet interactions, but close-in planets can experience such strong tidal effects that they affect their orbit, which drive migration and lead to a non-negligible impact on the orbital precession (Yee et al. 2020; Gomes et al. 2021). As discussed above, the measurement of TTVs can be a very efficient way of constraining the masses and eccentricities of close-in multi-planet systems. Agol et al. (2021) use the TTVs of the TRAPPIST-1 system to provide a better estimate of the masses of the seven planets, enabling the measurement of their bulk densities, and constraining their water content, thus refining the estimates of Grimm et al. (2018). Linking the planetary interior

structure with the TTVs can in principle be used for probing the planetary interior (e.g., for massive planets, see Ragozzine & Wolf 2009). Recently, Bolmont et al. (2020) have investigated this for the rocky planets of the TRAPPIST-1 system (Gillon et al. 2017). In particular, they estimate that the tidal effect on the TTVs for the planets b and c of the TRAPPIST-1 system can be as large as 10 s on a 3–4 yr timespan assuming that the planets have a liquid layer, which is at a potentially observable level, although it may be degenerate with the fundamental system parameters. Note that TTVs have also been used to constrain the migration history of the TRAPPIST-1 resonant chain (Teyssandier et al. 2022).

Numerous previous works modeled tidal interaction using equilibrium tide models such as the constant time lag model CTL (e.g., Hut 1981), which assumes the lag angle proportional to the tidal frequency, or the constant phase lag model CPL (e.g., Goldreich 1966), which sets the geometrical lag to be constant. Although those models are relevant for the equilibrium tides of weakly viscous objects like stars and giant gaseous planets (Alexander 1973), they do not reproduce the correct behavior for solid bodies (Henning et al. 2009; Efroimsky & Makarov 2013) in particular their rotation (Makarov & Efroimsky 2013). Recent studies have developed more realistic rheological responses such as the Maxwell model, the Andrade model, the Burgers model, and the Sundberg-Cooper model (e.g., Renaud & Henning 2018, for a comparative study). The Maxwell model is the simplest model and is broadly used (e.g., Correia et al. 2014). The Andrade model is more complex (Andrade 1910) but is known to better reproduce the response of rocky materials to tidal forcing (e.g., Efroimsky & Lainey 2007; Castillo-Rogez et al. 2011). Sundberg & Cooper (2010) have published a more complex rheological model which better fits the experimental data. It includes the correct background attenuation as in the Andrade model but with an additional feature: a second attenuation peak, which is observed in high-temperature experiments (also included in the Burgers model). Thus, the Andrade model could represent a compromise between the fidelity to reproduce the empirical response of rocky materials to tidal forcing and the model complexity.

Encapsulating the complex response of rocky planets to stress in consistent tidal models is important for understanding their evolution. These models require the use of either a decomposition of the tidal potential into Fourier harmonic modes as developed by Kaula (1964), or a time domain approach as proposed by Correia et al. (2014) and Gevorgyan et al. (2020). These developments allow the study of more complex and realistic rocky and icy bodies and could be used to investigate the effect of the presence of liquid layers and their complex behavior (e.g., Auclair-Desrotour et al. 2018).

Such developments are thus very promising for numerical simulations, as Walterová & Běhouňková (2020) show with

a semi-analytical model in 1D simulations. Further implementation work in N -body codes could allow the study of both the effects of planet–planet and tidal interactions on the dynamics of close-in planetary systems in a much more realistic way, improving our comprehension and characterization of potential habitable worlds.

3.3. The Challenge of Forming and Detecting Co-orbital Worlds

Co-orbital bodies (also known as trojans) reside in the gravitational wells exerted by two more massive bodies. The mathematical framework of this type of configurations is known since the formulation of the two-body problem in the eighteenth century by Joseph-Louis Lagrange (Lagrange 1772). The consequence of this composed gravitational field is the so-called Lagrangian points, five equilibrium locations resulting from the solution of this problem. Among them, only two Lagrangian points (L_4 and L_5 , located in an equilateral triangle disposition with the two bodies) are stable against small perturbations. Consequently, other smaller bodies can be trapped in these regions. Indeed, we see these configurations in our solar system, where six out of the eight planets host co-orbital bodies in their Lagrangian points (e.g., Wolf 1906; Innanen et al. 1991; Pittichova et al. 2003; Connors et al. 2011; Alexandersen et al. 2013; de la Fuente Marcos & de la Fuente Marcos 2014 for the case of Jupiter, Mars, Neptune, Earth, Uranus, and Venus, respectively). Of special interest is the case of Jupiter, where more than 10^5 trojans in each Lagrangian point are known and contain critical information about the formation and dynamical history of the solar system (e.g., Morbidelli et al. 2005). This has motivated the design and launch of the Lucy mission (Levison et al. 2017) to explore these intriguing bodies.

