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Building Modelling

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1. INTRODUCTION

Buildings are becoming more and more complex: the services installed are always more complicated, and are integrated with passive solutions, in order to reduce the energy consumption of the buildings. To lower this latter, the research is now focused on the development of advanced control systems to be introduced in the so called “Smart Buildings”. However, to implement these systems, it is necessary a deep knowledge of the building, and therefore a detailed model of this latter. The aim of this project is, then, to develop a model on Matlab/Simulink of two rooms of a school building, which will be the basis on which apply a control system aimed to increase the energy efficiency of the rooms analyzed.

1.1. Context

As discussed by Levy et al. (Lévy & Belaïd, 2018), a building model should draw on a set of variables relating not only to the properties of the building, the price of energy, climate or (in the case of residential buildings) to the household incomes and the number of household electrical appliances, but should refer also to the energy behavior of individuals or groups and to sociological, anthropological and geographical approaches, because they affect practices and lifestyles.

A building model is therefore the basis of a strategical reduction of the energy consumption, that puts its foundation on the integration of the occupant awareness, the use of more efficient building envelope and systems, the exploitation of renewable energy sources and the implementation of intelligent HVAC controls.

However, to achieve this result, a proper and accurate model of the considered building and of its systems must be defined. To do this, different methodologies can be applied. As for the building model, a detailed review of these latter is described by Atam and Helsen. (Atam, E., Helsen, 2016). Instead, for the building systems (and, particularly, the HVAC system) a critical discussion has been conducted by Afroz et al. (Afroz et al., 2018). Both of them analyse the following three main categories of approaches, considering their advantages and disadvantages:

- white-box approaches, where all the building components are modelled for heat conduction, convection and radiation on the basis of mass, energy and momentum transfer;
- black-box approaches, that are based only on data and do not require any physical knowledge of the system;
- grey-box approaches, that are the mix of the two latter.

Among them, Atam and Helsen mention as main possible applicable methods System Identification and RC Network approach and goes further by describing the possible methods suitable for large-scale models and for the management of humidity and CO₂ concentration. Afroz et al., instead, give an interesting and detailed application of a white-box method on different components of the HVAC system.

All these above-mentioned approaches are the instruments thanks to which the energy loads and consumptions can be finally modelled. As described by Fanti et al. (Fanti et al., 2018), it can be done in the environment of Matlab/Simulink, by taking into account both the total building energy consumptions and the respect of the comfort conditions. That is why the power consumed by each load, the set-points of each load, the available energy provided by the electric main grid and the renewable sources, the total power cost and the controller must be included inside the model.

The same importance must be given also to people, because they affect the characteristics of the indoor environment and interact with buildings to enhance their personal comfort. As discussed by Page et al. (Page et al., 2008), their model can be built considering people’s presence as an

inhomogeneous Markov chain interrupted by occasional periods of long absence, so to realistically reproduce key properties of occupant presence such as times of arrival and departure, periods of intermediate absence and presence as well as periods of long absence from the zone.

All these aspects here mentioned, as stated by Afroz et al., will allow improving the function of HVAC control system, and so a reduction of the building energy consumption and an improvement of both indoor air quality and thermal comfort.

1.2 Case study

The rooms studied are two classrooms (classroom 251 and 252) located at the first floor of the university, with the main axis in direction NE-SW. Their main dimensions are 7.30 x 10.70 m, with an internal free height of 3.22 m. The maximum capacity is of about 30 people.

The rooms are symmetric. Therefore, during the following discussion, the analysis will be discussed in detail only for the room 251. This latter is in contact with:

- the outside at NE and NW, through the external walls
- a similar heated room at SE, through an internal partition
- a heated corridor at SW, through an internal partition
- a heated class at the floor below
- the roof above.

The NW side is fully glazed starting from a height of 105 cm. Along this side, three openable parts are present, with a dimension of 80 x 110 cm.

The NE and the SW sides, instead, have only a glazed part in correspondence with the NW corner. In the first case, there is only a non-openable window 190cm wide starting from a height of 105 cm. In the second case, instead, there is a 190 x 204 cm glazed element wide and, next to it, there is a fully opaque door 150 cm wide and 204 cm high. Finally, the SE is fully opaque, with a door that allows the communication with the confining classroom.

2. LITERATURE REVIEW

In this section, we discuss the references and summarize their content.

2.1. The determinants of domestic energy consumption in France: Energy modes, habitat, households and life cycles

In this Section, we revise the work (Lévy & Belaïd, 2018).

Introduction

Whatever the (economic or engineering) methods employed, the models relating to energy consumption and demand draw on a set of variables relating to the properties of buildings, the price of energy, household incomes, the number of household electrical appliances, or climate (particularly, indoor temperature).

The great weakness of these models, however, is that they ignore consumer lifestyles, and therefore the energy use patterns of households. In other words, they treat the consumption patterns of users as governed purely by supply, reducing the individual to a mere energy consumer.

Attempts to include socio-demographic variables reveal results in which the age of individuals and household size emerge as decisive variables in understanding domestic consumption processes: by shifting the focus to the energy behavior of individuals or groups, sociological, anthropological and geographical approaches, the residential practices and lifestyles result to be significantly important to better understand energy consumption processes.

The essential goal of this article (which seeks to study the effect of household characteristics on the modes and intensities of French domestic energy consumption) is, then, to understand whether socio-demographic factors affect the typological profiles of domestic energy consumers, and whether the latter have a direct impact on the intensity of and trends in consumption.

The empirical analysis is divided into three stages: the first seeks to verify the correspondences between the combined use of domestic energy, the household profiles and the dwelling profiles, in order to establish consumer mode typologies. The second looks at the stabilities and variations over time of consumption in each of these types, endeavoring to distinguish between the role of household characteristics (consumption per head) and of dwellings (consumption per m²) in these intensities. The final stage seeks to identify the causes of these changes, notably by introducing a life-cycle based analysis.

The findings reveal sharp divergences between the factors affecting global consumption, consumption per m² and per person, due to the impact of the demographic characteristics of households, residential mobility and life cycles. Therefore, these findings demonstrate the flexibility over time of domestic energy consumption, which is still too often approached as a static variable solely associated with building characteristics, but that should be taken further through a longitudinal and multidisciplinary approach to energy consumption patterns.

First stage: a typology of domestic energy consumers in France

In the first step, in order to obtain a typology of consumers based on the combinations of energy types used by French households, a hierarchical cluster analysis (HCA) and multiple correspondence analysis (MCA, i.e., a generalization of factorial correspondence analysis to multivariate cases) were conducted. The distances calculated between the different households within the factorial axis space was used to classify the closest individuals and merge them, at each successive stage, using a proximity criterion called Ward's minimum variance method.

In the second phase, in order to analyse the morphological, social and demographic determinants of domestic energy consumption, a logistic regression was employed, in which the variable to be explained is low energy consumption per person and per m², and the explanatory variables are the socio-demographic characteristics of the households and of their habitat.

Finally, in the third and last stage, for the analysis of household consumption on a life-cycle basis, the age of the reference person was combined with household size. In this way, the major life cycles of households were identified, assuming linear family development (without separation, death or divorce). Four snapshots of life-cycle stages were then obtained, meaning that these stages can be observed from a transversal perspective on a given date, rather than from a longitudinal perspective.

Ultimately, it may be observed that changes in the distribution of consumer types related more to the mode of energy used than to the intensity of consumption. This then raises the question of the connection between changes in consumer types and changes in buildings and households, insofar as the former are indicative of the latter and, above all, insofar as the structure of households and of the housing stock changed little between 2002 and 2006, whereas that of consumer types changed significantly.

Second stage: the morphological, social and demographic determinants of domestic energy consumption

In the light of this, it was observed that consumption intensity per person increases with household size and diminishes with dwelling size. It might seem counterintuitive, especially if one begins with the assumption that big households occupy large dwellings, and that logically, total consumption should be distributed between family members and across housing space. It becomes less so if one considers that there is no automatic match between household size and dwelling size, and that small households can also live in large dwellings, for example when the children of a large family occupying a large dwelling leave the family home, while the parents stay.

Third stage: analysis of consumption in relation to life cycle

The role of life cycle on consumption provides a better explanation than household or dwelling size in isolation. Therefore, the major stages in the life-cycle snapshots of households in 2006 were marked out to conduct the study.

By distinguishing between consumption by working-class categories and consumption by higher class categories, the graph shows that average annual consumption per person is the same for both categories, whatever their stage in the life cycle. On the other hand, it is very clear that working-class households consume more energy per m² than higher class households, since for the same position in the life cycle, household size is greater and that they occupy smaller dwellings and because their houses are less energy-efficient than those of the higher-class categories.

Moreover, the older buildings consume more energy per m², and for the same place in the life cycle, the occupants of the older dwellings consume more per person than those in the more recent dwellings, probably because the latter are better insulated and more energy-efficient. And for equivalent social status and construction date, consumption per person is relatively stable for families, whatever the age of the reference person. However, consumption explodes when the household reference person is aged over 60, and when the household consists of just one person or a childless couple.

Finally, by distinguishing between those living in rented social housing and in private rented housing, it was shown that private tenants consume more energy per m² than social housing tenants.

2.2. Control-Oriented thermal modeling of multizone buildings: methods and issues

In this Section, we revise the work (Atam, E., Helsen, 2016).

Introduction

Residential and commercial buildings are responsible of around 40% of the energy consumption, mostly due to HVAC systems and to lighting. Considering the problems of climate change and of the limited amount of non-renewable energy sources, different strategies to reduce the energy consumption could be adopted. Among them, we can mention the implementation of intelligent HVAC controls, that can be done following many different approaches. The aim of the article is to give an overview of these latter, particularly describing their issues, the important aspects to keep in mind while modeling and giving some examples of their application.

Building modelling approaches

Three main categories of approaches aimed to modeling the thermal dynamics of buildings are:

- White-box
- Data driven
- Hybrid

White box methods are based on mass, energy and momentum transfer and all the building components are modelled for heat conduction, convection and radiation. Some examples are:

- CFD, that is not applicable to control domain;
- Zonal approach, which is still too complex;
- White multi-zone approach.

Data driven (or black-box) approaches, instead, require only data and do not need any physical knowledge of the system. Some examples are:

- time domain methods, like System identification (SI) and Machine Learning (ML);
- frequency domain methods.

Finally, hybrid approaches are a mix of the two latter and are applied when white and black box alone are not enough or are too complex. Some examples that can be mentioned are:

- Grey box methods, like Resistance Capacitance (RC) and LP models;
- Others, like response-factor models (RF).

Control-oriented building modelling methods

Control-oriented building modeling methods should be applicable to different building types and scales, work in different locations, be robust and updatable if a part of the building changes, be easily understandable and implementable, work with different HVAC equipment and have a satisfactory prediction performance.

