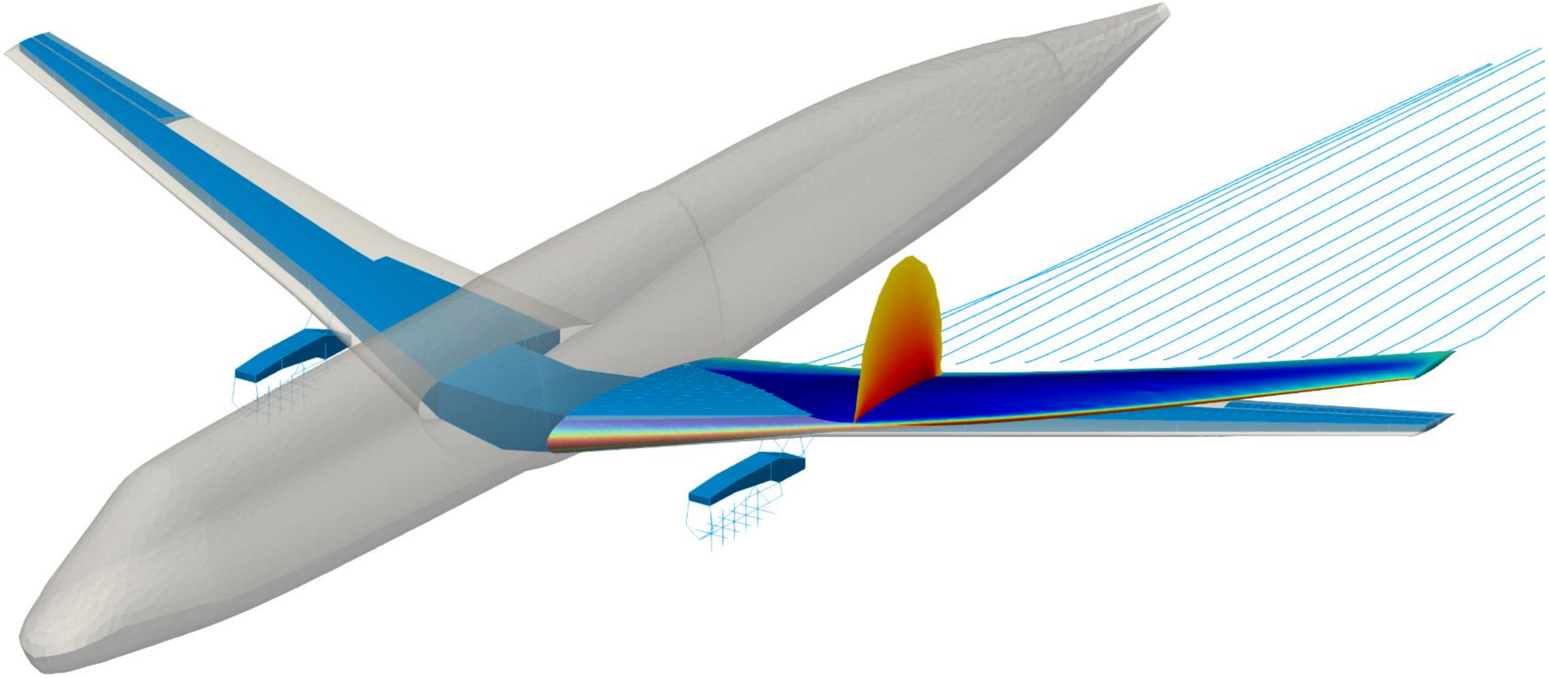
Aerothermodynamics of high-speed flows

Transonic flows

Adrien Crovato

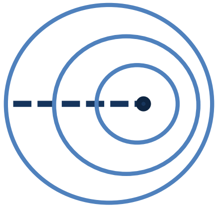


The von Karman Institute for Fluid Dynamics

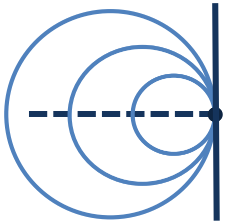
Liège, Q2 2023

LIÈGE université

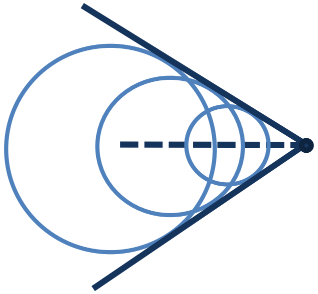
Flight speed and speed of sound



Subsonic



Sonic



Supersonic

http://www.chuckyeager.org/news/gen-chuck-yeager-describes-broke-sound-barrier/

The sound barrier


Spitfire


P38


DH 108

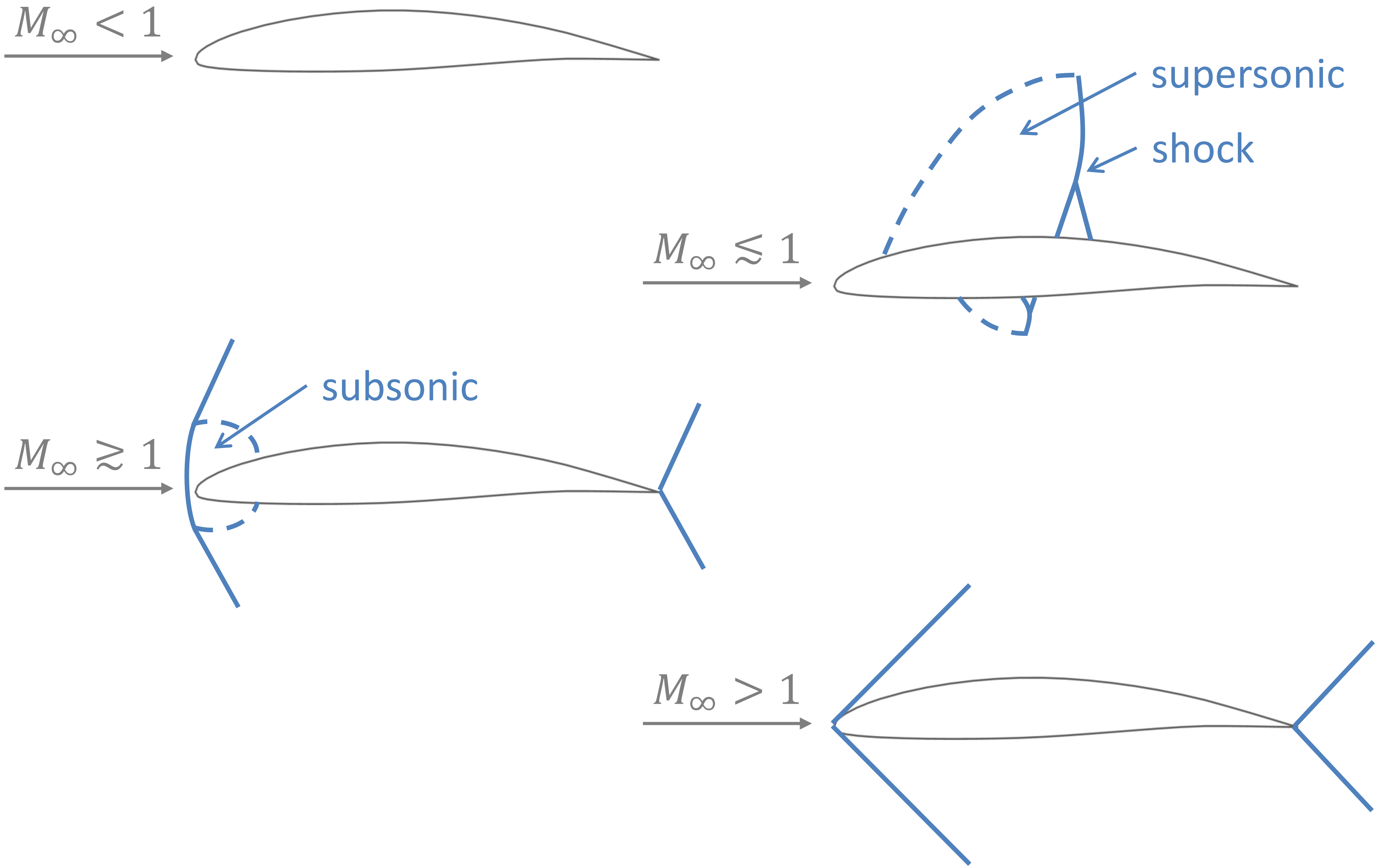
Images from Wikipedia

At high subsonic speeds, pilots could feel

- a strong resistance to further acceleration (drag divergence)
- the aircraft dropping (loss of lift)
- the aircraft responding in the opposite way as supposed to (control reversal)
- the aircraft becoming nose-heavy (Mach tuck)
- strong sustained (buffeting) or growing (flutter) vibrations

These observations led to the concept of a **barrier** that an aircraft had to **cross** in order to exceed the **speed of sound**.

Flow regimes



Flow regimes explained

Subsonic regime – $M_\infty < 1$

- Flow is smooth

Transonic regime – $M_\infty \lesssim 1$

- Flow is accelerated and deviated by the body
- Supersonic regions are created and terminated by normal shocks

Transonic regime – $M_\infty \gtrsim 1$

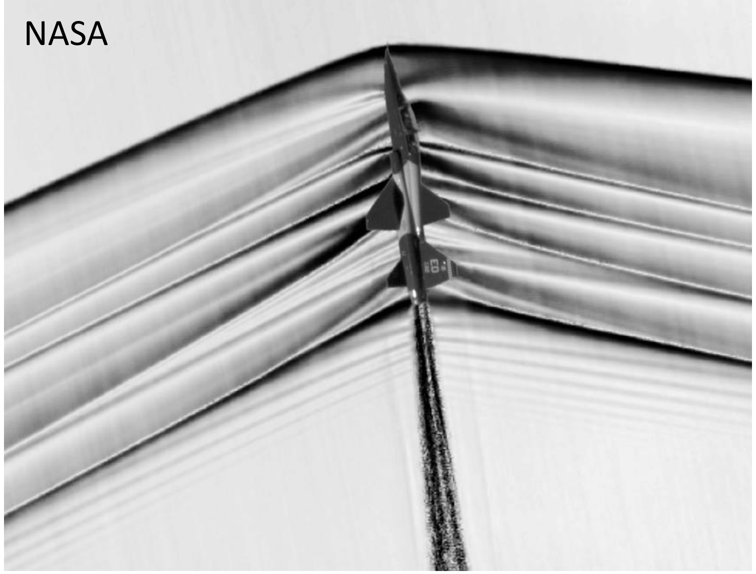
- Shocks are moved towards trailing edge
- Normal bow (detached) shock is created in front of leading edge

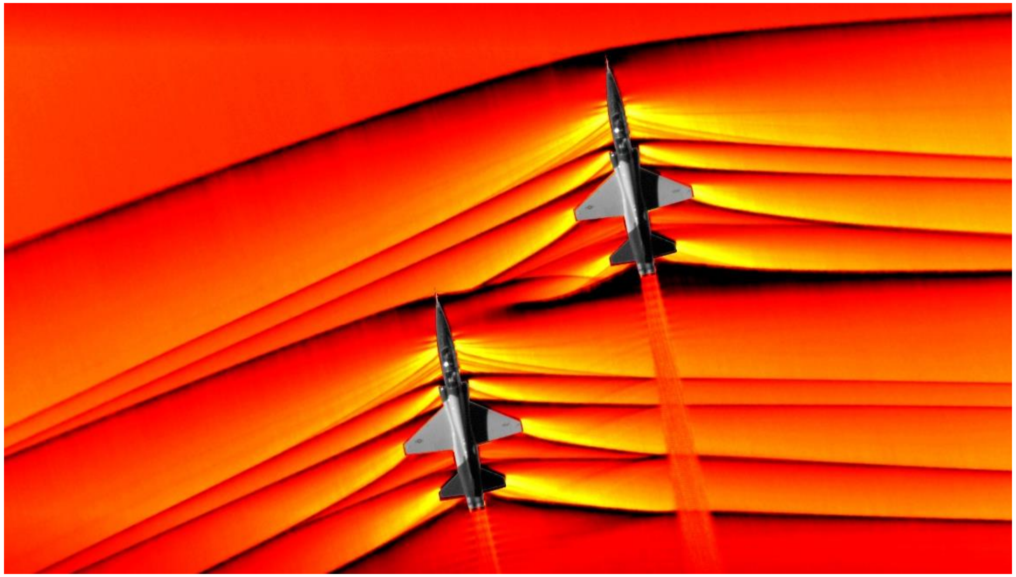
Supersonic regime – $M_\infty > 1$

- Trailing edge shocks become oblique
- Leading edge shock becomes attached and oblique