The analysis of the three-body problem (e.g., Laughlin & Chambers 2002, and references therein), provides interesting conclusions about the stability of these co-rotating objects. First, the stability of the co-orbital bodies is long-term and strong under the assumption of no additional perturbations. This implies that co-orbital bodies are long-term residents of these locations. Moreover, the condition for stability derived for a system composed of a main body of mass m_* (the star), a surrounding body with mass m_p (the planet), and a co-orbital body with mass m_t (the trojan) is given by the simple equation derived by Gascheau (1843) and reformulated later on by Laughlin & Chambers (2002) as:

$$\frac{m_p m_t + m_t m_* + m_p m_*}{(m_t + m_p + m_*)^2} < \frac{1}{27} \quad (1)$$

This criterion limits the stability of the system to configurations in which the sum of the trojan and planet masses is smaller than about 3.7% of the stellar mass. This simple equation implies that for solar-type stars, the stability criterion

is met even for pairs of planet and trojan both possibly having similarly large masses. However, the dynamics of planetary systems is more complex and other factors play a relevant role in the dynamics of these configurations. As such, additional restrictions apply in terms of the orbital eccentricity (e.g., Namouni et al. 1999; Mikkola et al. 2006; Leleu et al. 2017) or the orbital period and hence the influence of migration and dissipation (e.g., Leleu et al. 2019). Nevertheless, even with such restrictions, co-orbital bodies can still reach the planetary domain (e.g., Beaugé et al. 2007; Leleu et al. 2019).

Besides, one of the major restrictions on the mass of co-orbital planets so far and one of the most debated restrictions is the formation step. Two main formation mechanisms have been proposed for co-orbital bodies, the in situ growth within the Lagrangian points, and the capture of planets formed in other parts of the system and subsequently trapped into 1:1 resonances. The in situ formation scenario starts with a forming planet embedded in a protoplanetary disk. Laughlin & Chambers (2002) show through hydrodynamical simulations that a sub-Jovian planet accreting material through a vortical flow would accumulate material in the vicinity of the Lagrangian points. This large-scale vortical flow envelops the horseshoe region creating additional smaller vortices that can trigger the formation of planetary cores. Chiang & Lithwick (2005) probe that with such a formation scenario, they could reproduce the current population of Neptune trojans starting from small 20 cm pebbles. Beaugé et al. (2007) test further this scenario by performing N -body simulations including gas interactions with the trapped particles. In both gas-free and gas-rich scenarios, they manage to form a single terrestrial-like stable trojan, reaching as much as $0.6 M_{\oplus}$ (~ 6 times the mass of Mars). However, Lyra et al. (2009) find that the gravitational collapse of 1–20 cm particles leads to the formation of Earth-mass trojans in a relatively short amount of time of about 200 orbits. The alternative scenario is the capture of already formed planets into 1:1 resonances. Cresswell & Nelson (2008) simulate the hydrodynamical evolution of multi-planetary systems still embedded in a protoplanetary disk. In $>30\%$ of their simulations, at least a pair of protoplanets end up in a stable co-orbital configuration. Interestingly, they also find that only the cases that formed co-orbitals could retain other planets that would have otherwise been scattered out of the system. This scenario was revised by Leleu et al. (2019) in the particular case of low-mass stars by taking the generic dissipation of the disk and the evolution of the planet mass into account. They find that in 12% of their 880 simulated systems, a co-orbital is present in the final configuration. The formed co-orbitals usually end up in a tadpole configuration and mostly in resonance with other planets.