To satisfy these constraints, different possibilities are available. Particularly, we can mention System Identification (SI) and RC Network, that are respectively black and grey-box approaches.

SI is done according to the following steps:

- define a minimum-cardinality input set, with the identification of the dominant ones and their ranking;

- define an informative identification data set, where the measured outputs are due to the application of all the inputs to the system;
- decide if apply it to the whole building or to the single zone;
- define an appropriate model structure and order;
- implement an appropriate algorithm to identify the model.

However, time domain SI methods cannot be applied in general because they identify system matrixes and not specific parameters. The same is for frequency domain methods.

RC Network, instead, starts from the analogy between heat transfer and current flow in electrical circuits. In this case:

- we define a Resistance for each type of heat exchange (advective, conductive, convective and radiative) and the capacitance C of materials;
- we build the circuit, with heat inputs and disturbances at the appropriate nodes and RC values estimated from measured data or based on materials and construction properties.

Another approach that is mentioned is the Artificial Neural Network, that accurately predict multi-zone air temperatures when nonlinearities, uncertainties, delay times, time-varying aspects and a not uniform zone temperatures occur, but resulting in nonconvex optimization problems.

Control models for large-scale buildings

According to the application, there are three possible methods:

- Composable-zones-based
- Graph-theory-based
- Model-Order reduction

The composable-zones-based approach has a good single zone prediction, that however may result in a poor global prediction due to error propagation among zones. To apply this method, the following steps must be put in place:

- modular decomposition of the thermal dynamics of a single zone, with three main blocks (zone envelope, zone air, zone ventilation and airflow)
- construction of subsystem matrixes for each subcomponent
- couple the dynamics of the subcomponents to obtain a single-zone model
- couple single-zone models to obtain a multiple-zone model

To extend this method, the graph-theory-based approach has been developed: it divides large-scale multizone systems into groups of zones for which identification is possible, simpler and less time-consuming in order to guarantee that the identifiability of the groups of zones determines the identifiability of the whole building. This method, therefore, increases the computational speed, is more systematic and guarantees the identifiability of the subsystems and of the overall system. In addition, weakly interacting zones can be separately considered from the interacting ones that, instead, are grouped together. This facilitates the parameter identification problem.

Finally, if the models obtained are still large-scale (with, therefore, controllability, observability issues), the Model-Order reduction can be applied: it allows reducing the number of states and the order of the model, lowering also the computational time. However, these improvements have as a cost the loss of physical meaning of the state variables. This latter can be preserved thanks to the PROM (Parameterized Reduced-Order Modelling) technique, that allows also updating automatically

the low-order when a parameter changes in the large-scale one, automatically updating the effects of the parameters and representing a variety of building types. However, it makes increasing again the number of parameters and so the computational demand. That is why it is better to use it on single-zone models.

The importance of humidity and CO₂

Humidity and CO₂ are parameters to be considered, but that are very difficult to manage. A method that can be applied is the Model Predictive Control for multizone buildings, that allows the control of thermal conditions in multizone buildings with many advantages: if convex, it is a feedback control, it can be applied to MIMO systems and gives globally optimum results.

Provided a good prediction performance, the least sensitive should be chosen, because the sensitivity is strongly linked to the robustness of the closed-loop system.

2.3. Modeling techniques used in building HVAC control systems: A review

In this Section, we revise the work (Afroz et al., 2018).

The choice of the proper modelling technique is essential to conduct an appropriate HVAC analysis and to improve its control system. That is why this paper is critically reviews the current modelling techniques used to improve HVAC systems in terms of energy consumption, thermal comfort and indoor air quality, with the aim to summarize their strengths, weaknesses, applications, and performance, focusing on their applicability and ease of acceptance in practice and on the need for further research efforts.

Modelling processes in HVAC control system

In a modelling process (that leads to dynamic, nonlinear, and very high-order models), the most challenging and important part of the model development for a particular application is the process of identifying the model order and the optimum parameters (SI).

Most of the existing studies just applied trial and error method to determine the model structure and order. However, different techniques are available:

- the physics-based (or white-box/mathematical/forward)
- the data-driven (or black-box/empirical/inverse)
- the grey-box (or hybrid)

The physics-based approach generally leads to continuous and deterministic models. It is based on fundamental laws of energy (mass balance, heat transfer, momentum, and flow balance), from where a set of mathematical equations can be derived and solved. Distributed or lumped parameter methods can be applied. Particularly, based on ease of use, the lumped parameter method has shown superior performance over distributed parameter type.

Physics-based HVAC system models are suitable for the prediction and the analysis of the performance of HVAC system components through simulation. The dynamic ones are commonly developed for the slow-varying temperature and humidity processes (e.g., zone temperature dynamics, zone humidity dynamics, heating/cooling coil dynamics, etc.), while static models are implemented for the fast-moving dynamics of the system (e.g., mixed air temperature and carbon dioxide concentration in mixing box, and flow rate of air and water through damper and valve respectively) and energy consumption (fan or pump energy consumption).

Data-driven methods result in discrete and deterministic or stochastic models. They collect the system performance data from real practice and then establish a relationship between the input and output variables.

Finally, grey-box methods are a combination of white box and black box models. Here, the basic structure of the model is formed from physics-based methods while the model parameters are determined by using the parameter estimation algorithms on the measured data of the system. This approach is especially beneficial for control applications when the model is expressed in a suitable form, such as transfer function or state space.

Physics-based modelling technique

In the physics-based model, the following main components are analysed in detail: chiller, zone, heating and cooling coils.

The chiller rejects heat from a liquid through a vapor compression cycle or an absorption cooling cycle. It is composed by four elementary elements: evaporator, condenser, compressor, and expansion valve. It can be physically described using the evaporator load and the energy balance equations, and the load and energy balance equations. However, the model is not fully capable of predicting the modulating nature of the chiller.

Modelling zones is also a challenging task, especially when the number of zones becomes multiple. They can be described by the following equations:

- energy and mass balance governing equations of the zone:
- the rate of change of energy in the zone, equal to the difference between the energy transferred to the zone by either conduction or convection and the energy removed from the zone:
- the rate of change of energy in the walls, equal to the energy transferred to the walls due to the temperature difference between indoor and outdoor air.

Finally, the heating and cooling coils act as heat exchangers where air loses or gains heat from water passing through the coil. Their model is based on the energy balance equations and the mass balance equations. :

In addition to these components, also the humidifier, the cooling model and the mixing box model can be added.

Data-driven modelling approach

Inside the domain of the data-driven modelling approaches, multiple methods can be discussed:

- frequency domain models, where slow moving due to the substantial thermal inertia of the system can be modelled using the first and second order (over-damped) models with dead time.
- data mining algorithms, like Artificial Neural Network (ANN) and Support Vector Machine (SVM), that are capable of dealing complex and non-linear system dynamics. In particular, a significant reduction in energy consumption can be achieved by applying ANN model in building HVAC systems.
- fuzzy logic models, based on if-then-else statements, whose rules are expressed in the form of a table or database.
- statistical models, that describe how a sample of data can be generated from a massive dataset by following a particular trend. Some examples are single and multivariate regression, output error (OE), Box-Jenkins (BJ), autoregressive integrated moving average (ARIMA),

autoregressive exogenous (ARX), autoregressive moving average exogenous (ARMAX) and finite impulse response (FIR).

- State Space models, used for system identification, but for which very few literatures can be found in the domain of HVAC systems.
- geometric models, which use mathematical methods to model real objects thanks to computer graphics and computer-aided design (CAD). They represent a system through the use of two-dimensional (2D) or three dimensional (3D) geometric shapes such as curves, surfaces, and volumes.
- case-based reasoning, that is not very popular since the model suffers from the problems related to the unseen cases.
- stochastic models, which can deal with the random processes of the HVAC systems that act as random variables. Although a large amount of data is a prerequisite, they are characterized by a strong versatility, with the possibility to apply it to many physical processes and to approximate them to standard normal and uniform distributions.
- instantaneous models, that comprise a statistical and a pattern model to find the patterns in previous data similar to the current data. They allow food thermal load estimation as the weather condition are stable; however, on a unique weather day, thermal load estimation is not good.

2.4. A simulation and control model for building energy management

In this Section, we revise the work (Fanti et al., 2018).

Introduction

One of the main problems to be faced in the near future is the increase of the power demand. Indeed, in the next decade, power demand is estimated to rise by 19% and the existing infrastructures can increase their productivity by only 6%. In this context, this paper proposes a detailed model devoted to simulating the building appliance energy consumptions and to controlling the loads usage by a strategy that takes into account both the total building energy consumptions and the respect of the comfort conditions.

This demand reduction strategy is implemented by a Building Energy Management System (BEMS) and aims at monitoring the real-time building energy consumptions in order to avoid the overcoming of power provided by the electric grid and to respect the user comfort by an intelligent power reduction based on a priority list of the electric loads. Therefore, the control procedure ensures the aggregated power consumption does not exceed the available power profile and avoid curtailment, by managing the appliances according to the comfort preferences.

In this paper, the controller above described is integrated in a Matlab/Simulink tool, where appliances and renewable energy sources are modelled in a way that the simulation can not only be used as a framework to test the controller, but can also be suitable to sensitize the building occupants. Finally, the simulation and control model is validated by experimental data measured in a large size dwelling equipped with domestic appliances and renewable energy sources, where smart meters and WiFi smart plugs are used to collect energy consumption data and a PLC is used to apply the control strategy.

Simulation framework for appliances modelling and control

The Simulink model of the simulation and control systems includes 6 modules: the domestic loads (for which the state of functioning on/off is provided), the power consumed by each load, the set-points of each load, the available energy provided by the electric main grid and by renewable sources, the total power cost and the controller.

Going into details, the appliances and the renewable energy sources constitute the building micro-grid. As for the renewable sources, wind and solar renewable sources are modelled and integrated inside the system. Regarding the domestic loads, instead, they are characterised by both controllable and non-controllable appliances. In the first case, it is possible to switch off or partialize them using a controller. Some examples are HVAC system (heating and cooling mode), water heater, dishwasher, washing machine and dimmable lamps. On the other hand, the non-controllable appliances are passive loads that cannot be switched off or partialized, like ovens, TVs, PCs, irons, refrigerators and freezers.

Finally, the control unit of the BEMS is specified and modelled in a Petri Network (PN) framework and manages the appliances by respecting the user preferences.

HVAC system and water heater models

The HVAC system is modelled both in heating and cooling functioning mode. The paper provides details on the HVAC dynamics equation based on the physics considerations.

The paper also provides equations to model the heat dynamics.

