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SOME COMMENTS ON MINIJETS*

C. S. Kim, J. R. Cudell and F. Halzen
Physics Department, University of Wisconsin, Madison, Wisconsin 53706

ABSTRACT

When the collision energy of two hadrons increases the physics of an increasing fraction of minimum bias events is dictated by perturbative QCD. Gaisser,[1] Pancheri[2] and their collaborators have proposed that the *minijets* observed by UA1 signal the onset of this phenomenon. Whether minijets are responsible for the rise of the total cross sections is an unrelated issue. Their suggestion is much more radical, namely that all non-scaling features[3] of minimum bias events are related to hard scattering and calculable in perturbation theory. These include the rise of the rapidity plateau in the central region, the increase of $\langle p_T \rangle$, the appearance of a high multiplicity tail in the KNO distribution and the correlation between multiplicity and transverse momentum. We comment on the possibility of implementing this idea in a quantitative way avoiding the use of an unphysical minimum p_T cutoff routinely appearing in the present calculations.

Recent $p\bar{p}$ collider data have dramatically confirmed the existence of parton jets and their QCD origin. This study naturally concentrates on jets with the largest p_T. Their cross sections are calculable in perturbation theory. The physics issue introduced in the abstract involves on the contrary jets with p_T not much larger than $\langle p_T \rangle$. Calculations now involve $x = p_T/\sqrt{s}$ values as small as 10^{-3} and one has to reexamine the use of perturbation theory.[4] When x becomes small an increasing number of relatively soft partons are stacked into the colliding hadrons. Multiple parton interaction become likely and their emergence provides a physical low p_T cutoff for the application of perturbation theory. Following Humpert *et al.*[5] and Paver *et al.*[6] we estimate that multiple parton interactions do not** become competitive with perturbative cross section unless x is much

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** For a dissenting point of view, see T. Sjöstrand.[7] We question the assumption that p_{TMIN} is the same for different $\sqrt{s}$ values, an assumption which crucially affects the conclusions of this analysis.

smaller than 10^{-3}. It is interesting to note that for small x

$$\sigma_2 \simeq \frac{\sigma_1^2}{\pi R^2} \,. \tag{1}$$

Here σ_1 is the two-jet cross section while σ_2 represents the four-jet cross section resulting from 2 hard parton collisions in a single $p\bar{p}$ interaction. "πR^2" $\simeq 10 \sim 100\,mb$ is the "size" of the nucleon. Equation (1) is approximately true at small x as overall momentum conservation does not play a crucial role in the determination of σ_2. Therefore multiple parton interactions become important when $\sigma_1 \simeq \sigma_2 \simeq 10 - 100\,mb$, *i.e.*, the hard jet cross sections reach values exceeding $10\,mb$.

The previous remarks naturally lead to a calculation of the inclusive jet p_T-distribution for $p_T > 5$ GeV at $\sqrt{s} = 630$ GeV. The result is shown in Fig. 1a. Although hard jets are adequately described, the calculation falls short of the data for $p_T \lesssim 40$ GeV. We used DO1 structure functions[8] ($\Lambda = 0.2$) with all scales set to be $Q^2 = p_T^2$. We checked that our conclusion is not affected by the choice of structure functions. We also investigated the effect of alternative scale choices such as $\hat{s}$, $p_T^2/4$ and $2\hat{s}\hat{t}\hat{u}/(\hat{s}^2 + \hat{t}^2 + \hat{u}^2)$ in both $\alpha_s(Q^2)$ and structure functions. In Table I we show the integrated cross section

$$\sigma_J = \int_{p_{T\min}} \frac{d\sigma^{\mathrm{QCD}}}{dp_T}\, dp_T \tag{2}$$

for $p_{T\min} = 2 - 5$ GeV. The dependence on scale choices is illustrated. Again our preliminary conclusion is that $\sigma_J(p_T > 5\text{ GeV})$ is smaller than the measured values,[9] see Table I and Fig. 2.

Several guesses for the origin of this discrepancy can be made:

(*i*) a K-factor reflecting $O(\alpha_s^3)$ contributions (this *factor* can be a function of p_T)

(*ii*) neglect of multiple parton interactions or

(*iii*) distortion of the low p_T cross section by the UA1 jet–finding algorithm.

