Behaviour of minor-axis joints and 3-D joints

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Abstract

In a beam-to-column minor-axis joint, the beam is directly connected to the web of an H or I column, in the direction of the column minor-axis. The connection can be welded or bolted using connecting elements like endplates or cleats. Such a joint has a strongly non linear M-Φ curve where the relative rotation Φ is due to the deformation of the connecting elements and to the deformation of the column web in the direction of the column minor-axis. Failure mechanisms of the column web and the associated moment resistance are presented. Some topics on initial, secant and membrane stiffness as well as rotation capacity are also presented.

In a 3-D joint the behaviour of the major-axis joint is affected by the presence of the minor-axis joint. Interaction between minor- and major-axis joint resistance is discussed.

1. Introduction

The 3-D joint as shown in Fig. 1 has two different kinds of joints: a major-axis joint with the beam connected to the column flange and a minor-axis joint where the beam is often directly connected to the column web with no stiffeners. Interaction between major- and minor-axis joint behaviour must be evaluated, since they are not independent.

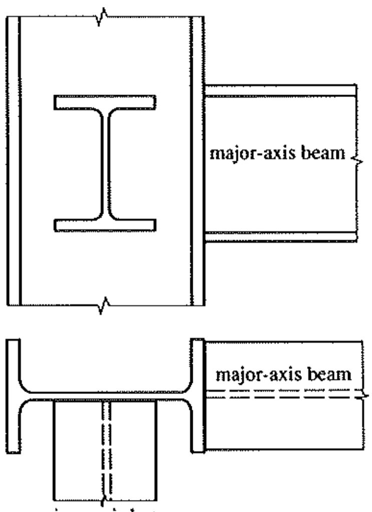

Fig. 1 - Minor- and major-axis joints in a 3-D joint

2. Minor-axis joint behaviour

Recent research on minor-axis joints [1] - [5] shows an increasing interest on its semi-rigid behaviour.

Minor-axis joints, with common types of beam-to-column connections (bolted or welded), have strongly non linear M - Φ curves (Fig. 2) where the relative rotation Φ is due to the deformation of the connecting elements and to the deformation of the column web (Fig. 3). Depending on the connection dimensions, a large variety of joint behaviour can be observed: from nominally pinned to strongly resistant joints.

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For design purposes we may define the following parameters (Fig. 2):

- moment resistance M_R - associated to failure mechanisms (1st order plastic analysis);
- secant stiffness S_j - depending on the moment;
- membrane stiffness S_m - due mainly to membrane stresses in the web (2nd order analysis);
- rotation capacity.

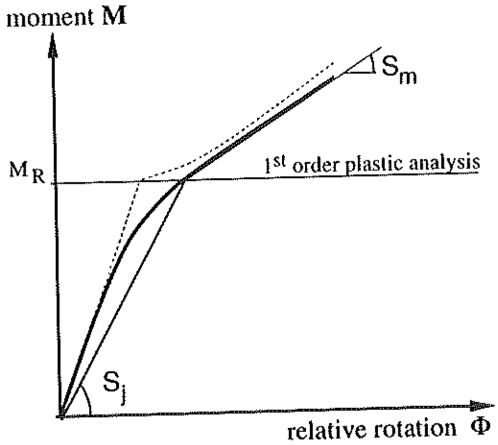

Fig. 2 - Typical M–Φ curve

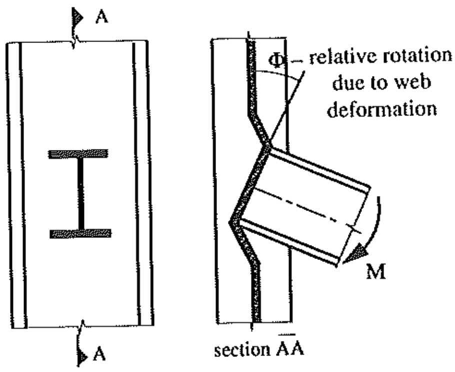

Fig. 3 - Minor-axis joint: global failure of the column web

3. Moment resistance and plastic failure mechanisms of the column web

For the evaluation of the plastic moment resistance we can identify 5 failure modes:

- Flexural modes:
  1) Local mechanism
  2) Global mechanism
  3) Mechanism under each bolt head (only for bolted connections)
- Modes with punching shear:
  4) Punching shear around each bolt head or around the compression or tension zone
  5) Combined flexural and punching shear mechanism

