



Envelope Reconstruction with Equivalent Wind Loads

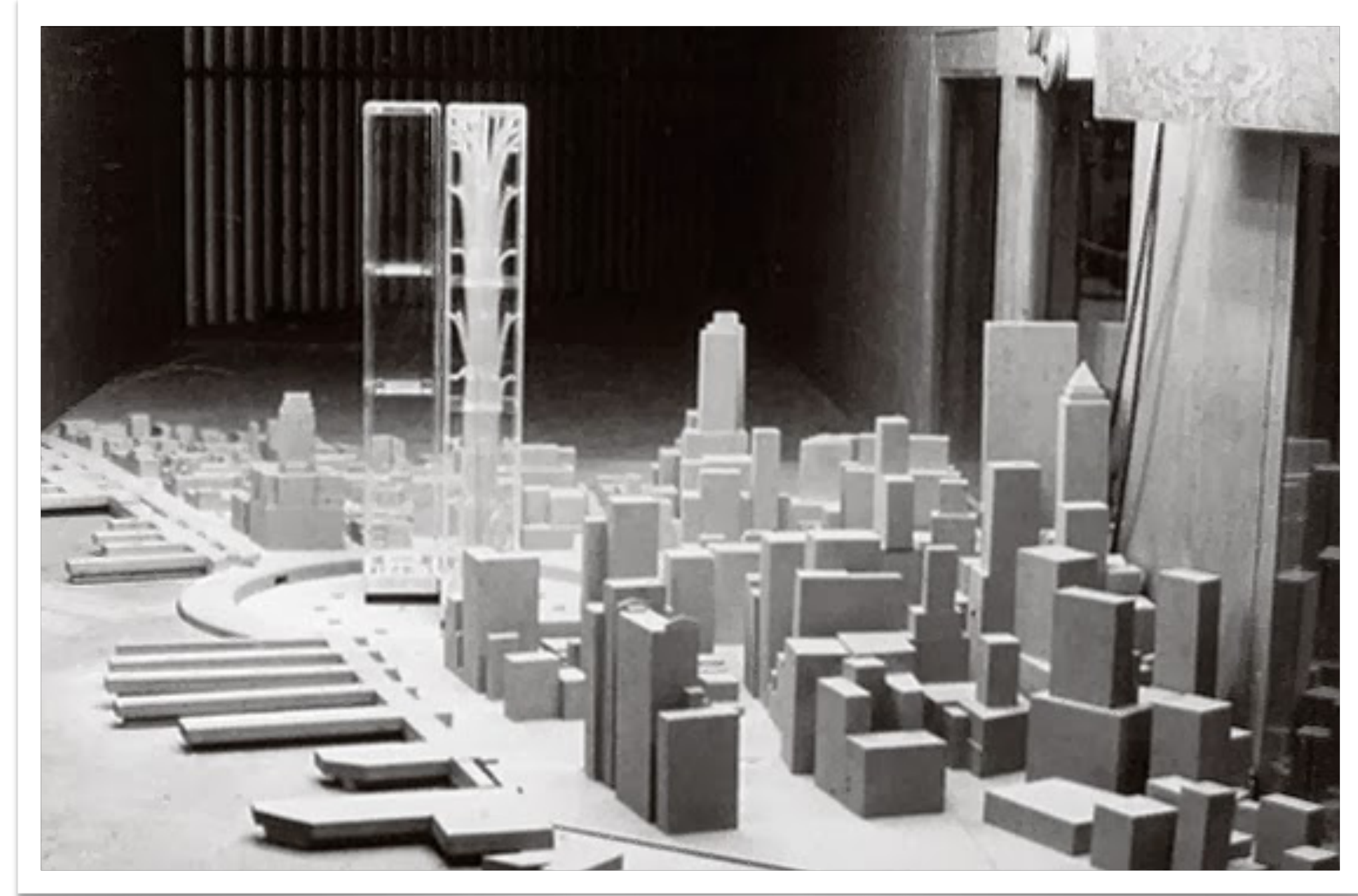
Vincent Denoël



August 27-31, 2023
FLORENCE - ITALY

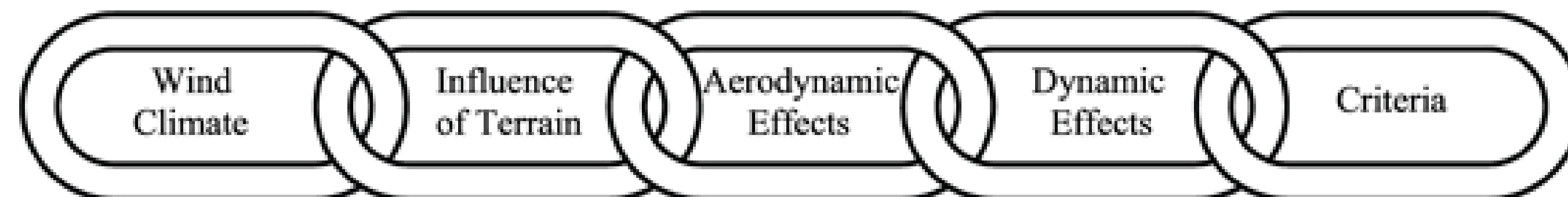
**16th International
Conference on
Wind Engineering**

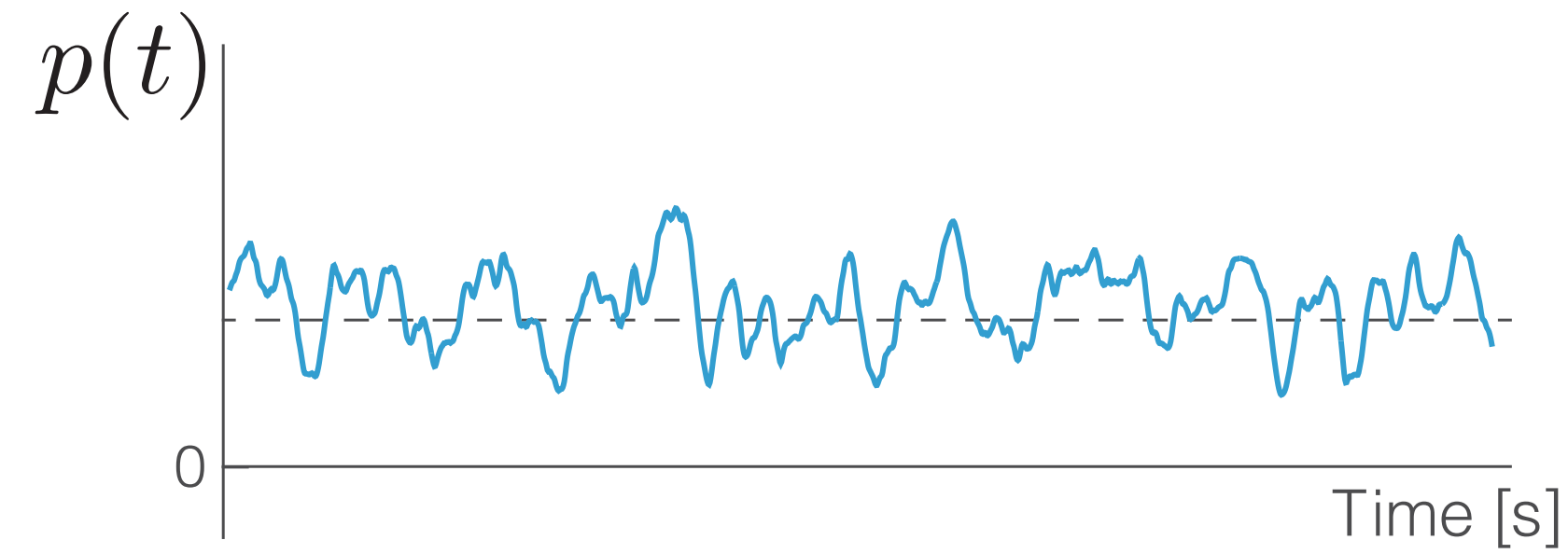




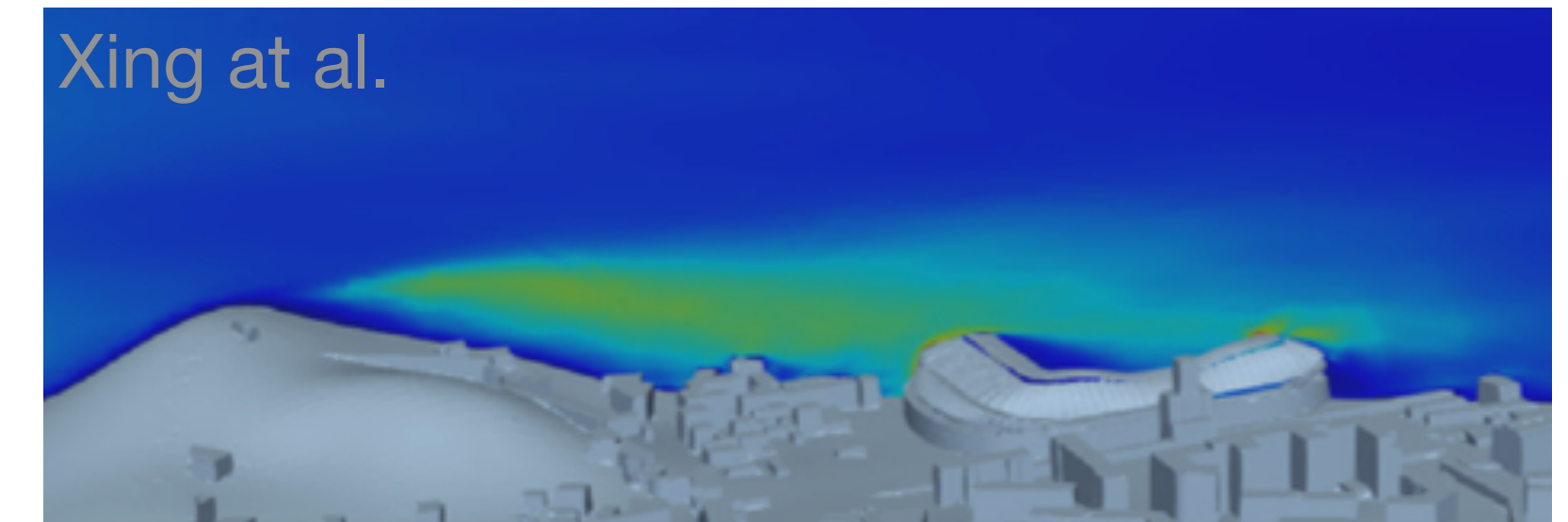
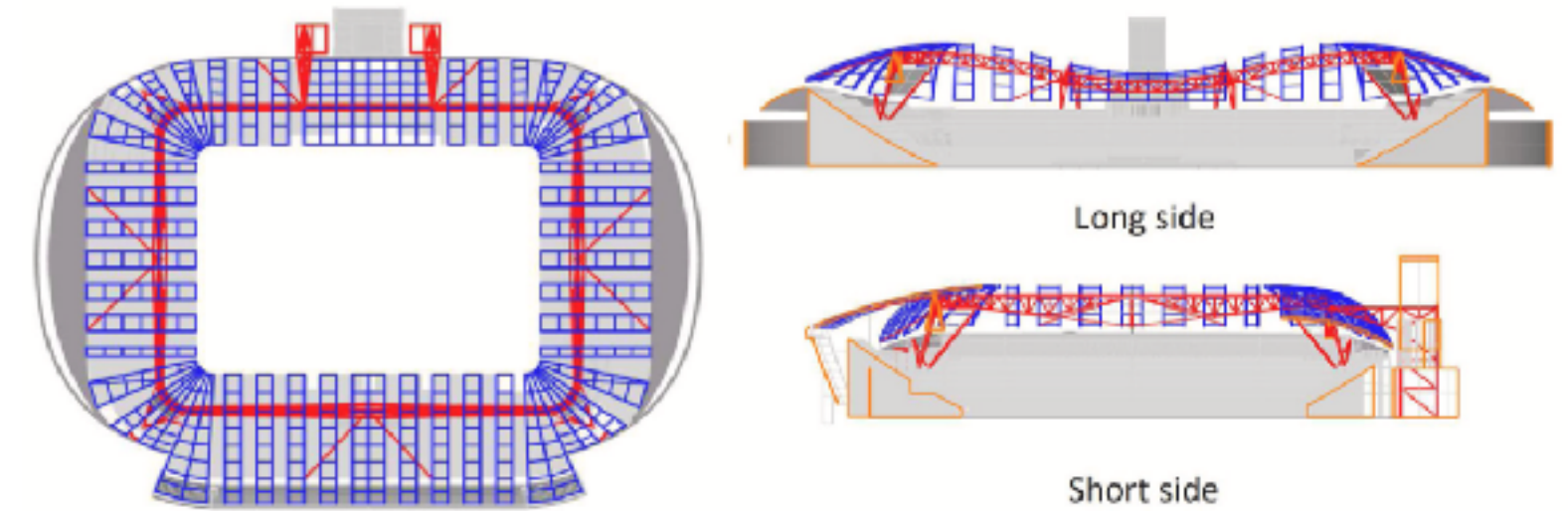
1964: Wind Tunnel Testing World Trade Center (Drs. J. E. Cermak and A.G. Davenport in the Colorado State University boundary layer wind tunnel)

THE ALAN G. DAVENPORT WIND LOADING CHAIN





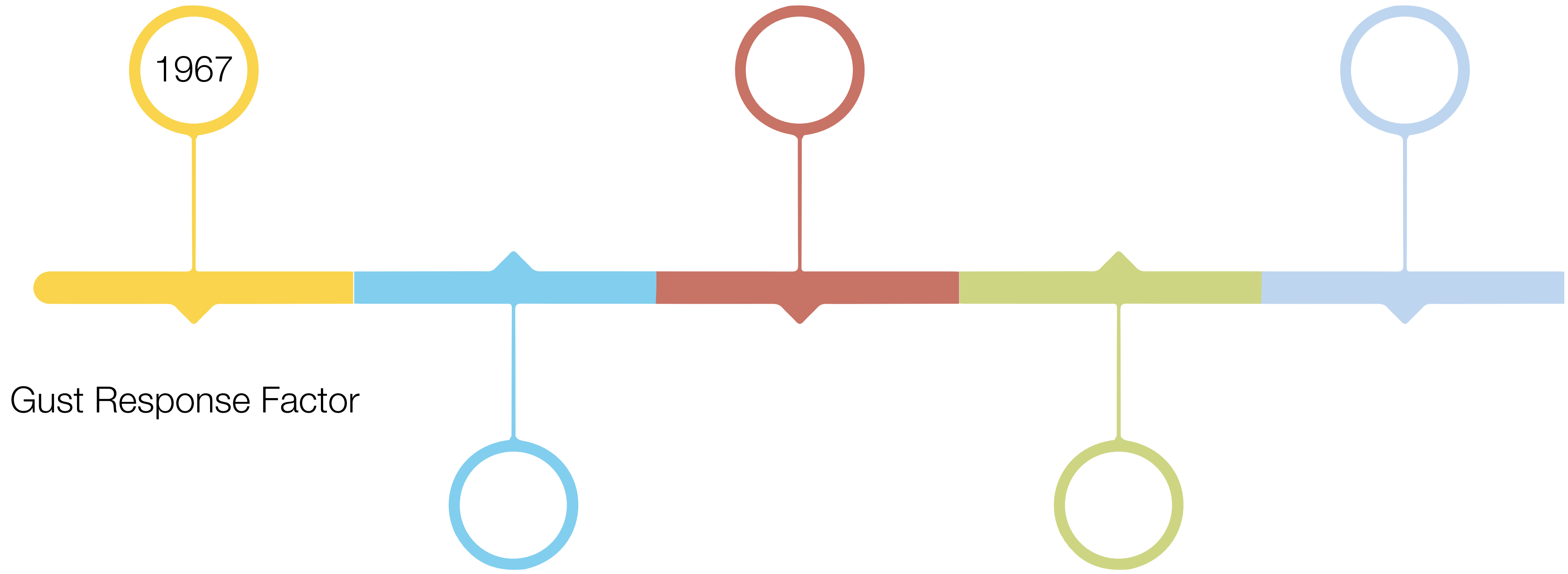
Wind tunnel testing



Computational Fluid Dynamics



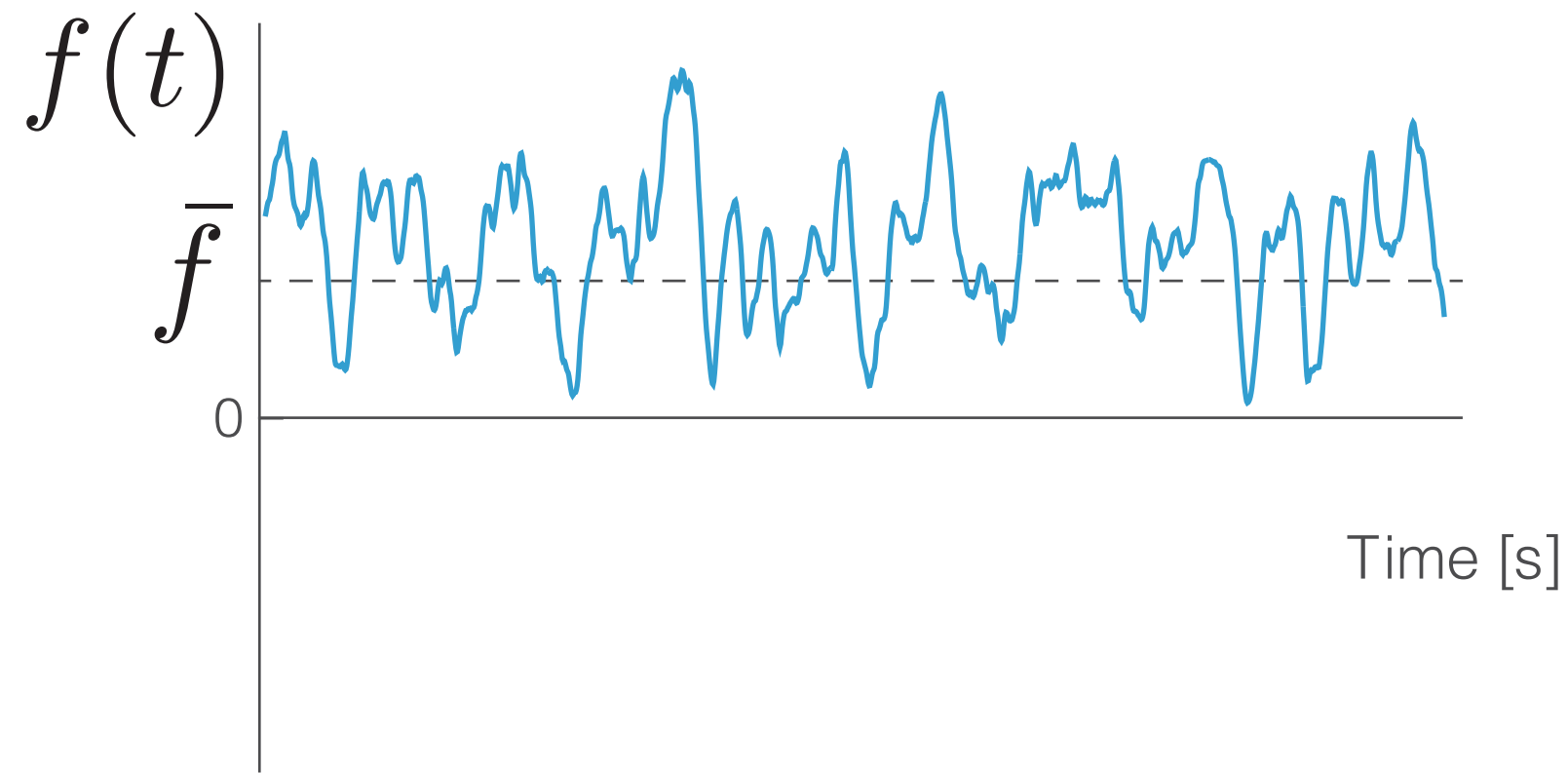
Part I : Equivalent Static Wind Loads



Gust Response Factor

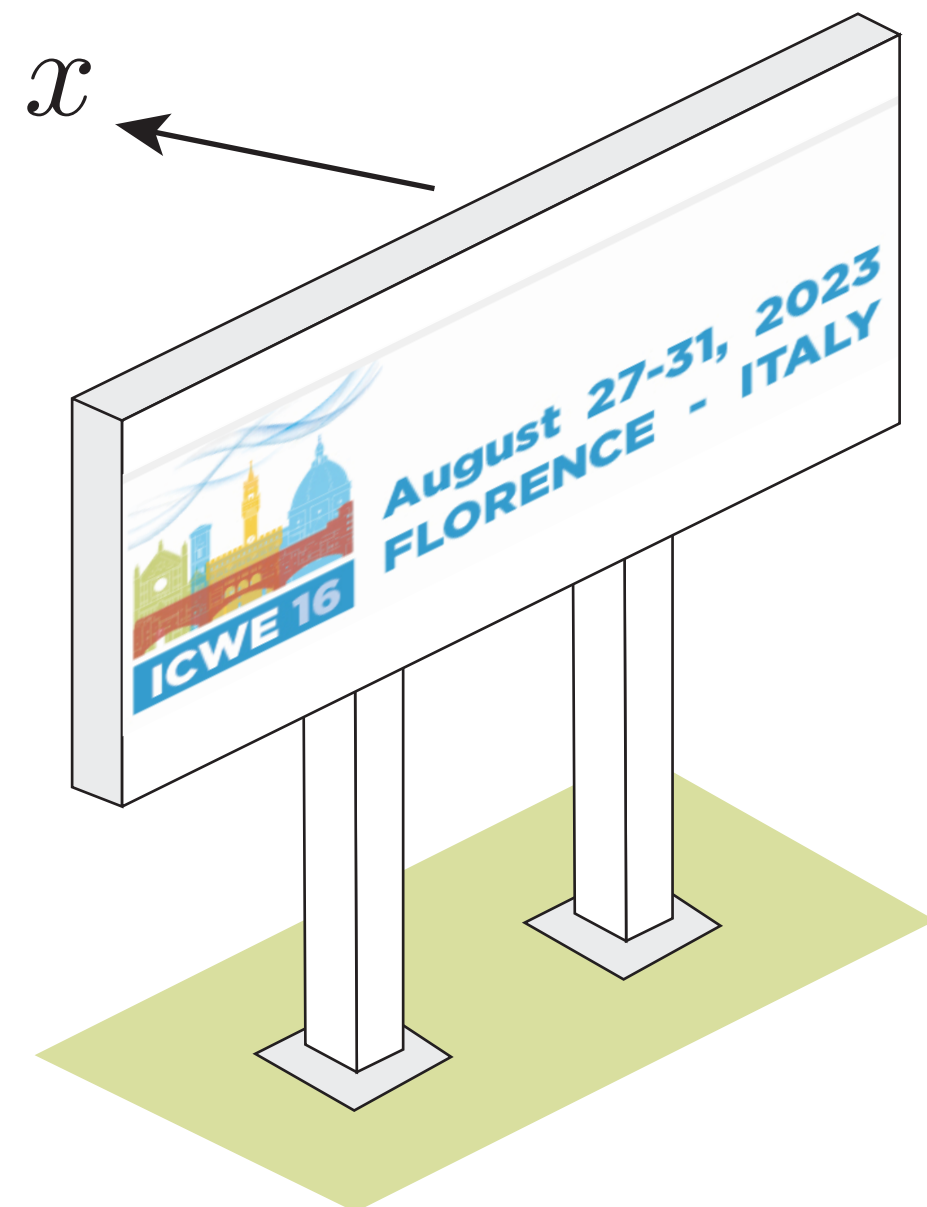
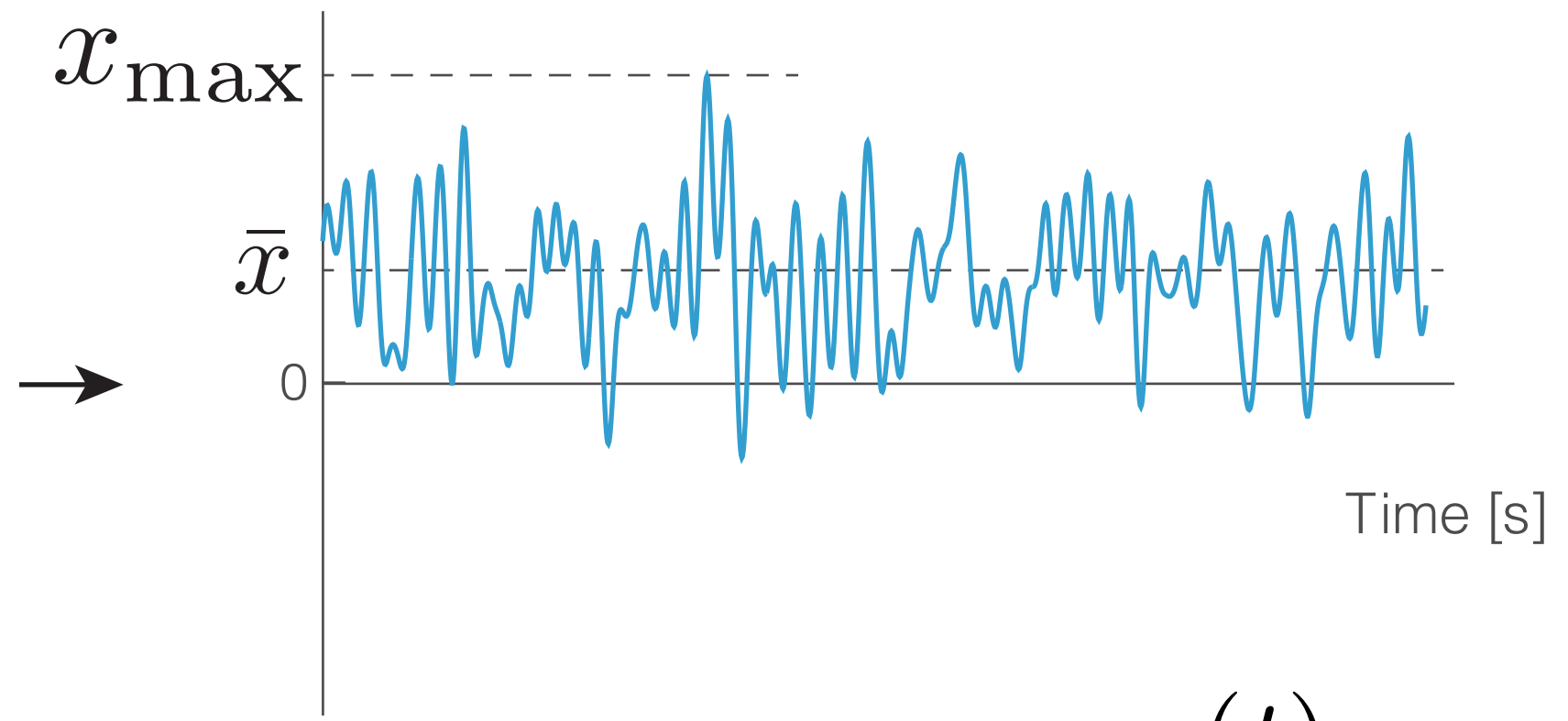
Davenport, 1964-1967

1-DOF Mass-spring oscillator - drag only



dynamic

$$m\ddot{x} + c\dot{x} + kx = f(t)$$



static

$$kx_{\max} = f_E$$

avoid ?

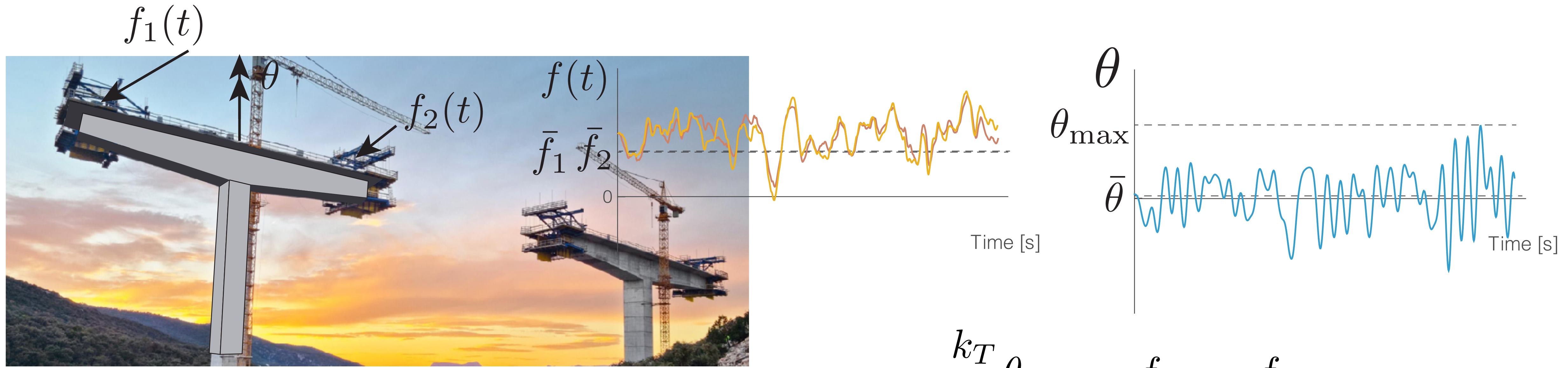
$$x_{\max} = \max_t x(t)$$

$$x_{\max} = \bar{x} + \Delta x_{\max} = \bar{x} + g \sigma_x$$

$$x_{\max} = \bar{x} \left(1 + g \frac{\sigma_x}{\bar{x}} \right) = \text{GRF } \bar{x}$$

$$f_E = k x_{\max} = \text{GRF } k \bar{x} = \text{GRF } \bar{f}$$

1-DOF torsional oscillator (balanced cantilever bridge)

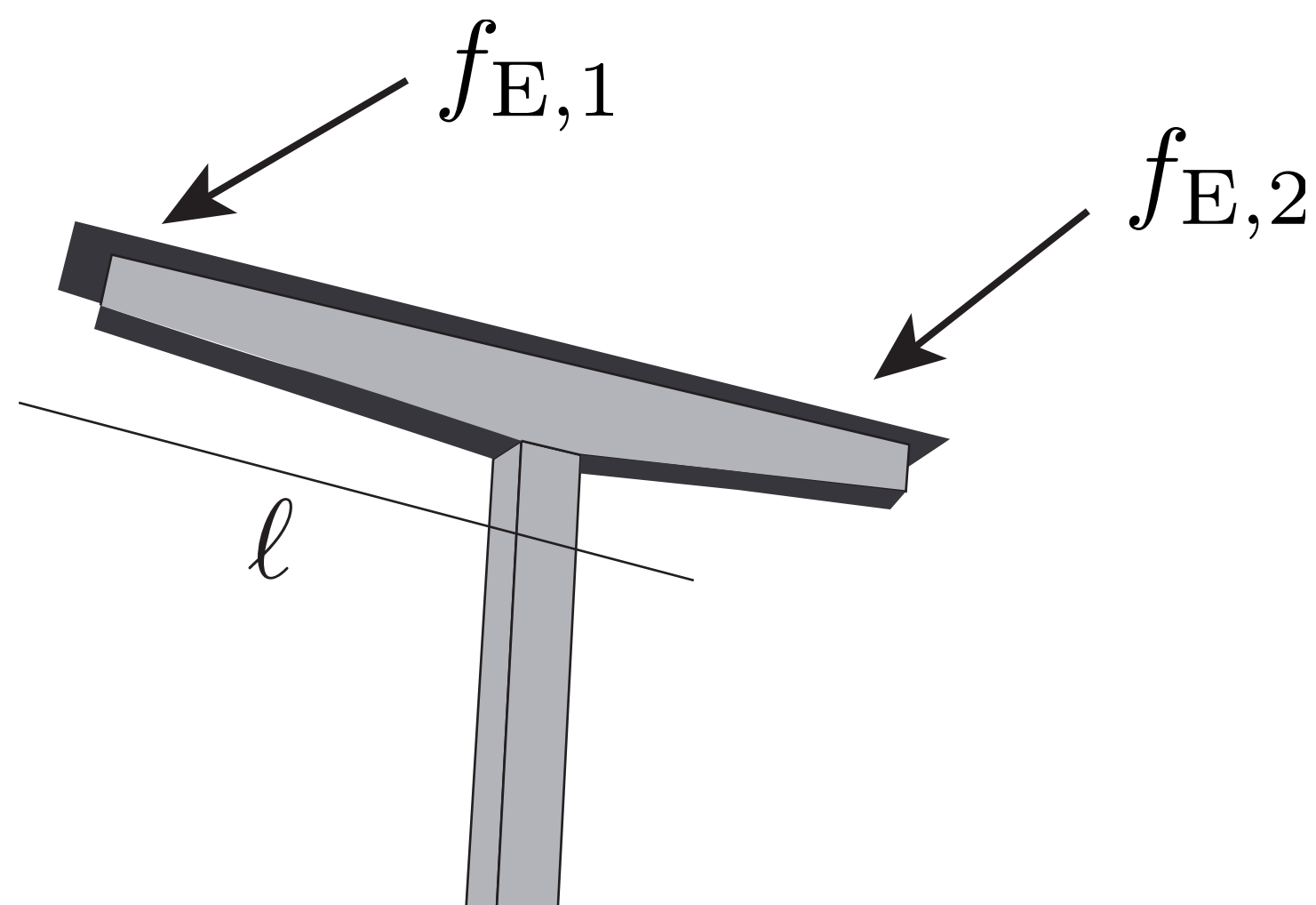


$$\frac{k_T}{\ell} \theta_{\max} = f_{E,2} - f_{E,1}$$

To use $\mathbf{f}_E \propto \bar{\mathbf{f}}$ doesn't work anymore !

GRF fails if small or zero average

However, we could use $f_{E,2} \neq f_{E,1}$ to create a torsional response



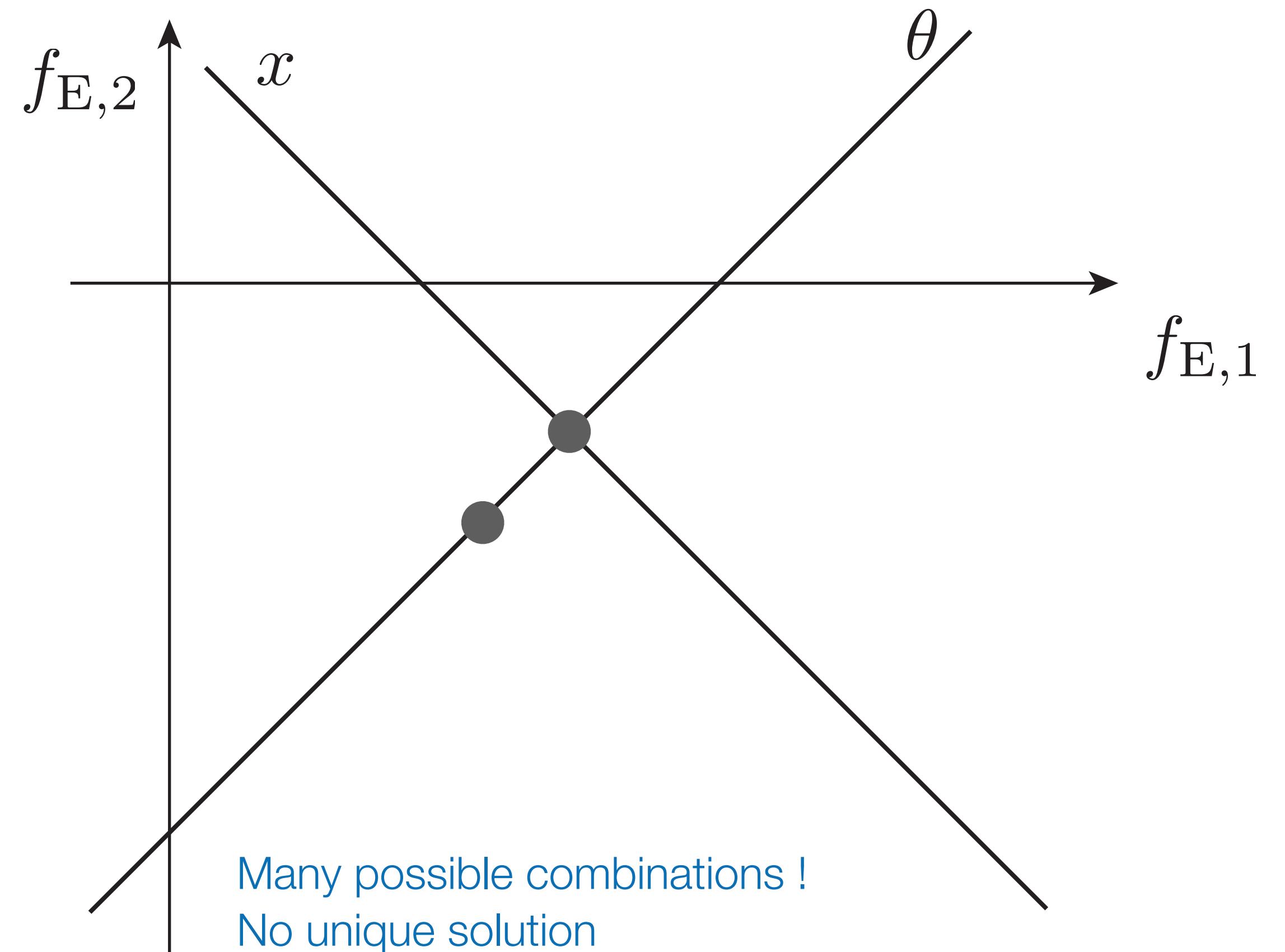
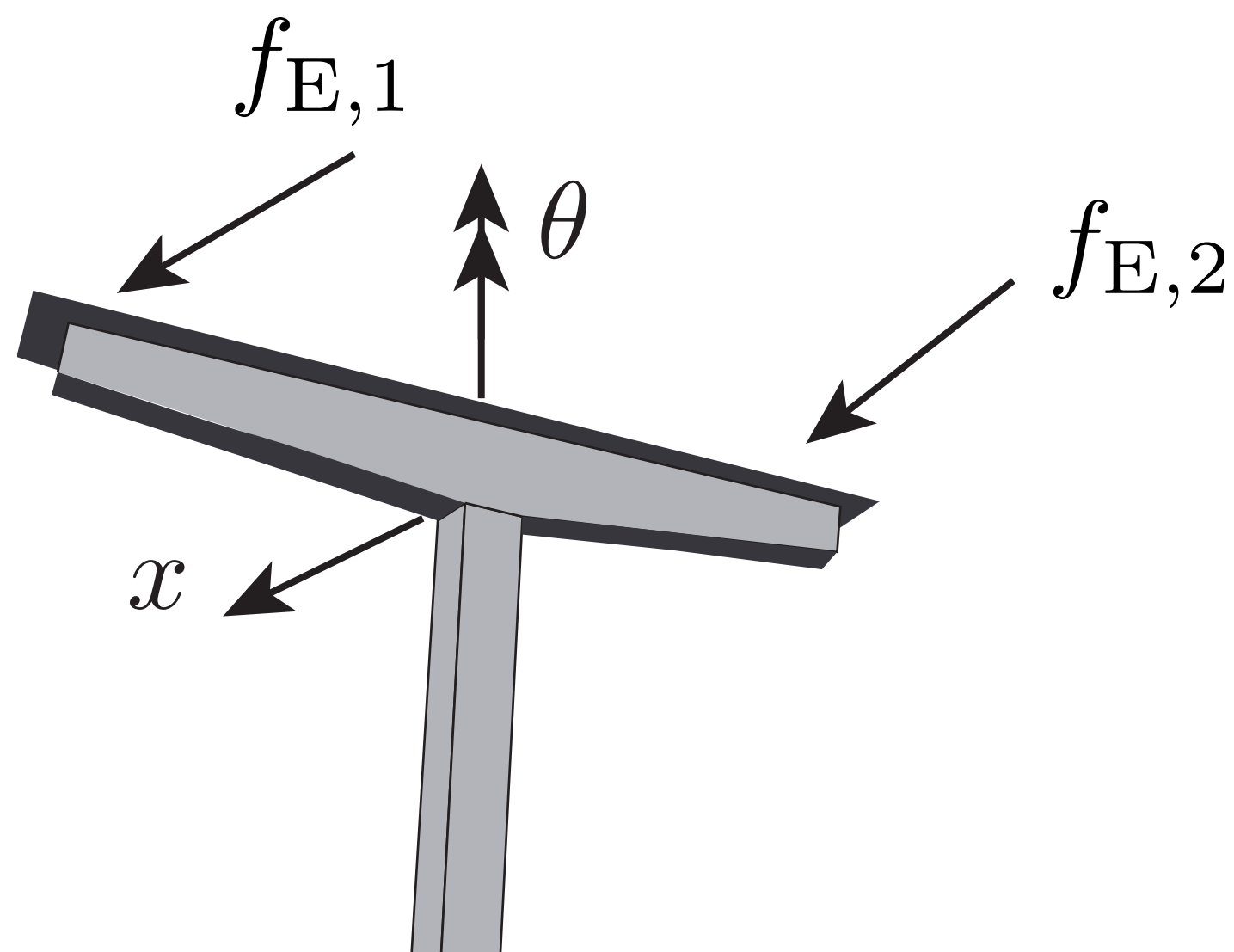
1-DOF torsional oscillator (balanced cantilever bridge)

