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SENSITIVITY OF STEEL BUILDING FRAMES TO JOINT PROPERTIES

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Summary
Present paper is aimed at describing the way in which structural joint response has to be idealized in view of an elastic linear global frame analysis.

1. Introduction

The elastic analysis of a steel frame with semi-rigid joints requires the idealization of the behaviour laws of the beam and column elements as well as of the structural joints.

The idealization of the response of the beam and column elements is achieved through the definition of the so-called flexural rigidity EI/L (I denotes the inertia and L the length of the considered element). As shown in figure 1, EI/L is expressed as the ratio between the applied bending moment and the resultant rotation ϕ.
The range of application of this idealized behaviour is the elastic domain.
In this respect, two different approaches can be followed:

- *Elastic verification of the cross-section*

  Further to the elastic frame analysis, the maximum applied bending moment in each element is compared to the maximum elastic moment resistance M_{eRd} of the cross-section (figure 1.a). M_{eRd} corresponds to the first onset of plasticity in the most stressed fiber of the cross-section.

- *Plastic verification of the cross-section*

  The plastic verification consists in limiting the value of the maximum applied bending moment in each element to the plastic moment resistance M_{plRd} of the

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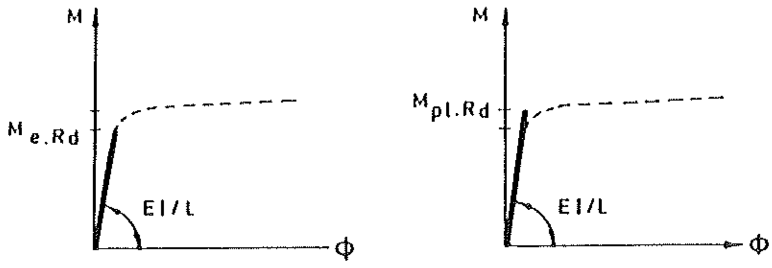
cross-section (figure 1.b).

The second approach requires Class 1 or 2 cross-sections (Eurocode 3, 1993) in which local plate buckling is likely to occur only after the plastic moment resistance is reached. $M_{e.Rd}$ and $M_{pl.Rd}$ values may be reduced, if necessary, to take into consideration any possible interaction with normal or shear forces in the cross-section.



a - Elastic verification

b - Plastic verification

Figure 1 - Linear idealizations of the beam and column flexural response

Quite similar idealizations are required for structural joints. Present paper is aimed at highlighting the specific character of such joint idealizations by referring to the new revised Annex J of Eurocode 3 (Revised Annex J, 1994) and to results of investigations recently carried out at the University of Liège.

2. Joint classification

In view of their modelling, joints have to be classified. For an elastic structural frame analysis, this classification is based on the joint rotational stiffness; three classes are identified:

- perfectly pinned joints
- semi-rigid joints
- perfectly rigid joints.

In fact, the concept of perfectly rigid or pinned joints is a pure theoretical view of mind which is nevertheless well useful to simplify the calculation of actual frame structures.

In practice, some real beam-to-column joints can yet be considered as pinned if their behaviour is such that the bending moment they can carry over is so law that it does not significantly influence the behaviour of the structure. In the same way, some actual structural joints can be considered as perfectly rigid if the relative rotation between the connected members is small enough not to significantly influence the behaviour of the frame.

So it could be of great help to have a classification system at one's disposal in order to see whether an actual joint can reasonably be considered as pinned or rigid or whether the joint semi-rigidity has to be taken into account in the frame design procedure.

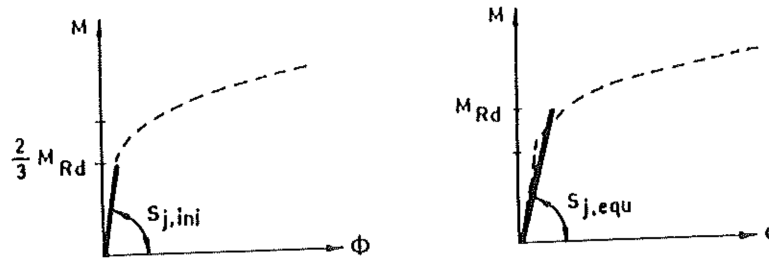
Such a classification system is described in the new revised EC3 Annex J for beam-to-column joints; it is simply based on the comparison of the initial joint stiffness $S_{j.ini}$ (defined in figure 2) with two boundaries (one between pinned and semi-rigid fields, one between semi-rigid and rigid fields). Some investigations are currently in progress at the University of Liège to derive more refined boundary values than those suggested in the new revised Annex J.