The latter possibility deserves careful investigation. Further phenomenological progress seems impossible without examining this effect. Phrased in a different way one should ask the question whether jets with $p_T < p_{T\min}$, characterized by large cross sections can contribute to (2) or to UA1's operational definition of (2). Fig. 2 compares the rise of the jet cross section for $p_{T\min} = 3$, 4 and 5 GeV with the increase of $\frac{1}{2}\sigma_{\mathrm{tot}}$ and σ_{inel} as measured[3] by UA4 and UA5.

UA1's minijets[9] are shown with a rapidity $|\eta| < 1.5$ cut not applied to the calculations or the other data points. Figure 2 suggests that $E_T > 5$ GeV UA1 jets are generated by $p_{T\min} > 3$ GeV QCD jets. These statements cannot be taken too literally. Some minijets could be fluctuations of the soft interactions which are also characterized by large cross sections. Using ISAJET we find further support for this idea by calculating the effect of partons with $p_T < 5$ GeV on the $E_T > 5$ GeV jet cross section as defined by the UA1 algorithm. We find that they can indeed enhance the measured cross section by a factor of 3 up to $\sum E_T \simeq 10$ GeV, see Fig. 1b.

As a further comment we propose a procedure to investigate the emergence of hard scattering physics in the features of minimum bias events. Any calculation of the minimum bias *particle* production cross section $d\sigma/dk_T$ should be subject to the following theoretical/experimental constraints:

$$(i) \qquad \frac{1}{\sigma}\frac{d\sigma}{dk_T} \xrightarrow[k_T \to 0]{} f(k_T) \tag{3}$$

with

$$f(k_T) = a e^{-a k_T^2} \tag{4}$$

and a determined by $\langle k_T \rangle \simeq 0.3$ GeV,

$$(ii) \qquad \frac{d\sigma}{dk_T} \xrightarrow[k_T \text{ large}]{} \frac{d\sigma^{\text{QCD}}}{dk_T} \tag{5}$$

and

$$\int dk_T \, \frac{d\sigma}{dk_T} = \langle n \rangle \, \sigma_{\text{tot}} \, . \tag{6}$$

In mathematical terms it is now a straightforward problem to write a mathematical distribution in k_T with given $k_T \to o$, $k_T \to \sqrt{s}/2$ limits and normalized to a known area given by (6). In practice we implemented as follows

$$\begin{aligned} \frac{d\sigma}{dk_T^2 dy_1 dy_2} &= \sigma_{\text{tot}} \frac{d\sigma^s}{dy_1 dy_2} \, f(k_T^2) \\ &+ \frac{2}{\pi} \int d^2 q_T \int \frac{dp_T^2}{2 q_T p_T} \frac{d\sigma^{\text{QCD}}}{dp_T^2 dy_1 dy_2} \, D\left(\frac{q_T}{p_T}\right) \left[f\left[(\vec{k}_T - \vec{q}_T)^2 \right] - f(q_T^2) \right] \, . \end{aligned} \tag{7}$$

Here p_T, k_T are, respectively, the transverse momentum of the jet and the secondary particle. The D-function[10] describes the jet $\to$ charged pion fragmentation. The soft cross section limit is guaranteed by the first term in (7). We fitted it to the data with $a = 8.7$ in Eq. (4) and

$$\frac{d\sigma^{s}}{dy_1 dy_2} = \frac{1}{16(y_{\max} - 2)^2} \frac{dN}{dy_1} \frac{dN}{dy_2} \tag{8}$$

with $\frac{dN}{dy} = 2$ except for a cutoff function near $y_{\max}$, $y_{\min}$. σ^{QCD} in (7) is calculated from leading order QCD as before. The two jets have rapidity y_1, y_2. Notice that no arbitrary cutoff $p_{T\min}$ appearing in the cross section as opposed to (2). The inclusive charged π distribution calculated from (4), (7) and (8) is shown in Fig. 3. This distribution explicitly exhibits the rapidity structure discussed in Ref. 1 *i.e.*, a scaling distribution with a rising component of jet origin in the central part of the plateau. The two–component structure is not obvious. It forms a smooth distribution in k_T and y.