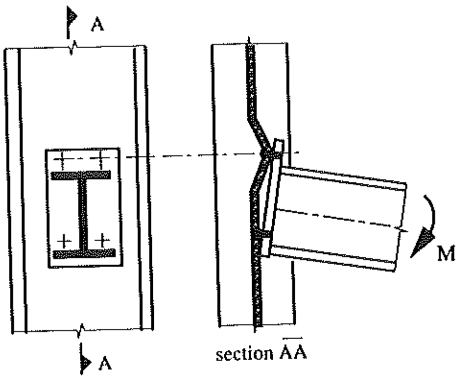

Fig. 4 - Local failure mechanism

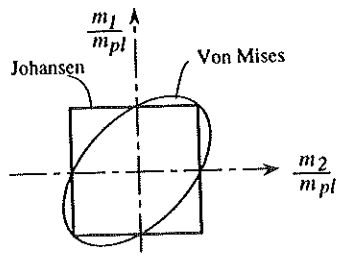

Fig. 5 - Yield criteria for a plate
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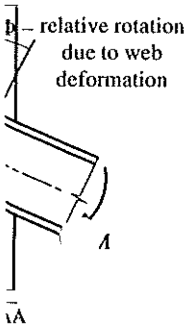

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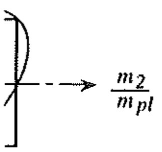

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VALIDITY RANGE: $b/L < 0.8$ **and** $0.7 \leq h/(L-b) \leq 10$

CONSTANTS:

$$m_{pl} = \frac{1}{4}\, t_w^2 f_y \qquad (1)$$

$$\alpha = \frac{4}{1 - b/L}\left(\pi\sqrt{1 - b/L} + 2\, c/L\right) \qquad (2)$$

$$k = \begin{cases} 1 & \text{if } (b+c)/L \geq 0.5 \\ 0.7 + 0.6\,(b+c)/L & \text{if } (b+c)/L \leq 0.5 \end{cases} \qquad (3)$$

LOCAL FAILURE:

$$F_{local} = m_{pl}\, \alpha\, k \qquad (4)$$

GLOBAL FAILURE:

$$F_{global} = \begin{cases} m_{pl}\left(\dfrac{2b}{h} + \dfrac{\alpha k}{2} + \pi + \dfrac{2h}{L-b}\right) & \text{if } \dfrac{h}{L-b} \geq 1 \\ m_{pl}\left(\dfrac{2b}{h} + \dfrac{\alpha k}{2} + \pi + 2\right) & \text{if } \dfrac{h}{L-b} \leq 1 \end{cases} \qquad (5)$$

PUNCHING SHEAR FAILURE:

- case: punching shear around n bolt heads: $F_{Q1} = n\, \pi\, d_m\, t_w \dfrac{f_y}{\sqrt{3}}$ (6a)

- case: punching shear around the rectangular area: $F_{Q1} = 2\,(b+c)\, t_w \dfrac{f_y}{\sqrt{3}}$ (6b)

COMBINED FLEXURAL AND PUNCHING SHEAR FAILURE:

$$F_{Q2} = \begin{cases} F_{local} & \text{if } t_w \leq L/20 \\ 4\, m_{pl}\left[\dfrac{\pi\sqrt{L\,(a+x)} + c}{a+x} + \dfrac{2\,c\,x + x^2}{\sqrt{3}\, t_w\,(a+x)}\right] & \text{if } t_w > L/20 \end{cases} \qquad (7)$$

where: $a = L - b$
and $x \geq 0$ determined by the iterative procedure:

$$x_{i+1} = -a + \sqrt{a^2 - 2\,ac + \frac{\sqrt{3}\, t_w}{2}\left[\pi\sqrt{L\,(a + x_i)} + 2\,c\right]} \qquad (8)$$

if $x_{i+1} < 0$ take: $F_{Q2} = F_{local}$

PLASTIC FAILURE LOAD:

$$F_{pl} = \min\left(F_{local}, F_{global}, F_{Q1}, F_{Q2}\right) \qquad (9)$$

Table 1. Determination of the failure load F_{pl} in the compression or tension zone of the column web in a minor-axis joint (see also § 3). For design purposes f_y must be replaced by f_y / γ_{M0}.