Once formed, migration is a major dynamical event in the evolution of planetary systems. Understanding its consequences for formed co-orbitals is critical to understand whether they exist or not in more mature systems and in which cases the

co-orbital resonance is disrupted. As mentioned before, during the formation phase, the presence of a forming giant planet induces a vortical flow within the disk. This generates a torque on the system that makes it migrate to the inner regions. Laughlin & Chambers (2002) show that the 1:1 resonance is kept under such forces. Cresswell & Nelson (2009) find that the co-orbital configuration remains stable before, during, and after the disk dispersal, including gap opening. However, the libration amplitude increases during the inward migration. This may lead to the disruption of co-orbital pairs in large-scale migrations, although other mechanisms can prevent this disruption, like resonance trapping with another inner/outer planet. Otherwise, in rapid migration situations, the co-orbital pair is very stable. On this same line, Pierens & Raymond (2014) show that similar-mass co-orbitals may be disrupted during large-scale migrations if the planet-to-trojan mass ratio is smaller than $m_p/m_t < 2$. The orbital radius for disruption depends on this mass ratio, with co-orbitals with higher ratios being disrupted at a larger orbital radius. Also, they find that if the leading planet is more massive, the co-orbital system is more stable, hence pointing to the L_5 point as the most promising site to locate large co-orbitals. The topic of migration was reviewed by Leleu et al. (2019), where they include the effects of disk dissipation and the evolution of the planet masses. They reach similar conclusions as those presented in previous works, with the inward migration and/or mass loss for the main planet leading to the disruption of the co-orbital configuration. Also, they find that if the co-orbitals migrate too close to the star, then the libration amplitude increases possibly leading to horseshoe orbits or even exiting the 1:1 resonance. Hence, hot Jupiters are not expected to host co-orbitals. On the positive side, the inward migration can generally retain the co-orbital configuration.

Based on all these considerations, we can think about habitability of co-orbital worlds. We have seen that planet migration down to 1 au could retain the co-orbital configuration. Most of the habitable zone planets are gaseous giants. If they host Earth-like trojans, these worlds could indeed be habitable (e.g., Dvorak et al. 2004; Funk et al. 2012). However, some destabilizing processes may occur in the Lagrangian point regions, like encounters with kilometer-sized objects, large librations producing variations in the irradiation levels, or the water content being damped by the presence of the more gravitationally attractive main planet. In any case, this is a plausible to-be-explored scenario. On the other side of the coin, if additional effects apply (e.g., tides, planet scattering, or collisions in the Lagrangian regions), the co-orbital can become unstable (Couturier et al. 2021). Interestingly, Kortenkamp & Hartmann (2016) find that in 85% of the cases, the unstable co-orbital ends up colliding with the main planet. This converts the Lagrangian points into reservoirs of bullets that can impact planets during and after their formation. These great impacts could indeed be the origin of the formation of large moons of

rocky planets, including our own Moon. Belbruno & Gott (1724) indeed demonstrate that this is a plausible scenario for the formation of our own Earth-Moon system.

Overall, co-orbital worlds of planetary size (even in the rocky regime) are plausible outcomes of planetary system formation and they might have a key influence on the final architecture of planetary systems. Searching for these bodies in exoplanetary systems and understanding their dynamics is both a challenge already started by both individual studies, such as Ford & Holman (2007), Janson (2013), and Hippke & Angerhausen (2015), and, recently, more ambitious endeavors, like the TROY project (Lillo-Box et al. 2018a, 2018b), and a possible missing piece of the puzzle of planet formation and evolution. Indeed, this team has recently presented in Balsalobre-Ruza et al. (2023) the first evidence for dust accumulation at the Lagrangian point L_5 of one of the planets in the only known planetary system that is currently in the formation process, PDS 70 (Keppler et al. 2018; Wang et al. 2021b; Benisty et al. 2021). Together with a previous detection of dust arch-like features in LkCa 15 by Long et al. (2022) resembling dust accumulation in the Lagrangian points of a non-detected planet in this system, these discoveries are starting points to understand the co-orbital conundrum and its role in planetary system formation and architecture.

3.4. Resonances and Planet-disk Interactions in Directly Imaged Exoplanetary Systems

Directly imaged exoplanets have been found in multiple systems, e.g., HR 8799 (Marois et al. 2008, 2010), β Pic (Lagrange et al. 2010), and PDS 70 (Keppler et al. 2018; Haffert et al. 2019). Commensurate period ratios have been inferred between the planets for HR 8799 (e.g., Konopacky et al. 2016a) and PDS 70 (Wang et al. 2021b), hinting at possible mean motion resonances which might stabilize these systems of massive Jovian planets, at least over the estimated age of the system. In particular, Goździewski & Migaszewski (2014) show based on stability arguments that the companions in the HR 8799 system shall be in the planetary mass regime. Another intriguing planetary system is HD 106906, which is composed of a central binary star (Lagrange et al. 2016) and of a widely separated Jovian planet (~ 700 au, Bailey et al. 2014), which appears misaligned with the plane of the circumstellar debris disk by ~ 20 deg (Kalas et al. 2015; Lagrange et al. 2016). To explain the formation of the planet, Rodet et al. (2017) propose a scenario involving formation in the circumbinary disk and inward migration followed by resonant ejection with the binary.

