An ADMM-inspired image reconstruction for Terahertz off-axis digital holography

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Abstract: We propose a reconstruction technique to off-axis Terahertz digital holography that jointly reconstructs the object field and the reference amplitude. The objective function combining data-fidelity and wavelet-based regularization terms is optimized with an ADMM-inspired approach.

1. Introduction

The Terahertz (THz) band (in 0.1-10 THz) owns the unique property of penetrating non-polar materials without ionization damage, giving to THz imaging technique high potential in security, biomedical and industrial non-destructive testing applications. Off-axis digital holography (DH), one of the first techniques in the coherent lensless imaging family implemented in the THz band [1], permits to reconstruct the complex object field $\boldsymbol{\psi}$ from one interference pattern recorded by a digital image sensor. Compared to the visible configuration, the off-axis THz DH suffers from a few particular problems that make the reconstruction task challenging [2]. Classical approaches, in particular the direct Fourier method, rely on filtering in the Fourier domain. Inverse problem (IP)-based methods are alternative approaches that provide reconstruction by minimizing a function promoting fidelity to the observations and specific image structures. In off-axis DH, the reconstruction problem is complicated since the reference field amplitude $\boldsymbol{\alpha}$ displays slow spatial variations due to the non-ideal holographic devices that are difficult to estimate. Earlier, we have proposed an IP-based reconstruction method [2] where, as in [3], the object field and the reference field amplitude are jointly estimated. This method demonstrated encouraging results compared to Fourier approaches but had limitations. First, the variability of $\boldsymbol{\alpha}$ on a sampled 2-D pixel grid leads to convergence problems preventing correct estimation of both solutions. Second, its application on real data produces severe artifacts emanating from the boundaries of the image throughout the whole image and resulting from the combination of periodic boundary condition (BC) assumption and the frame truncation by the camera. In this contribution, we present a new version in which the $\boldsymbol{\alpha}$ map is spatially constant and represented by a single value α_c; and, to alleviate the artifacts due to camera frame truncation in the real situations, we propose a pre-processing by apodization.

2. Method

The objective of off-axis DH reconstruction is to deduce from the measurements $\boldsymbol{y}$ the unknown complex image $\boldsymbol{\psi}$ and the unknown constant α_c; in other words, to solve the inverse problem related to the forward relation $\boldsymbol{y} = |\boldsymbol{A}_d\boldsymbol{\psi} + \alpha_c\boldsymbol{r}|^2 + \boldsymbol{n}$, where $\boldsymbol{A}_d$ is the instrumental point-spread-function (PSF) associated with the light propagation at distance d, $\boldsymbol{r}$ is the unit reference field and $\boldsymbol{n}$ represents the additive noise. Accordingly, the goal of off-axis DH reconstruction is to retrieve two estimates $\widetilde{\boldsymbol{\psi}}$ and $\widetilde{\alpha_c}$ minimizing the data-fidelity term $D(\boldsymbol{\psi}, \alpha_c) = \|\boldsymbol{y} - |\boldsymbol{A}_d\boldsymbol{\psi} + \alpha_c\boldsymbol{r}|^2\|_2^2$. Solving this problem is challenging. First, it is non-convex relatively to the unknowns and thus, D can have many local minima. Second, since the problem is in general ill-posed or ill-conditioned (*e.g.*, from the filtering properties of $\boldsymbol{A}_d$), adding a regularization is mandatory to select a meaningful solution among all compatible solutions. As in [2], we regularize the solution $\boldsymbol{\psi}$ in the wavelet domain by minimizing the ℓ_1-norm of its wavelet coefficients for promoting sparse solutions. After expressing the data-fidelity term D with respect to the wavelet coefficients $\boldsymbol{c}_\psi$ of $\boldsymbol{\psi}$ as $D(\boldsymbol{c}_\psi, \alpha_c) = \|\boldsymbol{y} - |\boldsymbol{A}_d\boldsymbol{W}^{-1}\boldsymbol{c}_\psi + \alpha_c\boldsymbol{r}|^2\|_2^2$, where $\boldsymbol{W}$ is the (orthonormal) discrete wavelet transform (DWT) matrix, the wavelet-based regularization of the off-axis DH reconstruction problem leads to the following minimization