$$\frac{k_T}{\ell} \theta_{\max} = f_{E,2} - f_{E,1}$$

$$\frac{k_B}{\ell} x_{\max} = f_{E,2} + f_{E,1}$$

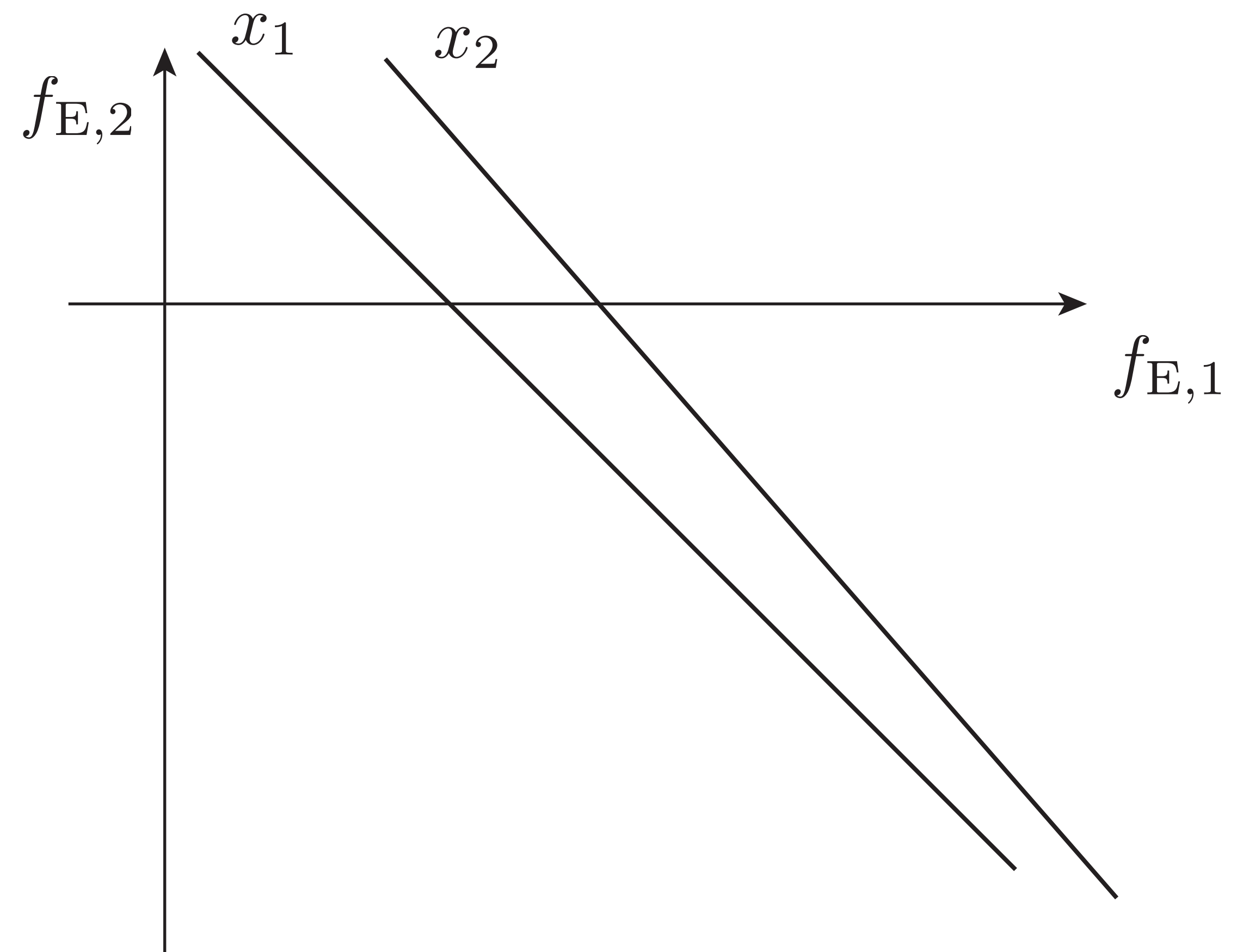
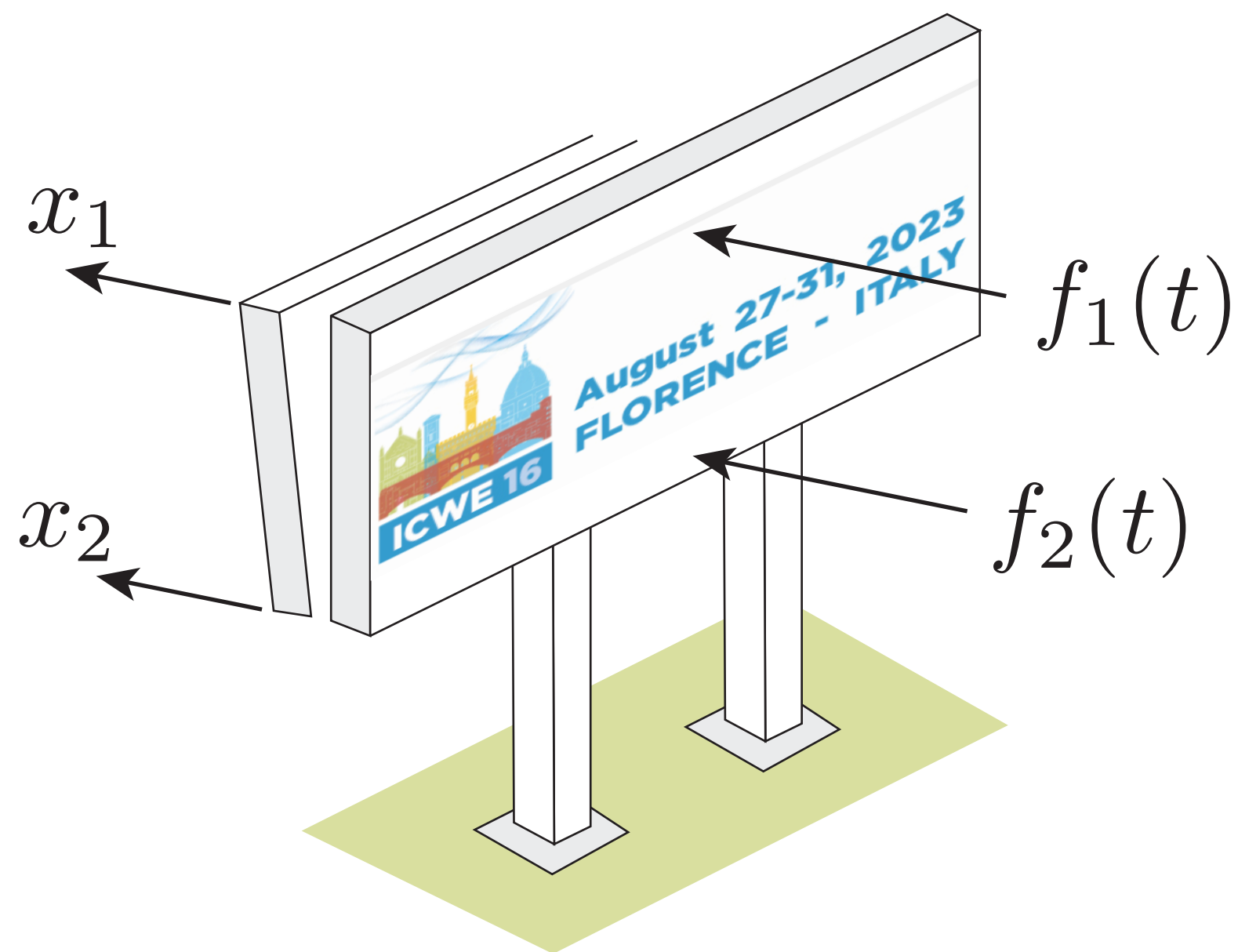
Nb. of LOADS = Nb. of responses

Problem is now properly closed
(but be careful again, this is an example)



(1) 2-DOF Mass-spring oscillator - drag only

Same problem, modeled as a 2-DOF structure (but $x_2(t) = \frac{1}{2}x_1(t) \quad \forall t$)





Static : replace dynamic analysis by static analysis

Equivalent (to what?) : should reproduce the same maximum response

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$$

$$\mathbf{z}(t) = \mathbf{A}\mathbf{x}(t)$$

$$\mathbf{z}_{\max} = \max_t \mathbf{z}(t)$$

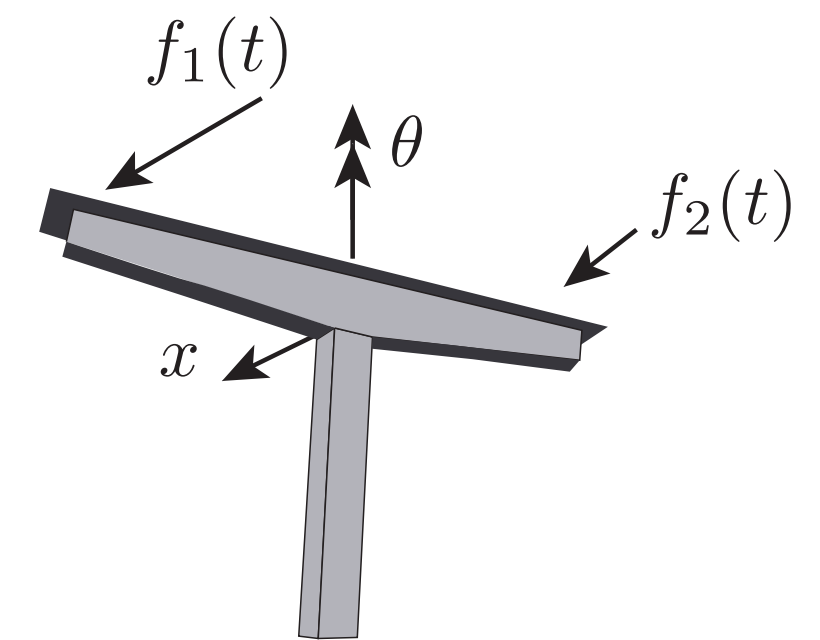
Find \mathbf{f}_E such that: $\mathbf{K}\mathbf{x}_E = \mathbf{f}_E$ and $\mathbf{A}\mathbf{x}_E = \mathbf{z}_{\max}$

Is the problem well posed ? Is the solution unique ?

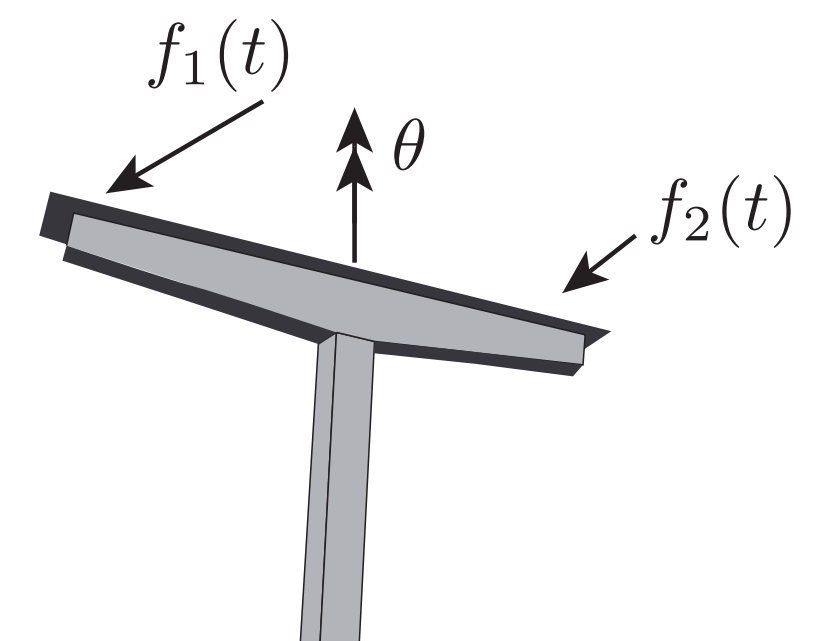
$$\mathbf{A}\mathbf{K}^{-1}\mathbf{f}_E = \mathbf{z}_{\max}$$

Nb. of *LOADS* = Nb. of *RESPONSES* \longleftrightarrow $\mathbf{A}\mathbf{K}^{-1}$ is square

Determined



Under-determined



Over-Determined



~~$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = \mathbf{f}$$~~

~~$$\mathbf{z}(t) = \Lambda \mathbf{x}(t) \quad \mathbf{z}_{\max} = \max_t \mathbf{z}(t)$$~~

~~Find \mathbf{f}_E such that: $K\mathbf{x}_E = \mathbf{f}_E$ and $\Lambda\mathbf{x}_E = \mathbf{z}_{\max}$~~

Informs about loading only

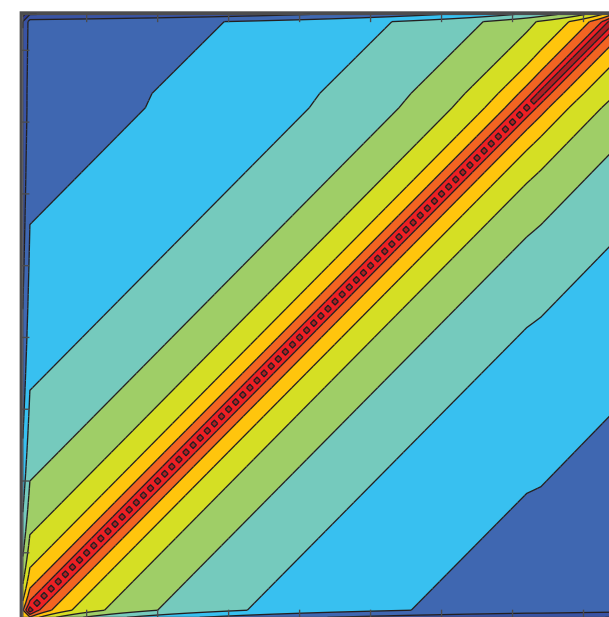
Average loading : $\bar{\mathbf{f}}$

Covariance of the loading : $\Sigma_{\mathbf{f}} = \mathbb{E} [\mathbf{f} \mathbf{f}^T]$

$$f_E^{\text{GRF}}$$

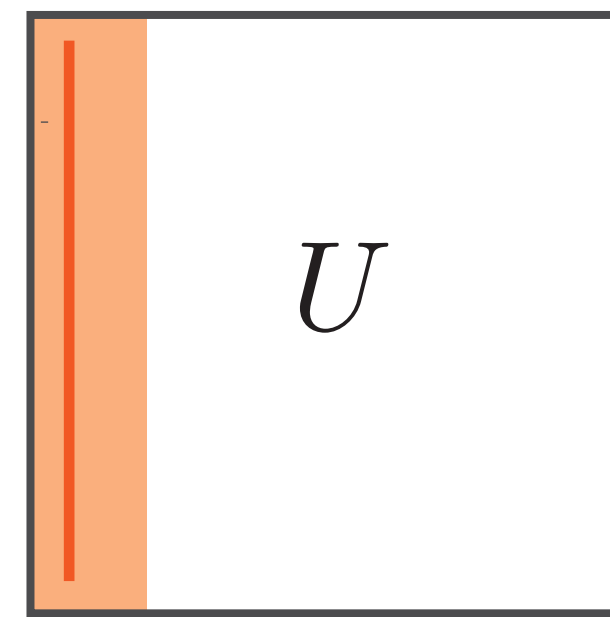
$$\mathbf{f}_E \propto \bar{\mathbf{f}}$$

$$f_E^{\text{CPT}}$$

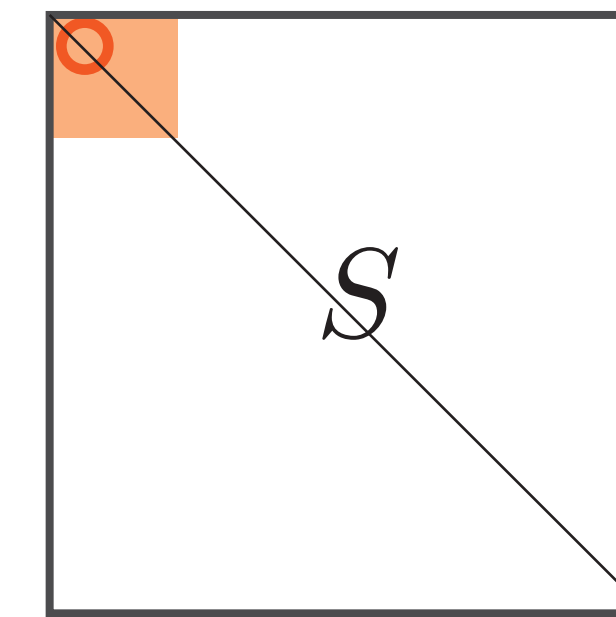


$\Sigma_{\mathbf{f}}$

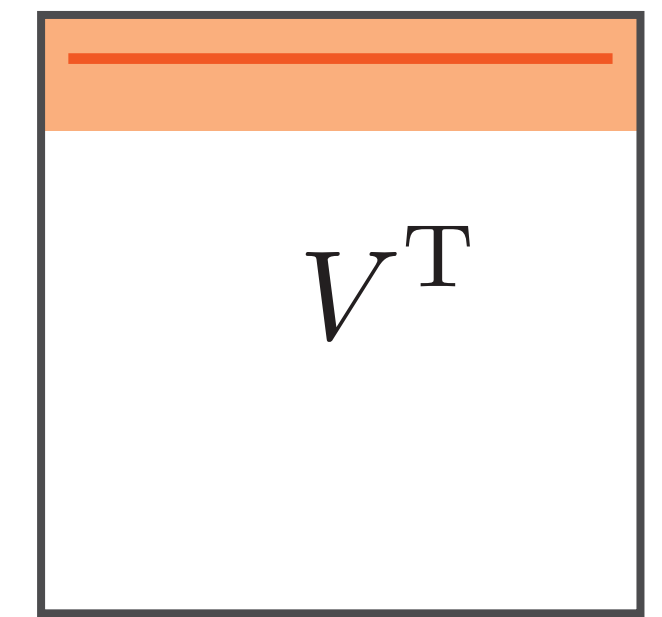
=



Eigen vectors

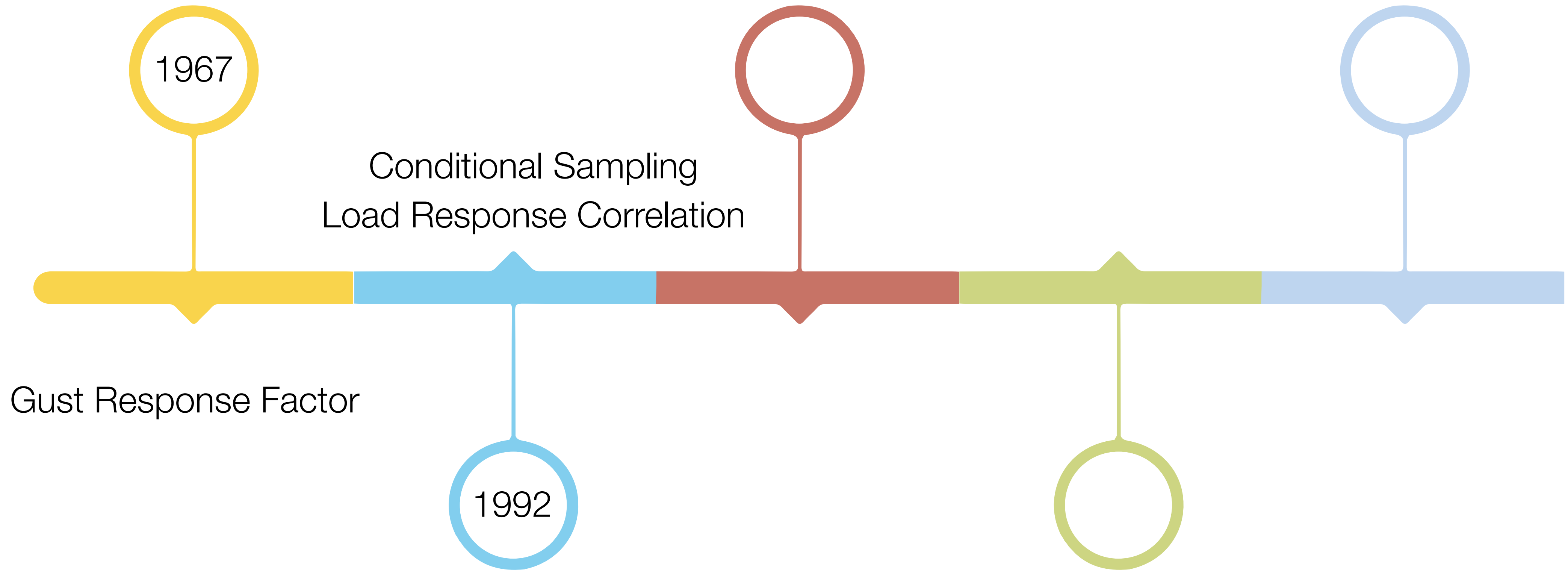


Eigen values





Davenport 1985; Holmes 1992;
Niemann @Bochum, 1992-95,
Holmes and Kasperski 1996;
Irwin 1998; Zhou et al. 2000



Davenport, 1967



~~$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$$~~

~~$$z(t) = \mathbf{a}^T \mathbf{x}(t)$$~~

$$z_{\max} = \max_t z(t)$$

$$z(t) = \mathbf{b}^T \mathbf{f}(t)$$

Find \mathbf{f}_E such that: ~~$\mathbf{K}\mathbf{x}_E = \mathbf{f}_E$~~ and ~~$\mathbf{a}^T \mathbf{x}_E = z_{\max}$~~

$$\mathbf{b}^T \mathbf{f}_E(t) = z_{\max}$$

- Memoryless I/O relation
- No dynamics
- Gaussian response
- (Unique peak factor)

Assume Gaussian response :

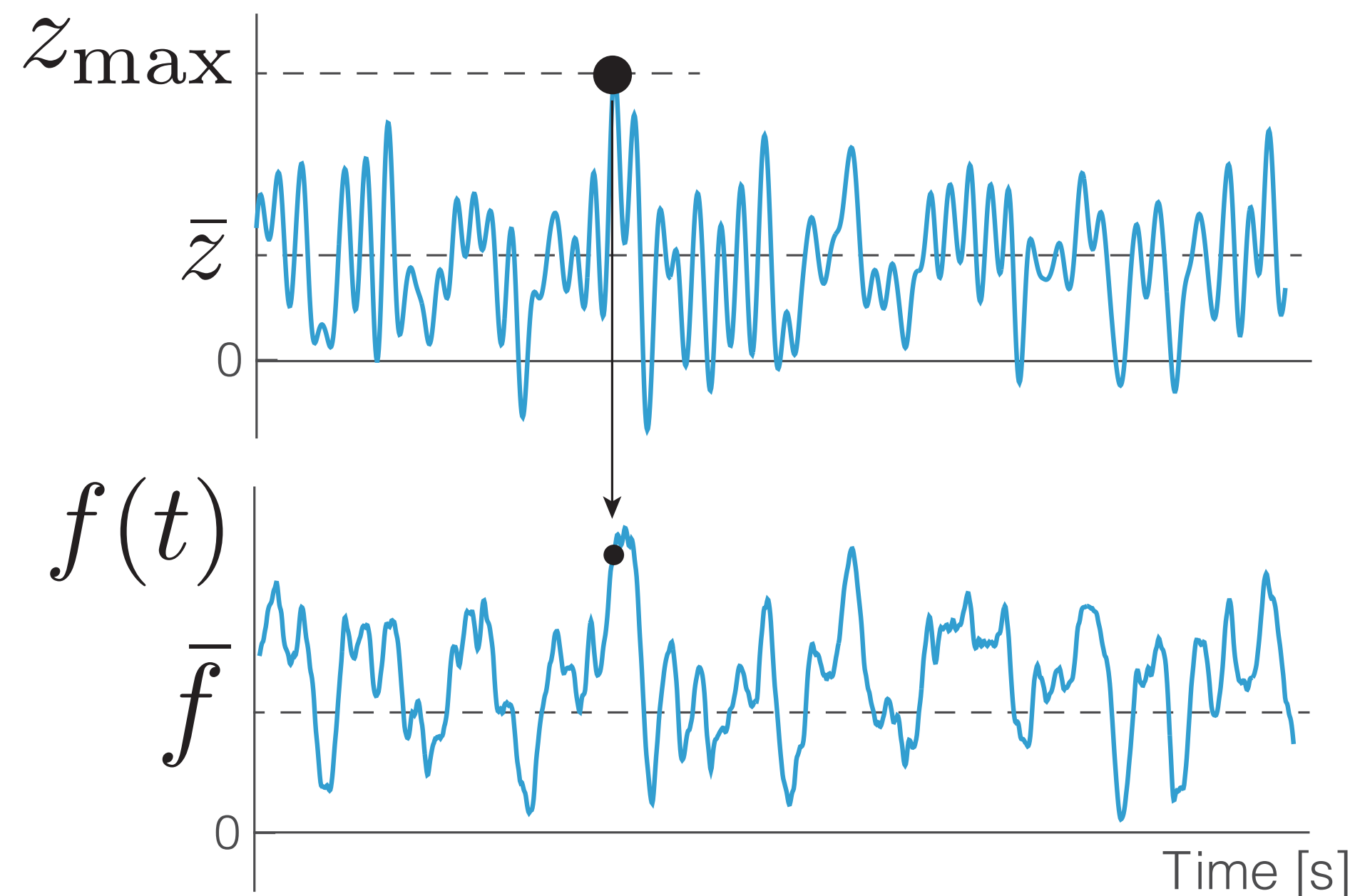
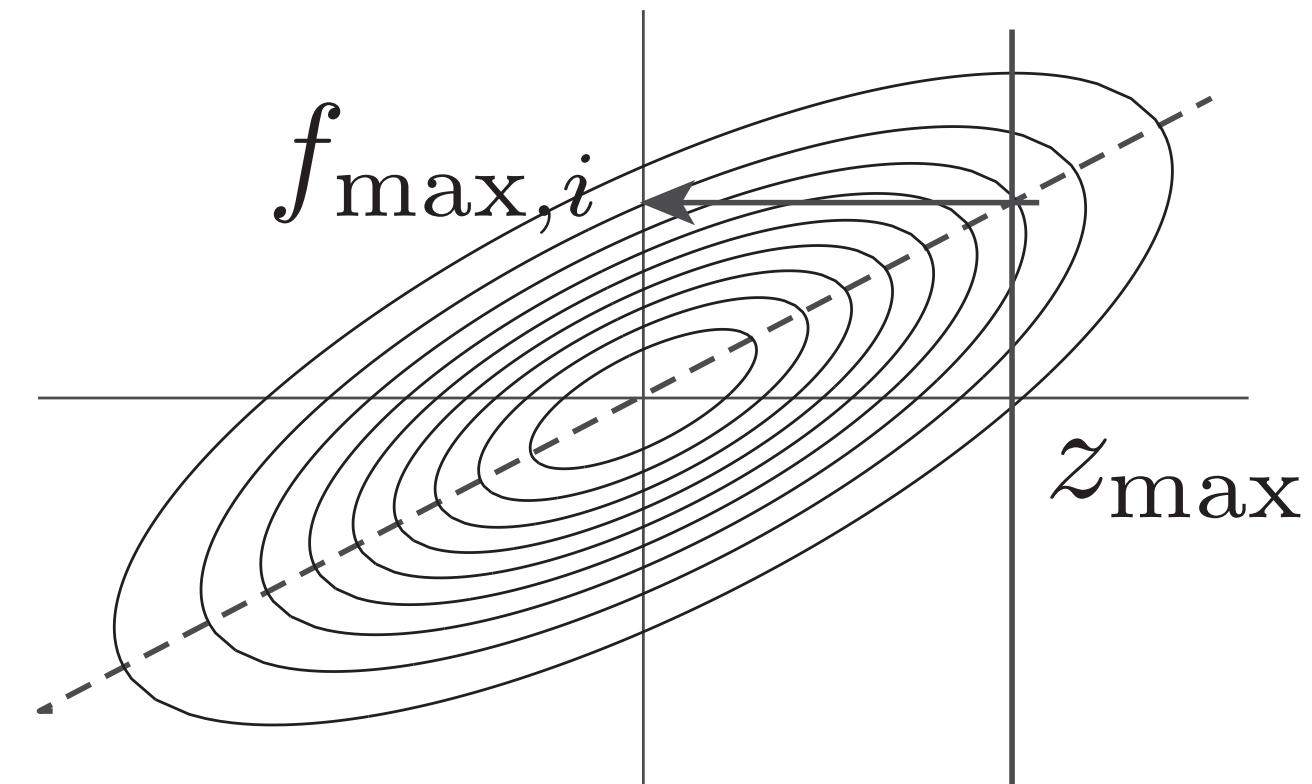
$$z_{\max} = g\sigma_z = g \frac{\sigma_z^2}{\sigma_z} = \frac{g}{\sigma_z} \sum_i \sum_j b_i b_j \text{COV}_{f,ij}$$

$$= \frac{g}{\sigma_z} \sum_i b_i \sum_j b_j \text{COV}_{f,ij} = g \sum_i b_i \rho_{zf,i} \sigma_{f,i}$$

Assume same peak factor :

$$f_{E,i}^{\text{LRC}} = \rho_{zf,i} f_{\max,i}$$

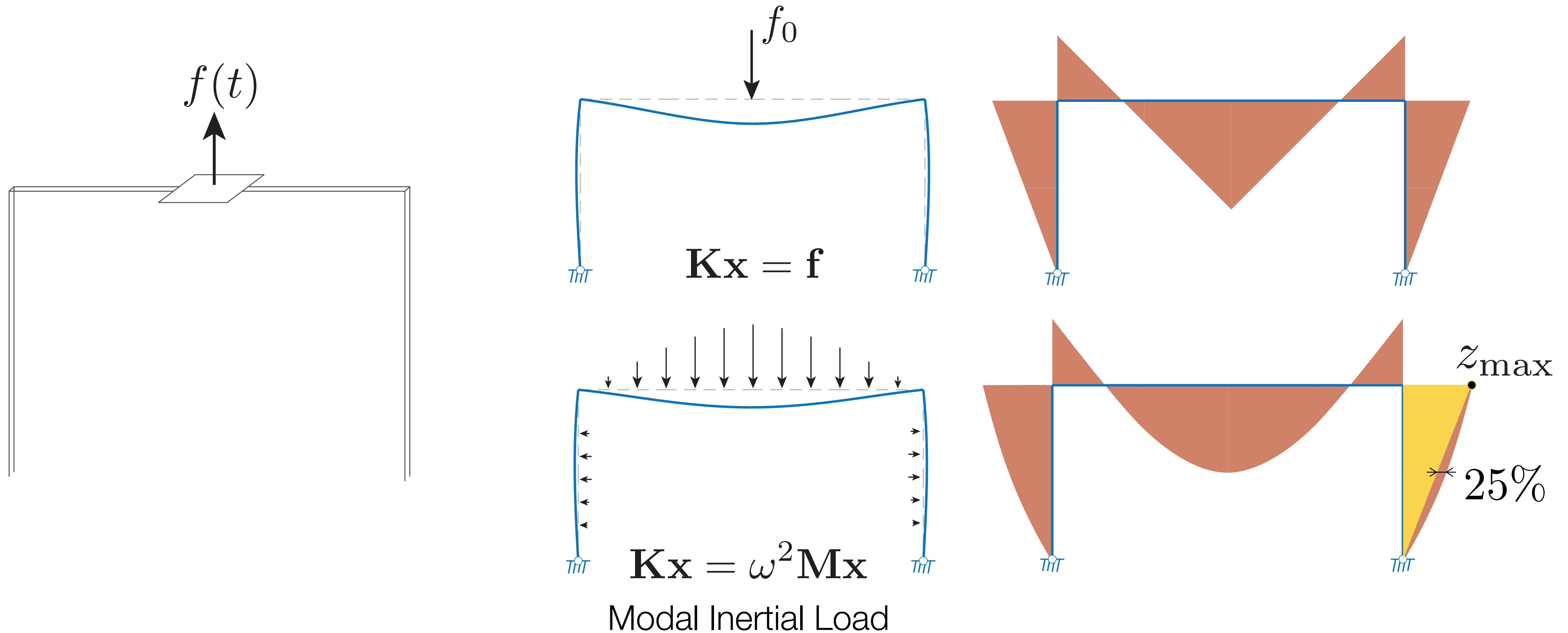
$$f_{E,i}^{\text{LRC}} = \rho_{zf,i} f_{\max,i}$$



- Memoryless I/O relation
- No dynamics
- Gaussian response
- (Unique peak factor)

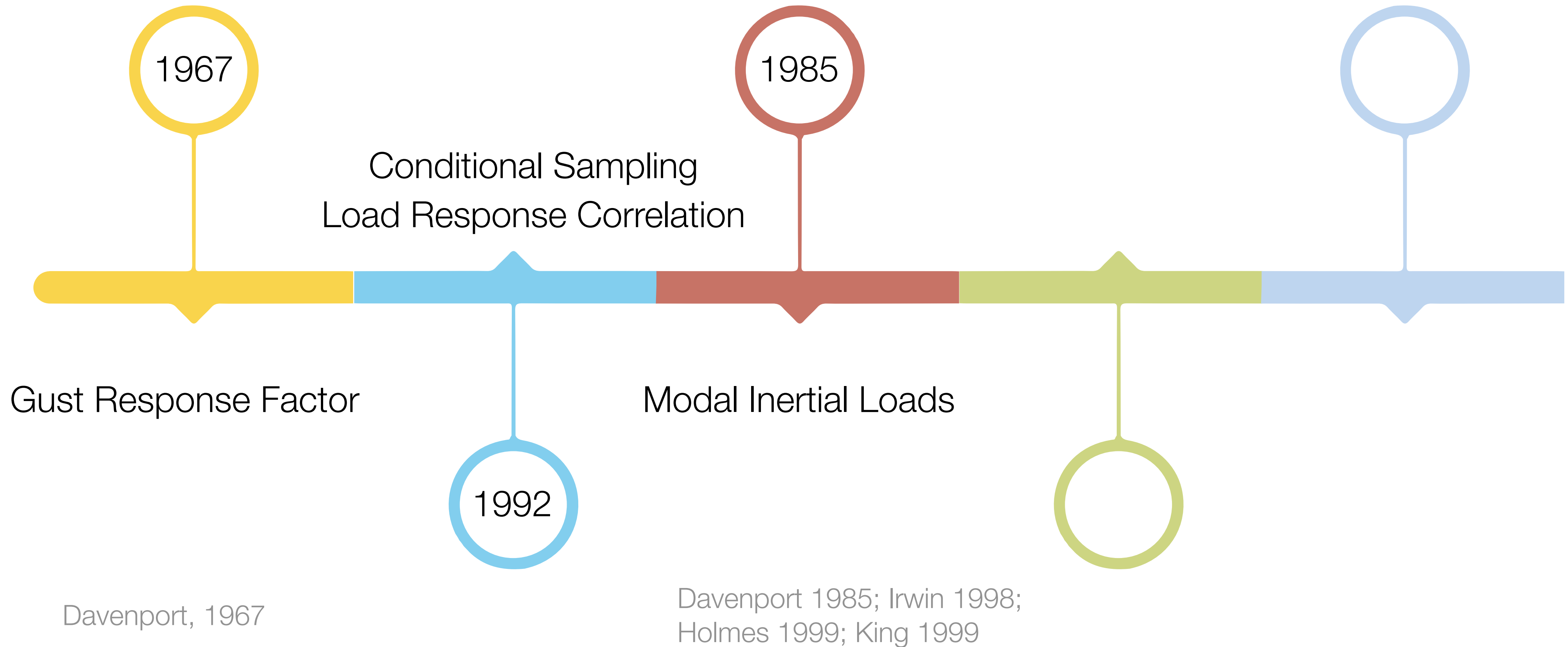
- Simple interpretation : most probable load
- Link with cond. sampling
- No overshooting of envelope (iif Gaussian)

The Load-Response Correlation method and the Conditional Sampling Technique have a drawback : equivalent loads are applied where pressures are measured





Davenport 1985; Holmes 1992;
Holmes and Kasperski 1996;
Irwin 1998; Zhou et al. 2000



$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$$

$$z(t) = \mathbf{a}^T \mathbf{x}(t) \simeq \mathbf{a}^T q(t) \phi_i \quad z_{\max} = \max_t z(t)$$

