Cenaero

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ETMM13

Analysis of space-time correlations for the two-dimensional periodic hill problem to support the development of wall models

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Overview

Problematic and Contextualization

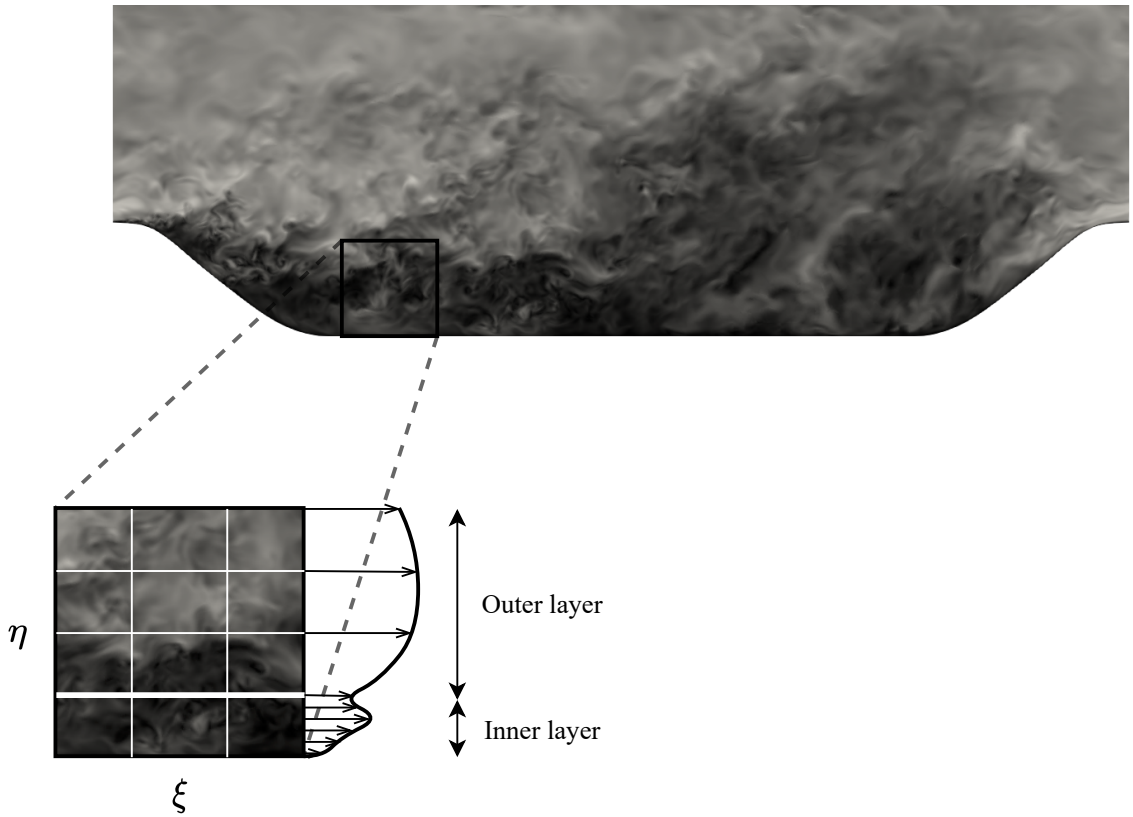
Periodic Hill Test Case

Pearson and Distance Correlations

Results, Main Conclusions and Future Work

Contextualization

In the context of Large Eddy Simulations (LES), a wall model (wm) should act ...



... as a driver for the wall shear stress boundary condition.

Contextualization

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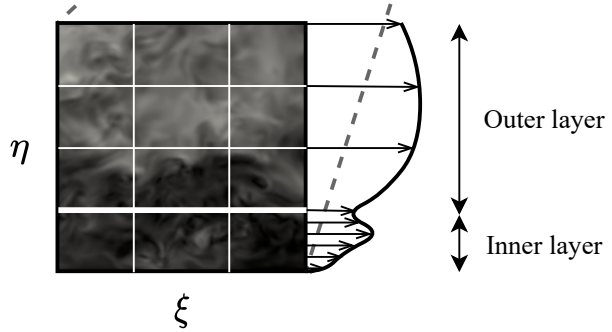
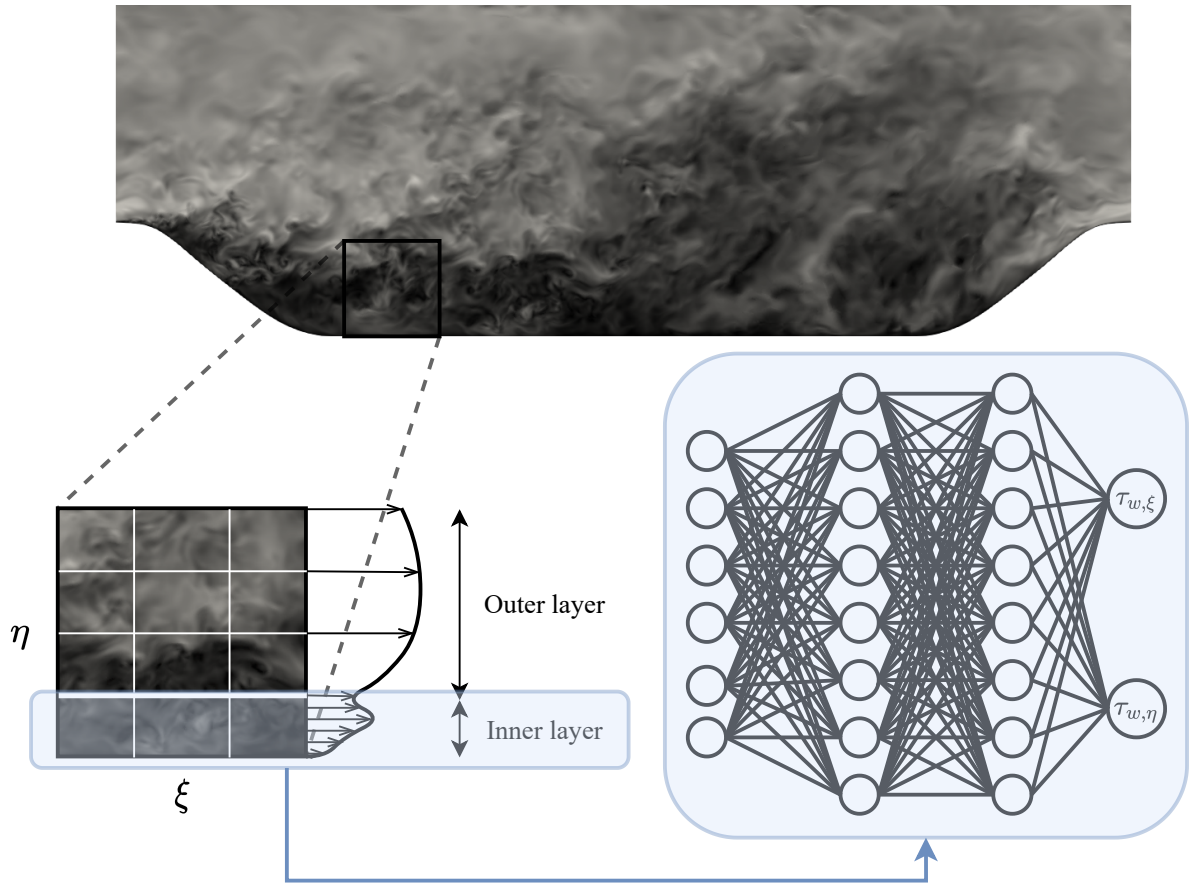