Direct imaging is a unique method to simultaneously study exoplanets and circumstellar disks, their birthplace environments, and their interactions. A large variety of features are observed in circumstellar disks even at an early stage (e.g., spirals, rings, gaps; Keppler et al. 2018). They are often used

to infer the presence of planets and their properties (e.g., Stolker et al. 2016). Yet, some of these features could be produced by a planet-free mechanism (e.g., Juhász et al. 2015). Giant planets and brown dwarfs have been detected around stars with debris disks, β Pic (Lagrange et al. 2010), HR 8799 (Marois et al. 2008, 2010), HD 95086 (Rameau et al. 2013), HD 106906 (Bailey et al. 2014), HR 2562 (Konopacky et al. 2016b), HD 206893 (Milli et al. 2017), and in young gas-rich protoplanetary disks (PDS 70; Keppler et al. 2018; Haffert et al. 2019). In some cases, there is evidence of disk shaping by the companion (e.g., HR 2562; Maire et al. 2018), while for other systems an additional companion seems needed to shape the disk (e.g., HD 206893; Delorme et al. 2017). The detection of brown-dwarf companions in the cavity of debris disks (HR 2562, HD 206893) suggests that they formed from the disk material. This poses some challenges to formation theories within circumstellar disks by gravitational instability because such massive companions are expected to be difficult to form and would require massive disks, non in-situ formation at wider separations followed by significant migration, and/or significant mass accretion after the formation (e.g., Maire et al. 2018, for HR 2562B). Brown dwarfs are thought to mainly form with a stellar-like mechanism (e.g., Chabrier et al. 2014). The number of detected systems with companions and circumstellar disks is increasing, which will allow for in-depth tests of planetary formation models inside circumstellar disks.

Observational constraints of a disk can be used to further constrain the orbital parameters of a detected companion responsible for its shaping when the orbital parameters are not well constrained. A major issue with hydrodynamical and N -body simulations of circumstellar disks is computing time, which does not allow for exploring large ranges of orbital parameters of the companion (e.g., Maire et al. 2018; Gonzalez et al. 2020). Empirical criteria for the clearing zone of a planet have been developed (e.g., Petrovich 2015; Lazzoni et al. 2018; Regály et al. 2018) and used to further constrain possible orbits given observational constraints on the orientation and cavity size of the disk (e.g., Delorme et al. 2017; Maire et al. 2018). Lazzoni et al. (2018) also use this approach in a statistical study of exoplanet demographics in debris disks with double belt architectures. Such a configuration is observed in the solar system and in a few directly imaged exoplanetary systems, such as HR 8799 (Su et al. 2009) and HD 95086 (Moór et al. 2013). Assuming that the gap between the disk belts is carved by planets, Lazzoni et al. (2018) show that multi-planet configurations are more compatible with SPHERE imaging detection limits because less massive planets are required and would stay undetected with the current instruments.

4. Prospects for Improving our Knowledge of the Architecture of Exoplanetary Systems

4.1. PLATO

The ESA mission PLATO (Rauer et al. 2014) is scheduled for launch in 2026. It is expected to detect and characterize a large number of transiting planets around bright stars ($V < 11$), including Earth-sized planets out to the habitable zone of stars as massive as solar-type stars. It will provide new constraints on planet occurrence rates, especially η_{\oplus} (the occurrence rate of habitable-zone rocky planets around Sun-like stars). Thanks to their brightness, PLATO targets will be amenable to RV follow-up, which will allow the measurement of the masses and eccentricities of the planets, and the detection of some possible additional non-transiting planets in the systems. PLATO will also perform asteroseismology, thus enabling the precise characterization of the host stars, including their age ($\sim 10\%$ age precision). This key asset will make it possible to decisively test and constrain planetary migration and dynamical evolution models, by studying planetary systems at different evolutionary phases. It will thus open up a new evolutionary understanding of planetary systems. Although the exact pointing strategy of PLATO still has to be confirmed, it is likely that it will reobserve the Kepler field, as well as some TESS systems. PLATO will considerably extend the baseline of observations for these systems, which will allow the study of TTVs over long timescales and the possible detection of long-term dynamical effects, such as orbital decay or precession.