We close with a comment on the total cross section

$$\sigma_{\rm tot} = \sigma_{e\ell} + \sigma_{\rm inel} \tag{9}$$

with

$$\sigma_{\rm inel} = \sigma_{\rm NSD} + \sigma_D \ . \tag{10}$$

As seen in Fig. 2 the jet cross section traces the increase with energy of the non-single diffractive cross section ($\sigma_{\rm NSD}$) to which it contributes. One could argue (*e.g.*, in a model[11] where $\sigma_{e\ell} \sim \frac{1}{2}\sigma_{\rm tot}$ and the diffractive cross section σ_D is small or has a weak energy dependence) that $\sigma_{\rm NSD} = \frac{1}{2}\sigma_{\rm tot}$ and therefore jets drive the increase of the total cross sections.[12] Although this question is peripheral to the discussion, it has stimulated a lot of interest. In Eq. (7) we also choose to normalize the cross section to $\sigma_{\rm tot}$. An increase in an inclusive cross section (*e.g.*, σ_J in (2)) does not necessarily result in a corresponding increase of $\sigma_{\rm tot}$. Mueller[4] phrases the issue in the following terms: a jet event only contributes to $\sigma_{\rm tot}$ if the jet is directly related to the primary origin of the interaction between the hadrons and not just a by–product of an interaction which occurred for an unrelated reason. *E.g.*, a hard two-jet event close to the edge of phase space contributes to $\sigma_{\rm tot}$, a jet radiated in the final state of a diffractive $p\bar{p}$ collision does not. It is therefore a relevant fact that minijet events seem to have identical event structure as hard QCD two-jet events,[9] only p_T is smaller. There is no event structure indicating a different origin of the interaction. The data suggest at present that the rising cross section in Fig. 2 do indeed contribute to $\sigma_{\rm tot}$.

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FIGURE CAPTIONS

Fig. 1a. Parton transverse momentum distribution at $y = 0$. The calculation is to leading order in QCD and structure functions[8] and strong coupling are evolved with $Q^2 = p_T^2$. All calculations are for 3 flavors and a parametrization of α_s with appropriate heavy quark thresholds.

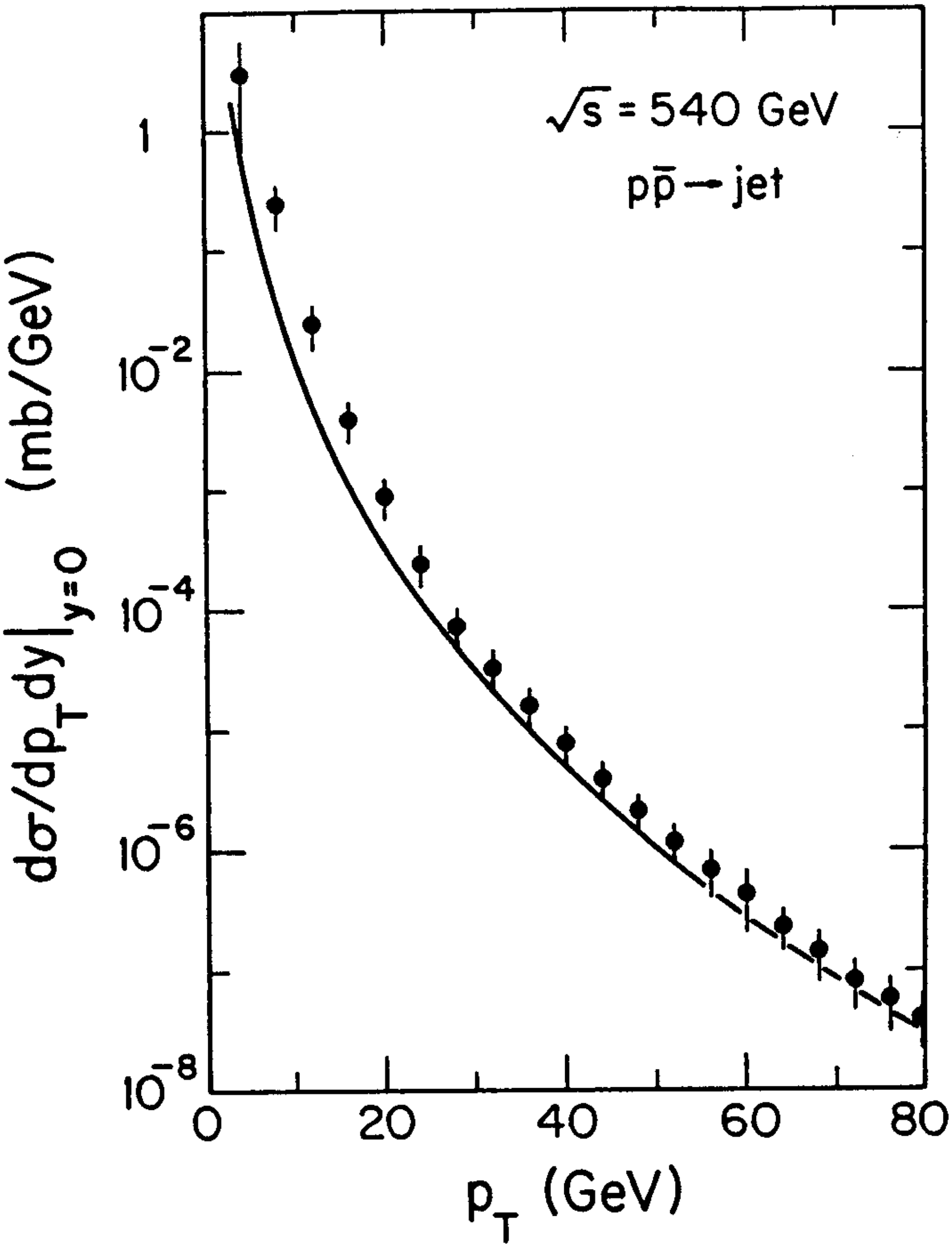
Fig. 1b. Jet transverse momentum distributions at zero pseudorapidity, generated by partons with p_T between 3 and 5 GeV, 5 and 10 GeV and summed over partons with p_T between 3 and 20 GeV. The calculation is done using ISAJET 5.20 and the ISAJET UA1-like jet-finding algorithm. All the parameters are identical to those of Fig. 1a.