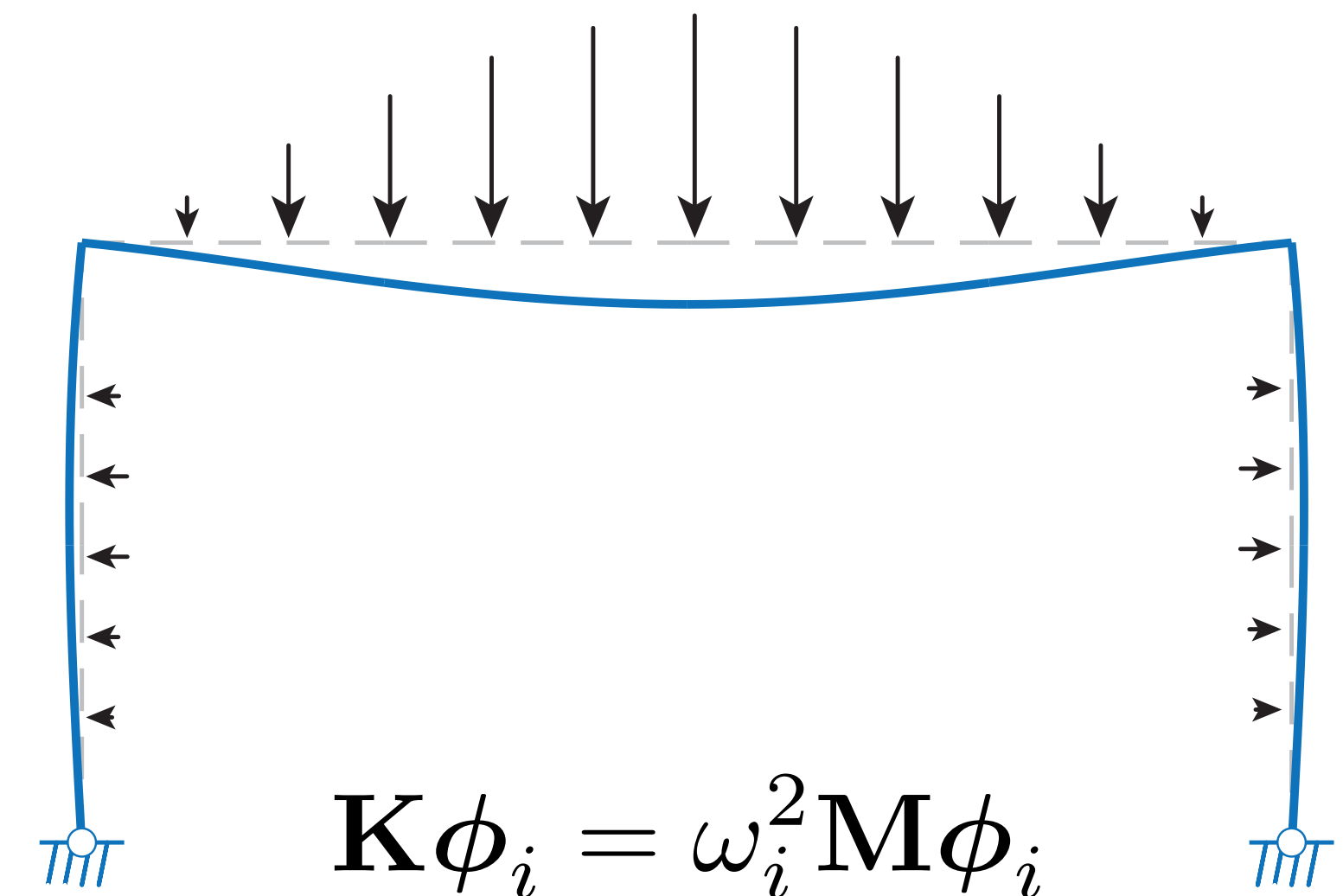
Find \mathbf{f}_E such that: $\mathbf{K}\mathbf{x}_E = \mathbf{f}_E$ and $\mathbf{a}^T \mathbf{x}_E = z_{\max}$

$$\sim \phi_i$$

$$\mathbf{f}_E^{\text{MIL}} = \mathbf{M}\phi_i$$

- Deformation = Mode shape
- Resonant Response
- Extension to multiple modes

Equivalent loads are applied where the mass is





LRC

MIL

Response

background

resonant

(wrong otherwise)

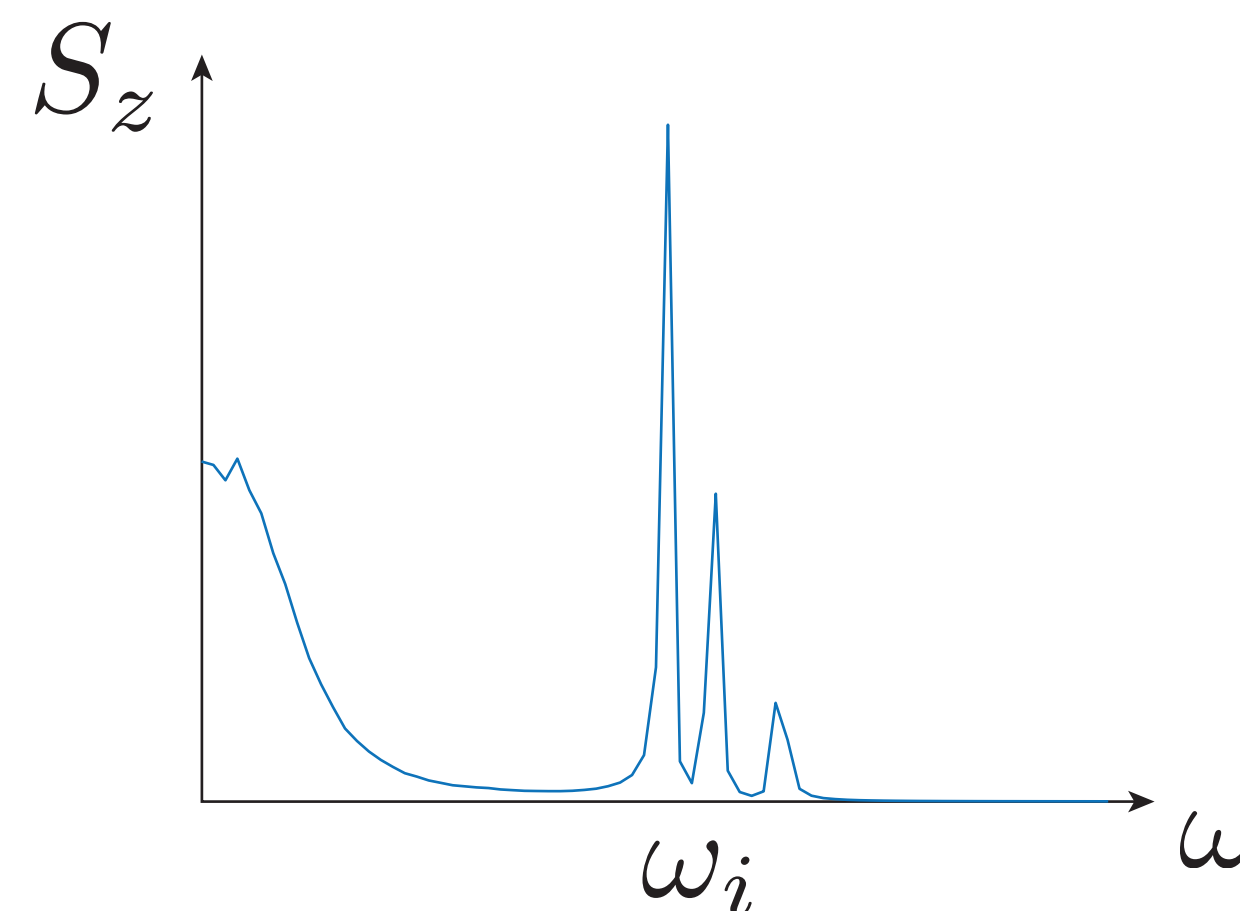
(wrong otherwise)

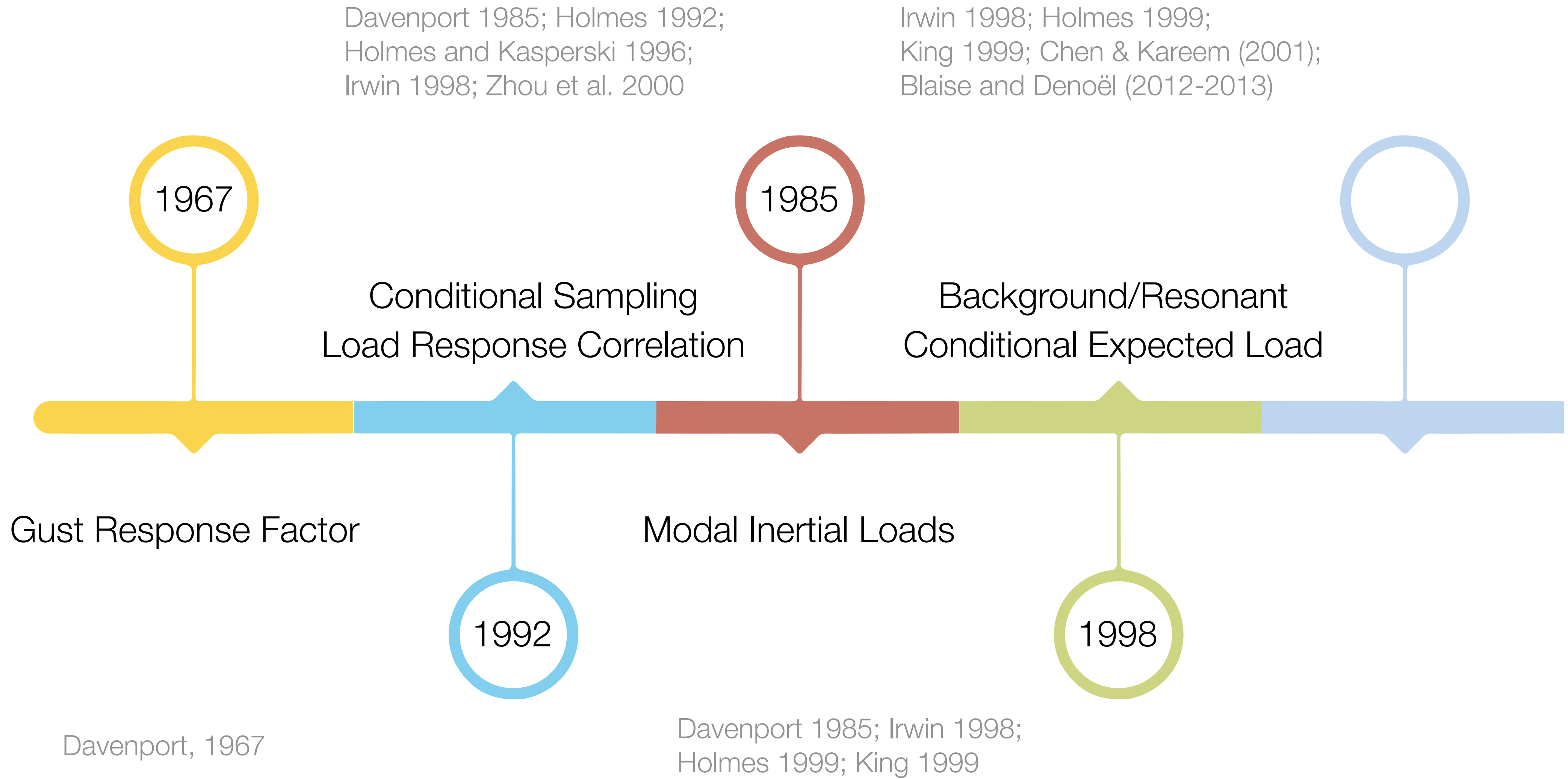
Load pattern

Most probable load
(conditional, Gaussianity)

Inertial loads
(conditional, Gaussianity)

Simple physical meaning





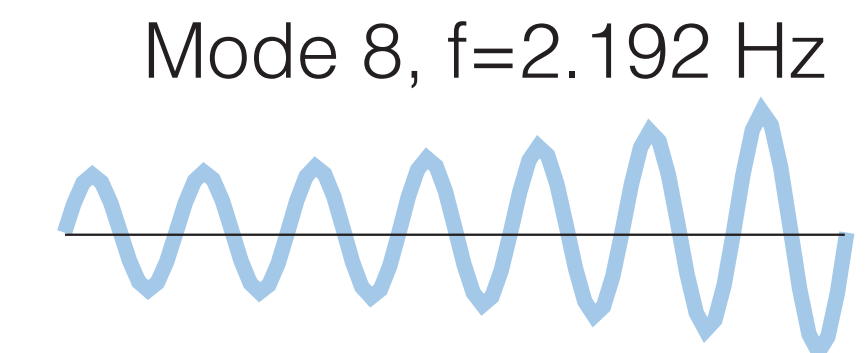
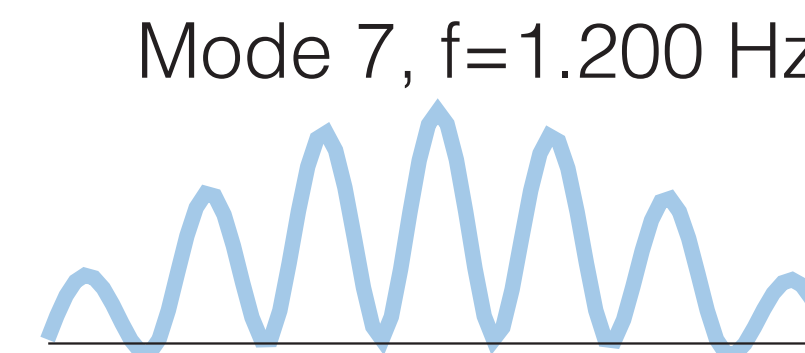
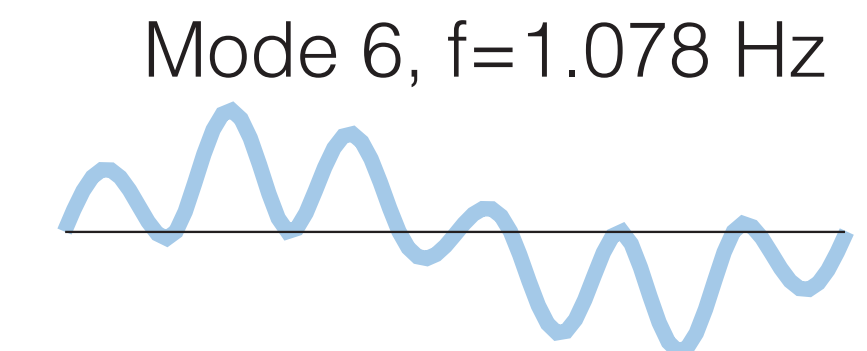
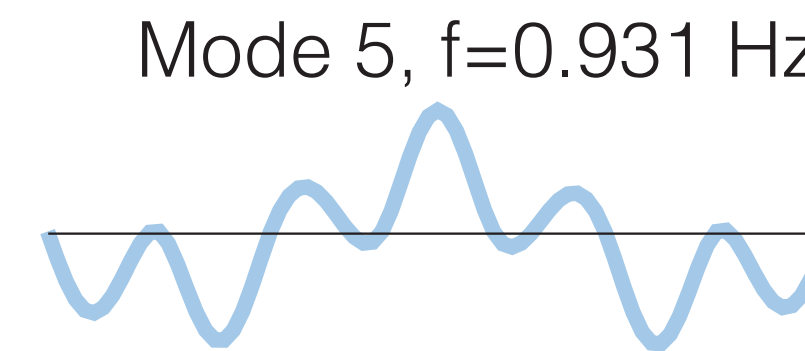
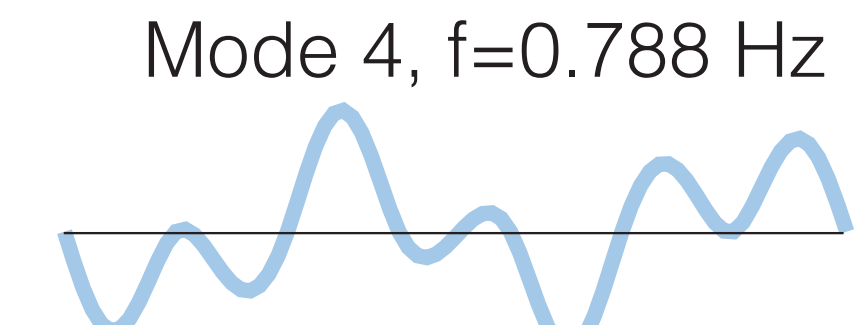
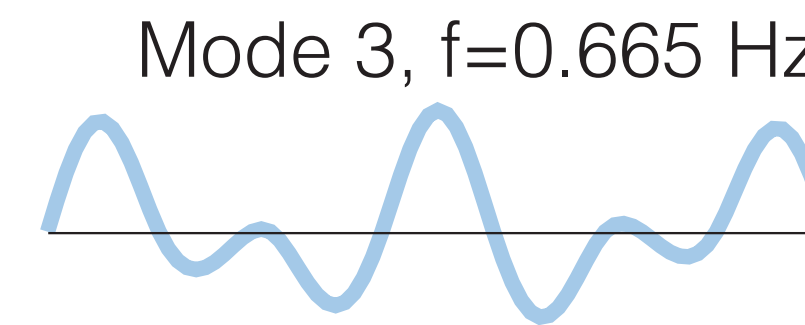
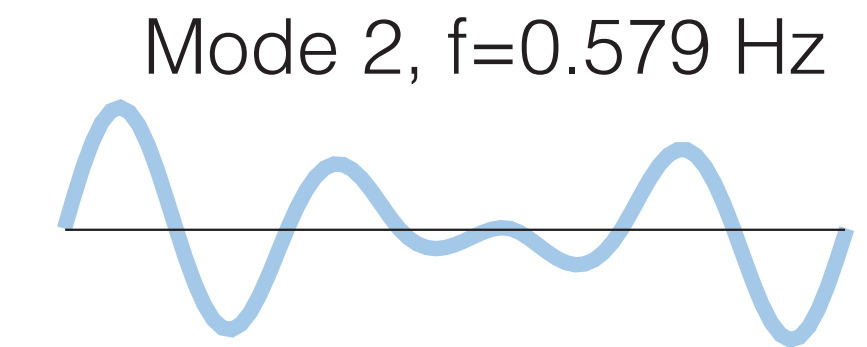
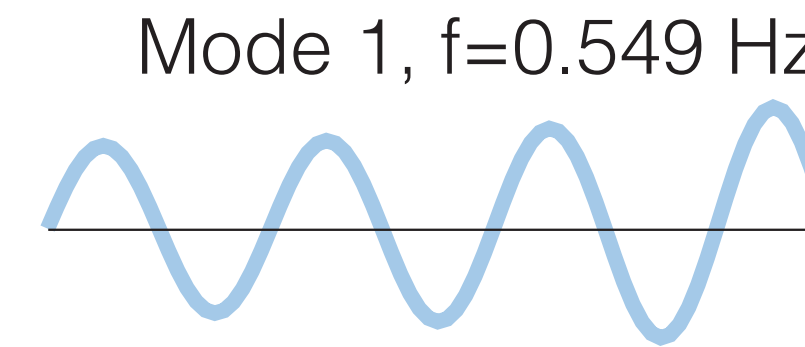


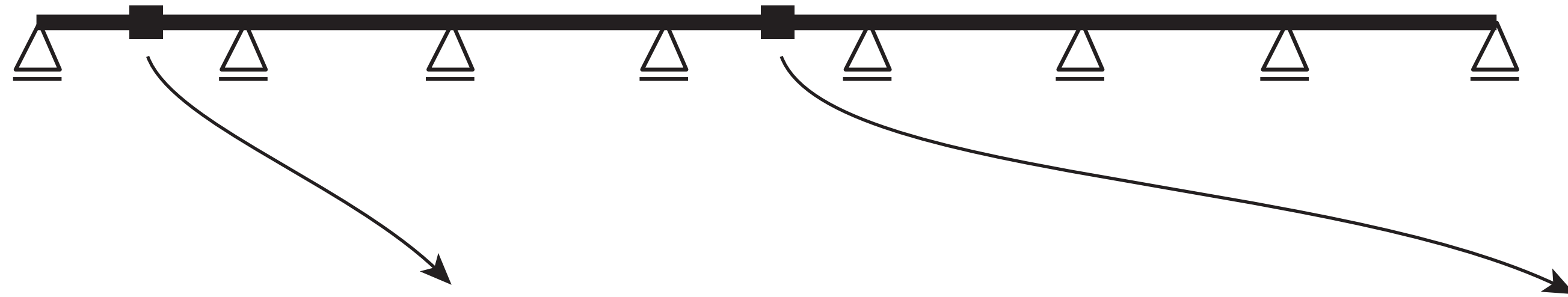
$$\mathbf{f}_E = W_b \mathbf{f}_E^{\text{LRC}} + \sum_i W_{r,i} \mathbf{f}_{E,i}^{\text{MIL}} \quad ; \quad W_b = \frac{\sigma_{z,b}}{\sigma_z} \quad ; \quad W_{r,i} = \sum_j \rho_{q,ij} \frac{\sigma_{z,r_j}}{\sigma_z}$$

- Weighted combination of LRC and MIL methods
- Degenerates into LRC and MIL
- In original formulation: nodal background & modal resonant
- Different variants :
 - without modal correlation (Holmes, 1999)
 - with modal correlation (Chen & Kareem, 2001)
 - including non-proportional damping (Blaise & Denoël, 2013)
 - ...

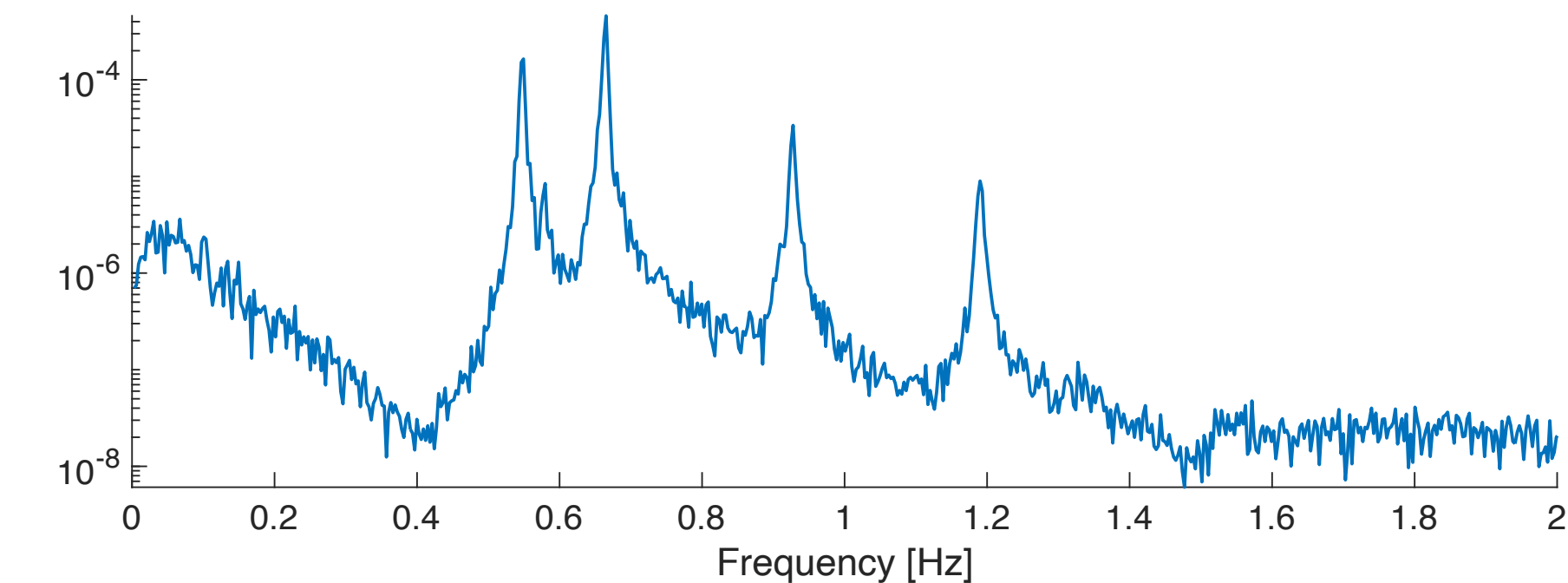
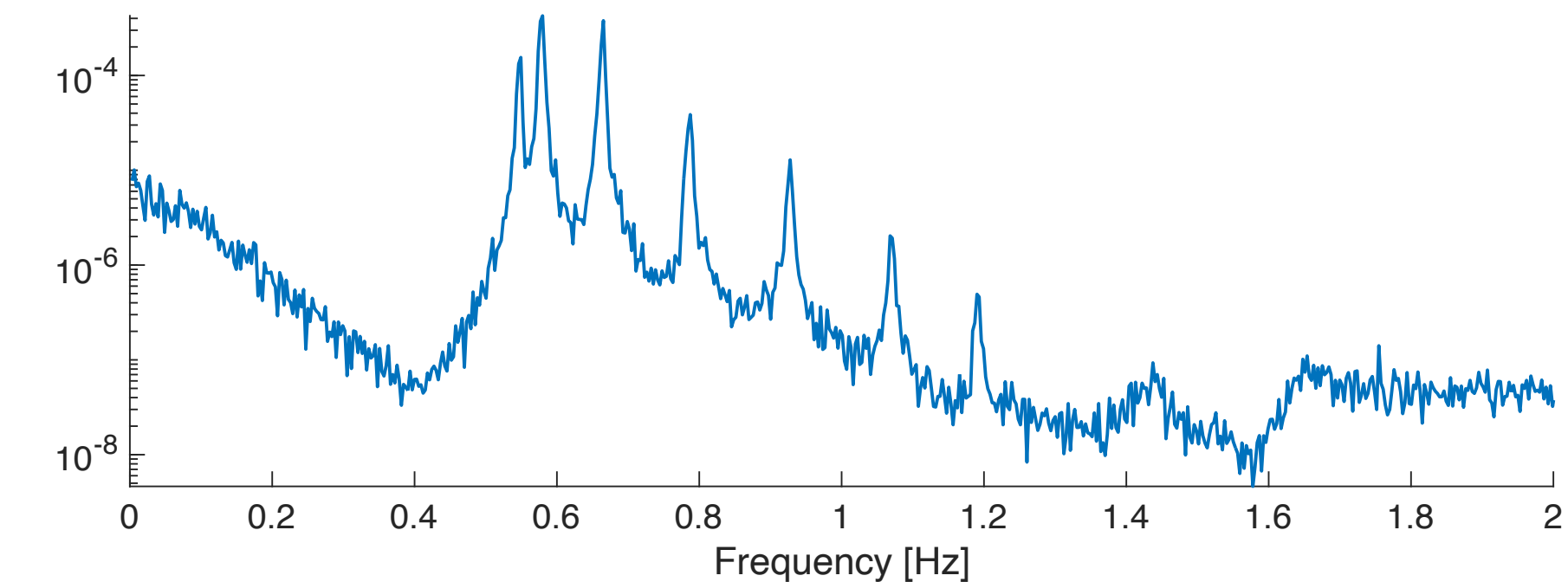
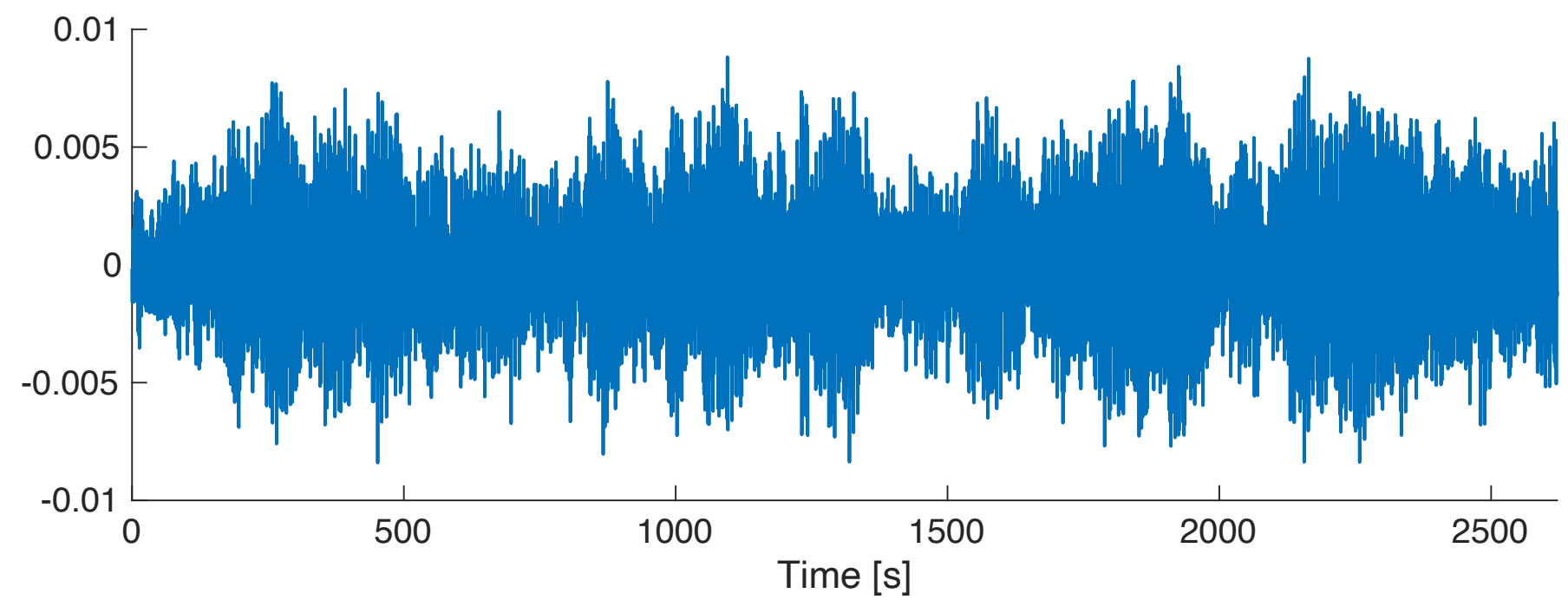
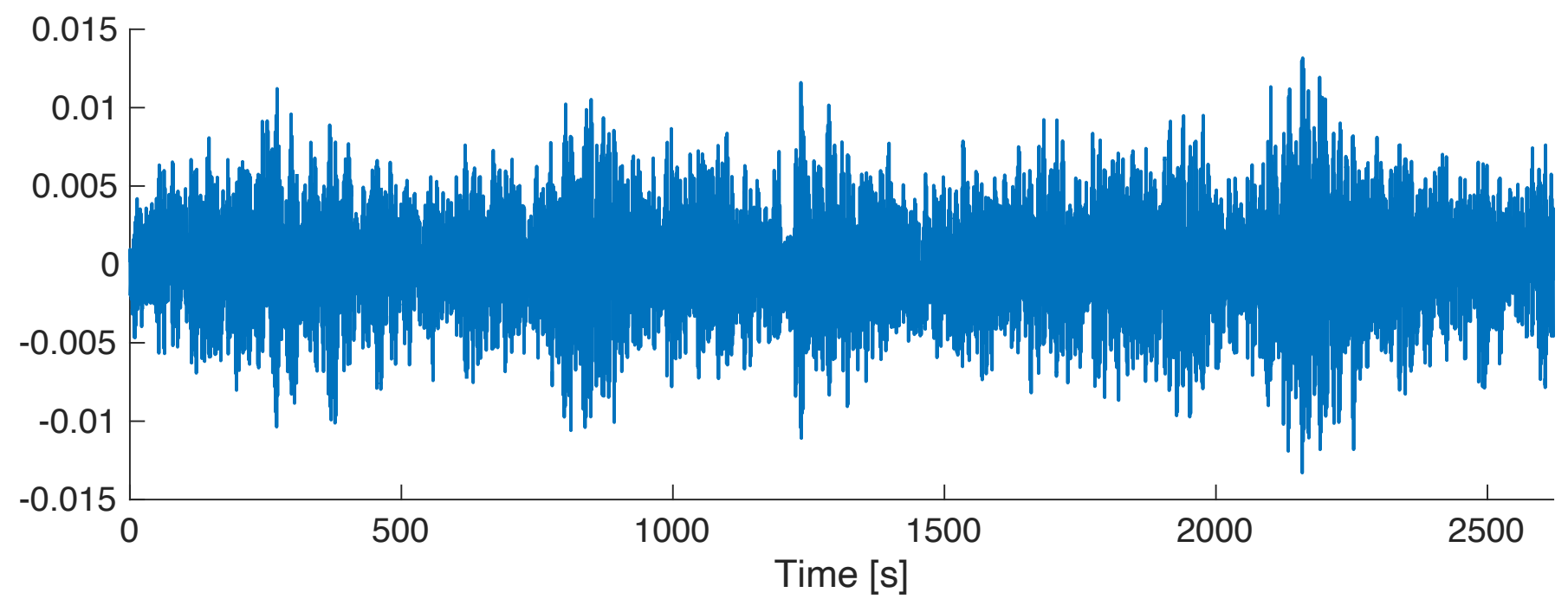


- 7-span viaduct (6@300m + 1@305m)
- Subject to turbulent wind :
 - Drag only
 - Spectrum : ENV1991-1.4, Annex B
 - Linearized loading (Gaussian)
- Model :
 - 84 Euler-Bernoulli finite elements
 - Mass = 10 to/m, $EI=10^{13}$ Nm²
- Analysis :
 - Time domain simulation
 - DT=0.04s, 65536 steps, T=2621 s

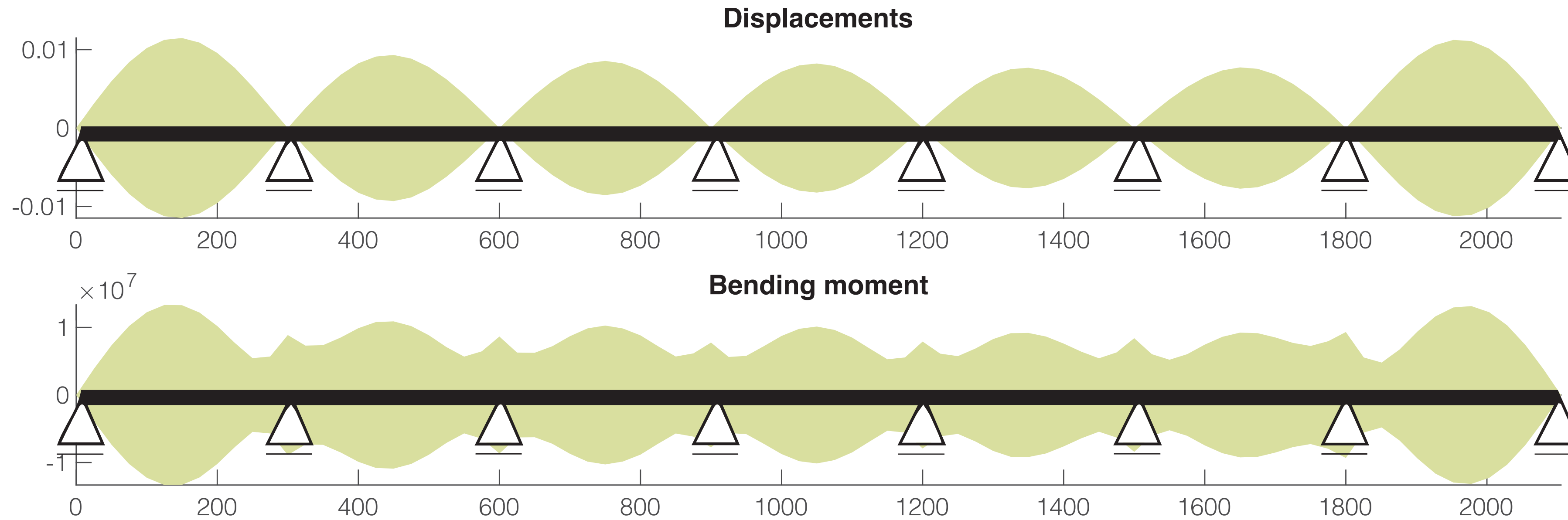




Responses are mostly resonant



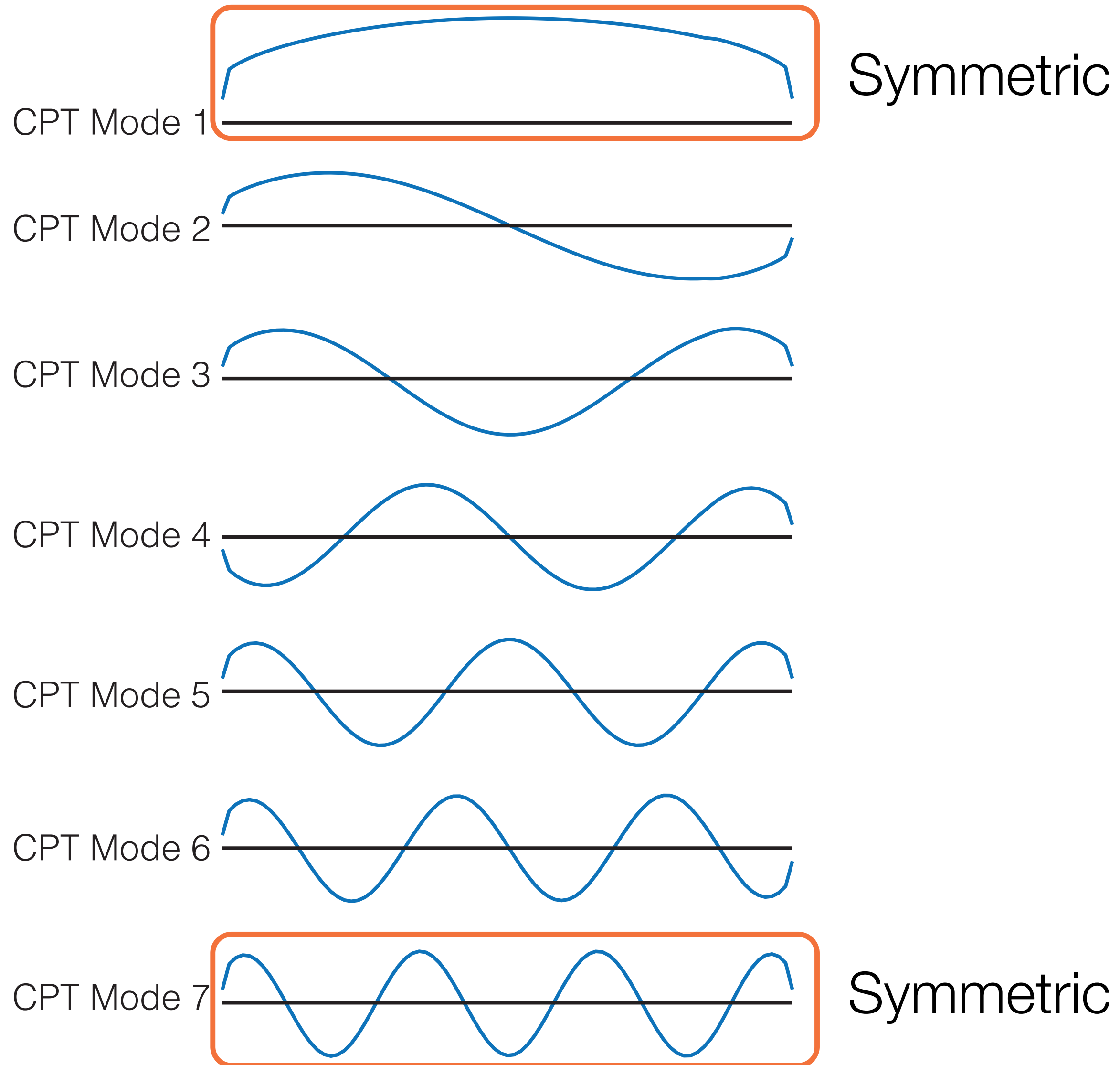
$$\mathbf{z} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \\ M_1 \\ \vdots \\ M_n \end{pmatrix}$$



Envelope = (Zero-mean) + 3.5 · (Standard deviation)

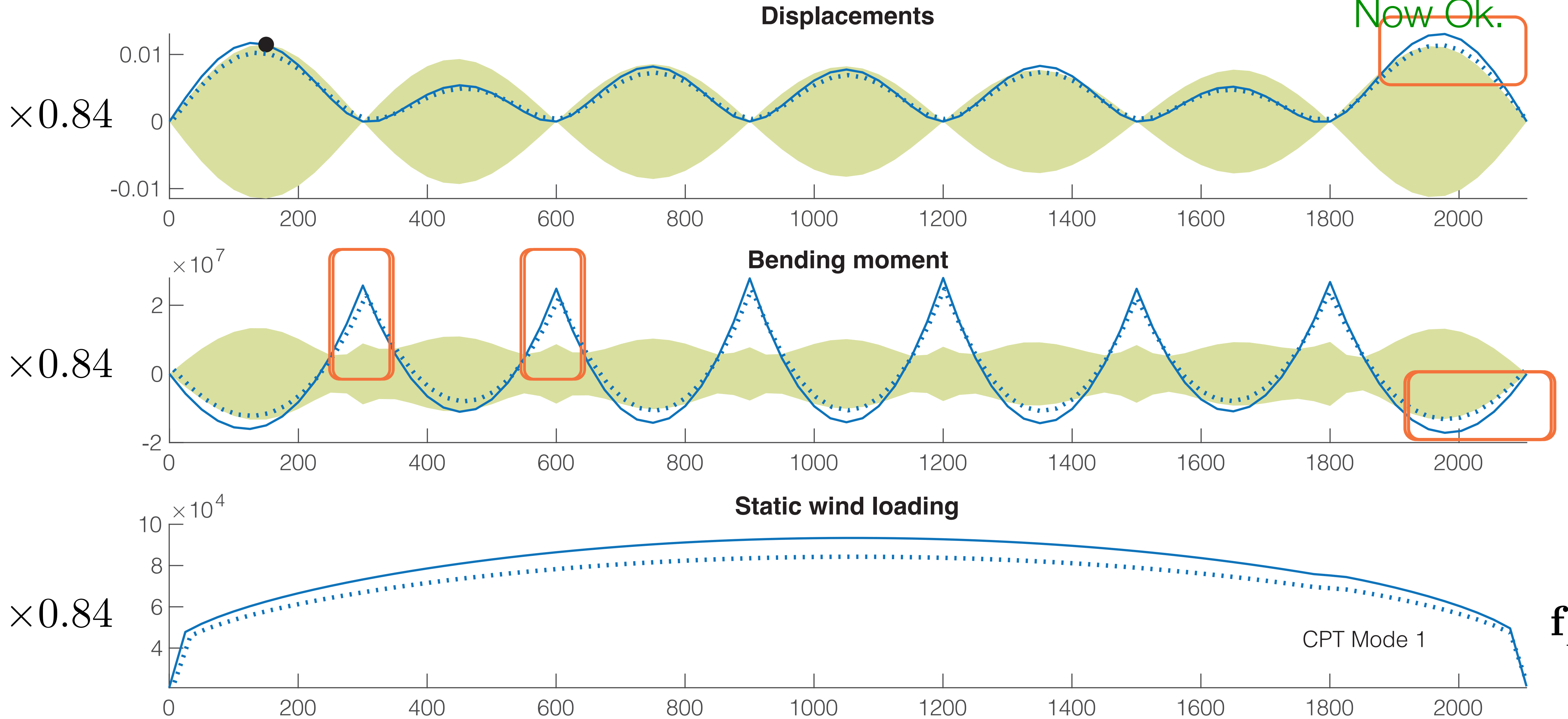
Next : (GRF), CPT, LRC, MIL, COMB

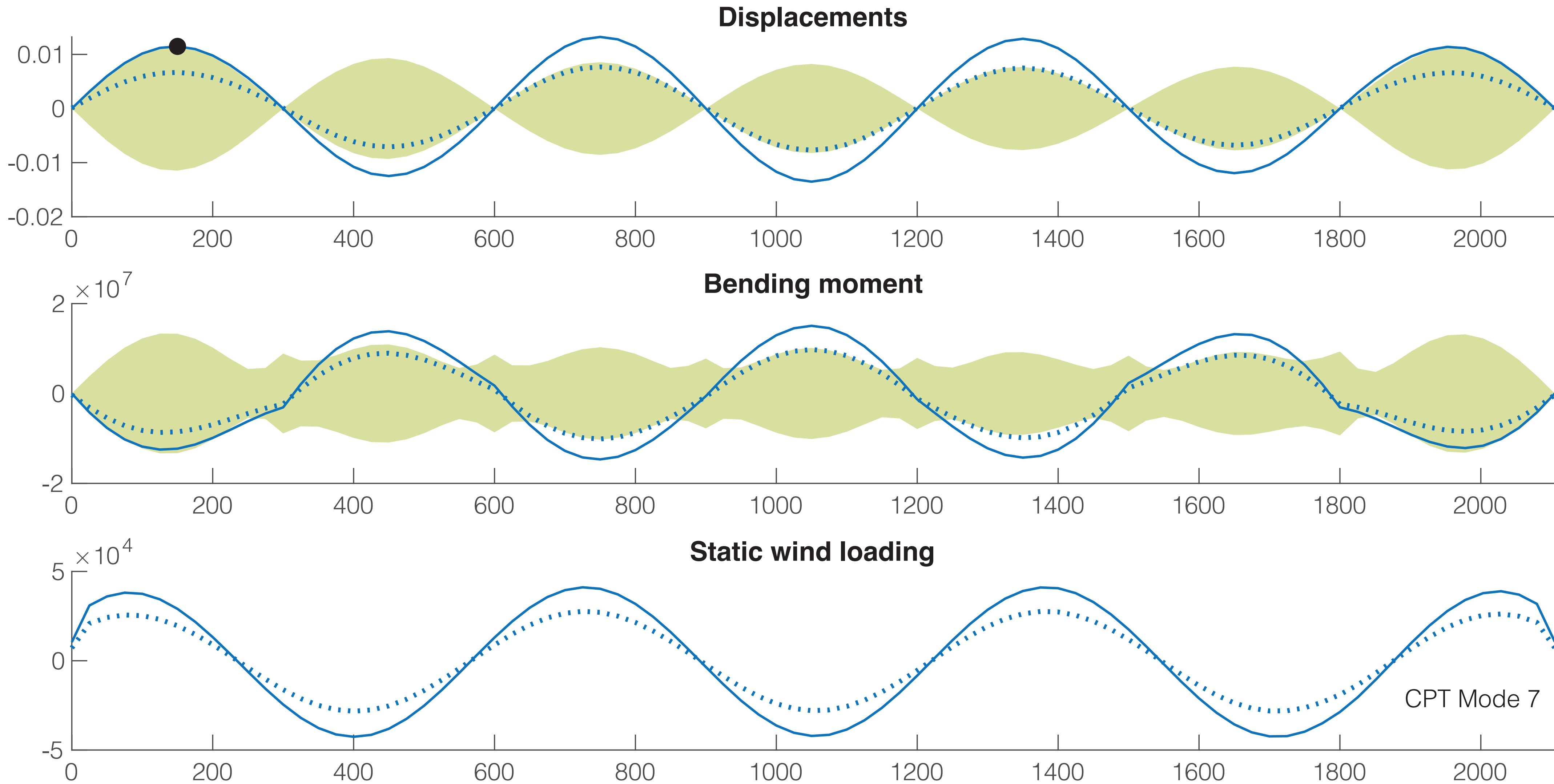
$$\mathbf{z} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \\ M_1 \\ \vdots \\ M_n \end{pmatrix}$$



Significant overshooting !