4.2. ESPRESSO and NIR RV Instrumentation

For about a decade, RV instruments were unable to routinely reach accuracies less than $\sim 1 \text{ m s}^{-1}$ and to access the regime of Earth-mass planets in the habitable zone of F-, G-, and K-dwarf stars (e.g., Fischer et al. 2016). The recent commissioning of extreme precision RV instruments designed to reach instrument accuracies approaching 10 cm s^{-1} and operating in the visible, e.g., ESPRESSO (Pepe et al. 2021), MAROON-X (Seifahrt et al. 2020), EXPRES (Jurgenson et al. 2016), NEID (Schwab et al. 2016), represents a stepping stone toward a complete picture of the architecture of nearby planetary systems within $\sim 1 \text{ au}$ around solar-type stars and bright M-dwarf stars. A strong synergy exists between these instruments and new NIR RV instrumentation to probe the architecture of nearby planetary systems around faint M-dwarf stars, such as CARMENES (Quirrenbach et al. 2018), SPIRou (Donati et al. 2020), and NIRPS (Bouchy et al. 2017). The latter can more easily access mid to late M dwarfs, which have been identified as the most promising targets to find and characterize Earth-sized planets.

ESPRESSO, the new high-resolution spectrograph of ESO's Very Large Telescope (VLT), has been fully operational since 2019 July (Pepe et al. 2021). It can be operated either with a

single Unit Telescope (UT) at a spectral resolving power of 140,000 or 190,000 over the 378.2–788.7 nm wavelength range, or with up to four UTs combined, thus turning the VLT into the equivalent of a 16 m telescope, with a resolving power of 70,000. With its instrument RV precision of $\sim 10 \text{ cm s}^{-1}$ and long-term thermo-mechanical stability, ESPRESSO opens a new parameter space in RV measurements and makes it possible to search for rocky planets out to the habitable zone of G-, K-, and M-dwarf stars, provided that they are quiet and sufficiently bright. In the framework of the guaranteed-time program of the ESPRESSO consortium, 45 targets fulfilling specific criteria were carefully selected for a blind search (Hojjatpanah et al. 2019). For these stars, the photon noise attained in a 15-minute integration time (or 30 minutes for the faintest targets) allows the detection of an Earth-mass planet orbiting inside the habitable zone. Lillo-Box et al. (2021) have recently reported the first detection from this survey, a planet with a minimum mass of $5.6 M_{\oplus}$ on a 5.1 days orbit around a late-type K-dwarf star at 13.6 pc.

Another important aspect of the scientific program of ESPRESSO is the RV follow-up of K2 and TESS low-mass transiting planets. In this context, its 2 mag gain and improved RV precision with respect to HARPS-like instruments enable robust mass measurements of the rocky planet population found by the two missions, with radii $\lesssim 2 R_{\oplus}$ and expected masses typically below $6\text{--}8 M_{\oplus}$. In the long term, this will allow the exploration of how the properties of these small planets (most notably their internal composition) correlate for example with their stellar irradiation, the stellar properties, and the architecture of their systems. As an example, ESPRESSO's capabilities and complementarity to TESS are demonstrated in a study of the L 98-59 system (Demangeon et al. 2021). L 98-59 is a nearby M-dwarf star around which TESS found three small transiting exoplanets with radii ranging from 0.8 to $1.6 R_{\oplus}$ and orbital periods between 2.25 and 7.45 days (Kostov et al. 2019). HARPS RVs allowed mass measurements for the two outer planets but were not able to detect the signal of the smallest, innermost planet (Cloutier et al. 2019). ESPRESSO RVs reveal that this planet actually has a mass of only $\sim 0.4 M_{\oplus}$, about half the mass of Venus. This is the least massive exoplanet measured using RVs (at the time of writing). This RV follow-up also reveals a fourth non-transiting planet with a minimum mass of $\sim 3 M_{\oplus}$ and an orbital period of 12.8 days, as well as some indications of a fifth non-transiting planet at ~ 23 days within the habitable zone. These results show the capability of ESPRESSO to not only measure precise masses of small transiting planets, but also to investigate the complex architecture of multi-planetary systems out to the habitable zone. In this context, the precision of ESPRESSO also makes it possible to assess the orbital alignment of small planets via their Rossiter–McLaughlin effect (e.g., Kunovac Hodžić et al. 2021), thus yielding useful constraints on their dynamical history.