$$(\widetilde{\boldsymbol{c}_\psi}, \widetilde{\alpha_c}) \in \operatorname*{argmin}_{\boldsymbol{c}_\psi, \alpha_c} D(\boldsymbol{c}_\psi, \alpha_c) + \lambda\|\boldsymbol{c}_\psi\|_1, \tag{1}$$

where $\lambda > 0$ controls the regularization strength. Despite the lack of guarantees regarding the convergence towards an optimal solution of the problem, as well as the unicity of that solution, we develop an algorithm inspired by convex optimization theory—the alternating direction method of multipliers (ADMM) [4]—to minimize the non-convex cost defining (1). Our algorithm alternates between two consecutive optimizations, one with respect to $\boldsymbol{c}_\psi$ and one with respect to $\boldsymbol{\alpha}_c$. The optimizations use projection operators derived from the one defined in [5] to minimize the data-fidelity term and one soft thresholding operator to minimize the regularization term. The projections operate not only on solution $\boldsymbol{\psi}$ but also on solution $\boldsymbol{\alpha}_c$.

3. Results

We have reconstructed synthetic and real data with the proposed method, referred to as method P, and for a comparison purpose, with direct Fourier method and method of Bourquard *et al.* [3], referred to as methods F and B, respectively. On the real data, all methods provide artifacts due to camera frame truncation. Pre-processing is thus applied before reconstruction for all methods to reduce these artifacts. Fig. 1 displays the results. Method F produces ringing artifacts resulting from the loss of high-frequency components. Methods F and B give reconstructions damaged with fringe artifacts. Method P provides improved image quality with better visibility of the patterns, on synthetic and real data. We conclude that method P outperforms the 2 other methods. Its only weakness is the production of blocky artifacts that are faintly visible.

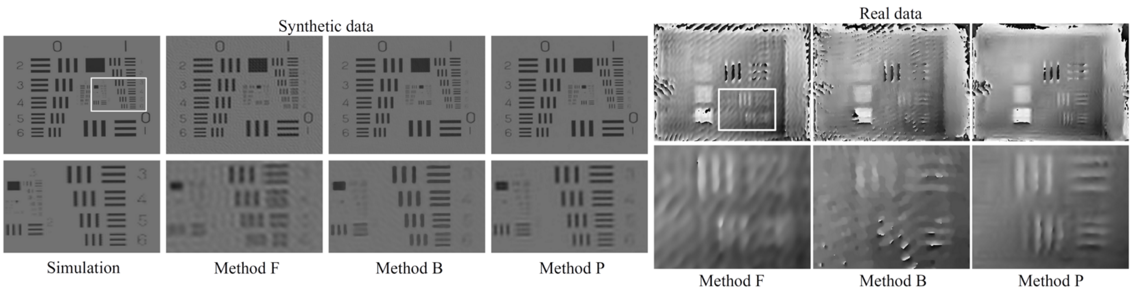

Fig. 1. Reconstruction of synthetic and real data (phase component)

4. Conclusions and future works

The proposed method jointly reconstructs the object field as well as the amplitude of the reference field. Experiments demonstrate improvements in terms of image quality such as gain in resolution and absence of fringe and ringing effects, compared to two other reconstruction methods, *i.e.*, the direct Fourier method and the method of Bourquard *et al.* [3]. For application in real situations, a pre-processing by apodization permits to reduce the artifacts due to camera frame truncation. Its limitation is to produce decrease in intensity in the border area of the reconstructions. A better solution would be to consider an unknown BC instead of a periodic BC. Some other future works will concern improvements for a better separation of solutions $\boldsymbol{\psi}$ and $\boldsymbol{\alpha}$ through a better modeling of $\boldsymbol{\alpha}$ and/or better prior information about $\boldsymbol{\alpha}$.

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