# Lateral-torsional buckling of beams made of monosymmetrical thin-walled sections

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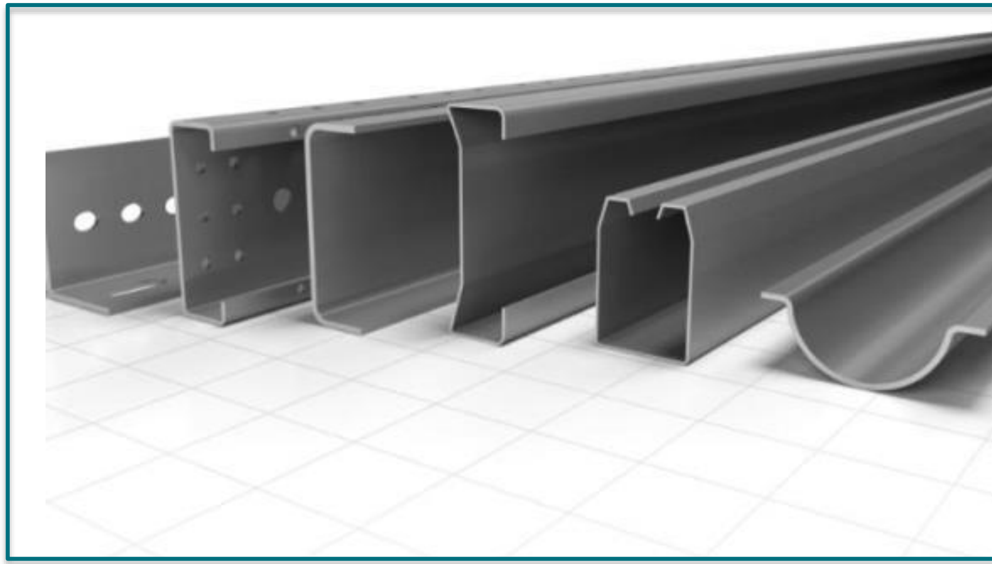
Common H/I sections



Various shape of thin-walled cold-formed sections

# Lateral-torsional buckling of beams made of monosymmetrical thin-walled sections

- Available design rules in the current European codes
- Studied cases
  - Beam over two simple supports subjected to four points bending
  - Beam over two simple supports subjected to constant bending
- Conclusion and perspectives

# Available design rules in the current codes

- EN1993-1-3 §6.2.4 (1)

**6.2.4 Lateral-torsional buckling of members subject to bending**

(1) The design buckling resistance moment of a member that is susceptible to lateral-torsional buckling should be determined according to EN 1993-1-1, section 6.3.2.2 using the lateral buckling curve b.

(2) This method should not be used for the sections that have a significant angle between the principal axes of the effective cross-section, compared to those of the gross cross-section.

- EN1993-1-1 §6.3.2.2

(2) $M_{cr}$ is based on gross cross sectional properties and takes into account the loading conditions, the real moment distribution and the lateral restraints.

- CEN/TR 1993-1-103

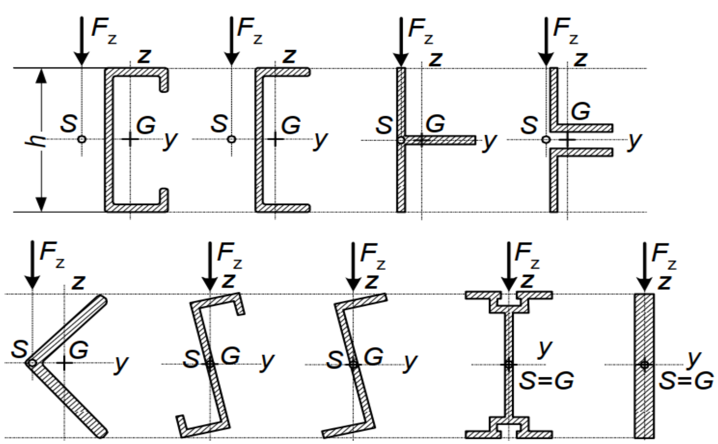
# Critical bending moment for symmetrical sections about their major axis