Assumptions: attached, turbulent
and at equilibrium (for many wm)
Strong pressure gradient or separation:
there is no equilibrium layer
nor constant-stress layer
Capability of current
non-equilibrium wm: not yet a proven success

... as a driver for the wall shear stress boundary condition.

Contextualization

To further address the **problematic**, we decide to use the tools provided by ...

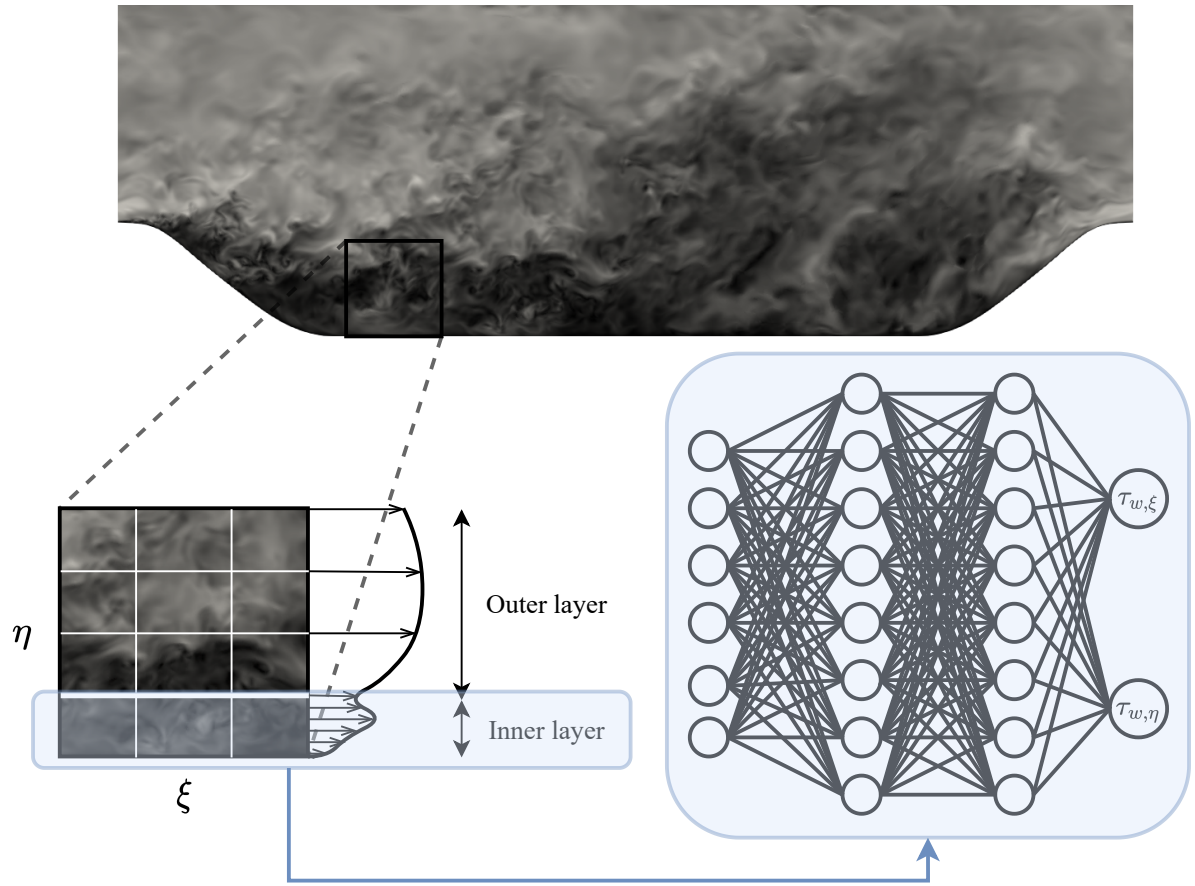


Problematic: finding a complex and dynamic relation between instantaneous flow fields, geometrical parameters and shear stress

... deep learning and **deep neural networks**.

Contextualization

To further address the **problematic**, we decide to use the tools provided by ...



New research subject:
need diagnostic tools
for pre-processing

... deep learning and **deep neural networks**.

Inspired from U. Piomelli. Wall-modeled large-eddy simulations: Present status and prospects. *Springer Netherlands*, 17, 2010.

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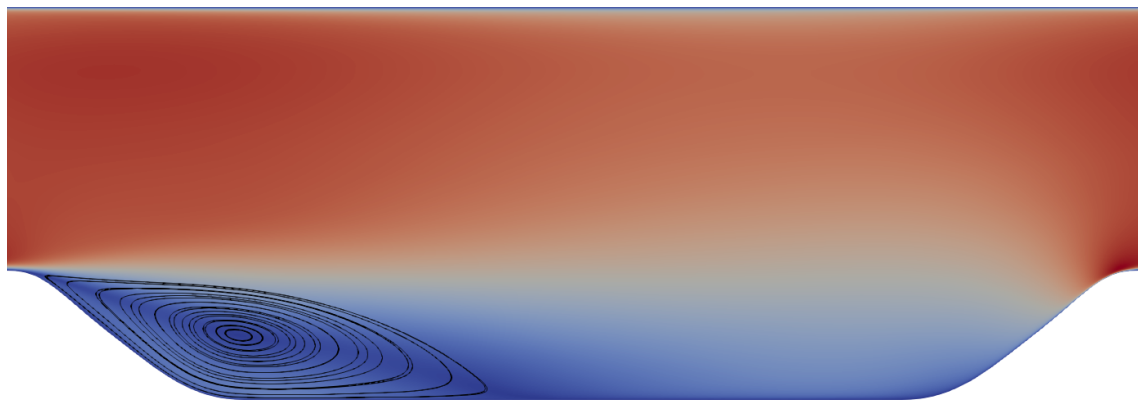
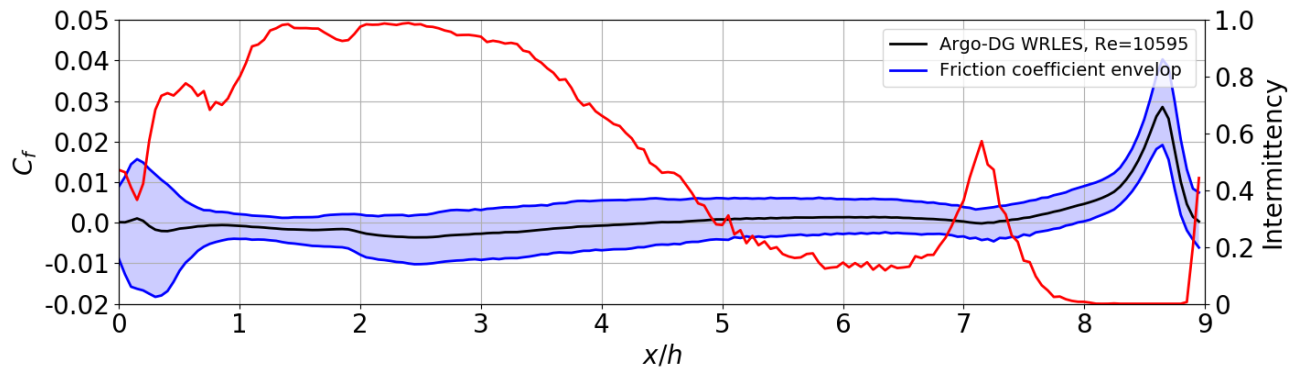
Two-dimensional Periodic Hill