3. Joint idealization

For joints classified as semi-rigid, the characteristic moment-rotation curve has to be linearly idealized (figure 2). The corresponding stiffness S_j characterizes the rotational spring which allows to take the semi-rigid behaviour of the joint into consideration when analyzing the structure.

As for beam and column elements, two approaches are to be considered:

- Elastic verification of the joint ;
- Plastic verification of the joint.



a - Elastic verification ($S_j = S_{j,ini}$)   b - Plastic verification ($S_j = S_{j,equ}$)

Figure 2 - Linear idealizations of the joint flexural response

3.1. Elastic verification of the joint

In case of an elastic verification of the joint, the linearized M-ϕ response is to be characterized by the initial elastic stiffness $S_{j,ini}$ (figure 2.a). The maximum bending moment to be reached in the joint is the elastic moment resistance which is defined in Eurocode 3 as equal to 2/3 of the so-called design moment resistance M_{Rd}. The latter can be considered as a pseudo-plastic moment of the joint.

This idealization may be referred to when considering serviceability or ultimate frame limit-states. It is anyway of particular interest for severe serviceability conditions; in such a case, the use of the initial stiffness is likely to reduce the transversal displacement of the beams and the lateral drift of the storeys and of the whole frame.

The designer may also take profit of the full resistance of the joint by accepting to reach M_{Rd} in the most loaded joint of the structure. The verification of the joint is then said "plastic".

3.2. Plastic verification of the joint.

In beam and column elements made of H or I sections, the ratio between $M_{pl.Rd}$ and $M_{e.Rd}$ (see figure 1) is so limited ($\approx$1,14) that the plastic verification of the most loaded section may be

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simply based on the diagram of internal forces resulting from the elastic global analysis in which the flexural behaviour of the members is characterized by the elastic stiffness EI/L. This is no more possible for joints where the use of the initial elastic stiffness $S_{j,ini}$ beyond the elastic limit 2/3 M_{Rd} leads to unacceptable overestimations of the joint stiffness and resistance properties.

An idealized joint response characterized by a reduced elastic stiffness has therefore to be substituted to the complex and non linear actual M-ϕ curve (figure 2.b), but in such a way that the resulting global frame response is not significantly affected compared to the actual one. This equivalent reduced stiffness $S_{j,equ}$ is to be used in the global frame analysis in view of a consistent determination of the internal forces in the frame.
The type of verification - elastic or plastic - is therefore seen to influence the joint stiffness which has to be introduced in the global frame analysis.

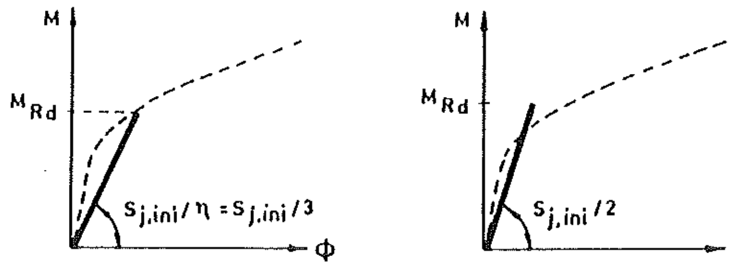
3.3. Equivalent elastic stiffness for plastic joint verification.

With a view to determine the "equivalent stiffness" $S_{j,equ}$, five different structures (one storey-one bay; one storey-two bays; three storeys-one bay; two storeys-three bays; four storeys-three bays) have been numerically studied (for different joint types) by means of the finite element program FINELG [FINELG, 1994] and this, in the two following cases:

- *exact numerical simulation*, i.e. actual non-linear behaviour of the joint;
- *numerical simulation with idealized joint response*, i.e. linearized joint response.

The equivalent stiffness recommended for joints in the old Annex J (Eurocode 3, 1993) is the secant stiffness corresponding to the design resistance M_{Rd} (see figure 3.a). In the new revised Eurocode 3 Annex J, on the other hand, a value of $S_{j,ini}/2$ is recommended (figure 3.b).

The secant stiffness corresponding to M_{Rd} is defined in the new revised Annex J as equal to $S_{j,ini}/\eta$ with, for instance, η equal to 3 for welded joints and bolted joints with end plates. In the present paper, η is taken as equal to 3 for simplicity.



a - Annex J of Eurocode 3, 1993 b - Revised Eurocode 3 Annex J, 1994

Figure 3 - Recommended linear idealizations of the joint response for plastic verification

For non-sway frames, the structural response under service loads and at collapse is seen to be not highly affected by the definition of the equivalent stiffness, $S_{j,ini}/3$ or $S_{j,ini}/2$. In both cases, the frame with idealized joint response exhibits a safe and relatively accurate response in comparison to the exact one.