Washing machine and dishwasher models

Washing machines and dishwashers are heterogeneous systems composed of mechanical parts, a hydraulic system, a thermal system and an electronic control system. To model such appliances, an asynchronous monophasic motor is considered in order to allow the rotation of the machine. In addition, a thermal system heats up the water according to the washing program selected by the user. The water heaters of washing machines and dishwashers to manage high temperatures are modelled as follows:

Dimmable lamp model

The dimmable lamp is controlled by a dimmer, i.e., an electronic regulator that is able to control the power absorbed by the lamp and vary its lighting intensity.

Non-controllable loads

With regards to the non-controllable loads, the paper provides equations to model the oven, the iron, the refrigerator, and the freezer.

The power consumption is assumed to be constant when the considered appliance is on.

Renewable energies

The wind and photovoltaic renewable energy sources are modelled considering their production forecasts for a 24 h time period. The forecasted power production and time data vectors are saved in a lookup table in the Simulink model for both renewable energy sources.

2.5. A generalized stochastic model for the simulation of occupant presence

In this Section, we revise the work (Page et al., 2008).

Each human being emits heat and “pollutants” (such as water vapor, carbon dioxide, etc.) and therefore directly modify the indoor environment. Occupants also interact with a building to enhance their personal comfort. It is for these reasons that their presence should be considered inside a building modelling procedure.

This paper describes an algorithm for the simulation of occupant presence by considering occupant presence as an inhomogeneous Markov chain interrupted by occasional periods of long absence. The resulting model has proven its capacity to realistically reproduce key properties of occupant presence such as times of arrival and departure, periods of intermediate absence and presence as well as periods of long absence from the zone.

Methodology

The model of occupancy is destined to deliver the metabolic heat gains and pollutants released by the occupants within the zone and to serve as an input for the use of windows, lighting appliances and other electrical and water appliances.

It is based on the hypothesis that the presence of occupants sharing the same zone can then be simulated by multiplying the obtained pattern by the total number of occupants (this case of collective behavior would correspond to the occupancy of a meeting-room), or by simulating each occupant separately and then adding the produced patterns of presence.

In addition, the probability of presence at a time step is assumed to be dependent on the state of presence at the previous time step.

Development

The desired model should be capable of generating a time series of zeros (absence) and ones (presence), that should not simply reproduce the pattern given as an input (the profile of probability of presence and the parameter of mobility), but create a pattern that never repeats itself while reproducing the statistics of the real world it is simulating.

The model was then based on the “inverse function method” (IFM) that can generate a sample (in our case a time series) of events from a given probability distribution function (PDF).

However, simple models were only capable of producing a Gaussian distribution around the average of the empirical data. This showed that, although the Markov chain model works well at reproducing periods of short absence and presence for one day, it needs to be complemented in order for the model to generate long periods of absence. These have been included by adding to the algorithm the possibility to start, at random, a period of long absence at each time step.

Algorithm

The model was implemented as a MATLAB script according to the following steps:

- given the probability of starting a period of long absence (derived from the number of long absences happening in a year, entered as an input), check whether the occupant starts a period of long absence or not by using the IFM;
- if so, determine the length of that absence given the distribution of the duration of periods of long absences (entered as an input) with the same method, during which period the occupant is considered to be absent;
- at his return, or if he did not start a long period of absence, we find ourselves in the case of the Markov chain of “usual daily” changes in state of occupancy;
- determine the next state of presence by the use of the IFM.

2.6 Conclusion

Considering what discussed in the papers previously analysed, it has been decided to build an RC model for the room considered in our project: in fact, knowing the detailed geometry of the room, the materials used for all its components and the physics governing the problem, it is possible to easily derive the values of all resistances and capacitances of the equivalent circuit.

In addition, it has been established to include a stochastic representation of people's presence inside the model: following what described by Page et al., it will allow realistically introducing the influence of people on our room parameters.

Finally, as for the HVAC system, it has been stated to start from the model described by Fanti et al. and arrange it according to our needs (avoiding, for example, the cost analysis).

3. RC Modelling

3.1. Brief description

There is an analogy between heat transfer in building components and current flow in electrical circuits. This analogy helps to derive the equations for thermal dynamics of building environments easily using circuit theory.

This is the reason why it has been adopted the RC modelling, which is a subcategory of grey-box modelling.

3.2. Advantages and disadvantages

The following main advantages of RC modelling have been identified:

- the identified parameters have some physical meaning, and hence their values can give some idea about the correctness of the identified model;
- since grey-box models are between white-box and black-box models, the time and effort required to obtain such models also, in general, are situated between white-box and black-box models;
- model validation through extra experiments may not be needed if the obtained RC values are reasonable.

The main disadvantages of RC modelling, instead, are:

- the determination of an appropriate model structure reflecting the dynamics of a subcomponent, which may not be easy;
- although there are some guidelines to assign the number of Rs and Cs for building components, for some complicated building structures, constructing an RC thermal network may be hard.

3.3. Methodology

Considering the relationship between building components and electrical circuits, an RC model can be interpreted according to the following analogies:

Electrical circuit	Thermal circuit
Voltage	Temperature
Current	Heat Flow
Electrical Conductance	Thermal Conductance
Electrical Resistance	Thermal Resistance
Electrical Capacitance	Thermal Capacitance
Current Source	Heat Source

where capacitance represents thermal capacity (i.e., the property of objects to describe its capability to store heat), while resistance represents thermal resistance (i.e., the property of objects or materials to resist the heat flow through it).

In addition, to understand an RC model, the following aspect must be kept in mind:

- the order of such kind of model is determined by the number of lumped capacitances (nodes);
- the physical interpretation of the parameters is dependent on how the building is divided into entities in the model.

Finally, the values of R and C are estimated based on samples of inputs and outputs by applying an identification algorithm, e.g., nonlinear regression algorithm, which typically minimizes a norm of either simulation errors or prediction errors. The boundaries on the parameters in the identification process are normally estimated from a rough description of the building geometry and materials.

The way in which the RC model is integrated in our building modelling project is represented by the scheme below.

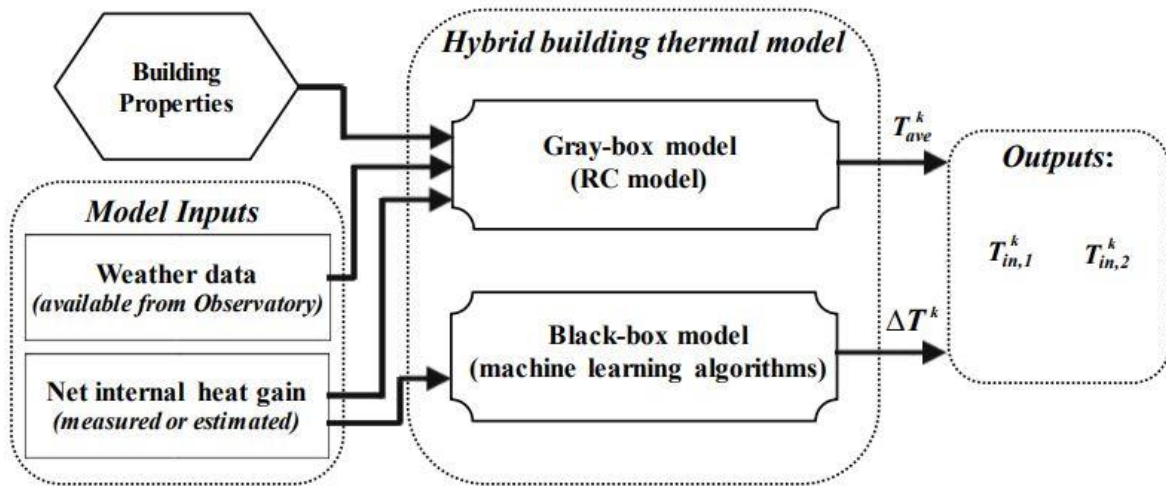


Figure 1 Integration of the RC model into a building modelling project

3.4. Equations

Using RC network, our building (MVP) can be modelled as follow:

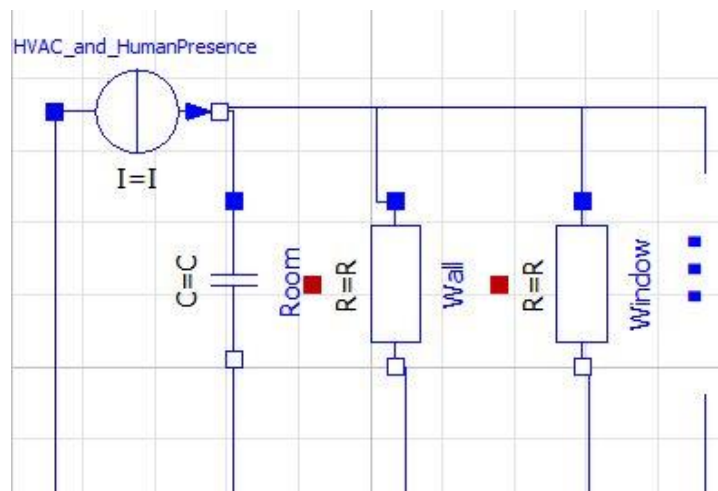


Figure 2 Model of an RC network of a building

Using the electrical-thermal analogy described above, we can determine that our current source (Heat flow source) will be the sum of the current (Heat flow) generated by the HVAC and the current (Heat flow) generated by the human presence.

Part of this current will be stored in the room, thus the presence of the capacitor, and part of it will be lost through the walls, windows, floor, ceiling, and doors. This loss is determined by the thermal resistance of each material of the building (e.g., concrete, wood), thus the presence of the resistances.

We can finally deduce the following equations:

$$i_{HVAC} + i_{Human} = C \cdot \frac{dV_{room}}{dt} + \sum_i I_i$$

$$I_i = (T_{room} - T_i)/R_i$$

$$V_{room} = \Delta V_{wall} = \Delta V_{window} = \Delta V_{ceiling} = \Delta V_{floor} = \Delta V_{door}$$

4. Building Analysis

In order to define the resistance values to introduce into the model, a preliminary analysis of the stratigraphy of the opaque and glazed elements has been performed.

4.1. Stratigraphy

Opaque elements

As for the opaque elements, we can distinguish the vertical (external walls and internal partitions) and horizontal ones (floor and roof).