- **Bi-periodic** flow evolving between two walls featuring a streamwise **constriction**[1]
- **Controlled** pressure gradient to match the bulk Reynolds number ($Re_b = 10595$) combined with a **low bulk Mach** number $M_b = 0.1$

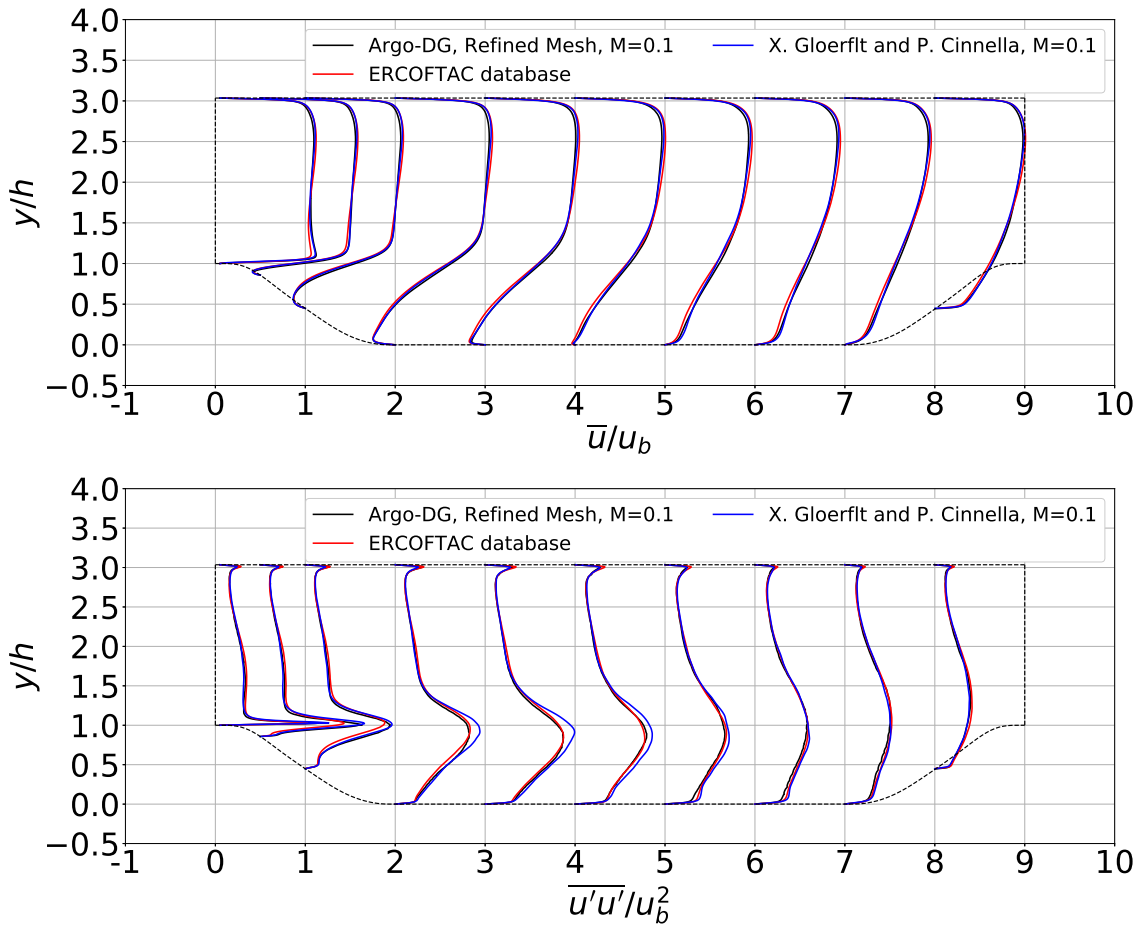




[1] https://www.kbwiki.ercoftac.org/w/index.php/Abstr:2D_Periodic_Hill_Flow

Two-dimensional Periodic Hill

Database: wrLES results obtained using Argo, a DG solver developed at Cenaero[2], are compared to literature and validated.



Order: 3
Hexahedra: 445005
Dof: 28,480,320
Nbr. of cores: 1152
Execution time/t_c: $\sim$ 8h
Averaging window: $38t_c$
CPU time: 350208h

[2]Frère, A. (2018).Towards wall-modeled Large-Eddy Simulations of high Reynolds number airfoils using a Discontinuous Galerkin method, Université catholique de Louvain.

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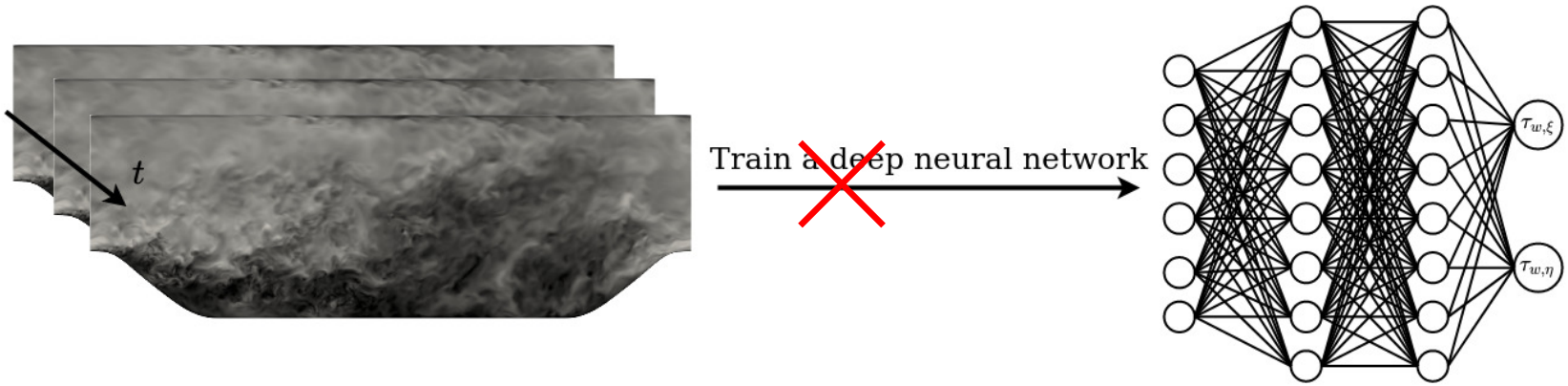
Space-time correlations

