PARAMETRIC STUDY OF THE NUMERICAL MODELLING
FOR SEMI-RIGID JOINTS

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1. INTRODUCTION

For sake of economy, beam-to-column bolted joints without any column web stiffener become a common practice (joints between H or I sections). Such a joint has an non-linear behaviour: when the beam is subject to bending the axes of the connected members do not rotate a same angle, what result in a relative rotation that is not proportional to the beam bending moment.

In a strong axis beam-to-column joint, two main sources of deformability are identified (Figure 1):

a) The deformation of the connection associated to the deformation of the connection elements (end plates, angles, bolts,...) to that of the column flange and to the load-introduction deformability of the column web;

b) The shear deformation of the column web associated mostly to the composite presence of forces P carried over by the beam(s) and acting on the column web at the level of the joint: these forces are statical equivalent to the beam moment M_b.

Figure 1 - Deformation of a strong axis joint

These components are illustrated in Figure 2 for the particular case of a joint between a single beam and a column. The deformability of the connection elements is concentrated into a single flexural spring located at the end of the beam (Figure 2.a). The associated behaviour is expressed in the format of a \( M_b - \theta \) curve.

The deformation of the ABCD column web panel is divided into:

- The load-introduction deformability which consists in the local deformation of the column web in both tension and compression zones of the joint (respectively a lengthening and a shortening) and which result in a relative rotation \( \phi \) between the beam and column axes; this rotation concentrates mainly along edge BC (Figure 2.b) and provides also a deformability curve \( M_b - \phi \).
The shear effect - due to shear force \( V_s \) - which results in a relative rotation \( \gamma \) between the beam and column axes (figure 2.c); this rotation makes it possible to establish a second deformability curve \( V_s = f(\gamma) \).

It is important to stress that the deformability of the connection (connection elements + load introduction) is only due to the forces carried over by the flanges of the beam(s) (beam moment(s) \( M_b \)), while the shear in a column web panel is the result of the combined action of these equal but opposite forces and of the shear forces in the column at the level of the beam flanges (shear force \( V_b \)) [1].

![Figure 1 - Joint deformability components](image)

**Figure 1 - Joint deformability components**

The difference between the loading of the connection and that of the column web in a specified joint requires, at a theoretical point of view, to account separately for both deformability sources when designing a building frame. However, doing so is only practicable when the frame is analysed by means of a sophisticated computer program allowing for the separate modelling of both deformability sources. In all other cases, the actual behaviour of the joints must be simplified by concentrating the whole deformability into a single flexural spring.

Based on a very large parametric study of braced and unbraced frames performed by means of the non-linear finite element program PINIELG, the present paper gives guidelines on how to define the spring characteristics in an accurate and safe manner.

**2. PINIELG FINITE ELEMENT PROGRAM**

PINIELG is a materially and geometrically non-linear finite element program which has been developed jointly at the University of Liège, Belgium, and at the Polytechnic Federal School of Lausanne, Switzerland. It is used to solve problems such as:
- Linear and non-linear instability with calculation of critical loads and instability modes;
- Calculation of eigenfrequencies and eigen modes, possibly taking into account the current stress state.

Its library is composed of various structural bar, plate, beam, space, membrane plate, thin and thick shells, springs, linear constraints, etc.

PINIELG program has been recently implemented to simulate accurately the non-linear behaviour of connections and sheared column web panels [2].

The flexural behaviour of the connection and of the adjacent beam as well as that of the column web panel are gathered into a single "plane beam + connection + sheared panel" finite element. Any type of non-linear response may be associated to the behaviour of the beam, of the connection and of the web panel respectively. Aforementioned element can be used according one of the three different manners sketched in figure 3.

![Figure 3 - Use of the finite element](image)

**Figure 3 - Use of the finite element**

It exhibits several superiorities:
- In contrast to many other approaches [3, 4, 5], it allows to fulfill equilibrium equations of the web panel;
- It makes possible an accurate and realistic picture of the actual macroscopic behaviour of the column web panel;
- It does not need a more refined discretization than that just required when rigid joints;
- The number of equations which have to be solved at each analysis iteration is not increased due to the non-linear self-rigidity of the joints.

**3. PARAMETRIC STUDY**

For daily practices, it cannot be expected to account separately for both the flexural behaviour of the connection and the sheared behaviour of the column web panel. Therefore, the possibility of concentrating the deformability of both connection and column web panel into a flexural spring located at the beam end must be contemplated (figures 4 and 5).

A matter of fact, how such springs affect the frame response can be reflected through appropriate design methods (see [2]).
joints, is consequently aimed at determining to which extent the ac-
etically complex behaviour of a joint (shear panel) - 1 or 2 conn-
ected - may be represented, with a sufficient accuracy, by isolated sp:l:
appropriate characteristics.