As for the external walls, starting from the outside we have:

- white concrete panels on inox substructure (3cm)
- air gap (4 cm)
- insulation layer in mineral wool (20 cm)
- concrete structure (20 cm)
- systems cavity (5 cm) (filled with PUR above windows)
- cardboard (2.5cm)
- finishing with painting (1.5 cm)

Regarding the internal walls, instead, we have one in contact with the corridor:

- painting (1.5 cm)
- concrete structure (20 cm)
- system's gap (5 cm)
- cardboard (2.5 cm)
- finishing with painting (0.5 cm)

and one with the room:

- finishing with painting (0.5 cm)
- cardboard panels (2.5 cm)
- steel structure with acoustic insulation (4.8 cm)
- cardboard panels (2.5 cm)
- finishing with painting (0.5 cm)

Moving to the horizontal elements, the floor stratigraphy is the following:

- linoleum finishing (dark coloured)
- concrete structure (alveolar slab + compression screed, total 31 cm)
- inspectable plenum (30 cm)
- counter ceiling panels in mineral fibres (2 cm)

Finally, as for the roof:

- aggregates finishing (4 cm)
- elastomeric waterproof membrane (0.3 mm)
- insulation in PUR (20 cm)
- vapour barrier (0.2 mm)
- concrete structure (alveolar slab + compression screed, total 31 cm)
- inspectable plenum (33 cm)
- counter ceiling panels in mineral fibres (2 cm)

Windows and doors

As for the glazed elements, all windows are characterised by a double glazing with aluminium frame. They are equipped with electrical sunshade screens.

The doors, instead, are fully opaque and are characterised by an HPL finishing on the room side and an MDF finishing on the other. Their thickness is respectively of 4mm and 8mm. The core of the door is filled with EPS insulation 2.5 cm thick.

Equipment

As for lighting, the counter ceiling is equipped with 24 light points. Instead, as for heating, two electrical radiators are positioned on the NW side.

4.2. Resistance values

Knowing the materials composing each building element, the unitary resistance values have been calculated for each stratigraphy. The detailed calculations are reported in the appendix (A1). Then, the dispersing surfaces have been calculated in order to find the final Resistance values of our rooms. The procedures performed to compute the dispersing surfaces and to find the final Resistance values for the two rooms are detailed in the Appendix (A2 and A3).

4.3. Capacitor value

The thermal capacity of each of the two rooms (which have equal dimensions) is given by the following formula:

$$C = m \cdot c_m$$

where:

- m is the mass of air contained in the room [kg]
- c_m is the specific heat [J/(kg·K)]

We assume to have a temperature of 19°C (292,15K) and a relative humidity ψ of 60% at a pressure of 1atm. To calculate the thermal capacity, we firstly calculate the density of the humid air as follows:

$$\rho = \frac{p_a}{R_a T} + \frac{p_v}{R_v T}$$

where:

- p_a : partial pressure of dry air [Pa]
- R_a : specific constant of dry air = 287,058 [J/(kg·K)]
- T : absolute temperature [K] = 292,15K
- p_v : pressure of water vapour [Pa]
- R_v : specific constant of water vapour = 461,495 [J/(kg·K)]

The partial pressures p_a and p_v are given by the following equations:

$$p_v = \varphi \cdot p_{sat} = \varphi \cdot 610.78 \cdot 10^{\frac{7.5 T - 2048.625}{T - 35.85}} = 0.6 \cdot 610.78 \cdot 10^{\frac{7.5 \cdot 292.15 - 2048.625}{292.15 - 35.85}} = 1318 \text{ Pa}$$

$$p_a = p - p_v = 101325 \text{ Pa} - 1318.3 \text{ Pa} = 100007 \text{ Pa}$$

where:

- ψ is the relative humidity

- p_{sat} is the pressure of saturation
- p is the ambient pressure

So, the density of the air is:

$$\rho = \frac{p_a}{R_a T} + \frac{p_v}{R_v T} = \frac{100007 Pa}{287,058 \frac{J}{kg \cdot K} \cdot 292.15 K} + \frac{1318 Pa}{461,495 \frac{J}{kg \cdot K} \cdot 292.15 K} = 1.202 \frac{kg}{m^3}$$

Secondly, we calculate the specific heat c_m as follows:

$$\begin{aligned} c_m &= c_{pa} + x \cdot c_{pv} = c_{pa} + \left(0.622 \cdot \frac{p_v}{p - p_v}\right) \cdot c_{pv} = \\ &= 1.005 \frac{kJ}{kg \cdot K} + \left(0.622 \cdot \frac{1318 Pa}{101325 Pa - 1318 Pa}\right) \cdot 1.87 \frac{kJ}{kg \cdot K} = 1.020 \frac{kJ}{kg \cdot K} \end{aligned}$$

where:

- c_{pa} is the specific heat of dry air = 1,005 [kJ/(kg·K)]
- x is the absolute humidity
- c_{pv} is the specific heat of water vapour = 1,87 [kJ/(kg·K)]

Being the dimensions of the room 7.30 x 10.70 x 3.22 m, its volume is 251.51 m³. So, concluding, the thermal capacity of the room is:

$$C = m \cdot c_m = \rho V \cdot c_m = 1.202 \frac{kg}{m^3} \cdot 251.51 m^3 \cdot 1.020 \frac{kJ}{kg K} = 308.36 \frac{kJ}{K}$$

4.4. Final RC values of the model

Considering what discussed in the previous paragraphs, a summary of the parameter values inserted into the model is reported in the table below.

Variable	Unit of measure	Room 1	Room 2
R_out	[K/W]	0.00937	0.00736
R_corr	[K/W]	0.02068	0.07456
R_int_room	[K/W]	0.00956	0.00956
C	[kJ/K]	308.36	308.36

5. Probabilistic model of the occupants

In order to model the presence of the students inside the room, a probabilistic model has been implemented, that describes the probability to find students inside the room. In this model, 0 indicates no probability to find students in the room, whereas with a probability of 1 it is certain to have students in the class. In order to implement the model, a typical school day has been considered, with the beginning of the morning lectures at 8.30 a.m., a pause of thirty minutes at 10 a.m., a lunch break from 12 a.m. to 1.30 p.m., and the afternoon classes until 5 p.m., with a thirty minutes break in the middle.

The graphical representation below represents the probabilistic curve of the school day described. From 7 p.m. to 7 a.m. the probability is negligible. Then, when the university opens, the probability linearly rises to 95% until 8.30 a.m., when the lecture starts, and remains constant until 10 a.m. Having a break of 30 minutes, between 10 and 10.15 a.m. the probability linearly lowers to 50% and comes back at the previous value by the end of the pause. The probability remains constant at 95% until the noon break, when the students progressively leave the room in order to have lunch. Therefore, the probability curve significantly decreases.

Following the same approach, the probabilistic model during the afternoon has a symmetric behavior with respect to the morning one, but with a slower decrease of the occupancy probability at the end of the lectures (between the end of the classes and the close of the university, there are two hours, and not one and half as in the morning).

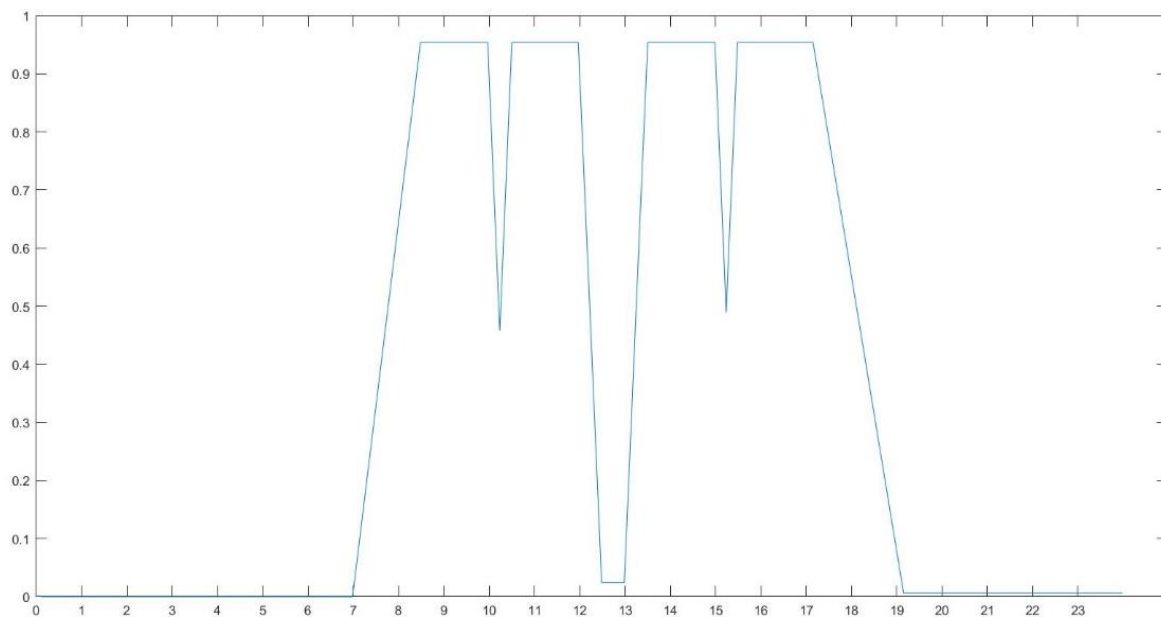


Figure 3 Probabilistic model of the occupants

6. Simulink model design

Our Simulink model is composed of 4 blocks: the model of the room 251, the model of the room 252, the HVAC system, and a block that models the heat gain of people.

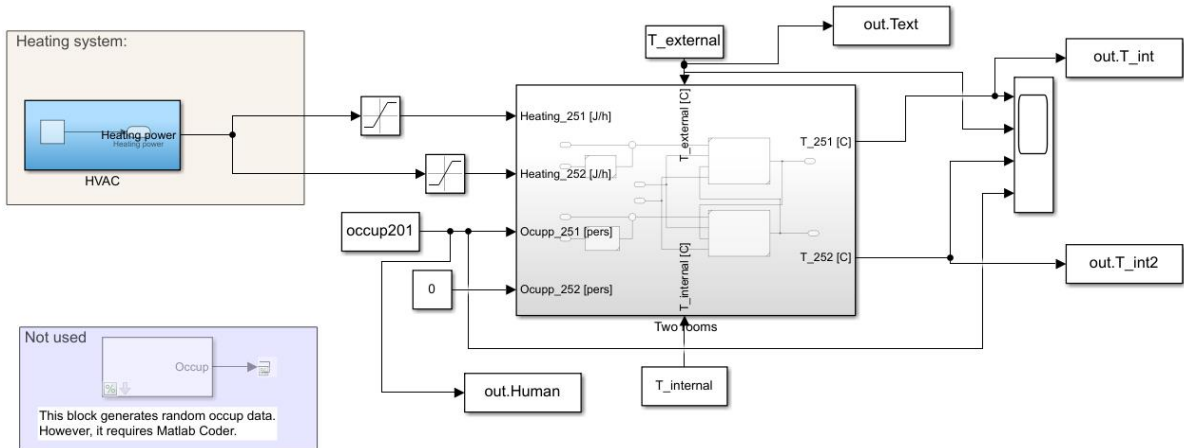


Figure 4 Simulink model of the rooms 251 and 252

6.1. Room 251 and 252

The blocks representing the models of room 251 and 252 take as an input the heat gain that come from HVAC and human presence in the room, the temperature outside, the temperature of the other room (for instance the block room 251 take as in input the temperature of the room 252), and the temperature of the corridor. It gives as an output the temperature inside the room.