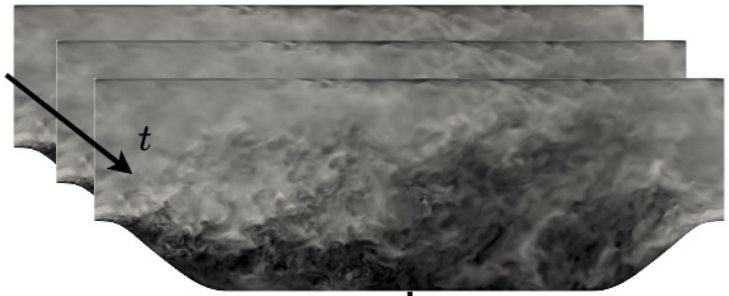


Reasons:

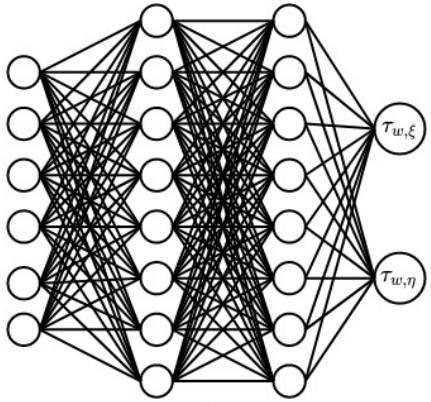
$\rightarrow$ need to select input and output labels
that have a strong relationship

$\rightarrow$ feature selection improves model performances
and reduce the computational cost of modeling

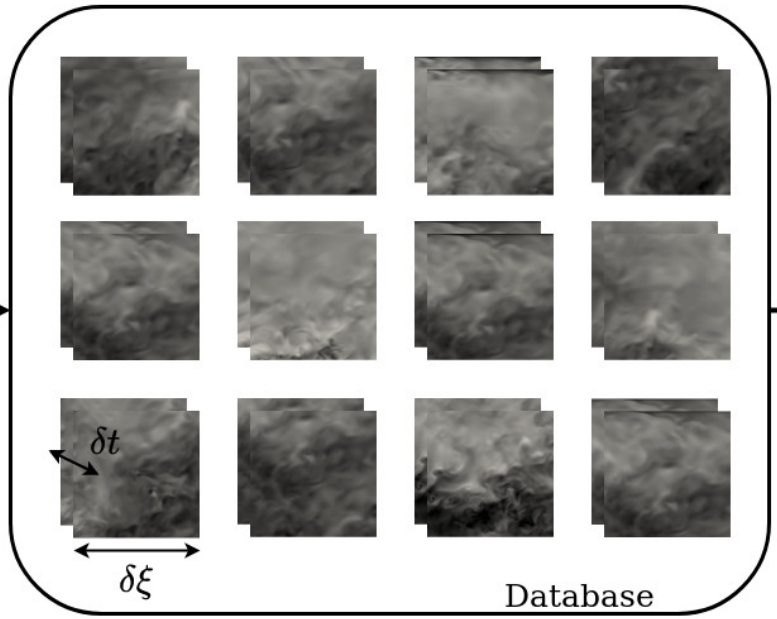
Space-time correlations



Train a deep neural network



Feature selection using correlations (Pearson and distance)



Definition:

δt = time delay
$\delta\xi$ = space shift
are two **quantities** to be deduced from correlations

Pearson's correlation (only detect **linear** relations)

$$r_{XY} := \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}}$$

where n is the number of samples and $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.

Spatio-temporal correlations - Formula

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where n is the number of samples and $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.

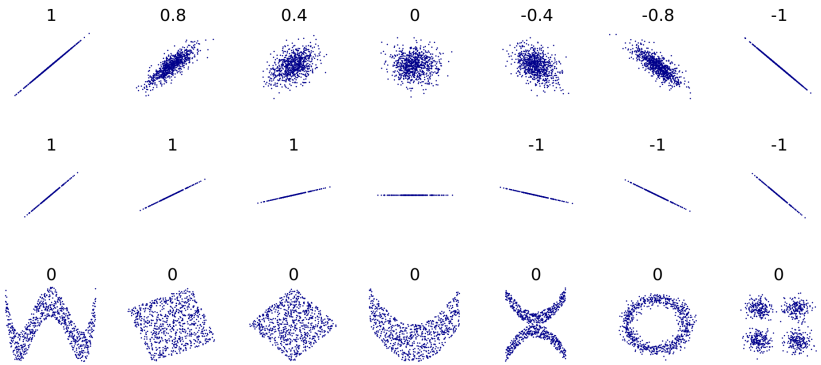
Distance correlation (measures both **linear and nonlinear** relations)

$$\mathrm{dCov}_n^2(X, Y) := \frac{1}{n^2}\sum_{j=1}^{n}\sum_{k=1}^{n} A_{j,k}B_{j,k}$$

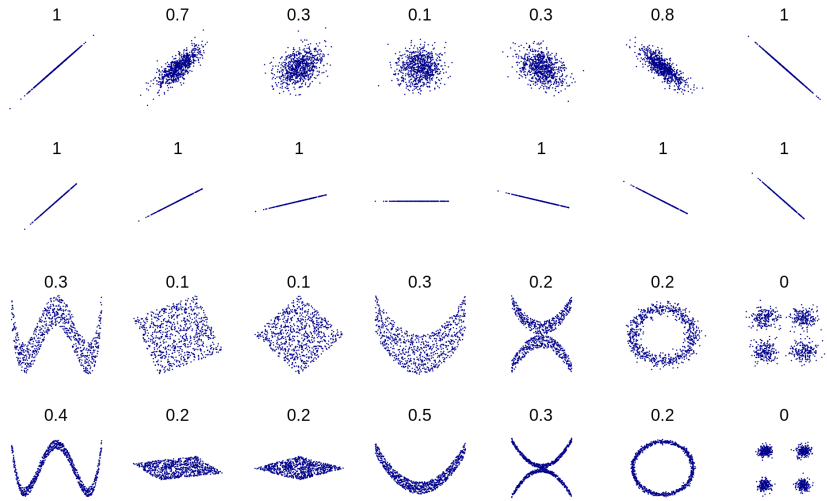
where $A_{j,k} := a_{j,k} - \overline{a}_{j.} - \overline{a}_{.k} + \overline{a}_{..}$ and $a_{j,k} = \|x_j - x_k\|$, same definition for $B_{j,k}$ but with y instead of x. $\overline{a}_{j.}$ is the j-th row mean, $\overline{a}_{.k}$ is the k-th column mean and $\overline{a}_{..}$ is the grand mean.