Figure 4 - Flexural characteristics of the springs

Figure 5 - Concentration of the joint deformability into flexural springs

Regarding the location of the spring, two possibilities arise (figure 6):
- either at the beam-to-column physical interface (points A), or
- at the intersection of both beam and column axes (points B).

The optimum location of the spring as well as the allowance for summing up
the joint deformability components cannot be demonstrated theoretically.
The parametric study, which consists in the numerical simulation of the
behaviour up to collapse of braced and unbraced frames with semi-rigid
c. Type B frame

d. Type D frame

Figure 7 - Different types of braced and unbraced frames
Three numerical modellings of joints are examined:

a. the deformabilities of both connection and sheared web panel are represented separately - type M3 (because being the most sophisticated modelling, the case is used as reference).

b. the deformabilities of both connection and sheared web panel are concentrated into a single spring located at the beam-to-column physical interface - type M4.

c. the deformabilities of both connection and sheared web panel are concentrated into a single spring located at the intersection of beam and column axes - type M5.

Preliminary calculations (carried out with the aim to define the program for the parametric study) have indicated that the deformability of the joints may be concentrated at the beam-to-column interface (points A in Figure 6), wherefrom model M4. In view to confirm these results, the numerical simulations relative to M3 and M4 modelings have been performed in the first instance.

The resulting curves are presented as follows:

- curves 'load factor A vs. mid-span vertical displacement A for the beam, the transverse displacement of which is the most important at each level of the frame loading' for braced and unbraced frames;
- curves 'load factor A vs. horizontal displacement V at the top' for unbraced frames.

These curves allow to control the influence of the simplified numerical modelling M4 on:

- the displacement of the beams and of the frame under service loads (k=1);
- the collapse load factor.

Two of these comparisons are plotted in figure 8. Others are reported in [2].

Figure 8 - Comparisons between M3 and M4 modelings

Except for type B unbraced frame with end-plate connections, the agreement between the curves relative respectively to 'exact' modelling M3 and simplified one M4 is almost perfect. The physical explanation of the discrepancy arising from the use of the simplified modelling for the unbraced frame B (Figure 8) is detailed in [2].

Figure 9 - Comparison relative to unbraced frame B with end-plate connections

Figure 10 - Characteristics of the joint with end-plate connection relative to unbraced frame B
4. CONCLUSIONS

It has been shown that the use of simplified numerical modelling does not lead to an accurate prediction of the actual response of braced and unbraced frames, except when the beam-to-column connections are almost fully rigid. In the latter case, the actual frame collapse load would be somewhat underestimated.

Numerical simulations using Mx simplified modelling are being presently in progress. The results, which should confirm the choice of Mx modelling in practice will be presented at the end of this year in the final report of the research launched jointly by M.S.M. Department of the University of Liège and ARBED Recherches in the frame of ECSC Research Contract (agreement N° 721C-SA/957).

Last, it is while stressing that Mx simplified modelling is fully representative of the actual joint behaviour when the column web panel is stiffened for shear. The joint deformability then consists in the sole connection deformability which is concentrated at the beam-to-column physical interface, in complete accordance with Mx modelling.

5. REFERENCES


DESIGN OF AXIALLY LOADED COMPRESSED ANGLES

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INTRODUCTION

Section E3 in the AISC LRFD Specification (Ref. 1) states that

"Singly symmetric and unsymmetric columns, such as angle or tee-shaped columns, may, ... may require consideration of the limit states of flexural-torsional and torsional buckling."

This requirement demands a fairly complicated procedure for the design of a common structural element which had been designed previously by much simpler method of flexural buckling.

This paper demonstrates on the basis of analysis and experiments that many angle-columns can be designed as before by the method of minor axis buckling, and that the present AISC procedure yields conservative results for shorter columns. It is also shown that the Q-factor method, which accounts for local buckling of the web. The two modes of flexural-torsional and unstiffened elements are designed (see Ref. 2). Finally an LRFD procedure is presented for the design of axially loaded angle columns.

ELASTIC BUCKLING

Flexural-torsional buckling involves both lateral translation and twisting of the cross section. For an unsymmetric shape, such as an unequal leg angle column, the two modes of flexural buckling (i.e., about the z and y axes, see Fig. 1)

\[ P_{y} = \frac{F_{c}}{L_{x}} \]  
(1)

\[ P_{x} = \frac{F_{c}}{L_{y}} \]  
(2)

and the mode of torsional buckling

\[ P_{t} = \left( \frac{F_{c}}{L_{y}} - F_{t} \right) \frac{1}{\gamma} \]  
(3)

are totally coupled through the cubic equation