- CEN/TR 1993-1-103 §5.1.3

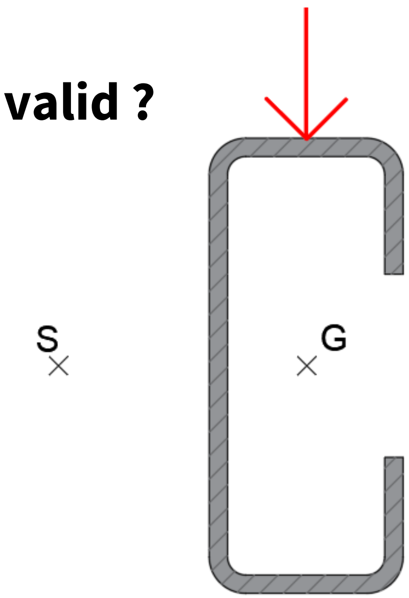
**5.2.2 Beams with uniform cross-sections, symmetrical about the major axis, point symmetrical and double symmetrical**

(1) For beams with uniform cross-sections, symmetrical about the major axis, point symmetrical and double symmetrical, loaded perpendicular to their major axis in the plane going through the shear centre, Figure 5.20, $z_j = 0$, thus:

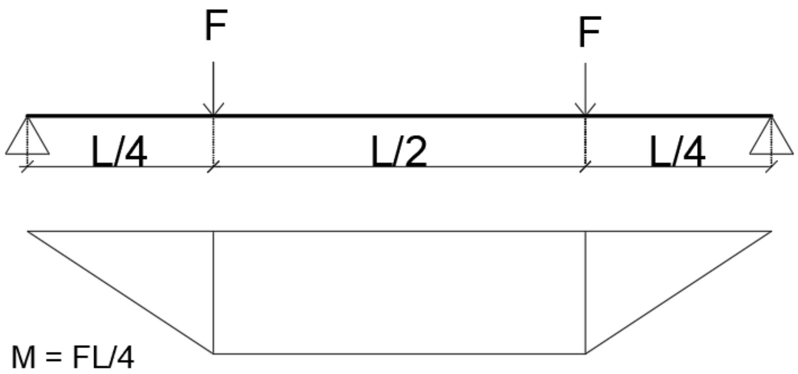
$$M_{cr} = C_1 N_{cr,z}\left[\sqrt{c^2 + \left(C_2 \cdot z_g\right)^2} + C_2 \cdot z_g\right] = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{(k_z \cdot L)^2} \cdot \left[\sqrt{\left(\frac{k_z}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(k_z \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + \left(C_2 \cdot z_g\right)^2} - C_2 \cdot z_g\right]$$