Fig. 2. Integral cross section of Eq. (2) for $p_{T\min} = 3$, 4 and 5 GeV. Also shown are experimental data for $(E_T)_{\rm jet} > 5$ GeV and $|\eta| < 1.5$, $\sigma_{\rm NSD}(\sqrt{s}) - \sigma_{\rm NSD}(200\ {\rm GeV})$ (squares), $\sigma_{\rm NSD}$ (900 GeV)/$\sigma_{\rm NSD}$(200 GeV) (arrow) and $\frac{1}{2}\sigma_{\rm tot}(\sqrt{s}) - \frac{1}{2}\sigma_{\rm tot}(200\ {\rm geV})$. The increase of $\sigma_{\rm NSD}$ and $\frac{1}{2}\sigma_{\rm tot}$ with energy should be similar if $\sigma_{el} \simeq \frac{1}{2}\sigma_{\rm tot}$ and $\sigma_{\rm SD} \ll \sigma_{\rm NSD}$ or if $\sigma_{\rm SD}$ varies slowly with $\sqrt{s}$.

Fig. 3. Inclusive charged pion distribution calculated in the two component model of Eqs. (7) and (8). The soft cross section is shown separately.

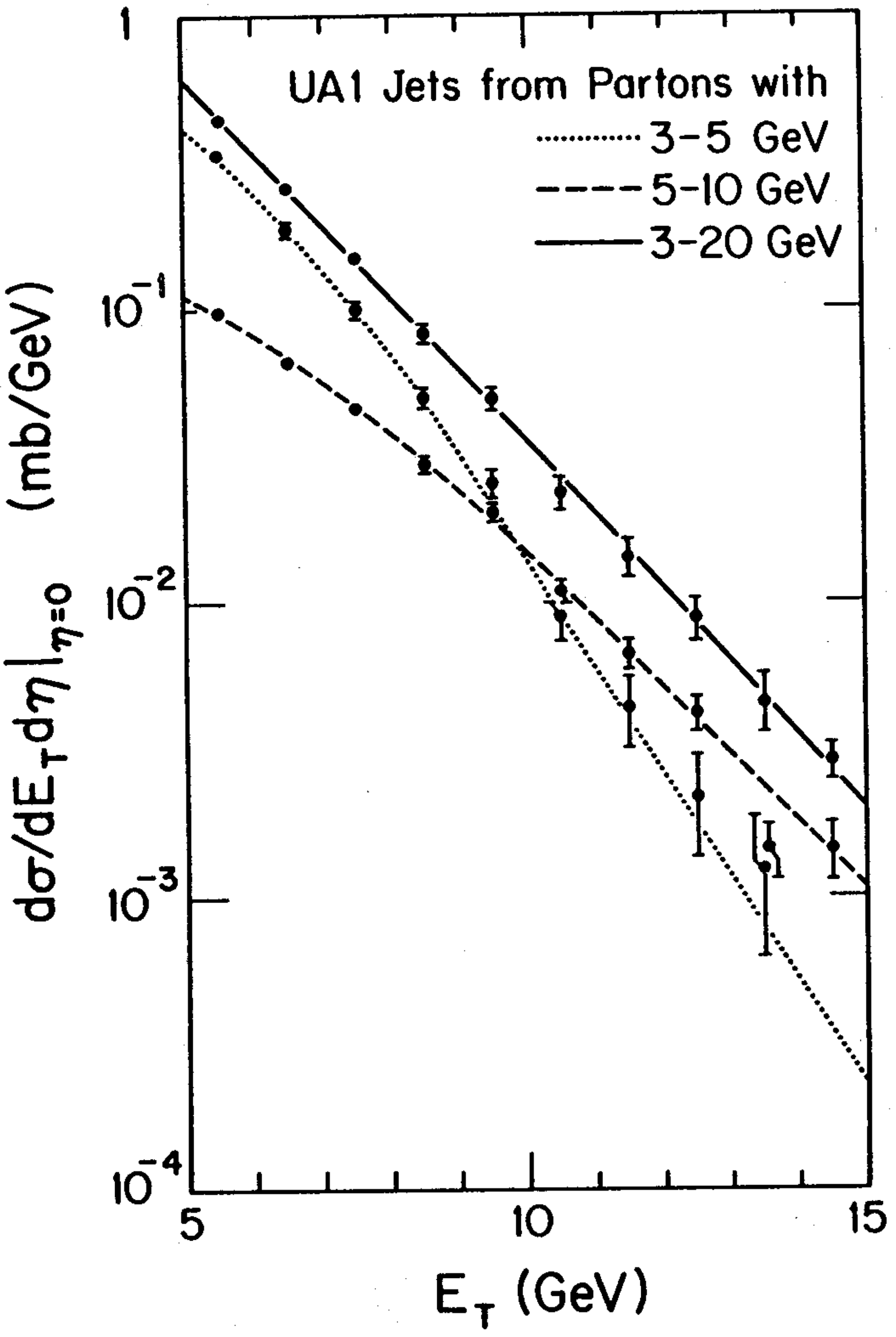
Table I. Value of the integral jet cross section defined by Eq. (2) in *mb* for various choices of Q^2 scale.

$\sqrt{s}$ (GeV)	Q^2 in DO1 and α_s	$P_{T\min}$ (GeV)			
		2	3	4	5
200	$\hat{s}$	9.1	2.0	0.6	0.22
	p_T^2	13.3	3.3	1.1	0.41
	$2\,\dfrac{\hat{s}\hat{t}\hat{u}}{\hat{s}^2+\hat{t}^2+\hat{u}^2}$	13.3	3.3	1.0	0.40
	$p_T^2/4$	22.0	4.6	1.5	0.64
500	$\hat{s}$	28.8	7.4	2.6	1.1
	p_T^2	29.3	9.5	3.8	1.7
	$2\,\dfrac{\hat{s}\hat{t}\hat{u}}{\hat{s}^2+\hat{t}^2+\hat{u}^2}$	30.1	9.6	3.8	1.7
	$p_T^2/4$	42.2	10.2	4.1	2.0
900	$\hat{s}$	57.4	16.1	6.1	2.7
	p_T^2	45.3	16.7	7.3	3.6
	$2\,\dfrac{\hat{s}\hat{t}\hat{u}}{\hat{s}^2+\hat{t}^2+\hat{u}^2}$	47.0	17.1	7.4	3.6
	$p_T^2/4$	59.0	15.4	6.8	3.7