Spatio-temporal correlations - Formula

Pearson's correlation[3]



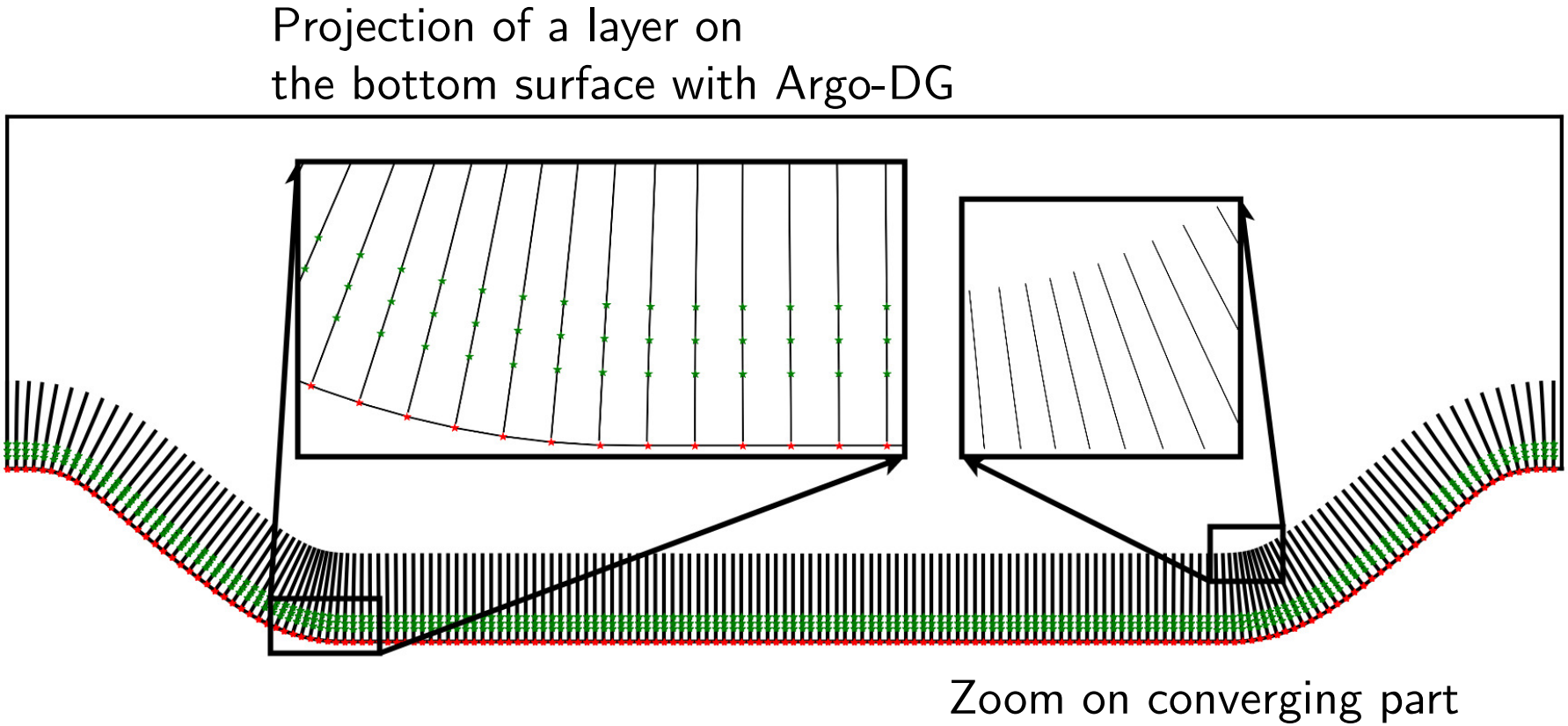
Distance correlation



[3] https://towardsdatascience.com/introducing-distance-correlation-a-superior-correlation-metric-d569dc8900c7

Space-time correlations: Hyperbolic Grid Generation

New probe grid based on hyperbolic grid generation:



... to prevent the normal lines of the probes from crossing.

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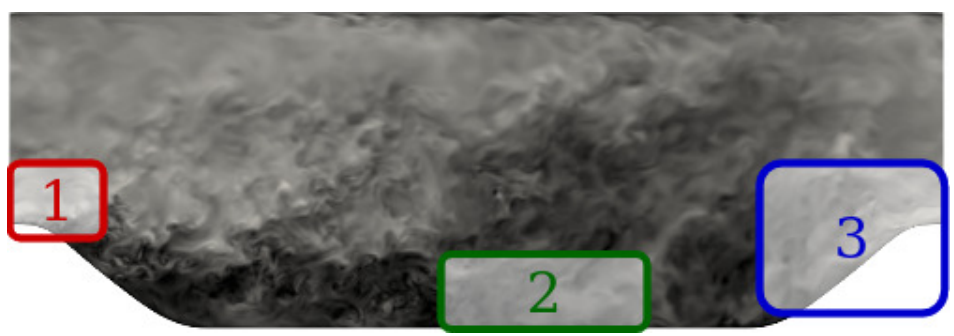
Space-time correlation - Relevant Combinations

	u_ξ	u_η	u_z	$\frac{\partial p}{\partial \xi}$	$\frac{\partial p}{\partial \eta}$	$\frac{\partial p}{\partial z}$
Separation (1) & Converging part (3)						
$\tau_{w,\xi}$	✓	✓	✗	✓	✗	✗
$\tau_{w,z}$	✗	✗	✓	✗	✗	✓
Reattachment (2)						
$\tau_{w,\xi}$	✓	✓	✗	✗	✗	✗
$\tau_{w,z}$	✗	✗	✓	✗	✗	✗