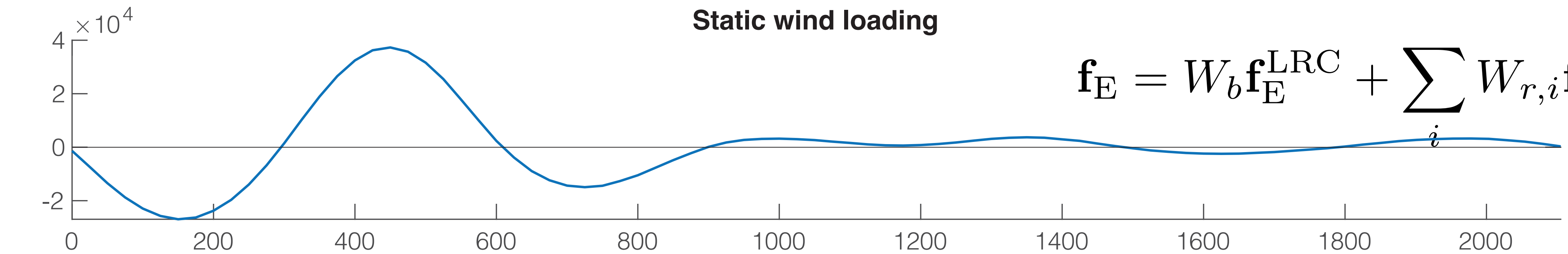
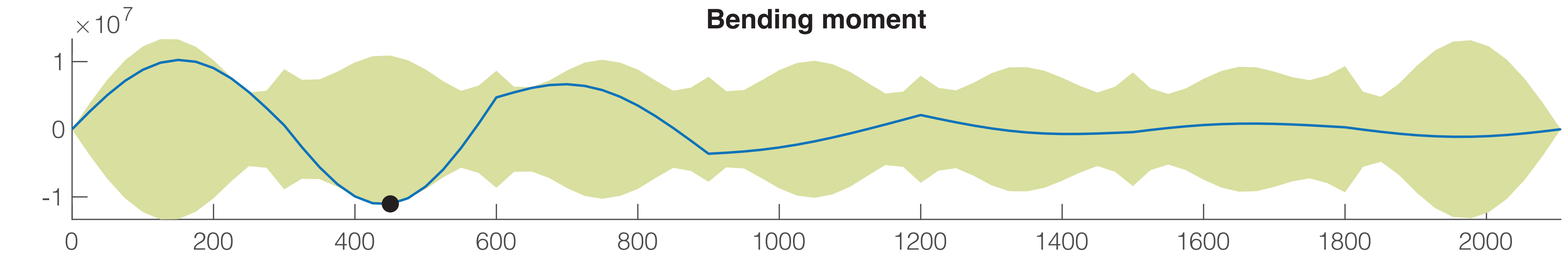
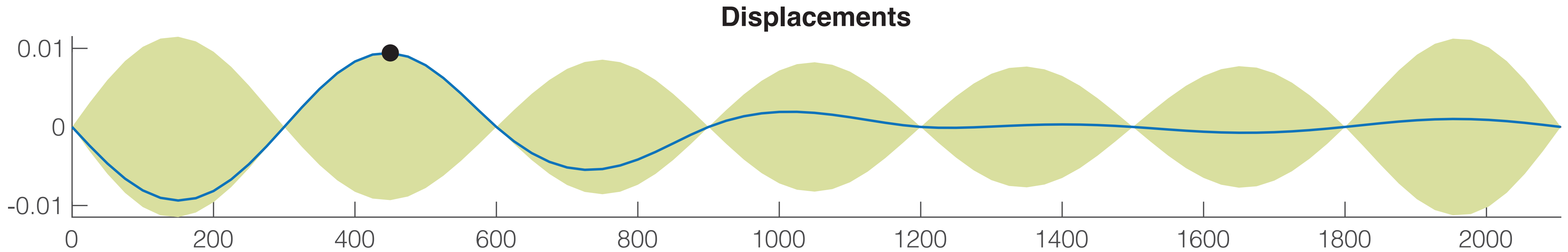
Now Ok.



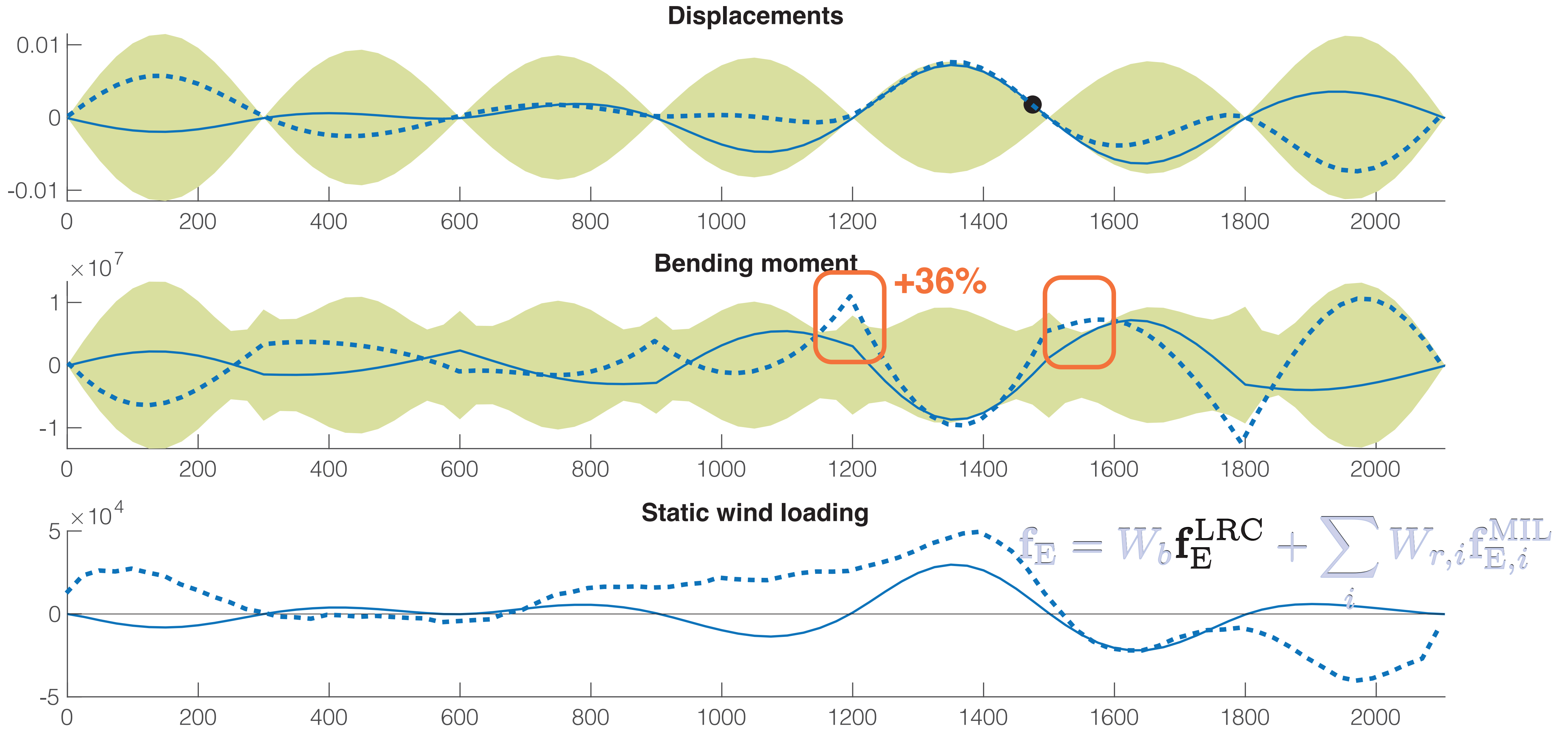


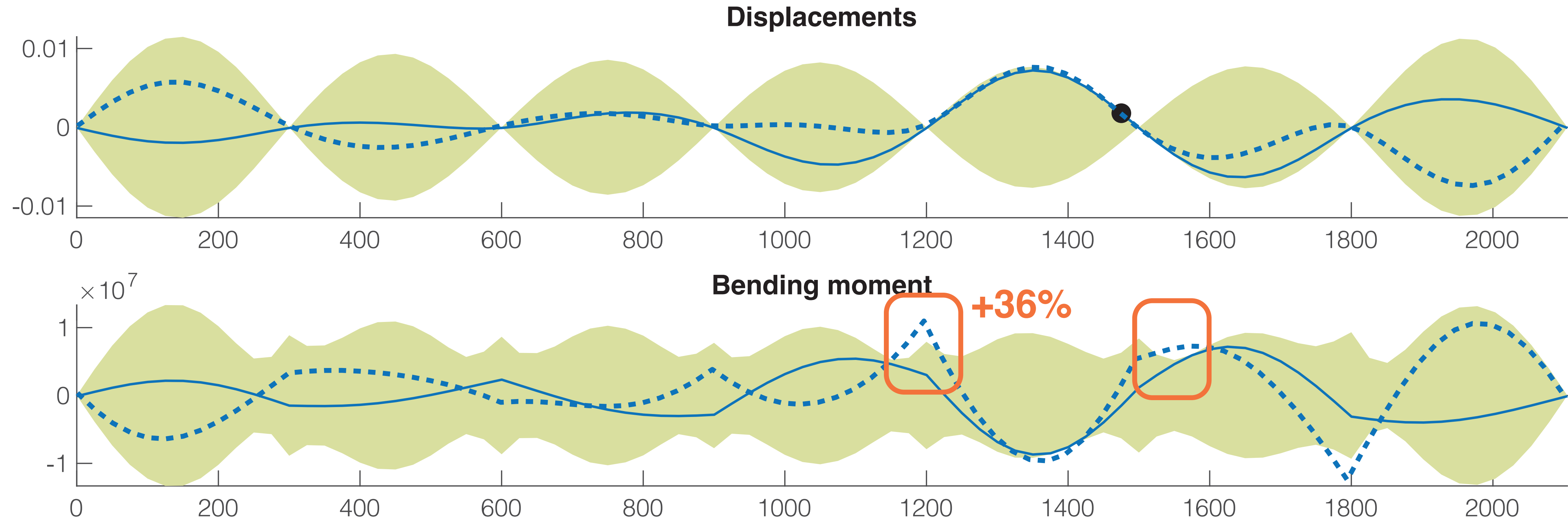
f_E^{CPT}

CPT Mode 7



$$\mathbf{f}_E = W_b \mathbf{f}_E^{\text{LRC}} + \sum_i W_{r,i} \mathbf{f}_{E,i}^{\text{MIL}}$$





LRC overshoots other responses than the one that is reconstructed

Solution : scale down the ESWL to avoid **overestimation**



	LRC	MIL	COMBINED
Response	background (wrong otherwise)	resonant (wrong otherwise)	both
Load pattern	Most probable load (conditional, Gaussianity)	Inertial loads (conditional, Gaussianity)	Conditional
Simple physical meaning	✓	✓	✓
No overshooting	✗	✗	✓

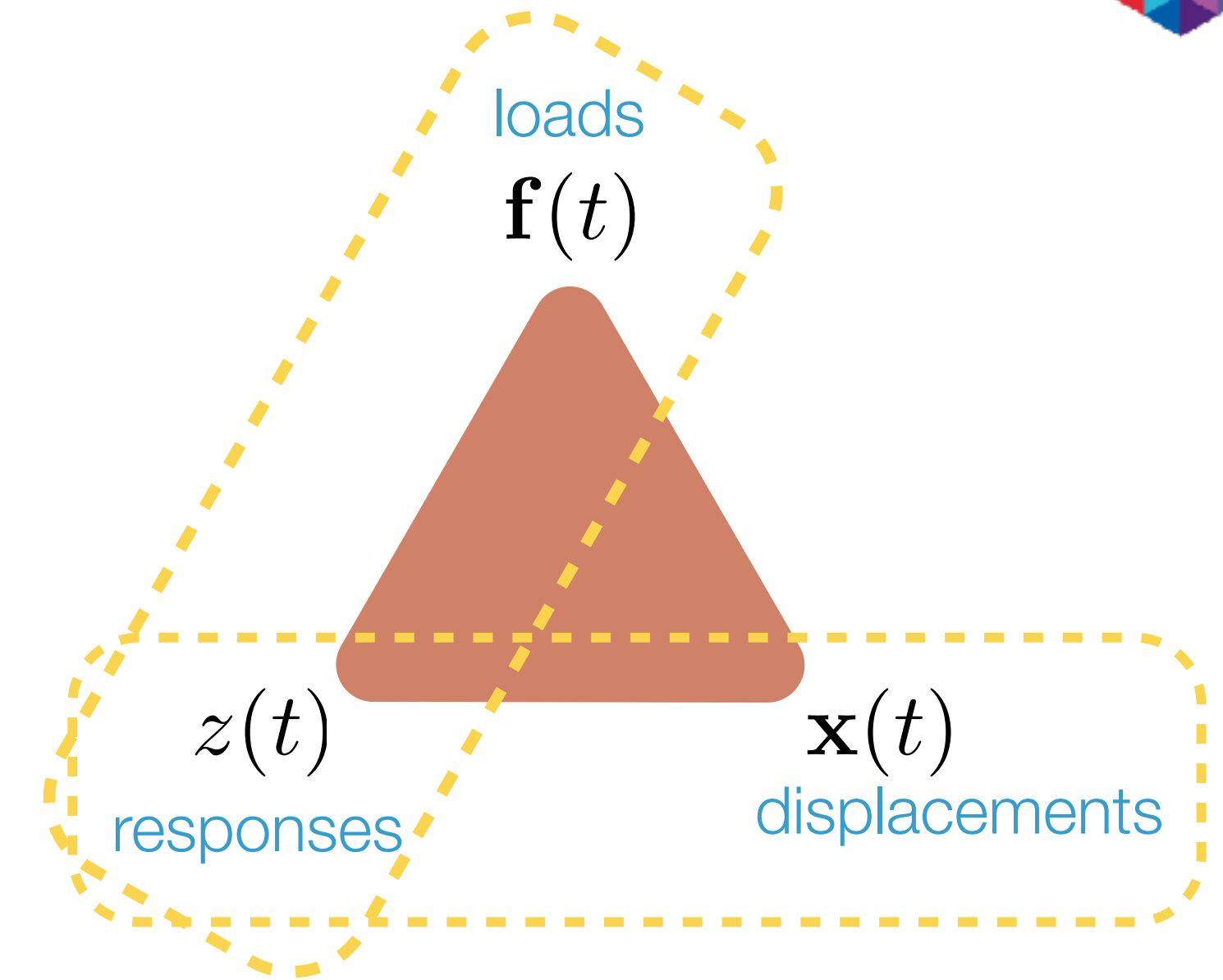
Possibility to extend



$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$$

$$z(t) = \mathbf{a}^T \mathbf{x}(t) \quad z_{\max} = \max_t z(t)$$

Find \mathbf{f}_E such that: $\mathbf{K}\mathbf{x}_E = \mathbf{f}_E$ and $\mathbf{a}^T \mathbf{x}_E = z_{\max}$



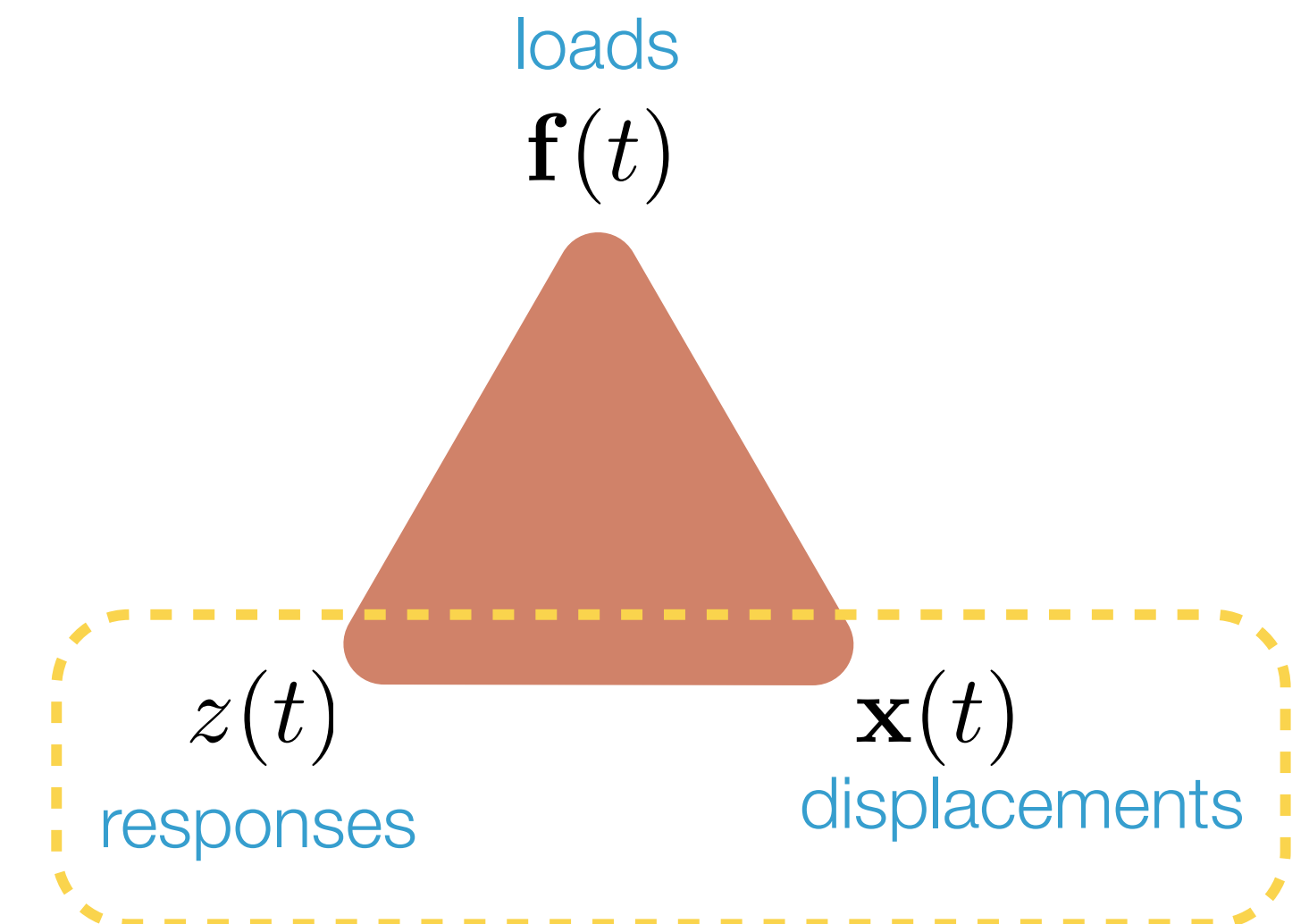
- in the LRC:
 - use of correlation between **loads** and **responses**
 - loads are proportional to structural **displacements** (static only)
- in the MIL :
 - modal contributions to the **response** are most probable inertial **loads**
 - loads are proportional to structural **displacements**, i.e. modes shapes (resonant only)

Equivalent wind loads: obtained from the most probable deformation of the structure when the response reaches its maximum value

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$$

$$z(t) = \mathbf{a}^T \mathbf{x}(t) \quad z_{\max} = \max_t z(t)$$

Find \mathbf{f}_E such that: $\mathbf{K}\mathbf{x}_E = \mathbf{f}_E$ and $\mathbf{a}^T \mathbf{x}_E = z_{\max}$



- 1. Solve equation of motion: compute $\mathbf{x}(t)$ from measured dynamic loads $\mathbf{f}(t)$
- 2. Compute **response** $z(t)$ from **displacements** $\mathbf{x}(t)$ (NB: possibly nonlinear)
- 3. Build a joint probabilistic model for the **response** and **displacements** (joint Gaussian if applicable, this is now a choice, not an assumption)
- 4. Find the most probable (or average) displacement field \mathbf{x}_E , conditioned upon the max. response
- 5. Determine the corresponding load distribution

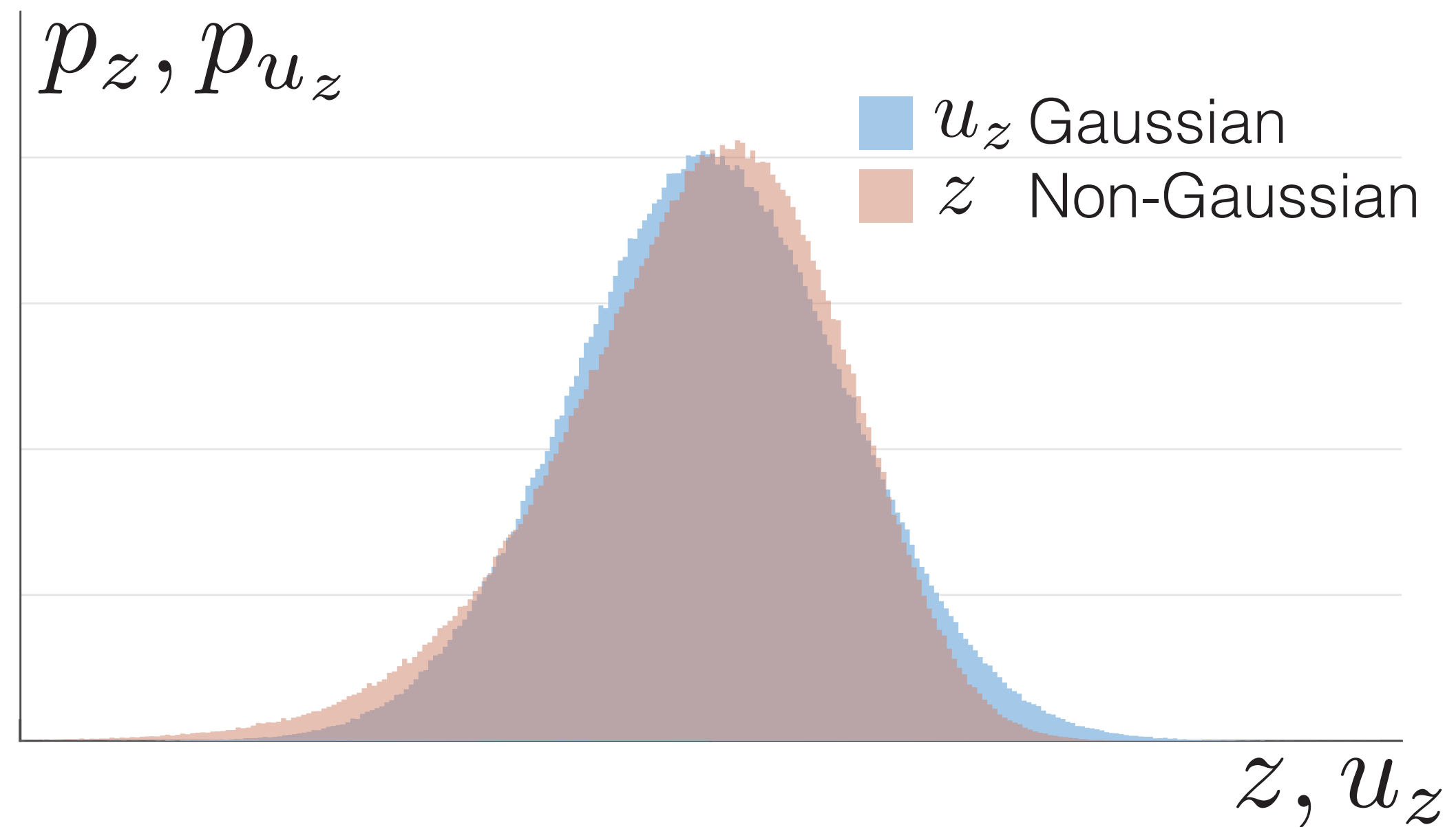
Equivalent wind loads: obtained from the most probable deformation of the structure when the response reaches its maximum value



z Response

1	2	3	4
μ_z	σ_z	$\gamma_{3,z}$	$\gamma_{4,z}$

$$z = \frac{\alpha_z}{b_z} \left(\frac{u_z^3}{3} + a_z u_z^2 + (b_z - 1) u_z - a_z \right)$$

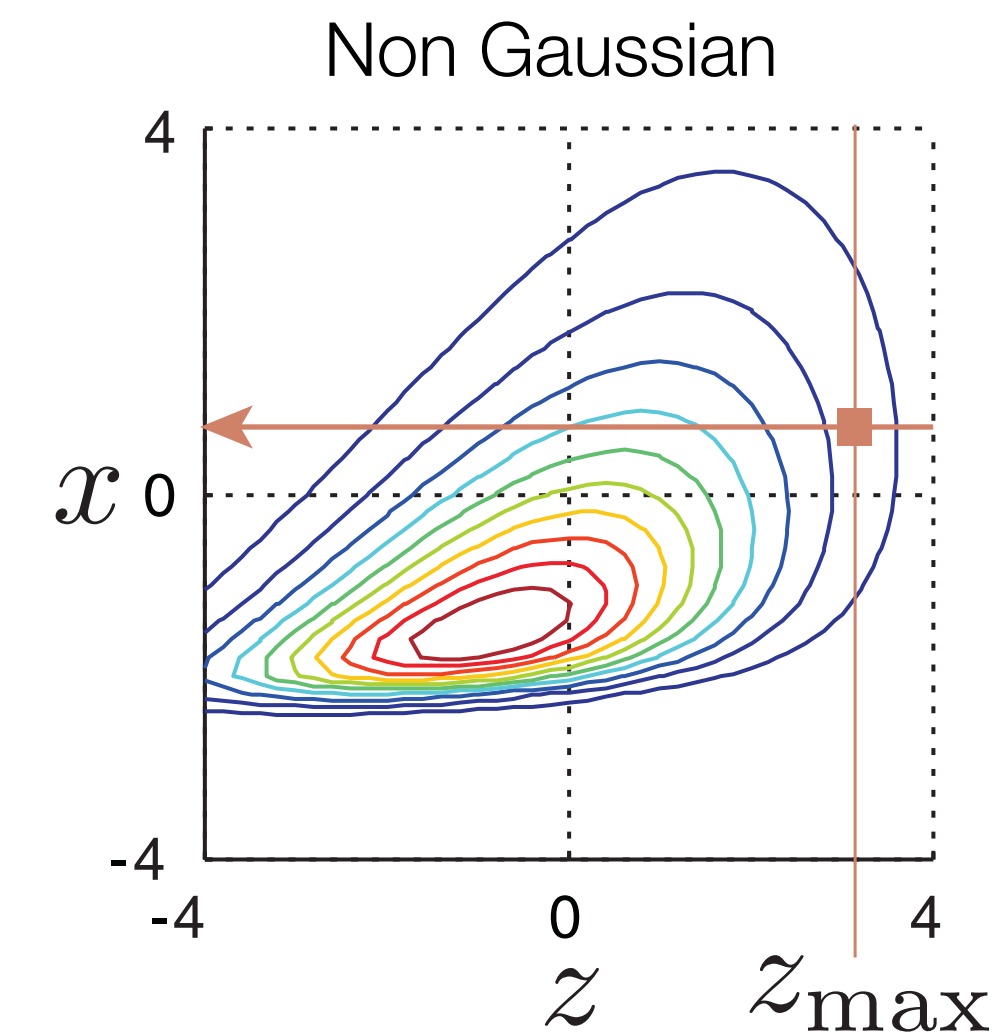
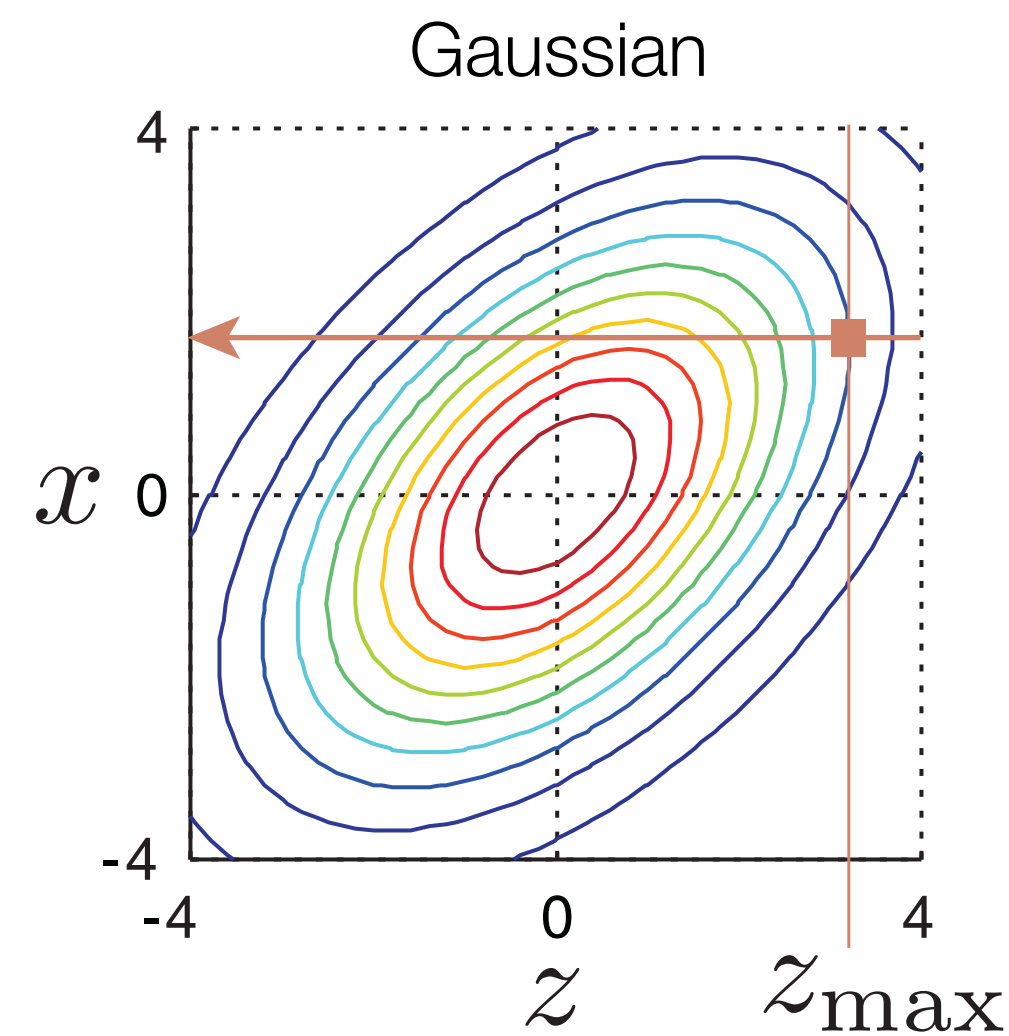




		z Response				
		0	1	2	3	4
x Displacement	0		μ_z	σ_z	$\gamma_{3,z}$	$\gamma_{4,z}$
	1	μ_x	ρ_{xz}			
	2	σ_x				
	3	$\gamma_{3,x}$				
	4	$\gamma_{4,x}$				

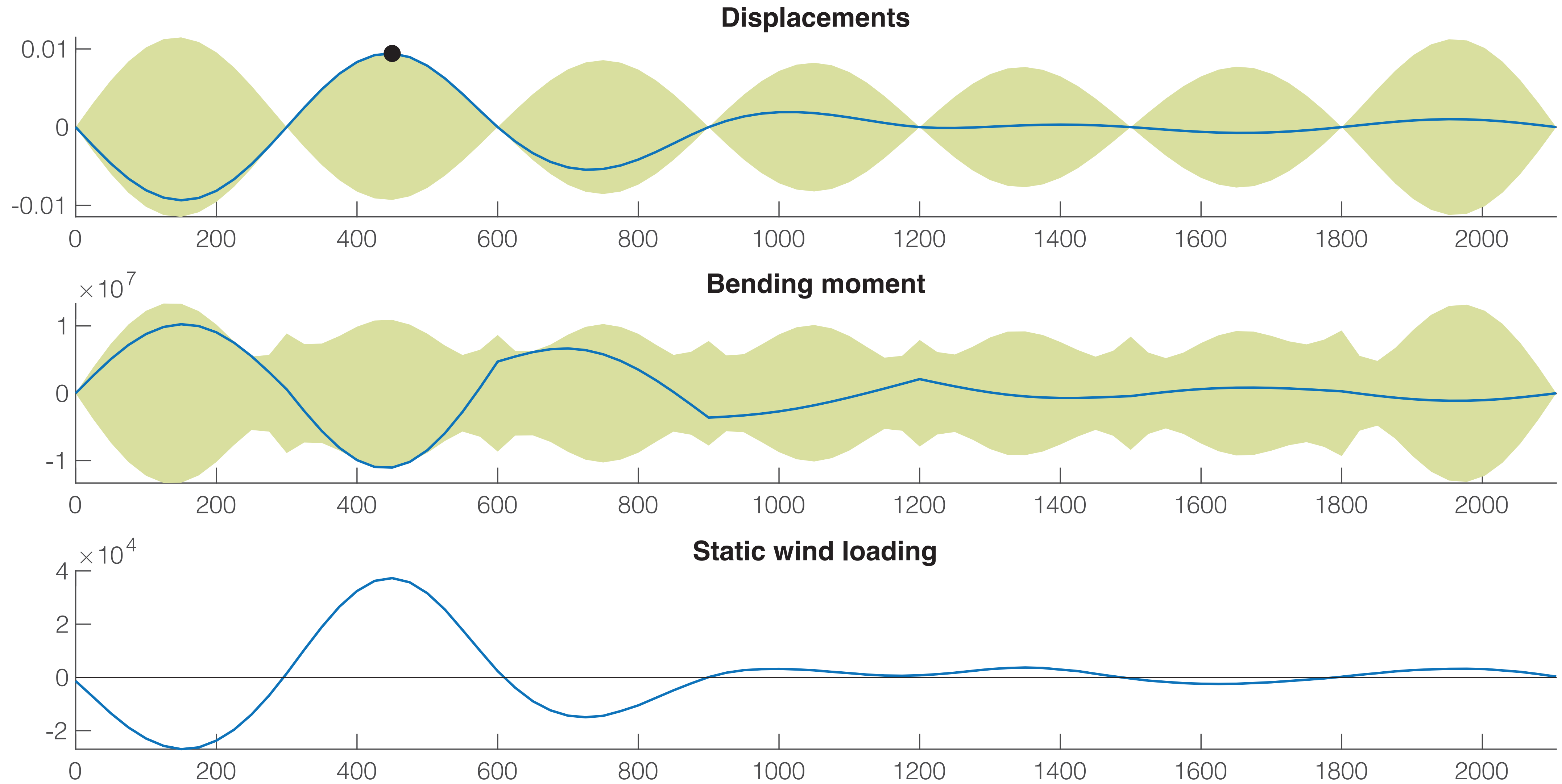
$$z = \frac{\alpha_z}{b_z} \left(\frac{u_z^3}{3} + a_z u_z^2 + (b_z - 1) u_z - a_z \right)$$