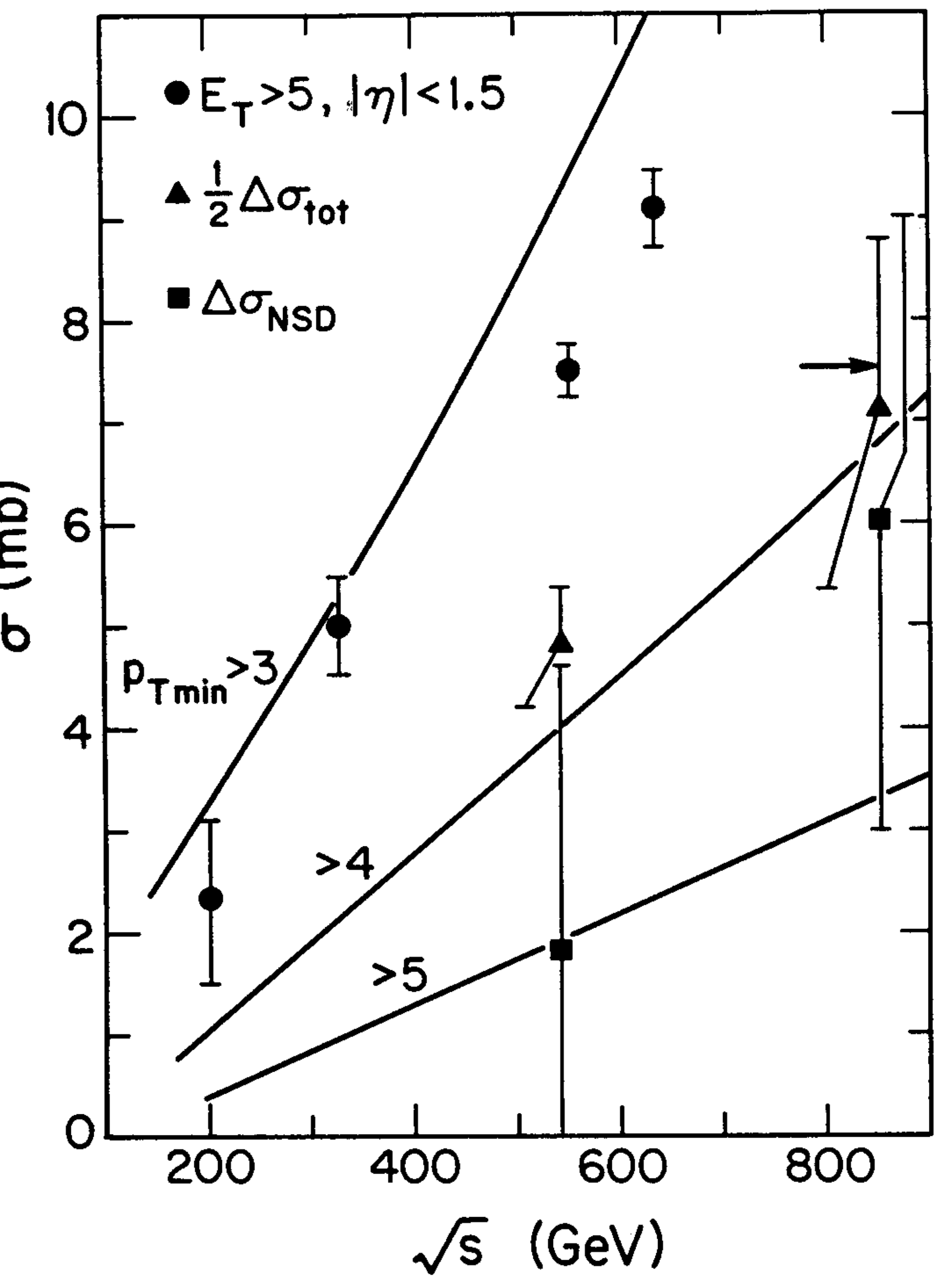
√s = 540 GeV
pp̄ → jet
1
10^-2
10^-4
10^-6
10^-8
dσ/dp_T dy|_y=0 (mb/GeV)
0
20
40
60
80
p_T (GeV)