Transonic flows consist of **mixed subsonic** and **supersonic** flow regions

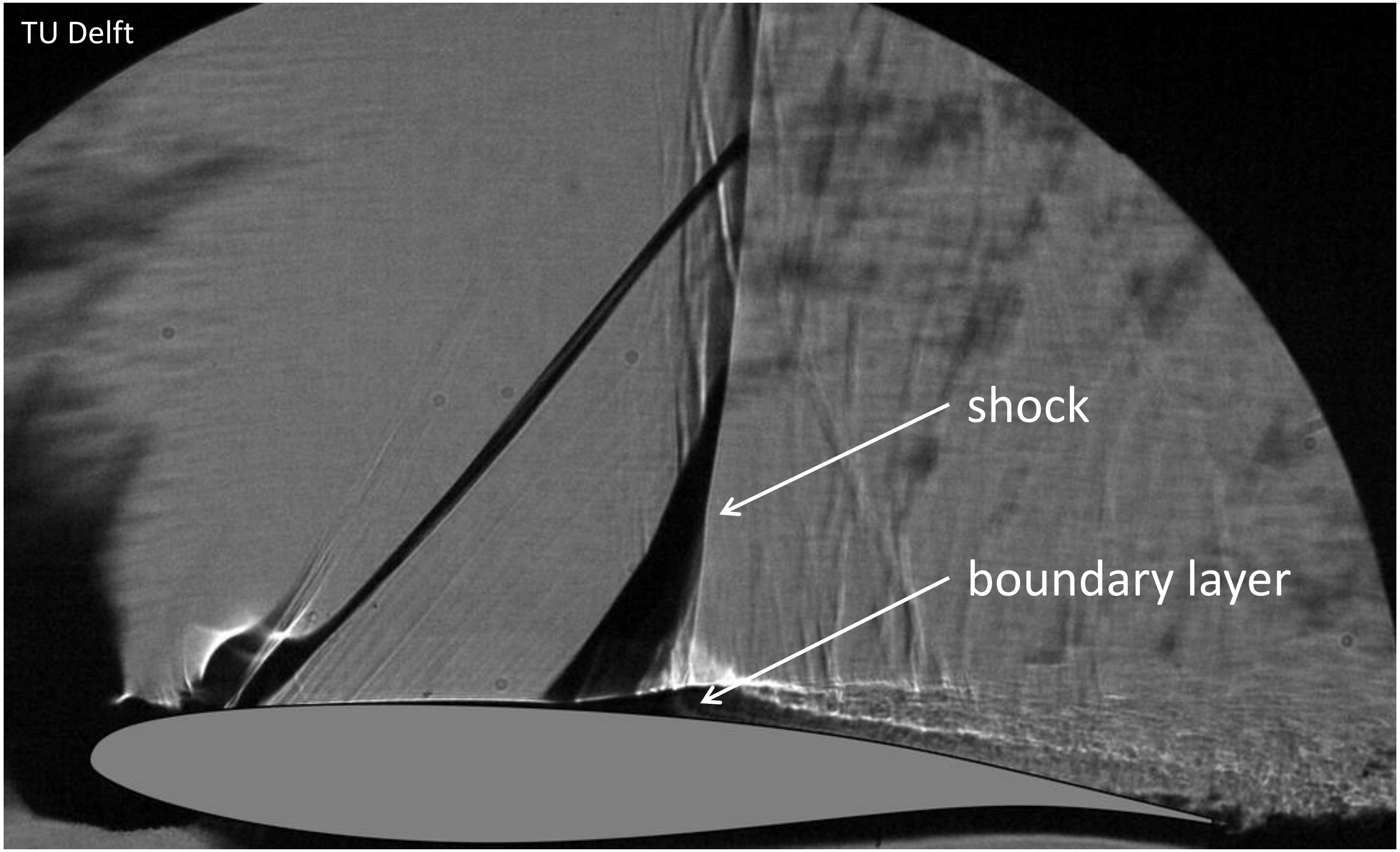
Transonic flows


NASA


https://www.youtube.com/watch?v=HekbC6Pl4_Y&ab_channel=RussellCroman

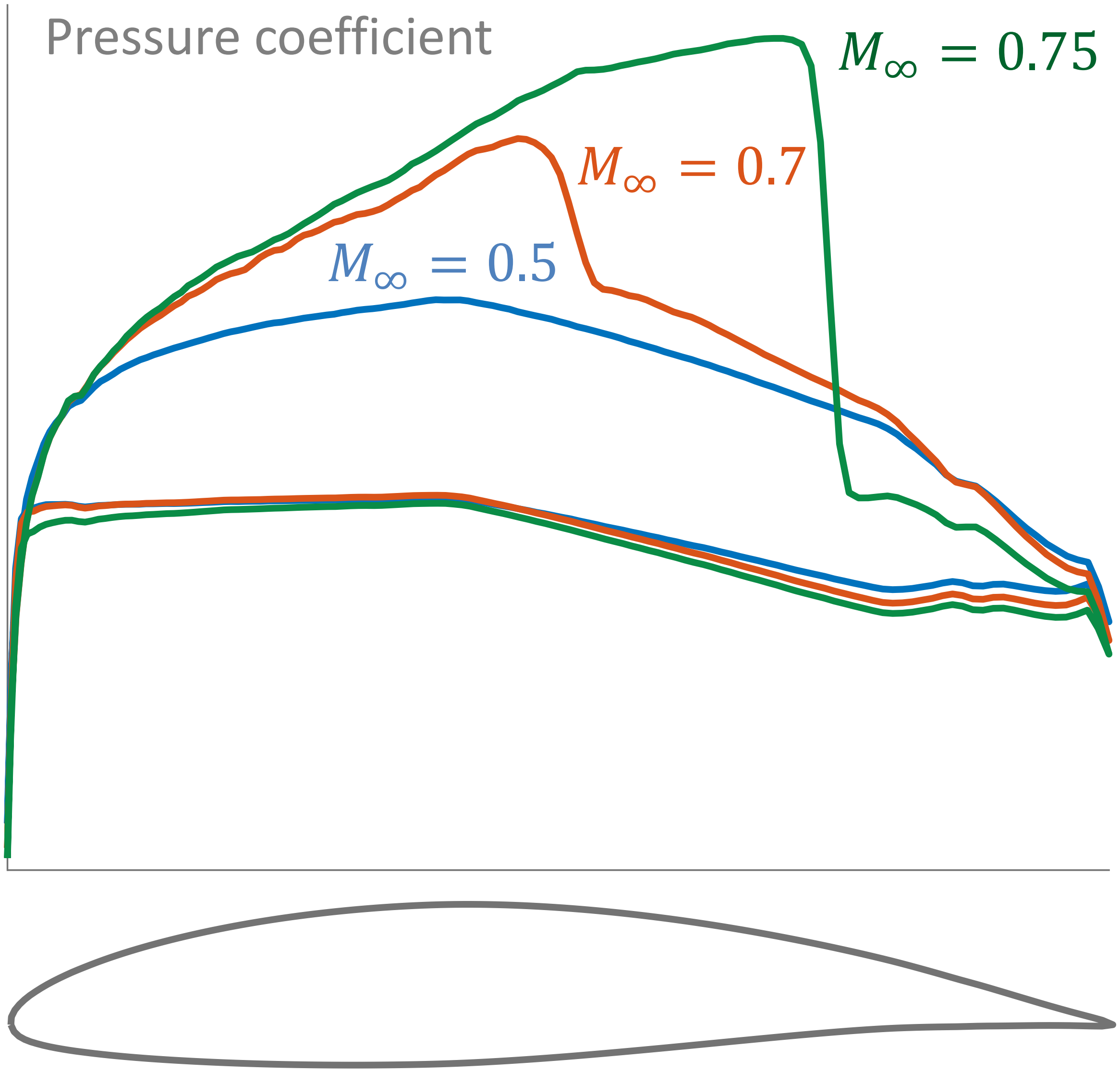

https://www.youtube.com/watch?v=uO4FckCAZtU&ab_channel=RealEngineering

Shock-induced separation



The **high adverse pressure gradient** created by **shocks** causes the **boundary layer** to **separate**!

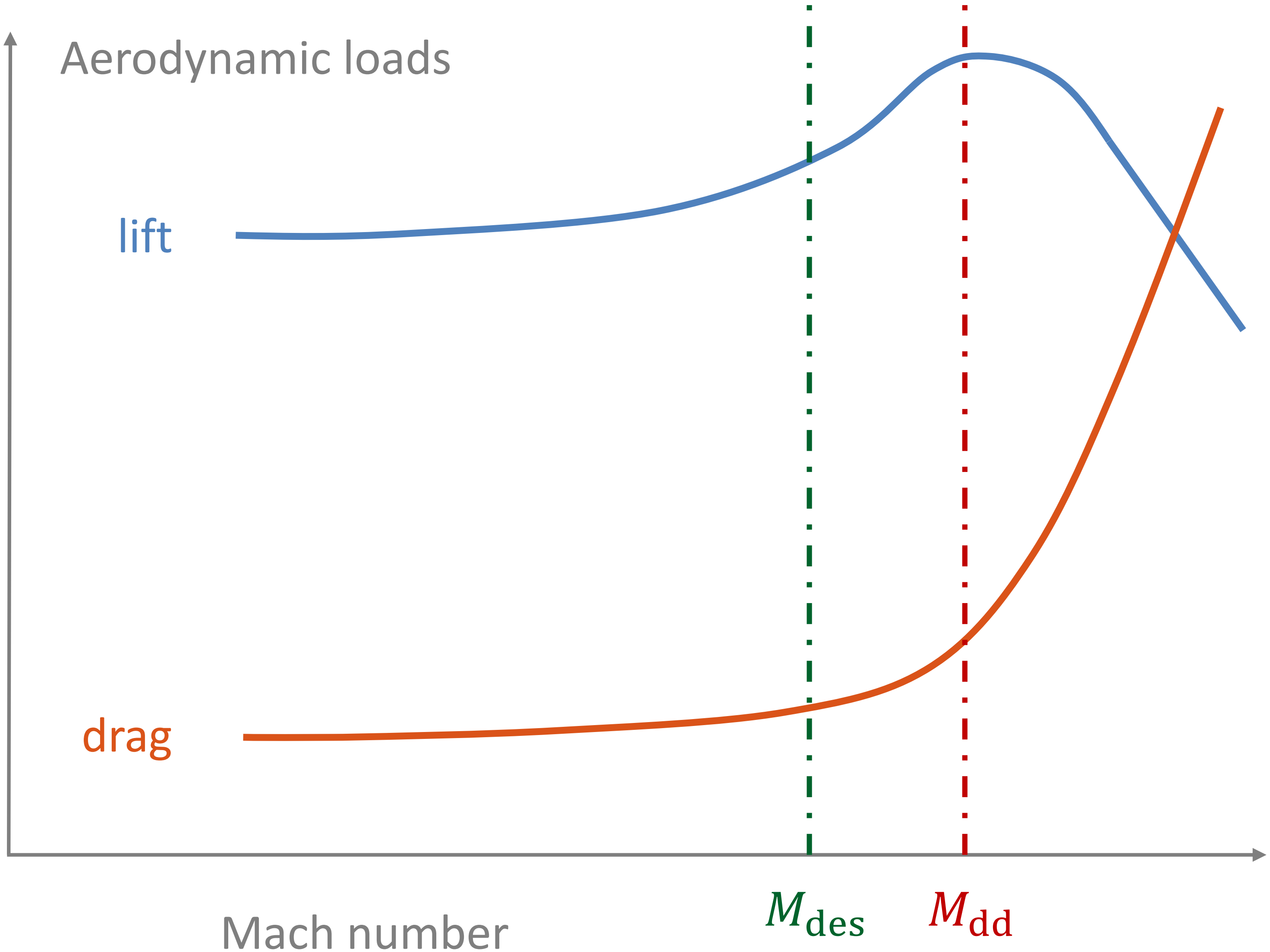
Flow properties across shockwaves



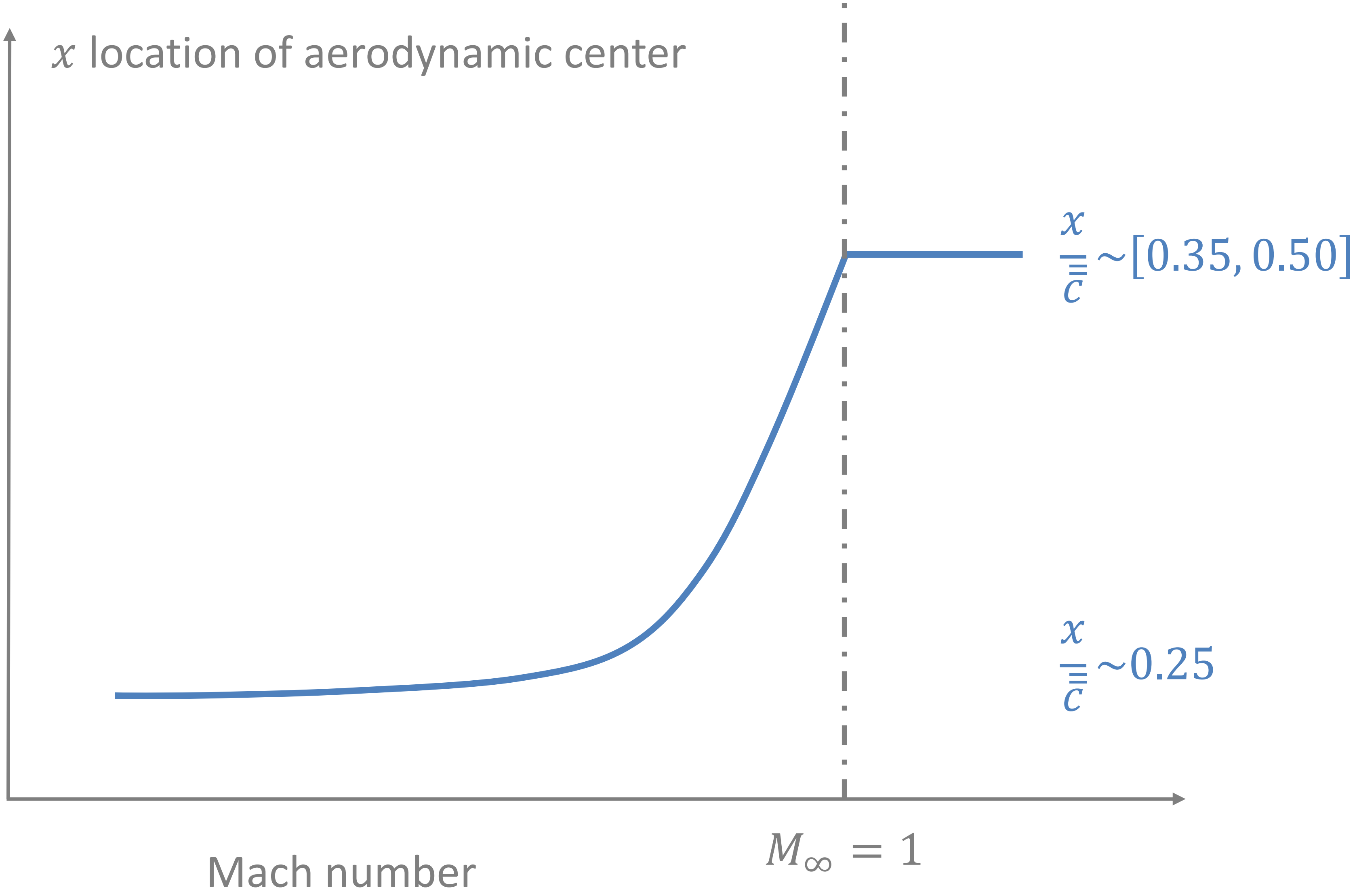
Across the shock

- static pressure ↑
- density ↑
- static temperature ↑
- total temperature =
- total pressure ↓
- entropy ↑