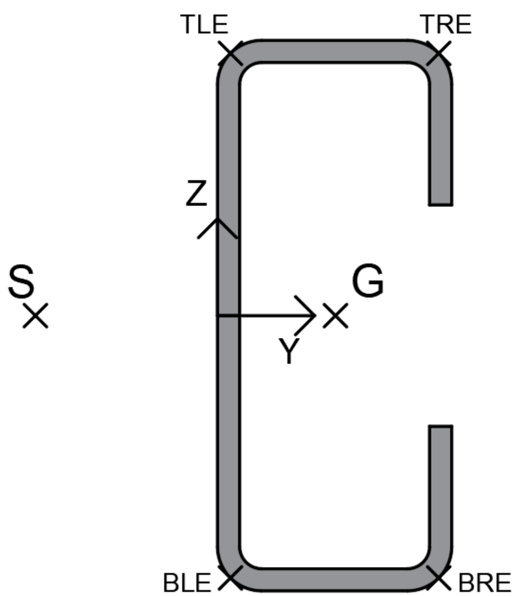

**Still valid ?**



# Case study 1 – Analytical evaluation



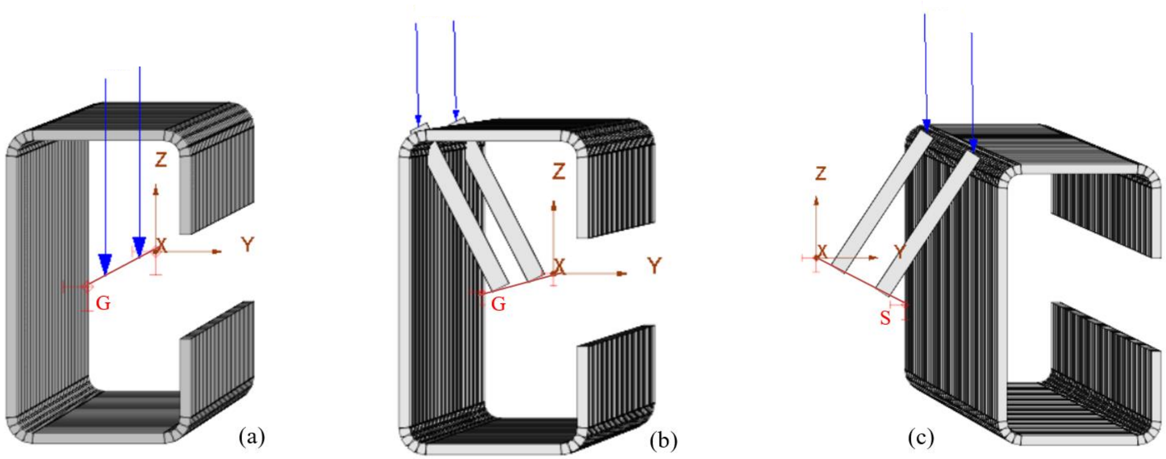

- L = 4,000 mm
- E = 210,000 Mpa




| N° | Position of the loading | $z_g$ | $C_1$ | $C_2$ | $k_z$ | $k_w$ | $M_{cr,an}$ [kNm] |
|---|---|---|---|---|---|---|---|
| 1 | S | 0 | 1.04 | 0.42 | 1 | 1 | 0.61 |
| 2 | G | 0 | 1.04 | 0.42 | 1 | 1 | 0.61 |
| 3 | TLE | $h/2$ | 1.04 | 0.42 | 1 | 1 | 0.52 |
| 4 | TRE | $h/2$ | 1.04 | 0.42 | 1 | 1 | 0.52 |
| 5 | BLE | $-h/2$ | 1.04 | 0.42 | 1 | 1 | 0.72 |
| 6 | BRE | $-h/2$ | 1.04 | 0.42 | 1 | 1 | 0.72 |

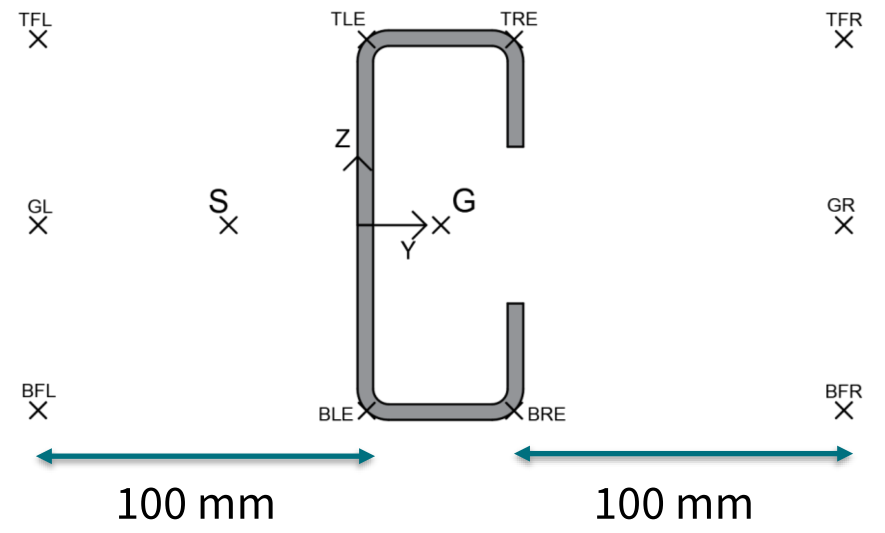
# Case study 1 – Numerical evaluation

- Linear buckling analysis performed with FINELG
- Beam elements
- Three situations considered
  - (a) SL and LL coincide
  - (b) SL through gravity centre, LL through one of the 6 before mentioned points
  - (c) SL through the shear centre, LL through one of the 6 before mentioned points



# Case study 1 – Numerical results

| N° | Sit. a | | Sit. b (axis at G) | | Sit. c (axis at S) | |
|---|---|---|---|---|---|---|
| | $M_{cr,n}$ [kNm] | $\frac{M_{cr,n}}{M_{cr,an}}$ | $M_{cr,n}$ [kNm] | $\frac{M_{cr,n}}{M_{cr,an}}$ | $M_{cr,n}$ [kNm] | $\frac{M_{cr,n}}{M_{cr,an}}$ |
| 1 | 0.61 | 0.99 | 0.61 | 0.99 | - | - |
| 2 | 0.61 | 0.98 | - | - | 0.61 | 0.99 |
| 3 | 0.51 | 0.98 | 0.51 | 0.98 | 0.51 | 0.99 |
| 4 | 0.51 | 0.98 | 0.51 | 0.98 | 0.51 | 0.98 |
| 5 | 0.71 | 0.98 | 0.71 | 0.98 | 0.71 | 0.98 |
| 6 | 0.71 | 0.98 | 0.71 | 0.98 | 0.71 | 0.98 |