4.3. Missing Pieces of Exoplanetary Systems: Moons

While moons orbiting exoplanets have not yet been confirmed, it is widely accepted that they exist. This assumption is based on the large number of moons orbiting the planets in the solar system. Moons are valuable puzzle pieces for understanding the solar system formation history and evolution (see e.g., Öpik 1960; Orgel et al. 2018), and even Earth’s habitability (e.g., Laskar et al. 1993). For this reason, significant efforts are being carried out to find them around exoplanets (e.g., Kipping et al. 2012, 2015; Kreidberg et al. 2019), unfortunately with negative or ambiguous results to date.

The interest in exomoons also extends to their potential habitability. Returning to the solar system analogy, icy moons orbiting giant planets such as Europa, Titan, and Enceladus are considered to be plausible locations for some forms of primitive biology (e.g., Kivelson et al. 2000; Waite et al. 2006; Stofan et al. 2007, for Europa, Titan, and Enceladus, resp.). Moreover, internal oceans might also exist in Triton (Hussmann et al. 2006), Ganymede (Kivelson et al. 2002), and Callisto (Zimmer et al. 2000), where future missions might explore the presence of biosignatures (Chela-Flores 2017). Extrapolating to exoplanetary systems, of particular interest are massive rocky exomoons orbiting temperate giant planets, where it has been suggested that life might arise and evolve over long timescales (e.g., Williams et al. 1997; Kaltenegger 2010; Heller & Barnes 2015). In this context, due to the observational advantage of monitoring temperate planets orbiting low-mass stars, there is an open debate about exomoons’ existence and their potential habitability when orbiting such planets. For example, the magnetic field generated by the planet may protect the hypothetical moon from the stellar flares produced by M dwarfs (Heller & Zuluaga 2013). In addition, such exomoons would be likely tidally locked to the planet, which would favor a uniform distribution of stellar irradiation (Trifonov et al. 2020). However, due to their proximity, gravitational perturbations exerted by the star and extra planets in the system could induce eccentricities that likely make any moon uninhabitable (Heller 2012). As it can be seen, while the field of habitable exomoons seems promising, due to the number of unknown parameters to date, it is still highly speculative.

Various methods have been proposed to detect exomoons; some examples study the asymmetries in the planet’s transit ingress and egress, through TTVs and TDVs (see e.g., Sartoretti & Schneider 1999; Kipping 2011a, 2011b; Heller 2014). All these methods require high-precision photometry. Also, Oza et al. (2019) suggest exogenic Na/K as possible tracers of exomoons. This approach requires high-precision spectrometry. Another challenge is the potential need for long observational baselines, as recent dynamical models suggest that to retain a rocky exomoon orbiting in a stable orbit the

most favorable scenario would be a giant planet ($M \gtrsim 10 M_{\oplus}$) with a long orbital period ($\gtrsim 50$ day) orbiting a low-mass star ($M_{\star} \lesssim 1.0 M_{\odot}$) (Sasaki et al. 2012; Dobos et al. 2021). The needs for high-precision photometry and long observational baselines make the detection of exomoons a very challenging task with the current technology, but will be reachable by the upcoming PLATO mission thanks to its high photometric performance and long-term observations (Rauer et al. 2014).

4.4. Gaia Exoplanet Catalog

The fourth release of Gaia data covering its nominal mission lifetime (5.5 yr) is expected in the mid-2020 s.⁴⁹ It will contain a catalog of detected exoplanets and also make the epoch astrometry publicly available. Eventually, when all 8+ yr of mission data will have been processed, Gaia is expected to discover thousands of brown-dwarf companions and exoplanets more massive than Saturn at separations of 2–8 au (e.g., Perryman et al. 2014; Holl et al. 2022) and also directly characterize accessible multi-planet systems and circumbinary planets (Sahlmann et al. 2015). The combination of the Gaia epoch astrometry with independent data, e.g., RV and imaging data (Section 2.3.2), will be extremely powerful for the comprehensive characterization of a multitude of exoplanets.