Lift drop and drag divergence



Aerodynamic center shift



Outline

Transonic aircraft design

- Sweep and thickness
- Supercritical airfoils
- Whithcomb’s rule

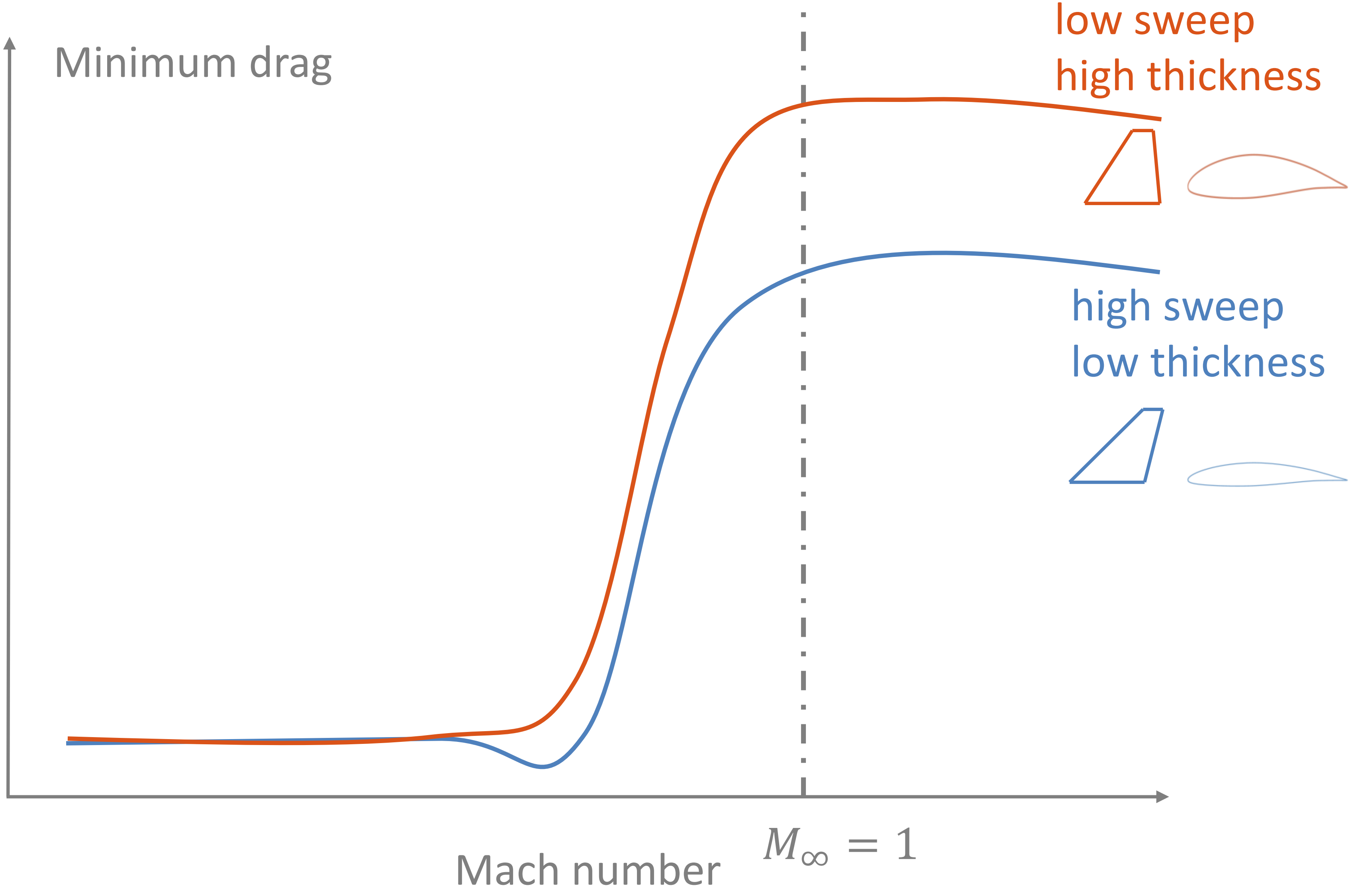
Mathematical and numerical modeling

- Levels of fidelity
- Potential equation
- Solution methods

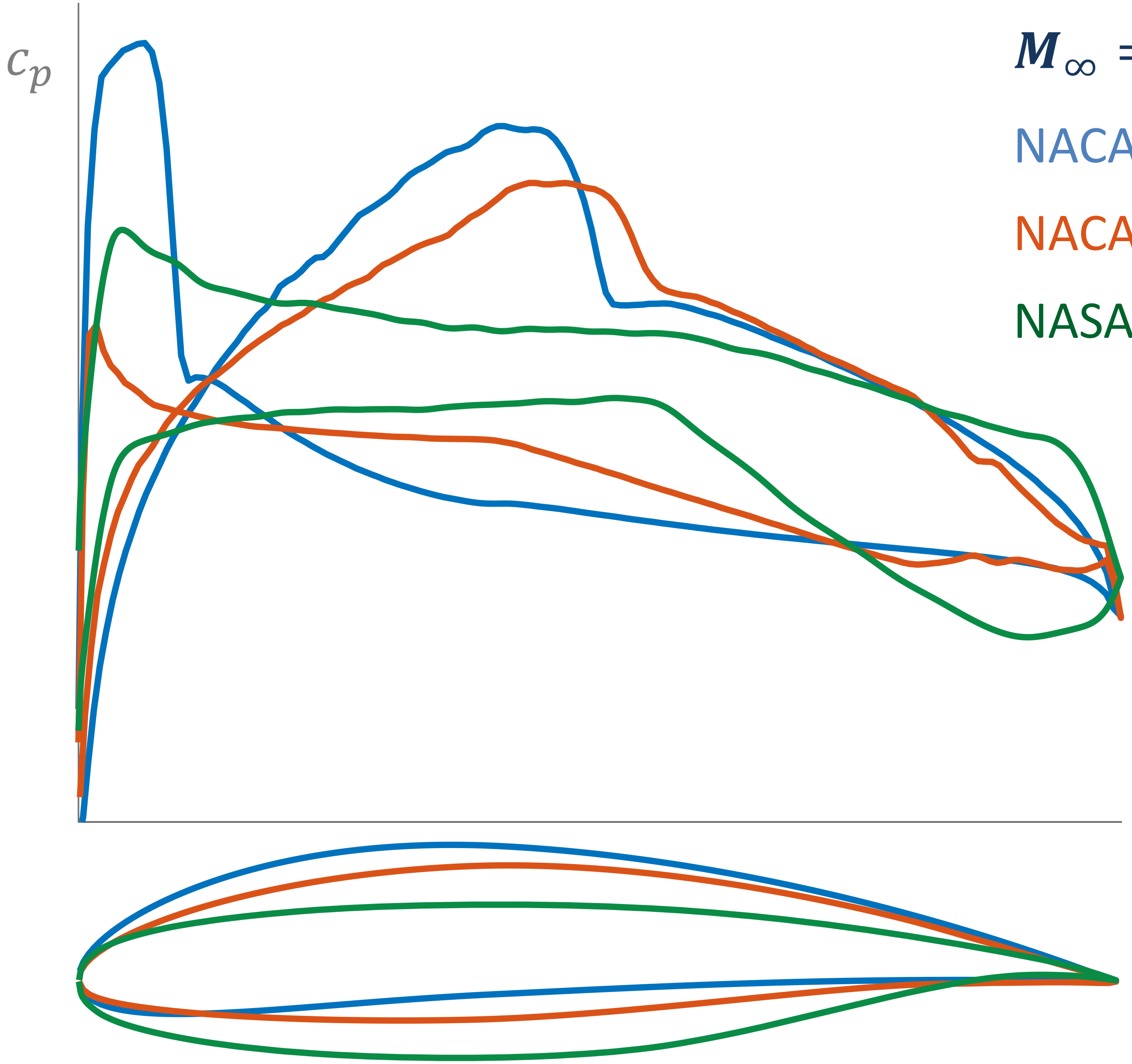
DARTFlo computer code

- Features
- Implementation
- Practical application

Wing sweep and airfoil thickness



Supercritical airfoils



$M_\infty = 0.72, \quad c_l = 0.4$

NACA 4410 $\alpha = -1.1°$

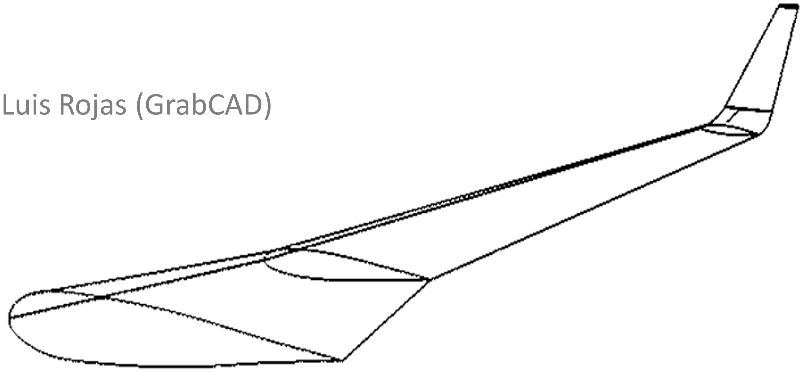
NACA 64A410 $\alpha = +0.2°$

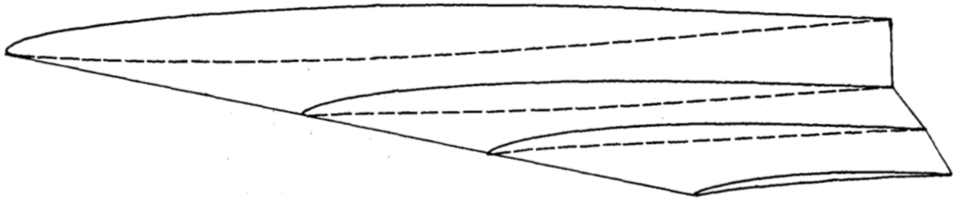
NASA SC(2)-0410 $\alpha = +0.0°$

Transonic wings


Wikipedia - B737


Wikipedia - F15C


Luis Rojas (GrabCAD)

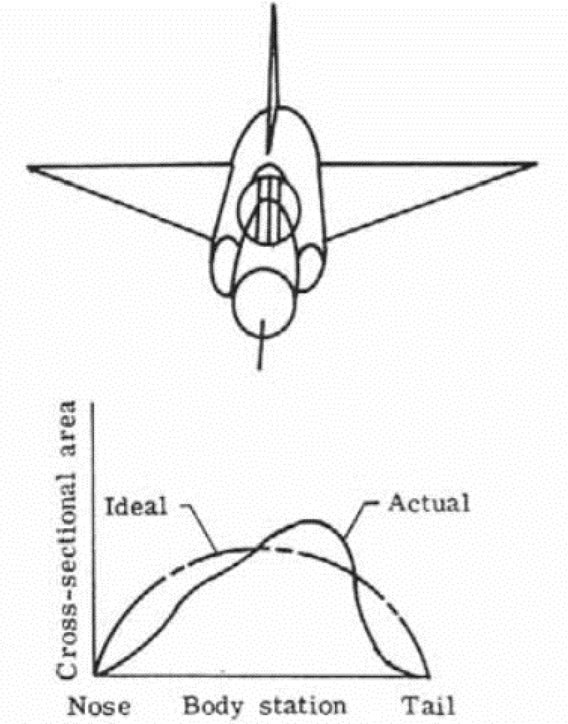

Pitt, 1989 (NASA)

$t/c \sim [10,15]$ %
$\Lambda \sim 25°$

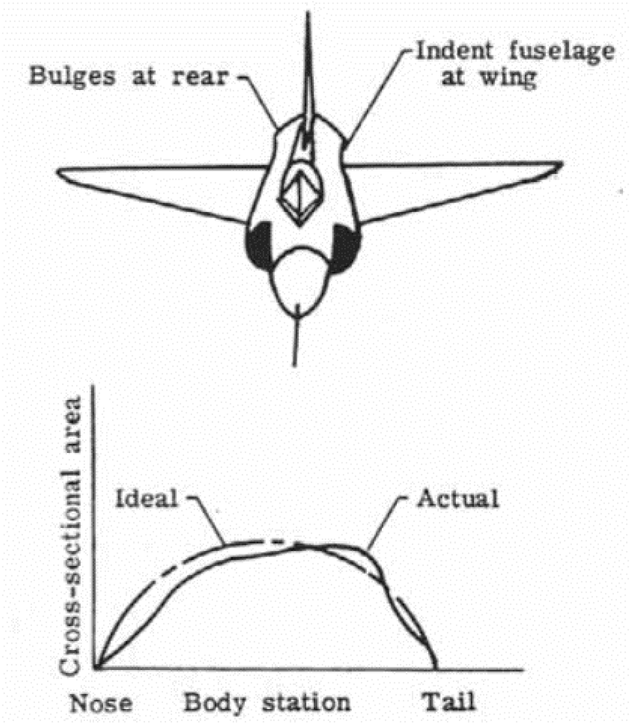
symmetric airfoil at root
cambered airfoil at tip

$t/c \sim [5,10]$ %
$\Lambda \sim 35°$

Whitcomb’s area rule



YF-102A



F-102A

Smoothen cross-sectional area is not always worth it.
Not widespread on today’s fighters.

Whitcomb’s area rule

Mirage 2000-5

Rafale

F106

Images from Wikipedia

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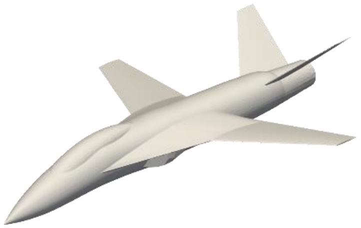
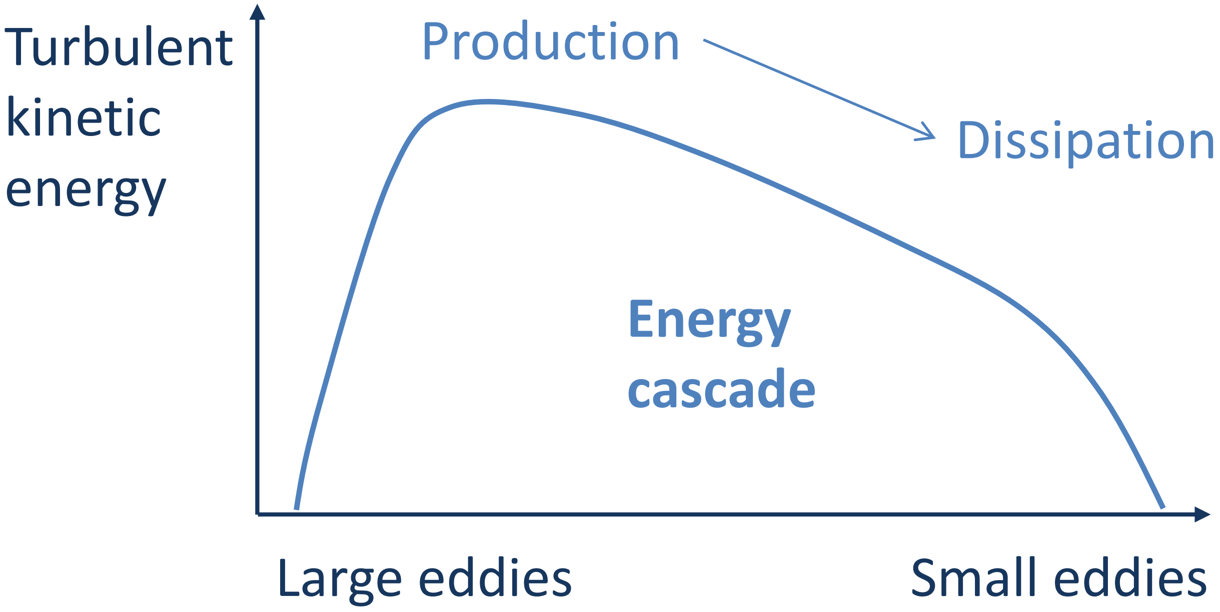
High-fidelity aerodynamic modeling



$Re \sim 10^7$

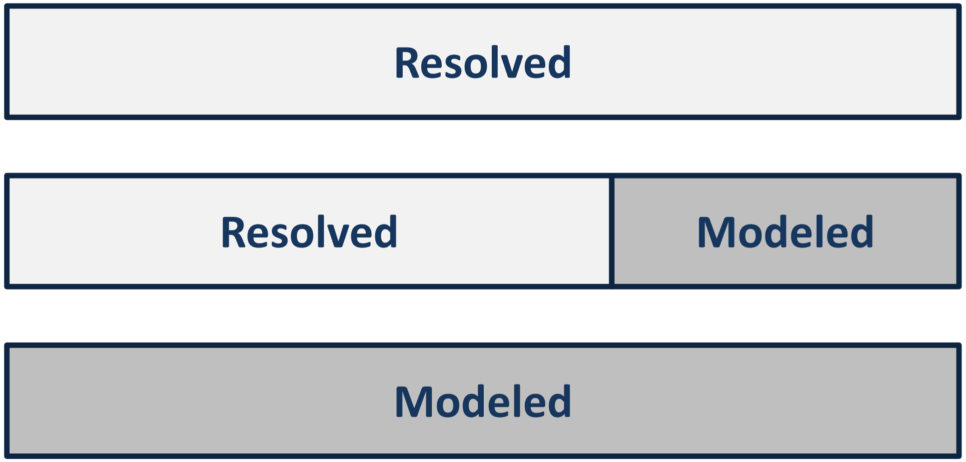
Flow is **turbulent**!