As mentioned before, in order to model the thermal behaviour of the rooms, we used the RC modelling method, where the capacity C represents the tendency of the room to store heat whereas the resistances represent the heat loss through thermal conductivity and convection. As for the resistances R , in particular, 3 main resistances have been defined in the block: the first one representing the heat loss with the outside, the second one representing the heat loss with the adjacent room, and the third one representing the heat loss with the corridor.

These latter are used to define the heat loss that stems from these 3 resistances, that is finally subtracted from the thermal heat gain to get the available heat in the room.

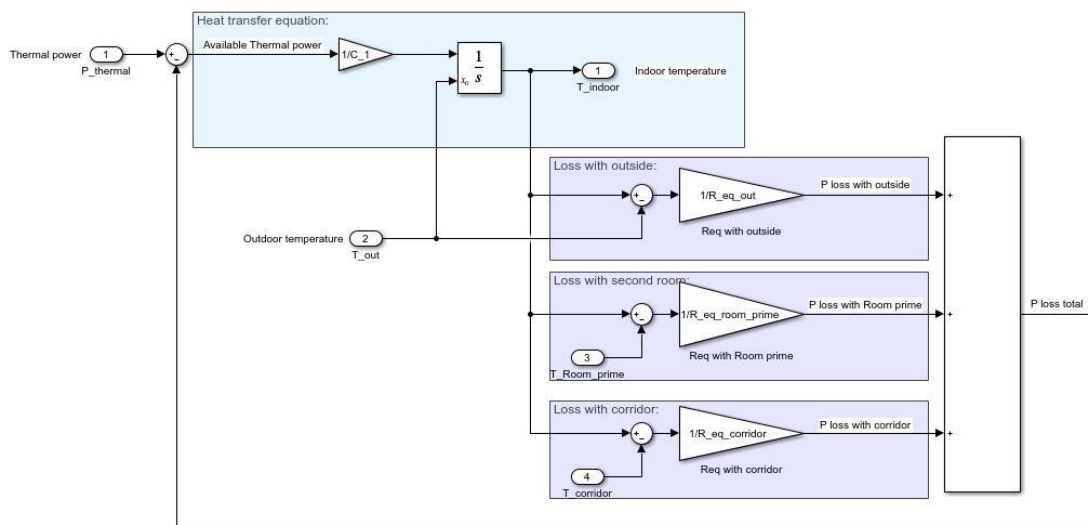


Figure 5 Simulink model of the Room 251

6.2. Heating system

Our heating system is modeled according to the following equation:

$$P = k \cdot \frac{dT}{dt}$$

It means that the heat provided by the HVAC is proportional to the difference between the desired (set-point) and measured indoor temperatures.

P is then saturated such as to respect physical constraints of the HVAC (i.e., maximum power that can be provided by the HVAC).

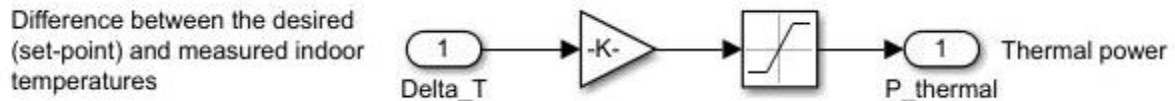


Figure 6 Simulink model of the HVAC system

6.3. Human presence function

This block is used to model the thermal gain provided by human present in the room.

As described in the section 5, it uses a stochastic algorithm to determine the number of people present in the room every hour during a day. Using this algorithm, we can then multiply the number of people by a gain in order to get the total heat gain. In particular, in order to define this latter, we have considered that a human being generates around 80 W of power as an average during the day.

7. Some illustrative simulations

7.1. Human presence

As explained above, we simulated the number of students in the room 251 during a week using a stochastic algorithm. Thus, we get the following graph, where on the y-axis the number of students is indicated:

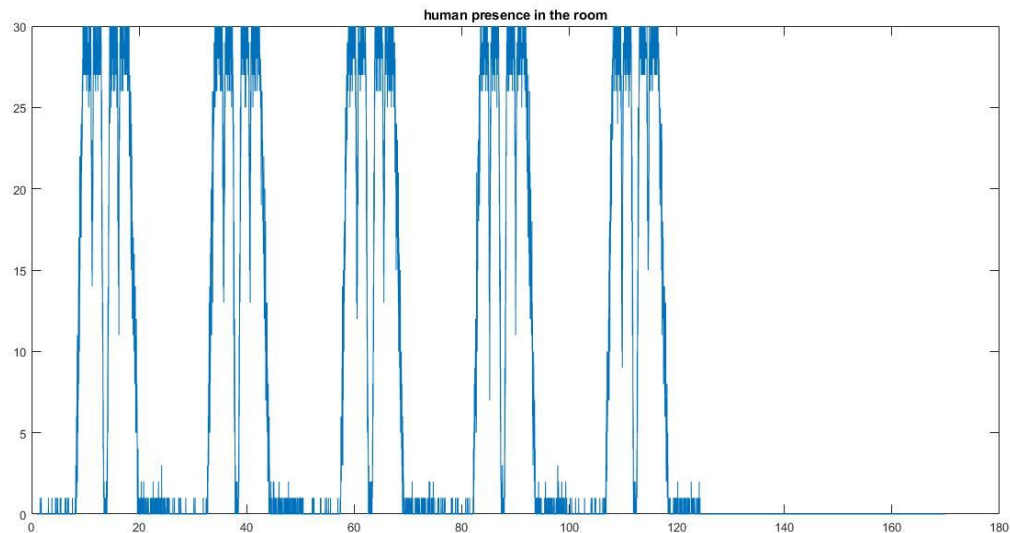


Figure 7 Human presence in the Room 251

7.2. External temperature

The external temperature data represents the temperature in Cesson-Sévigné in a week since the Campus of CentraleSupélec is located there. Its variation during the day is represented in the graph below.

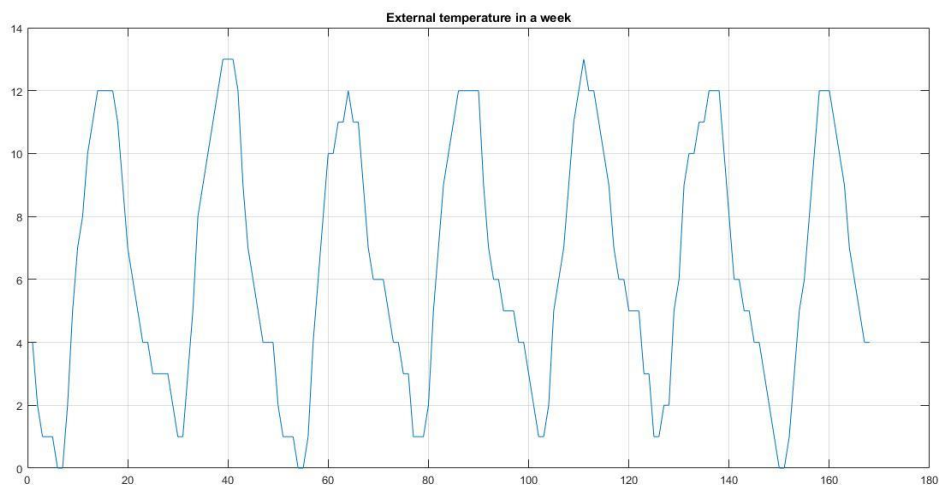


Figure 8 External temperature of Cesson-Sévigné

7.3. Temperature in room 251

We simulated to model without turning on the HVAC. As we can see in the graph below, the temperature in the 251 is affected by the external temperature (same shape of the graph) while being also affected by the human presence: in fact, we can notice a temperature drop in the last 2 days of

the week, due to the fact that no one is present in the room and we do not have any heat gain coming from students.

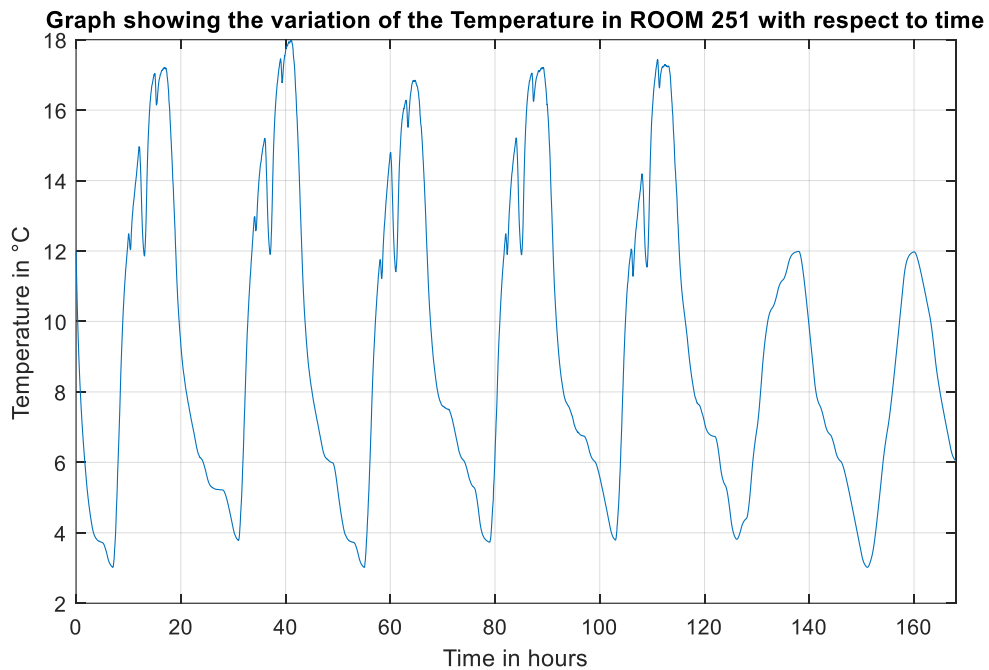


Figure 9 Temperature variation of the Room 251

7.4. Temperature in room 252

We simulated to model without turning on the HVAC. As we can see in the graph below, the temperature in the room 252 has almost the same shape as the one in room 251, but is lower in terms of temperature values than room 251. This is mainly due to the fact that there is not any human presence in this room and, therefore, we do not have any heat gain coming from students.