For sway frames, on the other hand, the numerical simulations show that the use of $S_{j,ini}/3$ is unsatisfactory; it leads to large and uneconomical overestimations of the frame displacements. The value of $S_{j,ini}/2$ is better, generally safe, but can sometimes lead to underestimations of the transverse displacements.

In the present paper, an original and simple formula for the assessment of the equivalent stiffness $S_{j,equ}$ is proposed. Its derivation is based on a two-step procedure described in the following paragraphs.

Step 1: Evaluation of the rotation Φ_{mean}

The rotation Φ_{mean} is defined as an estimation of the mean rotation in the joints acting at the ends of the considered beam. It is determined by similarity with the so-called "wind connection method" as far as it is assumed to result from the beam deformability only (the column being considered as rigid):

$$\phi_{mean} = \frac{qL_b^3}{24\ EI_b} - \frac{ML_b}{2\ EI_b} \qquad \text{in case of distributed load q} \qquad (1)$$

$$\phi_{mean} = \frac{PL_b^2}{16\ EI_b} - \frac{ML_b}{2\ EI_b} \qquad \text{in case of concentrated load P at mid-span} \qquad (2)$$

where I_b is the beam inertia;
L_b is the length of the beam;
M is the moment at beam end.

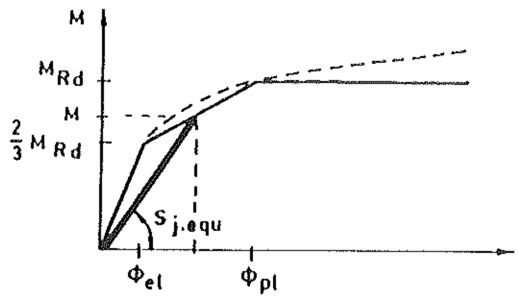
In the present procedure, the equivalent stiffness is defined as the actual secant stiffness corresponding to Φ_{mean} (figure 4) so $M = S_{j,equ} \cdot \Phi_{mean}$. By introducing this expression into equations (1) and (2), the following value of Φ_{mean} is derived:

$$\phi_{mean} = \frac{qL_b^2}{12\left(\frac{2\ EI_b}{L_b} + S_{j,equ}\right)} \qquad \text{distributed load q} \qquad (3)$$

$$\phi_{mean} = \frac{PL_b}{8\left(\frac{2\ EI_b}{L_b} + S_{j,equ}\right)} \qquad \text{concentrated load P} \qquad (4)$$

Step 2: Derivation of $S_{j,equ}$

For each value of ϕ_{mean}, the corresponding moment M may de estimated by referring to the approximated tri-linear moment-rotation curve (see figure 4). The equivalent stiffness $S_{j,equ}$ can then be derived by expressing $S_{j,equ}$ as the ratio between M and Φ_{mean}. In this procedure, three different cases have to be identified:



$$\Phi_{el} = \frac{2\ M_{Rd}}{3\ S_{j,ini}}$$

$$\Phi_{pl} = \frac{\eta\ M_{Rd}}{S_{j,ini}} = \frac{3\ M_{Rd}}{S_{j,ini}}$$

Figure 4 - Definition of $S_{j,equ}$ as dependent of M and Φ_{mean}

$\Phi_{mean} \leq \Phi_{el}$

In this elastic range of joint behaviour:

$$S_{j,equ} = S_{j,ini} \tag{5}$$

$\Phi_{el} < \Phi_{mean} \leq \Phi_{pl}$

The straight line characterizing the elastic-plastic part of the tri-linear design joint response in Figure 4 has the following equation:

$$\frac{M - \frac{2}{3} M_{Rd}}{M_{Rd} - \frac{2}{3} M_{Rd}} = \frac{\phi_{mean} - \phi_{el}}{\phi_{pl} - \phi_{el}} \tag{6}$$

By replacing M by $S_{j,equ} \cdot \Phi_{mean}$ and Φ_{mean} by expressions (3) and (4), the following values of $S_{j,equ}$ may be obtained:

$$S_{j,equ} = \frac{qL_b^2\ S_{j,ini} + 96\ \frac{EI_b}{L_b}\ M_{Rd}}{7\ qL_b^2 - 48\ M_{Rd}} \quad \text{distributed load q} \tag{7}$$

$$S_{j,equ} = \frac{PL_b\ S_{j,ini} + 64\ \frac{EI_b}{L_b}\ M_{Rd}}{7\ PL_b - 32\ M_{Rd}} \quad \text{concentrated load P at mid-span} \tag{8}$$

These two expressions of $S_{j,equ}$ can be written in the following format:

$$S_{j,equ} = \frac{S_{j,ini}\ M_{fixed} + 4\ R\ M_{Rd}}{7\ M_{fixed} - 4\ M_{Rd}} \quad (9)$$

where M_{fixed} is the end moment at the fixed ends of a symmetrically laoded beam;
R is the beam stiffness ($2EI_b/L_b$).