The moment M transmitted by the beam to the column web is first decomposed in a couple of forces F acting in the compression and tension zones. Table 1 synthesises the expressions that can be used for the practical determination of the plastic failure load F_{pl}, assuming that F acts on a rectangle of dimensions $b \times c$ as exemplified in § 3.1.

Plastic mechanisms, briefly described in the following, are obtained by Johansen yield line method and are optimised using log-spiral fans (Figs. 6i, 6iii, or 8). The plastic load F associated to each mechanism is first obtained in the form of non-linear equations. An approximate solution, explicit in F, is proposed for practical design. This second solution is close enough to the first one if geometrical parameters fall within the validity range referred in Table 1. Final expressions also include the correction factor k, Eq. (3), that introduces the influence of the yield criteria on the plastic load; k is evaluated by numerical simulations performed with the finite element program FINELG [6] that uses the Von Mises yield criterion, instead of the square yield criterion (Fig. 5) used in the Johansen yield line method (see [5]).

Often there is no axial force in the beam – the compression and tension forces are equal – and it may be assumed that the compression and tension rectangles, resisting to the same force F_{pl}, have the same dimensions $b \times c$. In this case, the procedure in Table 1 is applied only once. The plastic moment resistance is then

$$M_R = F_{pl}\ h$$

where h is the distance between centres of the two rectangles.

3.1 Local failure

A local failure means that the yield line pattern is localised only in the compression zone or in the tension zone (Fig. 4 and 6). It is assumed that the load F acts on a rigid rectangle with dimensions $b \times c$. These dimensions are, in the case of a welded connection, defined by the perimeter of the welds around the beam flange. For a bolted connection like that shown in Fig. 4, with the yield mechanism represented in Fig. 6i, an equivalent rectangle should be defined. The dimensions $b \times c$ of this equivalent rectangle, as shown in Fig. 6iii, are a function of the mean diameter of the bolt head d_m (see Fig. 6iv).

With the yield line pattern of Fig. 6i, a first solution for the failure load is given by

$$F = \frac{4\,\pi\, m_{pl}}{1 - \dfrac{a^*}{L - b_1}} \left(1 + \frac{4}{\pi}\,\mathrm{cotg}\,\theta + \mathrm{cotg}^2\,\theta\right)$$

where $\quad a^* = d_m\, e^{-\frac{\pi}{2}\mathrm{cotg}\,\theta}\ ; \qquad b_1 = b_0 + d_m\left(1 - e^{-\frac{\pi}{2}\mathrm{cotg}\,\theta}\right)$

and θ is the solution of the equation $\dfrac{b_1}{L - b_1} = 2\,\mathrm{cotg}\,\theta\ e^{\frac{\pi}{2}\mathrm{cotg}\,\theta}$.

An approximate solution, explicit in F, is the simplified formula $F = m_{pl}\,\alpha$. Introducing the correction factor k, the expression (4) proposed in Table 1 is finally obtained.