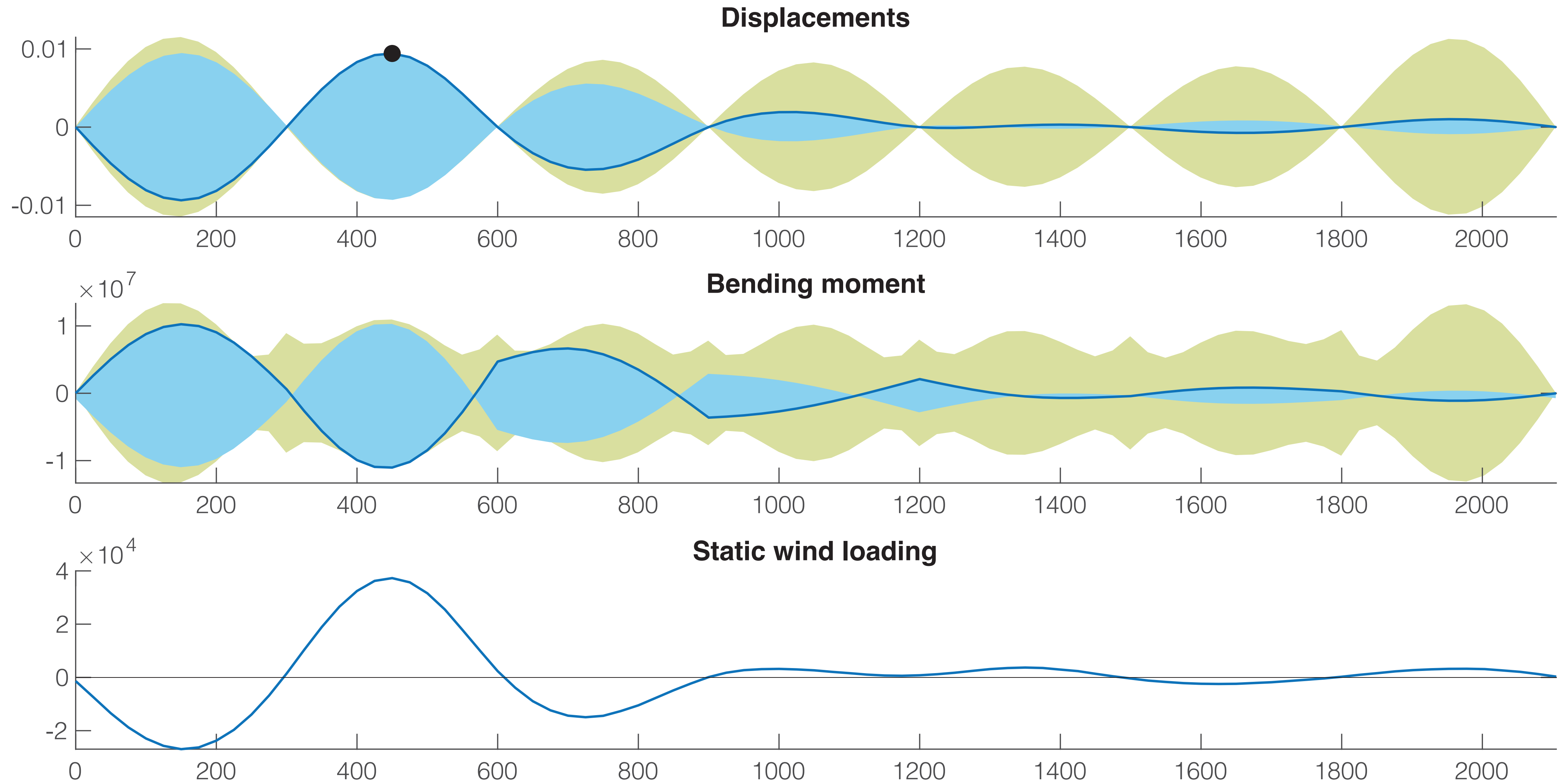
$$x = \frac{\alpha_x}{b_x} \left(\frac{u_x^3}{3} + a_x u_x^2 + (b_x - 1) u_x - a_x \right)$$

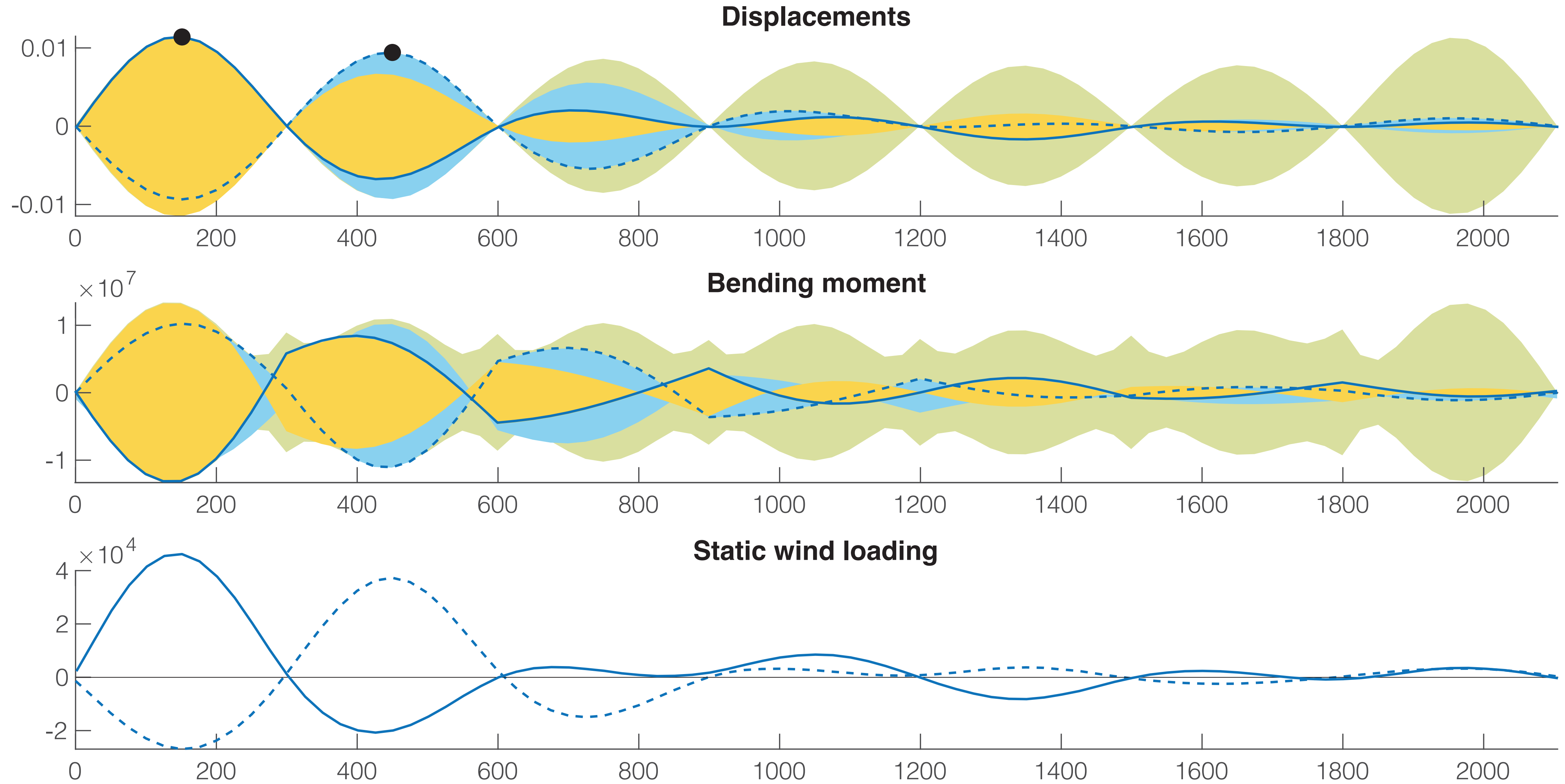


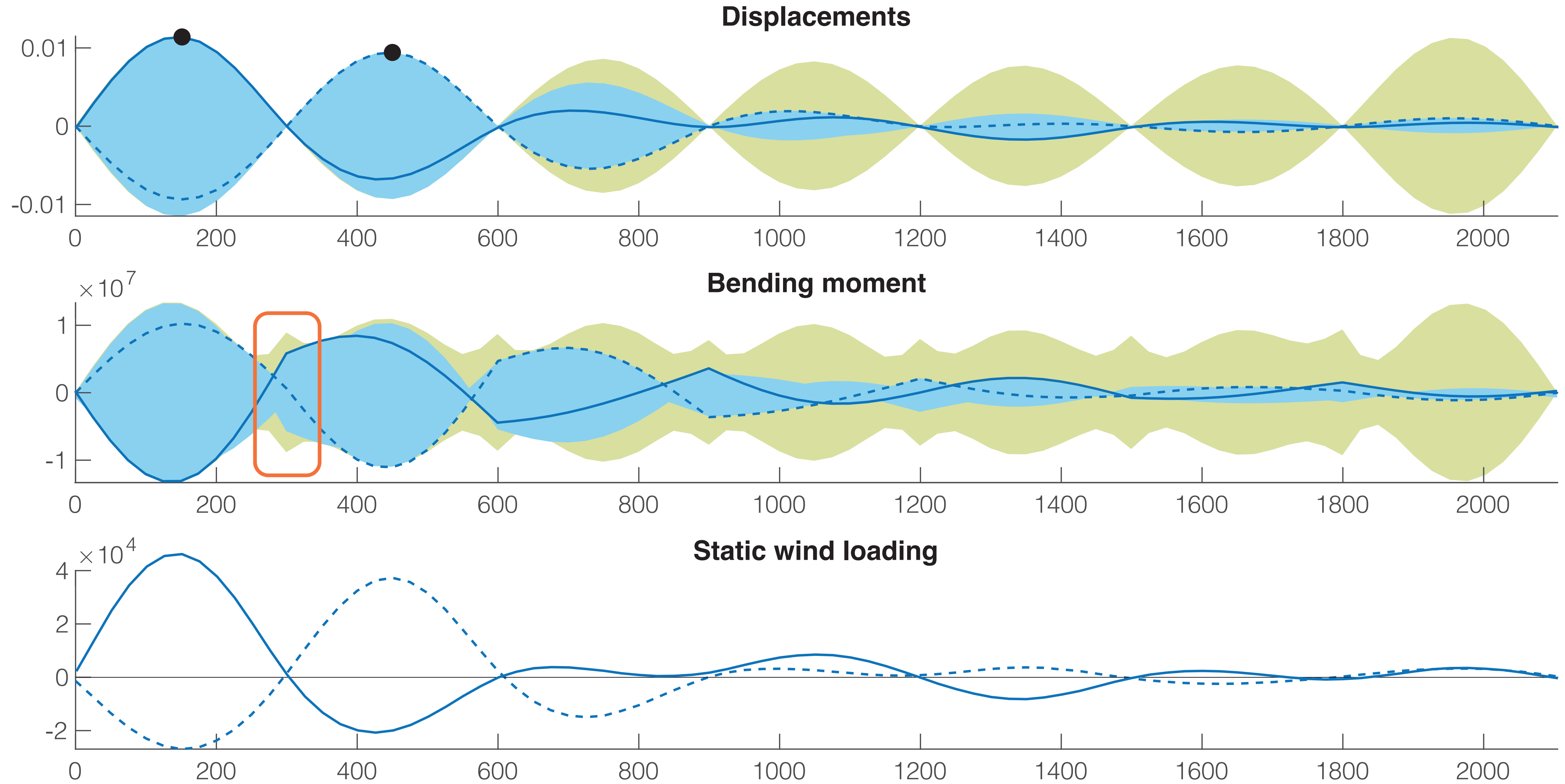


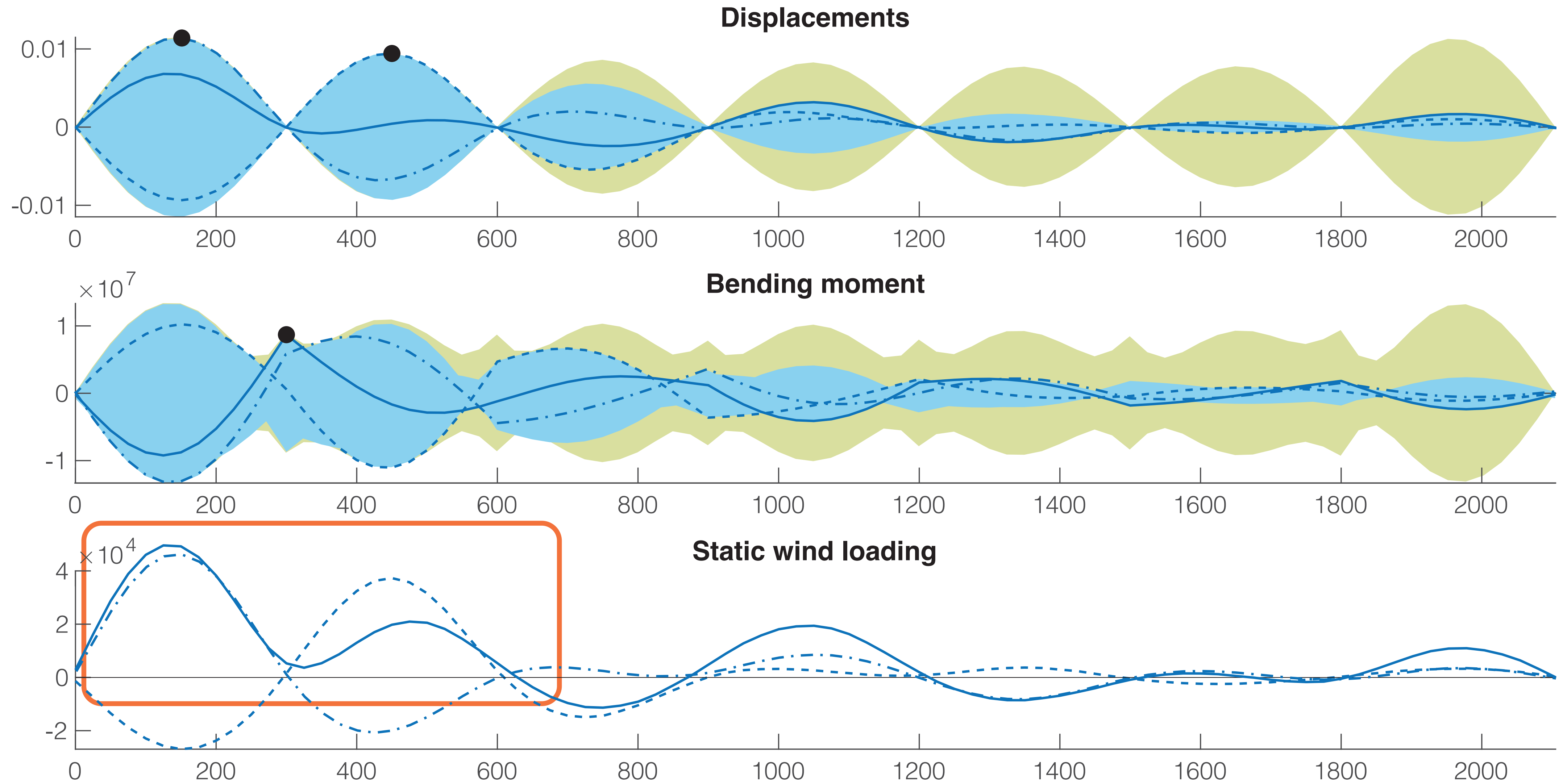
Part II : Envelope Reconstruction Problem

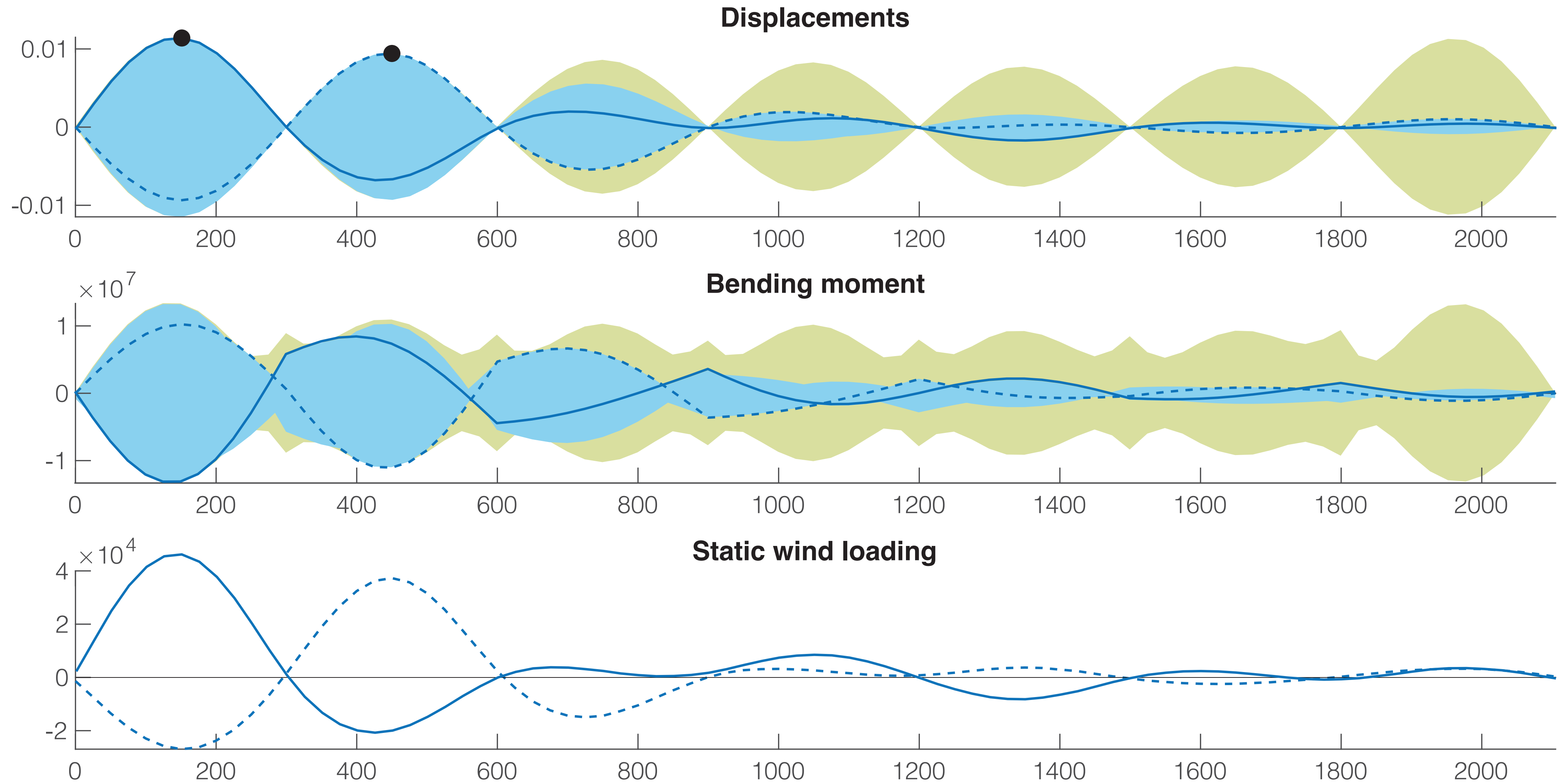


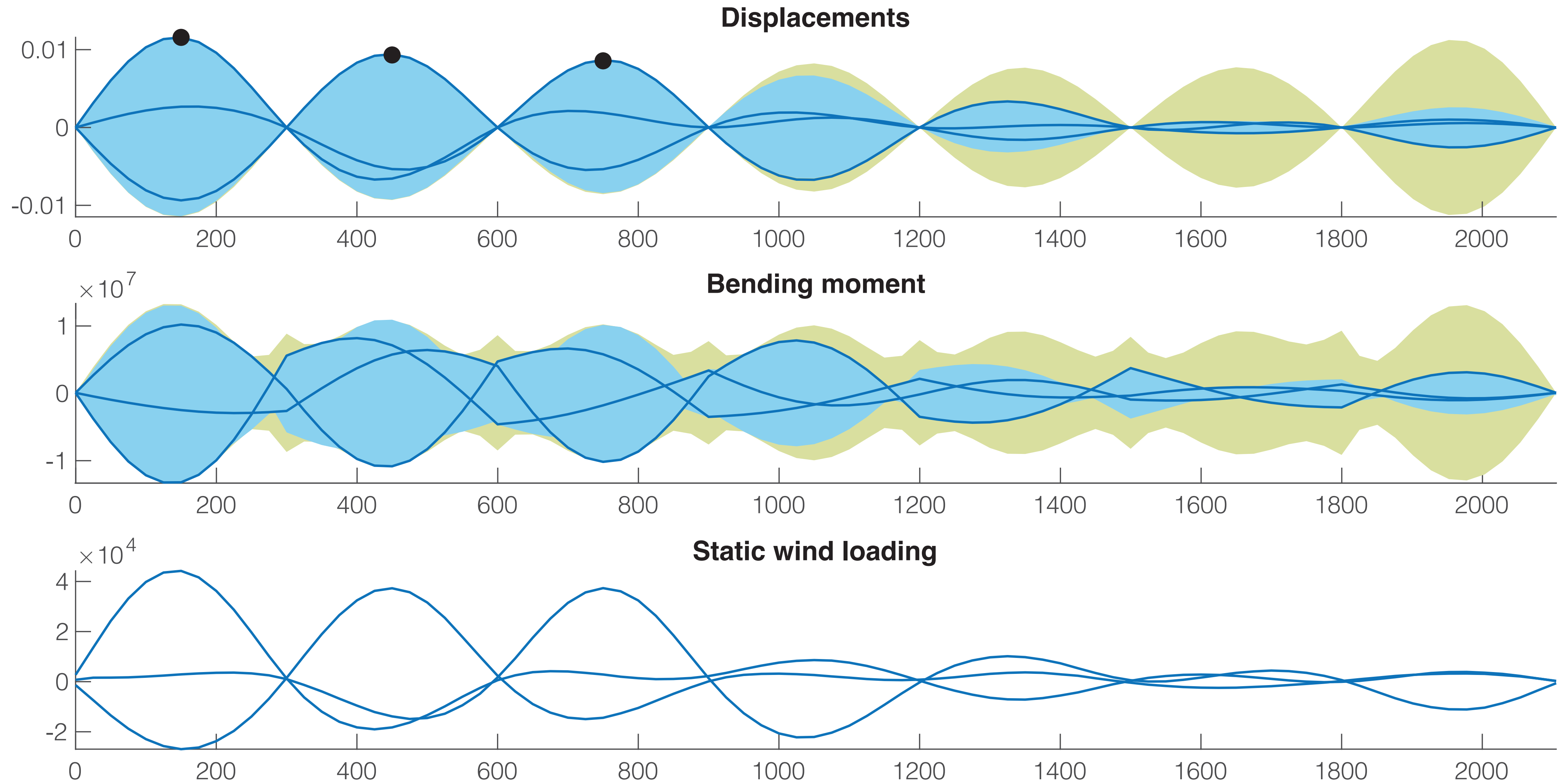


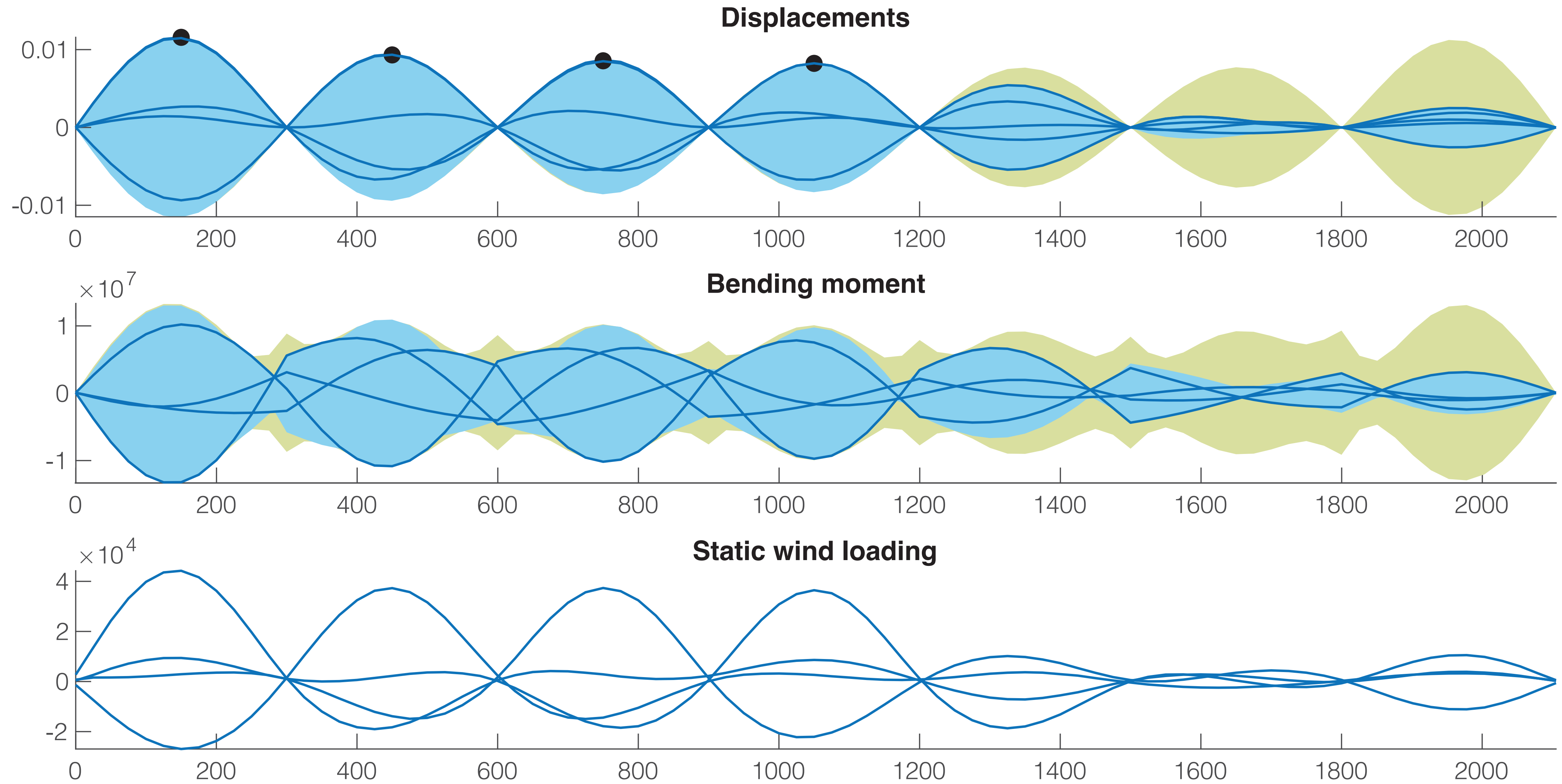


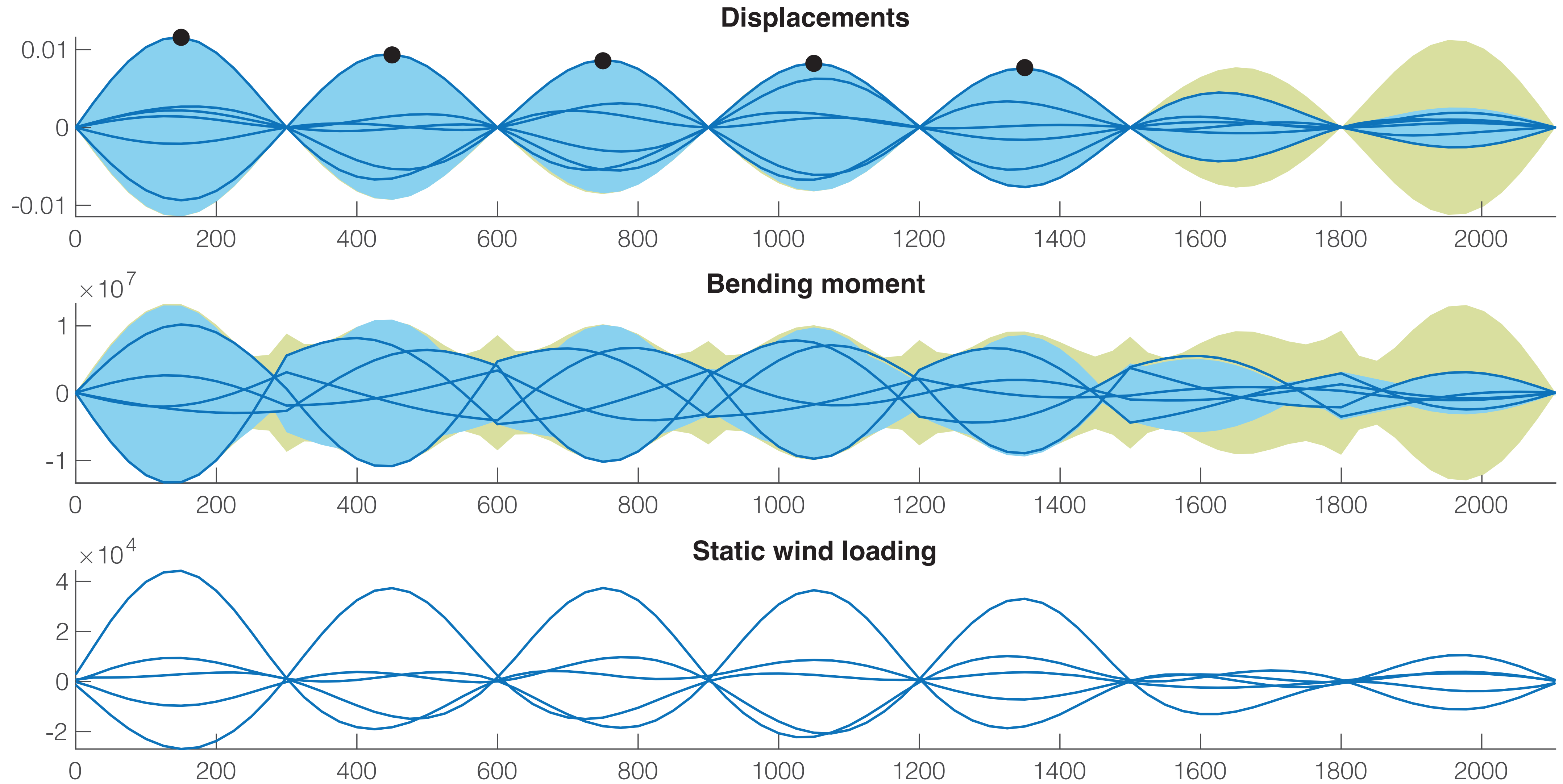


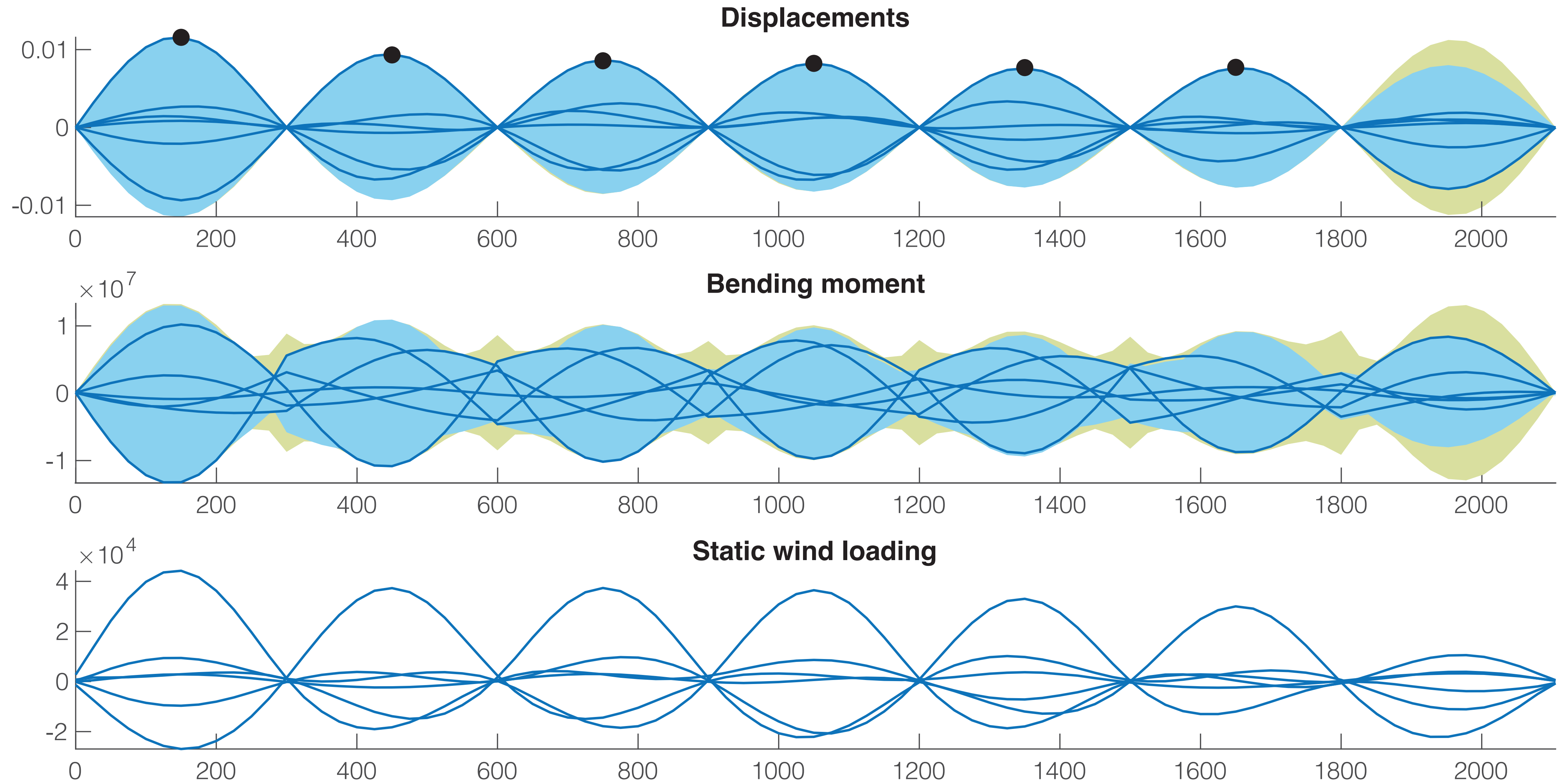


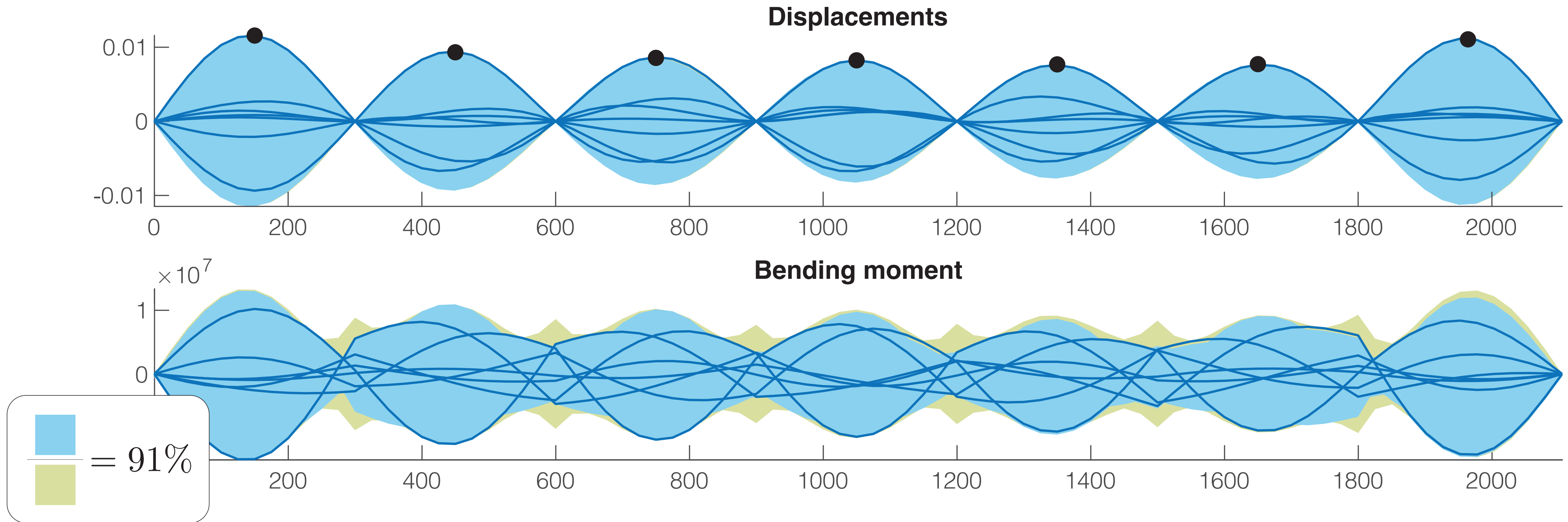






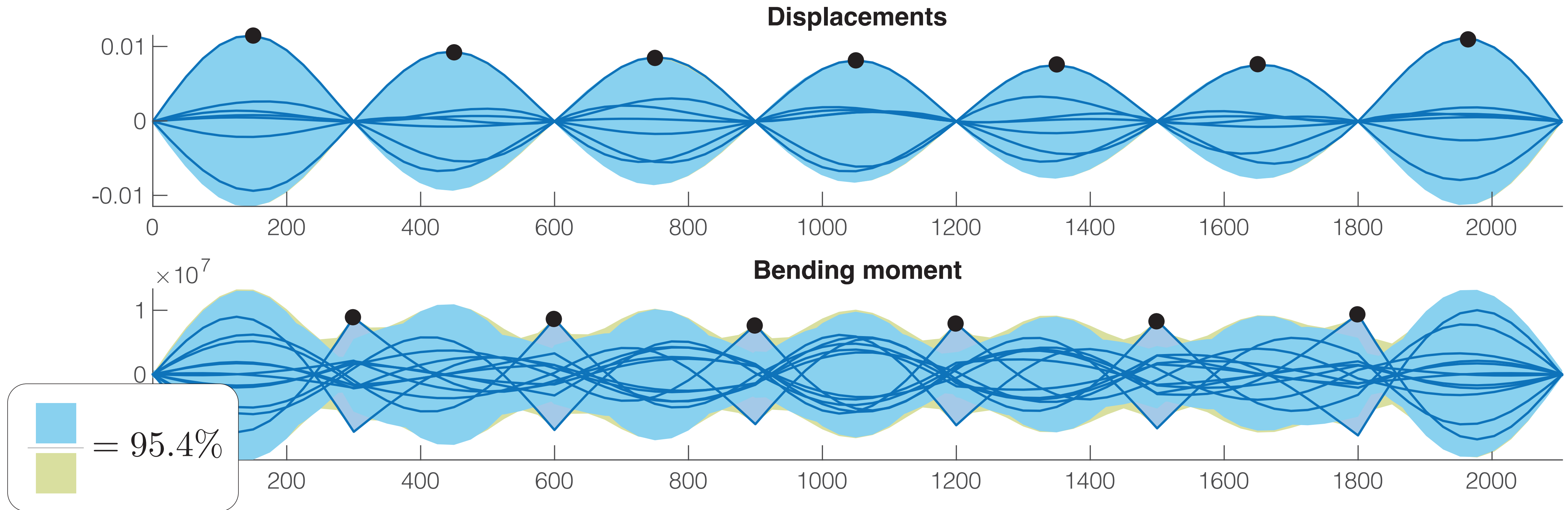






Reconstruction of the envelopes with mid-span displacements only

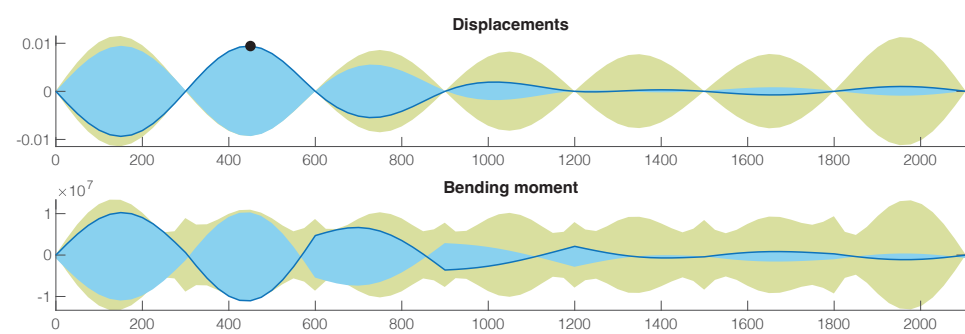
Don't try to reconstruct everything with displacements only



Reconstruction of the envelopes with : mid-span displacements
and bending moments on supports

We used 13 ESWL.

