



L'Open-source rend Sage :
l'informatique au service des mathématiciens

Open-source educates future Sages:
Computers as a tool for mathematicians

- 1 Before the fifties
- 2 A shift of paradigm
 - Computer assisted proofs
 - Computer generated examples
- 3 The council of Sages

The unique feature that sets mathematics apart from other sciences, from philosophy, and indeed from all other forms of intellectual discourse, is the use of rigorous proof.

Steven G. Krantz
2007

A bit of history . . .

300 BCE Euclid structures the thought process.

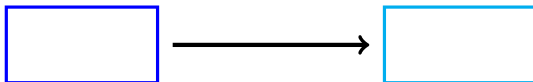


Figure: Logical derivation

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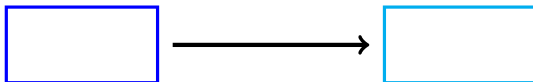
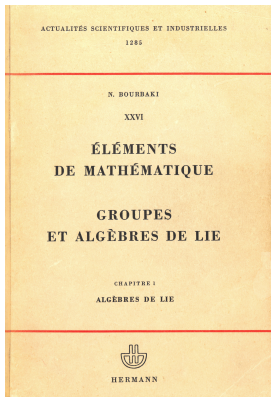


Figure: Logical derivation

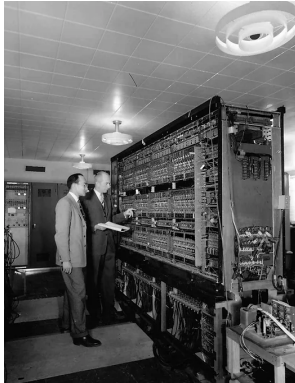
→ 19th c. Mathematics are being used throughout Europe. Establishing the soundness of this school of thought becomes necessary.

Two attempts



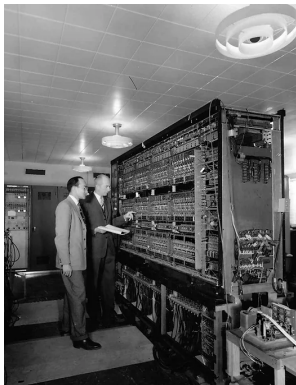
- **David Hilbert (1862–1943)** is the last mathematician to be proficient in “all” fields of mathematics. Bertrand Russel and Kurt Gödel disproved his approach.
- **Nicolas Bourbaki (1935–)** rewrites the entirety of mathematics. Their proofs omit no detail and are based on elementary constructions. They are however weakened by the absence of drawings and examples.

A META defining apparition



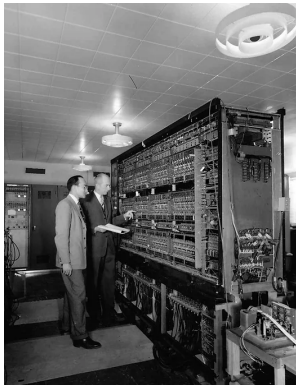
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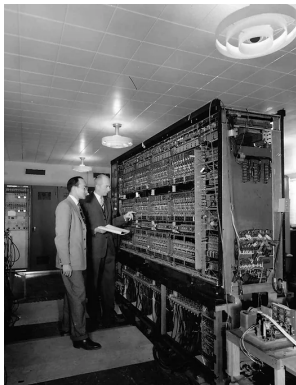
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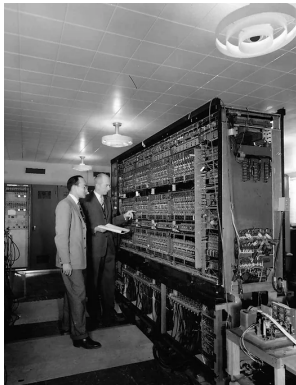
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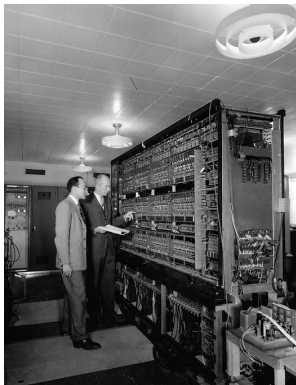
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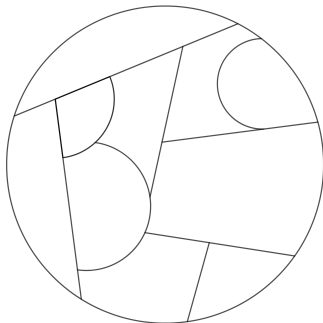
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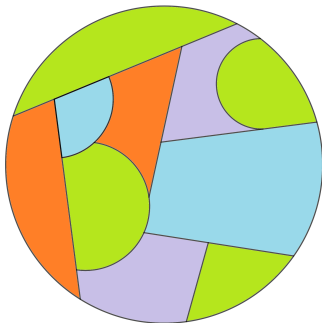
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- Proved with the use of a computer in 1976 by Kenneth Appel and Wolfgang Haken.
- In 1981, a Ulrich Schmidt noticed a mistake in their proof. It was corrected in the following two weeks.
- Forced mathematicians to face the inevitable questions about a relevant use of computers in research.

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→ This calls for open-source software that supplies mathematical tools.

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- For scientific collaboration, yes !
- For strategic reasons, maybe not ??

A specific example

My research is at the intersection between *Group Theory* and *Combinatorics on Words*. One of the results I found can be stated as follows :

Theorem (Leroy, Gheeraert, S.)

*The rank of the **return groups** of **eventually dendric word** admits an upper bound fixed by the complexity of the shift.*

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The use of open-software was necessary to the discovery of the theorem.

Introduction to SageMath

SageMath (or Sage) is an open-source software aiming to be an alternative Maple, Mathematica or MATLAB released in 2005. It covers a wide variety of mathematical subjects. The development of Sage is collaborative work by volunteers.



Assets of Sage

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2 "http://www.w3.org/TR/xhtml1/DTD/
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Thank you for your attention