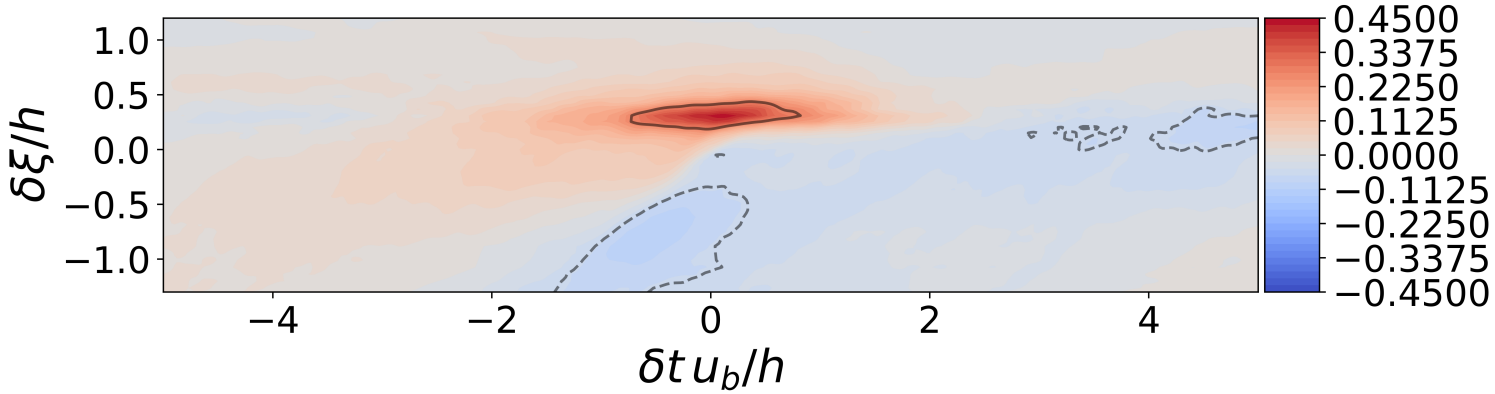


Space-time correlation - Near separation

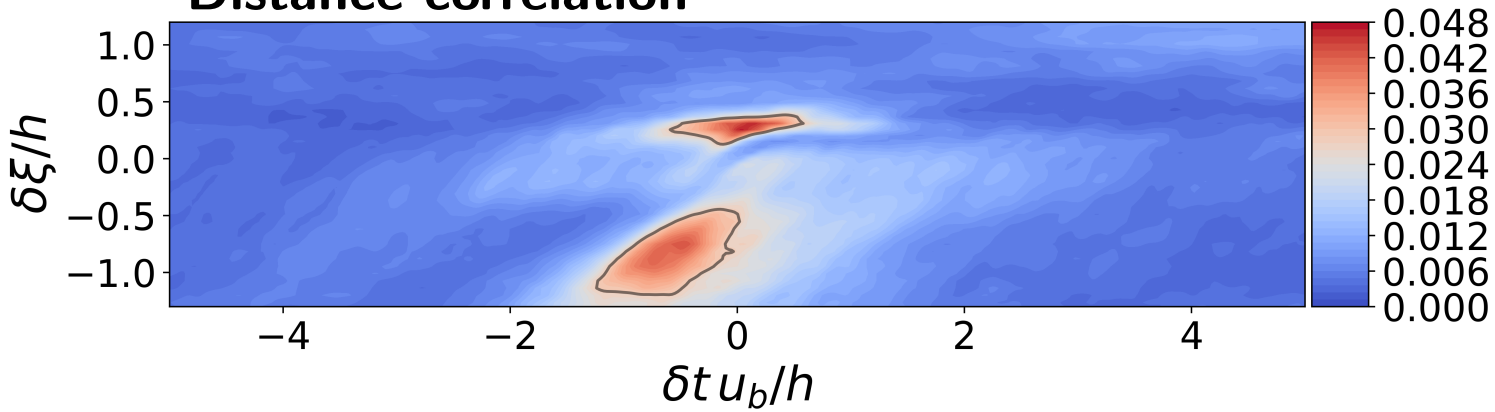
$(\xi, \eta)/h = (0.05, 0.10)$

Correlation between $\tau_{w,\xi}$ and u_ξ

Pearson's correlation



Distance correlation



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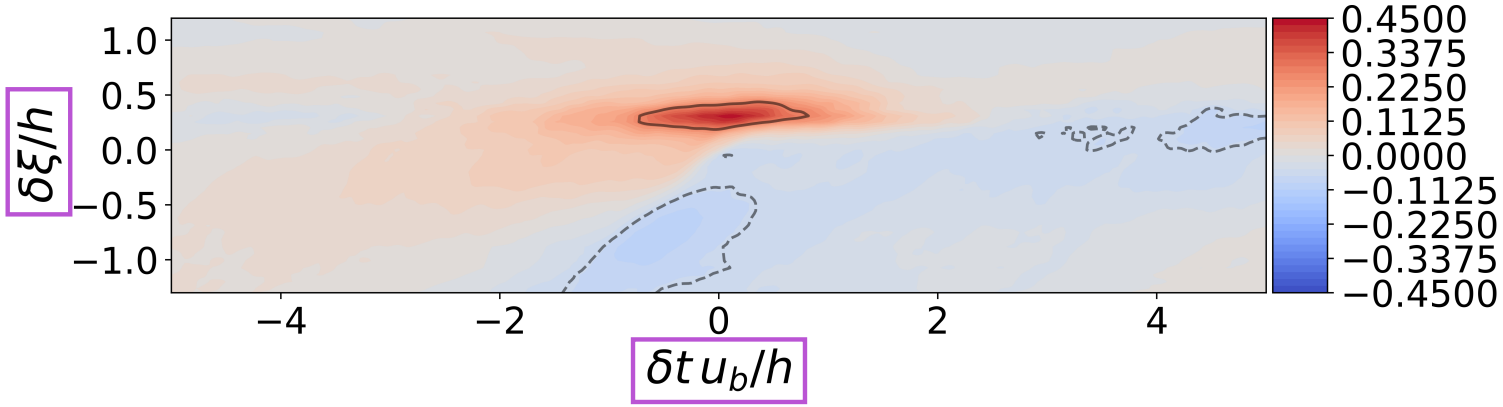