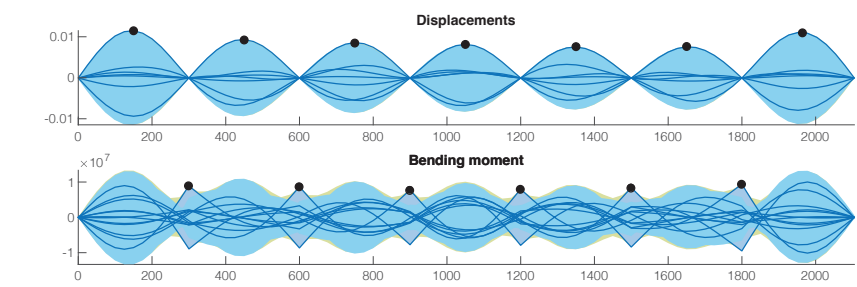
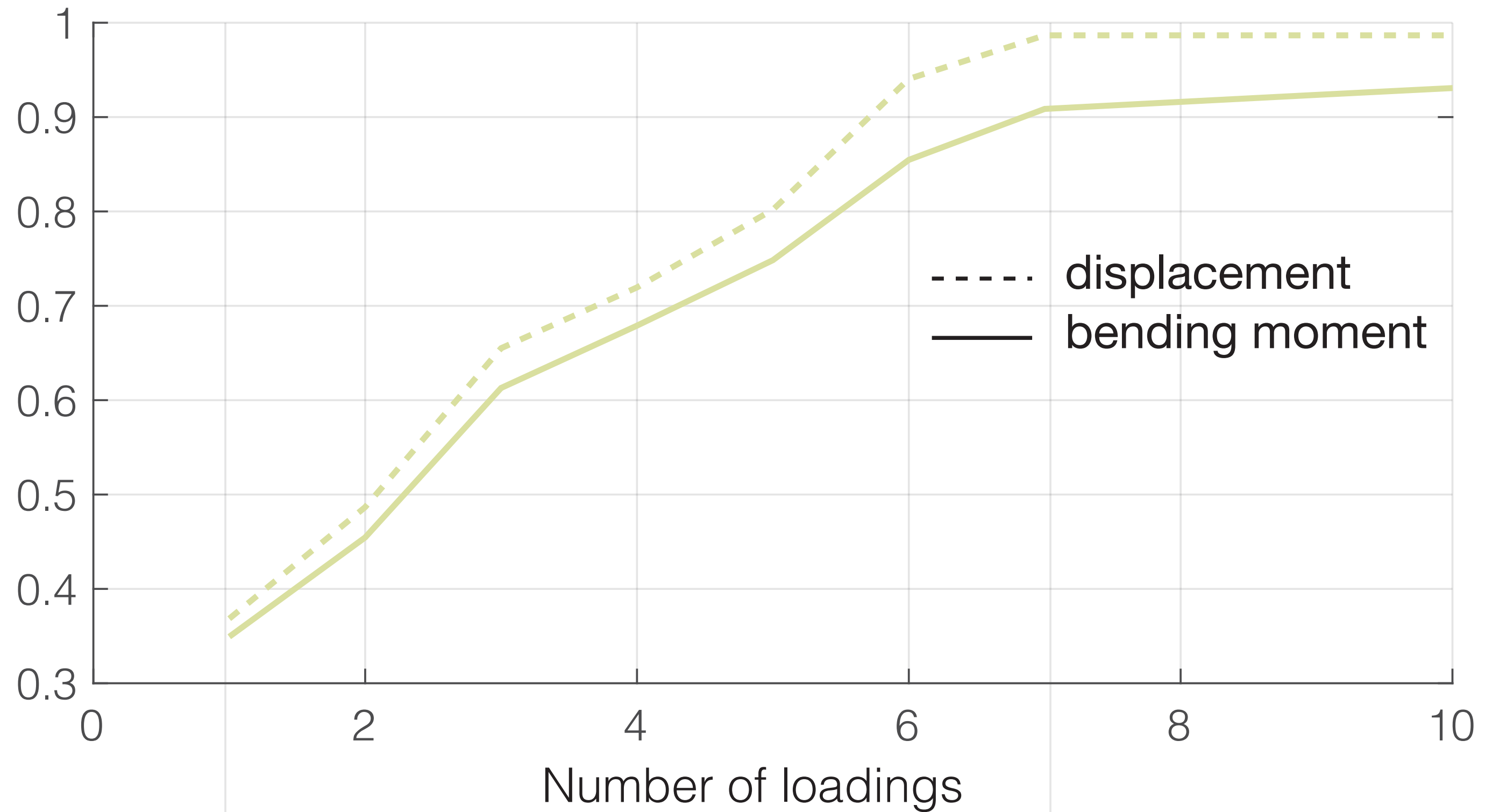
1. Did we do a good job ? (NB: strength vs. loading)



(1 ESWL)



Reconstruction rate

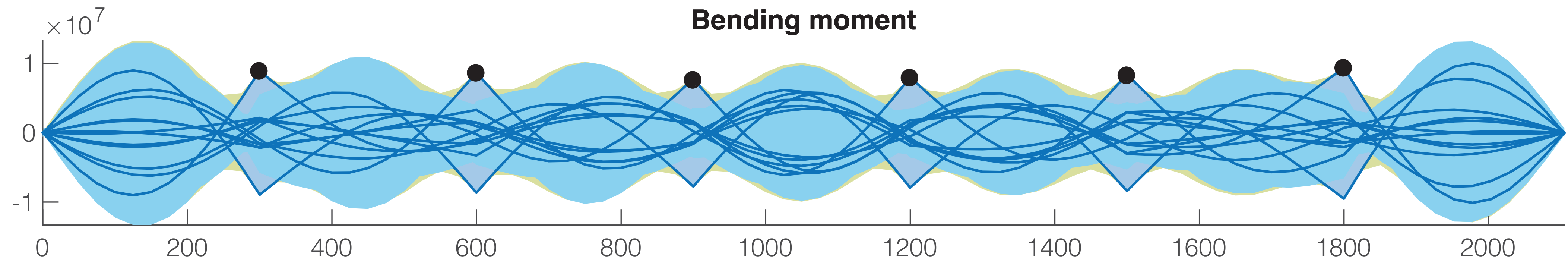


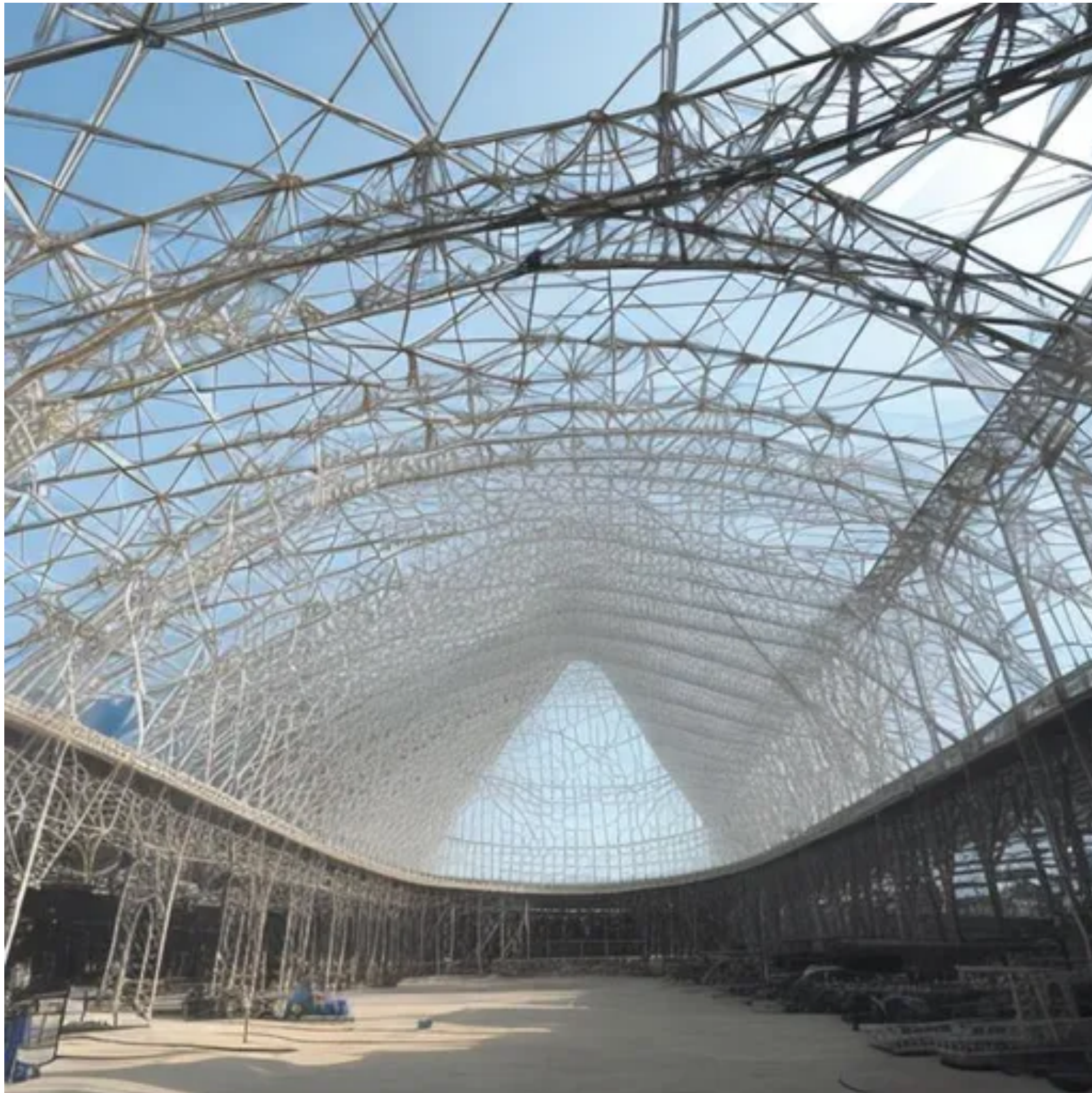
(13 ESWLs)

We used 13 ESWL.

1. Did we do a good job ? (NB: strength vs. loading)
2. Could we be more efficient ? Use 12 ESWL ? Or less ?
3. Nb. of ESWL vs. complexity. Why 13 ESWL ?
4. Difficulty to identify the « best » responses

$$\mathbf{z} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \\ M_1 \\ \vdots \\ M_n \end{pmatrix}$$





Envelope : (generous) set of structural responses \mathbf{z}_{\max} : axial forces, bending moments, displacements, ground reactions, ...

Reconstruction : isn't that clear ? (no matter origin of envelop — dynamic or not)

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$$

$$\mathbf{z}(t) = \mathbf{A}\mathbf{x}(t) \quad \mathbf{z}_{\max} = \max_t \mathbf{z}(t)$$

Find \mathbf{f}_E such that: $\mathbf{K}\mathbf{x}_E = \mathbf{f}_E$ and $\mathbf{A}\mathbf{x}_E = \mathbf{z}_{\max}$

Find $\{\mathbf{f}_{E,i}\}$ such that, for $\mathbf{K}\mathbf{x}_{E,i} = \mathbf{f}_{E,i}$, $\mathbf{A}\mathbf{x}_{E,i} = \mathbf{z}_{\max,i}$,

$$\text{and } \hat{\mathbf{z}}_{\max} = \max_{i=1,\dots,n} |\mathbf{z}_{\max,i}|$$

$$\|\mathbf{z}_{\max} - \hat{\mathbf{z}}_{\max}\| \text{ is minimum}$$



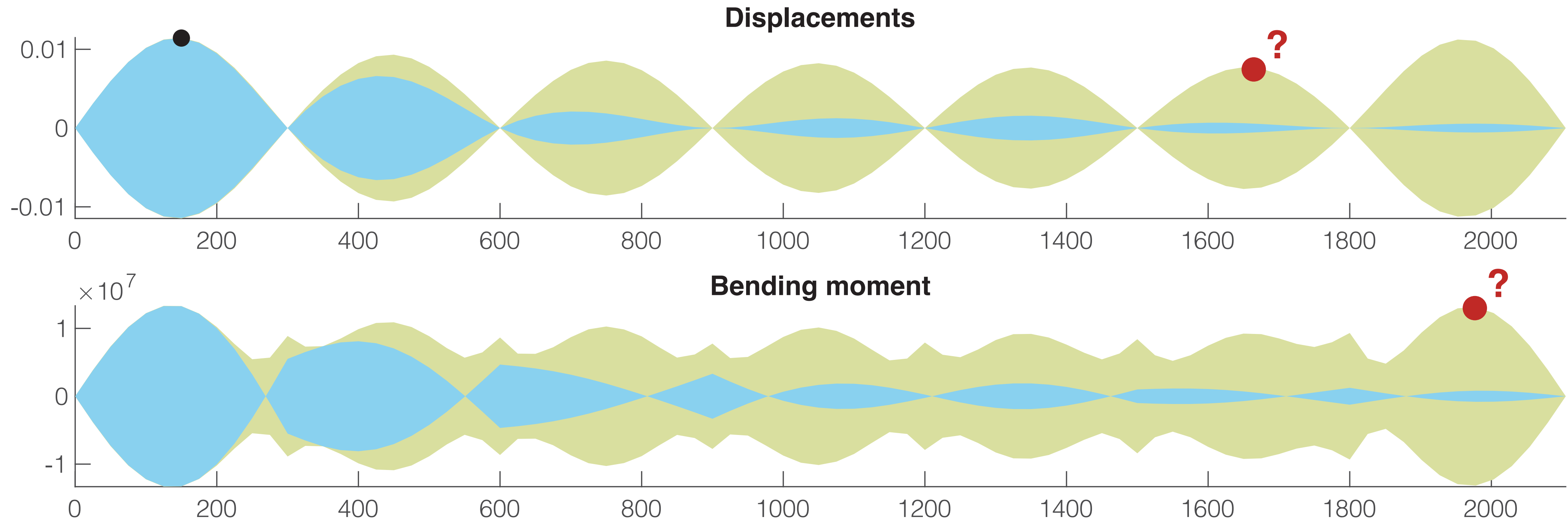
Envelope : (generous) set of structural responses : axial forces, bending moments, displacements, ground reactions, ...

Which envelope ? Absolute, Relative values, Units (!?), Consider strength.

Constraints : the $\{\mathbf{f}_{E,i}\}$ should look realistic
the number of ESWL should not be too large (but it depends on the complexity)

Freedom : maybe allow for some over-estimation / under-estimation

$$\text{Find } \{\mathbf{f}_{E,i}\} \text{ such that, for } \mathbf{K}\mathbf{x}_{E,i} = \mathbf{f}_{E,i}, \quad \mathbf{A}\mathbf{x}_{E,i} = \mathbf{z}_{\max,i},$$
$$\text{and } \hat{\mathbf{z}}_{\max} = \max_{i=1,\dots,n} |\mathbf{z}_{\max,i}|$$
$$\|\mathbf{z}_{\max} - \hat{\mathbf{z}}_{\max}\| \text{ is minimum}$$

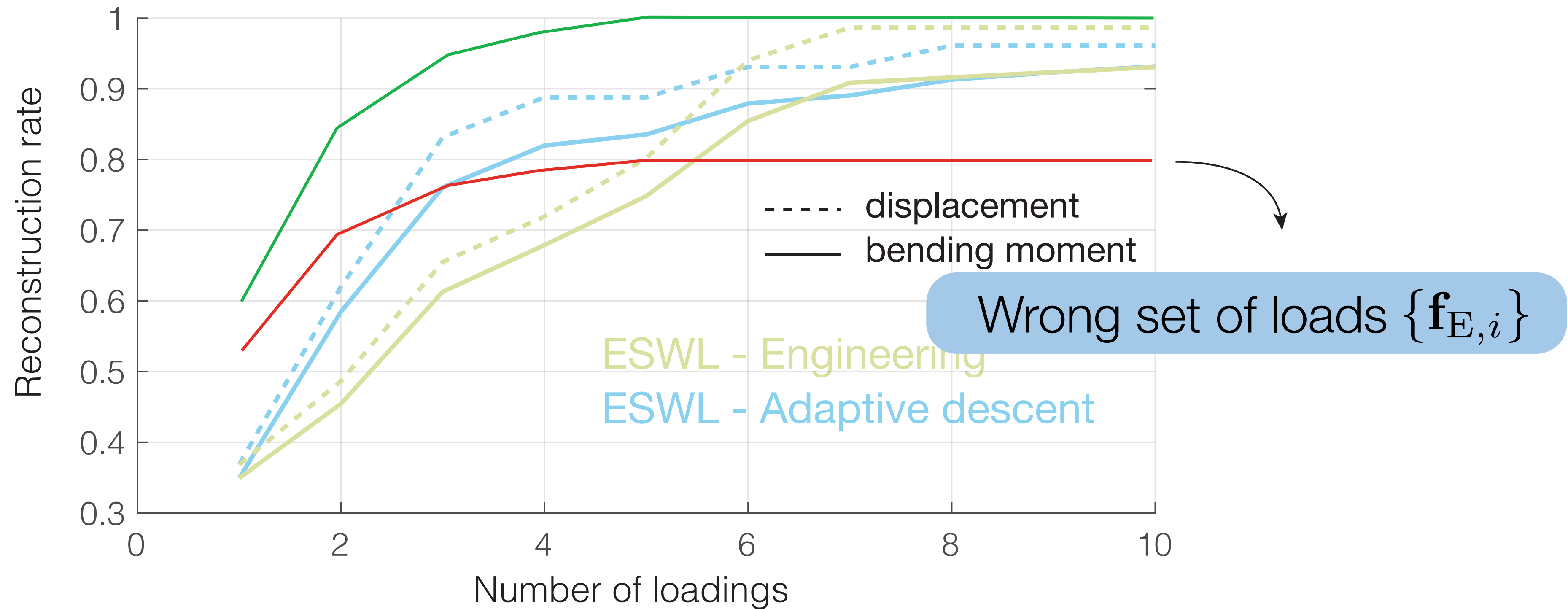


Step 1 : consider 1 major response & associated ESWL

Step 2 : find worst reconstructed response

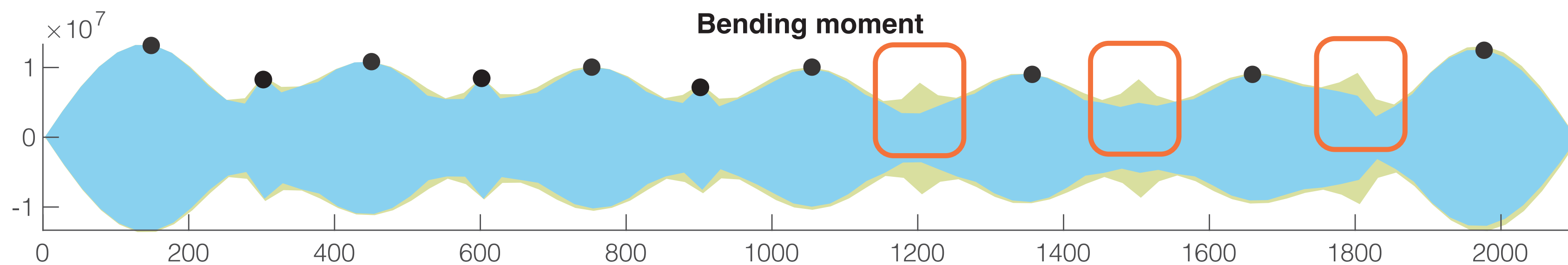
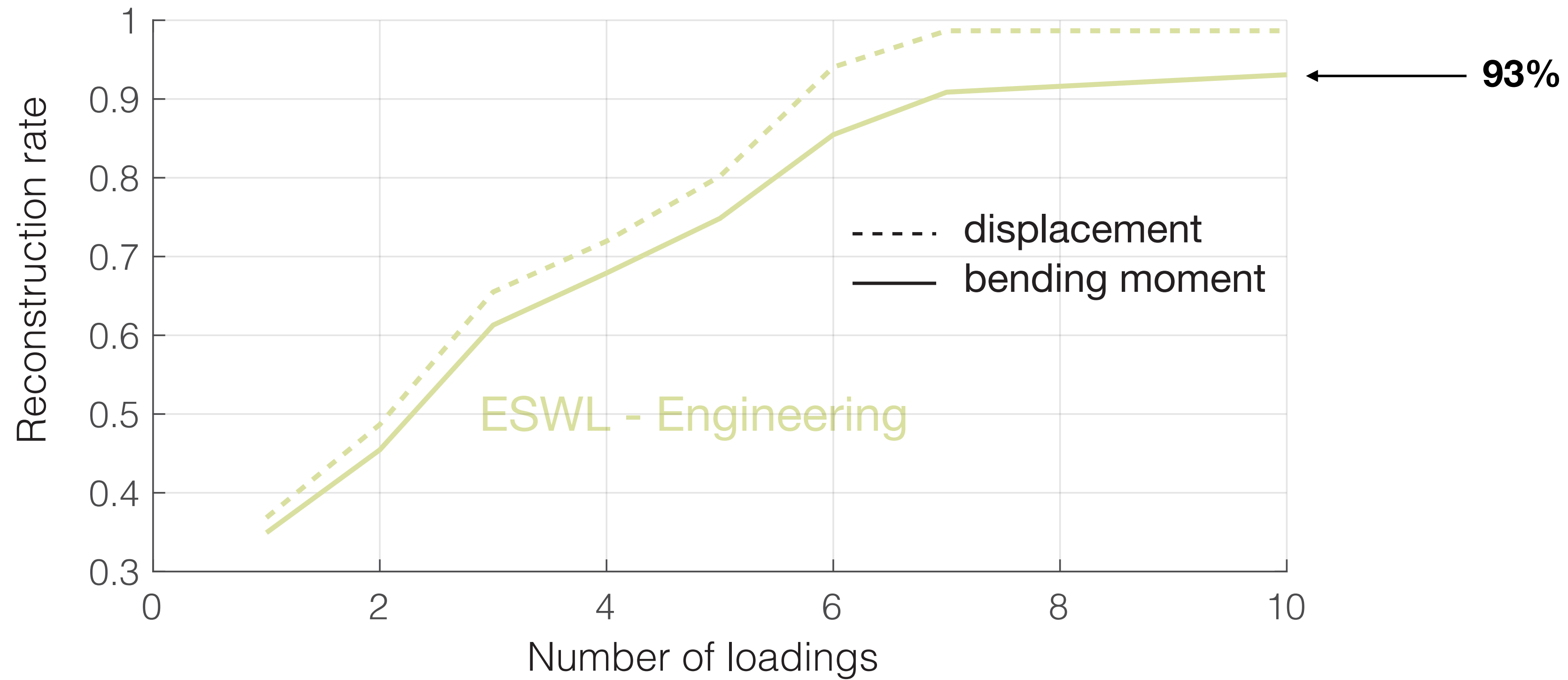
Step 3 : define new ESWL & iterate

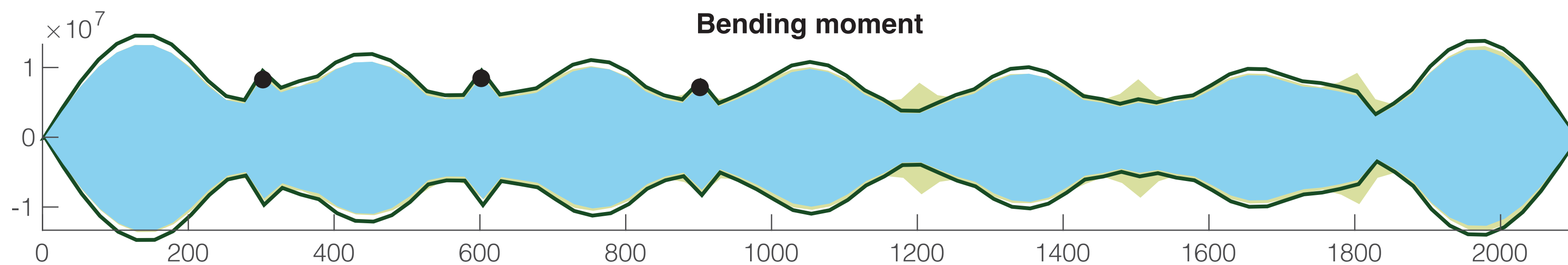
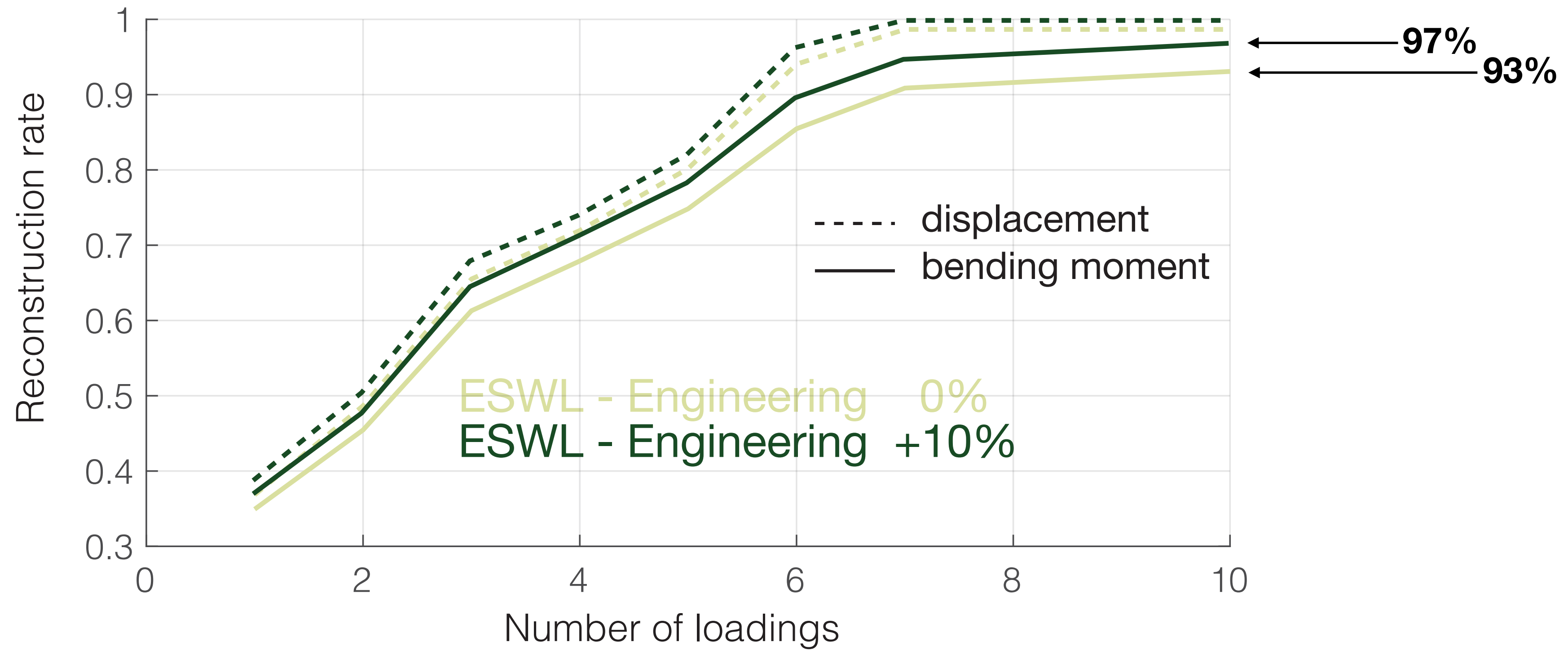
NB: needs a starting point

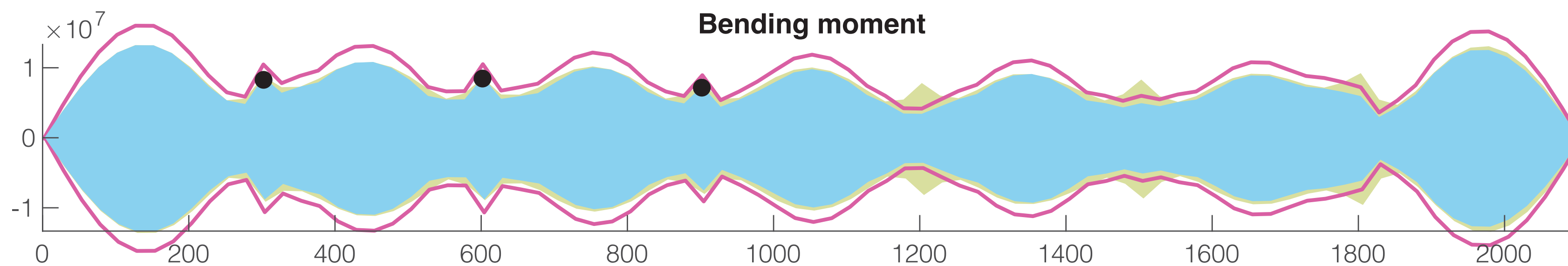
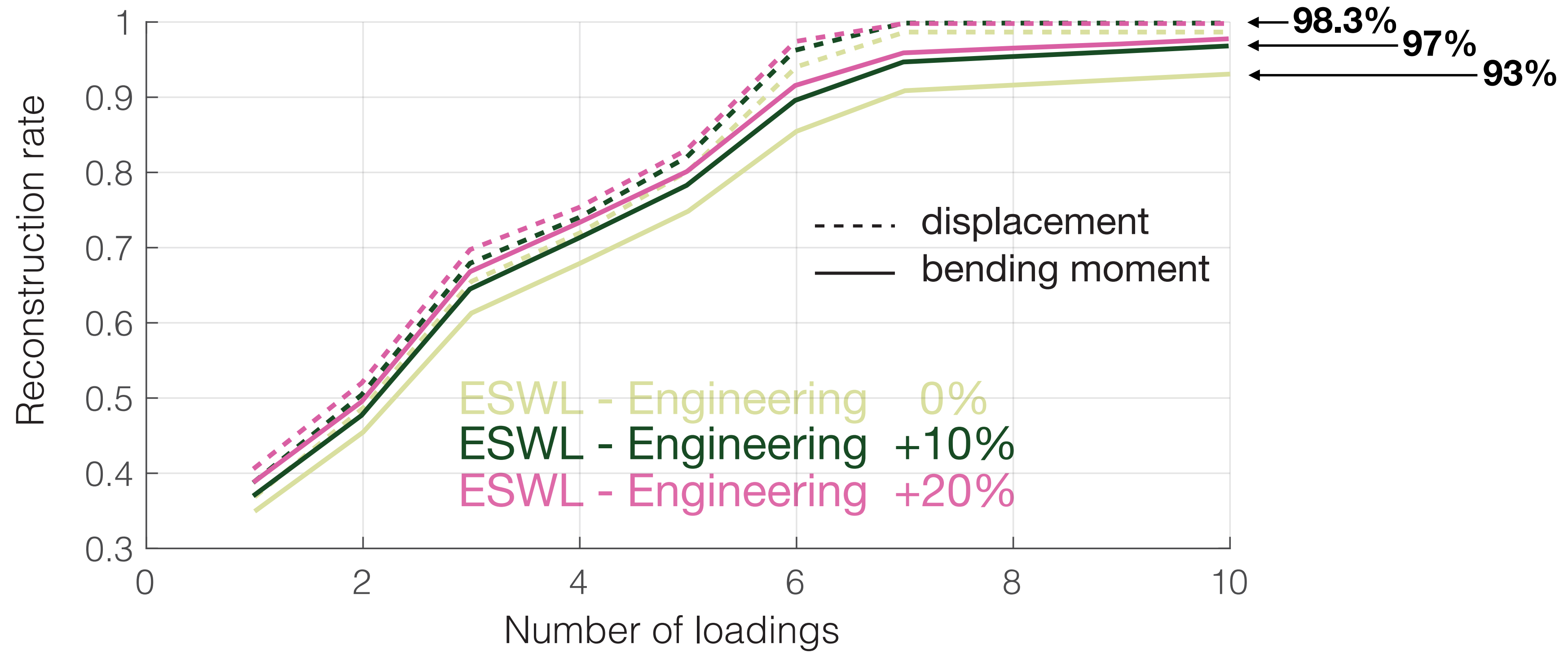


Quality of a good reconstruction algorithm :

- fast convergence
- high terminal convergence rate









Find $\{\mathbf{f}_{E,i}\}$ such that, for $\mathbf{K}\mathbf{x}_{E,i} = \mathbf{f}_{E,i}$, $\mathbf{A}\mathbf{x}_{E,i} = \mathbf{z}_{\max,i}$,

$$\text{and } \hat{\mathbf{z}}_{\max} = \max_{i=1,\dots,n} |\mathbf{z}_{\max,i}|$$

~~$\|\mathbf{z}_{\max} - \hat{\mathbf{z}}_{\max}\|$ is minimum~~

Envelope is reconstructed :

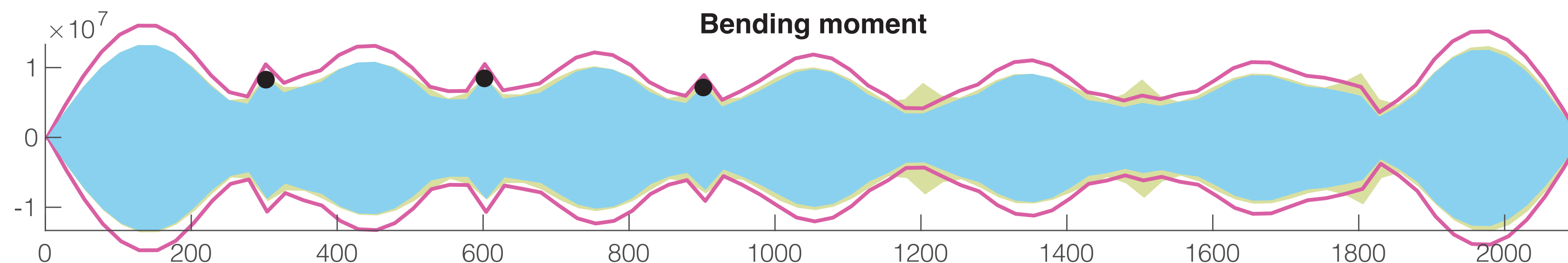
- within chosen/agreed over- and under-estimations (10%-20%)
- faster convergence rate

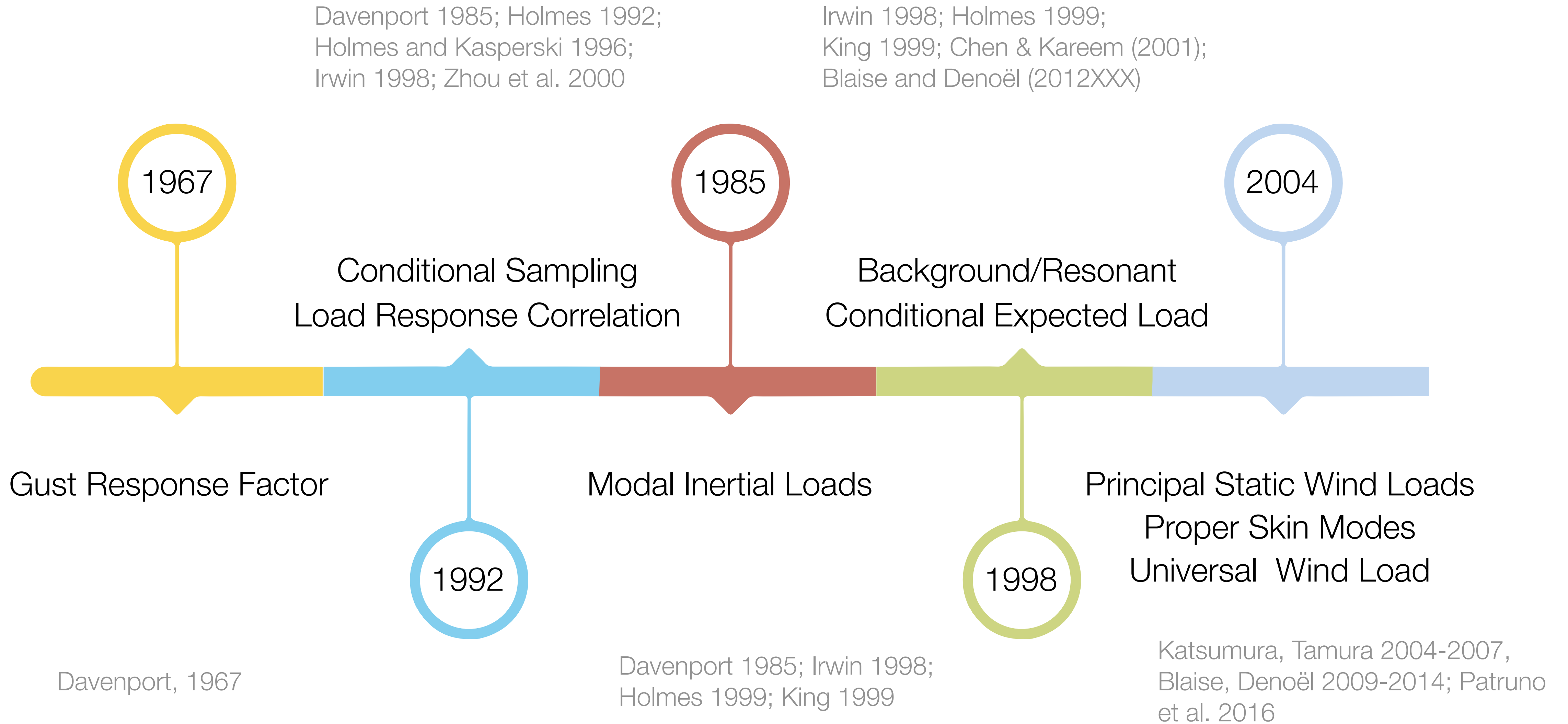
Underestimation :