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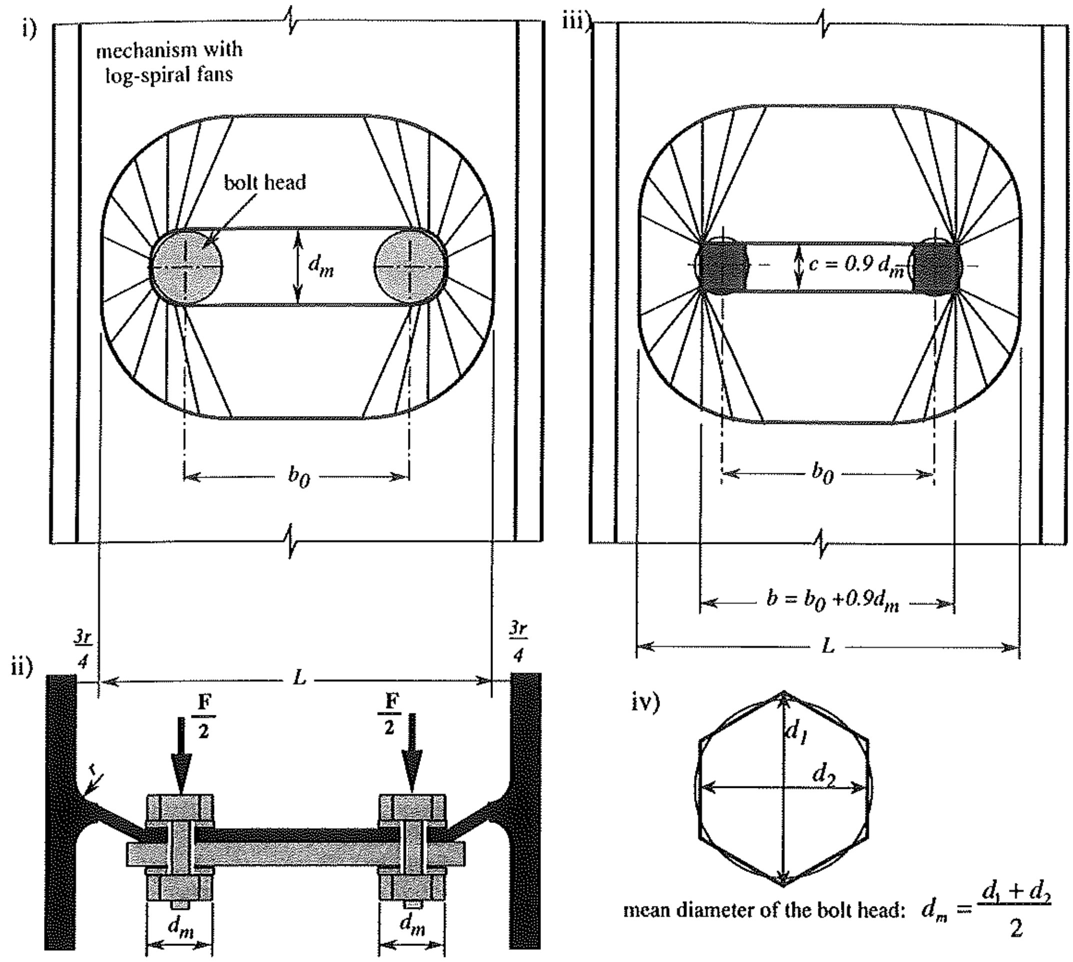

Fig. 6 - Local mechanism for bolted connection:
i) yield line pattern
ii) section view
iii) yield line pattern for equivalent rectangle $b \times c$
iv) mean diameter of the bolt head (or nut)

Comparison with other analytical solutions

Packer *et al.* [7] propose the two failure mechanisms of Figs. 7i and 7ii with the corresponding failure load represented in the diagram of Fig. 7iii. These two solutions are unsafe: comparing with proposed solutions, they overestimate the plastic load with a significant error.

3.2 Global failure

Global failure is a mechanism which includes both compression and tension zones (Fig. 3). For such a mechanism, the failure load can be determined by Eq. (5) of Table 1. Details on the background of this simplified equation can be found in Ref. [5].