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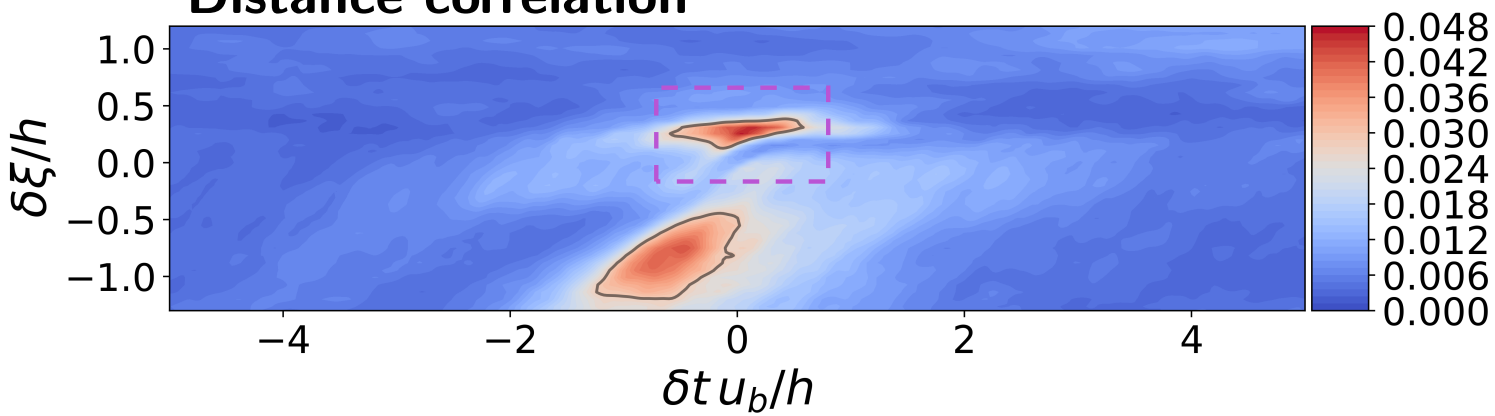
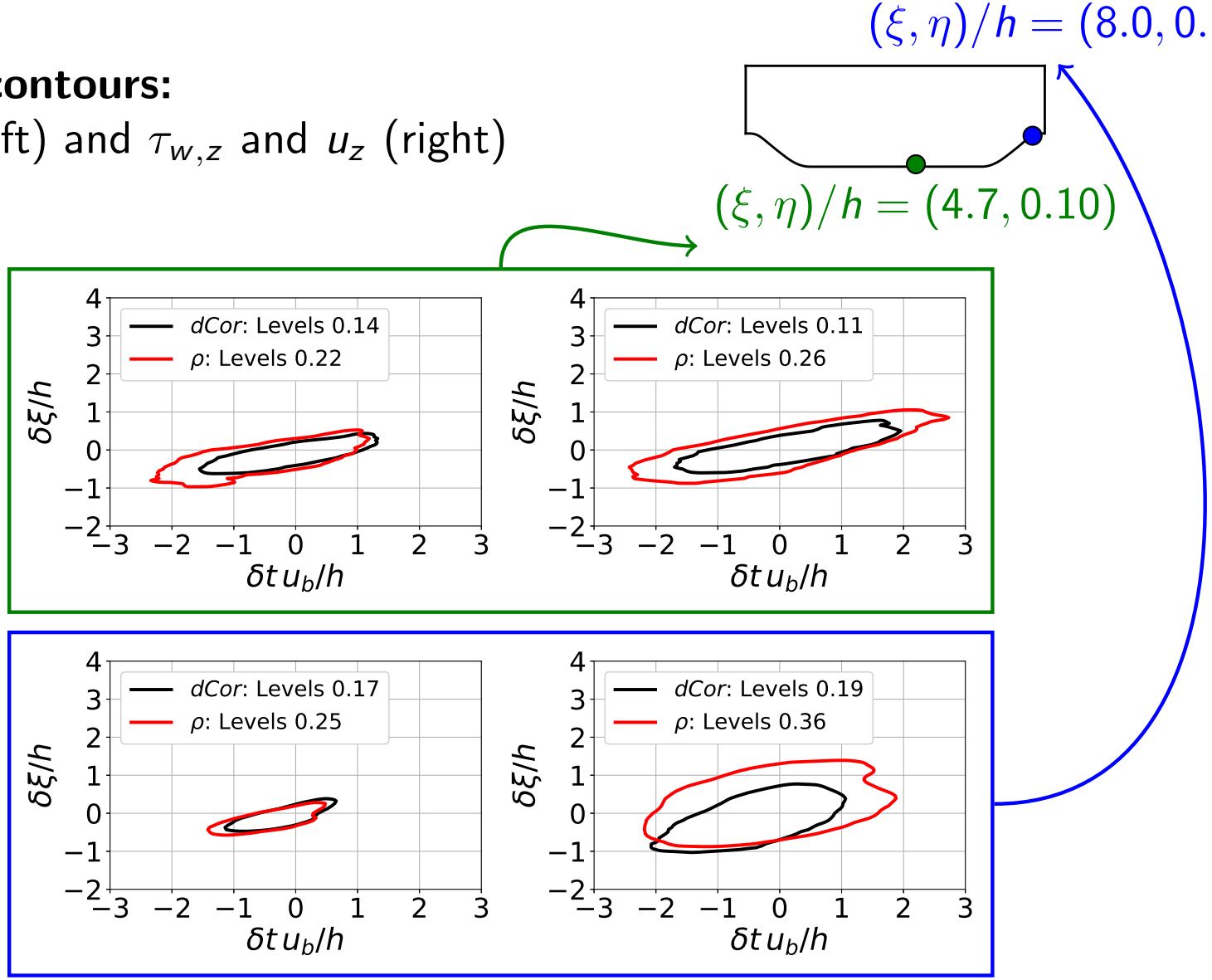


Instantaneous and local correlation is **insignificant**: **shifted** in $\delta\xi > 0$.