$\Phi_{mean} > \Phi_{pl}$

In this plastic range of behaviour, $S_{j,equ}$ is taken as equal to the secant joint stiffness corresponding to M_{Rd} so:

$$S_{j,equ} = S_{j,ini}\ /\ \eta = S_{j,ini}\ /\ 3 \quad (10)$$

For practical applications, the procedure is simply used as follows:

- $S_{j,equ}$ is calculated through the following formula:

$$S_{j,equ} = \frac{S_{j,ini} + (2\eta - 1)\ R\ \alpha}{(3\eta - 2) - 2\ (\eta - 1)\ \alpha} \quad (11)$$

which generalizes formula (9) for any value of η and the obtained value is considered as correct if:

$$S_{j,ini}\ /\ \eta \leq S_{j,equ} \leq S_{j,ini} \quad (12)$$

In expression (11), α is defined as M_{Rd}/M_{fixed}.

- If condition (12) is not fulfilled and $S_{j,equ}$ calculated through formula (11) is greater than $S_{j,ini}$, then $S_{j,equ} = S_{j,ini}$;

- If condition (12) is not fulfilled and $S_{j,equ}$ calculated through formula (11) is lower than $S_{j,ini}/\eta$, then $S_{j,equ} = S_{j,ini}/\eta$;

4. Accuracy of the proposed formula

To verify this evaluation procedure, the transversal displacement V under service loads of the five studied non-sway frames obtained through two types of numerical simulations:
- with actual joint response
- with linear idealized joint response

have been compared.

These comparisons (Figure 5) have been performed, for all the five considered structures, with different joint types (extended end plates, flush end plates, ...) so justifying the number of dots reported in Figure 5.

It can be seen that the majority of the calculated displacements differ of less than 5 % from the actual ones. This accuracy is of primary importance but in view of a practical application, the simple format of the proposed approach has also to be highlighted.

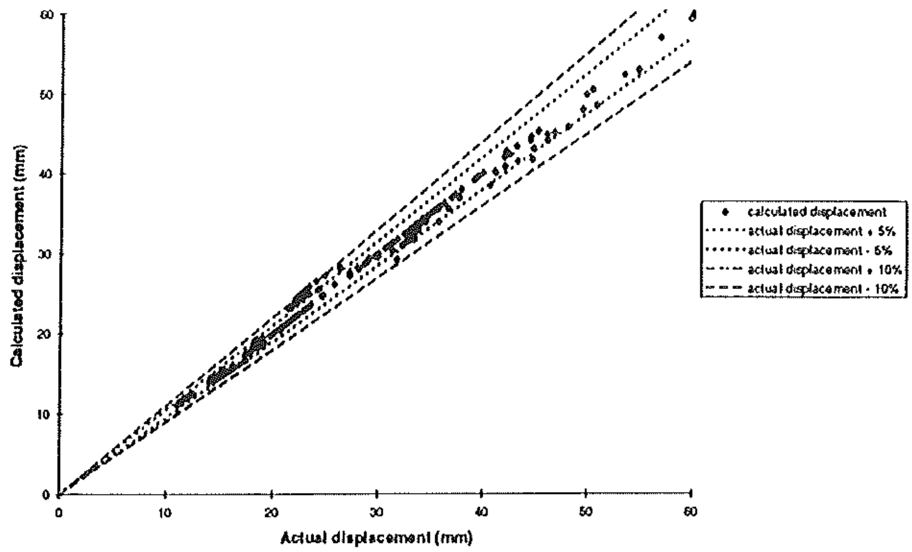


Figure 5 -Comparisons between actual and calculated frame transverse displacements under service loads.

5. Conclusions

In view of a structural elastic linear analysis, the joint M-ϕ curves have to be linearized. Further to the frame analysis, the sufficient resistance of the joints has to be checked. In this respect, two classical check procedures - the elastic one and the plastic one - are presented. The way to linearize the M-ϕ curves in both cases is shown and a simple and validated procedure to derive the elastic stiffness to be used for joint plastic verification is given.

6. Acknowledgement

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7. References

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