Direct Numerical Simulation

Large Eddy Simulation

Reynolds-Averaged Navier-Stokes



Aerodynamic modeling for aircraft design

RANS equations

- Subsonic
- Supersonic
- Transonic
- Viscous

Euler equations

- Subsonic
- Supersonic
- Transonic
- **Inviscid**

Full potential equation

- Subsonic
- Supersonic
- **~Transonic**
- Inviscid

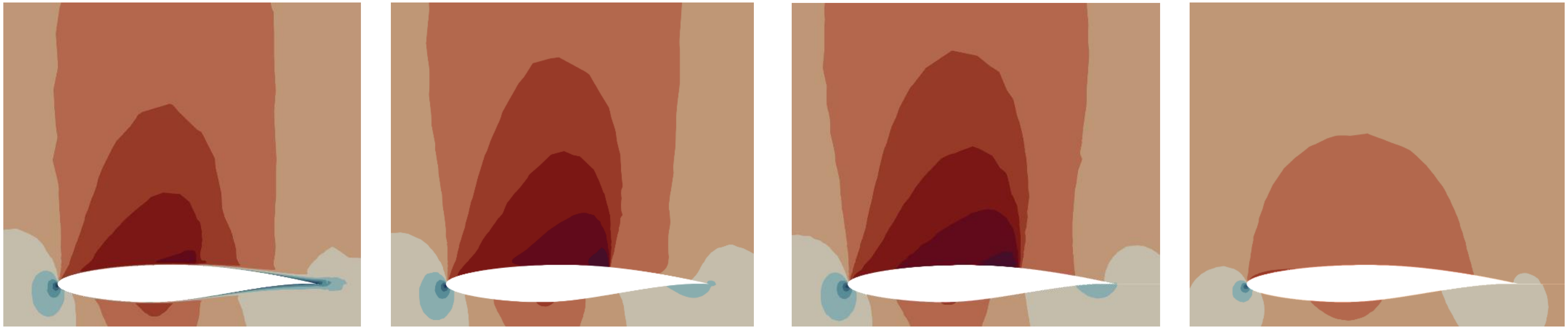
Linear potential equation

- **~Subsonic**
- **~Supersonic**
- ~~Transonic~~
- Inviscid

Inviscid

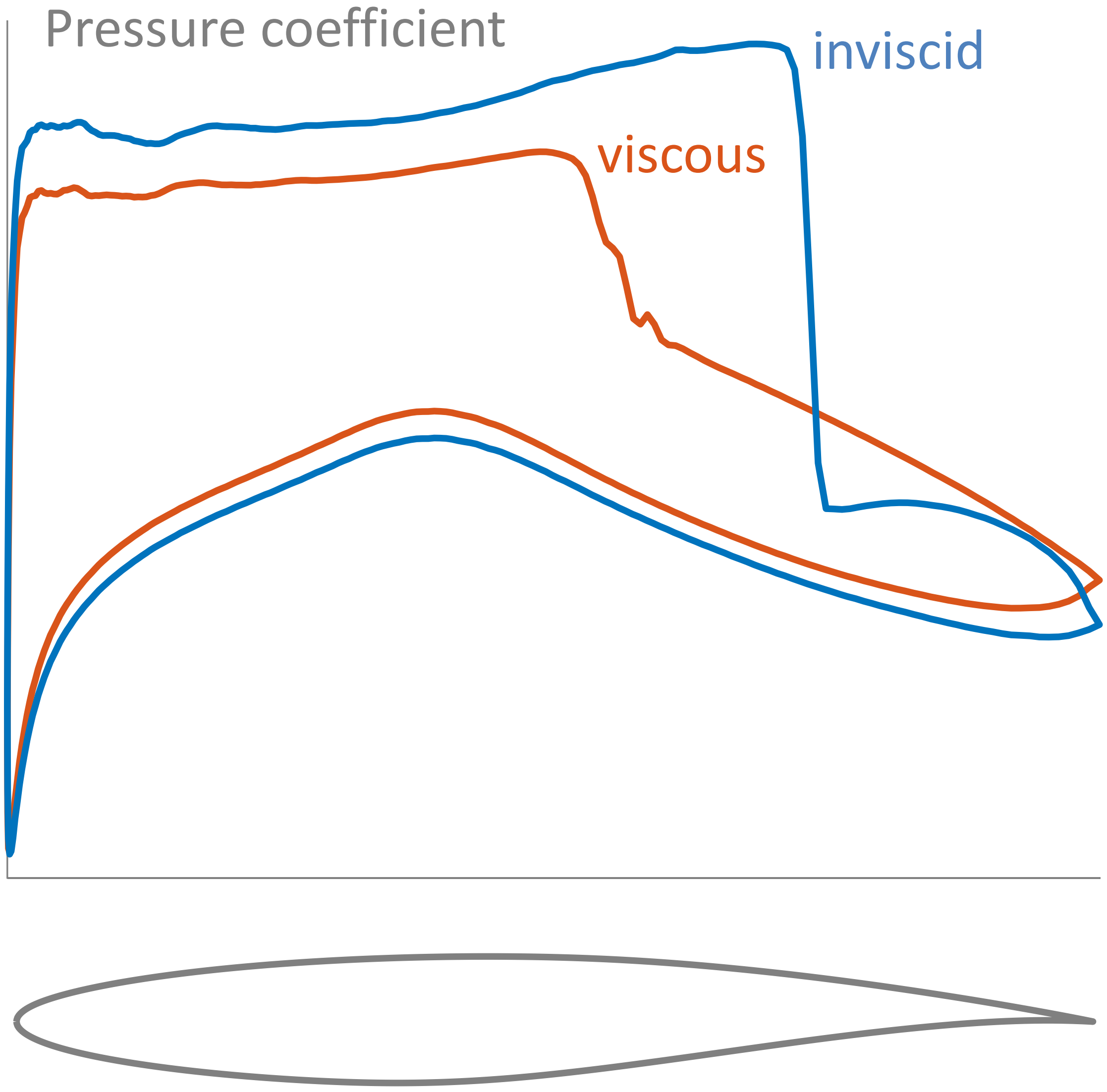
Isentropic

Linear



Mach number

Shock and boundary layer interaction

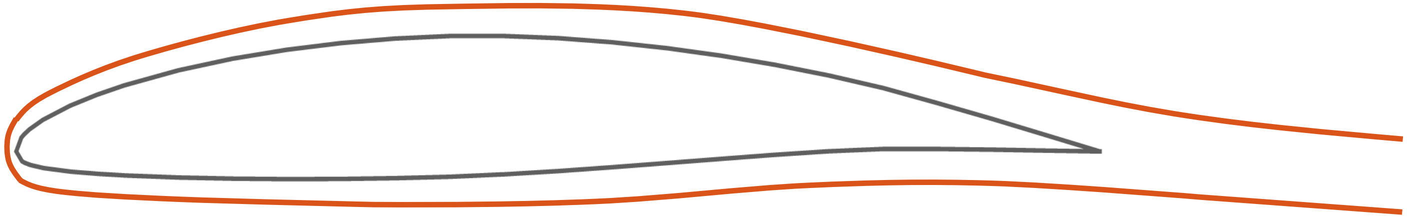


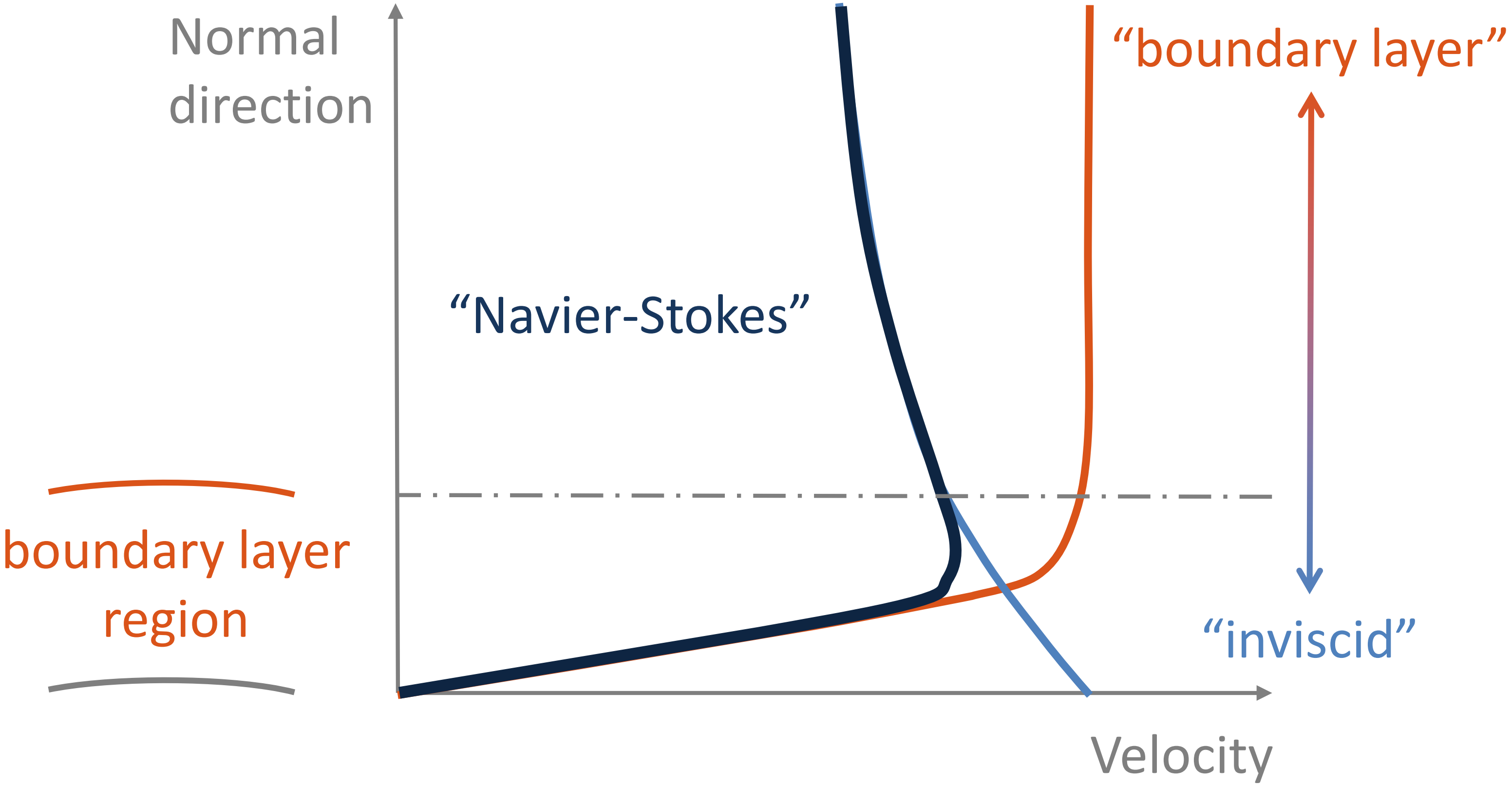
No friction in inviscid flow

- stronger shock ← higher total pressure gradient
- aft location ← later compression

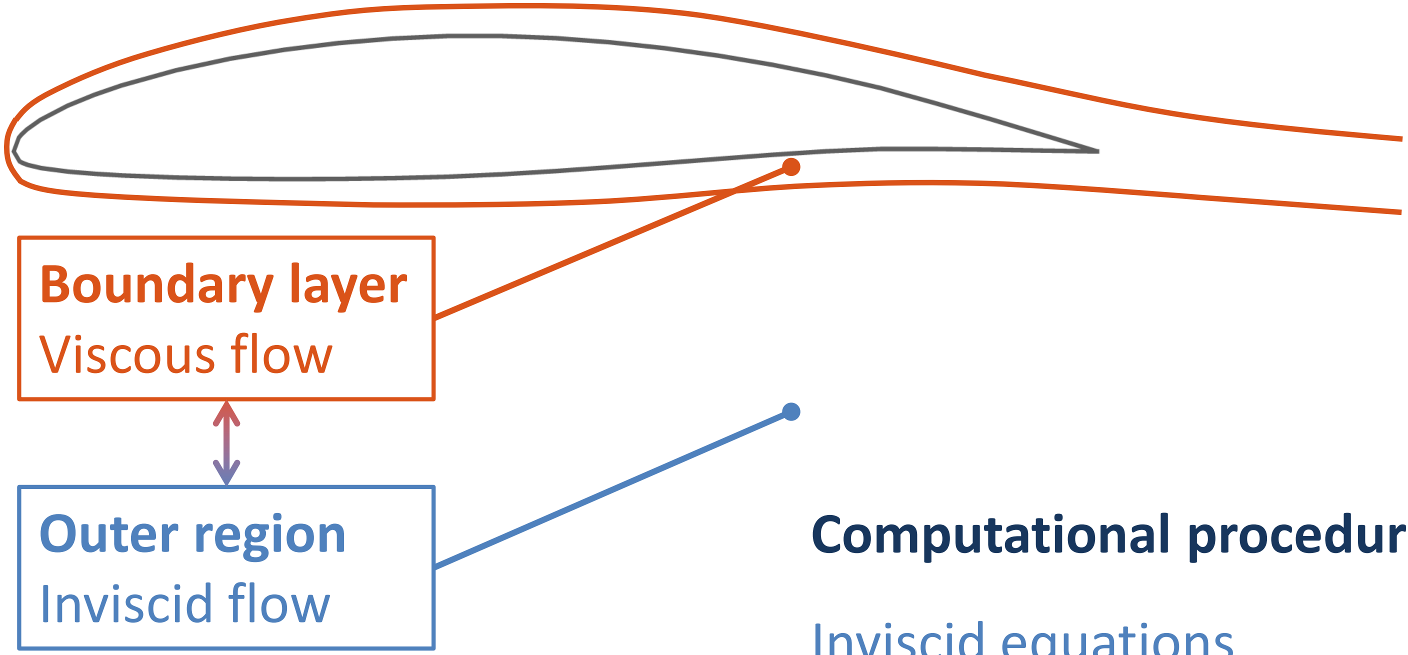
Boundary layer must be taken into account for **accurate predictions**!

Viscous-inviscid interaction – principle





Viscous-inviscid interaction – procedure



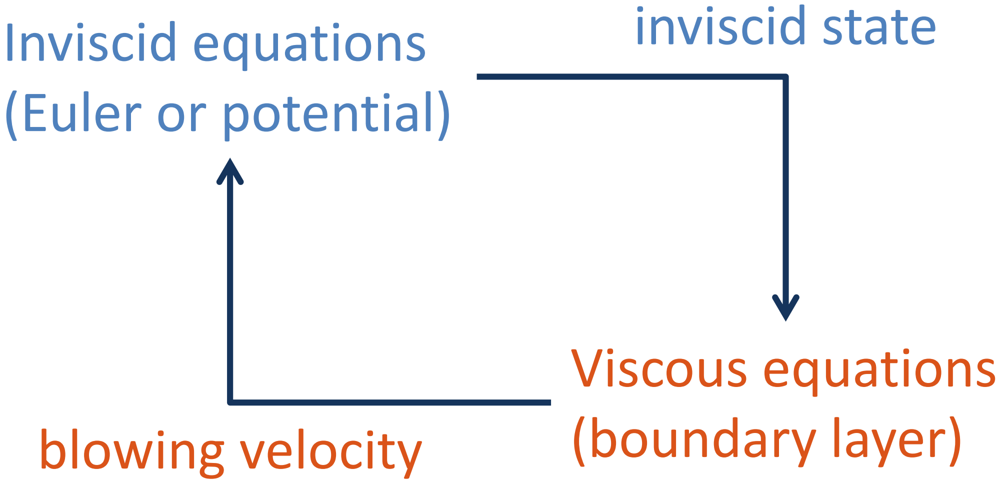
Boundary layer

Viscous flow

Outer region

Inviscid flow

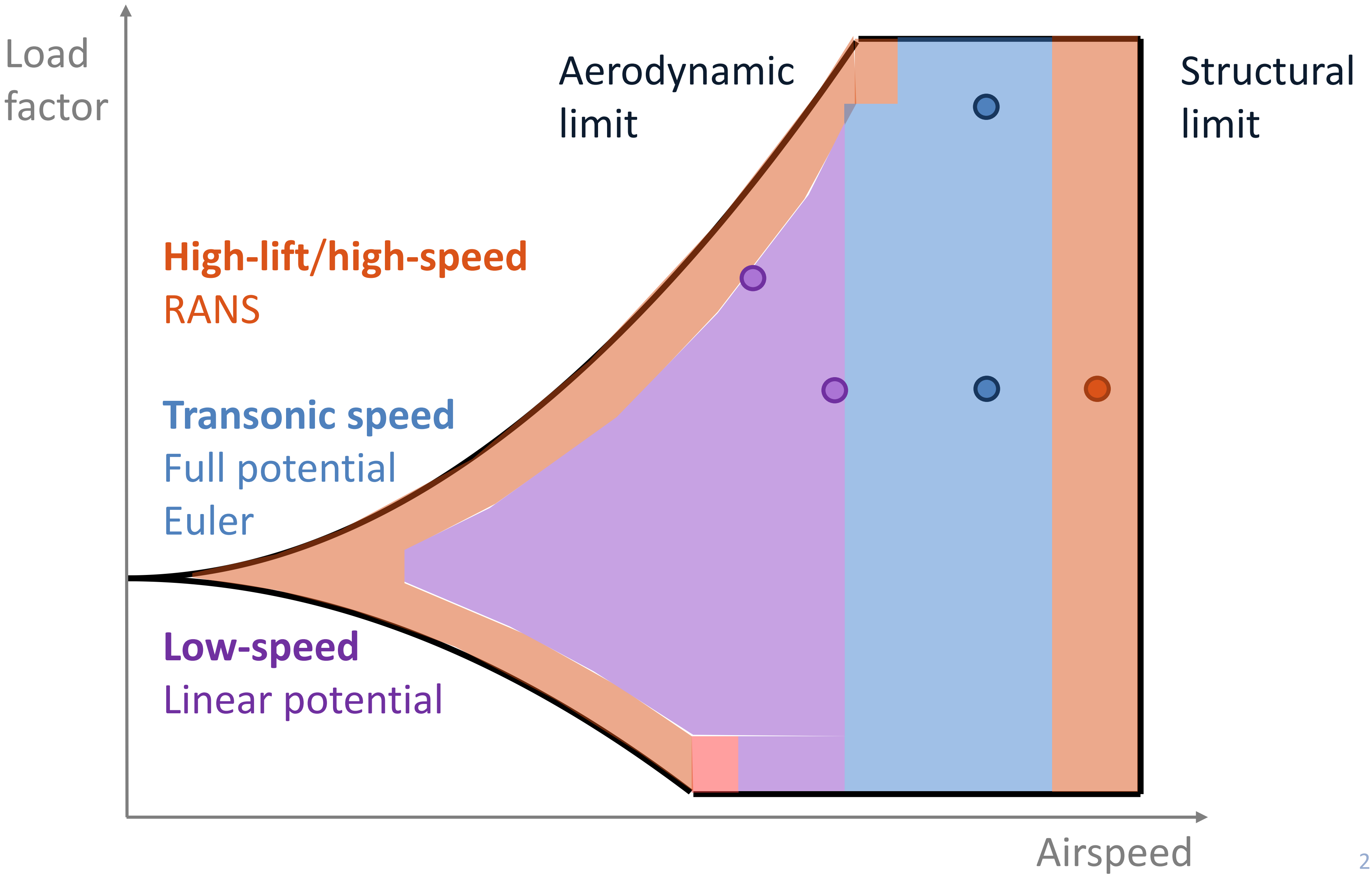
Computational procedure



Inviscid equations
(Euler or potential)

Viscous equations
(boundary layer)

Range of validity



High-lift/high-speed
RANS

Transonic speed
Full potential
Euler

Low-speed
Linear potential

The potential flow equation

$$\nabla \cdot (\rho \nabla \phi) = 0$$

$$\rho \sim \rho(\nabla \phi)$$

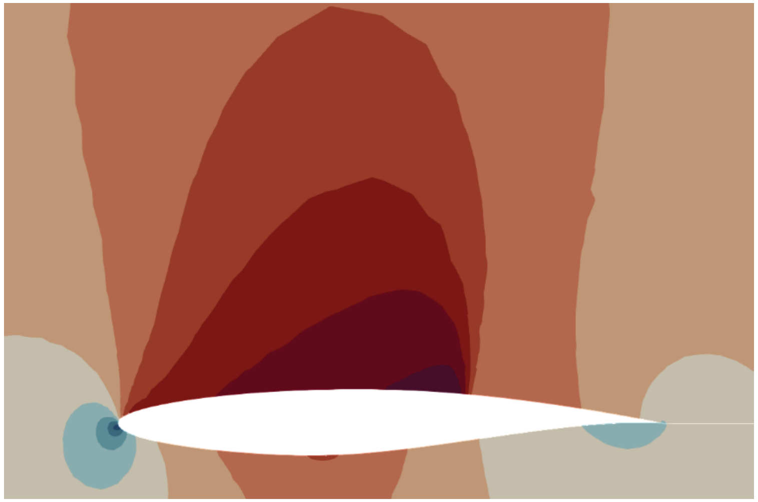
Linear

- Scaled incompressible
- Subsonic or supersonic
- **Not transonic**

$$\rho = \rho(\nabla \phi)$$

Full (nonlinear)

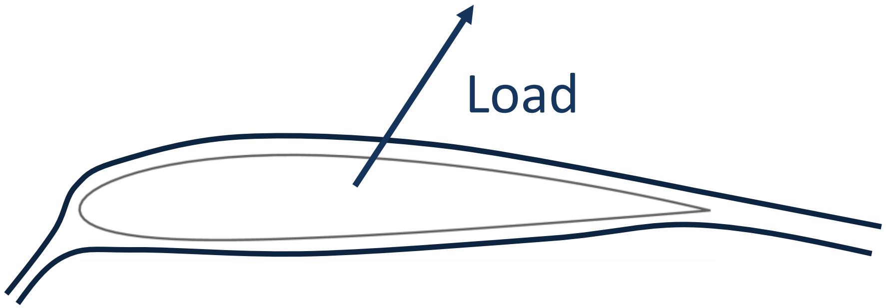
- Compressible
- Subsonic or supersonic
- **Weak transonic**



Challenges of potential flows – loads

Real fluid

- viscous, finite velocity
- no turn around sharp corners

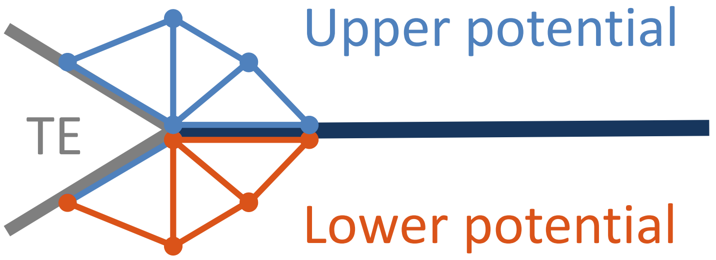


Irrotational flow

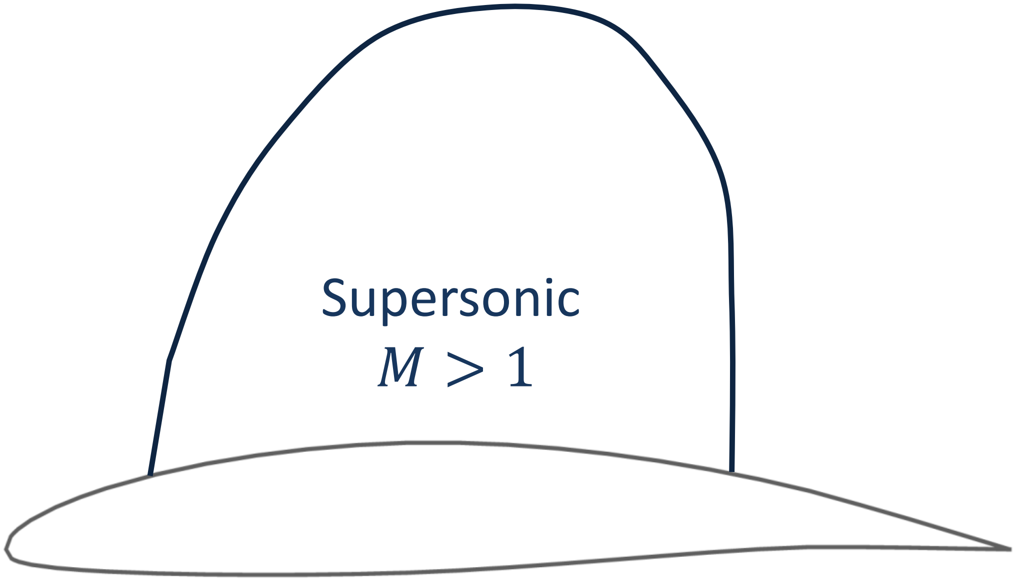
- no deviation
- no load

Numerical model

- prescribe wake
- enforce continuity in physical variables



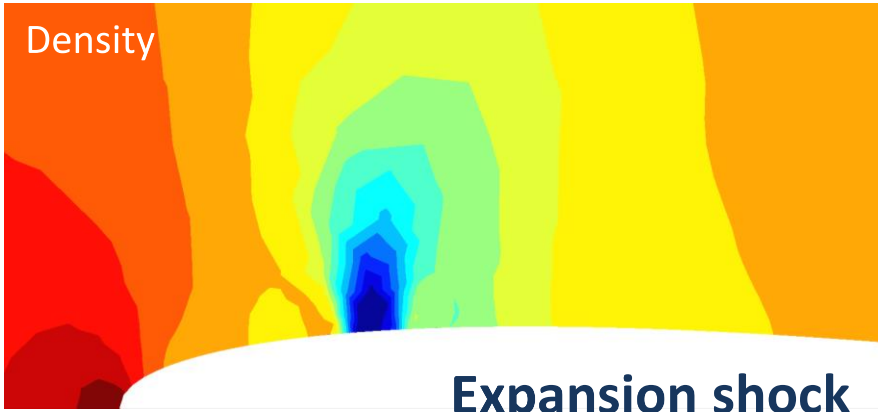
Challenges of potential flows – stabilization