# Case study 1 – Numerical results

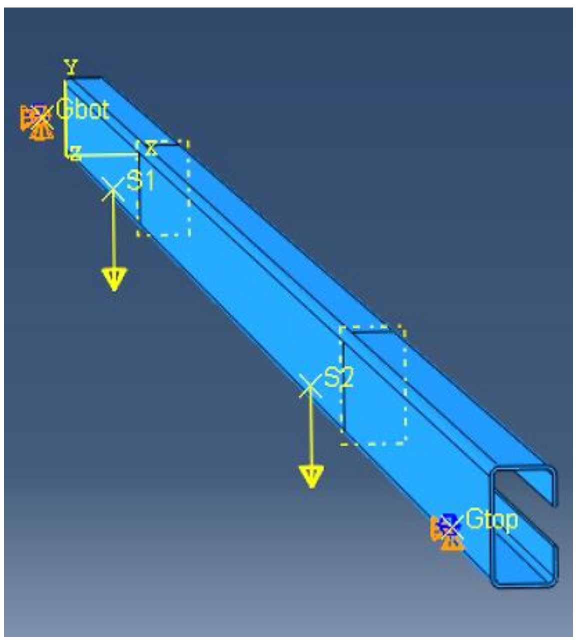
| N° | Sit. a | | Sit. b (axis at G) | | Sit. c (axis at S) | |
|---|---|---|---|---|---|---|
| | $M_{cr,n}$ [kNm] | $\frac{M_{cr,n}}{M_{cr,an}}$ | $M_{cr,n}$ [kNm] | $\frac{M_{cr,n}}{M_{cr,an}}$ | $M_{cr,n}$ [kNm] | $\frac{M_{cr,n}}{M_{cr,an}}$ |
| 1 | 0.61 | 0.99 | 0.61 | 0.99 | - | - |
| 2 | 0.61 | 0.98 | - | - | 0.61 | 0.99 |
| 3 | 0.51 | 0.98 | 0.51 | 0.98 | 0.51 | 0.99 |
| 4 | 0.51 | 0.98 | 0.51 | 0.98 | 0.51 | 0.98 |
| 5 | 0.71 | 0.98 | 0.71 | 0.98 | 0.71 | 0.98 |
| 6 | 0.71 | 0.98 | 0.71 | 0.98 | 0.71 | 0.98 |
| 7a | 0.60 | 0.98 | 0.61 | 0.99 | 0.61 | 0.99 |
| 7b | 0.60 | 0.98 | 0.60 | 0.98 | 0.6 | 0.98 |
| 8a | 0.51 | 0.99 | 0.51 | 0.99 | 0.51 | 0.99 |
| 8b | 0.51 | 0.98 | 0.51 | 0.98 | 0.50 | 0.98 |
| 9a | 0.71 | 0.98 | 0.71 | 0.98 | 0.71 | 0.98 |
| 9b | 0.71 | 0.98 | 0.71 | 0.98 | 0.71 | 0.98 |

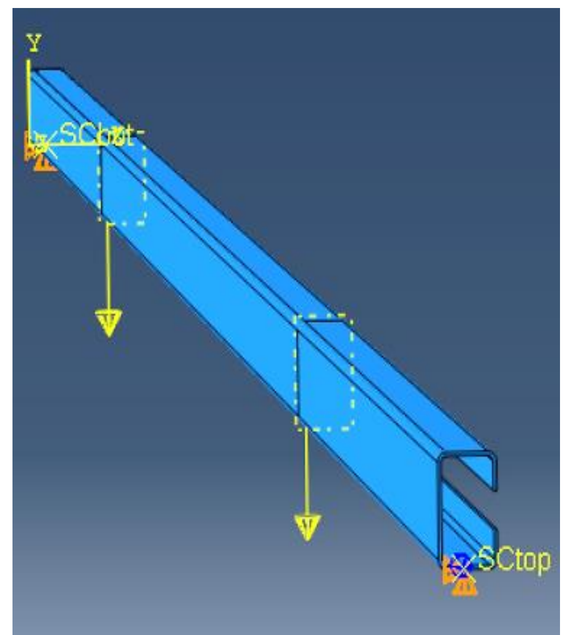
| | Sit. A | Sit. B | Sit. C |
|---|---|---|---|
| Mean$\left(\frac{M_{cr,n}}{M_{cr,an}}\right)$ | 0.98 | 0.98 | 0.98 |
| Standard deviation [%] | 0.36 | 0.34 | 0.34 |