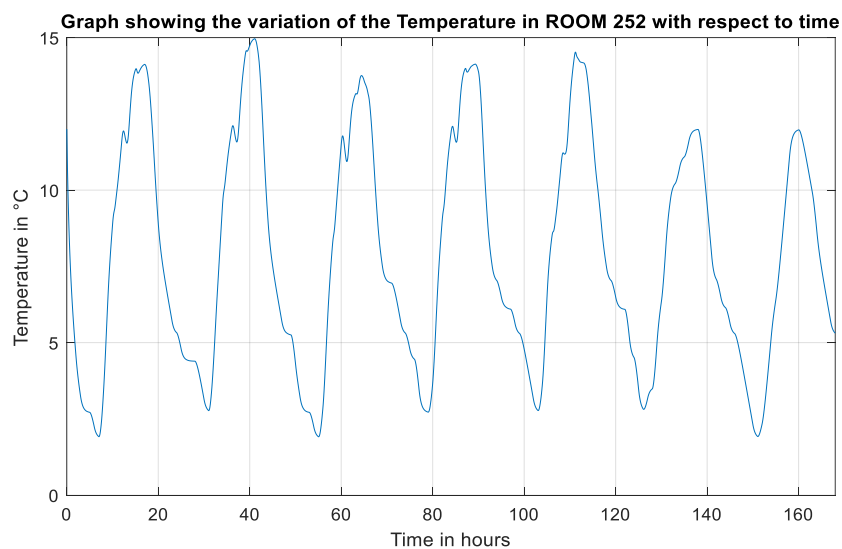


Figure 10 Temperature variation of the Room 252

8. Conclusion

Designing a building model is a complex operation, that needs to consider a significant number of variables. As we have seen, in particular, a decisive role is played by the human presence, that significantly impacts on the heat gains. However, this aspect is also the most difficult to predict, either in terms of number, or in terms of typology of occupants (e.g. males, females, age, activity and metabolism).

Thanks to the model we have realised, it is possible to partially manage this latter problem, thanks to the possibility to change manually the human heat gain per person. However, the model could certainly be improved, for example considering the solar radiation, the solar exposition and the humidity, which play a significant role either in the design and control of an HVAC system, or to guarantee a proper level of comfort for the user.

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Appendix

A1. Calculation of the unitary resistances

In the tables below, the resistance of each building component has been calculated, considering the stratigraphy listed in the section “Building analysis - Stratigraphy”.

External wall – CODE EW1			
Layer	Thickness [m]	Density [kg/m ³]	Thermal conductivity λ [W/(m·K)]
OUTDOOR			
Cement board	0.030	1280	0.350
Air gap	0.040	1	0.218
Mineral wool	0.200	100	0.035
Concrete structure	0.200	2200	1.610
Air gap	0.050	1	0.273
Cardboard	0.025	900	0.210
Finishing	0.015	1400	0.700
INDOOR			
Resistance [m²·K/W]			6.60

External wall [top window] – CODE EW2			
Layer	Thickness [m]	Density [kg/m ³]	Thermal conductivity λ [W/(m·K)]
OUTDOOR			
Cement board	0.030	1280	0.350
Air gap	0.040	1	0.218
Mineral wool	0.100	100	0.035
PUR insulation	0.100	32	0.032
Concrete structure	0.200	2200	1.610
PUR insulation	0.050	32	0.032
Cardboard	0.025	900	0.210
Finishing	0.015	1400	0.700
INDOOR			
Resistance [m²·K/W]			8.25

Internal wall [corridor] – CODE IW1			
Layer	Thickness [m]	Density [kg/m ³]	Thermal conductivity λ [W/(m·K)]
INDOOR			
Finishing	0.015	1400	0.700
Concrete structure	0.200	2200	1.610
Air gap	0.050	1	0.273
Cardboard	0.025	900	0.210
Finishing	0.005	1700	1.470
INDOOR			
Resistance [m²·K/W]			0.72

Internal wall [room] – CODE IW2			
Layer	Thickness [m]	Density [kg/m ³]	Thermal conductivity λ [W/(m·K)]
INDOOR			
Finishing	0.005	1700	1.470
Cardboard	0.025	900	0.210
Mineral wool (acoustic)	0.048	2200	1.610
Cardboard	0.025	900	0.210
Finishing	0.005	1700	1.470
INDOOR			
Resistance [m²·K/W]			1.88

Floor – CODE F1			
Layer	Thickness [m]	Density [kg/m ³]	Thermal conductivity λ [W/(m·K)]
INDOOR			
Linoleum	0.003	1200	0.170
Concrete structure	0.310	2300	2.300
Plenum	0.300	1	1.287
Mineral fibres boards	0.020	280	0.041
INDOOR			
Resistance [m²·K/W]			1.15

Roof – CODE R1			
Layer	Thickness [m]	Density [kg/m ³]	Thermal conductivity λ [W/(m·K)]
OUTDOOR			
Aggregates	0.040	1500	0.700
Elastomeric waterproof membrane	0.0032	1000	0.200
PUR insulation	0.200	40	0.032
Vapour barrier	0.0002	940	0.400
Concrete structure	0.310	2300	2.300
Plenum	0.330	1	2.031
Mineral fibres boards	0.020	280	0.041
INDOOR			
Resistance [m²·K/W]			7.25

Window 1 [0.9m x 1.86m] – CODE W1	
Layer	Thickness [m]
OUTDOOR	
Float glass	0.006
Gap - Argon	0.016
Float glass	0.008
INDOOR	
Resistance [m²·K/W]	
0.26	

Window 2 [1.80m x 1.86m] – CODE W2	
Layer	Thickness [m]
OUTDOOR	
Float glass	0.006
Gap - Argon	0.016
Float glass	0.008
INDOOR	
Resistance [m²-K/W]	0.28

Window 3 [2.30m x 1.86m] – CODE W3	
Layer	Thickness [m]
OUTDOOR	
Float glass	0.006
Gap - Argon	0.016
Float glass	0.008
INDOOR	
Resistance [m²-K/W]	0.27

Window 4 [1.90m x 2.04m] – CODE W4	
Layer	Thickness [m]
INDOOR	
Float glass	0.006
Gap - Air	0.016
Float glass	0.008
INDOOR	
Resistance [m²-K/W]	0.35

Door 1 [1.40m x 2.04m] – CODE D1	
Layer	Thickness [m]
INDOOR	
HPL finishing	0.004
EPS insulation	0.025
MDF finishing	0.008
INDOOR	
Resistance [m²-K/W]	0.56

Door 2 [0.90m x 2.04m] – CODE D2	
Layer	Thickness [m]
INDOOR	
HPL finishing	0.004
EPS insulation	0.025
MDF finishing	0.008
INDOOR	
Resistance [m²-K/W]	0.52

A2. Calculation of the dispersing surfaces

Computation criteria

Regarding horizontal surfaces, they must include:

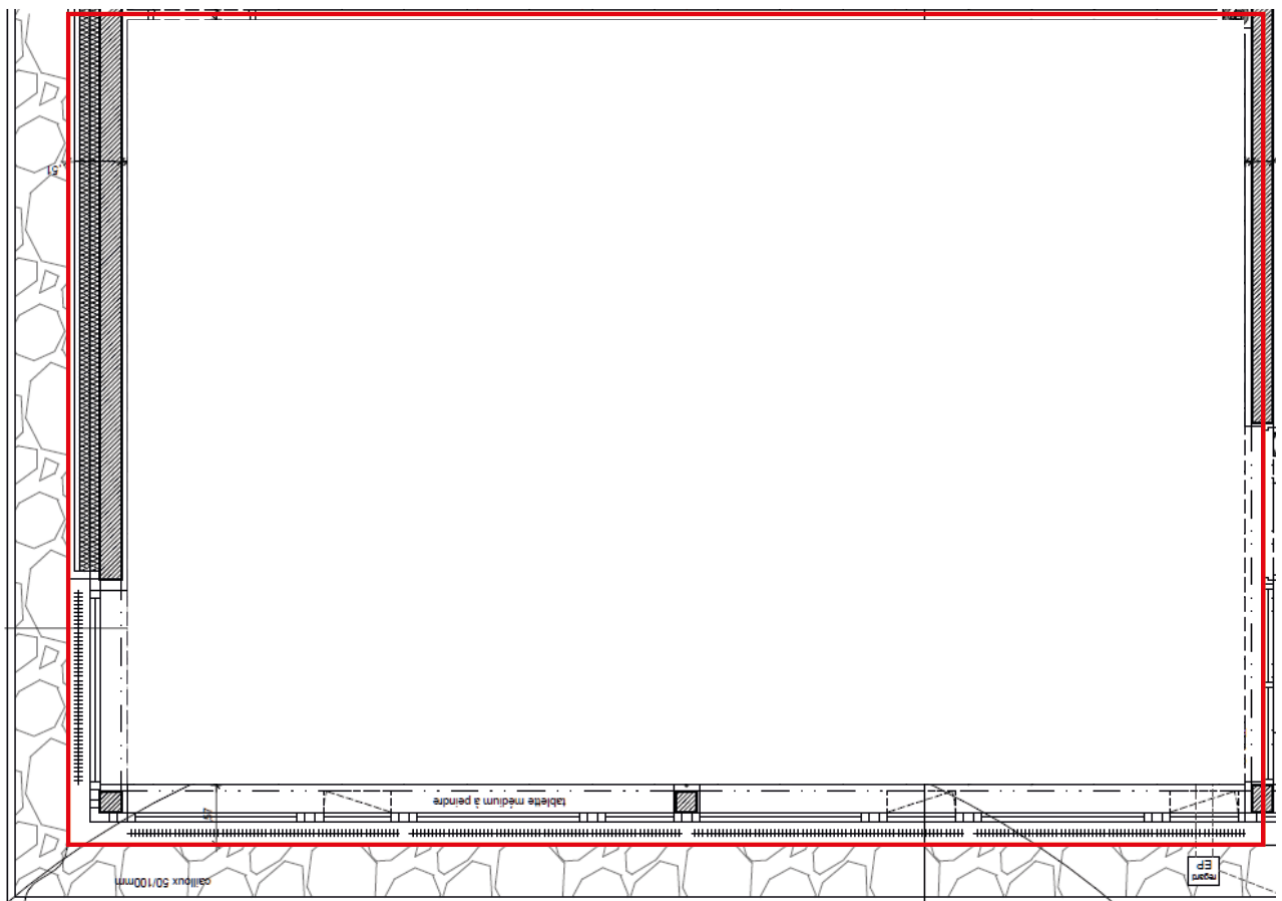
- the whole thickness of the walls if these latter are in contact with the outside or with a non-heated zone;
- half thickness of the walls if these latter are in touch with another heated zone.