Fig. 1a



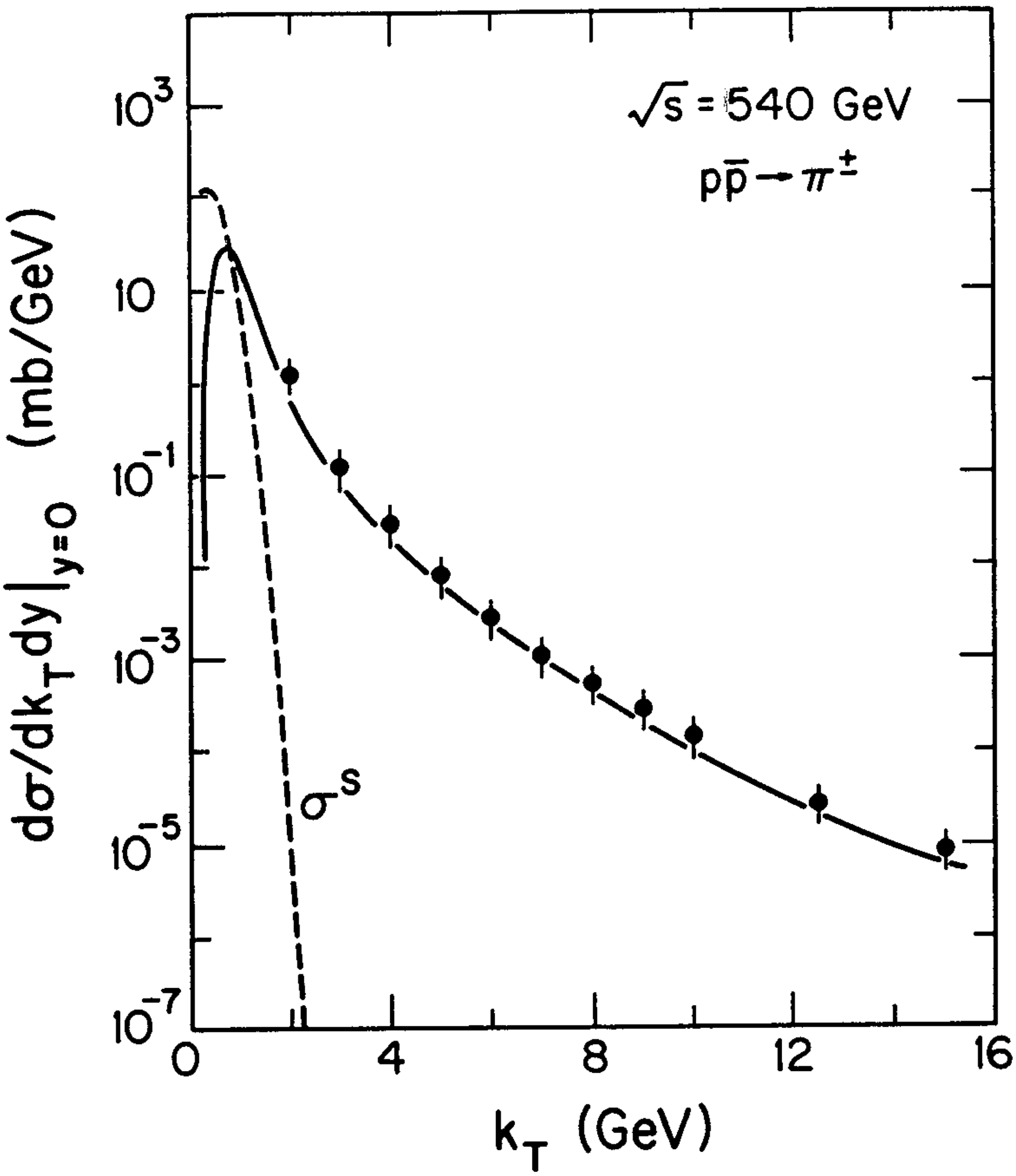
1
UA1 Jets from Partons with
3-5 GeV
5-10 GeV
3-20 GeV
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$d\sigma/dE_T d\eta|_{\eta=0}$ (mb/GeV)
5
10
15
E_T (GeV)

Fig. 1b



E_T >5, |η| < 1.5
½ Δσ_tot
Δσ_NSD
10
8
6
4
2
0
σ (mb)
p_Tmin > 3
>4
>5
200
400
600
800
√s (GeV)

Fig. 2



$\sqrt{s}$ = 540 GeV
$p\bar{p} \rightarrow \pi^{\pm}$
10^3
10
10^{-1}
10^{-3}
10^{-5}
10^{-7}
$d\sigma/dk_T dy|_{y=0}$ (mb/GeV)
σ^S
0
4
8
12
16
k_T (GeV)

Fig. 3