Supersonic
$M > 1$

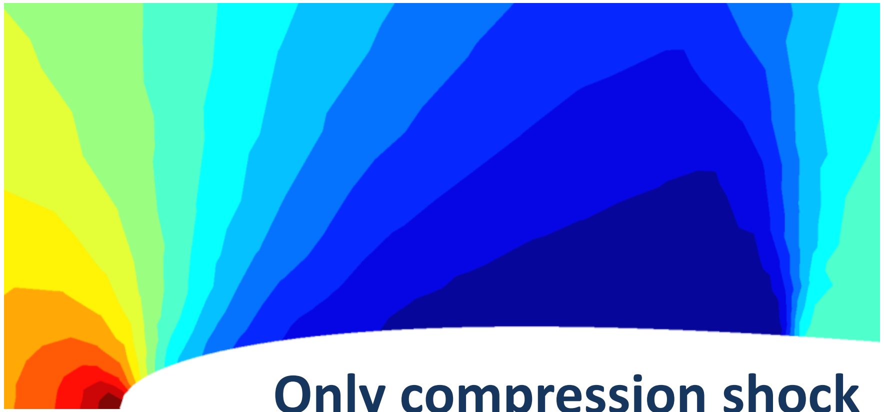
Subsonic
$M < 1$

Without stabilization



Expansion shock

With stabilization



Only compression shock

Transonic stabilization techniques

Full potential equation in conservative and non-conservative forms

$$\partial_i(\rho\partial_i\phi) = 0 \;\Rightarrow\; \underbrace{\left(a^2 - u_i^2\right)}\,\partial_{ii}\phi - 2u_iu_j\,\partial_{ij}\phi = 0$$

$+, M < 1$ elliptic (**uniform** direction of propagation)

$-, M > 1$ hyperbolic (**preferred** direction of propagation)

Physical and **mathematical** change
must be reflected in **numerical** scheme!

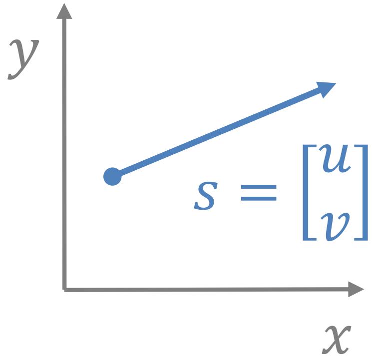
Main stabilization techniques

- 1971, Murman & Cole: automatic central-to-upwind scheme switch
- 1974, Jameson & Caughey: rotated differences and artificial viscosity
- 1989, Eberle et al.: artificial density

Transonic stabilization techniques

Two-dimensional full potential equation

$$\partial_i(\rho\partial_i\phi) = 0 \Leftrightarrow \nabla\cdot\begin{bmatrix}\rho u\\ \rho v\end{bmatrix} = 0 \Leftrightarrow \nabla\cdot\begin{bmatrix}f\\ g\end{bmatrix} = 0$$



Murman & Cole idea

$$f \leftarrow f + \nu\Delta f$$
$$g \leftarrow g$$

upwinding in x direction only

Jameson & Caughey idea

$$f \leftarrow f + \nu F$$
$$g \leftarrow g + \nu G$$

upwinding in x and y directions

Eberle et al. idea

$$\rho \leftarrow \rho - \nu\frac{\partial\rho}{\partial s}\Delta s$$

upwinding in x and y directions

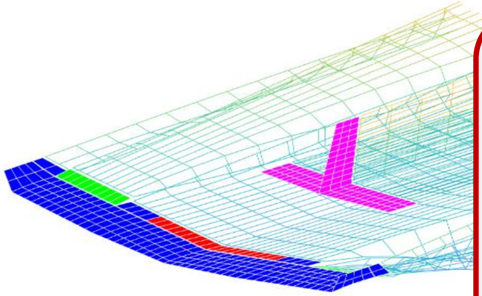
Numerical methods

Boundary Element Method

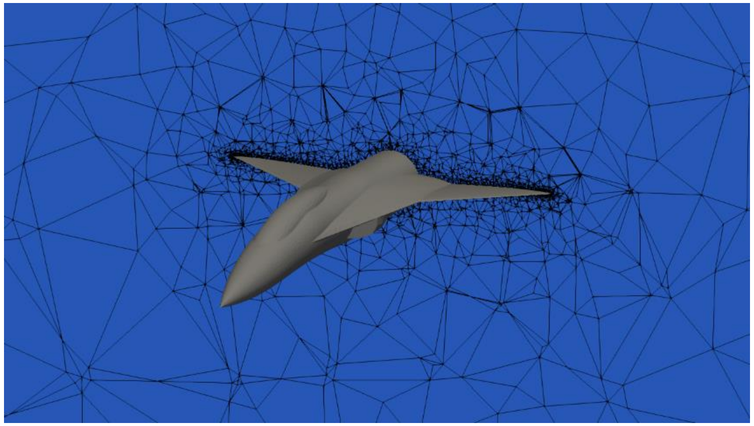
- Only boundary is discretized
- Linear equations only
- Panel/lattice/particle methods

Field Method

- Whole field is discretized
- Linear and nonlinear equations
- Finite volume/element methods



Current industrial practice for aeroelastic computations



Field panel method

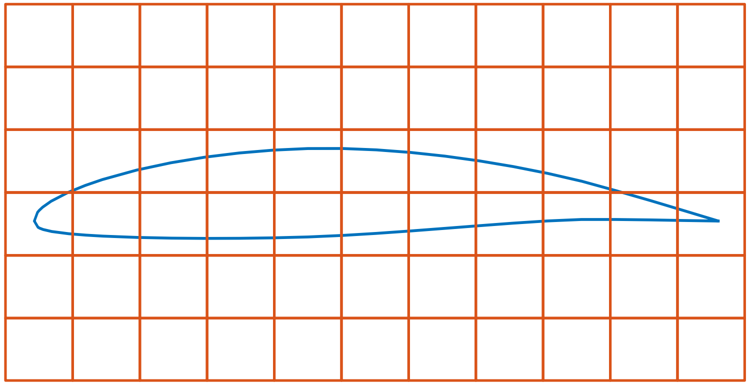
Combination

Boundary element method

- linear part
- on the wing surface

Field method

- nonlinear part
- in the field



Advantages

- extension to panel method
- simple grid generation

Disadvantages

- high memory requirement
- disagreement in literature

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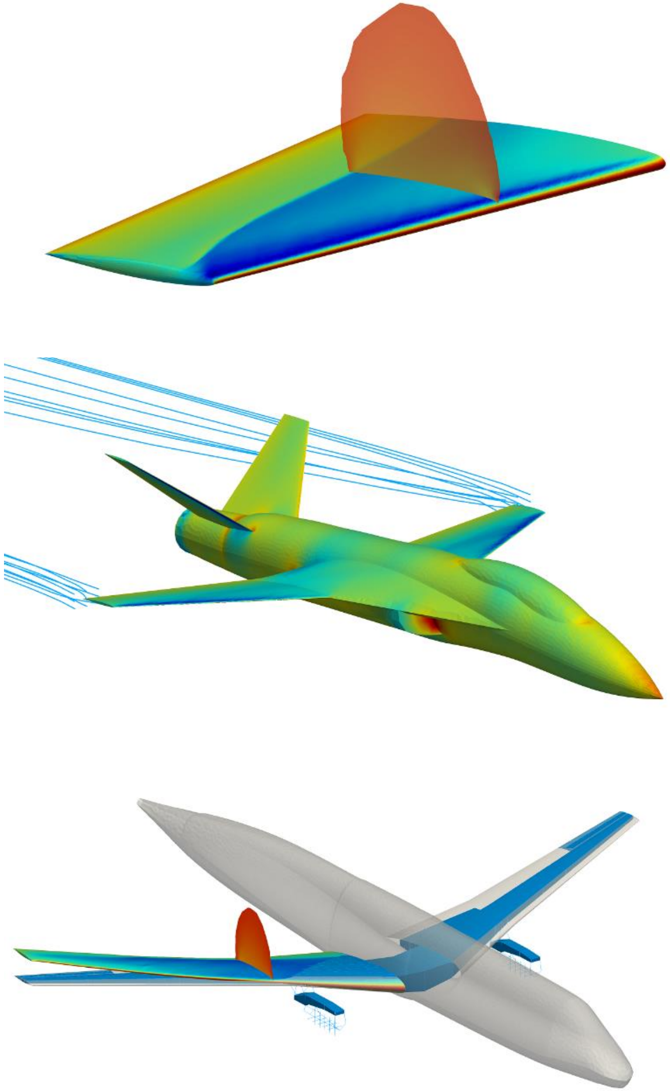
DART



Discrete Adjoint for Rapid Transonic Flows

- Steady full potential formulation
- Finite element discretization
- Unstructured tetrahedral grid
- Mesh morphing
- Analytical discrete adjoint
- Viscous-inviscid interaction
- C++ with python API

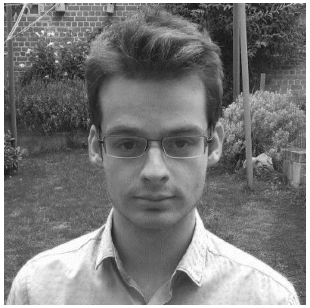
Performance (712Ke – 4.3GB @ 3.4GHz)

- Solution – 100 s
- Morphing – 25 s
- Gradient – 45 s

https://gitlab.uliege.be/am-dept/dartflo

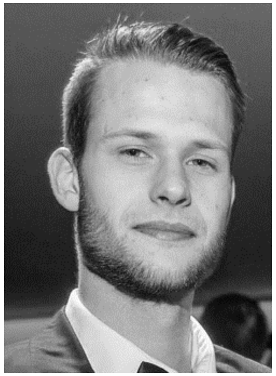
Acknowledgements

Lead developer



Adrien Crovato

Current developers



Paul Dechamps

Romain Boman

Former collaborators

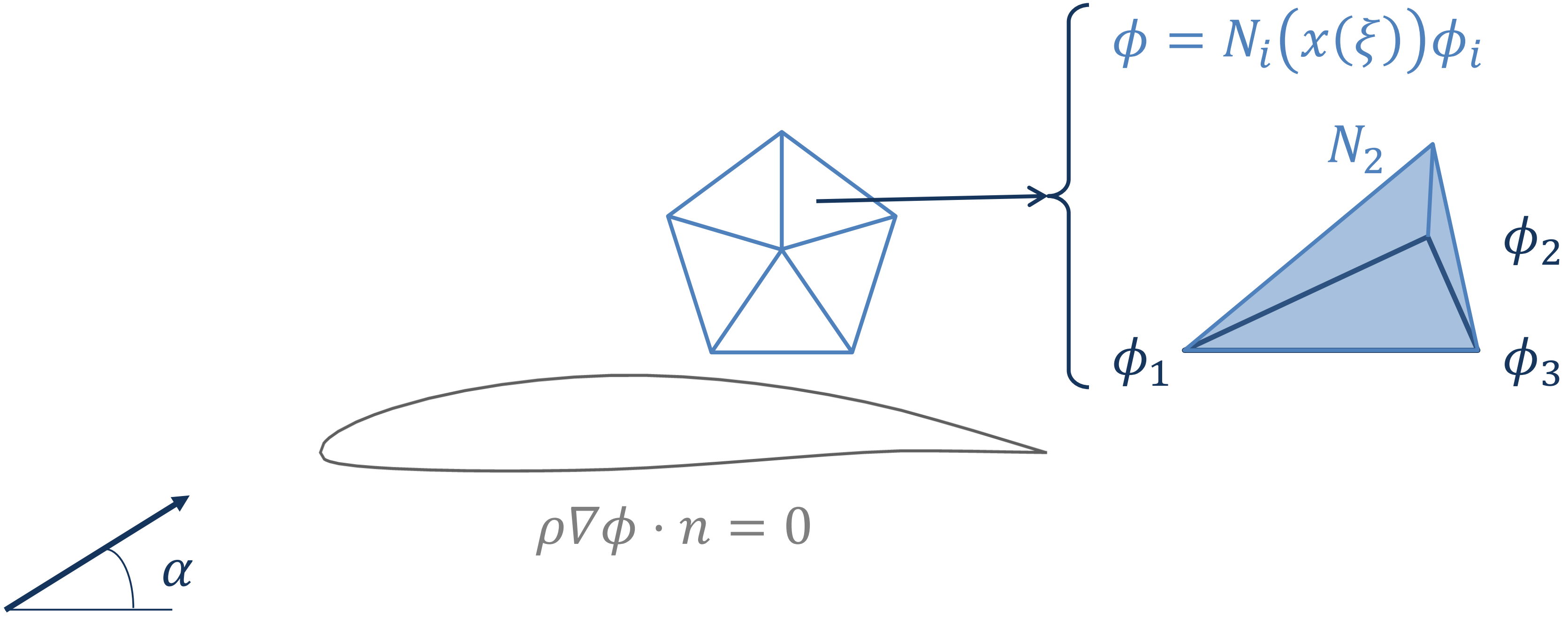
Amaury Bilocq

Guillaume Brian

Guillem Batlle i Capa

Finite element discretization

$$\int_{\Omega} \rho \nabla\phi \cdot \nabla\psi \, dV - \int_{\Gamma} \rho \nabla\phi \cdot n \, \psi \, dS = 0$$



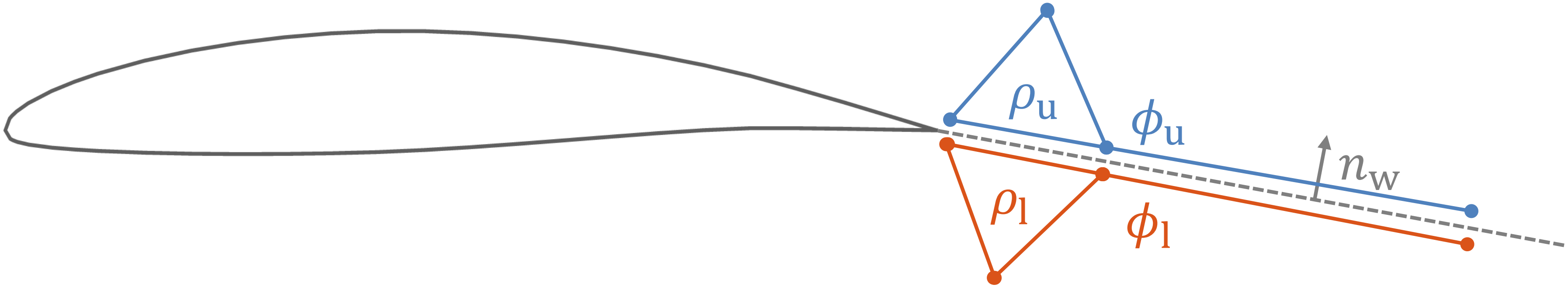
$$\phi = N_i\big(x(\xi)\big)\phi_i$$

$$\rho \nabla\phi \cdot n = 0$$

$$U_\infty = [\cos\alpha, \sin\alpha]$$

$$\nabla\phi \cdot n = U_\infty \cdot n$$

Wake modeling



Formulation

$$\rho_{\mathrm{u}} \nabla_{\mathrm{n}} \phi_{\mathrm{u}} = \rho_{\mathrm{l}} \nabla_{\mathrm{n}} \phi_{\mathrm{l}} \quad \rightarrow \int_{\Gamma_{\mathrm{wake}}} \psi \, [[\rho \nabla \phi \cdot n]] \, dS = 0$$

$$p_{\mathrm{u}} = p_{\mathrm{l}} \quad \rightarrow \int_{\Gamma_{\mathrm{wake}}} \left(\psi + \frac{h}{2} U_{\infty} \cdot \nabla \psi \right) [[|\nabla \phi|^2]] \, dS = 0$$