Three extreme theoretical cases to study the influence of the horizontal position of the load

# Case study 1 – Numerical evaluation

- Linear buckling analysis performed with ABAQUS
- Solid elements (linear and quadratic)
- One situation considered
  - (a) SL and LL coincide

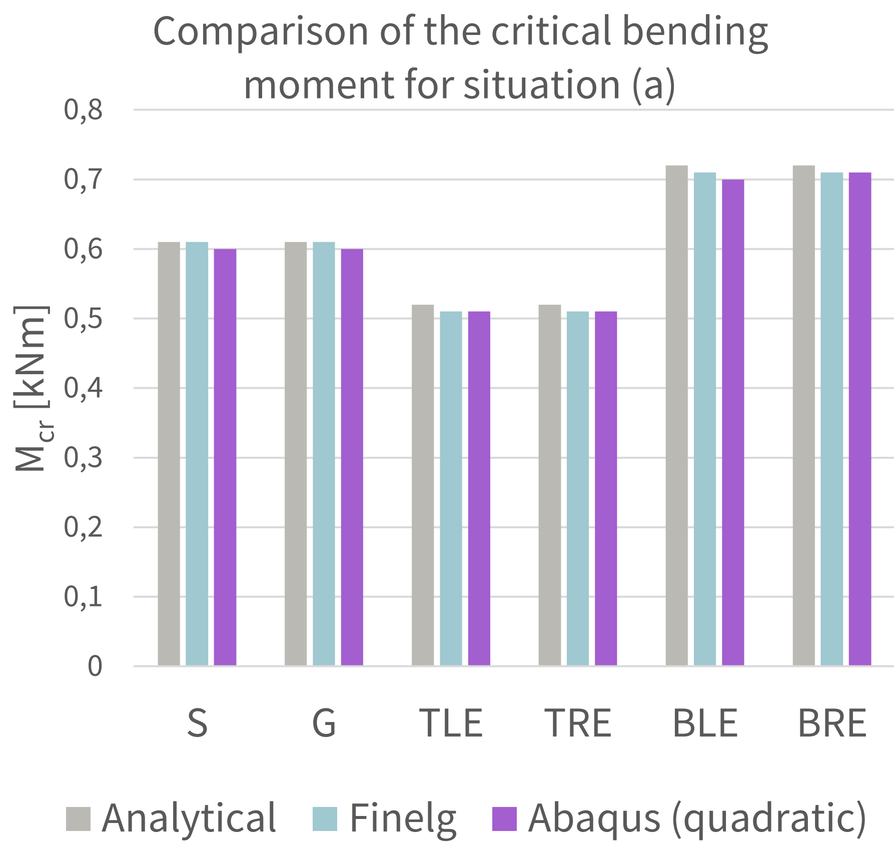


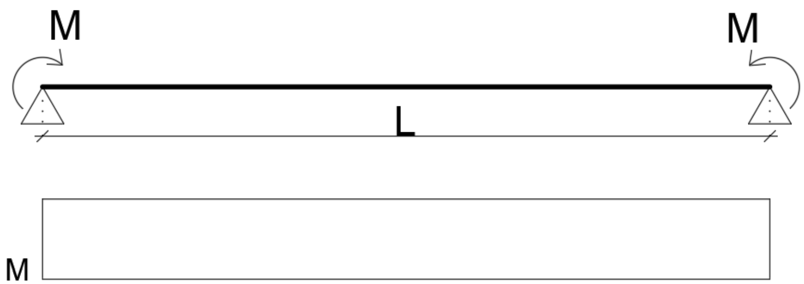
# Case study 1 – Numerical results

| N° | Position of loading | Linear FEM | | Quadratic FEM | |
|---|---|---|---|---|---|
| | | $M_{cr,FEM}$ [kNm] | $\frac{M_{cr,FEM}}{M_{cr,an}}$ | $M_{cr,FEM}$ [kNm] | $\frac{M_{cr,FEM}}{M_{cr,an}}$ |
| 1 | S | 0.59 | 0.96 | 0.60 | 0.98 |
| 2 | G | 0.59 | 0.96 | 0.60 | 0.97 |
| 3 | TLE | 0.50 | 0.96 | 0.51 | 0.98 |
| 4 | TRE | 0.50 | 0.95 | 0.51 | 0.98 |
| 5 | BLE | 0.69 | 0.95 | 0.70 | 0.97 |
| 6 | BRE | 0.69 | 0.96 | 0.71 | 0.98 |