Gaia will provide a large sample of young giant exoplanets around nearby stars with separations suitable for imaging follow-up. Despite having targeted several hundreds of young stars, direct imaging surveys have shown a paucity of substellar companions at separations beyond ~ 10 au, with measured frequencies of $\sim 10\%$ for giant planets and $\sim 1\%$ for brown dwarfs (e.g., Nielsen et al. 2019; Vigan et al. 2021). With its all-sky survey, Gaia will provide a complete census of young exoplanets at separations of a few au from nearby stars for targeted direct imaging surveys therefore increasing the yield of the direct imaging technique. Imaging follow-up of Gaia-detected exoplanets with current ground-based telescopes equipped with high-contrast imaging instruments, JWST, the coronagraphic instrument onboard the Nancy Grace Roman Space Telescope (formerly the Wide Field Infrared Space Telescope; Kasdin et al. 2020), and extremely large telescopes will be valuable to measure their luminosity and/or to improve their orbital parameters and mass. To maximize the chances of success of imaging follow-up, predicting the observing epochs when the angular separations of the companions to the stars are large will be critical.

A large sample of exoplanets with measured luminosity, mass, and age will allow for in-depth tests of the models of thermal evolution of giant exoplanets and brown dwarfs (e.g., Saumon & Marley 2008). Better age constraints are expected to come from distance measurements from Gaia, and gyrochronology and asteroseismology measurements from TESS and

⁴⁹ <https://www.cosmos.esa.int/web/gaia/release>

soon PLATO. Testing evolutionary models may allow the calibration of theoretical mass–luminosity–age relations. This may lead to revise the mass estimates of the majority of directly imaged exoplanets and brown dwarfs. Mass–luminosity–age relations are strongly affected by the mass of the planetary cores (Mordasini 2013) and the physics of the accretion process (Mordasini et al. 2017; Marleau et al. 2019). Testing the theoretical relations may provide clues on these parameters. By characterizing exoplanets in younger systems and at wider separations to stars compared to the RV and transit techniques, Gaia and direct imaging will also allow for a more complete understanding of the formation and evolution of the architectures of exoplanetary systems.

4.5. RV Measurements of Young Exoplanets in Wide Orbits with Medium-/high- resolution Spectrometry

RV measurements are fundamental to constrain the motion perpendicular to the plane of the sky and lift the ambiguities in the longitude of the ascending node and argument of the periastron inherent to direct imaging and astrometry. Besides, RV measurements coupled with direct imaging data allow for mass measurements of companions (e.g., Nowak et al. 2020).

RV measurements of young stars are challenging (e.g., Lagrange et al. 2013). However, measuring the RV of the companions due to their orbital motion around the stars could be feasible provided they can be angularly separated, because the RVs are expected to be larger than the stellar RV ($\sim 10 \text{ km s}^{-1}$ versus $\sim 10 \text{ m s}^{-1}$ for a Jupiter analog orbiting a Sun analog). Pioneering experiments on the directly imaged giant exoplanet β Pic b (Snellen et al. 2014) and brown dwarf GQ Lup B (Schwarz et al. 2016) with high-resolution spectrometry ($R \sim 100,000$) have demonstrated the feasibility of the technique. In addition to RV measurements of the companions, they have provided measurements of their rotation period ($15.4 \pm 1.7 \text{ km s}^{-1}$ and $\sim 8 \text{ hr}$ for β Pic b, resp.). They have been followed by observations of the directly imaged Jovian exoplanets HR 8799bcd (Ruffio et al. 2019, 2021) and HIP 65426b (Petrus et al. 2021) and of the super-Jupiter κ And b (Wilcomb et al. 2020) with medium-resolution integral field spectrometry ($R \sim 5000$). A major difficulty in measuring the RV of young exoplanets outlined in Ruffio et al. (2019) and Petrus et al. (2021) is the need for a precise absolute stellar RV. Such a measurement can be challenging if the star has an early spectral type and is a fast rotator (the majority of directly imaged exoplanets orbit stars with spectral types earlier than G, e.g., www.exoplanet.eu; Schneider et al. 2011). The sample of measurements is expected to increase with new instruments, e.g., NIRSpec (Ferruit et al. 2022) and MIRI (Wright et al. 2015) onboard the JWST, and CRIRES+ (Dorn et al. 2014), ERIS (Davies et al. 2018), KPIC (Mawet et al. 2016), HARMONI (Thatte et al. 2016), and METIS (Brandl et al. 2018) on

ground-based telescopes. First science results from KPIC on the HR 8799 exoplanets including an RV measurement of HR 8799e are presented by Wang et al. (2021a).