Transonic stabilization

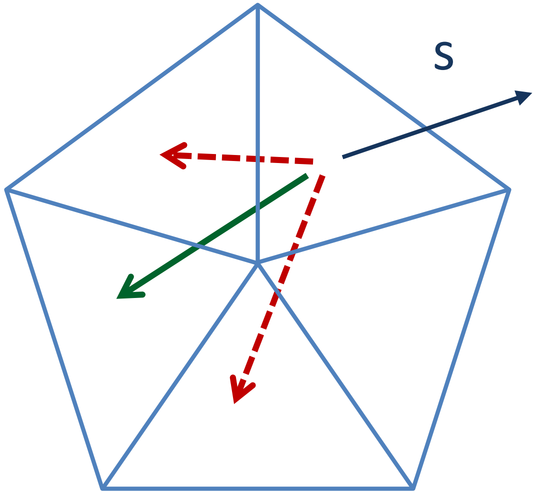
Density upwinding

+

Newton-Raphson procedure

$$\rho \leftarrow \rho - \nu \frac{\partial \rho}{\partial s} \Delta s$$

$$\rho \leftarrow \rho - \nu \overleftarrow{\Delta} \rho$$



Line search

$$F(\phi) = 0 \Rightarrow \frac{\partial F}{\partial \phi} \Delta \phi + F \approx 0$$

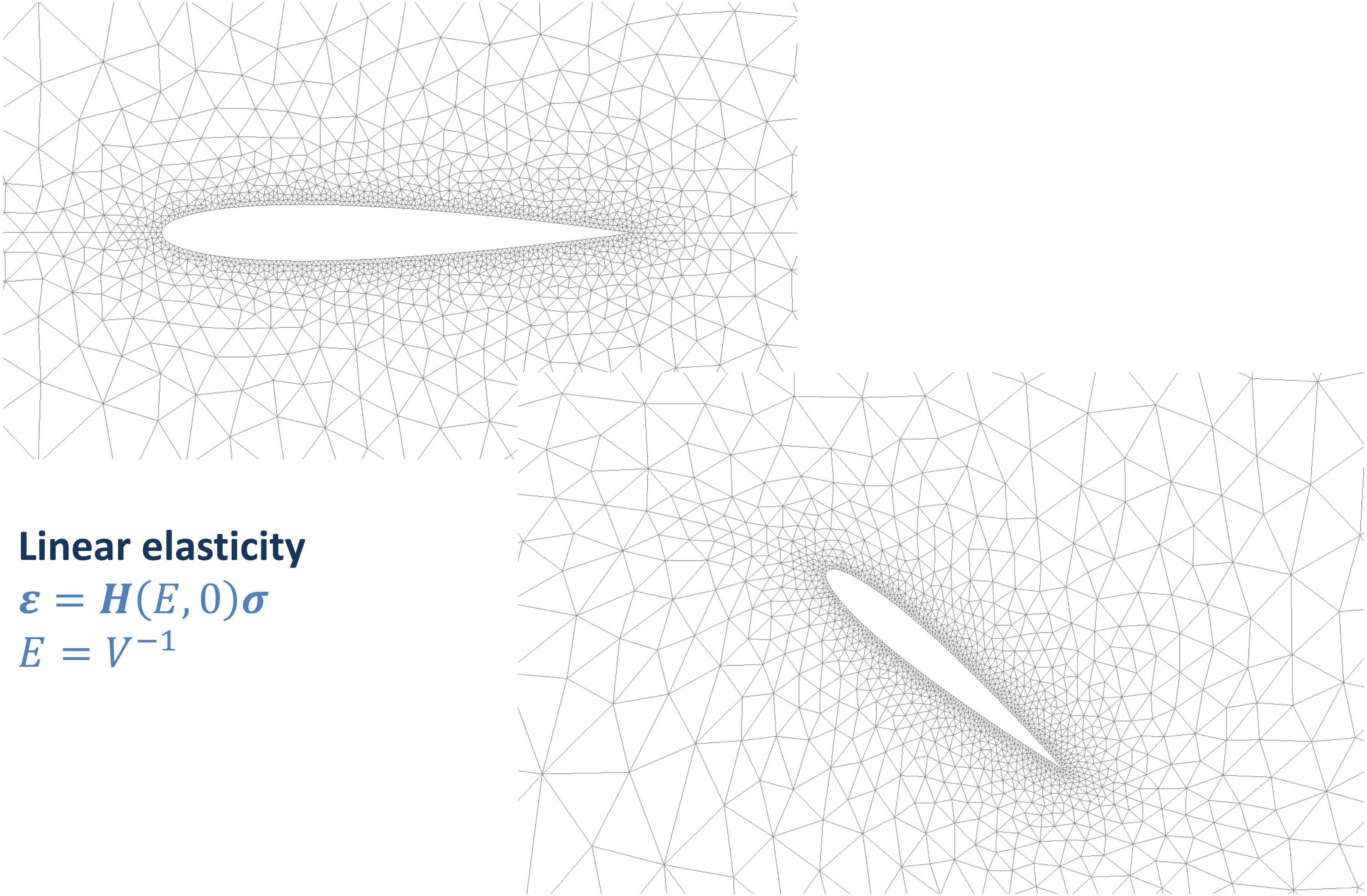
$$\phi_{\mathrm{new}} = \phi_{\mathrm{old}} + \lambda \Delta \phi$$

Adaptive viscosity

$$\nu = \boldsymbol{\nu}_{\mathbf{c}\downarrow} \left(1 - \frac{\boldsymbol{M}_{\mathbf{c}\uparrow}^{2}}{M_{\mathrm{e}}^{2}} \right)$$

$$M_{\mathrm{e}} = \max[1, M, M_{\mathrm{u}}]$$

Mesh morphing

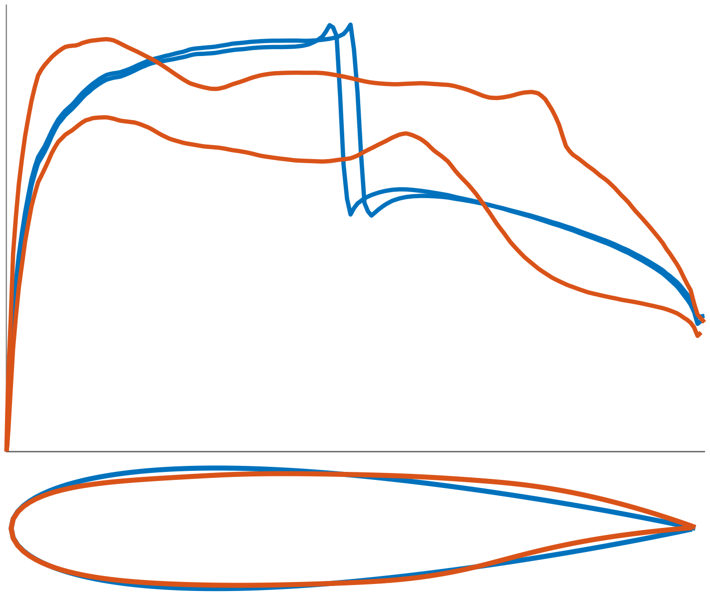


Linear elasticity

$$\boldsymbol{\varepsilon} = \boldsymbol{H}(E,0)\boldsymbol{\sigma}$$

$$E = V^{-1}$$

Discrete adjoint formulation



Aerodynamic shape optimization

$$\min_{y,\alpha} c_d$$

$$c_l = c_l^\star$$

$$d_p F(u;p) \to 0$$

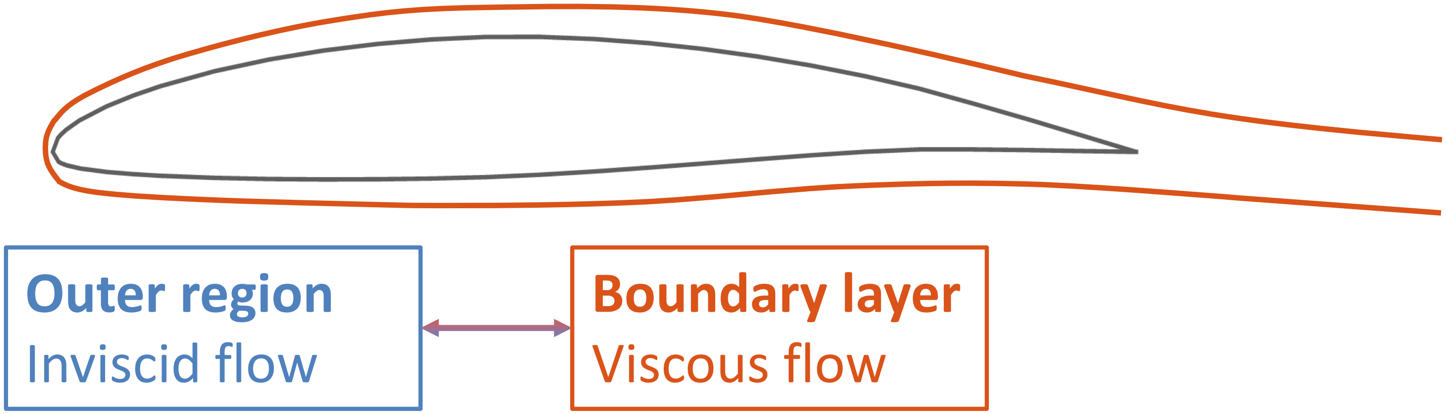
$$R(u;p) = 0$$

$$\begin{cases} R\big(u(p+\delta \mathrm{p})\big) = 0 \\ d_p F = \Delta \left\{ \dfrac{F\big(u(p+\delta p)\big)}{\delta p} \right\} \end{cases}$$

$$d_p F = \partial_p F - \underbrace{\partial_u F \, \partial_u R^{-1}}_{\partial_u R^{\mathrm{T}} \lambda = \partial_u F^{\mathrm{T}}} \partial_p R$$

Adjoint: independent on no. design variables

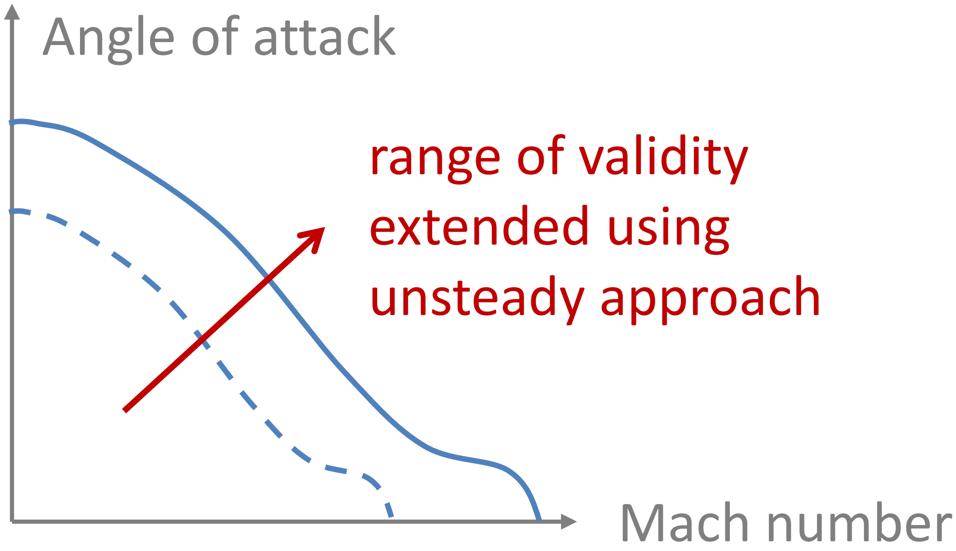
Viscous-inviscid interaction



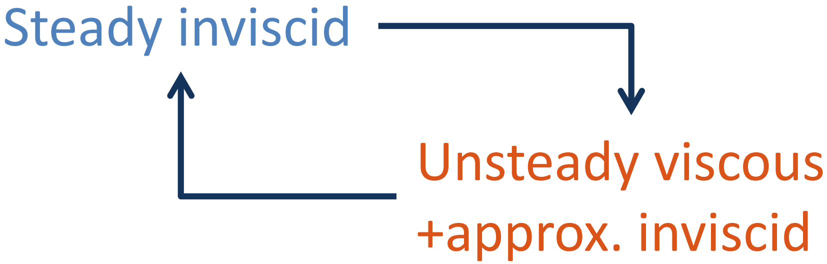
Outer region
Inviscid flow

Boundary layer
Viscous flow

Quasi-simultaneous pseudo-unsteady approach



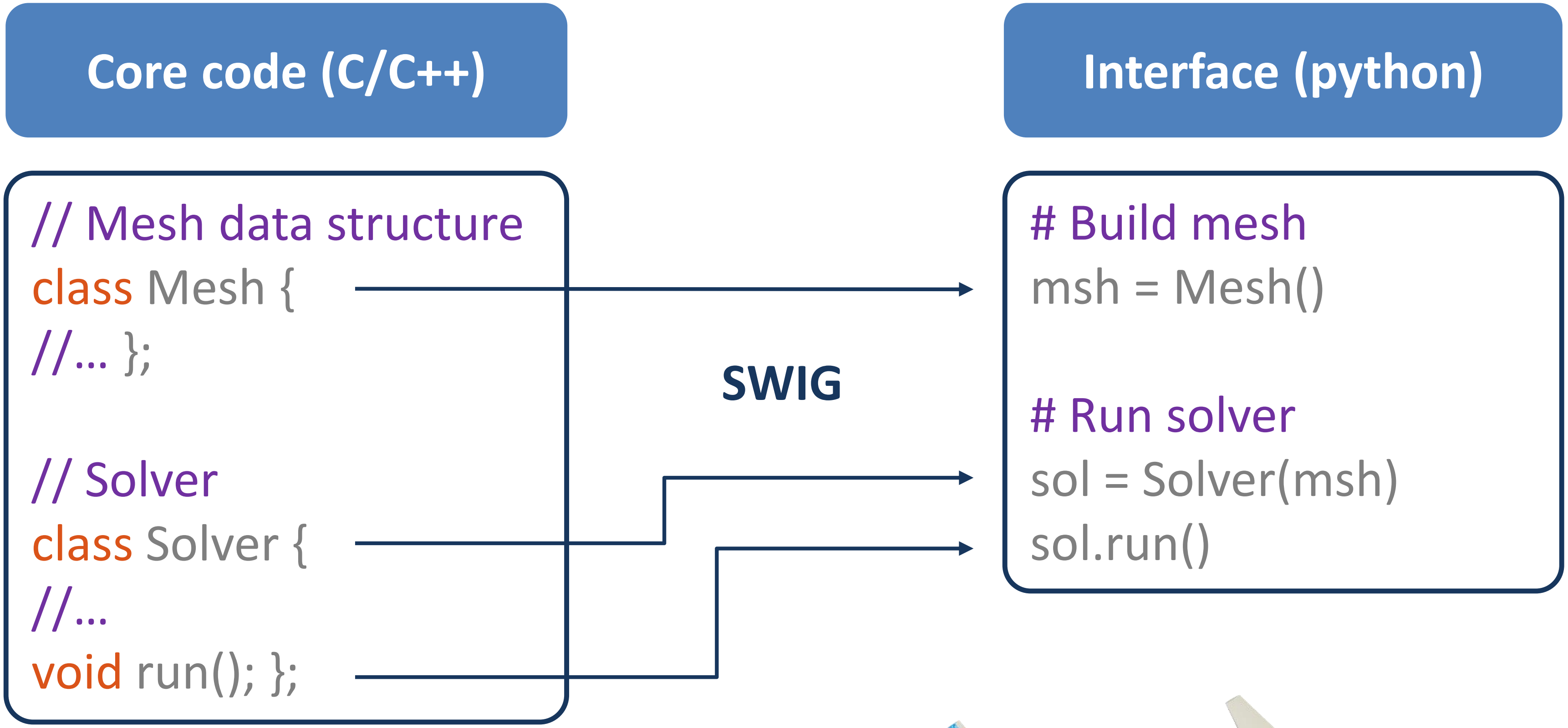
range of validity
extended using
unsteady approach



Steady inviscid

Unsteady viscous
+approx. inviscid

C++/Python languages



Core code (C/C++)

```
// Mesh data structure
class Mesh {
//… };

// Solver
class Solver {
//…
void run(); };
```

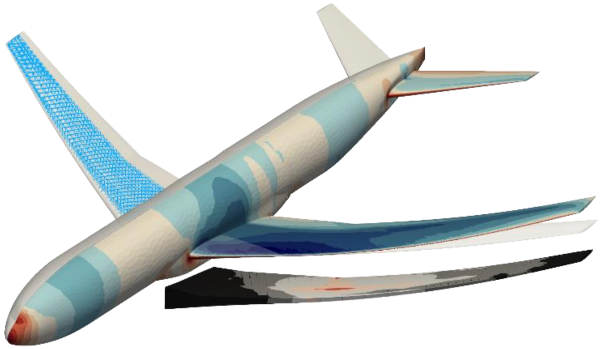
SWIG

Interface (python)