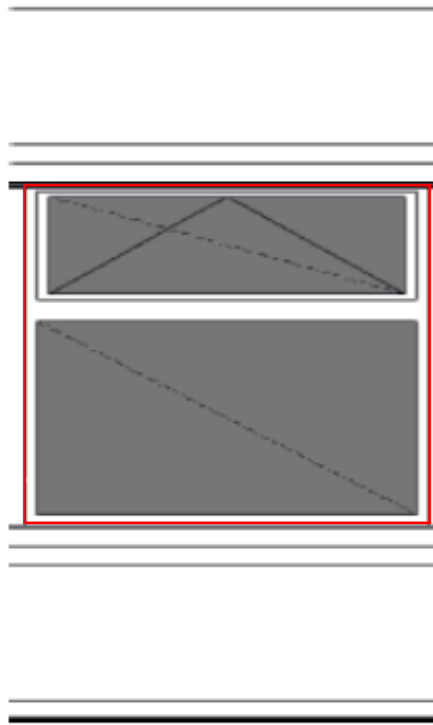
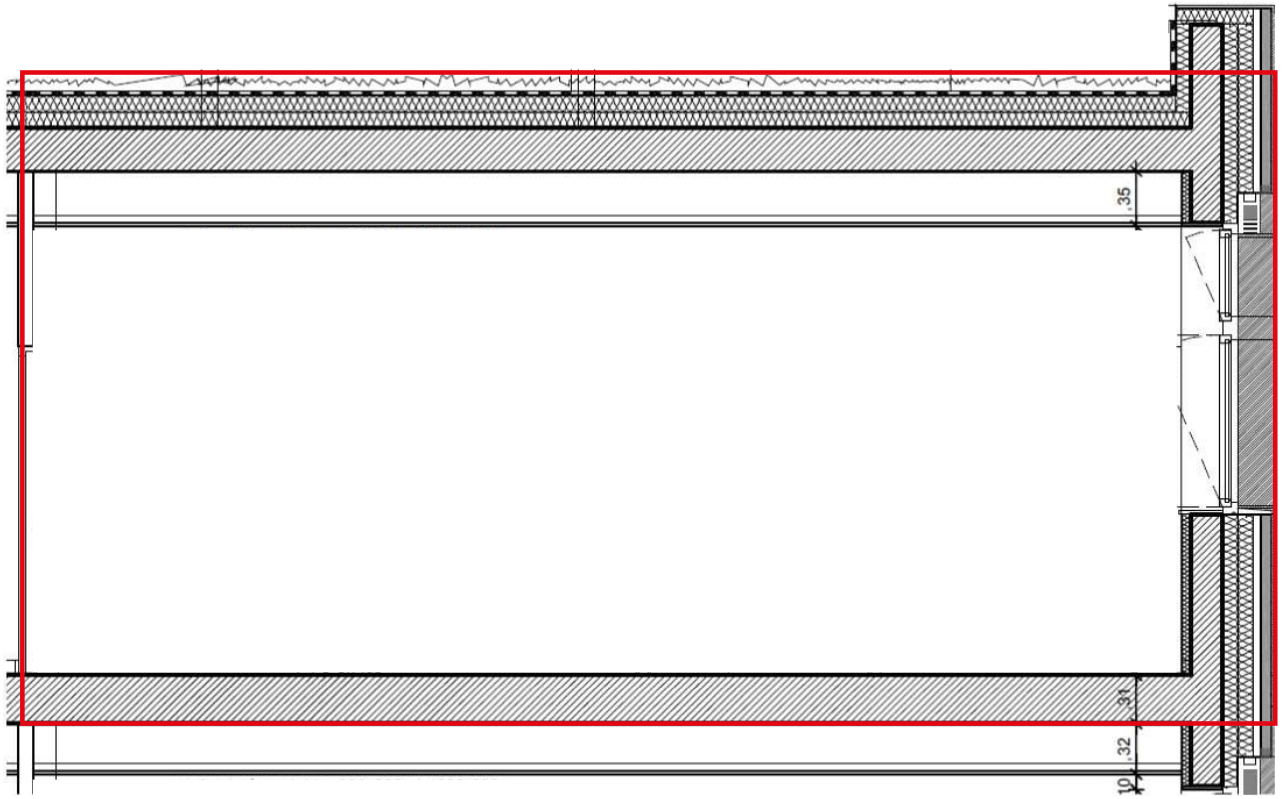
As for vertical surfaces, they must include:

- the entire thickness of the floors if these latter are in touch with the outside, the ground or a non-heated zone;
- half thickness of the floors if these latter are in touch with another heated zone.

Finally, for windows and doors the gross area must be considered.

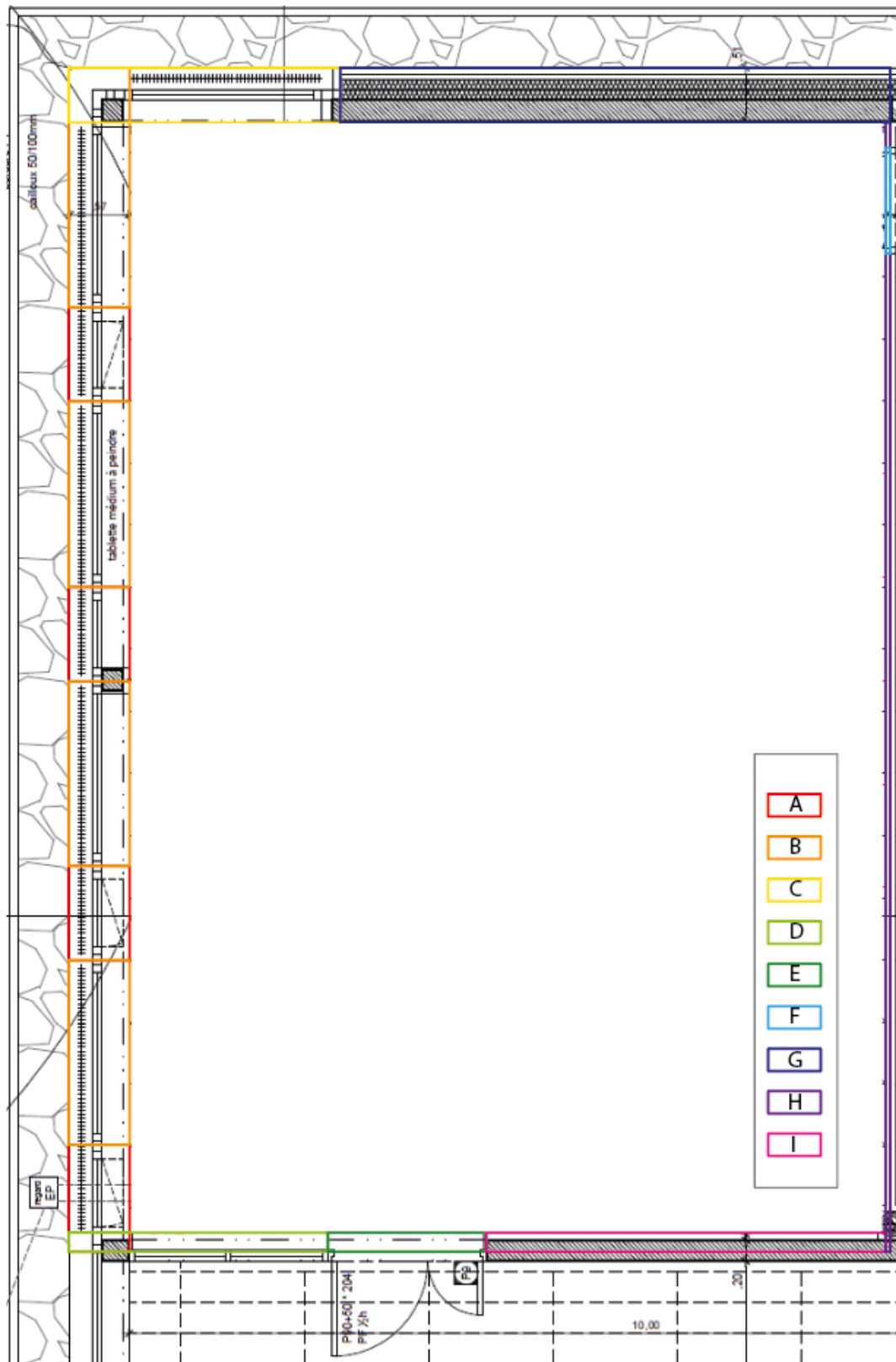
An example for each category of technical element is represented below.



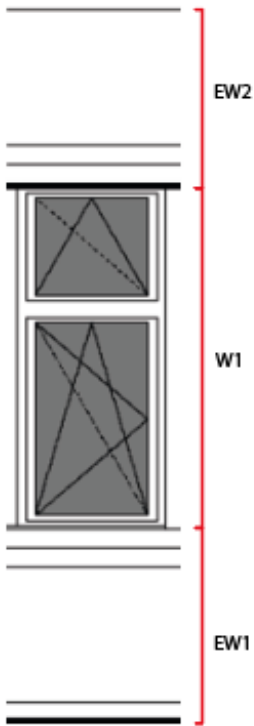


Computation of the dispersing areas

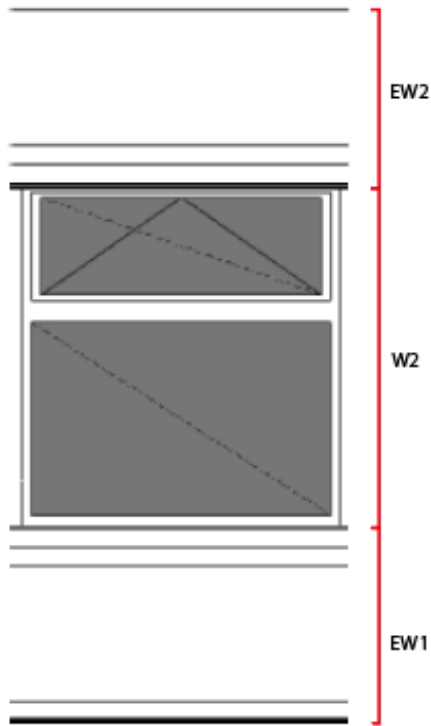
In order to calculate the dispersing areas of each typology of technical element, the vertical surfaces of the room have been divided into homogeneous parts. The subdivision and the characteristics of each sub-zone are reported in the figures below.