| | Linear FEM | Quadratic FEM |
|---|---|---|
| Mean$\left(\frac{M_{cr,fem}}{M_{cr,an}}\right)$ | 0.96 | 0.98 |
| Standard deviation [%] | 0.39 | 0.19 |



# Case study 2




- L = 4000 mm
- E = 210,000 Mpa

- Analytical approach

$$M_{cr} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{(k_z \cdot L)^2} \cdot \left[ \sqrt{\left(\frac{k_z}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(k_z \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + (C_2 \cdot z_g)^2} - C_2 \cdot z_g \right]$$

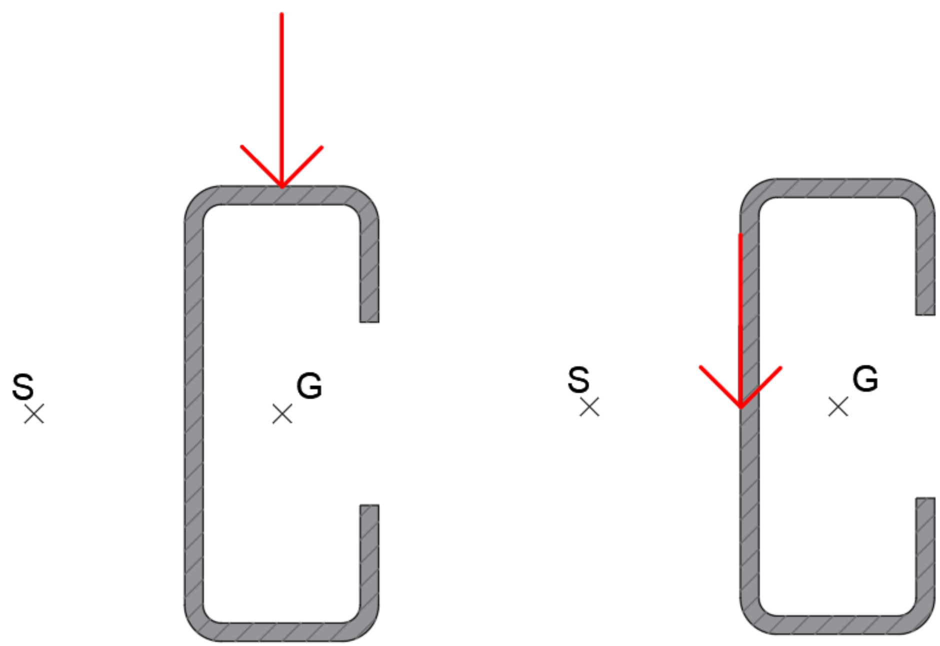
$\rightarrow$ $M_{cr} = 0.59$ kNm

- Numerical approach → Numerical analysis using Abaqus with quadratic solid elements

| N° | Position of loading | Analytical formula $M_{cr,an}$ [kNM] |
|---|---|---|
| 1 | S | 0.59 |
| 2 | G | 0.59 |
| 3 | TLE | 0.59 |
| 4 | TRE | 0.59 |
| 5 | BLE | 0.59 |
| 6 | BRE | 0.59 |

# Conclusions and perspectives

- Equation in CEN/TR 1993-1-103 predicts with high accuracy the critical bending moment for monosymmetrical section about the major axis not only for the case where the section is loaded through the shear centre




**Still valid ? → YES !**

# Conclusions and perspectives

- Equation in CEN/TR 1993-1-103 predicts with high accuracy the critical bending moment for monosymmetrical section about the major axis not only for the case where the section is loaded through the shear centre
- The vertical position of the load strongly influences the results, in contrast with the horizontal position
- The two software where different finite elements have been used provide results quite close to the analytical ones
- The use of solid quadratic elements gives better results compared to linear elements or beam elements
- Parametric studies and experimental campaign are planned

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Thank you for your attendance!

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