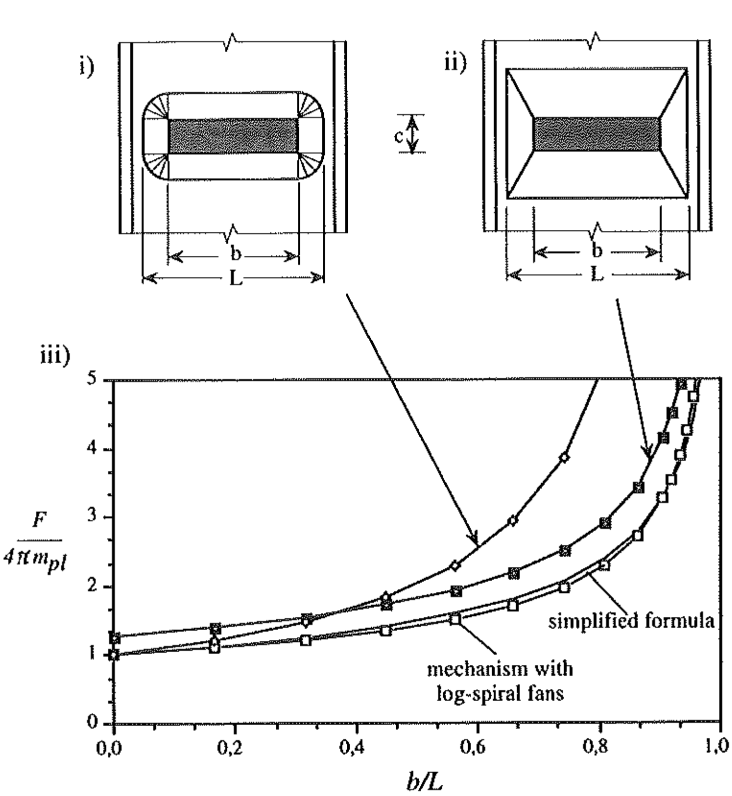

Fig. 7 - Plastic load for local mechanism with c = 0
i) Yield pattern with circular fans (Packer et al.)
ii) Yield pattern with only straight lines (Packer et al.)
iii) Comparison between proposed solutions and Packer's ones.

3.3 Failure under each bolt head

The theoretical assumption that bolts act like point loads leads to the possibility of a failure mechanism under each bolt (or nut). However, it can be shown [5] that this failure mode never governs the failure of current I-section column webs if the actual dimension of a bolt head and the limitations of the geometrical parameters included in Table 1 (validity range) are taken into account .

The check for this failure mode is therefore not included in Table 1.

3.4 Punching shear failure

In the case of a bolted connection, as shown in Fig. 4, the tension force is applied by the bolts. Assuming a punching shear line around each bolt head or nut (mean diameter d_m, Fig. 6iv) and the shear yield stress $f_y / \sqrt{3}$ (Von Mises), the punching shear failure load for n bolts in tension is given by Eq. (6a).

If the punching perimeter is a rectangle with dimensions $b \times c$ Eq. (6b) should be used.

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3.5 Combined flexural and punching shear failure

A failure mechanism presents not only flexural yield lines (thick lines in Fig. 8) but also punching shear yield lines (dotted lines in Fig. 8). This partial punching can be observed experimentally.

Packer *et al.* [7] proposed similar combined flexural and punching shear failure modes, using straight lines or circular fans, instead of the optimised mechanism of Fig. 8 that uses log-spiral fans.

The failure load F_{Q2} is given by Eq. (7) and an iterative procedure, Eq. (8), will be necessary to find x , if $t_w > L/20$. This iterative procedure is very fast (usually 3 or 4 iterations are enough).

However, for thinner webs ($t_w \leq L/20$) punching shear may be neglected:

$$F_{Q2} \approx F_{local}$$

and then, no check for the combined flexural and punching shear failure mode will be necessary.

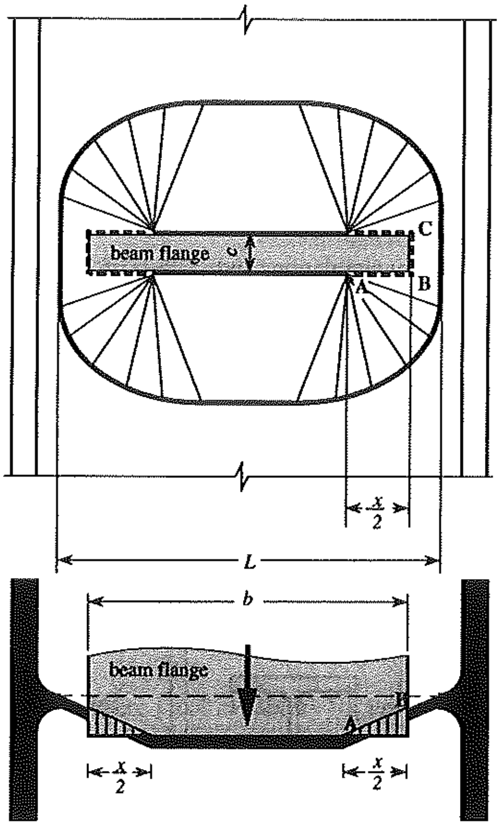

Fig. 8 - Combined flexural and punching shear failure

4. Rotational stiffness and rotation capacity of minor-axis joints

Simple elastic flexibility formulae given in Ref. [8] for rectangular hollow section joints can be adapted to evaluate the initial rotational stiffness S_i due to web deformation in minor-axis joints. However, bilinear M - Φ behaviour should be defined using the secant stiffness S_j , which depends on a large number of geometrical parameters - column, connection, type of failure. A parametric study is in progress at the University of Coimbra with a non-linear finite element program, in order to find reliable design formulae for S_j as well as S_m

A large rotation capacity was obtained in all the 12 minor-axis joints tested at the University of Liège [4]. Theoretical plastic moment resistances were determined and compared to test results [5]. In all the cases the failure mechanism corresponds to either a global or local failure mode of the column web. Generally, these flexural failure mechanisms allow to develop large rotation capacities. The web deformation alone gives a rotation capacity of at least 0.04 rad.