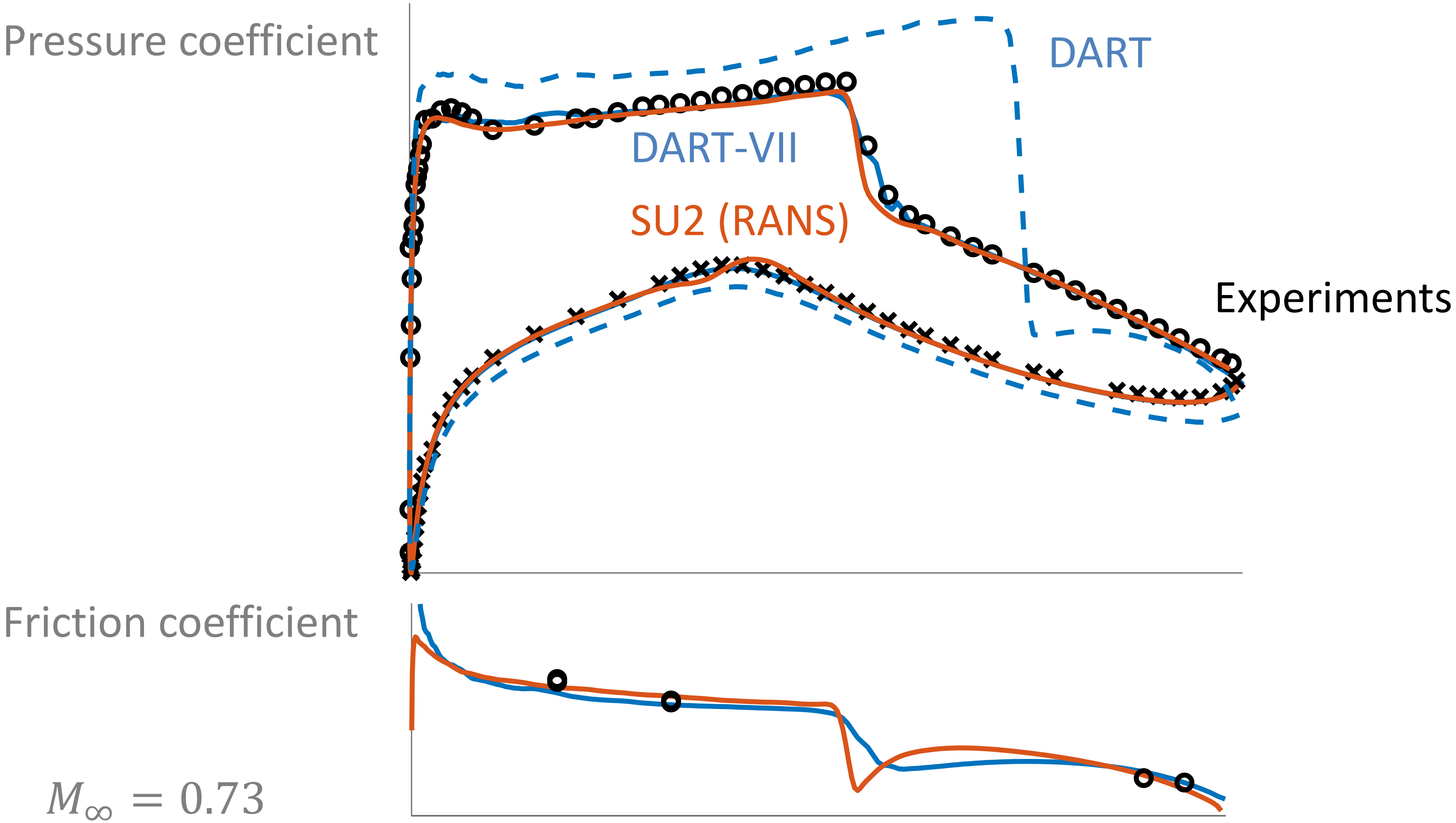
```
# Build mesh
msh = Mesh()

# Run solver
sol = Solver(msh)
sol.run()
```

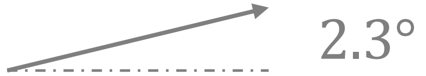
- ✓ Efficient
- ✓ User-friendly
- ✓ Modular



Two–dimensional viscous analysis



$M_\infty = 0.73$
$Re = 6.5\ \mathrm{M}$



Three–dimensional aeroelastic analysis

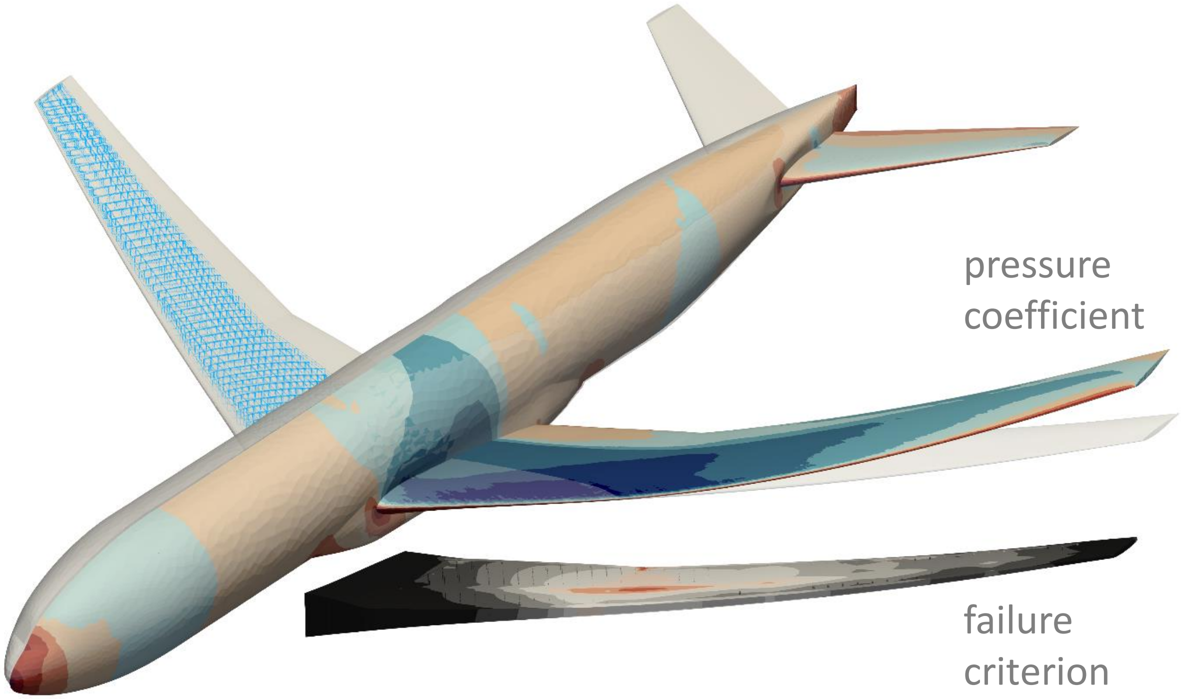
NASA CRM

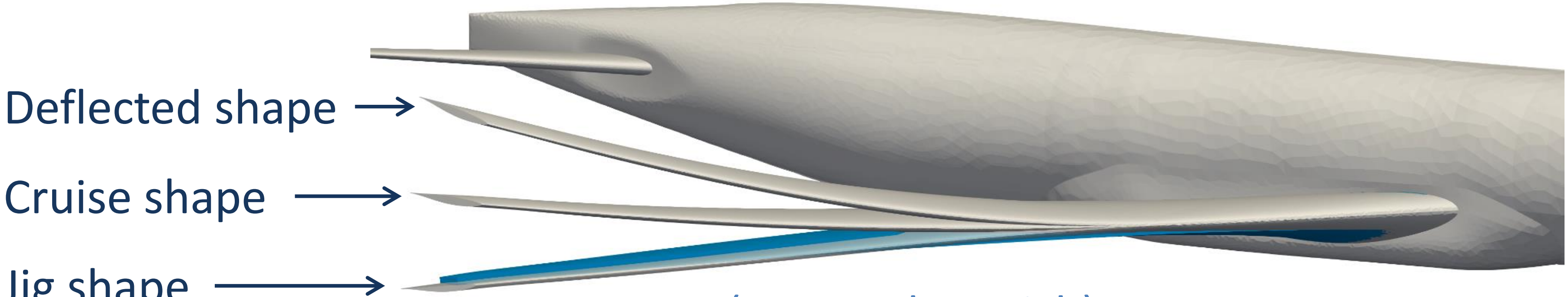
Cruise

$$\mathrm{M}_\infty = 0.85 \; - \; \mathrm{FL}\,370$$
$$n = 1.0 \; (C_L = 0.5)$$

Maneuver

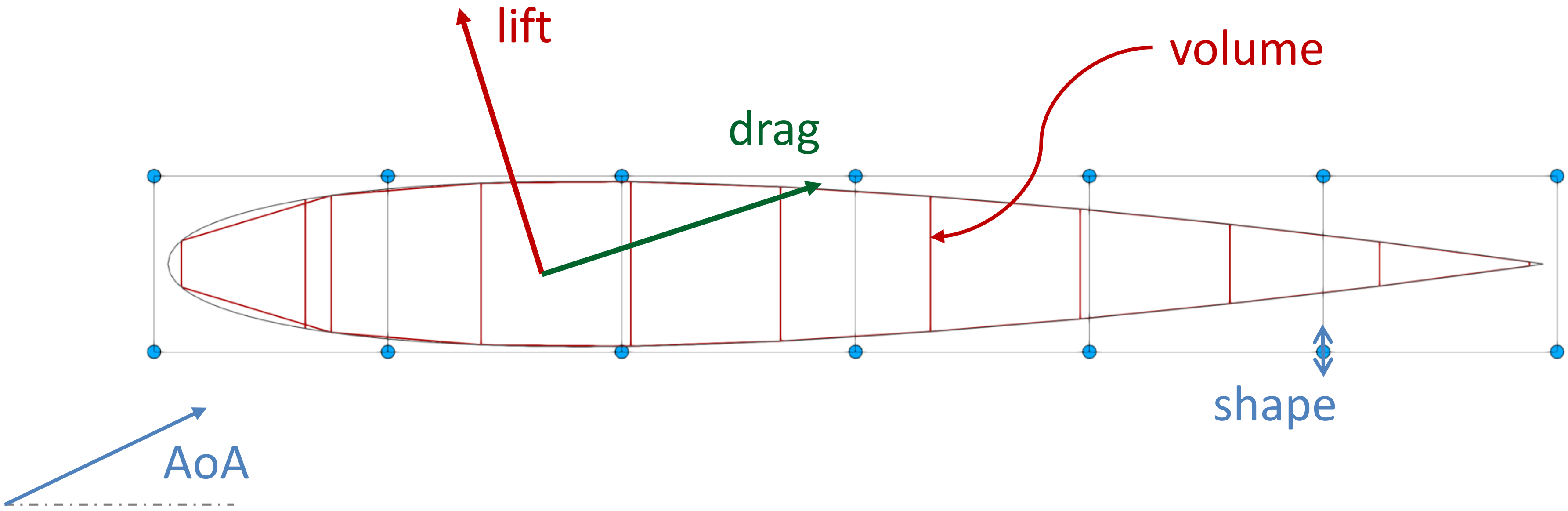
$$\mathrm{M}_\infty = 0.85 \; - \; \mathrm{FL}\,200$$
$$n = 2.5$$





uCRM-9 (MDO Lab UMich)