5. Summary and Outlook

RV and TTV mass measurements of mature and close-in exoplanetary systems seem to be overall in good agreement for the few systems for which independent values from the two methods have been reported so far. As new compact planetary systems are found by TESS around bright stars, it is however important to perform more comparative studies between the two methods, to better understand their possible biases and limitations, in parallel to a simultaneous fit to both data sets which will yield the best constraints on the system parameters (e.g., Judkovsky et al. 2022). For the latter, using a photodynamical model to fit the RVs and the whole transit photometry (i.e., not “just” the TTVs) can reveal more about the system architecture than an N -body fit to the RVs and TTVs only. Photodynamical modeling is computationally intensive and can be challenging for non-experts, but the ongoing development of some publicly available tools should make this approach increasingly accessible to the community. We note that the new approach by Judkovsky et al. (2022) seems to be less computationally demanding.

Relative astrometry of directly imaged systems is now reaching milliarcsecond precisions and better thanks to dedicated observing strategies and calibration plans. Direct imaging, Hipparcos-Gaia astrometry, and RVs are providing the orbit and mass for a growing sample of young massive Jovian planets and of brown dwarfs in the outer regions of exoplanetary systems ($\gtrsim 5 \text{ au}$). The field will expand significantly with the publication of the Gaia exoplanet catalog, which will include mass measurements. Direct imaging will be critical to measure the luminosity of Gaia exoplanets with well-characterized system ages to test the formation and evolution models of non-irradiated giant planets in details (e.g., physics of the accretion shock, atmospheric opacities, clouds). Direct imaging is also moving toward measuring the inclination, mass, and luminosity of RV exoplanets thanks to ground-based extremely large telescopes. Measurements of non-Keplerian motions in multiple directly imaged systems to get planet masses will expand with the GRAVITY instrument. RV measurements of young exoplanets to get mass and/or 3D orbits will expand thanks to new medium- and high-resolution spectrometers fed by adaptive optics on ground-based telescopes.

Direct imaging also provides critical insights into planet-disk interactions by studying simultaneously planets and circumstellar disks. There is evidence in several systems for resonant processes, which could explain their architecture and stability, and for disk shaping. The sample of detected planet-disk systems is growing, which will allow for statistical analyses of

the underlying mechanisms producing disk features and of their relations or not with perturbing planets.

Linking the formation and evolution processes that might take place in planetary systems with present-day observations (mostly of mature and close-in systems) represents a crucial point to understanding the diversity of system architectures found. However, since planetary systems cannot be observed over their entire lives but at a particular point of their evolution, establishing this link is challenging. To this end, two main strategies are used; observing many systems at different evolutionary phases to get some general trends and evolutionary patterns, or combining current observations of a given system and theoretical models to infer its past. However, these two strategies still have some difficulties that hinder estimating a likely dynamical history of planetary systems. Consequently, our understanding of planetary system architectures is still unmaturing, and despite all the configurations discovered, more exotic structures may exist; for example, co-orbital worlds have been theoretically explored as potential outcomes of planetary system formation. Several mechanisms are proposed to form co-orbital bodies of planetary sizes, and this configuration is found to be stable over long timescales in the Lagrangian point L_5 . While their existence has not been proven so far, they might play a key role in the final architecture of planetary systems, and observational efforts are being carried out to find them. Current observational techniques are more sensitive to close-in planets. In such scenarios, tidal interactions between the planets and the host star might influence the evolution of their orbit. However, while the most used models to study these effects (CTL and CPL) reproduce well the behavior of weakly viscous objects, such as stars and giant planets, a more realistic description is needed for solid bodies. In this context, the Andrade model is of particular interest, which relies on a more complex rheological model that better reproduces the empirical response of rocky materials. The future implementation of such a model in an N -body integrator will allow deriving a more realistic dynamical evolution of terrestrial planets in closely packed systems.

The future of orbital and dynamical studies of exoplanetary systems is bright with a large flow of new data coming from both ground and space which will feed the studies: e.g., TESS, CHEOPS (Benz et al. 2021), ESPRESSO, JWST, Gaia, PLATO, the Nancy Grace Roman Space Telescope, the ELTs, as well as upgraded instruments on 8–10 m class telescopes such as ERIS, CRIRES+, GRAVITY+ (Eisenhauer 2019), SPHERE+ (Boccaletti et al. 2020), and GPI 2.0 (Chilcote et al. 2020). These new data will help answering major questions which remain on the formation, evolution, and fate of exoplanetary systems.

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