5. Ultimate resistance of 3-D joints.

In 3-D joints, two kinds of interaction between major- and minor-axis resistance are referred hereafter.

5.1 Interaction V - F

In a 3-D joint (Fig. 9) with no minor-axis joint action, $F = 0$, the shear force V due to the major-axis joint must be compared to the plastic shear resistance V_{pl} of the column. However, in 3-D joints with $F \neq 0$, there is an interaction between minor-axis force resistance $F_{V.pl}$ and the shear resistance $V_{F.pl}$. Theoretical bounds of this interaction were obtained using an approximate Ilyushin yield criterion for shells; these bounds define the shaded area of Fig. 10. Numerical simulations and 18 tests of 3-D joints performed at the University of Liège show that theoretical bounds are conservative [10].

In the prediction of the resistance of the column cross-section within the length of the 3-D joint under the combined effect of the axial force (N), biaxial moment (M_y , M_z) and shear forces (V_y , V_z), the reduced plastic shear resistance should be used. For example, in §5.4.9 of Eurocode 3, the plastic shear resistance V_{pl} should be replaced by the reduced plastic shear resistance $V_{F.pl}$.

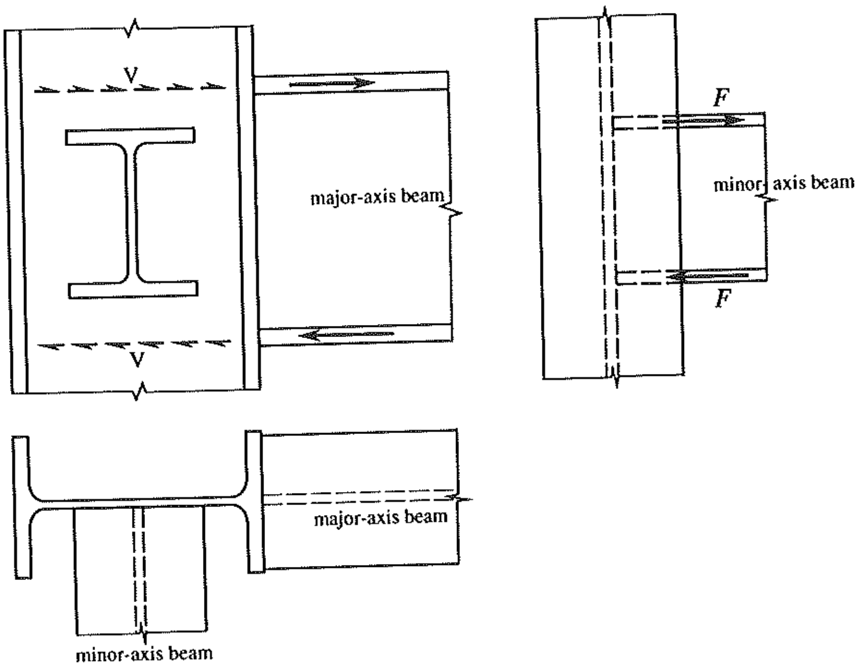

Fig. 9 - 3-D joint. V - the shear force in the joint panel, as in a major-axis joint.
F - the compression or tension force equivalent to minor-axis moment.