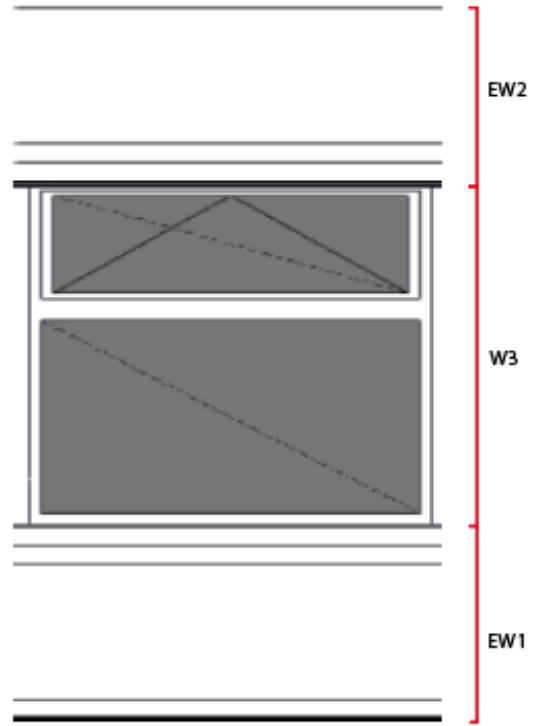
PART A



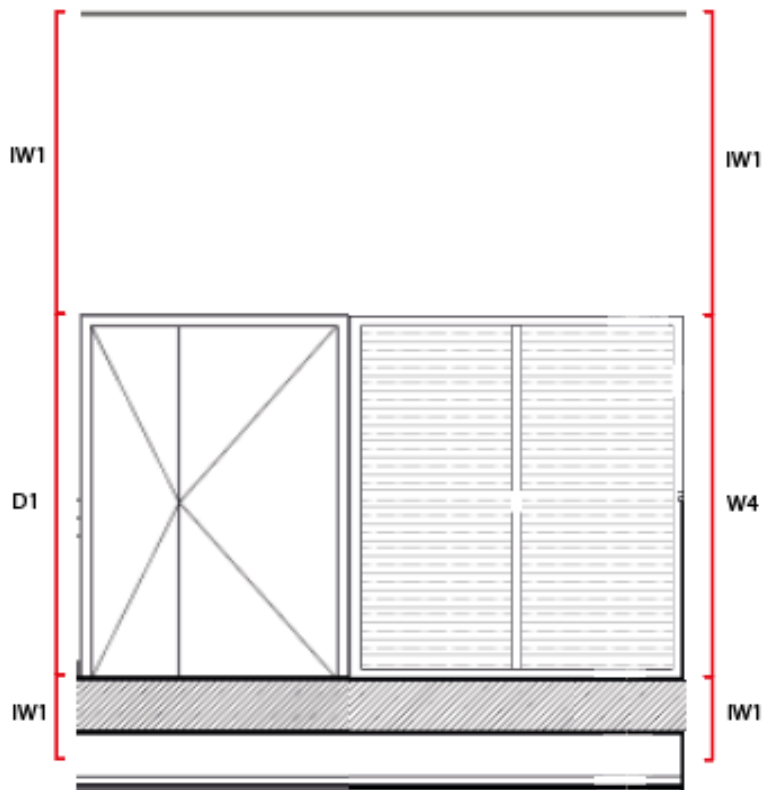
PART B



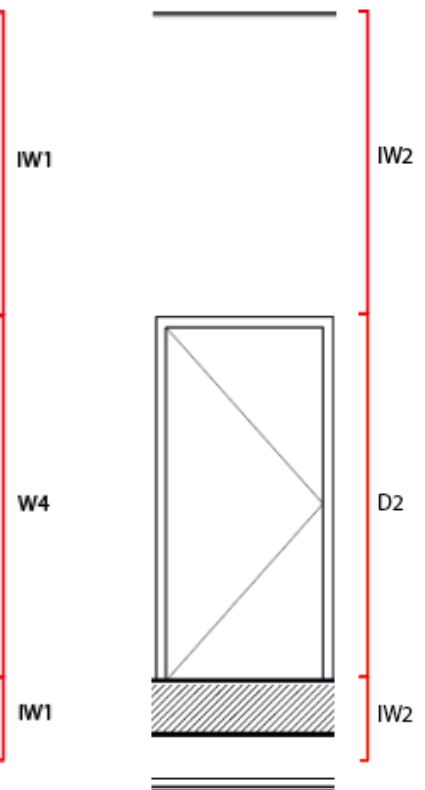
PART C



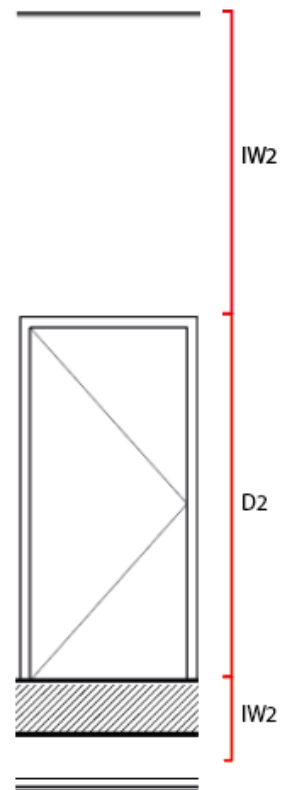
PART E



PART D



PART F



PART G

PART H

PART I



After having divided the vertical surfaces of the room, the dispersing areas of each technical element have been calculated considering the orientation of the surfaces. This process will also allow considering the different irradiance on each surface.

The results obtained for each of the two rooms are reported in the tables below.

ROOM 1	NE	NW	SW	SE	Horizontal
OPAQUE ELEMENTS	[m ²]				
External wall [EW1]	21.82	15.58	-	-	-
External wall [EW2]	2.07	10.30	-	-	-
Internal wall [IW1]	-	-	23.16	-	-
Internal wall [IW2]	-	-	-	41.93	-
Floor [F1]	-	-	-	-	90.64
Roof [R1]	-	-	-	-	90.64
GLAZED ELEMENTS					
Window 1 [W1]	-	6.70	-	-	-
Window 2 [W2]	-	13.40	-	-	-
Window 3 [W3]	4.28	-	-	-	-
Window 4 [W4]	-	-	3.88	-	-
DOORS					
Door 1 [D1]	-	-	2.86	-	-
Door 2 [D2]	-	-	-	1.84	-

ROOM 2	NE	NW	SW	SE	Horizontal
OPAQUE ELEMENTS	[m ²]				
External wall [EW1]	21.82	-	9.83	15.58	-
External wall [EW2]	2.07	-	3.24	10.30	-
Internal wall [IW1]	-	-	5.98		-
Internal wall [IW2]	-	41.93	-	-	-
Floor [F1]	-	-	-	-	90.64
Roof [R1]	-	-	-	-	90.64
GLAZED ELEMENTS					
Window 1 [W1]	-	-	3.35	6.70	-
Window 2 [W2]	-	-	3.35	13.40	-
Window 3 [W3]	4.28	-	-	-	-
Window 4 [W4]	-	-	-	-	-
DOORS					
Door 1 [D1]	-	-	2.86	-	-
Door 2 [D2]	-	1.84	-	-	-

A3. Calculation of the resistance values

Knowing the unitary resistances and the dispersing surfaces, it is possible to calculate the resistance values that will be introduced into the model. They are identified considering the heat losses towards the outside, the neighbour rooms and the corridor. To define the final resistance towards the outdoor, the following calculations have been performed:

$$R_{out} = \frac{1}{\frac{1}{R_{NE}} + \frac{1}{R_{NW}} + \frac{1}{R_{Roof}}} = 0.009375 \frac{K}{W}$$

where:

$$R_{NE} = \frac{1}{\frac{A_{EW1}}{R_{EW1}} + \frac{A_{EW2}}{R_{EW2}} + \frac{A_{W3}}{R_{W3}}} = \frac{1}{\frac{21.82 \text{ m}^2}{6.60 \frac{\text{m}^2\text{K}}{W}} + \frac{2.07 \text{ m}^2}{8.25 \frac{\text{m}^2\text{K}}{W}} + \frac{4.28 \text{ m}^2}{0.27 \frac{\text{m}^2\text{K}}{W}}} = 0.059054 \frac{K}{W}$$

$$R_{NW} = \frac{1}{\frac{A_{EW1}}{R_{EW1}} + \frac{A_{EW2}}{R_{EW2}} + \frac{A_{W1}}{R_{W1}} + \frac{A_{W2}}{R_{W2}}} = \frac{1}{\frac{15.58 \text{ m}^2}{6.60 \frac{\text{m}^2\text{K}}{W}} + \frac{10.30 \text{ m}^2}{8.25 \frac{\text{m}^2\text{K}}{W}} + \frac{6.07 \text{ m}^2}{0.26 \frac{\text{m}^2\text{K}}{W}} + \frac{13.40 \text{ m}^2}{0.28 \frac{\text{m}^2\text{K}}{W}}} = 0.012947 \frac{K}{W}$$

$$R_{Roof} = \frac{R_{Roof}}{A_{Roof}} = \frac{7.25 \frac{\text{m}^2\text{K}}{W}}{90.64 \text{ m}^2} = 0.079987 \frac{K}{W}$$

The same procedure is valid for the resistance towards the neighbour rooms and the corridor:

$$R_{corr} = R_{SW} = 0.020678 \frac{K}{W}$$

$$R_{SW} = \frac{1}{\frac{A_{IW1}}{R_{IW1}} + \frac{A_{W2}}{R_{W2}} + \frac{A_{D1}}{R_{D1}}} = \frac{1}{\frac{23.16 \text{ m}^2}{0.72 \frac{\text{m}^2\text{K}}{W}} + \frac{3.88 \text{ m}^2}{0.35 \frac{\text{m}^2\text{K}}{W}} + \frac{2.86 \text{ m}^2}{0.56 \frac{\text{m}^2\text{K}}{W}}} = 0.020678 \frac{K}{W}$$

$$R_{int_room} = \frac{1}{\frac{1}{R_{SE}} + \frac{1}{R_{Floor}}} = 0.009555 \frac{K}{W}$$

$$R_{SE} = \frac{1}{\frac{A_{IW2}}{R_{IW2}} + \frac{A_{D2}}{R_{D2}}} = \frac{1}{\frac{41.93 \text{ m}^2}{1.88 \frac{\text{m}^2\text{K}}{W}} + \frac{1.84 \text{ m}^2}{0.52 \frac{\text{m}^2\text{K}}{W}}} = 0.038697 \frac{K}{W}$$

$$R_{Floor} = \frac{R_{Floor}}{A_{Floor}} = \frac{1.15 \frac{\text{m}^2\text{K}}{W}}{90.64 \text{ m}^2} = 0.012688 \frac{K}{W}$$

The same procedure is applied for the room 252, obtaining the following results:

$$R_{out} = 0.00736 \frac{K}{W}$$

$$R_{corr} = 0.074556 \frac{K}{W}$$

$$R_{int_room} = 0.009555 \frac{K}{W}$$