Two–dimensional shape optimization



$M_\infty = 0.8$

NACA 0012

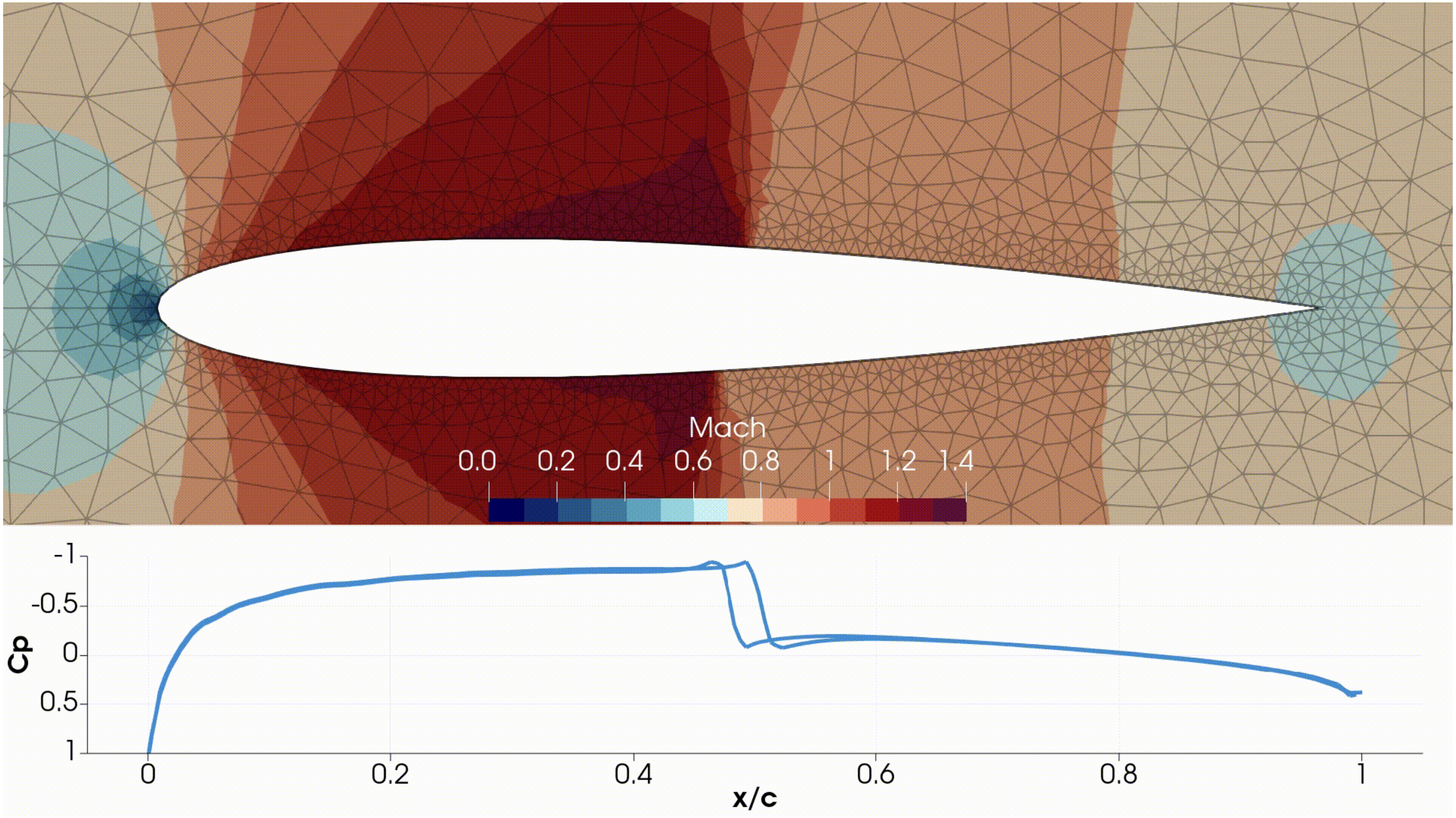
min drag

w.r.t. AoA, shape

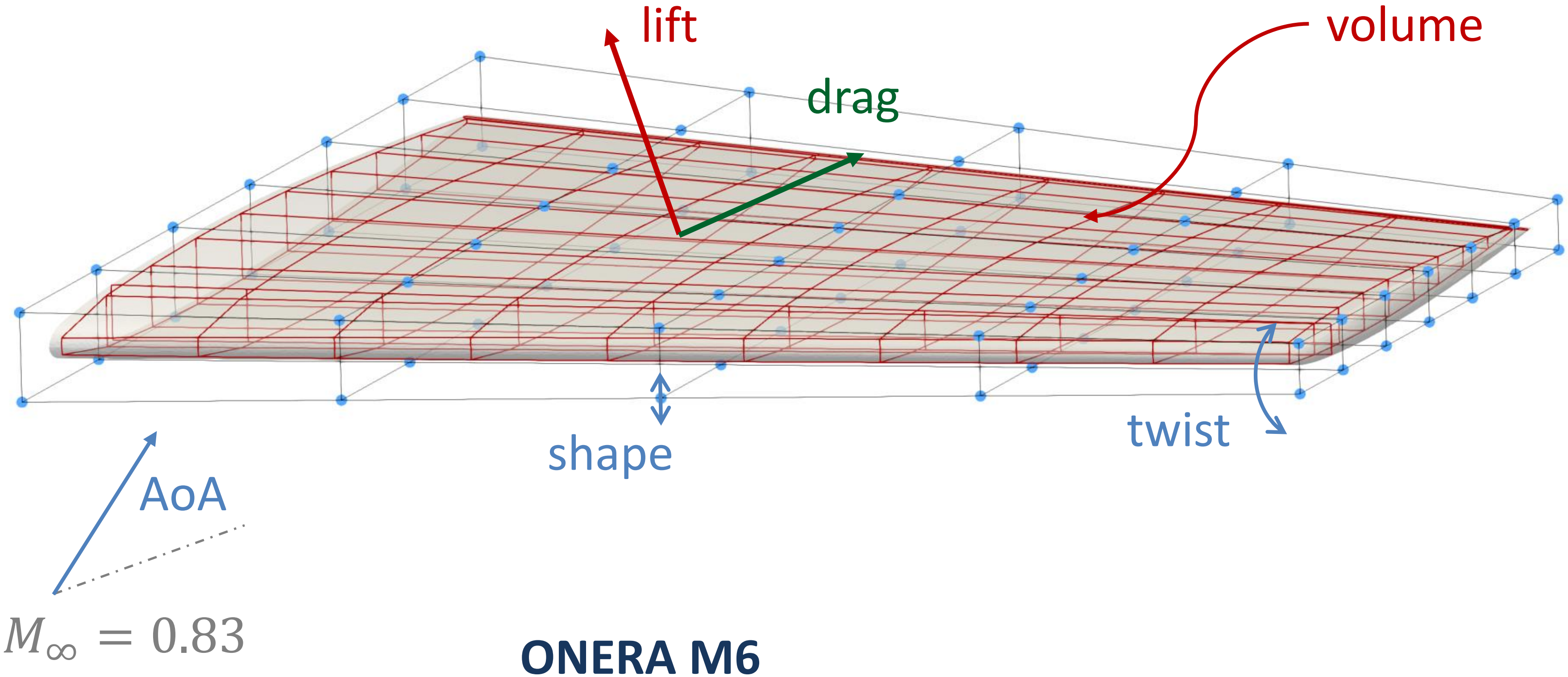
s.t. lift

internal volume

Two–dimensional shape optimization



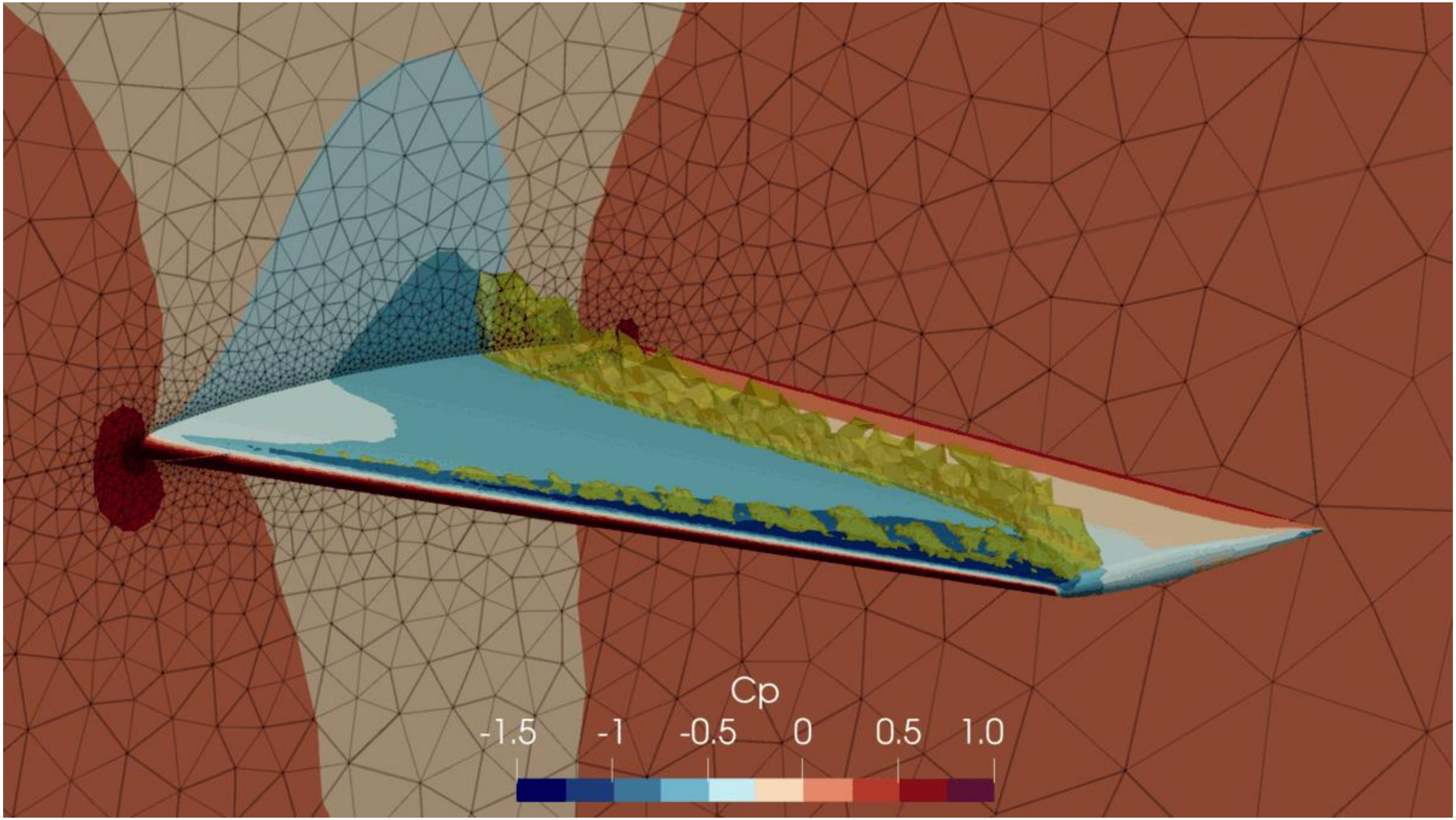
Three–dimensional shape optimization



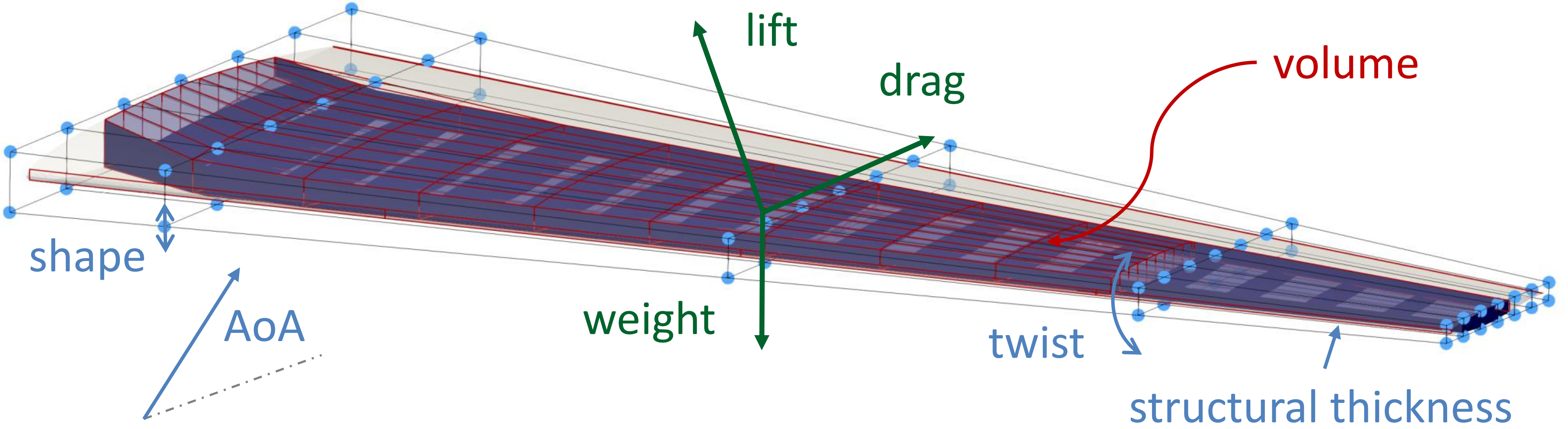
ONERA M6

min drag
w. r. t. AoA, shape, twist
s. t. lift
internal volume

Three–dimensional shape optimization



Three–dimensional aeroelastic optimization



RAE

Cruise

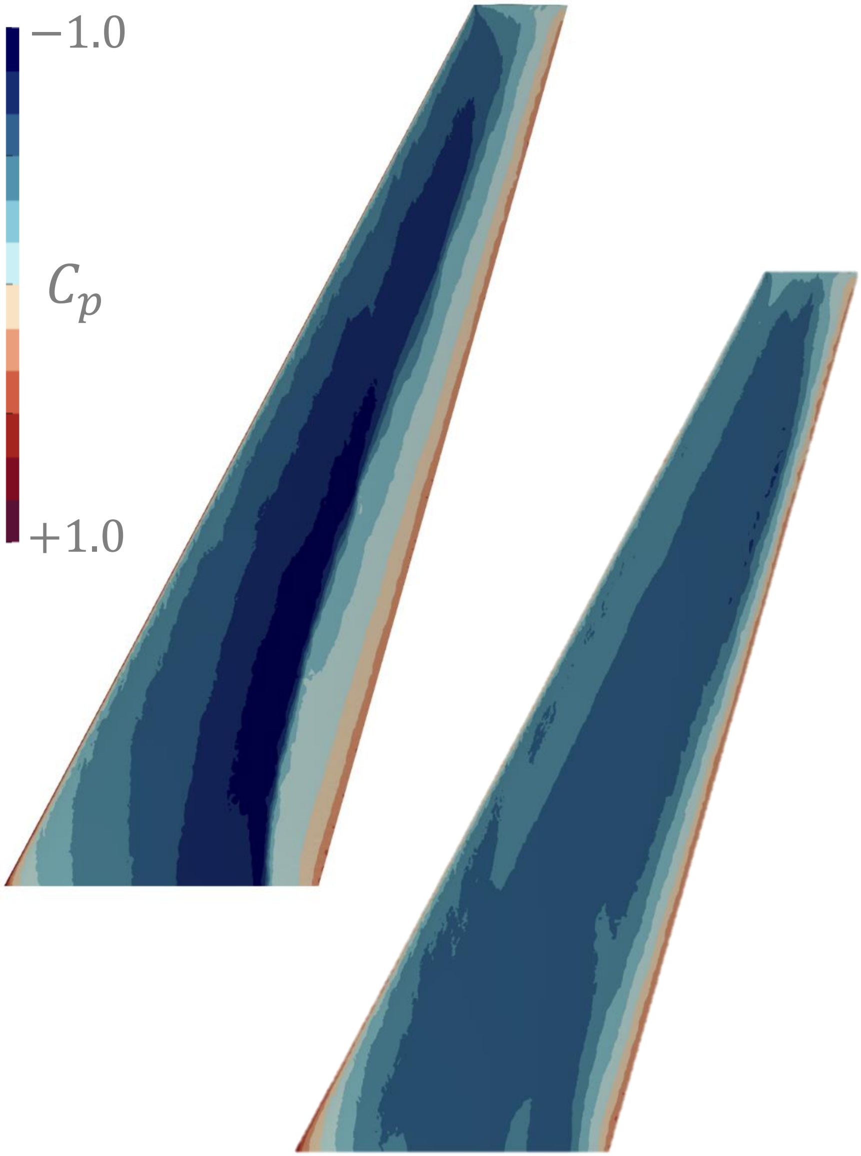
$M_\infty = 0.82 - \text{FL } 350$

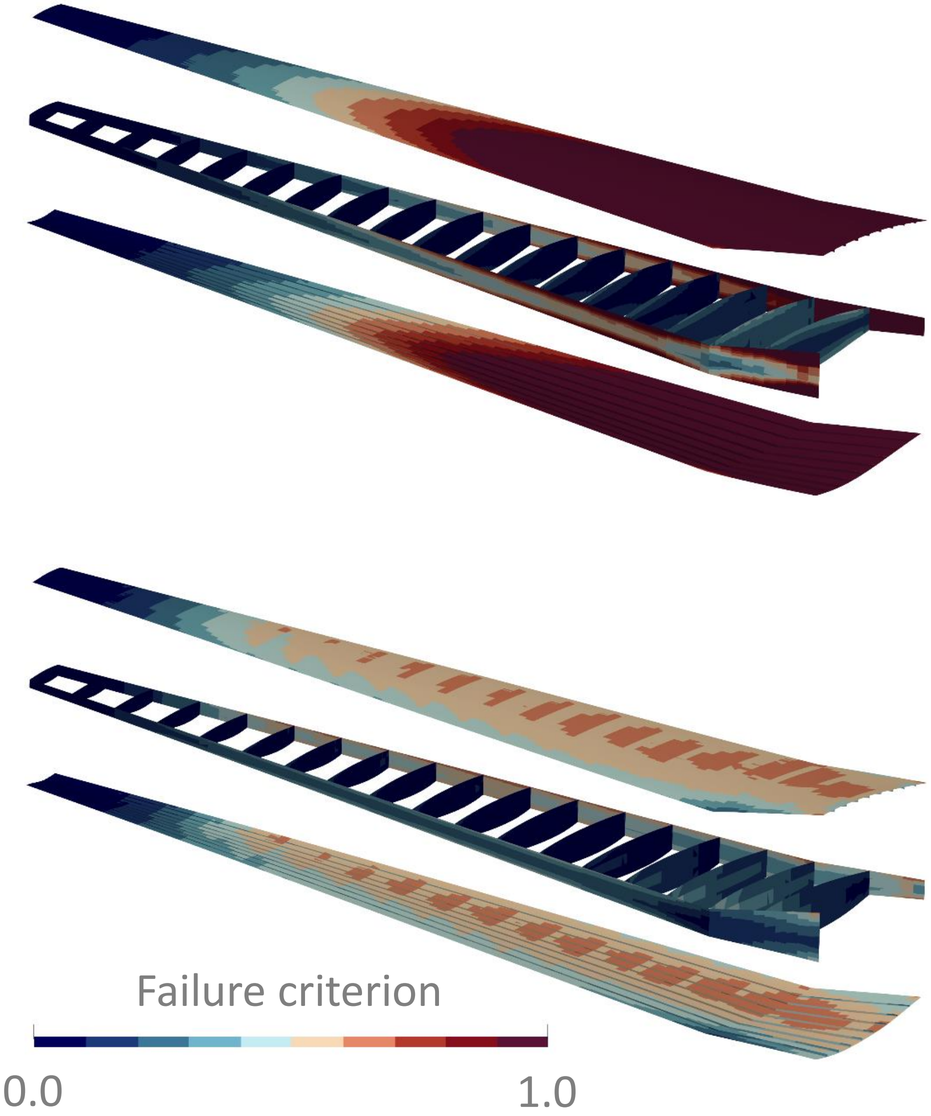
Maneuver

$M_\infty = 0.78 - \text{FL } 200$

$$\begin{array}{ll} \text{min} & \text{fuel} = \text{Breguet}(\text{lift}, \text{drag}, \text{weight}) \\ \text{w.r.t.} & \text{AoA}, \text{shape}, \text{twist}, \text{structural thickness} \\ \text{s.t.} & \text{load factor} \\ & \text{internal volume} \\ & \text{structural adjacency} \\ & \text{structural failure} \end{array}$$

Three–dimensional aeroelastic optimization





Conclusion

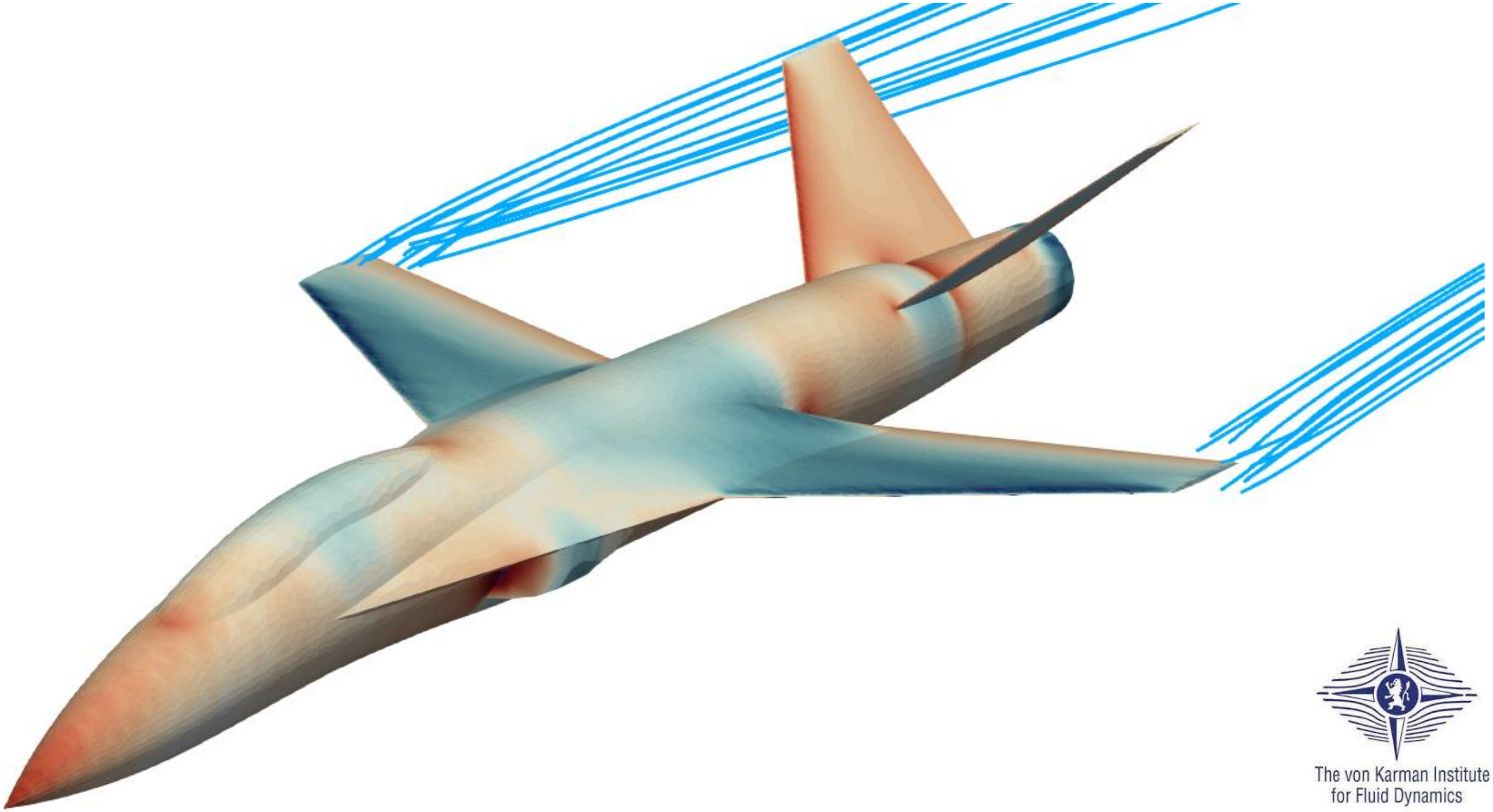
Key points

- **Transonic flows** consist of **mixed subsonic** and **supersonic** flow regions
- **Transonic flows** are **nonlinear** and must be **understood** and **modeled** properly for the **aircraft design** to be **efficient** and **robust**
- The **full potential** flow equation, coupled to the **boundary layer equations**, is the **lowest level-of-fidelity** that can be used to model **transonic flows**
- Our in-house software **DART** is designed to **quickly** solve **transonic flows** for **aerostructural optimization** applied to **preliminary aircraft design**

Aerothermodynamics of high-speed flows

Transonic flows

Adrien Crovato – Liège, Q2 2023



The von Karman Institute
for Fluid Dynamics

https://acrovato.github.io

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