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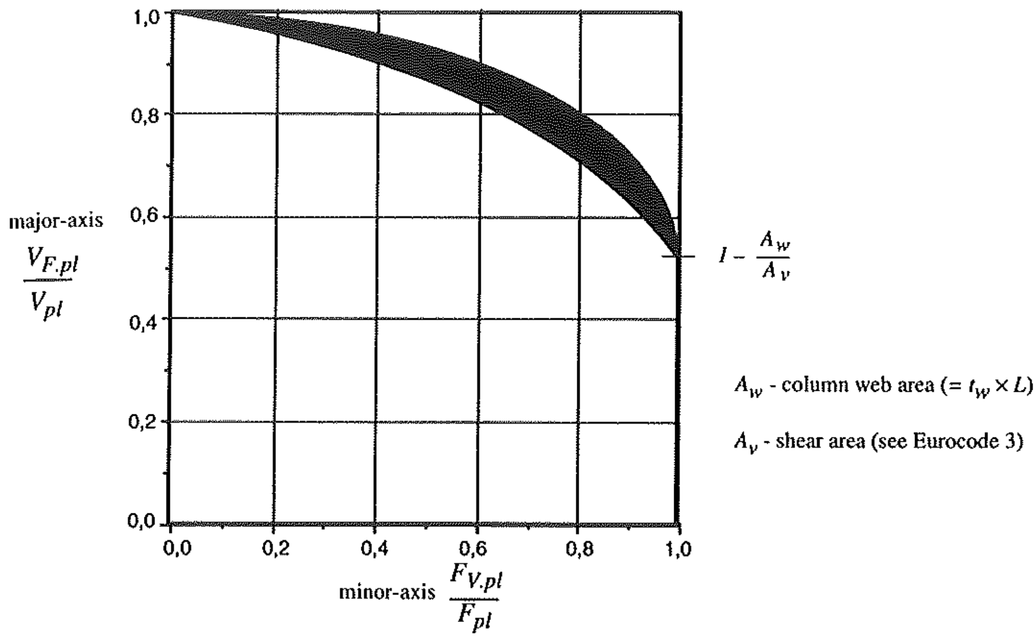

Fig. 10 - Interaction V-F in 3-D joints (see Fig. 9)
V_{pl} - plastic shear resistance of the column in major-axis joint; $V_{F.pl}$ - reduced value
F_{pl} - plastic force resistance of the column web in minor-axis joint; $F_{V.pl}$ - reduced value

5.2 Local buckling of the column web

Fig. 11 shows a typical loading case of a 3-D joint where local instability (or crushing) of the column web can govern the joint failure in the shaded area of Fig. 11. With no minor-axis joint, the buckling resistance of the web is usually obtained by considering the web as a virtual compression member with an effective breadth b_{eff} obtained from, for example, Eq. (5.79) of Eurocode 3. However, in 3-D joints, the minor-axis joint force, acting perpendicular to the plan of the web, reduces the buckling resistance. Consequently, the buckling resistance of the virtual member should be evaluated as a member subject to combined bending and axial compression.

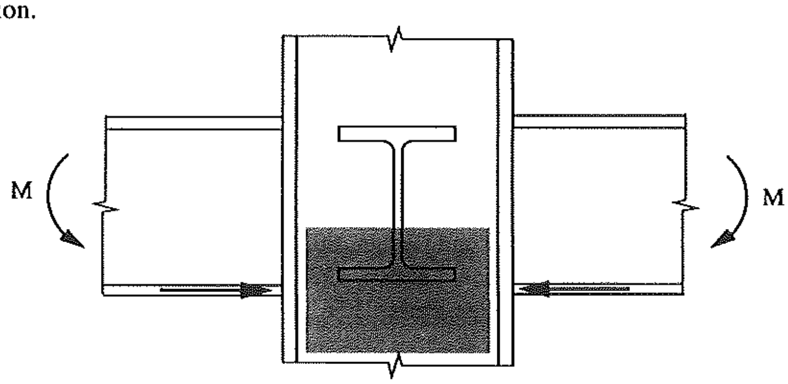

Fig. 11 - Loading case of a 3-D joint. Local instability of the column web (shaded area) is influenced by the minor-axis joint force

6. Conclusions

This paper gives an overview of the research on minor-axis joints and 3-D joints resulting from the collaboration between the University of Liège and the University of Coimbra.

A design method to evaluate the plastic moment resistance of the column web in minor-axis joints is suggested in Table 1. Section 3 describes failure mechanisms for a better understanding of the method.

The papers also refers the current research on the stiffness of minor-axis joints. Concerning 3-D joints, two important cases of interaction between minor- and major-axis resistance are discussed. However more research on the global behaviour of 3-D joints is needed, specially the influence of 3-D actions on the secant stiffness of individual major- or minor-axis joint

Acknowledgements

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