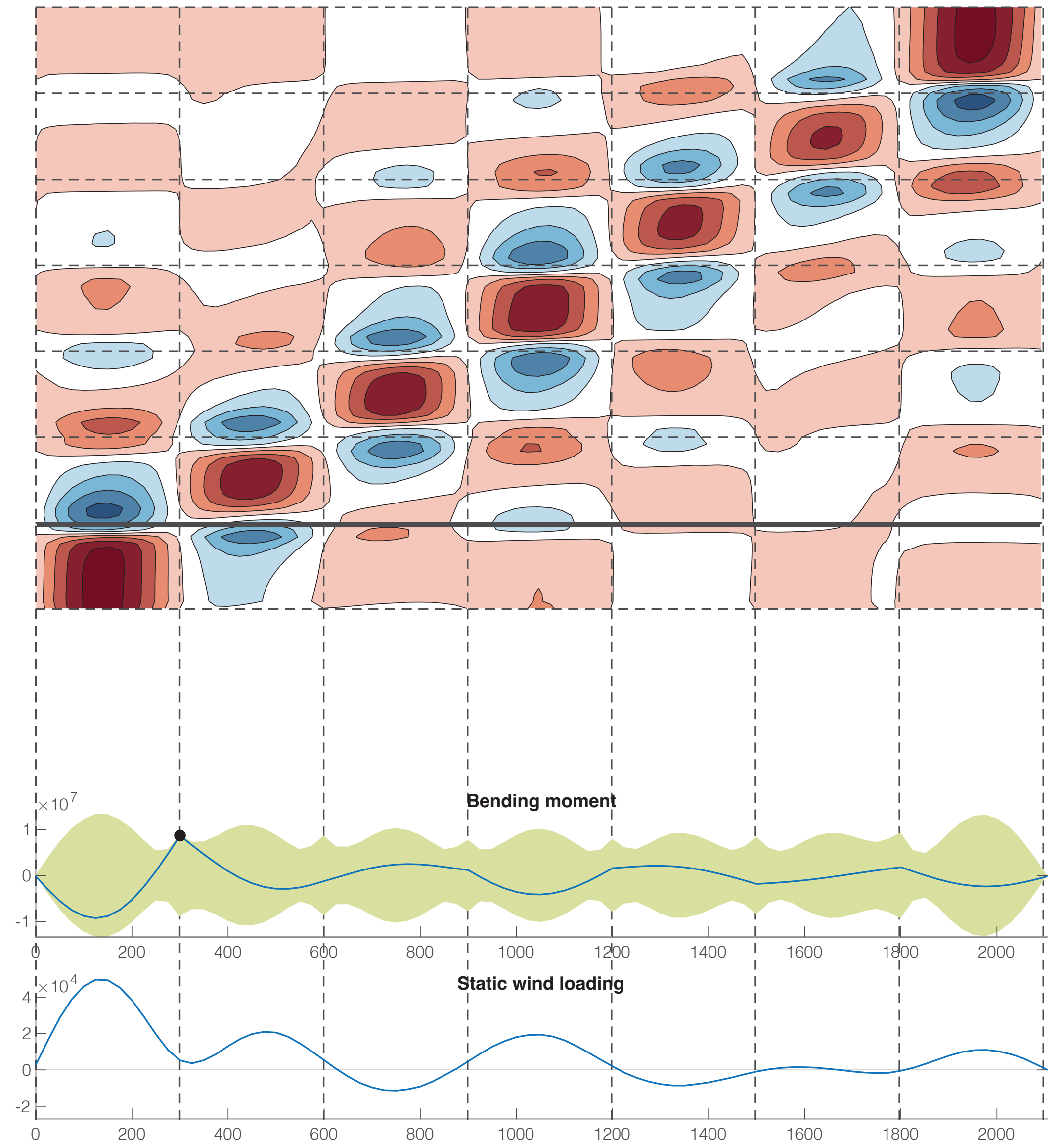
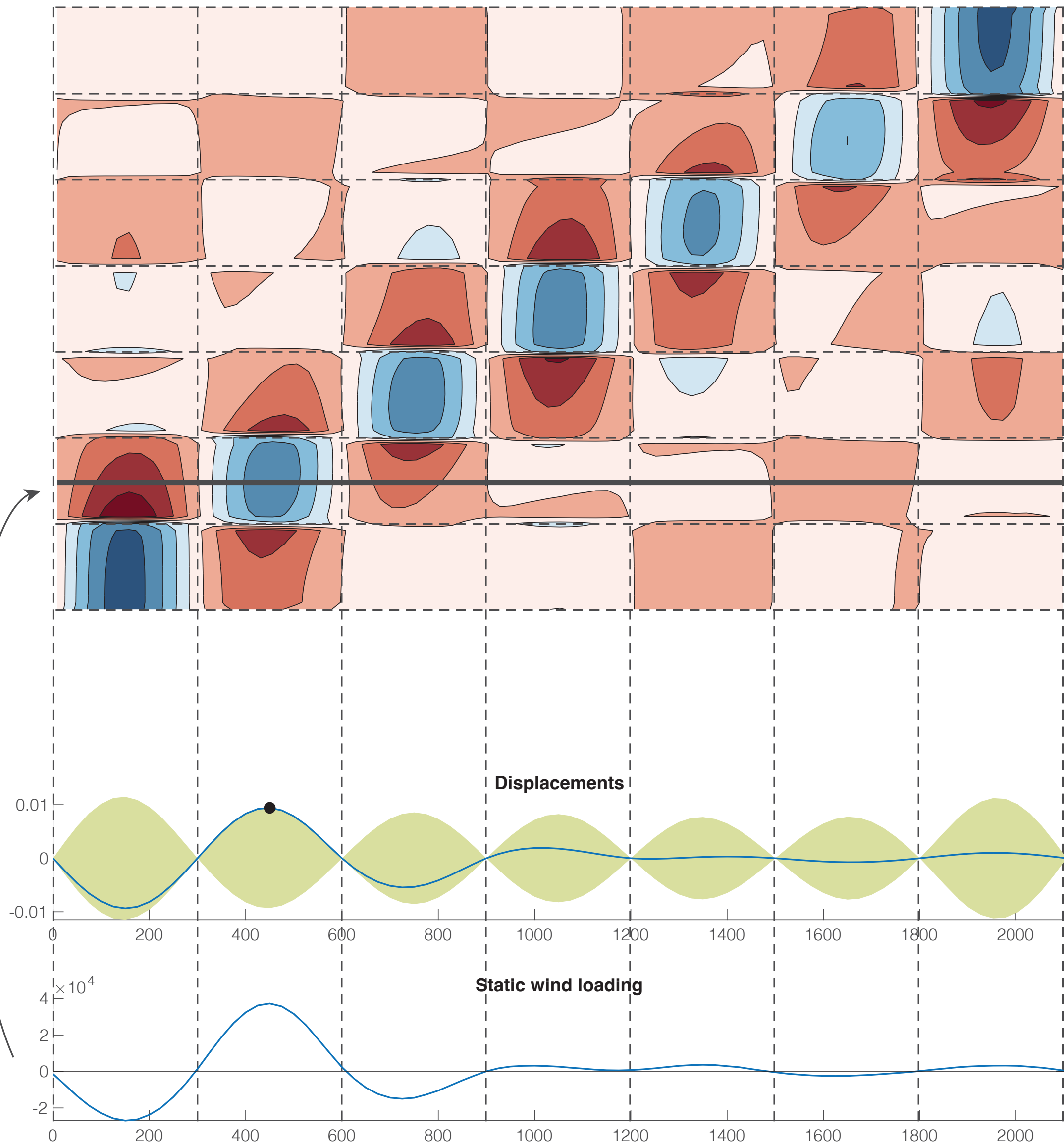
- could be accepted if smaller or secondary elements (but then, don't include them in responses)
- to be discussed with client

Overestimation :

- pay attention to excessive costs
- to be discussed with client







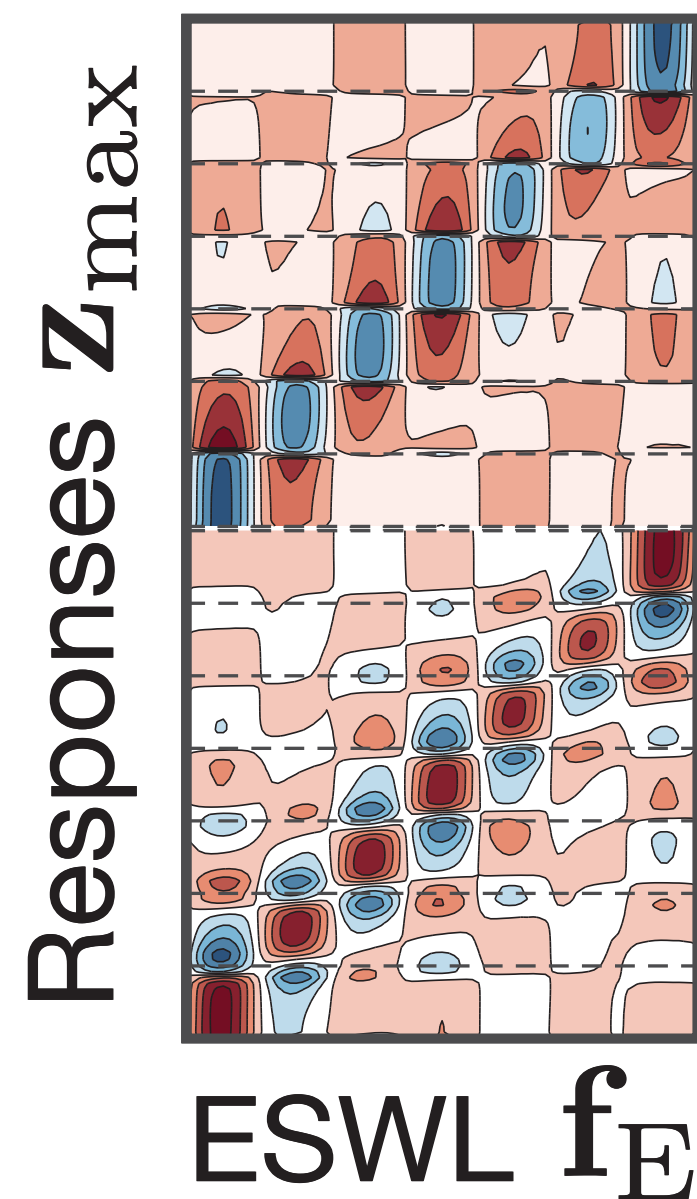
$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}^{\text{CPT}}$$

$$\mathbf{z}(t) = \mathbf{A}\mathbf{x}(t)$$

$$\mathbf{z}_{\max} = \max_t \mathbf{z}(t)$$

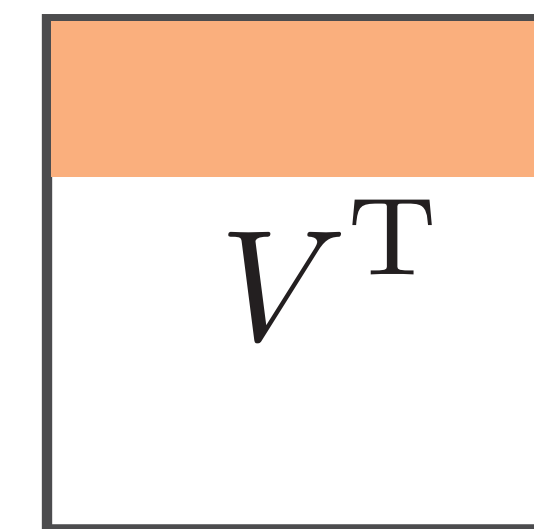
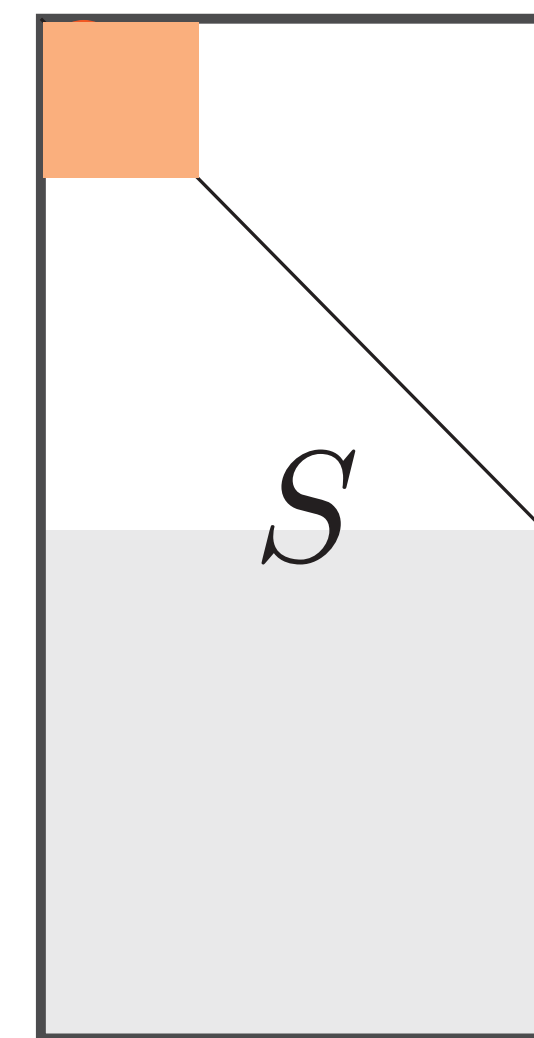
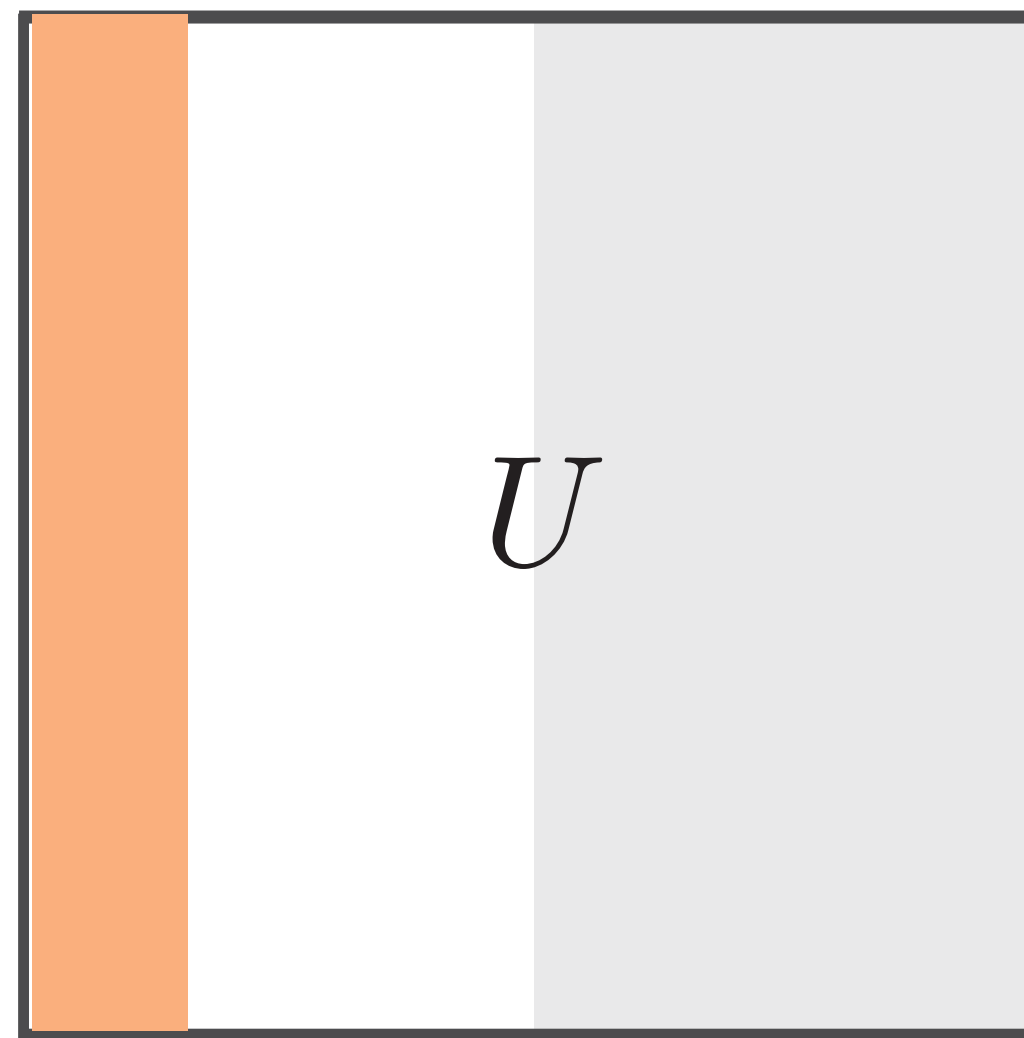
$$\mathbf{A}\mathbf{K}^{-1}\mathbf{f}_E = \mathbf{z}_{\max}$$

Find \mathbf{f}_E such that: $\mathbf{K}\mathbf{x}_E = \mathbf{f}_E$ and $\mathbf{A}\mathbf{x}_E = \mathbf{z}_{\max}$



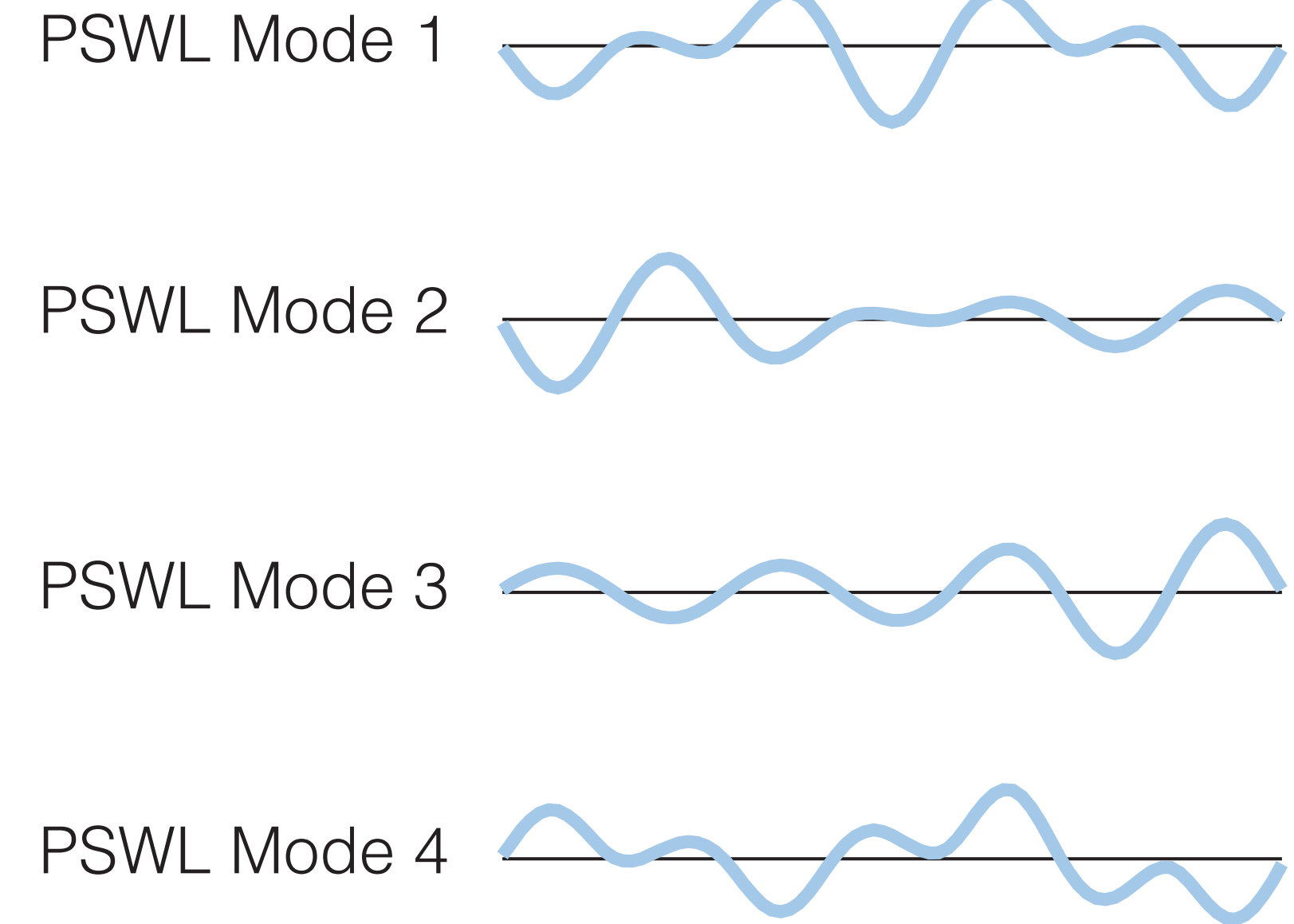
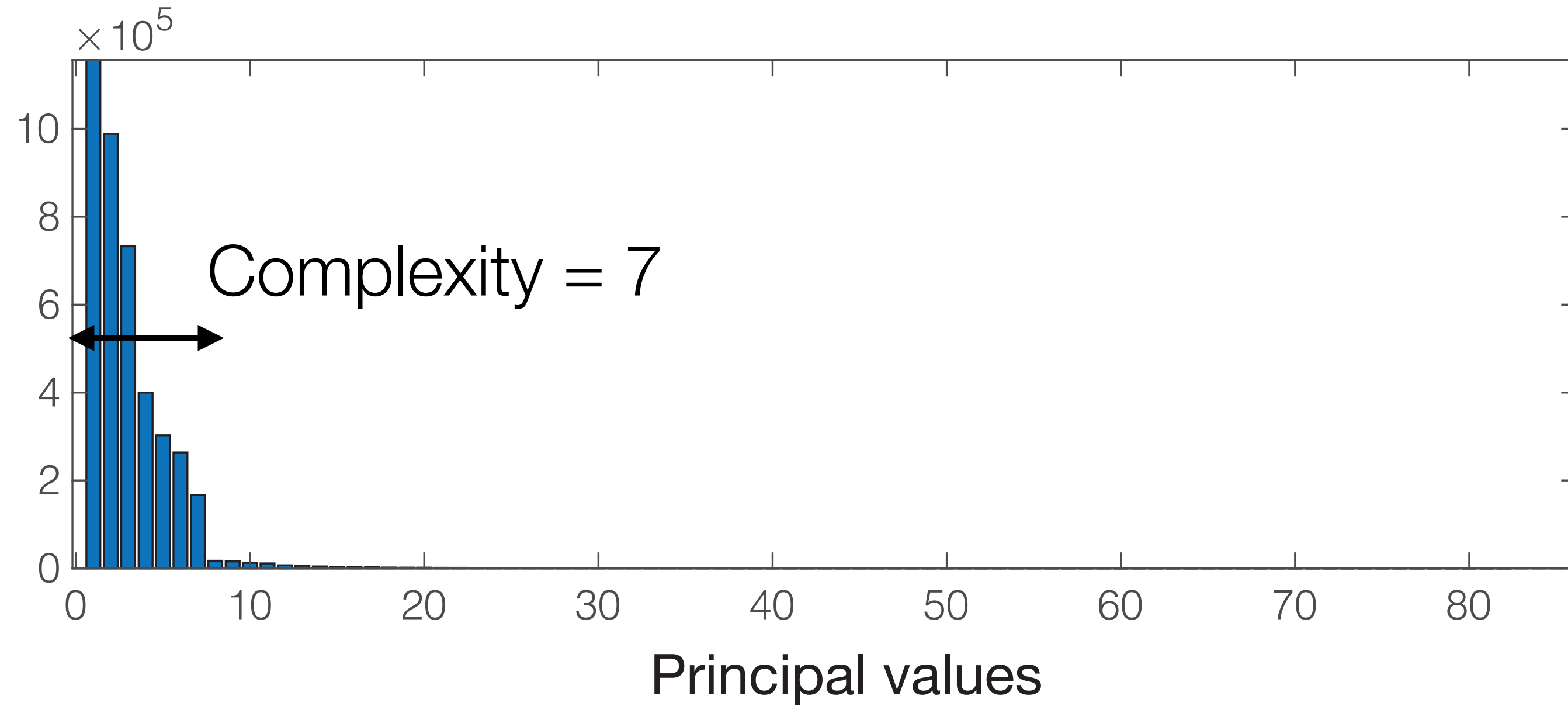
=

Principal Responses



PSWL

Pr. values



Complexity = 1



Complexity = 2

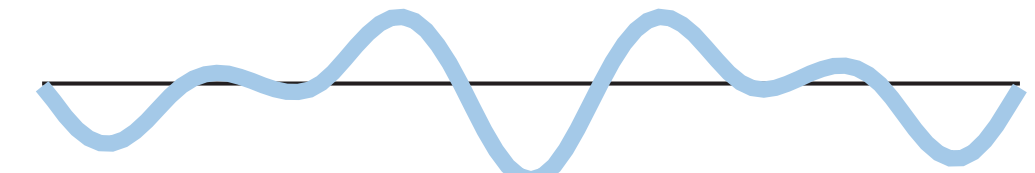


Complexity ~ 100-500

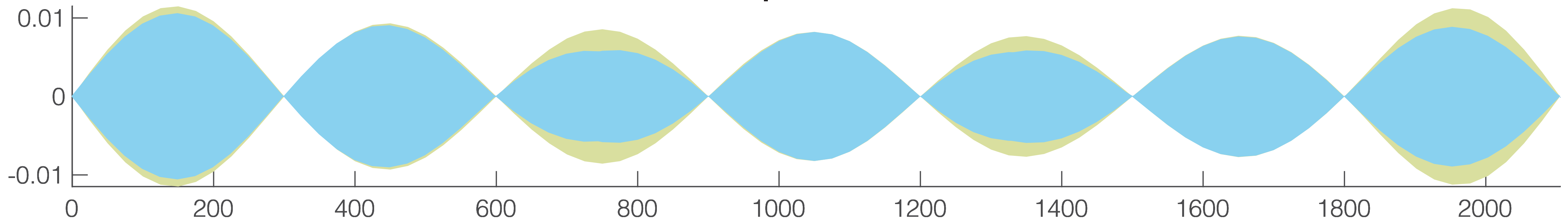
- PSWLs are affected by the nature of the response, i.e. Background or Resonant
- PSWLs, **as such**, are not necessarily optimum to reconstruct the envelope

PSWL 1 & 2 & 3 & 4

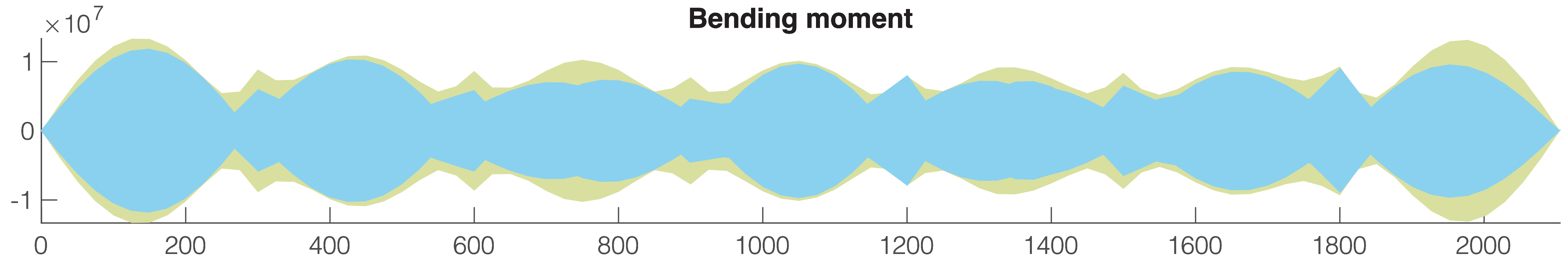
PSWL Mode 1



Displacements



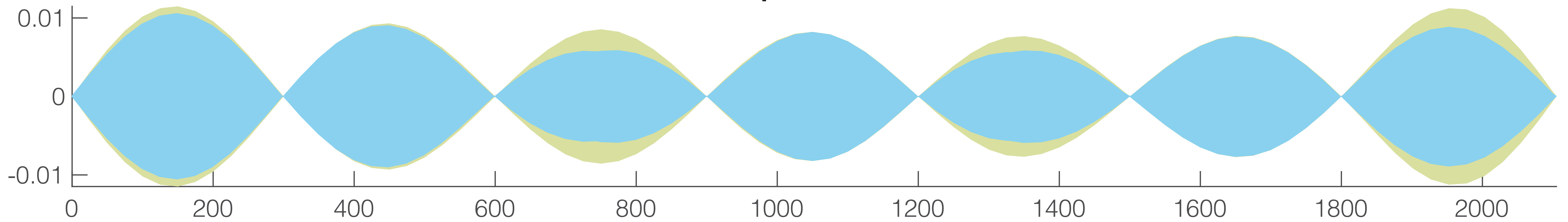
Bending moment



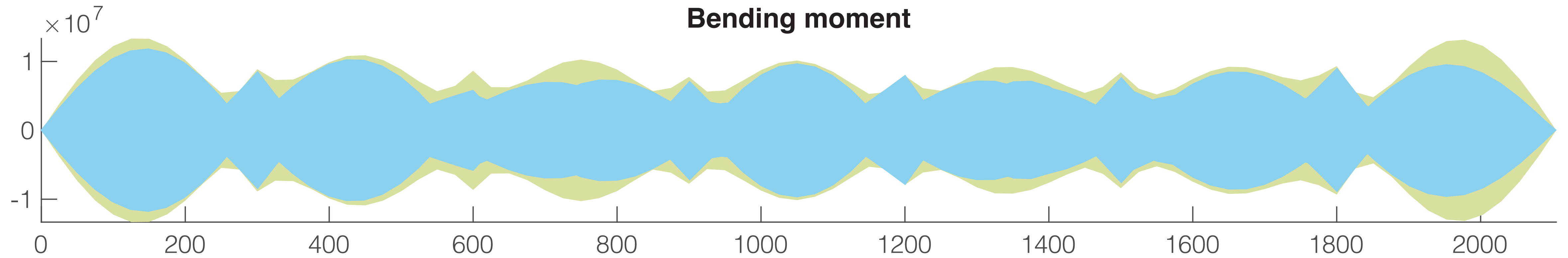


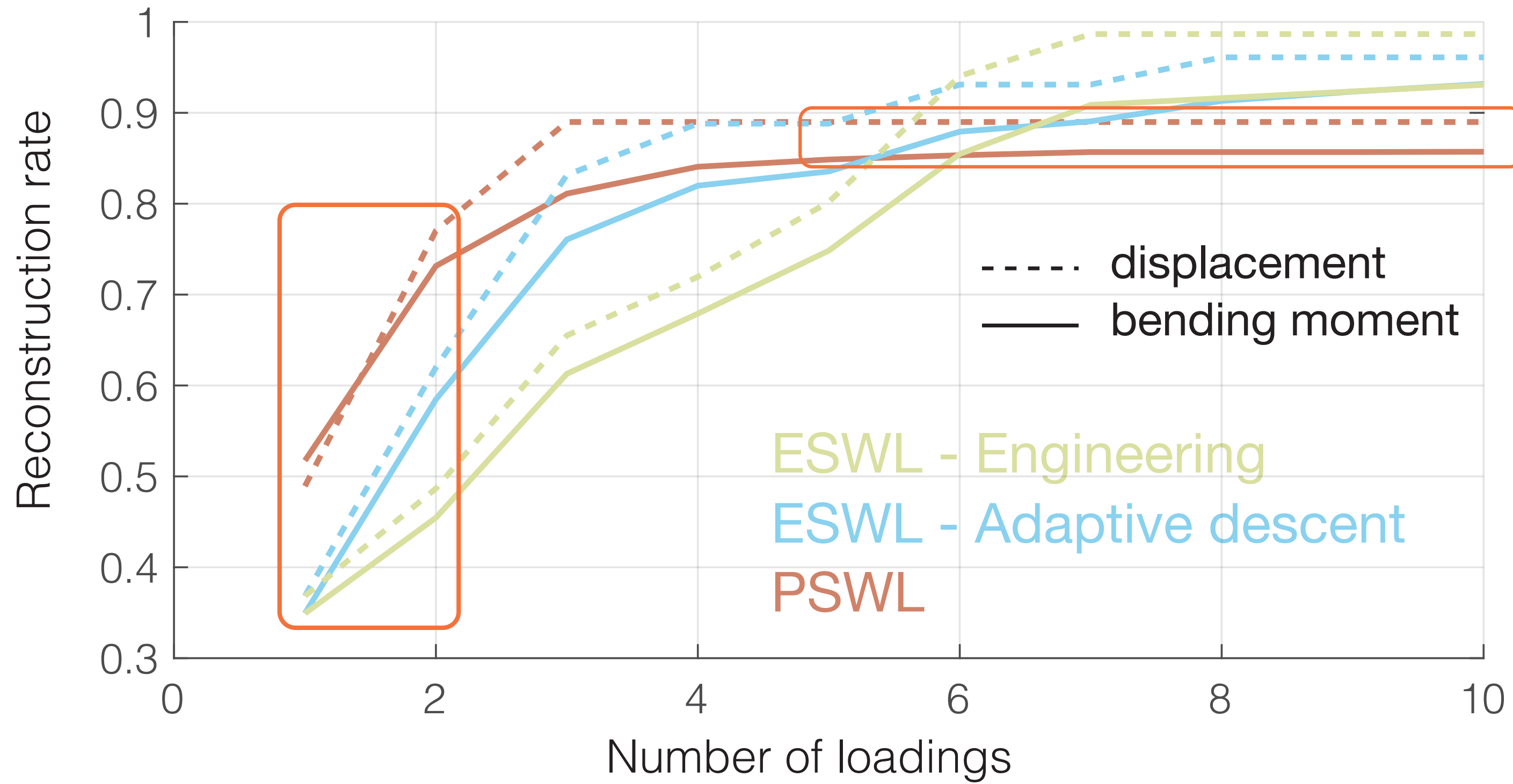
PSWL 1 to 7

Displacements

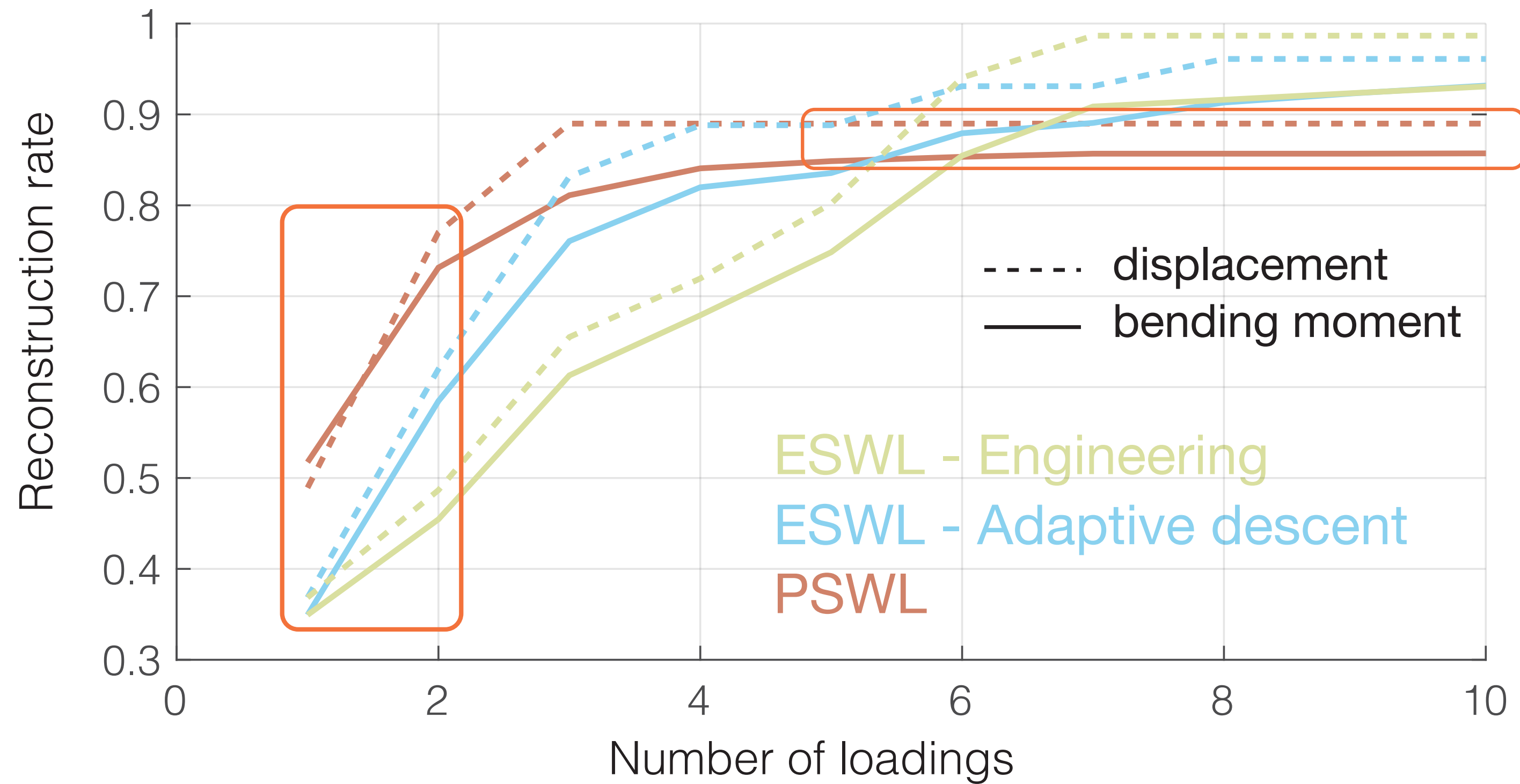


Bending moment



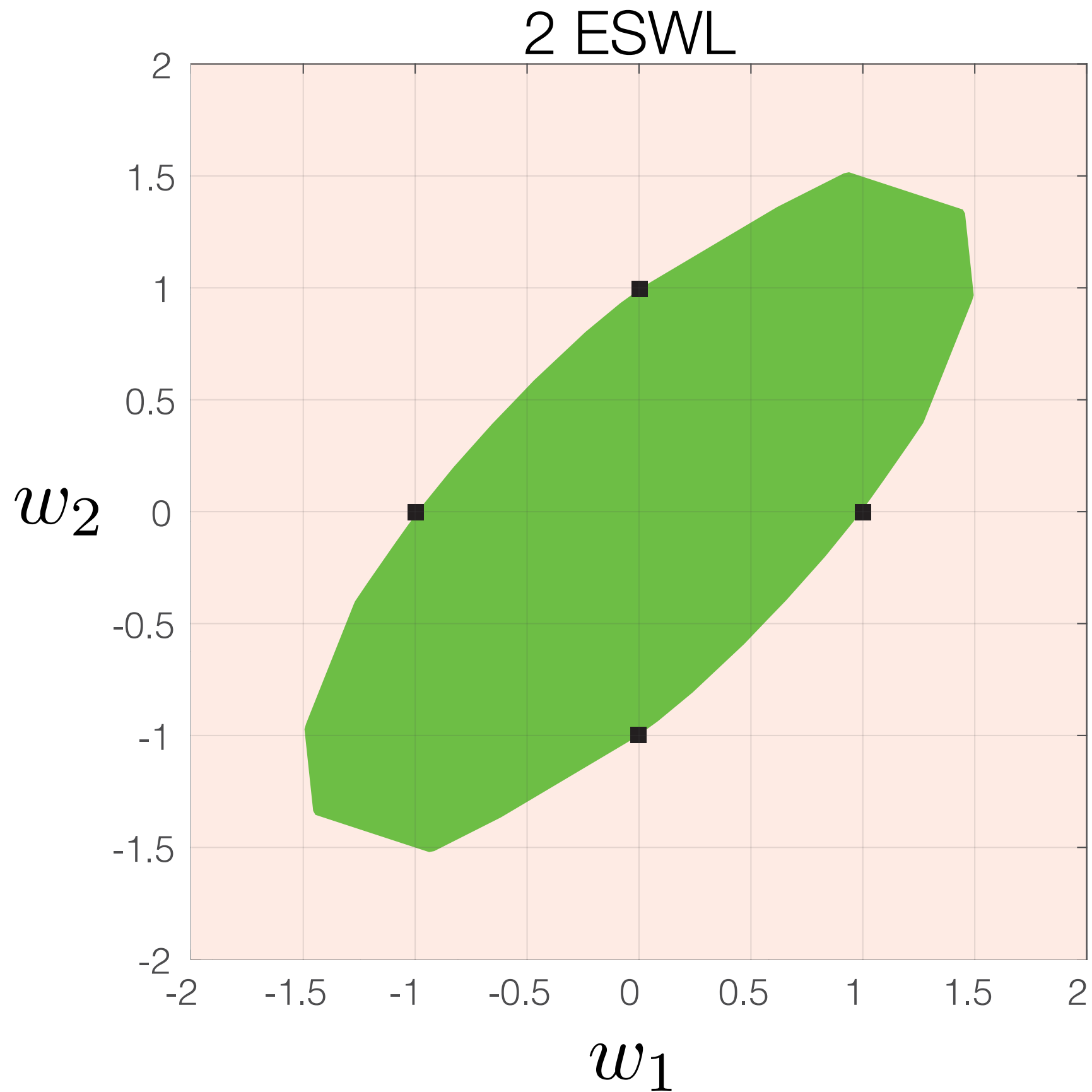


Observations : very good start, as expected
 poor terminal reconstruction rate

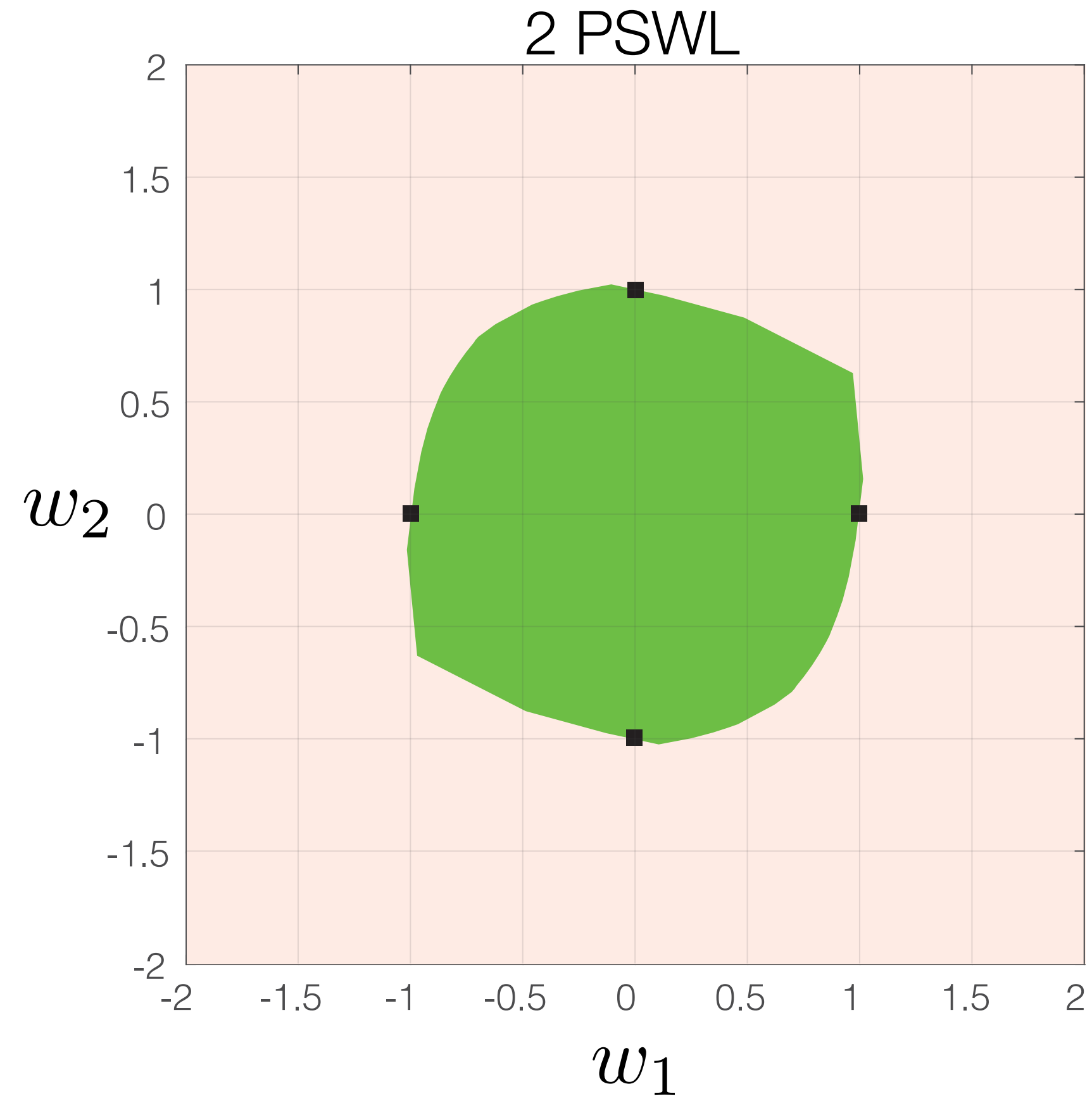


- Solutions :
- (i) allow some over-estimation - TBD with clients
 - (ii) use some PSWLs, then Adaptive descent
 - (iii) **(linearly) combine PSWLs**