Correlation Contours - Reattachment & Converging Part

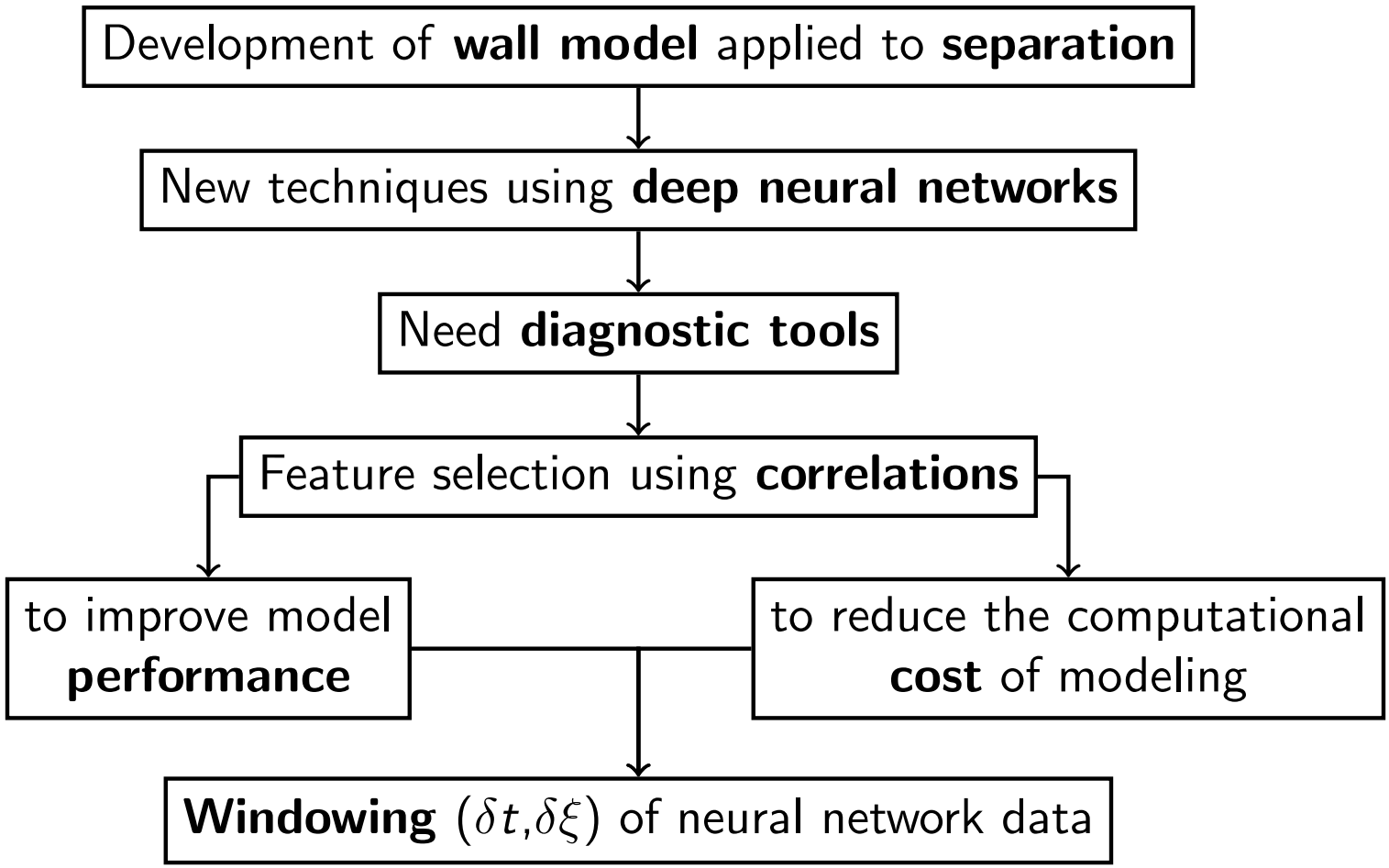
Correlations contours:

$\tau_{w,\xi}$ and u_ξ (left) and $\tau_{w,z}$ and u_z (right)

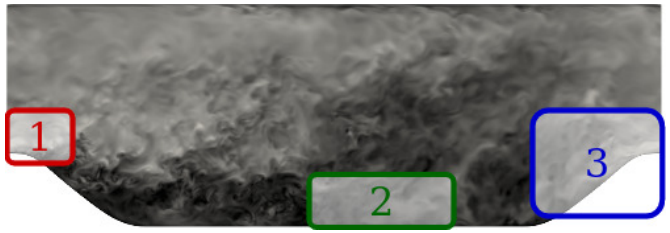


Instantaneous and **local** information is sufficient,
upstream information can be used if **convection** delay is considered

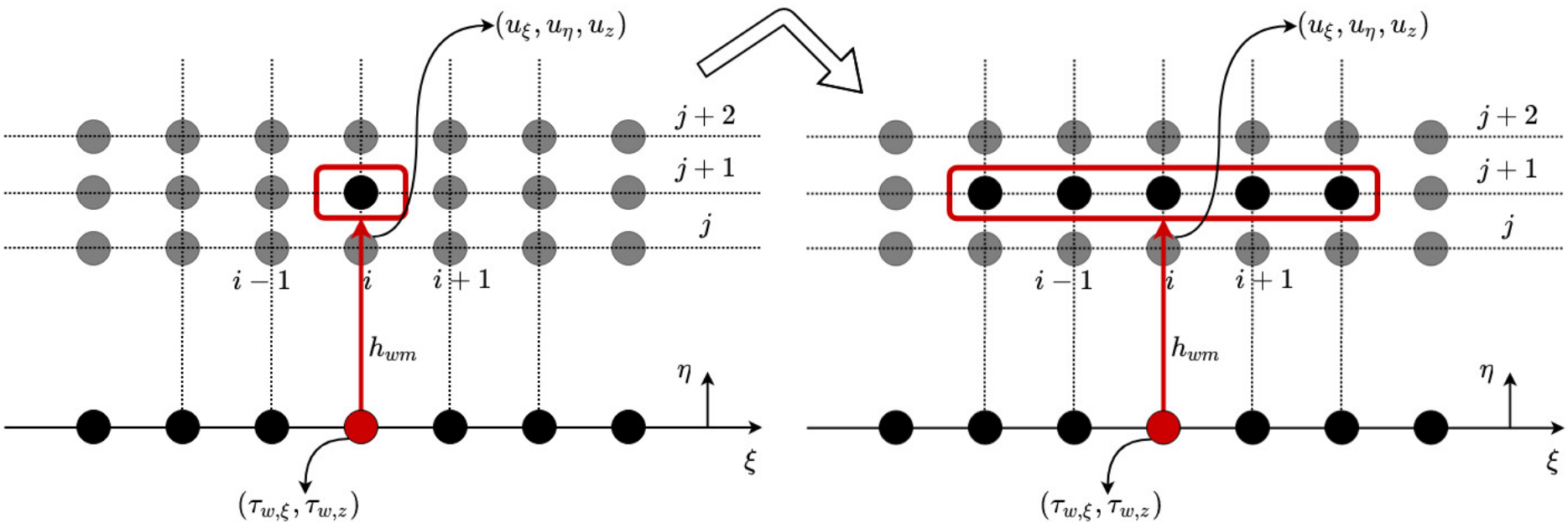
Summary


Development of **wall model** applied to **separation**
New techniques using **deep neural networks**
Need **diagnostic tools**
Feature selection using **correlations**
to improve model **performance**
to reduce the computational **cost** of modeling
Windowing $(\delta t, \delta \xi)$ of neural network data

Main Conclusions



- purely **local** and **instantaneous** information is sufficient for **attached** flows
- upstream information can be used **if a convection delay** is considered
- for **separated** flow, information has to be sought **up- and downstream**
- need to **enlarge** the domain of dependence of the wall model in both space ($\delta\xi$) and time (δt)



Future Work

- **Defining** other interesting test cases featuring separation;
- **Training** a densely connected feedforward neural network for the prediction of the wall shear stress components on data extracted from wrLES results;
- **Playing** with δt and $\delta \xi$ to observe their impact on the neural network model prediction;
- **Implementing** such a model in Argo DG;

Future Work

Architecture:	fork NN with $[400, 200, 200, 200, 100, 100, 50]$ neurons/layers
Loss function:	Mean Square Error (MSE)
Optimizer:	Adam with learning rate 0.001
Outputs:	$\tau_{w,\xi}$ and $\tau_{w,z}$

Preliminary results of a data-driven wall model trained on data extracted from the periodic hill (wrLES with Argo-DG):