Acceptable range of values for combination coefficients

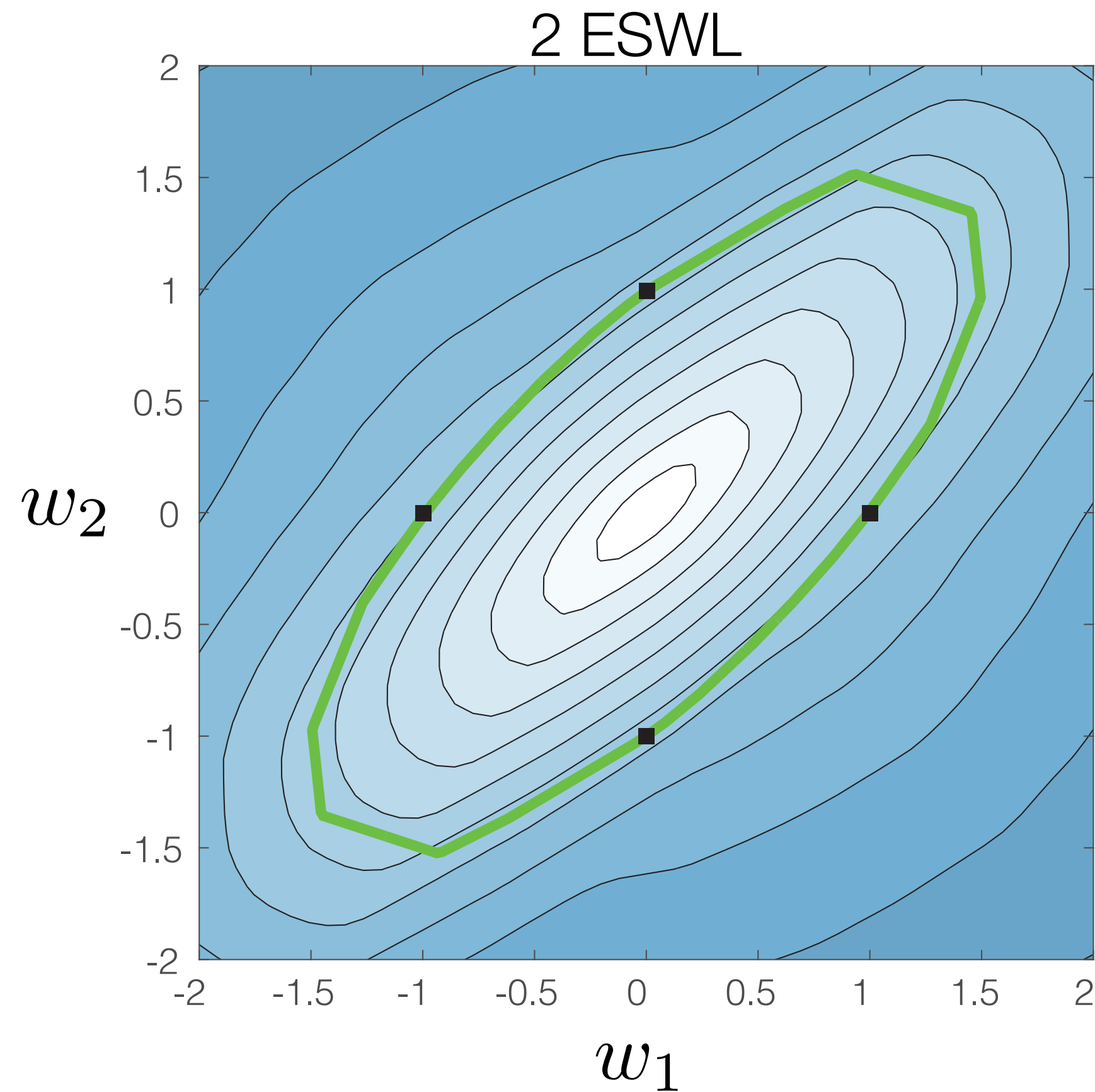


$$\mathbf{f}_E = w_1 \mathbf{f}_{E,1} + w_2 \mathbf{f}_{E,2}$$

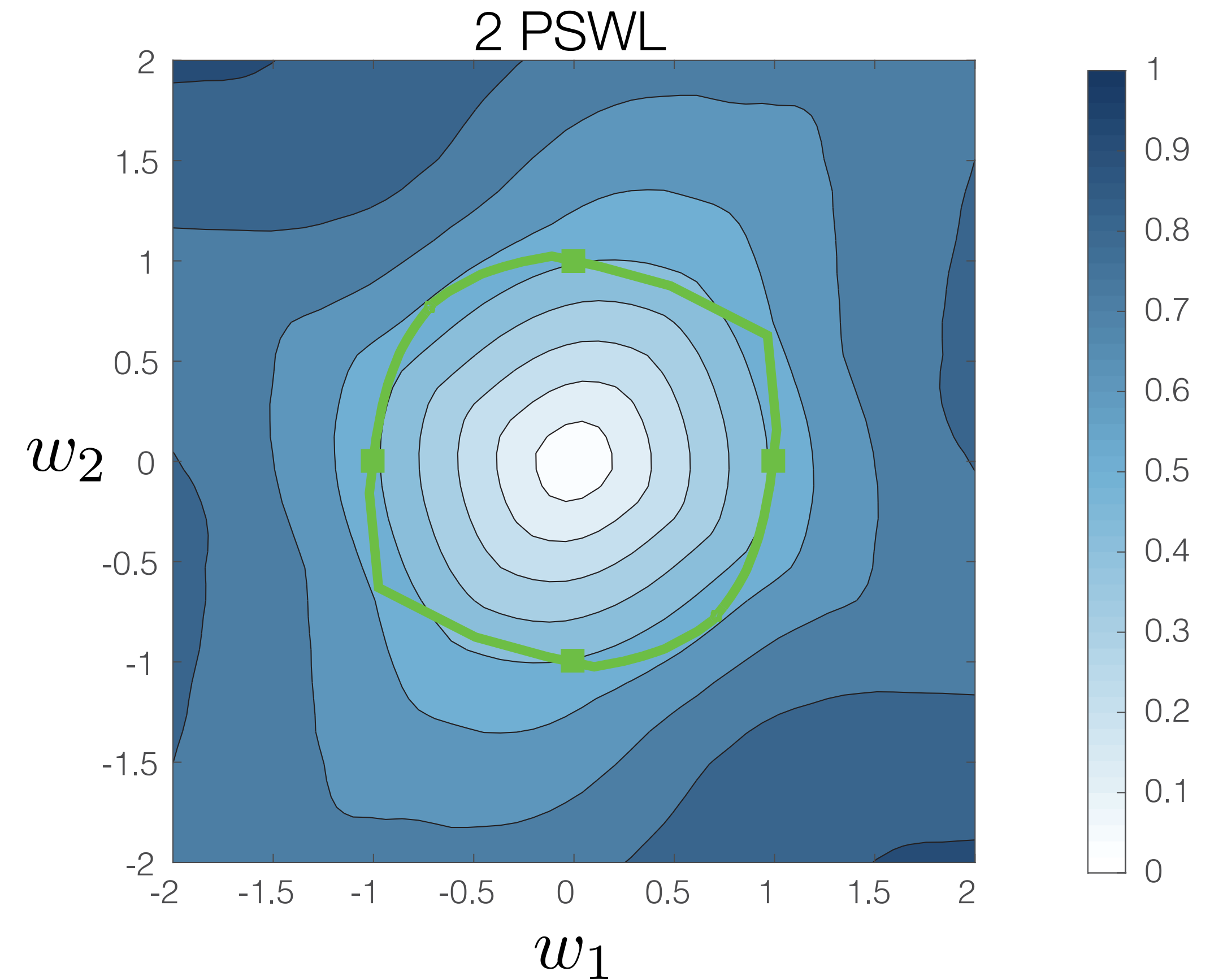


$$\mathbf{f}_E = w_1 \mathbf{f}_{E,1}^{\text{PSWL}} + w_2 \mathbf{f}_{E,2}^{\text{PSWL}}$$

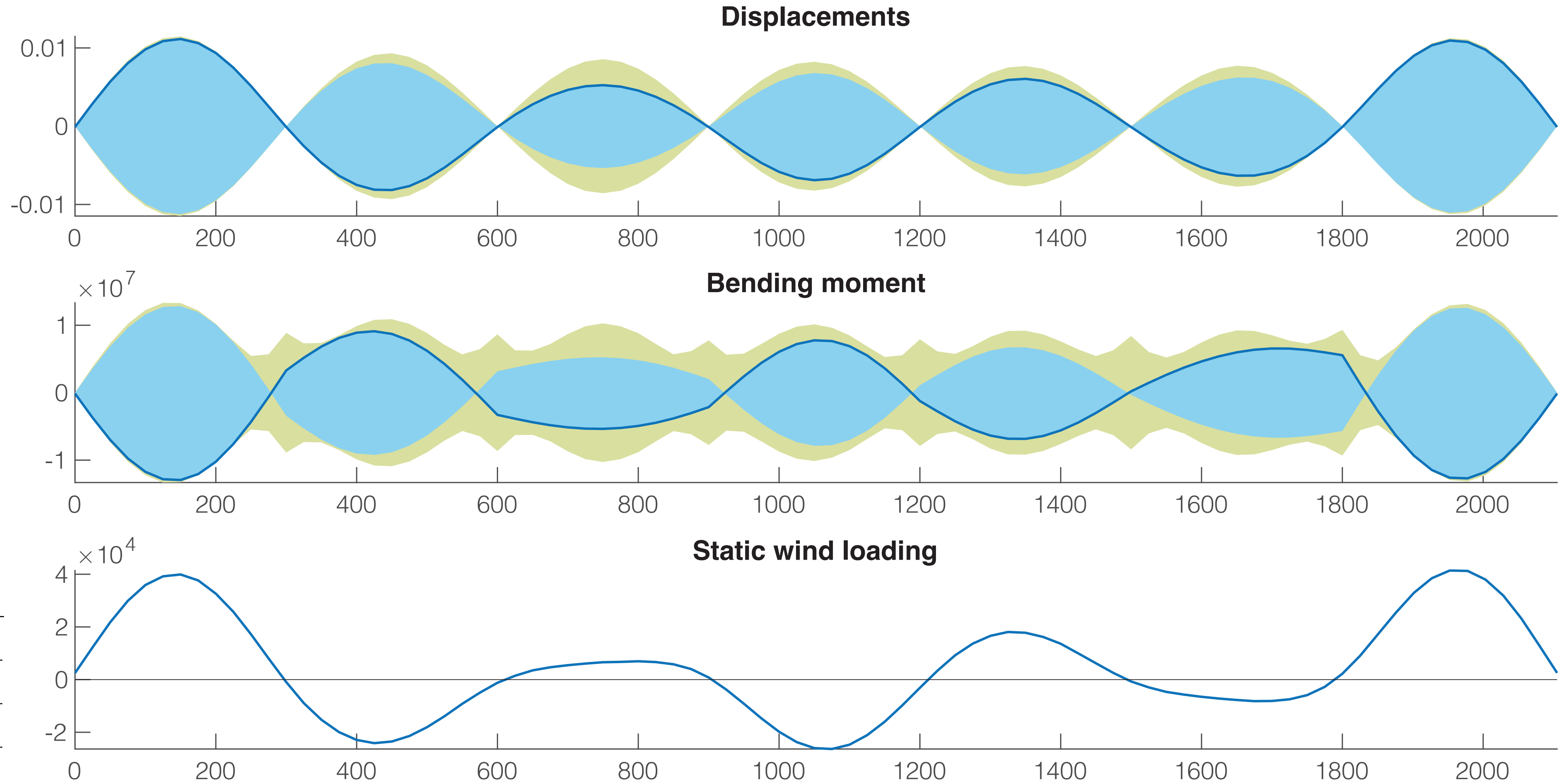
Reconstruction rate



$$\mathbf{f}_E = w_1 \mathbf{f}_{E,1} + w_2 \mathbf{f}_{E,2}$$



$$\mathbf{f}_E = w_1 \mathbf{f}_{E,1}^{\text{PSWL}} + w_2 \mathbf{f}_{E,2}^{\text{PSWL}}$$



Comb. 1

-0.16

-0.45

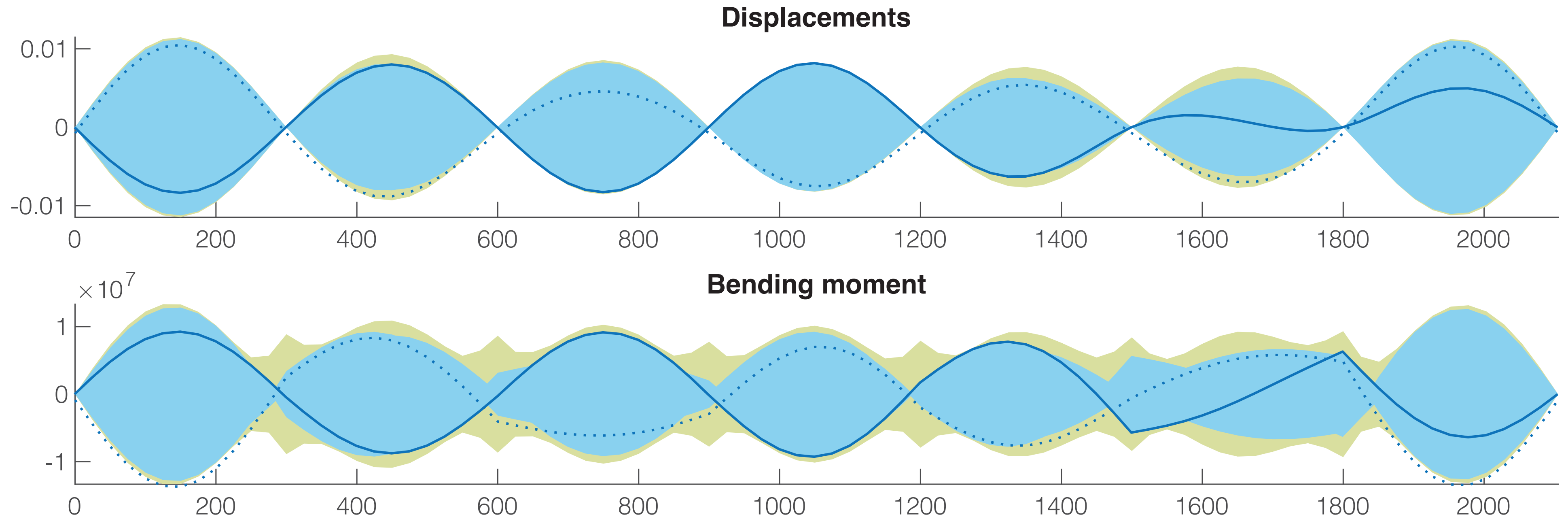
1.00

-0.04

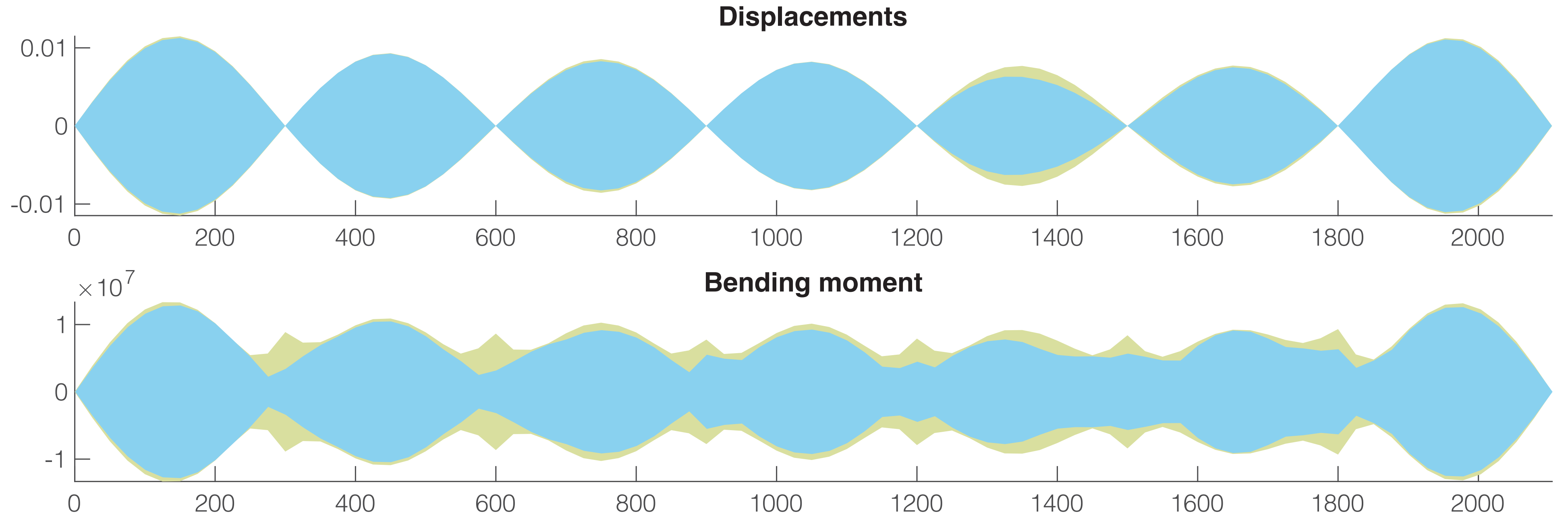
0.31

-0.38

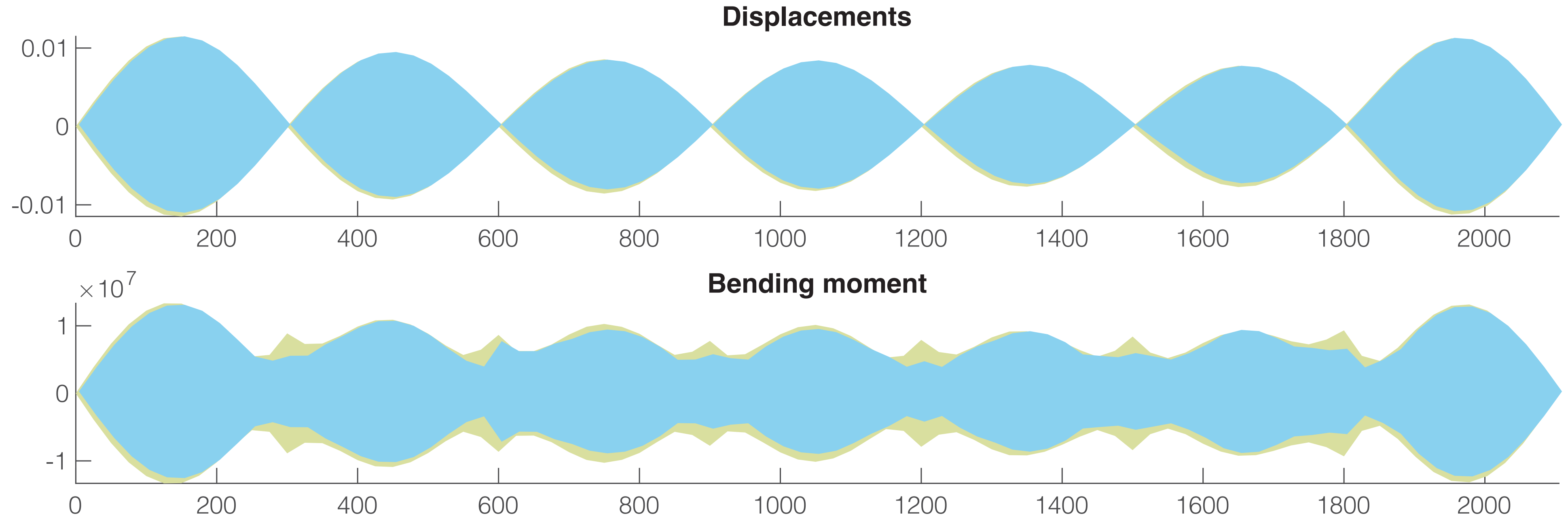
-0.14



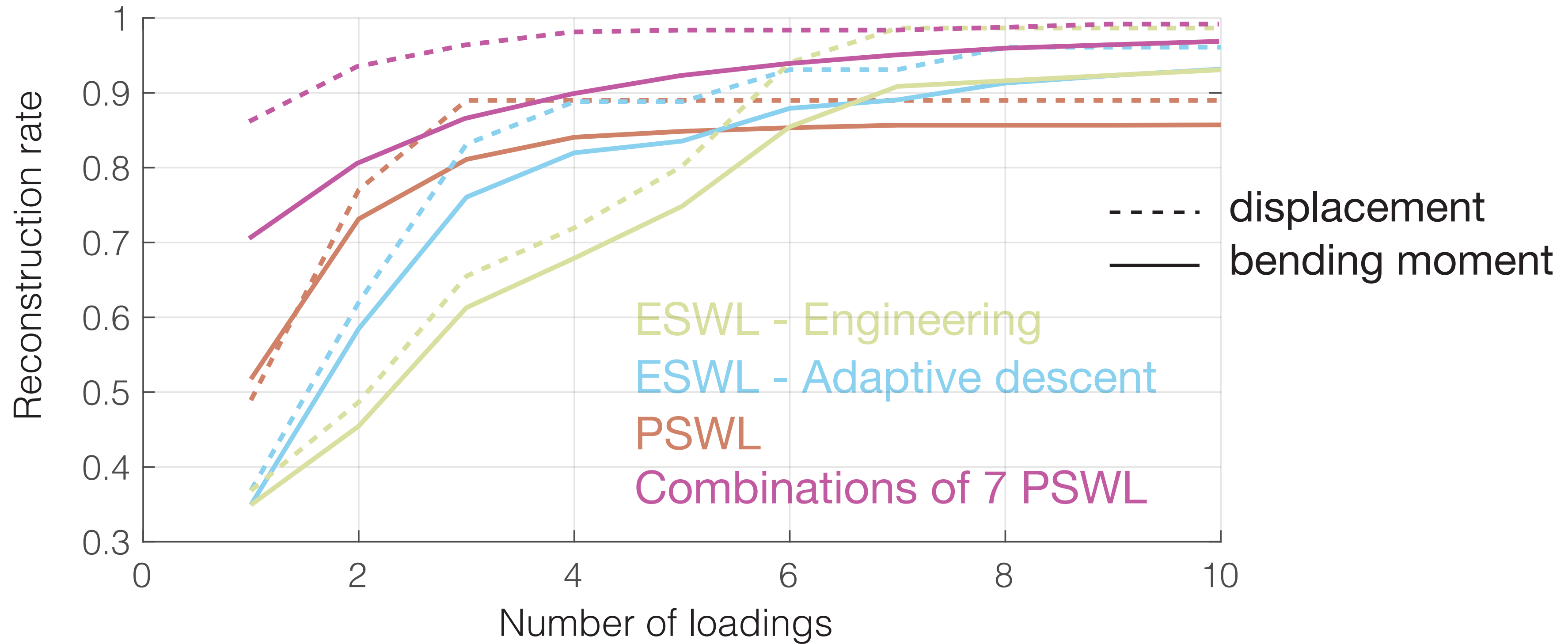
Comb. 1	Comb. 2
-0.16	-0.92
-0.45	1.00
1.00	-0.50
-0.04	-0.47
0.31	-0.08
-0.38	0.05
-0.14	-0.05



Comb. 1	Comb. 2	Comb. 3
-0.16	-0.92	-0.05
-0.45	1.00	1.00
1.00	-0.50	0.18
-0.04	-0.47	0.16
0.31	-0.08	-0.10
-0.38	0.05	0.02
-0.14	-0.05	0.21



Comb. 1	Comb. 2	Comb. 3	Comb. 4	...	Comb. 10
-0.16	-0.92	-0.05	0.60
-0.45	1.00	1.00	0.77
1.00	-0.50	0.18	0.75
-0.04	-0.47	0.16	0.44
0.31	-0.08	-0.10	1.00
-0.38	0.05	0.02	0.16
-0.14	-0.05	0.21	0.16



Advantages :

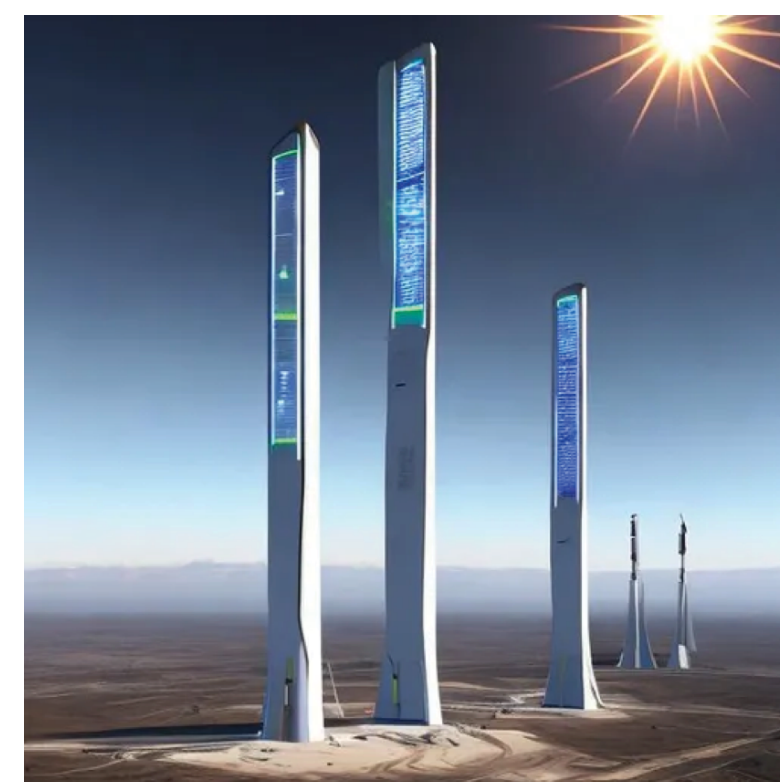
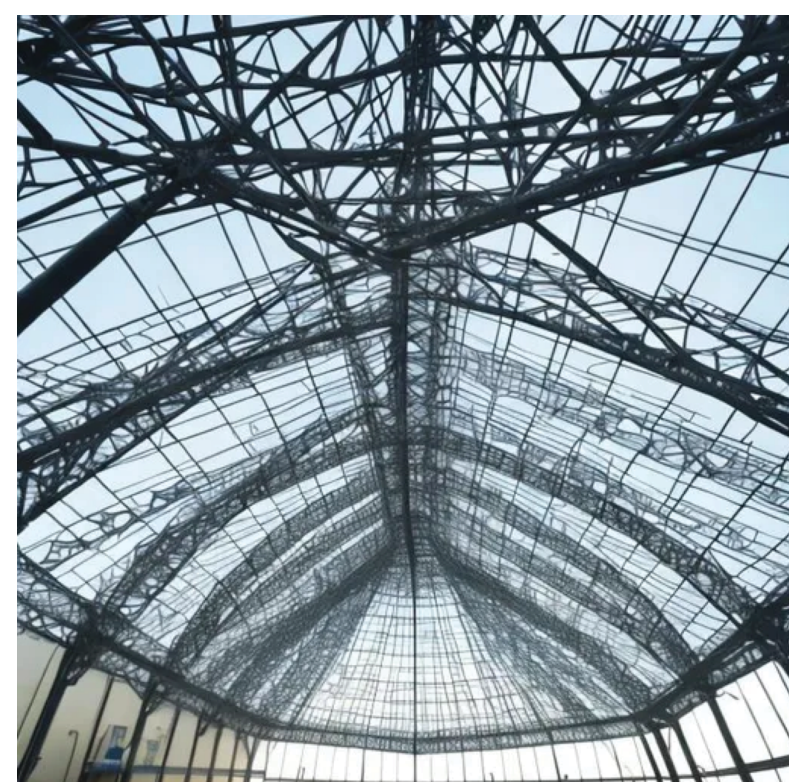
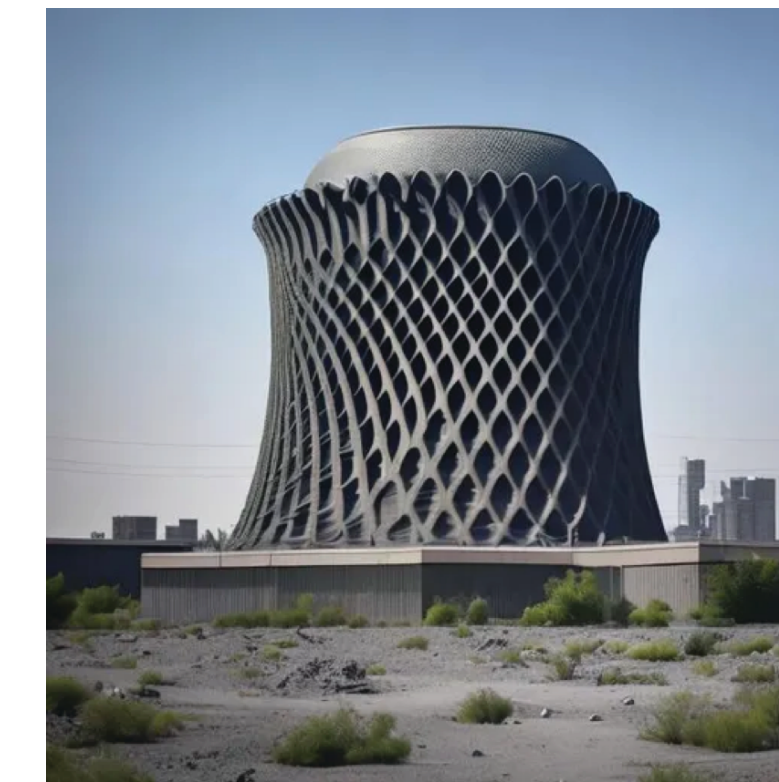
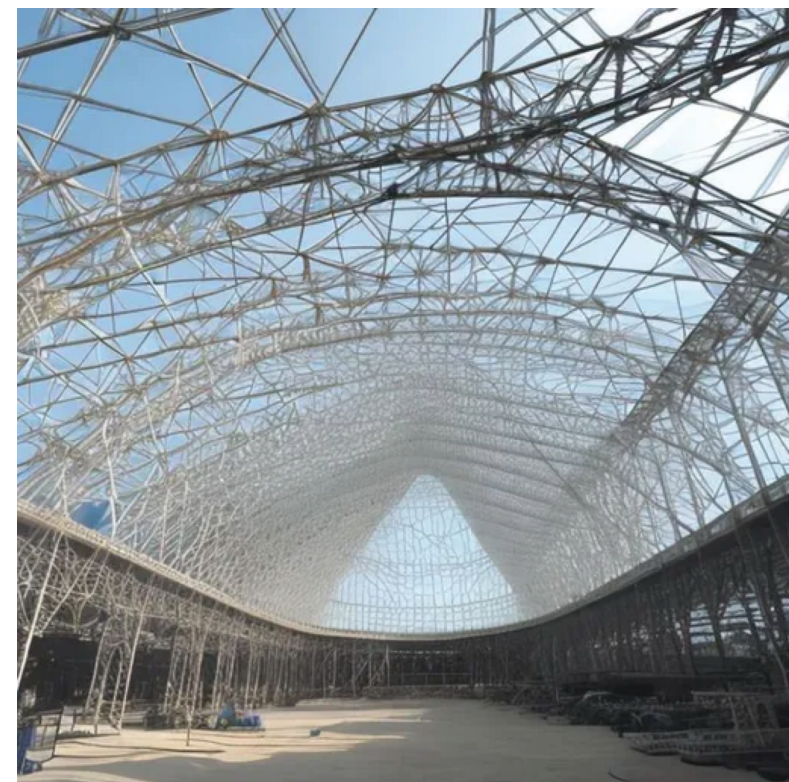
- provide to client / design engineers a minimum number of load distributions that are realistic
- provide a number of combination coefficients
- simple and efficient (no over-estimation required)

Challenges : Large roofs,
Silos,
Cooling tower,
Bridges, ...

- Non-Symmetric envelopes
- Nonlinear responses, $\mathbf{z}(t) = \mathbf{A}\mathbf{x}(t)$
- Design/Extreme values
- Several wind incidences

See beyond buffeting

Traditional « engineering solutions » (vs. AI ...)





The Wind Tunnel Lab

Thomas Andrienne

PhD students

N. Blaise, S. Hippola, J. Heremans, M. Esposito Marzino, T. Canor, H. Vanvinckenroye, M. Geuzaine, F. Rigo, K. Theunissen

Visiting scholars

Academic partners

Public funding

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FRIA, National Fund for Industrial and Agricultural Research

SPW, Walloon Public Services

WT partners

CSTC, Nantes
Politecnico di Milano

Industrial partners

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Editors

IJWEIA, W&S, Wind MDPI

Illustrations

OpenArt

How did we end up here ?
What do we want ?
What do we need ?

ESWL

Envelope Reconstruction

Grazie!

Envelope Reconstruction
with Equivalent Wind Loads

Vincent Denoël



August 27-31, 2023
FLORENCE - ITALY

16th International
Conference on
Wind Engineering

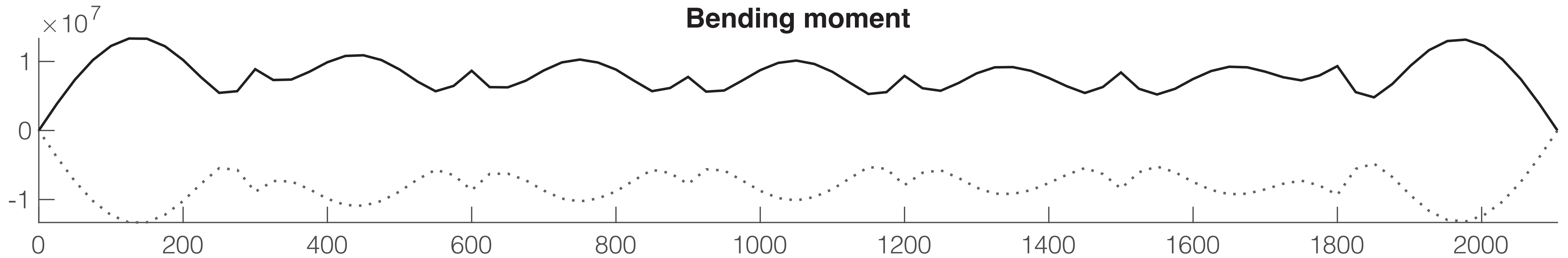




Backup slides



$$M' = T \quad ; \quad M'' = p$$



Second derivative of bending moment

